SOLUTION MANUAL FOR

ENGINEERING MECHANICS STATICS

SIXTH EDITION

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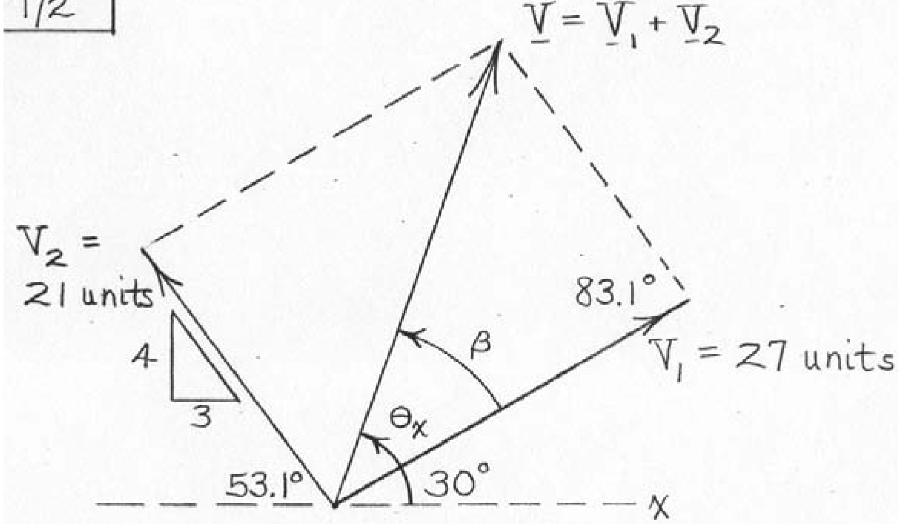
1/1 $V = \sqrt{V_x^2 + V_y^2} = \sqrt{36^2 + 15^2} = 39$

$\cos\theta_x = \frac{V_x}{V} = \frac{-36}{39}$, $\underline{\theta_x = 157.4°}$

$\cos\theta_y = \frac{V_y}{V} = \frac{15}{39}$, $\underline{\theta_y = 67.4°}$

$\underline{n} = \frac{\underline{V}}{V} = \frac{-36\underline{i} + 15\underline{j}}{39} = \underline{-0.923\underline{i} + 0.385\underline{j}}$

1/2



Graphically, $V = \underline{32 \text{ units}}$, $\underline{\theta_x = 70°}$

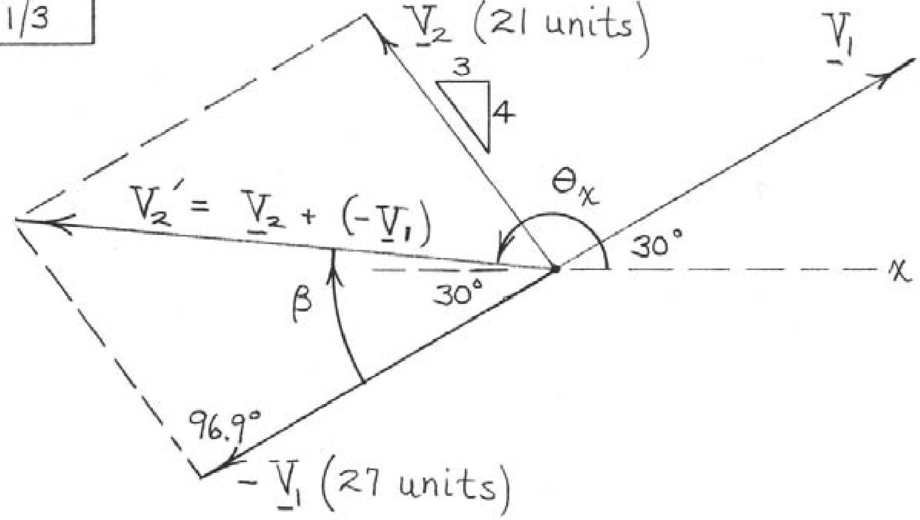
Algebraically, $V^2 = 27^2 + 21^2 - 2(27)(21)\cos 83.1°$

$$V = \underline{32.2 \text{ units}}$$

$$\frac{\sin\beta}{21} = \frac{\sin 83.1°}{32.2}, \quad \beta = 40.4°$$

$$\theta_x = \beta + 30° = 40.4° + 30° = \underline{70.4°}$$

1/3



Graphically, $\underline{V' = 36 \text{ units}}$, $\underline{\theta_x = 175°}$

Algebraically, $V'^2 = 27^2 + 21^2 - 2(27)(21)\cos 96.9°$

$\underline{V' = 36.1 \text{ units}}$

$\frac{\sin\beta}{21} = \frac{\sin 96.9°}{36.1}$, $\beta = 35.2°$

$\theta_x + \beta = 210°$, $\theta_x = 210 - 35.2° = \underline{174.8°}$

1/4

$$F = \sqrt{160^2 + 80^2 + 120^2} = 215 \text{ lb}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \quad \underline{\theta_x = 42.0°}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \quad \underline{\theta_y = 68.2°}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \quad \underline{\theta_z = 123.9°}$$

1/5

$$m = \frac{W}{g} = \frac{1000}{32.2} = \underline{31.1 \text{ slugs}}$$

$$m = 31.1 \text{ slugs} \left(\frac{14.594 \text{ kg}}{\text{slug}} \right) = \underline{453 \text{ kg}}$$

1/6

$$F = W = \frac{Gm_1m_2}{r^2},$$

where $G = 6.673(10^{-11})\ m^3/(kg \cdot s^2)$

$m_1 = 80\ kg$

$m_2 = 5.976(10^{24})\ kg$

and $r = (6371 + 250)(10^3)\ m$

Substitute these numbers & obtain $\underline{W = 728\ N}$

U.S. units : $W = 728\ N\left(\frac{1\ lb}{4.4482\ N}\right) = \underline{163.6\ lb}$

1/7 $W = (125 \text{ lb})\left(\frac{4.4482 \text{ N}}{\text{lb}}\right) = \underline{556 \text{ N}}$

$$m = \frac{W}{g} = \frac{125}{32.2} = \underline{3.88 \text{ slugs}}$$

$$m = \frac{W}{g} = \frac{556}{9.81} = \underline{56.7 \text{ kg}}$$

1/8 $A = 8.67, \quad B = 1.429$

$(A+B) = 8.67 + 1.429 = \underline{10.10}$

$(A-B) = 8.67 - 1.429 = \underline{7.24}$

$(AB) = (8.67)(1.429) = \underline{12.39}$

$(A/B) = 8.67/1.429 = \underline{6.07}$

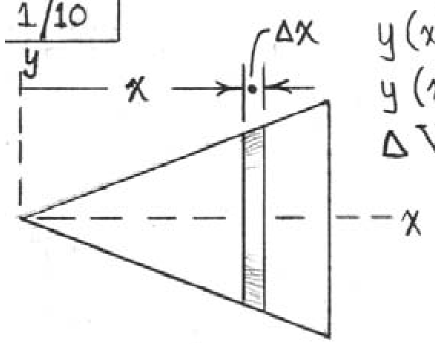
1/9

$$F = \frac{G m_e m_s}{d^2} = \frac{3.439(10^{-8})(1)(333{,}000)(4.095 \cdot 10^{23})^2}{(92.96 \cdot 10^6 \cdot 5280)^2}$$

$$= \underline{7.97(10^{21})\text{ lb}}$$

$$F = 7.97(10^{21})\text{ lb}\left(\frac{4.4482\text{ N}}{\text{lb}}\right) = \underline{3.55(10^{22})\text{ N}}$$

1/10



$$y(x) = \frac{r}{h}x$$

$$y(x+\Delta x) = \frac{r}{h}(x+\Delta x)$$

$$\Delta V = V(x+\Delta x) - V(x)$$

$$= \frac{1}{3}\pi[y(x+\Delta x)]^2[x+\Delta x] - \frac{1}{3}\pi[y(x)]^2[x]$$

$$= \frac{1}{3}\pi\left[\frac{r}{h}(x+\Delta x)\right]^2[x+\Delta x] - \frac{1}{3}\pi\left[\frac{r}{h}x\right]^2[x]$$

$$= \frac{1}{3}\pi\frac{r^2}{h^2}\left[(x+\Delta x)^3 - x^3\right]$$

$$= \frac{1}{3}\pi\frac{r^2}{h^2}\left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3\right]$$

$$= \frac{\pi r^2}{h^2}\left[x^2\Delta x + x(\Delta x)^2 + \frac{1}{3}(\Delta x)^3\right]$$

In the limit as $\Delta x \to dx$, the higher-order terms drop out.

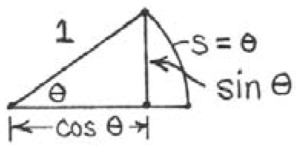
1/11 $20^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.3491$ rad

$\sin 20^\circ = 0.3420$

Percent error is $\frac{0.3420 - 0.3491}{0.3420}(100) = \underline{2.06\%}$

$\tan 20^\circ = 0.3640$

Percent error is $\frac{0.3640 - 0.3491}{0.3640}(100) = \underline{4.09\%}$



The approximation $\sin\theta \cong \theta$ involves the approximation that the arclength $s = \theta$ is the vertical side of the triangle. The approximation that $\tan\theta \cong \theta$ involves, in addition, the approximation that 1 is the horizontal side of the triangle.

2/1

$$\begin{cases} F_x = -800 \sin 35^\circ = \underline{-459 \text{ N}} \\ F_y = 800 \cos 35^\circ = \underline{655 \text{ N}} \end{cases}$$

$$\underline{\mathbf{F} = -459\mathbf{i} + 655\mathbf{j} \text{ N}}$$

2/2

$$\underline{F} = 600(\cos 30^\circ \underline{i} - \sin 30^\circ \underline{j})$$

$$= \underline{520\underline{i} - 300\underline{j} \text{ lb}}$$

Scalar components: $\begin{cases} F_x = 520 \text{ lb} \\ F_y = -300 \text{ lb} \end{cases}$

Vector components: $\begin{cases} \underline{F}_x = 520\underline{i} \text{ lb} \\ \underline{F}_y = -300\underline{j} \text{ lb} \end{cases}$

2/3

$$\underline{F} = 4.8\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right)$$

$$= -2.88\underline{i} - 3.84\underline{j} \text{ kN}$$

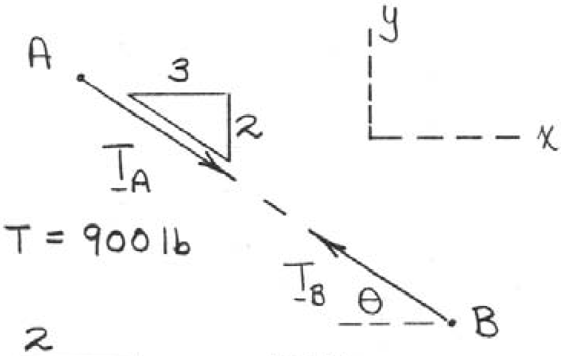
2/4

$$\underline{F} = F\underline{n}_{AB} = 4800\left[\frac{45\underline{i} + 30\underline{j}}{\sqrt{45^2+30^2}}\right]$$

$$= 3990\underline{i} + 2660\underline{j} \text{ lb}$$

Scalar components: $\begin{cases} F_x = 3990 \text{ lb} \\ F_y = 2660 \text{ lb} \end{cases}$

2/5



$T_A = T_B = T = 900 \text{ lb}$

$$\sin\theta = \frac{2}{\sqrt{2^2+3^2}} = 0.555$$

$$\cos\theta = \frac{3}{\sqrt{2^2+3^2}} = 0.832$$

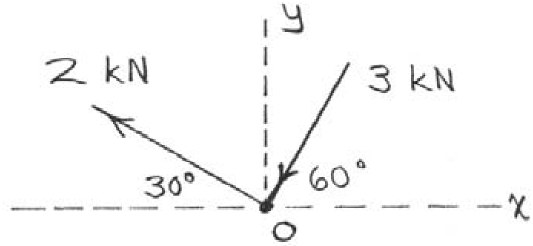
$$\underline{T}_A = 900(0.832\underline{i} - 0.555\underline{j}) = \underline{749\underline{i} - 499\underline{j} \text{ lb}}$$

$$\underline{T}_B = -\underline{T}_A = \underline{-749\underline{i} + 499\underline{j} \text{ lb}}$$

2/6

$$\underline{F} = 1800\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right) = \underline{-1080\underline{i} - 1440\underline{j}\ \text{N}}$$

2/7



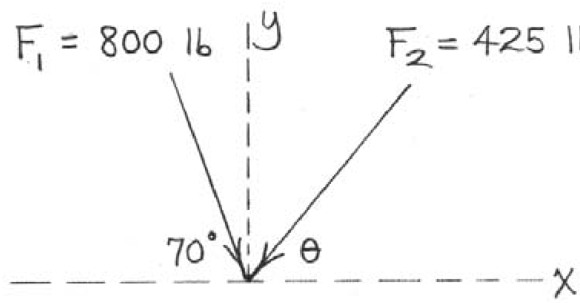
$$R_x = \Sigma F_x = -2\cos 30° - 3\cos 60° = -3.23 \text{ kN}$$

$$R_y = \Sigma F_y = 2\sin 30° - 3\sin 60° = -1.598 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{3.61 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = \underline{206°}$$

2/8



$R_x = \Sigma F_x = 800 \cos 70° - 425 \cos \theta = 0$

$\underline{\theta = 49.9°}$

$R_y = \Sigma F_y = -800 \sin 70° - 425 \sin 49.9°$

$= -1077$ lb

So $\underline{R = 1077 \text{ lb}}$

2/9

$$P_x = -260\left(\frac{12}{13}\right) = -240 \text{ N}$$

$$P_y = -260\left(\frac{5}{13}\right) = -100 \text{ N}$$

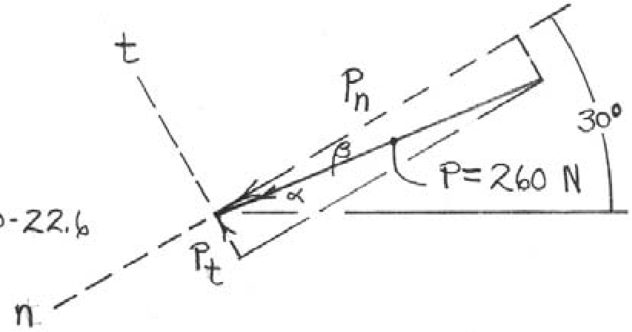
2/10

$\alpha = \tan^{-1} \frac{5}{12}$

$= 22.6°$

$\beta = 30 - \alpha = 30 - 22.6$

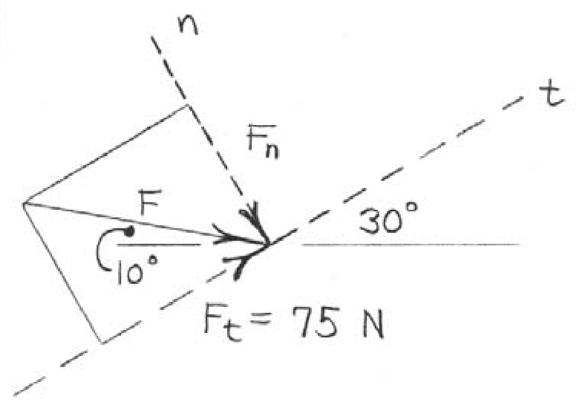
$= 7.38°$



$P_n = P \cos\beta = 260 \cos 7.38° = \underline{258 \text{ N}}$

$P_t = P \sin\beta = 260 \sin 7.38° = \underline{33.4 \text{ N}}$

2/11



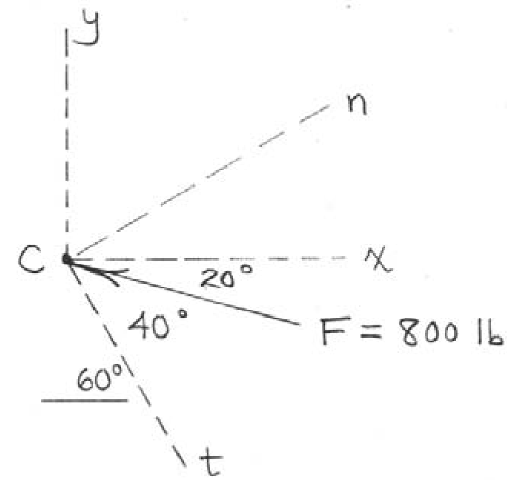
$$\frac{|F_n|}{F_t} = \tan 40°, \quad |F_n| = F_t \tan 40° = 75 \tan 40°$$

$$F_n = \underline{-62.9 \text{ N}}$$

$$\frac{F_t}{F} = \cos 40°, \quad F = \frac{F_t}{\cos 40°} = \frac{75}{\cos 40°}$$

$$= \underline{97.9 \text{ N}}$$

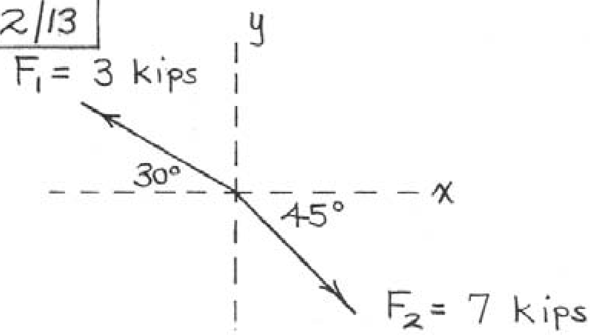
2/12



$$\begin{cases} F_x = -800 \cos 20° = \underline{-752 \text{ lb}} \\ F_y = 800 \sin 20° = \underline{274 \text{ lb}} \end{cases}$$

$$\begin{cases} F_n = -800 \sin 40° = \underline{-514 \text{ lb}} \\ F_t = -800 \cos 40° = \underline{-613 \text{ lb}} \end{cases}$$

2/13

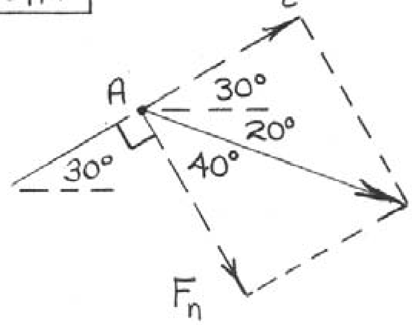


$R_x = \Sigma F_x = -3 \cos 30° + 7 \cos 45° = 2.35$ kips

$R_y = \Sigma F_y = 3 \sin 30° - 7 \sin 45° = -3.45$ kips

$\underline{R} = 2.35\underline{i} - 3.45\underline{j}$ kips

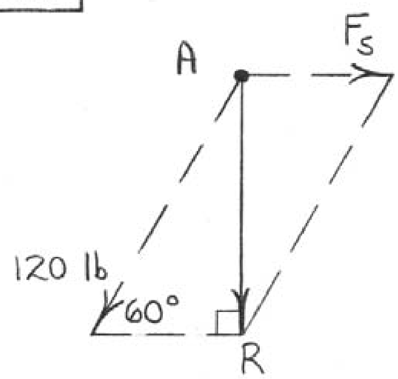
2/14



$F_t = 2 \cos 50° = \underline{1.286 \text{ kN}}$

$F_n = 2 \sin 50° = \underline{1.532 \text{ kN}}$

2/15



$$\cos 60° = \frac{F_s}{120}$$

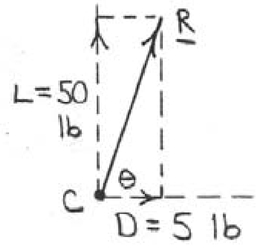
$$\underline{F_s = 60 \text{ lb}}$$

$$\sin 60° = \frac{R}{120}$$

$$\underline{R = 103.9 \text{ lb}}$$

2/16

$$\frac{L}{D} = \frac{50}{D} = 10 \;; \quad D = 5 \text{ lb}$$



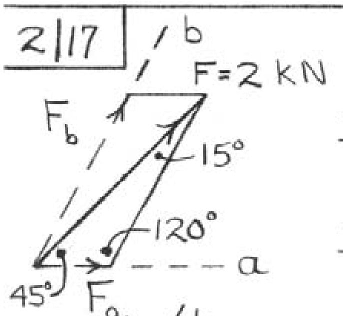
$$R = \sqrt{L^2 + D^2} = \sqrt{50^2 + 5^2}$$

$$= \underline{50.2 \text{ lb}}$$

$$\theta = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{50}{5}\right)$$

$$= \underline{84.3°}$$

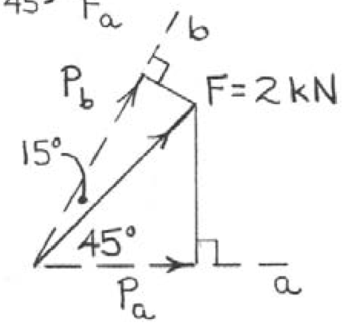
2/17



Components

$$\frac{\sin 120^\circ}{2} = \frac{\sin 15^\circ}{F_a}, \quad \underline{F_a = 0.598 \text{ kN}}$$

$$\frac{\sin 120^\circ}{2} = \frac{\sin 45^\circ}{F_b}, \quad \underline{F_b = 1.633 \text{ kN}}$$

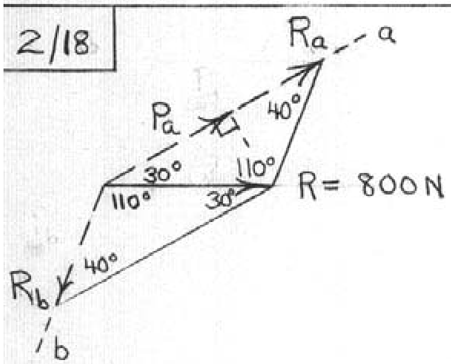


Projections

$$P_a = 2 \cos 45^\circ = \underline{1.414 \text{ kN}}$$

$$P_b = 2 \cos 15^\circ = \underline{1.932 \text{ kN}}$$

2/18



Law of sines :

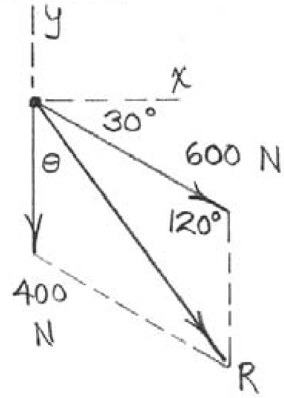
$$\frac{800}{\sin 40°} = \frac{R_a}{\sin 110°} = \frac{R_b}{\sin 30°}$$

$\underline{R_a = 1170 \text{ N}}$

$\underline{R_b = 622 \text{ N}}$

Projection $P_a = R\cos 30° = 800 \cos 30° = \underline{693 \text{ N}}$

2/19 Law of cosines:



$$R^2 = 400^2 + 600^2 - 2(400)(600)\cos 120°$$

$$\underline{R = 872 \text{ N}}$$

Law of sines:

$$\frac{600}{\sin\theta} = \frac{872}{\sin 120°}, \quad \underline{\theta = 36.6°}$$

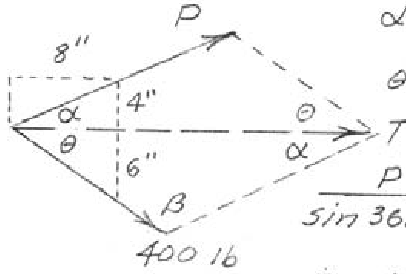
$$R_x = \Sigma F_x = 600 \cos 30° = 520 \text{ N}$$

$$R_y = \Sigma F_y = -600 \sin 30° - 400 = -700 \text{ N}$$

So $\underline{\underline{R} = 520\underline{i} - 700\underline{j} \text{ N}}$

$$\left(\begin{aligned} \text{Check: } \quad R &= \sqrt{520^2 + 700^2} = 872 \text{ N} \checkmark \\ \theta &= \tan^{-1}\frac{520}{700} = 36.6° \checkmark \end{aligned}\right)$$

2/20



$\alpha = \tan^{-1}\frac{4}{8} = 26.57°$

$\theta = \tan^{-1}\frac{6}{8} = 36.87°$

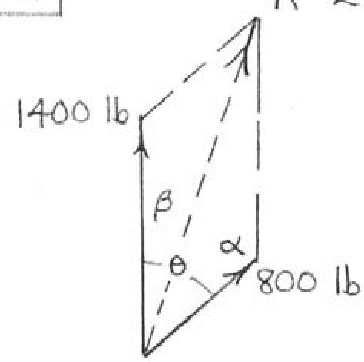
$\beta = 180 - (\alpha + \theta) = 116.57°$

$$\frac{P}{\sin 36.87°} = \frac{400}{\sin 26.57°}$$

$$P = 400\,\frac{0.6}{0.4472} = \underline{537\text{ lb}}$$

$$\frac{T}{\sin 116.56°} = \frac{400}{\sin 26.57°},\quad T = 400\,\frac{0.8944}{0.4472} = \underline{800\text{ lb}}$$

2/21



Law of cosines: $2000^2 = 1400^2 + 800^2 - 2(1400)(800)\cos\alpha$

With $\alpha = 180 - \theta$ and $\cos(180 - \theta) = -\cos\theta$:

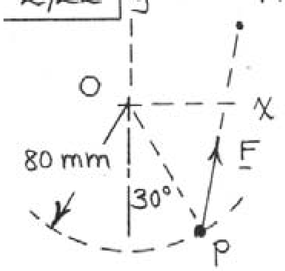
$2000^2 = 1400^2 + 800^2 + 2(1400)(800)\cos\theta$

$\underline{\theta = 51.3°}$

Law of sines: $\dfrac{800}{\sin\beta} = \dfrac{2000}{\sin(180° - 51.3°)}$

$\underline{\beta = 18.19°}$

2/22



A (60, 40) mm

The coordinates of P are

$$(80 \sin 30^\circ, -80 \cos 30^\circ) =$$

$$(40, -69.3) \text{ mm}$$

$$\underline{n}_{PA} = \frac{\underline{PA}}{\overline{PA}} = \frac{(60-40)\underline{i} + (40 - (-69.3))\underline{j}}{\sqrt{20^2 + 109.3^2}}$$

$$= 0.1800\underline{i} + 0.984\underline{j}$$

The magnitude of $\underline{F}$ is

$$F = kx = 1200 \left[\frac{\overline{PA} - 100}{1000}\right] = 1200\left[\frac{111.1 - 100}{1000}\right]$$

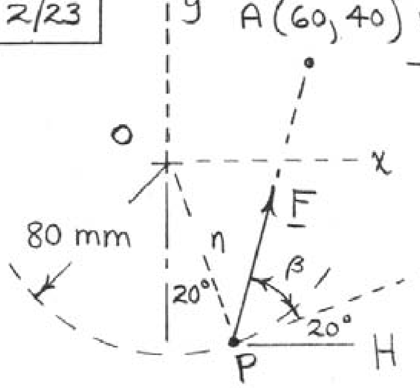
$$= 13.32 \text{ N}$$

Then $\underline{F} = F\underline{n}_{PA} = 13.32\,(0.1800\underline{i} + 0.984\underline{j})$

$$= 2.40\underline{i} + 13.10\underline{j} \text{ N}$$

So $\begin{cases} F_x = 2.40 \text{ N} \\ F_y = 13.10 \text{ N} \end{cases}$

2/23



The x-y coordinates of P are

$(80 \sin 20°, -80 \cos 20°)$

or $(27.4, -75.2)$ mm

$$\overline{PA} = \left[(60-27.4)^2 + (40-(-75.2))^2\right]^{1/2} = 119.7 \text{ mm}$$

$$F = kx = 1200\left[\frac{119.7-100}{1000}\right] = 23.7 \text{ N}$$

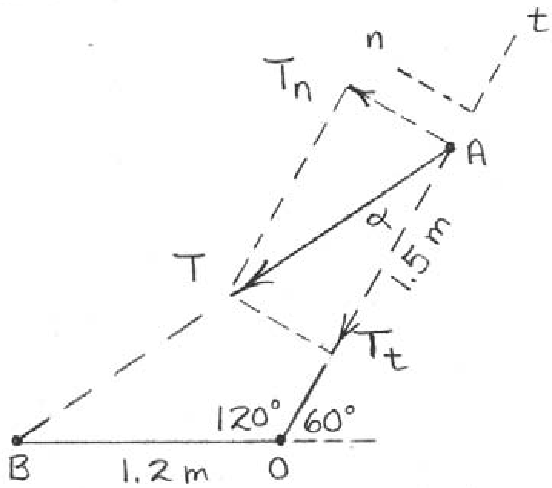
The angle APH is

$$\alpha = \tan^{-1}\left[\frac{40+75.2}{60-27.4}\right] = 74.2°$$

Then $\beta = \alpha - 20° = 74.2 - 20 = 54.2°$

$$\begin{cases} F_t = F\cos\beta = 23.7 \cos 54.2° = \underline{13.84 \text{ N}} \\ F_n = F\sin\beta = 23.7 \sin 54.2° = \underline{19.18 \text{ N}} \end{cases}$$

2/24



$\overline{AB}^2 = 1.2^2 + 1.5^2 - 2(1.2)(1.5)\cos 120°$

$\overline{AB} = 2.34$ m

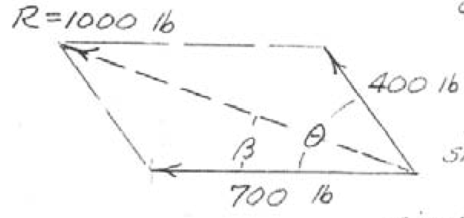
$\frac{\sin\alpha}{1.2} = \frac{\sin 120°}{2.34}$, $\alpha = 26.3°$

$T_n = T\sin\alpha = 750\sin 26.3° = \underline{333 \text{ N}}$

$T_t = -T\cos\alpha = -750\cos 26.3° = \underline{-672 \text{ N}}$

2/25 Law of cosines: $\overline{1000}^2 = \overline{400}^2 + \overline{700}^2 + 2(400)(700)\cos\theta$

$\cos\theta = 0.6250$, $\underline{\theta = 51.3^\circ}$



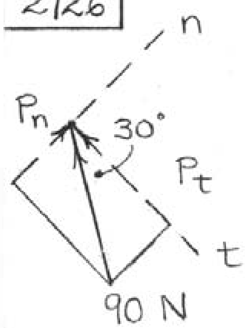
Law of sines:

$$\frac{1000}{\sin(180-51.3)^\circ} = \frac{400}{\sin\beta}$$

$$\sin\beta = \frac{400}{1000}\,0.7806 = 0.312$$

$$\underline{\beta = 18.19^\circ}$$

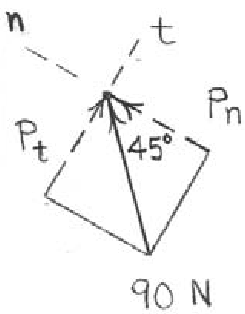
2/26



BC

$$P_t = -90 \cos 30° = \underline{-77.9 \text{ N}}$$

$$P_n = 90 \sin 30° = \underline{45.0 \text{ N}}$$

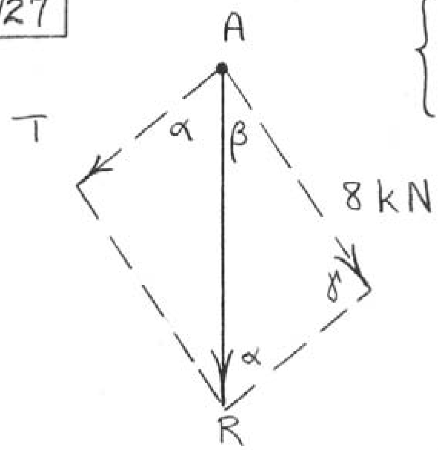


AB

$$P_t = 90 \sin 45° = \underline{63.6 \text{ N}}$$

$$P_n = 90 \cos 45° = \underline{63.6 \text{ N}}$$

2/27



$$\begin{cases} \alpha = \tan^{-1} \dfrac{50}{40} = 51.3° \\ \beta = \tan^{-1} \dfrac{40}{60} = 33.7° \end{cases}$$

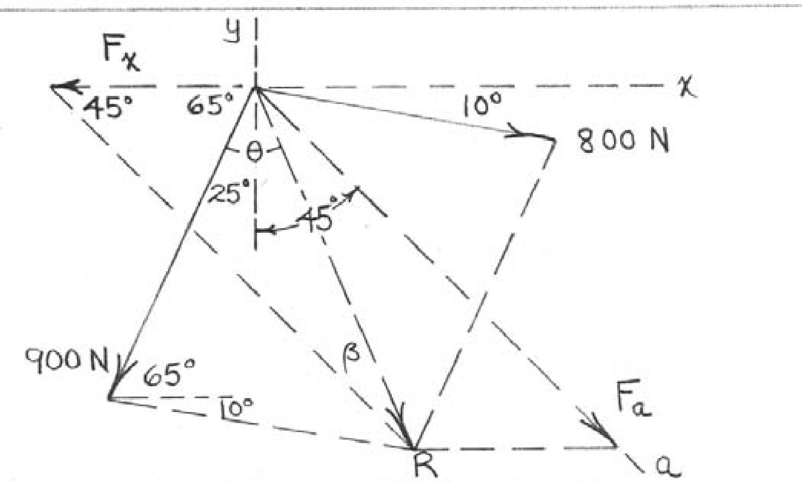
$$\gamma = 180 - \alpha - \beta = 95.0°$$

$$\frac{\sin \beta}{T} = \frac{\sin \alpha}{8}$$

$$\underline{T = 5.68 \text{ kN}}$$

$$\frac{\sin \gamma}{R} = \frac{\sin \alpha}{8}, \qquad \underline{R = 10.21 \text{ kN}}$$

2/28



Law of cosines: $R = \sqrt{900^2 + 800^2 - 2(900)(800)\cos 75^\circ} = 1038\text{ N}$

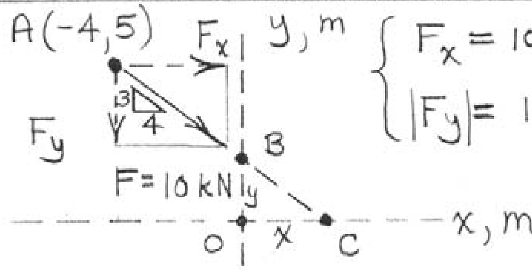
Law of sines: $\dfrac{1038}{\sin 75^\circ} = \dfrac{800}{\sin\theta}$, $\theta = 48.1^\circ$

$\beta = 180^\circ - 45^\circ - (65^\circ + 48.1^\circ) = 21.9^\circ$

$\dfrac{1038}{\sin 45^\circ} = \dfrac{F_x}{\sin 21.9^\circ}$ $\qquad$ $\dfrac{F_a}{\sin(65^\circ + 48.1^\circ)} = \dfrac{1038}{\sin 45^\circ}$

$\underline{F_x = 547\text{ N}}$ $\qquad$ $\underline{F_a = 1350\text{ N}}$

2/29



$$\begin{cases} F_x = 10\left(\frac{4}{5}\right) = 8 \text{ kN} \\ |F_y| = 10\left(\frac{3}{5}\right) = 6 \text{ kN} \end{cases}$$

$\circlearrowright + \; M_O = 8(5) - 6(4) = \underline{16 \text{ kN}\cdot\text{m CW}}$

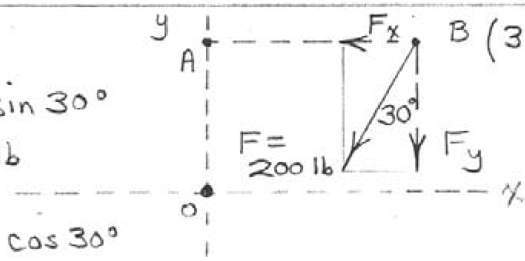
By similar triangles, $\frac{4+x}{5} = \frac{4}{3}$, $x = 2.67$ m

$\frac{y}{2.67} = \frac{3}{4}$, $y = 2$ m

So the two intercept points are $(x, y) =$

$\underline{(2.67, 0)}$ m and $\underline{(0, 2) \text{ m}}$.

2/30



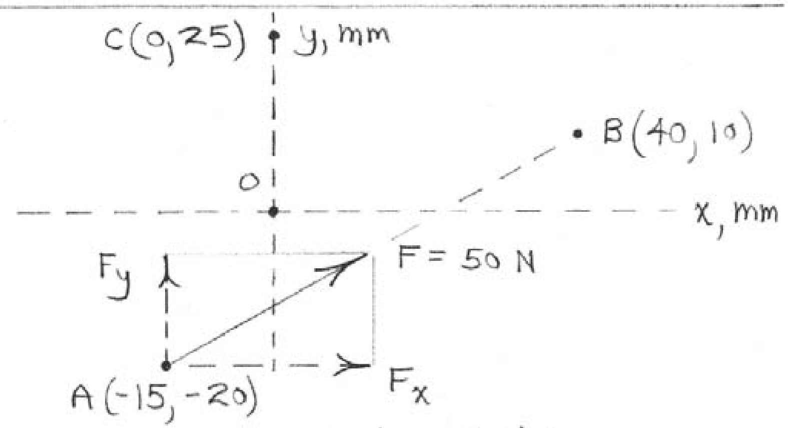
$|F_x| = 200 \sin 30°$
$= 100$ lb

$|F_y| = 200 \cos 30°$
$= 173.2$ lb

$+\!\curvearrowright M_A = 173.2(35) = 6060$ lb-in. (505 lb-ft) CW

$+\!\curvearrowright M_O = 173.2(35) - 100(25)$
$= 3560$ lb-in. (297 lb-ft) CW

2/31



$$\underline{F} = F\underline{n}_{AB} = 50\left[\frac{55\underline{i} + 30\underline{j}}{\sqrt{55^2 + 30^2}}\right]$$

$$= 43.9\underline{i} + 23.9\underline{j} \text{ N}$$

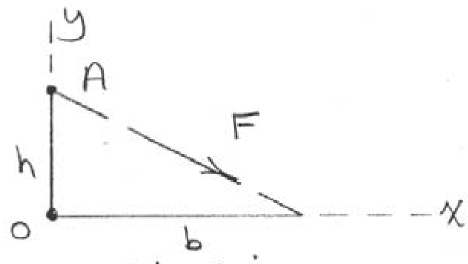
$$M_o = 43.9(15) - 23.9(20) = \underline{179.6 \text{ N}\cdot\text{mm CCW}}$$

$$\underline{M}_C = \underline{r}_{CB} \times \underline{F}$$

$$= (40\underline{i} - 15\underline{j}) \times (43.9\underline{i} + 23.9\underline{j})$$

$$= \underline{1616\underline{k} \text{ N}\cdot\text{mm}} \quad \text{or} \quad \underline{1.616\underline{k} \text{ N}\cdot\text{m}}$$

2/32



$$\underline{F} = F\left[\frac{b\underline{i} - h\underline{j}}{\sqrt{b^2+h^2}}\right]$$

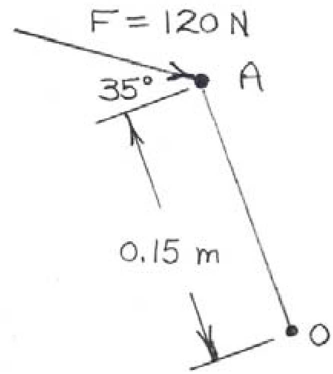
Acting at A:

$$\curvearrowright + M_O = \frac{Fb}{\sqrt{h^2+b^2}}(h) = \underline{\frac{Fbh}{\sqrt{h^2+b^2}} \quad CW}$$

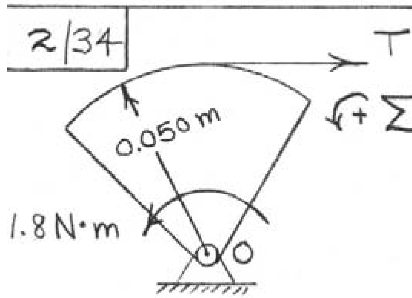
2/33

$+\circlearrowright M_O = 120 \cos 35^\circ (0.15)$

$= \underline{14.74 \text{ N}\cdot\text{m CW}}$



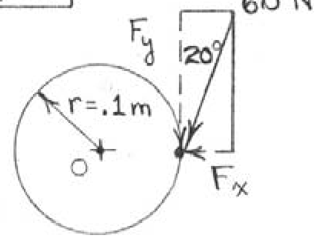
2/34



$$\circlearrowleft + \sum M_O = 1.8 - T(0.050) = 0$$

$$\underline{T = 36 \text{ N}}$$

2/35

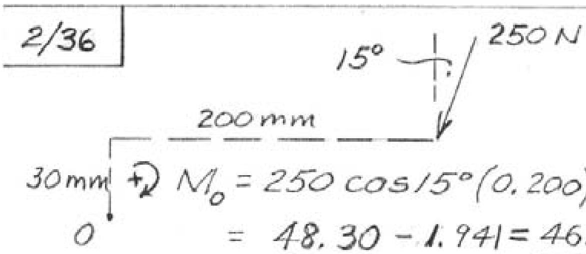


$$+\circlearrowleft M_O = r F_y$$

$$= (0.1)(60 \cos 20°)$$

$$= \underline{5.64 \text{ N}\cdot\text{m}}$$

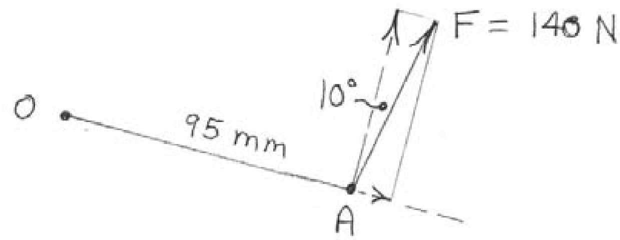
2/36



$$+\circlearrowleft\ M_O = 250\cos 15°(0.200) - 250\sin 15°(0.030)$$

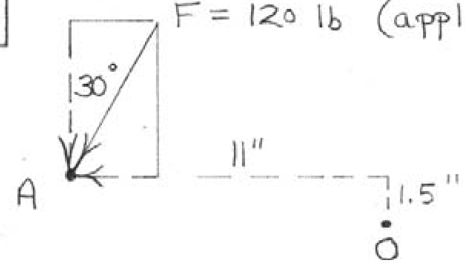
$$= 48.30 - 1.941 = \underline{46.4\ \text{N}\cdot\text{m}}$$

2/37



$$\curvearrowleft\!+\ M_O = (140 \cos 10°)(0.095) = \underline{13.10\ \text{N}\cdot\text{m CCW}}$$

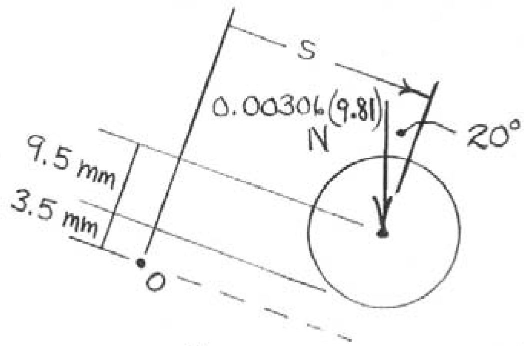
2/38



$$\circlearrowleft + \; M_O = 120 \cos 30° (11) + 120 \sin 30° (1.5)$$

$$= 1233 \text{ lb-in. or } 102.8 \text{ lb-ft CCW}$$

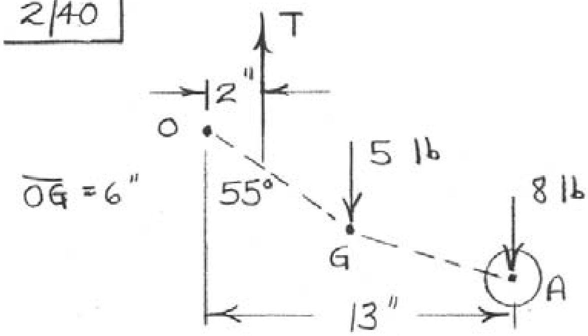
2/39



$$\circlearrowleft + \quad M_O = 0.00306(9.81)\left[s \cos 20° + (9.5+3.5)\sin 20°\right]$$

$$= 0.1335 + 0.0282s \text{ N·mm} \quad (s \text{ in mm})$$

2/40



The combined moment about O of the 5-lb and 8-lb weights is

$$+\circlearrowright M_O = 5\,(6 \sin 55°) + 8(13) = 128.6 \text{ lb-in.}$$
(CW)

$$+\circlearrowright \Sigma M_O = 0: \; -T(2) + 128.6 = 0$$

$$T = 64.3 \text{ lb}$$

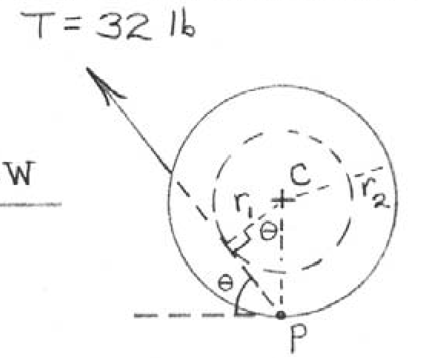
2/41

$\curvearrowright{+}\ M_C = Tr_1 = 32(5)$

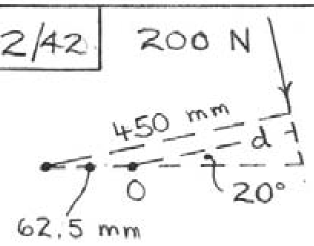
$= \underline{160 \text{ lb-in. CW}}$

$\cos\theta = r_1/r_2 = 5/8$

$\underline{\theta = 51.3^\circ}$



2/42



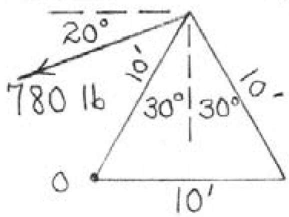
$d = 450 - 62.5 \cos 20°$

$= 391 \text{ mm}$

$\circlearrowleft + \; M = Fd = 200(0.391)$

$= \underline{78.3 \text{ N}\cdot\text{m}}$

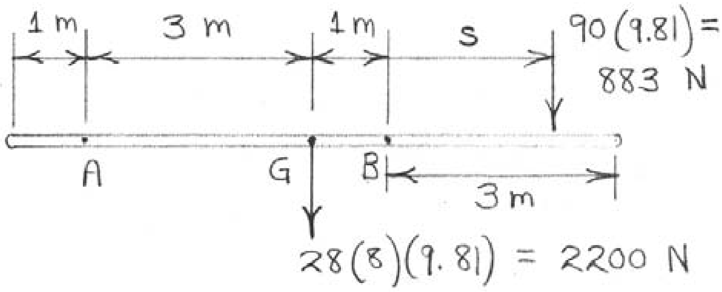
2/43



$\overset{+}{\curvearrowleft}$ $M_O = 780 \cos 20° \, (10 \cos 30°)$

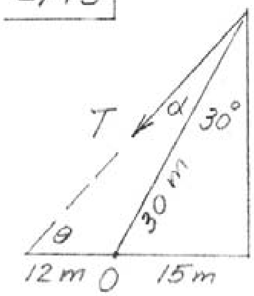
$- 780 \sin 20° \, (5) = \underline{5010 \text{ lb-ft}}$

2/44



$\circlearrowleft + \; M_B = 2200(1) - 883s = 0, \quad \underline{s = 2.49 \text{ m}}$

2/45



$$\theta = \tan^{-1}\frac{30(0.866)}{12+15} = 43.90^\circ$$

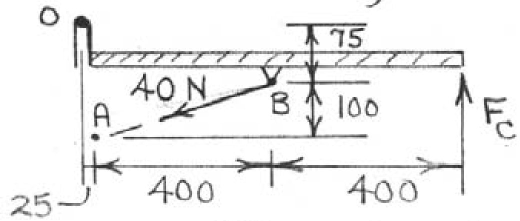
$$\alpha = 90^\circ - (30^\circ + 43.90^\circ) = 16.10^\circ$$

$$M_O = 72 \text{ kN}\cdot\text{m}$$

$$= T \sin 16.10^\circ\,(30) = 8.32T$$

$$T = \frac{72}{8.32} = \underline{8.65 \text{ kN}}$$

2/46 (Dim. in mm)



$AB = \sqrt{400^2 + 100^2}$

$= 412$ mm

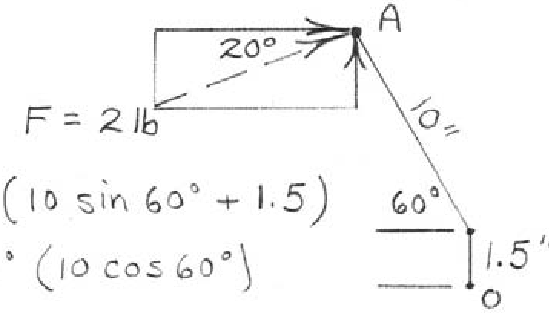
$\circlearrowleft + \; M_O = \left(\frac{400}{412} \cdot 40\right)(75) + \left(\frac{100}{412} \cdot 40\right)(425)$

$= 7030$ N·mm or $\underline{7.03 \text{ N·m}}$

$\circlearrowleft + \; \Sigma M_O = 0: -F_C(825) + 7030 = 0$

$\underline{F_C = 8.53 \text{ N}}$

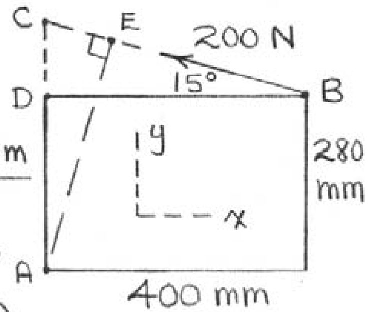
2/47



$$+\circlearrowright M_O = 2\cos 20^\circ\,(10\sin 60^\circ + 1.5) + 2\sin 20^\circ\,(10\cos 60^\circ)$$

$$= \underline{22.5 \text{ lb-in. CW}}$$

2/48



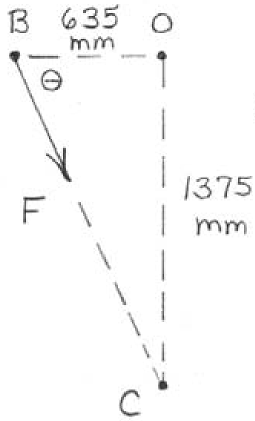
(1) $M_A = 200 \cos 15° \,(0.280)$
$+ 200 \sin 15° \,(0.400) = \underline{74.8 \text{ N·m}}$

(2) $AE = 0.280 \cos 15° + 0.400 \sin 15°$
$= 0.374$ m ; $M_A = 200\,(0.374)$
$= \underline{74.8 \text{ N·m}}$

(3) Apply force at C
$CD = 0.400 \tan 15° = 0.1072$ m
$CA = 0.280 + 0.1072 = 0.387$ m
$M_A = (200 \cos 15°)(0.387) = \underline{74.8 \text{ N·m}}$

(4) $\underline{M}_A = \underline{r} \times \underline{F} = (0.400\underline{i} + 0.280\underline{j}) \times 200(-\cos 15°\underline{i}$
$+ \sin 15°\underline{j}) = \underline{74.8\underline{k} \text{ N·m}}$

2/49



$$F = kx = 60\left(\sqrt{0.635^2 + 1.375^2} - 0.740\right)$$

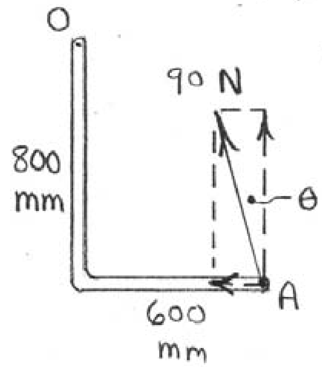
$$= 46.5 \text{ N}$$

$$\theta = \tan^{-1}\frac{1.375}{0.635} = 65.2°$$

$$\curvearrowleft + \; M_O = 46.5 \sin 65.2° \, (0.635)$$

$$= \underline{26.8 \text{ N·m CCW}}$$

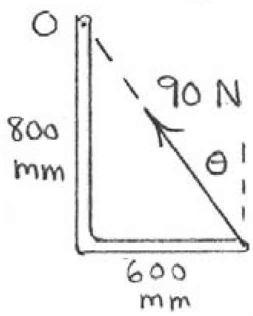
2/50



(a) $\theta = 15°$

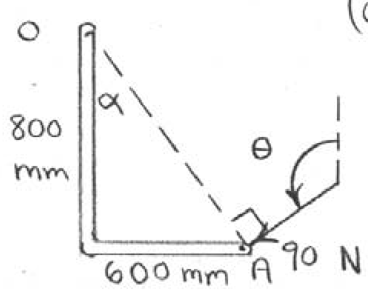
$$\curvearrowleft M_O = 90 \cos 15° (0.6) - 90 \sin 15° (0.8)$$

$$= \underline{33.5 \text{ N·m}}$$



(b) $\theta = \tan^{-1}\left(\frac{600}{800}\right) = \underline{36.9°}$

$\underline{(\text{or } 217°)}$

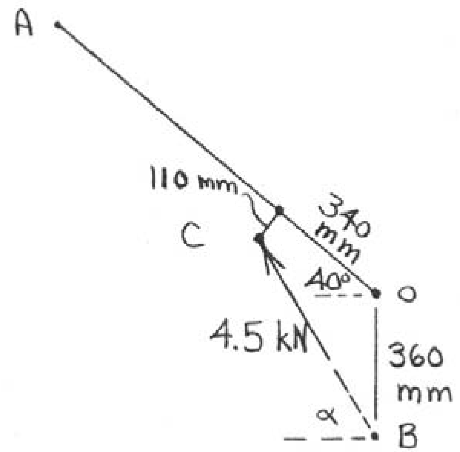


(c) $F \perp OA$, so

$\theta = 90° + \alpha$, $\alpha = \tan^{-1}\frac{600}{800} = 36.9°$

So $\theta = \underline{126.9°}$ (or $\underline{307°}$)

2/51

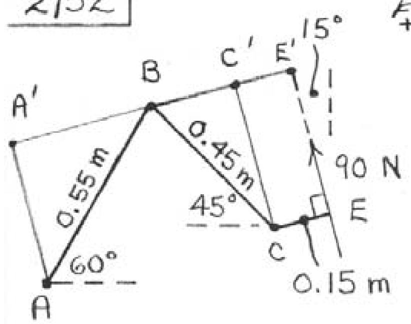


$$\alpha = \tan^{-1}\left[\frac{360 + 340 \sin 40° - 110 \sin 50°}{340 \cos 40° + 110 \cos 50°}\right]$$

$$= 56.2°$$

$$\circlearrowright^{+} M_O = 4.5\,(0.360 \cos 56.2°) = \underline{0.902 \text{ kN}\cdot\text{m CW}}$$

2/52



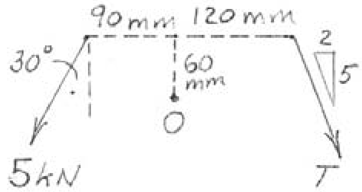
$$\circlearrowleft + \quad M_C = F(\overline{CE}) = 90(0.15) = \underline{13.50\ \text{N·m}}$$

$$M_B = F(\overline{BE'})$$
$$= 90(0.15 + 0.45 \sin 30°)$$
$$= \underline{33.8\ \text{N·m}}$$

$$M_A = F(\overline{A'E'})$$
$$= 90(0.15 + 0.45 \sin 30° + 0.55 \sin 45°)$$
$$= \underline{68.8\ \text{N·m}}$$

2/53 $M_O = 5\left[(\cos 30°)90 + (\sin 30°)60\right]$

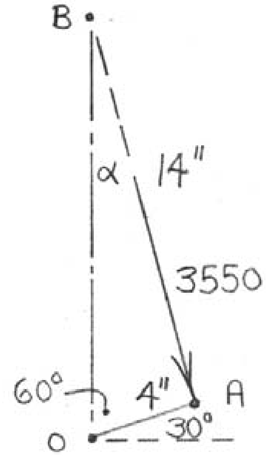
$- T\left[\frac{5}{\sqrt{29}}(120) + \frac{2}{\sqrt{29}}(60)\right] = 0$



$539.7 - 133.7T = 0$, $\underline{T = 4.04 \text{ kN}}$

$\sqrt{2^2+5^2} = \sqrt{29}$

2/54



Law of sines: $\dfrac{4}{\sin\alpha} = \dfrac{14}{\sin 60°}$

$\alpha = 14.33°$

$\overline{BO} = 14\cos 14.33° + 4\cos 60°$

$= 15.56$ in.

Consider 3550 lb acting at B:

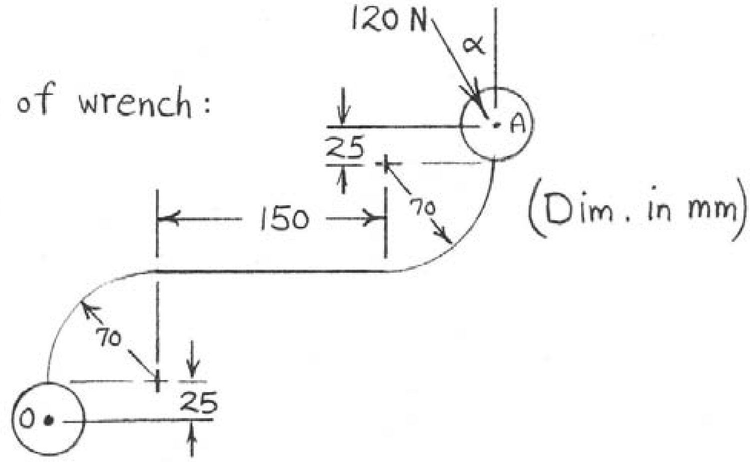
$+\circlearrowright M_o = (3550 \sin 14.33°)(15.56)$

$= \underline{13{,}670 \text{ lb-in.}}$

$(\text{or } M_o = \underline{1139 \text{ lb-ft}})$

2/55

Elements of wrench:



$\alpha = 30°$:

$$\stackrel{+}{\circlearrowright} M_o = 120 \cos 30° [70 + 150 + 70]$$

$$+ 120 \sin 30° [25 + 70 + 70 + 25] = 41\,500 \text{ N}\cdot\text{mm}$$

or $M_o = 41.5 \text{ N}\cdot\text{m CW}$

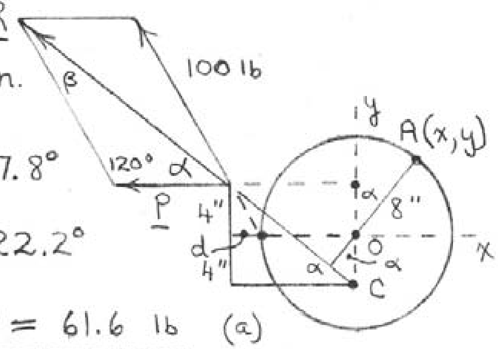
For maximum M_o:

$$\alpha = \tan^{-1}\left[\frac{25+70+25+70}{70+150+70}\right] = 33.2°$$

$$(M_o)_{max} = 120\sqrt{(25+70+25+70)^2 + (70+150+70)^2}$$

$$= 41\,600 \text{ N}\cdot\text{mm} \text{ or } 41.6 \text{ N}\cdot\text{m CW}$$

► 2/56



$d = \dfrac{4}{\tan 60°} = 2.31$ in.

$\alpha = \tan^{-1}\left(\dfrac{8}{d+8}\right) = 37.8°$

$\beta = 180° - (120° + \alpha) = 22.2°$

$\dfrac{P}{\sin\beta} = \dfrac{100}{\sin\alpha}$, $\underline{P = 61.6 \text{ lb}}$ (a)

$\dfrac{R}{\sin 120°} = \dfrac{100}{\sin\alpha}$, $\underline{R = 141.3 \text{ lb}}$ 37.8° (b)

$x = 8 \sin\alpha = \underline{4.90}$ in., $y = 8\cos\alpha = \underline{6.32 \text{ in.}}$ (c)

$+\circlearrowleft M_A = Rd = 141.3\,(8 + 4\cos\alpha) = \underline{1577 \text{ lb-in.}}$ (d)

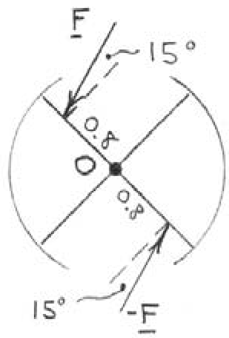
2/57 $\overset{+}{\curvearrowright}$ $M = Fd = 80(1.4) = \underline{112 \text{ lb-in. CW}}$

$\underline{(\text{or } 9.33 \text{ lb-ft CW})}$

2/58 $\underline{F} = 60\left(-\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\right) = \underline{-36\underline{i} + 48\underline{j} \text{ N}}$

$\curvearrowleft + \; M_C = 48(50) = 2400 \text{ N}\cdot\text{mm} = \underline{2.4 \text{ N}\cdot\text{m CCW}}$

2/59



$$\circlearrowleft\!+\; M_O = \Sigma Fd$$

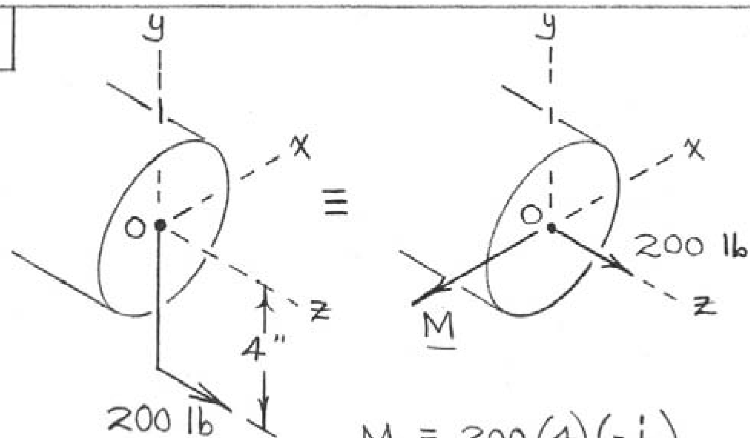
$$25 = 2F(\cos 15°)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$

2/60 $\underline{\underline{R} = 6\underline{j} \text{ kN}}$ @ $x = \frac{400}{6000} = 0.0667$ m

or $\underline{x = 66.7 \text{ mm}}$

2/61

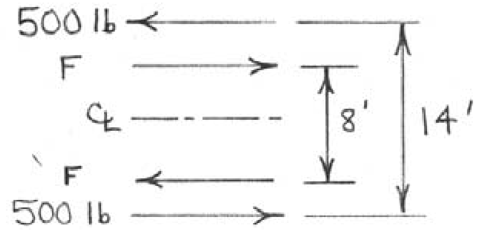


$$\underline{M} = 200(4)(-\underline{i})$$

$$= \underline{-800\underline{i} \text{ lb-in.}}$$

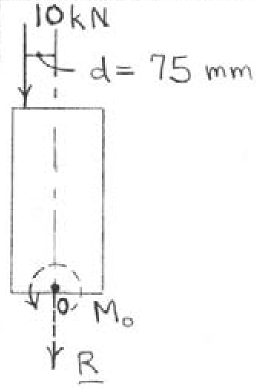
2/62

$$\curvearrowleft\!+\ \sum M = 500(14) - F(8) = 0$$



$$\underline{F = 875 \text{ lb}}$$

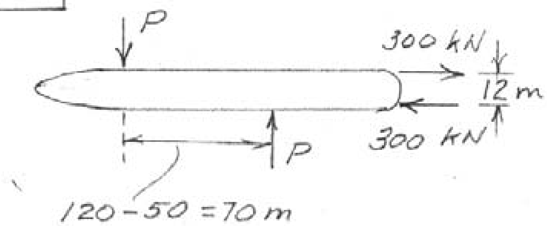
2/63



$$\begin{cases} R = \underline{10\ \text{kN}} \quad (\downarrow) \\ M_O = Fd = (10)(0.075) \end{cases}$$

$$= \underline{0.75\ \text{kN}\cdot\text{m}} \quad (\circlearrowleft)$$

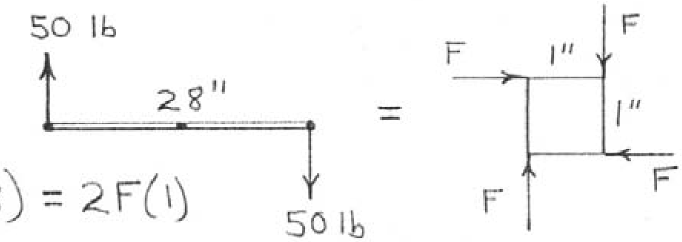
2/64



$70P = 300(12)$

$\underline{P = 51.4 \text{ kN}}$

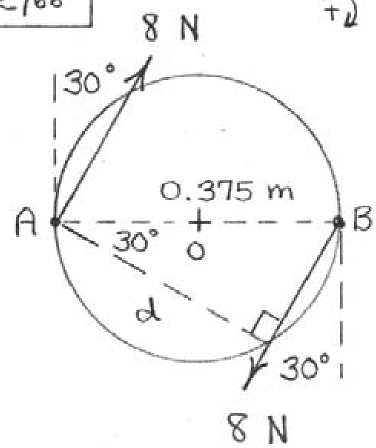
2/65



$M = 50(28) = 2F(1)$

$\underline{F = 700 \text{ lb}}$

2/66



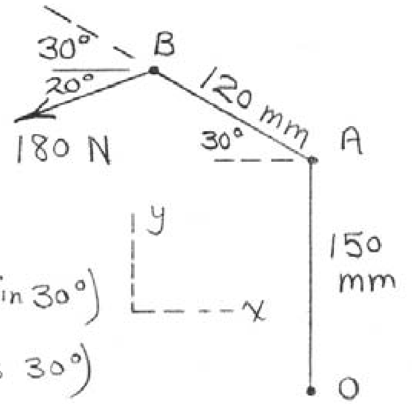
$\overset{+}{\circlearrowright}$ $M = Fd = 8\,(0.375 \cos 30°)$

$= \underline{2.60\ \text{N}\cdot\text{m}\ \ \text{CW}}$

2/67

$$\underline{F} = 180\left(-\cos 20^\circ \underline{i} - \sin 20^\circ \underline{j}\right)$$

$$= \underline{-169.1\,\underline{i} - 61.6\,\underline{j}\ \text{N}}$$

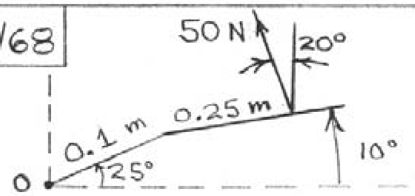


$$\circlearrowleft M_O = 180 \cos 20^\circ (150 + 120 \sin 30^\circ)$$

$$+ 180 \sin 20^\circ (120 \cos 30^\circ)$$

$$= 41\,900\ \text{N}\cdot\text{mm} \quad \text{or} \quad \underline{41.9\ \text{N}\cdot\text{m}\ \ \text{CCW}}$$

2/68



Use principle of moments.

$$\circlearrowleft + \Sigma M_O = 50 \cos 20° [0.1 \cos 25° + 0.25 \cos 10°]$$

$$+ 50 \sin 20° [0.1 \sin 25° + 0.25 \sin 10°]$$

$$= 17.29 \text{ N}\cdot\text{m}$$

Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \ \ \angle 110° \\ M_O = 17.29 \text{ N}\cdot\text{m} \ \circlearrowleft \end{cases}$$

2/69

$$\underline{F} = 250(\sin 10° \underline{i} + \cos 10° \underline{j})$$

$$= \underline{43.4\underline{i} + 246\underline{j} \text{ N}}$$

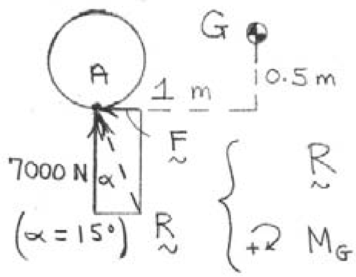
$$\circlearrowright\!+ \; M_O = 250\left[\cos 10°(0.235) + \sin 10°(0.050)\right]$$

$$= \underline{60.0 \text{ N·m CW}}$$

2/70

$$\tan 15^\circ = \frac{F}{7000}$$

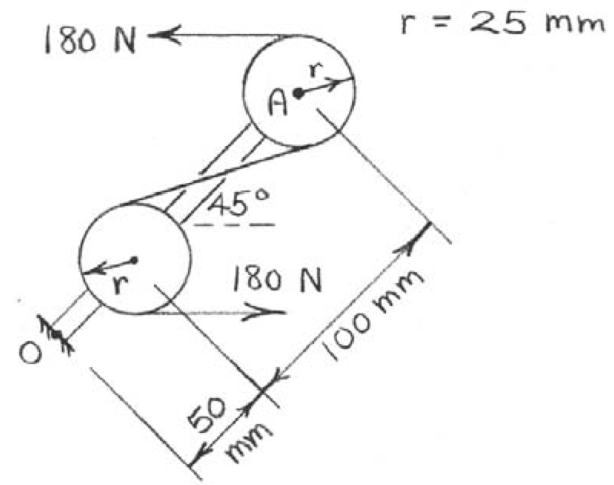
$$F = 1876 \text{ N}$$



At G:

$$\begin{cases} \mathbf{R} = \Sigma \mathbf{F} = \underline{7250 \text{ N}} \quad \underline{\angle 105^\circ} \\ +\circlearrowleft M_G = 7000(1) + 1876(.5) \\ \quad = \underline{7940 \text{ N}\cdot\text{m}} \end{cases}$$

2/71

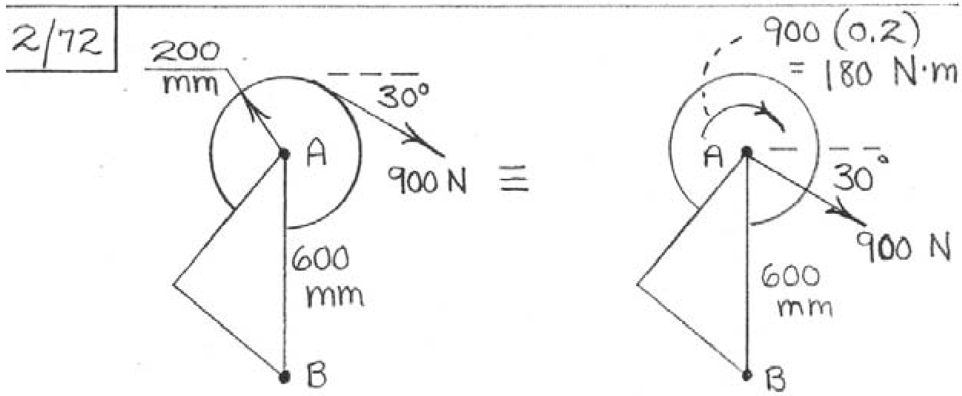


The system at O is a couple.

$\curvearrowleft\!+\; M = Fd = 180\,(100\sin 45^\circ + 25 + 25)$

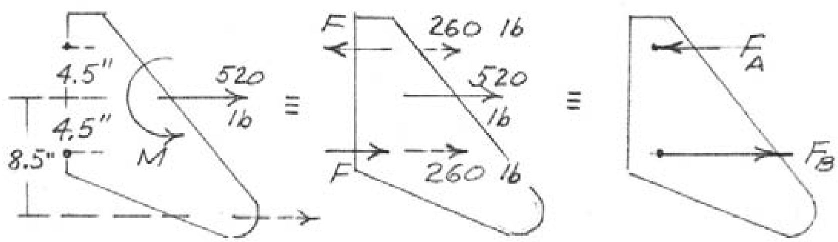
$= 21\,700\ \text{N}\cdot\text{mm}$ or $\underline{21.7\ \text{N}\cdot\text{m CCW}}$

2/72



$$+\circlearrowright\ M_B = 180 + 900\cos 30^\circ(0.6) = \underline{648\ \mathrm{N\cdot m\ CW}}$$

2/73



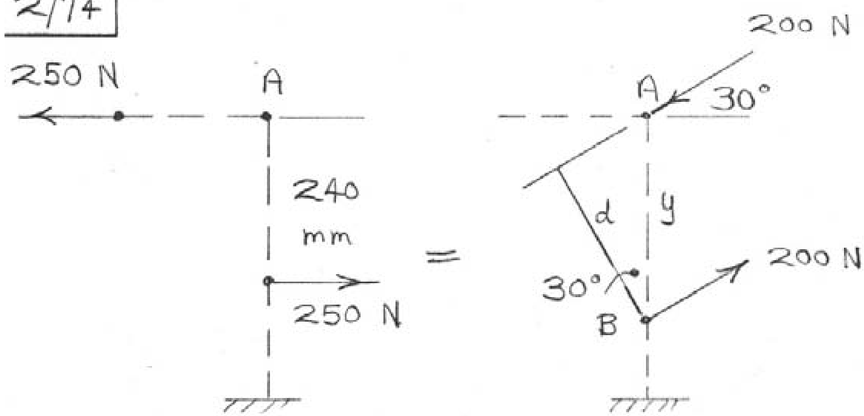
$M = Fd = 520(8.5) = 4420$ lb-in.

$F(9) = 4420, \quad F = 491$ lb

$F_A = 491 - 260 = \underline{231 \text{ lb}}$

$F_B = 491 + 260 = \underline{751 \text{ lb}}$

2/74

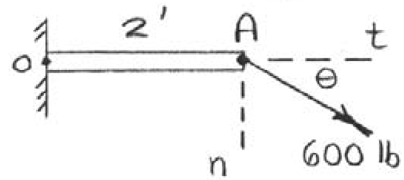


Equal couples :

$$\circlearrowleft^{+}\ 250(240) = 200(y \cos 30^\circ)$$

$$\underline{y = 346 \text{ mm}}$$

2/75



The equivalent force-couple system at O is

$R_t = 600 \cos\theta$ (lb)

$R_n = 600 \sin\theta$ (lb)

$\circlearrowleft + M_O = 2(600 \sin\theta) = 1200 \sin\theta$ (lb-ft)

Constraints:

$$\begin{cases} 600\cos\theta \le 550, & \theta \ge 23.6° \\ 600 \sin\theta \le 550, & \theta \le 66.4° \\ 1200 \sin\theta < 1000, & \theta \le 56.4° \end{cases}$$

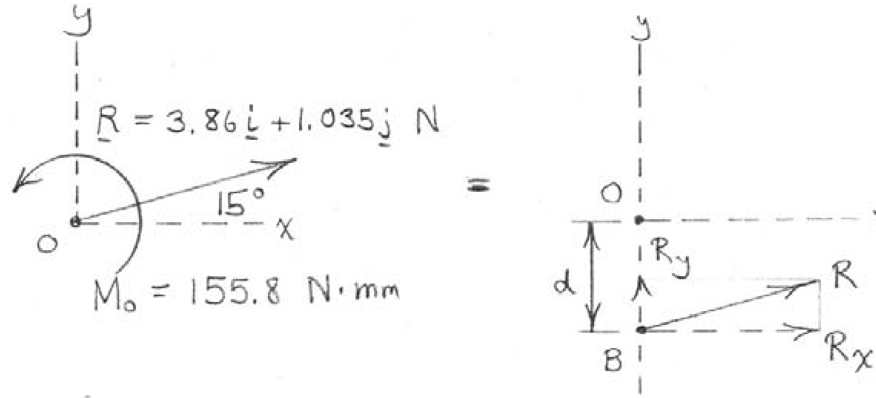
All considered, $\underline{23.6° \le \theta \le 56.4°}$

2/76 At O:

$\underline{R} = 4(\cos 15° \underline{i} + \sin 15° \underline{j}) = 3.86\underline{i} + 1.035\underline{j}$ N

$\curvearrowleft + M_O = 300 - 4\cos 15°(40) + 4\sin 15°(10)$

$= 155.8$ N·mm CCW



Condition: $R_x d = M_O$

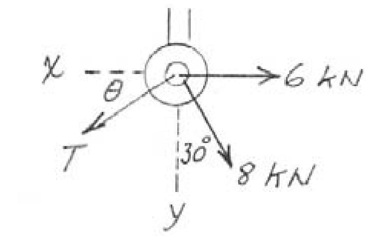
$3.86\, d = 155.8$, $d = 40.3$ mm

So $\underline{y = -40.3 \text{ mm}}$

2/77

$R = R_y = 15 = T \sin\theta + 8\cos 30^\circ$

$R_x = 0 = T\cos\theta - 6 - 8\sin 30^\circ$



So $T\sin\theta = 8.07$

$T\cos\theta = 10$

Divide & get $\theta = \tan \frac{8.07}{10}$

$\underline{\theta = 38.9^\circ}$

$T = \frac{10}{\cos 38.9^\circ} = \underline{12.85 \text{ kN}}$

$\boxed{2/78}$ $R_x = \Sigma F_x = 60 \cos 40° + 50 \sin 20° - 30 \cos 20°$

$= 34.87 \text{ kN}$

$R_y = \Sigma F_y = 60 \sin 40° + 40 - 50 \cos 20° + 30 \sin 20°$

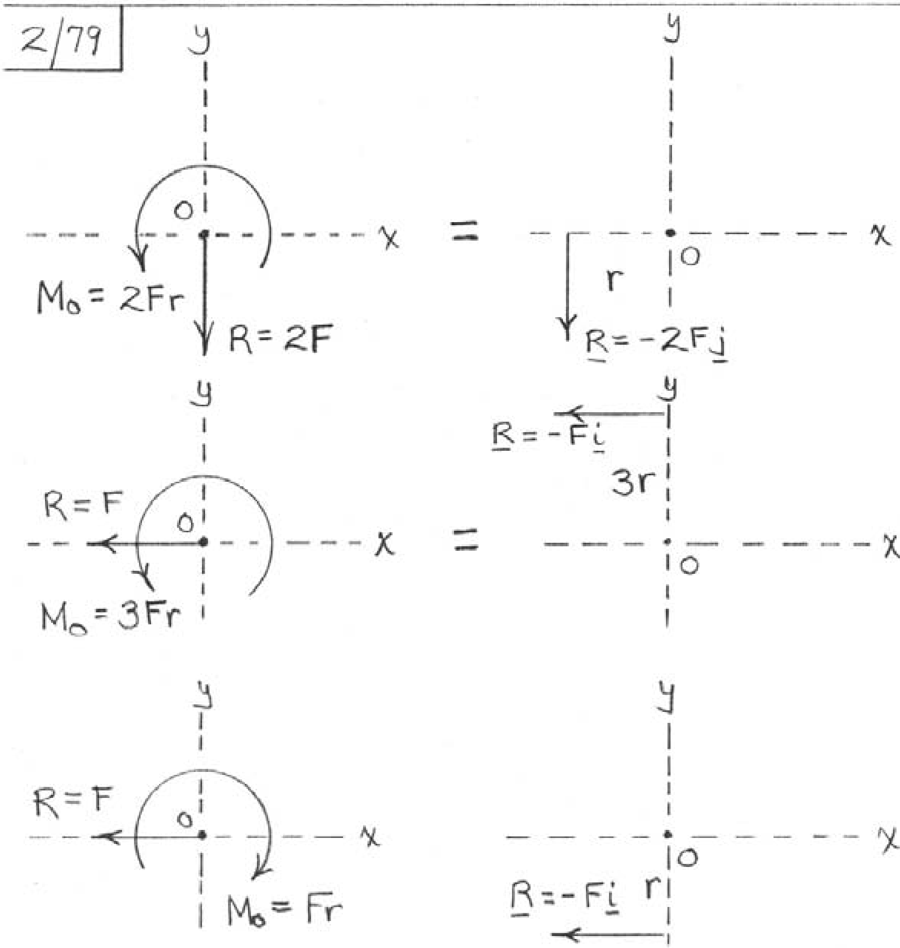
$= 41.84 \text{ kN}$

$R = \sqrt{34.87^2 + 41.84^2} = \underline{54.5 \text{ kN}}$

$\theta_x = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{41.84}{34.87} = \underline{50.2°}$

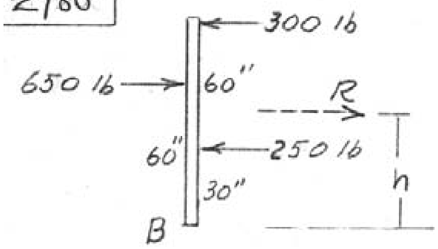
$\underline{\underline{\mathbf{R} = 34.9\mathbf{i} + 41.8\mathbf{j} \text{ kN}}}$

2/79



y
x
O
$M_O = 2Fr$
$R = 2F$
=
y
x
O
r
$\underline{R} = -2F\underline{j}$
y
x
O
$R = F$
$M_O = 3Fr$
=
y
$\underline{R} = -F\underline{i}$
3r
x
O
y
x
O
$R = F$
$M_O = Fr$
y
x
O
$\underline{R} = -F\underline{i}$
r

2/80



$R = \Sigma F = 650 - 250 - 300$

$= 100$ lb

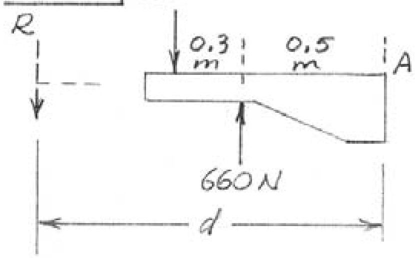
$Rh = \Sigma M_B$;

$100h = 650(60) - 300(90)$

$-250(30)$

$h = 45$ in.

2/81



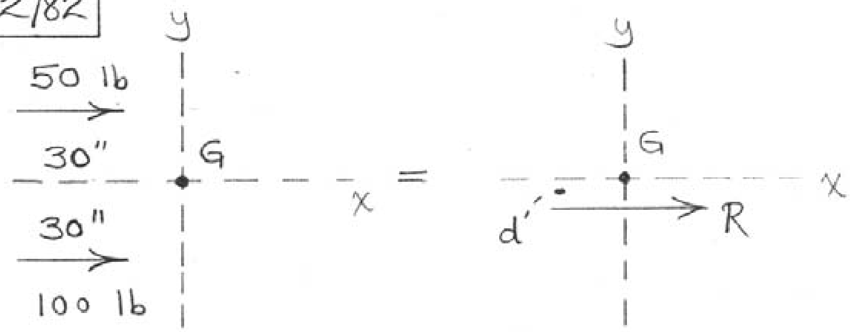
$R = \Sigma F = 680 - 660 = 20\ N$

$Rd = \Sigma M_A$

$20d = 680(0.8) - 660(0.5)$

$d = 10.70\ m$ to the left of A

2/82



$\xrightarrow{+} R = \Sigma F = 50 + 100 = 150 \text{ lb}$ (to the right)

$\curvearrowleft M_G = 100(30) - 50(30) = 1500 \text{ lb-ft CCW}$

$= Rd = 150d, \quad d = 10 \text{ in.}$

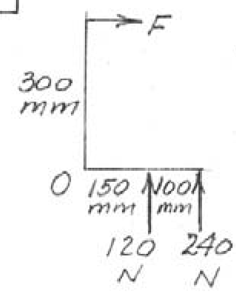
So the y-intercept of the standalone force resultant is $\underline{y = -10 \text{ in.}}$ The effect is to propel the truck forward and rotate it CCW about G.

2/83 $M_o = 0$, so

$\circlearrowleft_{+} M - 400(0.150 \cos 30°) - 320(0.300) = 0$

$\underline{M = 148.0 \text{ N}\cdot\text{m}}$

2/84



$\Sigma M_O = 0$ for R to pass through O

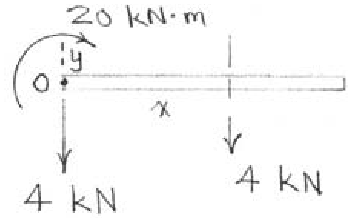
$300F - 120(150) - 240(250) = 0$

$\underline{F = 260 \text{ N}}$

2/85 Force – Couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (6-10)\,\underline{j} = -4\underline{j} \text{ kN}$$

$$\circlearrowleft + \; M_O = 6(3) - 10(9) + 52 = -20 \text{ kN}\cdot\text{m}$$

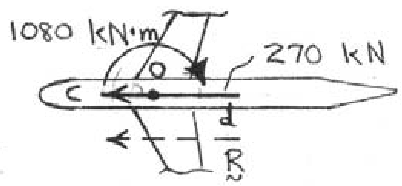


$$x = \frac{M_O}{R} = \frac{20}{4}$$

$$= \underline{5 \text{ m}} \quad \text{(on beam!)}$$

2/86 Force - Couple system at point O:

$$\begin{cases} R = 3(90) = 270 \text{ kN } (\leftarrow) \\ +\circlearrowright M_O = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{cases}$$

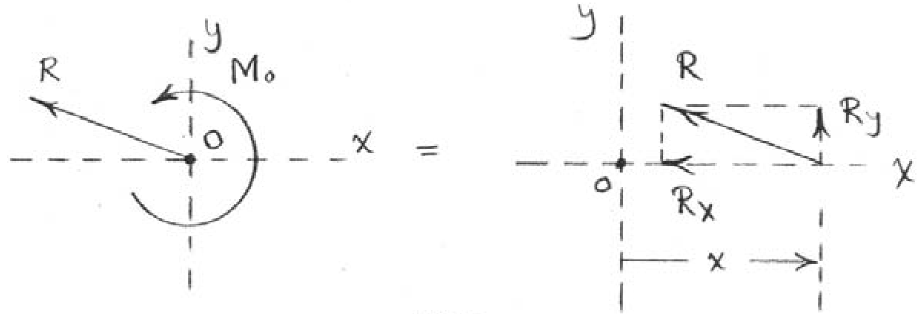


$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= \underline{4 \text{ m}}$$

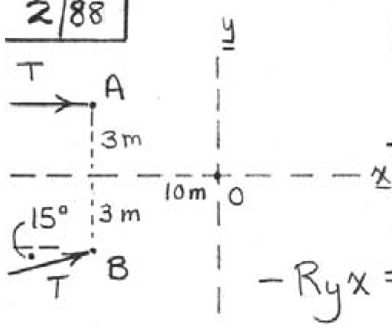
2/87 $\underline{\underline{R} = -50\underline{i} + 20\underline{j} \text{ lb}}$

$\circlearrowleft + \; M_O = -40(10) + 60(20) + 50(10) = 1300 \text{ lb-in.}$



$R_y x = M_O, \quad x = \frac{1300}{20} = \underline{65 \text{ in. (off pipe)}}$

2/88



$\underline{R} = \Sigma \underline{F} = T\underline{i} + T(\cos 15°\underline{i} + \sin 15°\underline{j})$

$= 1.966T\,\underline{i} + 0.259T\,\underline{j}$

$\stackrel{+}{\curvearrowleft} M_O = 3T - T\cos 15°(3)$

$+ T\sin 15°(10) = \underline{2.69T}$

$-R_y x = M_O: \quad -0.259T(x) = 2.69T$

$\underline{x = -10.39 \text{ m}}$

2/89 Equivalent force - couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-25 + 20 \sin 30°)\, \underline{i}$$
$$+ (-30 - 20 \cos 30°)\, \underline{j} = \underline{-15\,\underline{i} - 47.3\,\underline{j} \text{ kN}}$$

$$\curvearrowleft\!+ M_o = 25(5) - 30(9) - (20 \cos 30°)9$$
$$- (20 \sin 30°)5 = -351 \text{ kN}\cdot\text{m}$$

For final location of $\underline{R}$:

$$\underline{r} \times \underline{R} = \underline{M}_o \,, \quad (x\underline{i} + y\underline{j}) \times (-15\underline{i} - 47.3\,\underline{j})$$
$$= -351\, \underline{k}$$

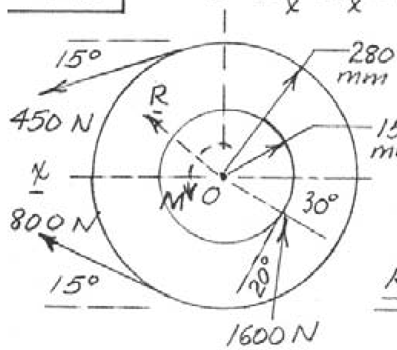
$$-47.3x + 15y = -351$$

Axis intersections : $\underline{x = 7.42 \text{ m}, \; y = -23.4 \text{ m}}$

2/90

$R_x = \Sigma F_x = (800 + 450)\cos 15° - 1600 \sin(30° - 20°)$

$= 1207 - 278 = 929.6 \text{ N}$



$R_y = \Sigma F_y = 1600 \cos 10°$

$+ (800 - 450)\sin 15°$

$= 1576 + 90.6 = 1666 \text{ N}$

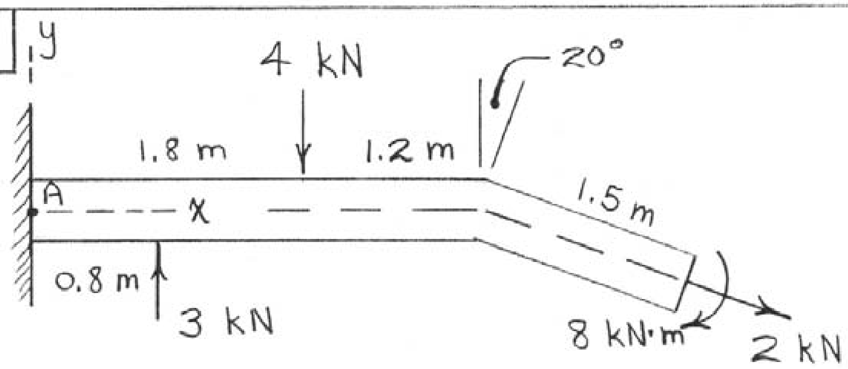
$\underline{R} = 930\underline{i} + 1666\underline{j} \text{ N}$

$M = \Sigma M_O \; (\circlearrowleft +); \; M = 1600 \cos 20°(0.150) + (450 - 800)0.280$

$= 225.5 - 98.0 = \underline{127.5 \text{ N}\cdot\text{m CCW}}$

so unit is speeding up in CCW dir.

2/91



$R_x = \Sigma F_x = 2 \cos 20° = 1.879$ kN

$R_y = \Sigma F_y = 3 - 4 - 2 \sin 20° = -1.684$ kN

$\underline{\mathbf{R} = 1.879\mathbf{i} - 1.684\mathbf{j}\ \text{kN}}$, $R = \sqrt{1.879^2 + 1.684^2} = \underline{2.52\ \text{kN}}$

$\circlearrowright^{+}$ $M = \Sigma M_A = 4(1.8) - 3(0.8) + 2 \sin 20°(3.0) + 8$

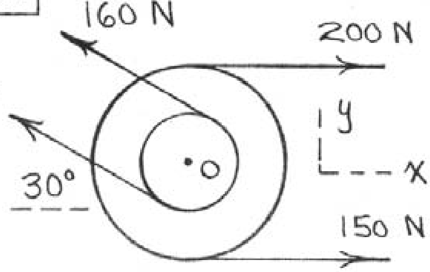
$= \underline{14.85\ \text{kN·m}\ \ \text{CW}}$

2/92 $\Sigma M_O = 0$ since $\underline{R}$ passes through O.

$40(8) + 60(4) - 5P\cos 20° = 0$, $\underline{P = 119.2 \text{ lb}}$

Moment of 40-lb & 60-lb forces unaffected by θ so result for P is not dependent on θ.

2/93



$$\begin{cases} r_1 = 0.1\text{ m} \\ r_2 = 0.2\text{ m} \end{cases}$$

$$+\circlearrowleft M_O = 0: \quad 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$$

$$\underline{T = 60\text{ N}}$$

$$R_x = \Sigma F_x = 200 + 150 - (160 + 60)\cos 30° = 159.5\text{ N}$$

$$R_y = \Sigma F_y = (160 + 30)\sin 30° = 110\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7\text{ N}}$$

$$\theta = \tan^{-1}(R_y/R_x) = \underline{34.6°}$$

$\boxed{2/94}$ $\underline{\underline{R} = 45\underline{i} - 15\underline{j} \text{ lb}}$

$\circlearrowleft M_A = 25(30) + 15(60) = 1650 \text{ lb-in.}$

or $\underline{\underline{M}_A = -1650\underline{k} \text{ lb-in.}}$

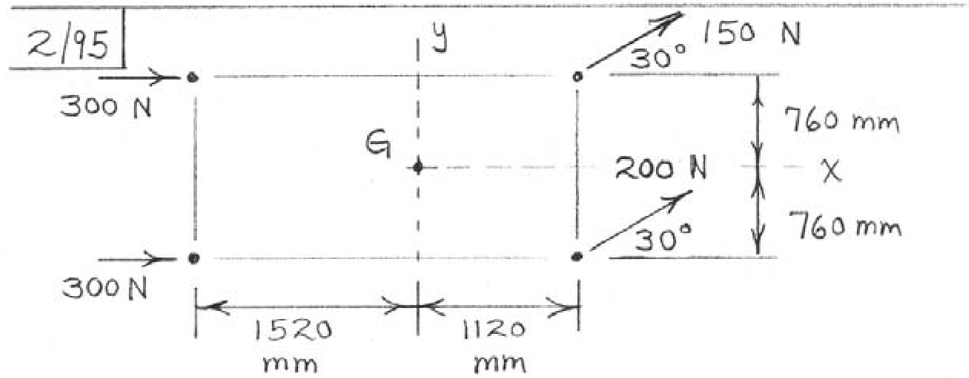
For final line of action, $\underline{r} \times \underline{R} = \underline{M}_A$

$(x\underline{i} + y\underline{j}) \times (45\underline{i} - 15\underline{j}) = -1650\underline{k}$

$\Rightarrow -15x - 45y = -1650$ or $\underline{y = -\frac{1}{3}x + \frac{110}{3} \text{ in.}}$

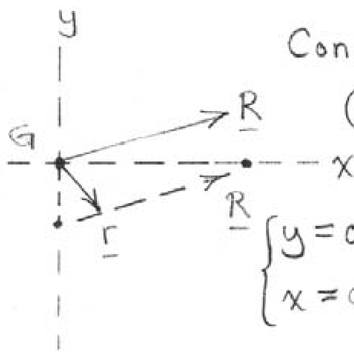
(Axis intercepts: $x = 110''$, $y = 110/3\,''$)

2/95



$$\underline{R} = \Sigma \underline{F} = 2(300\underline{i}) + 150(\cos 30°\underline{i} + \sin 30°\underline{j})$$
$$+ 200(\cos 30°\underline{i} + \sin 30°\underline{j}) = \underline{903\underline{i} + 175\underline{j}\ \text{N}}$$

$$\curvearrowleft + \; M_G = -150\cos 30°(760) + 150 \sin 30°(1120)$$
$$+ 200 \cos 30°(760) + 200 \sin 30°(1120)$$
$$= 229\,000\ \text{N·mm or } 229\ \text{N·m CCW}$$



Condition: $\underline{r} \times \underline{R} = \underline{M}_G$

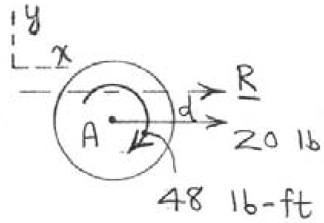
$$(x\underline{i} + y\underline{j}) \times (903\underline{i} + 175\underline{j}) = +229\underline{k}$$
$$175x - 903y = +229$$
$$\begin{cases} y = 0: & x = 1.308\ \text{m} \\ x = 0: & \underline{y = -0.253\ \text{m}} \end{cases}$$

2/96 Force - Couple System at point A:

$\underline{R} = \Sigma \underline{F} = -500\underline{j} + 60\underline{i} - 100\underline{j} - 40\underline{i} + 600\underline{j}$

$= \underline{20\underline{i} \text{ lb}}$

$\circlearrowleft + \; M_A = 2 - 40\left(\frac{15}{12}\right) = -48 \text{ lb-ft}$



$Rd = M_A$

$d = \frac{M_A}{R} = \frac{48}{20}$

$= \underline{2.40 \text{ ft}}$

2/97 Use $\overset{y}{\uparrow}\!\overset{+}{\circlearrowleft}\!\rightarrow x$ system at G:

$$\underline{R} = \Sigma \underline{F} = (80 + 40 + 40 + 50 \sin 30^\circ)\underline{i} + (50 \cos 30^\circ + 70)\underline{j}$$

$$= \underline{185\,\underline{i} + 113.3\,\underline{j} \text{ lb}}$$

$$M_G = 70(66) + 50 \sin 30^\circ (36) = 5520 \text{ lb-in.}$$

$$= \underline{460 \text{ lb-ft}} \quad (\circlearrowleft)$$

For line of action of resultant:

$$\underline{r} \times \underline{R} = \underline{M}_G$$

$$(x\underline{i} + y\underline{j}) \times (185\underline{i} + 113.3\underline{j}) = 460\,\underline{k}$$

$$113.3x - 185y = 460$$

$$\underline{x = 4.06 \text{ ft}} \text{ when } y = 0.$$

2/98 For a zero force - couple system at point O :

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30^\circ + F_D \sin 30^\circ)\,\underline{i} + (50 - 10 - 100 - 50 + F_B + F_C \cos 30^\circ + F_D \cos 30^\circ)\,\underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30^\circ (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}, \quad \underline{F_B = 98.9 \text{ N}}$$

2/99 $\underline{F} = F\underline{n}$

$$= 60\left[\frac{40\underline{i} - 50\underline{j} + 110\underline{k}}{\sqrt{40^2 + 50^2 + 110^2}}\right]$$

$$= \underline{18.86\underline{i} - 23.6\underline{j} + 51.9\underline{k} \text{ N}}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-23.6}{60}, \quad \underline{\theta_y = 113.1^\circ}$$

$\boxed{2/100}$ $\underline{T} = T\underline{n}_{BA}$

$$\underline{T} = 12\left[\frac{-35\underline{i} + 25\underline{j} + 60\underline{k}}{\sqrt{35^2 + 25^2 + 60^2}}\right]$$

$$= \underline{-5.69\underline{i} + 4.06\underline{j} + 9.75\underline{k} \quad \text{kN}}$$

2/101

$$\underline{F} = F\underline{n}_{AB}$$

$$= 750\left[\frac{-40\underline{i} + 70\underline{j} + 65\underline{k}}{\sqrt{40^2 + 70^2 + 65^2}}\right]$$

$$= -290\underline{i} + 507\underline{j} + 471\underline{k} \text{ lb}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{-290}{750}, \qquad \underline{\theta_x = 112.7°}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{507}{750}, \qquad \underline{\theta_y = 47.5°}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{471}{750}, \qquad \underline{\theta_z = 51.1°}$$

2/102 $F_z = 5 \cos 40° = 3.83$ kN

$F_h = 5 \sin 40° = 3.21$ kN

$F_x = -3.21 \sin 35° = -1.843$ kN

$F_y = 3.21 \cos 35° = 2.63$ kN

So $\underline{F} = -1.843\underline{i} + 2.63\underline{j} + 3.21\underline{k}$ kN

$F_{OA} = \underline{F} \cdot \underline{n}_{OA}$

$= (-1.843\underline{i} + 2.63\underline{j} + 3.21\underline{k}) \cdot (\cos 30°\underline{i} + \sin 30°\underline{j})$

$= -0.280$ kN (as a scalar)

$\underline{F}_{OA} = F_{OA}\,\underline{n}_{OA}$

$= -0.280(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= -0.243\underline{i} - 0.1401\underline{j}$ kN

$\boxed{2/103}$ $\underline{F} = F\underline{n} = 300\left[\frac{4\underline{i} - 8\underline{j} - 8\underline{k}}{\sqrt{4^2 + 8^2 + 8^2}}\right]$

$= \underline{300\left[\frac{1}{3}\underline{i} - \frac{2}{3}\underline{j} - \frac{2}{3}\underline{k}\right] \text{ lb}}$

$\underline{F_x = 100 \text{ lb}}$, $\underline{F_y = -200 \text{ lb}}$, $\underline{F_z = -200 \text{ lb}}$

2/104 $\underline{T} = T\underline{n}_{AB} = 2.4\left(\frac{2\underline{i} + \underline{j} - 5\underline{k}}{\sqrt{2^2 + 1^2 + 5^2}}\right)$

$$= \underline{0.876\,\underline{i} + 0.438\,\underline{j} - 2.19\,\underline{k} \text{ kN}}$$

Projection $T_{AC} = \underline{T} \cdot \underline{n}_{AC}$

$$= (0.876\underline{i} + 0.438\underline{j} - 2.19\underline{k}) \cdot \left(\frac{2\underline{i} - 2\underline{j} - 5k}{\sqrt{2^2 + 2^2 + 5^2}}\right)$$

$$= \underline{2.06 \text{ kN}}$$

2/105

$$\underline{T} = T\frac{\underline{CD}}{\overline{CD}} = 1.2\,\frac{1.5\underline{i} + 3\underline{j} - 4.5\underline{k}}{\sqrt{1.5^2 + 3^2 + 4.5^2}}$$

$$= \underline{0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k} \ \text{kN}}$$

The two indicated coordinate systems are equivalent for the question at hand.

2/106

$$T_{GF} = \underline{T} \cdot \underline{n}_{GF}$$

$$= (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) \cdot \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$$

$$= \underline{0.978 \text{ kN}}$$

2/107 $\underline{T} = T\underline{n}_{AB} = 10\left[\dfrac{4\underline{i} - 7.5\underline{j} + 5\underline{k}}{(4^2 + (-7.5)^2 + 5^2)^{1/2}}\right]$

$= \underline{10\ (0.406\,\underline{i} - 0.761\,\underline{j} + 0.507\,\underline{k})\ \text{kN}}$

$\cos\theta_x = 0.406, \quad \theta_x = 66.1°$

$\cos\theta_y = -0.761, \quad \theta_y = 139.5°$

$\cos\theta_z = 0.507, \quad \underline{\theta_z = 59.5°}$

$\boxed{2/108}$ The coordinates of point B are

$$(x_B, y_B, z_B) = (1.6, -0.8 \sin 30°, 0.8 \cos 30°)$$
$$= (1.6, -0.4, 0.693) \text{ m}$$

The position vector $\underline{BC}$ is

$$\underline{BC} = (0-1.6)\underline{i} + (0.7-(-0.4))\underline{j} + (1.2-0.693)\underline{k}$$
$$= -1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k} \text{ m}$$

The unit vector which characterizes $\underline{BC}$ is

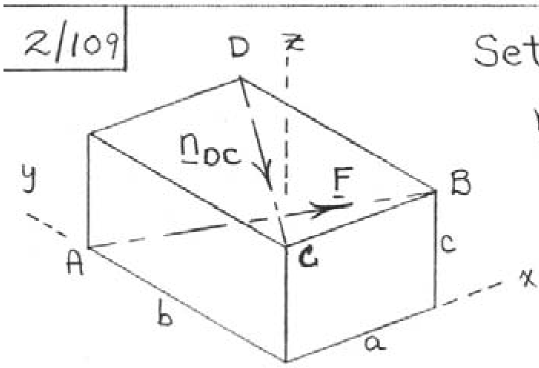
$$\underline{n}_{BC} = \frac{-1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k}}{\sqrt{1.6^2 + 1.1^2 + 0.507^2}}$$

$$= -0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k}$$

Then $\underline{T} = T\underline{n}_{BC}$

$$= 750(-0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k})$$
$$= \underline{-598\underline{i} + 411\underline{j} + 189.5\underline{k} \text{ N}}$$

2/109



Set up x-y-z axes shown

$$\underline{n}_{DC} = \frac{\underline{DC}}{\overline{DC}}$$

$$= \frac{-a\underline{i} - b\underline{j}}{\sqrt{a^2+b^2}}$$

$$\underline{F} = F\underline{n}_{AB} = F\frac{\underline{AB}}{\overline{AB}}$$

$$= F\frac{a\underline{i} - b\underline{j} + c\underline{k}}{\sqrt{a^2+b^2+c^2}}$$

$$F_{DC} = \underline{F}\cdot\underline{n}_{DC} = \frac{(b^2-a^2)F}{\sqrt{(a^2+b^2)}\sqrt{(a^2+b^2+c^2)}}$$

2/110

$$\underline{F} = F\underline{n}_{OM}$$

$$= F\left[\frac{\frac{a}{2}\underline{i} + b\underline{j} + \frac{c}{2}\underline{k}}{\sqrt{\frac{a^2}{4} + b^2 + \frac{c^2}{4}}}\right]$$

$$= F\left[\frac{a\underline{i} + 2b\underline{j} + c\underline{k}}{\sqrt{a^2 + 4b^2 + c^2}}\right]$$

$$F_{AE} = \underline{F} \cdot \underline{n}_{AE}$$

$$= F\left[\frac{a\underline{i} + 2b\underline{j} + c\underline{k}}{\sqrt{a^2 + 4b^2 + c^2}}\right] \cdot \left[\frac{-a\underline{i} - b\underline{j}}{\sqrt{a^2 + b^2}}\right]$$

$$= -\frac{F(a^2 + 2b^2)}{\sqrt{a^2 + b^2}\sqrt{a^2 + 4b^2 + c^2}}$$

2/111 $\underline{F} = F\underline{n}_{AB} = 200\left[\dfrac{-12\underline{i} + 24\underline{j} + 8\underline{k}}{\sqrt{12^2 + 24^2 + 8^2}}\right]$

$$= -85.7\underline{i} + 171.4\underline{j} + 57.1\underline{k} \quad \text{lb}$$

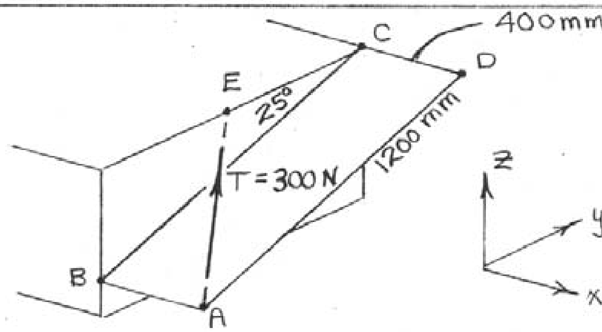
$\underline{OC} = 12\underline{i} + 24\underline{j}$ in.

The angle θ between $\underline{F}$ and $\underline{OC}$ is

$$\theta = \cos^{-1}\frac{\underline{F}\cdot\underline{OC}}{F(\overline{OC})} = \cos^{-1}\left[\frac{-85.7(12) + 171.4(24)}{200\left(\sqrt{12^2+24^2}\right)}\right]$$

$$= \underline{54.9^\circ}$$

2/112



$$\underline{T} = T\underline{n}_{AE} = 300\left[\frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}}\right]$$

$$= 300\left[-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}\right] \text{ N}$$

$\underline{n}_{BC} = \cos 25°\,\underline{j} + \sin 25°\,\underline{k}$

Carry out $T_{BC} = \underline{T} \cdot \underline{n}_{BC}$ to obtain

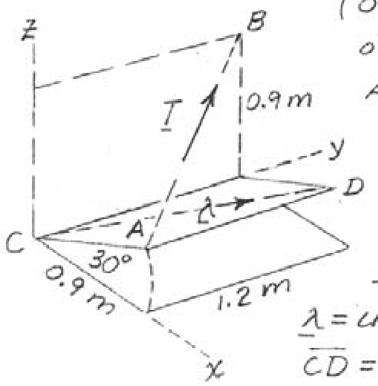
$$\underline{T_{BC} = 251 \text{ N}}$$

2/113 $T = 150\ N$

Coordinates of A are

$(0.9\cos 30°,\ 0,\ 0.9\sin 30°)$

or $(0.779, 0, 0.45)$ m



$\overline{AB} = \sqrt{\overline{0.779}^2 + \overline{1.2}^2 + (\overline{0.9 - 0.45})^2}$

$= 1.50$ m

$\ell = -0.779/1.5,\ m = 1.2/1.5,\ n = \frac{0.45}{1.5}$

$\underline{T} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$ N

$\underline{\lambda}$ = unit vector along CD

$\overline{CD} = \sqrt{\overline{0.9}^2 + \overline{1.2}^2} = 1.5$ m

$\ell = 0.779/1.5,\ m = 1.2/1.5,\ n = 0.45/1.5$

$\lambda = \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$

$T_{CD} = \underline{T} \cdot \underline{\lambda} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k}) \cdot \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$

$= \frac{100}{2.25}(-\overline{0.779}^2 + \overline{1.2}^2 + \overline{0.45}^2) = \frac{100}{2.25}(-0.6075 + 1.44 + 0.2025)$

$= \frac{400}{9}(1.035) = \underline{46.0\ N}$

▶2/114 The position of point A is

$$\underline{r}_A = 10\cos 15° \underline{i} + L\underline{j} + 10\sin 15° \underline{k}$$
$$= 9.66\underline{i} + L\underline{j} + 2.59\underline{k} \text{ in.}$$

[L = distance from O to disk center]

$$\underline{r}_B = 8\cos 30° \underline{i} + (L+36)\underline{j} - 8\sin 30° \underline{k}$$
$$= 6.93\underline{i} + (L+36)\underline{j} - 4\underline{k} \text{ in.}$$

$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = -2.73\underline{i} + 36\underline{j} - 6.59\underline{k} \text{ in.}$$

$$r_{AB} = \sqrt{2.73^2 + 36^2 + 6.59^2} = 36.7 \text{ in.}$$

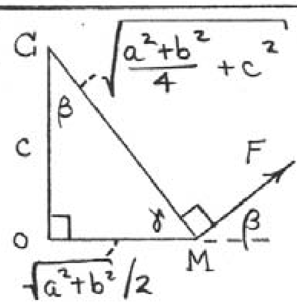
unstretched length is $\sqrt{(8-10)^2 + 36^2} = 36.1$ in.

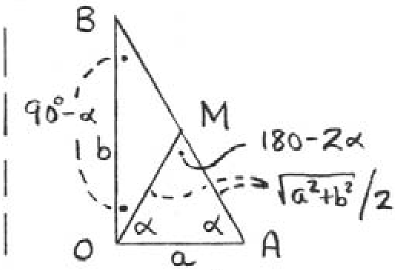
The spring force is $F = k\delta = 15(36.7 - 36.1) = 9.66$ lb

As a vector: $\underline{F} = F\underline{n}_{AB} = F\dfrac{\underline{r}_{AB}}{r_{AB}}$

$$\underline{F} = 9.66\left[\frac{-2.73\underline{i} + 36\underline{j} - 6.59\underline{k}}{36.7}\right]$$
$$= \underline{-0.719\underline{i} + 9.48\underline{j} - 1.734\underline{k} \text{ lb}}$$

2/115





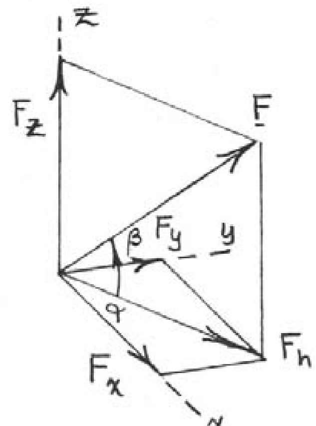
$$\tan\gamma = \frac{c}{\sqrt{a^2+b^2}/2} = \frac{2c}{\sqrt{a^2+b^2}}$$

$$\gamma + 90° + \beta = 180°$$

$$\beta = 90° - \gamma = 90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}$$

$$\tan\alpha = \frac{b}{a}$$

$$\begin{cases} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}} \end{cases}$$



$$\begin{cases} F_z = F\sin\beta \\ F_h = F\cos\beta \\ F_x = F_h\cos\alpha = F\cos\beta\cos\alpha \\ F_y = F_h\sin\alpha = F\cos\beta\sin\alpha \end{cases}$$

Now simplify $\sin\beta$ & $\cos\beta$ expressions:

$$\sin\beta = \sin\left[90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \sin 90° \cos\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right] - \cancel{\cos 90°}^{\,0} \sin\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+4c^2}}$$

(Also see that $\sin\beta = \cos\gamma$!)

$$\cos\beta = \sin\gamma = \frac{c}{\sqrt{\frac{a^2+b^2}{4}+c^2}} = \frac{2c}{\sqrt{a^2+b^2+4c^2}}$$

Finally,

$$F_x = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{a}{\sqrt{a^2+b^2}} = \frac{2acF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

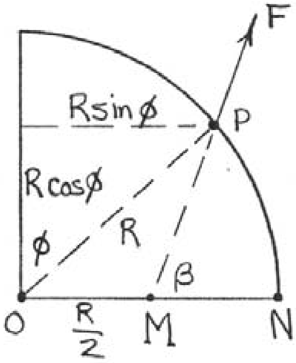
$$F_y = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{b}{\sqrt{a^2+b^2}} = \frac{2bcF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

$$F_z = F\sqrt{\frac{a^2+b^2}{a^2+b^2+4c^2}}$$

▶2/116

$F_x = F_{xy}\cos\theta$, $F_y = F_{xy}\sin\theta$

$F_z = F\sin\beta$, $F_{xy} = F\cos\beta$



$$\tan\beta = \frac{R\cos\phi}{R\sin\phi - \frac{R}{2}} = \frac{2\cos\phi}{2\sin\phi - 1}$$

So $\sin\beta = \dfrac{2\cos\phi}{\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2}}$

$$\cos\beta = \frac{2\sin\phi - 1}{\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2}}$$

Note that $\sqrt{(2\sin\phi-1)^2 + (2\cos\phi)^2} = \sqrt{5 - 4\sin\phi}$

So $\underline{F} = F\left[\cos\theta\cos\beta\,\underline{i} + \sin\theta\cos\beta\,\underline{j} + \sin\beta\,\underline{k}\right]$

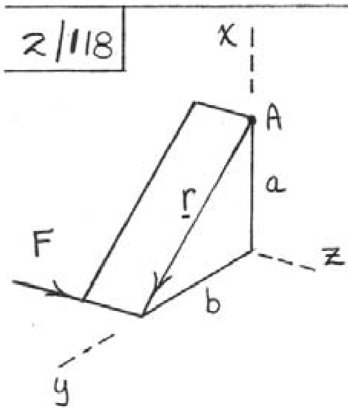
$$= \frac{F}{\sqrt{5-4\sin\phi}}\left[(2\sin\phi - 1)(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}) + 2\cos\phi\,\underline{k}\right]$$

$\boxed{2/117}$ By inspection, $\underline{\mathbf{M}_o = F(c\mathbf{j} - b\mathbf{k})}$

Or, $\mathbf{M}_o = \mathbf{r} \times \mathbf{F} = (b\mathbf{j} + c\mathbf{k}) \times F\mathbf{i}$

$= \underline{F(c\mathbf{j} - b\mathbf{k})}$ ✓

2/118



$$\underline{M}_A = \underline{r} \times \underline{F}$$

$$= (-a\underline{i} + b\underline{j}) \times F\underline{k}$$

$$= \underline{F(b\underline{i} + a\underline{j})}$$

2/119 (a) $\underline{M}_o = \underline{FL\underline{i}}$

(b) $\underline{M}_o = FL\underline{i} + FD\underline{k} = \underline{F(L\underline{i} + D\underline{k})}$

2/120

$$\underline{M}_z = (P\cos 30^\circ) d\,\underline{k}$$

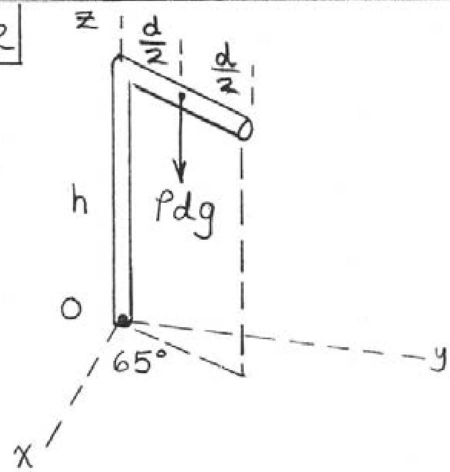
$$= (6\cos 30^\circ)(40)\,k$$

$$= \underline{208\,\underline{k}\ \text{lb-in.}}$$

2/121 $\underline{M} = -150(0.250 + 0.250)\underline{i} + 150(0.150)\underline{j}$

$= -75\underline{i} + 22.5\underline{j}$ N·m

2/122



$$\underline{M}_O = -\rho d g\left(\frac{d}{2}\sin 65°\right)\underline{i} + \rho d g\left(\frac{d}{2}\cos 65°\right)\underline{j}$$
$$= \underline{\rho g d^2\left(-0.453\,\underline{i} + 0.211\,\underline{j}\right)}$$

2/123 $\underline{M}_o = \underline{r} \times \underline{F}$

$$= (-6\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \times (-400\underline{j})$$

$$= \underline{480\underline{i} + 2400\underline{k} \ \ \text{N}\cdot\text{m}}$$

2/124 $\underline{M}_o = \underline{r}_{OA} \times \underline{F}$

$$= (1.5\underline{j} + 0.75\underline{k}) \times 4(-\cos 30° \underline{i} + \sin 30° \underline{j})$$

$$= -1.5\underline{i} - 2.60\underline{j} + 5.20\underline{k} \text{ lb-in.}$$

$\boxed{2/125}$ From the solution to Prob. 2/108, the force is $\underline{\mathbf{R} = \mathbf{T} = -598\mathbf{i} + 411\mathbf{j} + 189.5\mathbf{k}\ \text{N}}$

The moment associated with the couple is

$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{T}$, where $\mathbf{r}_{OC} = 0.7\mathbf{j} + 1.2\mathbf{k}$ m

Carry out the cross product to obtain

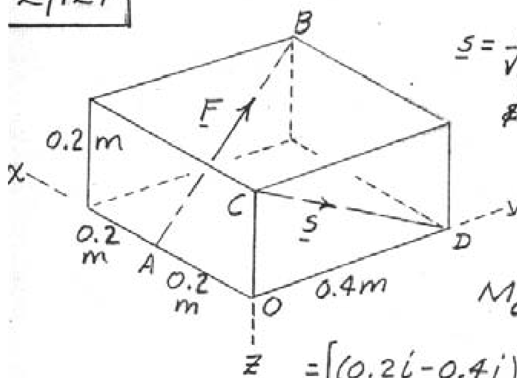
$$\underline{\mathbf{M}_O = -361\mathbf{i} - 718\mathbf{j} + 419\mathbf{k}\ \text{N}\cdot\text{m}}$$

$\boxed{2/126}$ $\underline{M} = \underline{r} \times \underline{F}$

$= -0.5\underline{i} \times 400\left(\cos 15°\underline{j} + \sin 15°\underline{k}\right)$

$= \underline{51.8\underline{j} - 193.2\underline{k}\ \text{N}\cdot\text{m}}$

2/127



$\mathbf{s} = \frac{1}{\sqrt{5}}(2\mathbf{j} + \mathbf{k})$

& $\mathbf{F} = \frac{F}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

Take $\mathbf{r} = \overrightarrow{DA}$

$= 0.2\mathbf{i} - 0.4\mathbf{j}$ m

$M_{CD} = \mathbf{r} \times \mathbf{F} \cdot \mathbf{s}$

$= \left[(0.2\mathbf{i} - 0.4\mathbf{j}) \times \frac{F}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} - \mathbf{k})\right] \cdot \frac{1}{\sqrt{5}}(2\mathbf{j} + \mathbf{k})$

So

$$50 = \frac{F}{\sqrt{30}} \begin{vmatrix} 0.2 & -0.4 & 0 \\ 1 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{F}{\sqrt{30}}(0.8 + 0.4) = \frac{1.2F}{\sqrt{30}}, \quad F = \frac{50\sqrt{30}}{1.2} = \underline{228 \text{ N}}$$

2/128

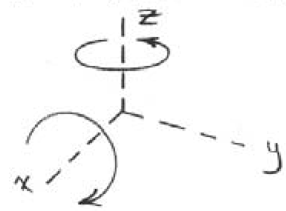
$$\underline{M}_O = 100\,(0.185 \sin 15^\circ)\,\underline{i} - 100\,(0.185 \cos 15^\circ)\,\underline{j}$$

$$= 4.79\underline{i} - 17.87\underline{j} \quad \text{N·m}$$

2/129 $\underline{M} = 1.2(40)\underline{k} - 1.2(50)\underline{i}$

$= \underline{-60\underline{i} + 48\underline{k} \quad \text{lb-in.}}$

The spacecraft will begin to rotate about its x- and z axes.



2/130

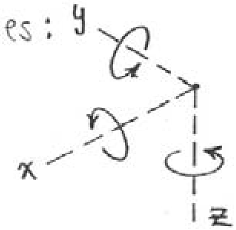
$$M_O = (250 \sin 60°)12 + (250 \cos 60°) \sin 40° (8 - 4.2)$$

$$= \underline{2900 \text{ lb-in.}}$$

2/131 $\underline{M} = (1700)(2)\underline{i} - (1700)(30)\underline{j} - (1700)(30)\underline{k}$

$= 3400\,\underline{i} - 51\,000\,\underline{j} - 51\,000\,\underline{k}$ N·m

The orbiter would acquire rotational motion about all three axes:



$\boxed{2/132}$ $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$, $\underline{r}_{OB} = 6\underline{i} + 13\underline{j}$ m

$$\underline{T} = T\underline{n}_{AB} = 24\left[\frac{6\underline{i} - 5\underline{j} - 30\underline{k}}{\sqrt{6^2 + 5^2 + 30^2}}\right]$$

$$= 4.65\underline{i} - 3.87\underline{j} - 23.2\underline{k} \text{ kN}$$

Carry out $\underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{\underline{M}_O = -302\underline{i} + 139.4\underline{j} - 83.6\underline{k} \text{ N}\cdot\text{m}}$$

2/133 $\underline{M}_O = 8(12)\underline{i} - 8(9)\underline{k}$

$= 96\underline{i} - 72\underline{k}$ lb-in.

The large moment about the x-axis is an undesirable characteristic of this wrench.

2/134 $\underline{F = 16 \text{ lb}}$ (up) will make $(M_o)_x$ zero.

The net moment about O is then

$$\underline{M}_o = [16(4) - 8(9)]\underline{k} = \underline{-8\underline{k} \text{ lb-in.}}$$

Comment: This wrench should be used only when access considerations make its use absolutely necessary!

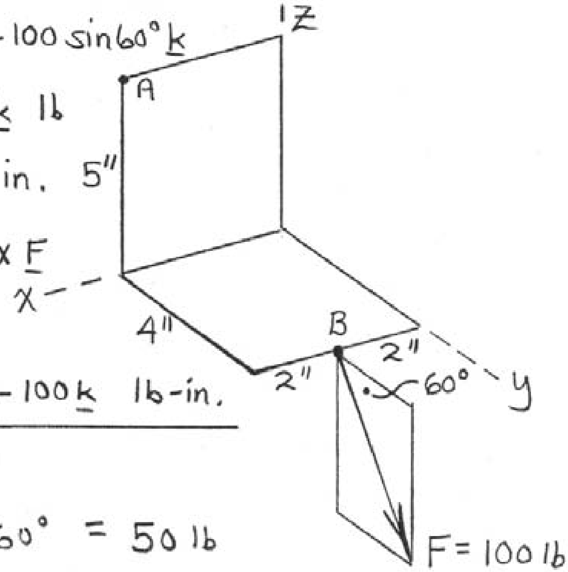
2/135 $\underline{F} = 100\cos 60°\,\underline{j} - 100\sin 60°\,\underline{k}$

(a) $= 50\underline{j} - 86.6\underline{k}$ lb

$\underline{r}_{AB} = -2\underline{i} + 4\underline{j} - 5\underline{k}$ in.

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{F}$ to obtain

$\underline{M_A = -96.4\underline{i} - 173.2\underline{j} - 100\underline{k}}$ lb-in.



(b) $\begin{cases} F_x = 0 \\ F_y = 100\cos 60° = 50 \text{ lb} \\ F_z = -100\sin 60° = -86.6 \text{ lb} \end{cases}$

$(M_A)_x = 50(5) - 86.6(4) = -96.4$ lb-in.

$(M_A)_y = -86.6(2) = -173.2$ lb-in.

$(M_A)_z = -50(2) = -100$ lb-in.

2/136

$$\underline{M}_o = 0\underline{i} - (200)(0.2 + 0.125 \sin 20°)\underline{j}$$

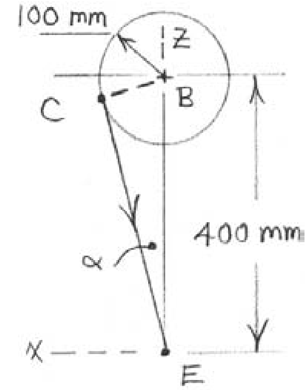
$$-200\,(0.125 \cos 20° - 0.070)\,\underline{k}$$

$$= \underline{-48.6\underline{j} - 9.49\underline{k} \quad \text{N}\cdot\text{m}}$$

There would be no z-component of $\underline{M}_o$ if

$d \cos 20° - 70 = 0$, $\underline{d = 74.5 \text{ mm}}$

2/137 Wheel detail:



$$\sin\alpha = \frac{\overline{BC}}{\overline{BE}} = \frac{100}{400}$$

$$\alpha = 14.48^\circ$$

So $\underline{T} = T\underline{n}_{CE}$

$$= 80\left[-\cos\alpha\,\underline{k} - \sin\alpha\,\underline{i}\right]$$

$$= -20\underline{i} - 77.5\underline{k} \text{ N}$$

$\underline{M}_o = \underline{r}_{OE} \times \underline{T}$, where $\underline{r}_{OE} = 0.2\underline{j}$ m

Carry out to obtain

$\underline{M}_o = -15.49\underline{i} + 4\underline{k}$ N·m

2/138 $\underline{M}_O = \underline{r}_{OB} \times \underline{F}$, where

$\underline{r}_{OB} = \underline{OA} + \underline{AB}$

$= 4\underline{k} + 16 \cos 10° (\cos 20° \underline{i} + \sin 20° \underline{j})$

$+ 16 \sin 10° \underline{k}$

$= 14.81\underline{i} + 5.39\underline{j} + 6.78\underline{k}$ in.

$\underline{F} = 80(-\cos 20° \underline{j} + \sin 20° \underline{i})$

$= 27.4\underline{i} - 75.2\underline{j}$ lb

Carry out $\underline{r}_{OB} \times \underline{F}$ to obtain

$\underline{M}_O = 510\underline{i} + 185.5\underline{j} + 1261\underline{k}$ lb-in.

2/139

$$\underline{T} = T\left[\frac{-0.35\underline{i} - 0.45\cos 20^\circ \underline{j} + (0.4 + 0.45\sin 20^\circ)\underline{k}}{\sqrt{(0.35)^2 + (0.45\cos 20^\circ)^2 + (0.4 + 0.45\sin 20^\circ)^2}}\right]$$

$$= 143.4\left[-0.449\underline{i} - 0.542\underline{j} + 0.710\underline{k}\right] \text{ N}$$

Moment of this force about the x-axis is

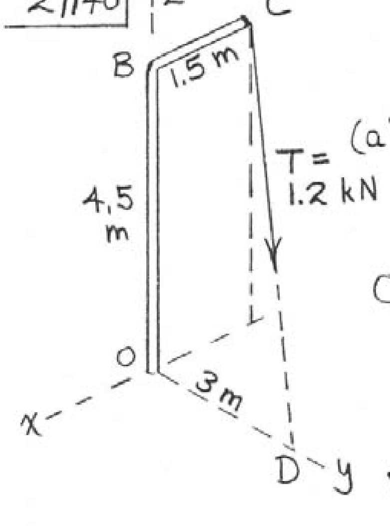
$$M_{O_x} = (0.710)(143.4)(0.45\cos 20^\circ) - 0.542(143.4)(0.45\sin 20^\circ) = \underline{31.1 \text{ N}\cdot\text{m}}$$

The moment of the weight W of the 15-kg plate about the x-axis is

$$(M_{O_x})_W = -15(9.81)\frac{0.45\cos 20^\circ}{2} = \underline{-31.1 \text{ N}\cdot\text{m}}$$

The moment of $\underline{T}$ about the line OB is <u>zero</u>, because $\underline{T}$ intersects OB.

2/140



From the solution to Prob. 2/95,

$\underline{T} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}$ kN

(a) $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$, $\underline{r}_{OD} = 3\underline{j}$ m

$\underline{M}_O = -2.89\underline{i} - 0.962\underline{k}$ kN·m

Could also use

$\underline{M}_O = \underline{r}_{OC} \times \underline{T}$, where

$\underline{r}_{OC} = -1.5\underline{i} + 4.5\underline{k}$ N

Result for $\underline{M}_O$ is the same.

(b) About z-axis:

$\underline{M}_z = -1.5\, T_y \underline{k} = -1.5\,(0.641)\underline{k} = \underline{-0.962\underline{k}\ \text{kN·m}}$

Also, $\underline{M}_z = \underline{M}_O \cdot \underline{k}\, \underline{k} = (-2.89\underline{i} - 0.962\underline{k}) \cdot \underline{k}\, \underline{k}$

$= \underline{-0.962\underline{k}\ \text{kN·m}}$

2/141 Using the coordinates of the figure:

$$\underline{M}_A = \underline{r} \times \underline{F} \quad , \quad \underline{F} = -1.8\,\underline{k} \text{ lb}$$

$$\underline{r} = [(2+1)\cos 30°]\,\underline{i} + 3\,\underline{j} + [(2+1)\sin 30°]\,\underline{k} \text{ in.}$$

$$\therefore \quad \underline{M}_A = -5.40\,\underline{i} + 4.68\,\underline{j} \text{ lb-in.}$$

$$\underline{M}_{AB} = (\underline{M}_A \cdot \underline{n}_{AB})\,\underline{n}_{AB} \,, \quad \underline{n}_{AB} = \cos 30°\,\underline{i} + \sin 30°\,\underline{k}$$

$$\therefore \quad \underline{M}_{AB} = -4.05\,\underline{i} - 2.34\,\underline{k} \text{ lb-in.}$$

2/142 $\underline{M}_O = \underline{r}_{OA} \times \underline{F}$

$$\underline{r}_{OA} = (0.050 + 0.130 \sin 60°)\underline{i} + (-0.140 - 0.130 \cos 60°)\underline{j} + 0.150k$$

$$= 0.1626\underline{i} - 0.205\underline{j} + 0.150\underline{k} \text{ m}$$

$$\underline{F} = 600(\cos 45° \sin 60° \underline{i} - \cos 45° \cos 60° \underline{j} + \sin 45° \underline{k})$$

$$= 600(0.612\underline{i} - 0.354\underline{j} + 0.707\underline{k})$$

$$= 367\underline{i} - 212\underline{j} + 424\underline{k} \text{ N}$$

Carry out $\underline{M}_O = \underline{r}_{OA} \times \underline{F}$ to obtain

$$\underline{M}_O = -55.2\underline{i} - 13.86\underline{j} + 40.8\underline{k} \text{ N}\cdot\text{m}$$

2/143

$$\begin{cases} R_x = \Sigma F_x = -7 \text{ kN} \\ R_y = \Sigma F_y = 4 - F_3 \cos\theta = -5 \text{ kN} & (1) \\ R_z = \Sigma F_z = F_3 \sin\theta = 6 \text{ kN} & (2) \end{cases}$$

(1) : $F_3 \cos\theta = 9$

(2) : $F_3 \sin\theta = 6$

Divide Eq. (2) by Eq. 1: $\tan\theta = \frac{2}{3}$

$\underline{\theta = 33.7°}$

Then $\underline{F_3 = 10.82 \text{ kN}}$

$R = \sqrt{7^2 + 5^2 + 6^2} = \underline{10.49 \text{ kN}}$

2/144

$$\begin{cases} \underline{R} = -3F\underline{k} \\ \underline{M}_o = -\frac{\sqrt{3}}{2}bF\underline{i} \end{cases}$$

$\underline{R} \cdot \underline{M}_o = 0$ so $\underline{R} \perp \underline{M}_o$

2/145 The given loads form two couples, each of which has an associated moment which is in the x-direction. So

$$\underline{R} = \Sigma \underline{F} = \underline{0}$$

$$\underline{M}_o = Fb\underline{i} + F\left(b\frac{\sqrt{3}}{2}\right)\underline{i}$$

$$= \underline{Fb\left(1+\frac{\sqrt{3}}{2}\right)\underline{i}}$$

The resultant of the system is a couple.

2/146 $\underline{R} = (1.2 - 1.2 - 1.2)\underline{j} = -1.2\underline{j}$ lb

$\underline{M}_G = 1.2(3)(20)\,\underline{k} + (1.2 - 1.2 - 1.2)(25)\,\underline{i}$

$\underline{M}_G = -30\underline{i} + 72\underline{k}$ lb-in.

2/147 $\underline{R} = \Sigma \underline{F} = \underline{-8\underline{i} \text{ kN}}$

$\underline{M}_G = 50(10)\underline{k} + 8(6)\underline{j} + 8(40)\underline{k}$

$= \underline{48\underline{j} + 820\underline{k} \text{ kN}\cdot\text{m}}$

2/148 $\underline{R} = (200 + 800)\underline{i} + 1200(\cos 10°\underline{j} - \sin 10°\underline{i})$

$$= \underline{792\underline{i} + 1182\underline{j} \text{ N}}$$

$$\underline{M}_O = [(200-800)(0.1) + (1200 \cos 10°)(0.075)]\underline{k}$$

$$+[-(200+800)(0.220+0.330) + 1200\sin 10°(0.220)]\underline{j}$$

$$+[1200 \cos 10° (0.220)]\underline{i}$$

$$= \underline{260\underline{i} - 504\underline{j} + 28.6\underline{k} \text{ N·m}}$$

2/149 $\underline{R} = \Sigma\underline{F} = 600(\sin 30°\,\underline{j} + \cos 30°\,\underline{k})$

$$+ 800(-\sin 45°\,\underline{j} + \cos 45°\,\underline{k})$$

$$= \underline{-266\,\underline{j} + 1085\,\underline{k}\ \ \text{N}}$$

$$\underline{M}_o = -0.080\,\underline{i} \times 600(\sin 30°\,\underline{j} + \cos 30°\,\underline{k})$$

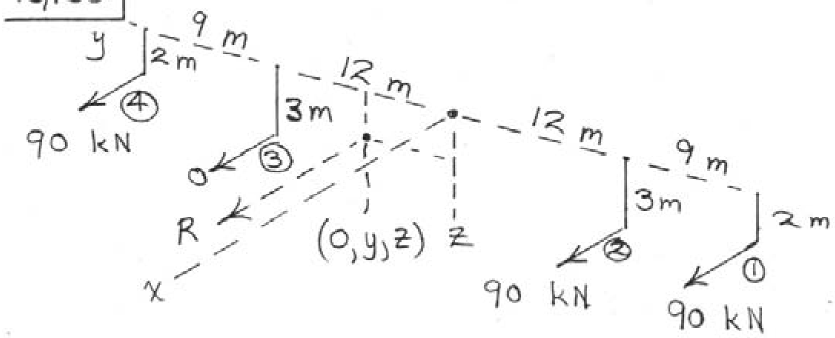
$$+ 0.160\,\underline{i} \times 800(-\sin 45°\,\underline{j} + \cos 45°\,\underline{k})$$

$$= \underline{-48.9\,\underline{j} - 114.5\,\underline{k}\ \ \text{N}\cdot\text{m}}$$

$\underline{R}$ is not perpendicular to $\underline{M}_o$, because

$\underline{R}\cdot\underline{M}_o \neq 0.$

2/150



$R = \Sigma F = 3(90) = 270 \text{ kN}$

$\Sigma M_z = -Ry: \; 90(21) + 90(12) - 90(21) = -270y$

$\underline{y = -4 \text{ m}}$

$\Sigma M_y = Rz: \; 2(90)(2) + 90(3) = 270z$

$\underline{z = 2.33 \text{ m}}$

$\boxed{2/151}$ The two 160-N forces constitute a couple $160(0.250)\underline{j} = 40\underline{j}$ N·m

$\underline{R} = \Sigma\underline{F} = \underline{120\underline{i} - 180\underline{j} - 100\underline{k}\ \text{N}}$

$\underline{M} = \Sigma\underline{M}_A = [120(0.25) + 100(0.3) + 40]\underline{j} + 50\underline{k}$ N·m

$= \underline{100\underline{j} + 50\underline{k}\ \text{N·m}}$

$\boxed{2/152}$ At O: $\underline{R} = \Sigma \underline{F} = (200 + 400)\underline{j} = \underline{600\underline{j} \text{ lb}}$

$\underline{M}_o = 600(8)\underline{k} + 400(3)\underline{i} = \underline{1200\underline{i} + 4800\underline{k} \text{ lb-ft}}$

$\underline{R} \cdot \underline{M}_o = 0 \Rightarrow \underline{R} \perp \underline{M}_o$ (Loading system can be represented by single force)

Let P have coordinates $(x, 0, z)$ and let $\underline{R}$ act at P.

$\underline{r}_{OP} \times \underline{R} = \underline{M}_o$: $(x\underline{i} + z\underline{k}) \times 600\underline{j} = 1200\underline{i} + 4800\underline{k}$

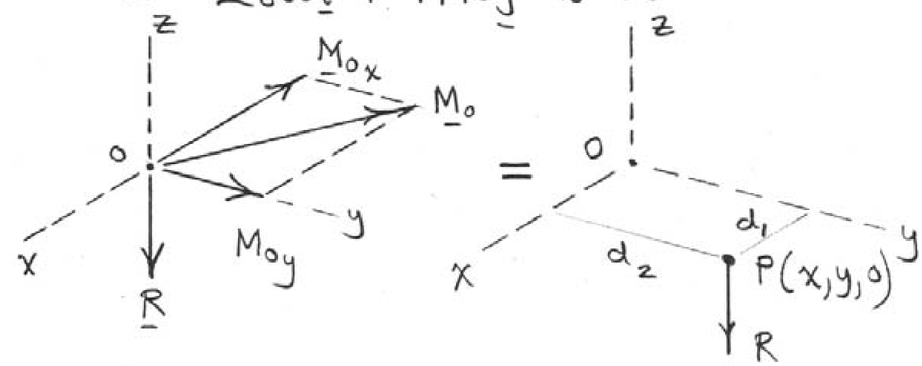
$$600x\,\underline{k} - 600z\,\underline{i} = 1200\underline{i} + 4800\underline{k}$$

$$\Rightarrow \underline{x = 8 \text{ ft}, \quad z = -2 \text{ ft}}$$

2/153 $\underline{R} = \Sigma \underline{F} = -50\underline{k} - 20\underline{k} = \underline{-70\underline{k}\ \text{lb}}$

$\underline{M}_o = 20(12\underline{j} - 20\underline{i}) + 50(24\underline{j} - 48\underline{i})$

$= -2800\underline{i} + 1440\underline{j}$ lb-ft



Conditions on P :

$R d_1 = |M_{o_y}|$, $d_1 = \dfrac{1440}{70} = 20.6$ in.

$R d_2 = |M_{o_x}|$, $d_2 = \dfrac{2800}{70} = 40$ in.

So the coordinates of P are

$(x, y) = \underline{(20.6, 40)\ \text{in.}}$ (Resultant is a standalone force

2/154 $R_x = -120\ N, \quad R_y = 0, \quad R_z = -160\ N$

$R = \sqrt{\overline{120}^2 + \overline{160}^2} = 200\ N, \quad \underline{R} = -120\underline{i} - 160\underline{k}\ \ N$

$M_x = 25 - 160(0.2) = -7\ N\cdot m$

$M_y = 160(0.075) - 120(0.100 - 0.075) = 9\ N\cdot m$

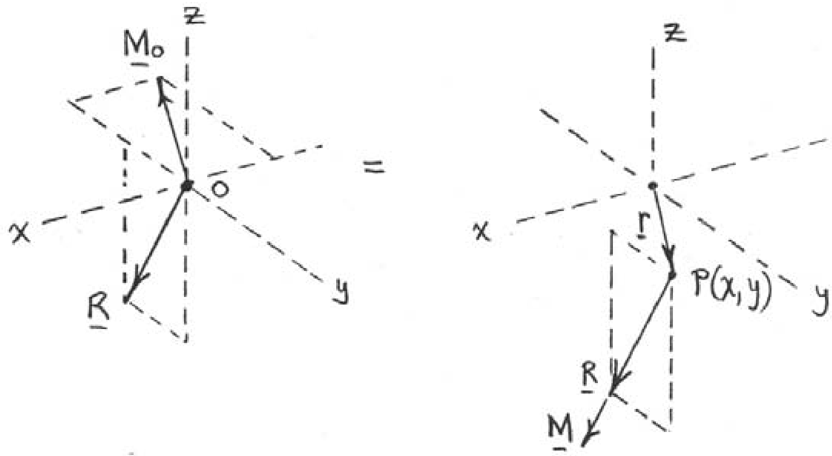
$M_z = 120(0.2) = 24\ N\cdot m$

$M = \sqrt{7^2 + 9^2 + \overline{24}^2} = 25.5\ N\cdot m$

$\underline{M} = -7\underline{i} + 9\underline{j} + 24\underline{k}\ \ N\cdot m$

2/155

$$\begin{cases} \underline{R} = -20\underline{j} - 40\underline{k} \text{ lb} \quad (= 44.7(-0.447\underline{j} - 0.894\underline{k})) \\ \underline{M}_o = -40(1.4)\underline{i} - 40(8)\underline{j} \\ \quad = -56\underline{i} - 320\underline{j} \text{ lb-in.} \end{cases}$$



$$\underline{M}_o: -56\underline{i} - 320\underline{j} = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + y\underline{j}) \times (-20\underline{j} - 40\underline{k}) + M(-0.447\underline{j} - 0.894\underline{k})$$

Equate coefficients:

$$\begin{cases} \underline{i}: -56 = -40y \\ \underline{j}: -320 = 40x - 0.447M \\ \underline{k}: 0 = -20x - 0.894M \end{cases}$$

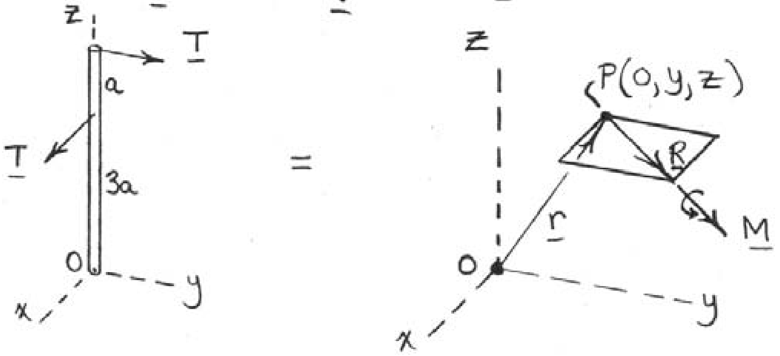
Solution:

$x = -6.4$ in.

$y = 1.4$ in.

2/156 $\underline{R} = \Sigma \underline{F} = T\underline{i} + T\underline{j} = \sqrt{2}T\left[\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right]$

$\Sigma \underline{M}_O = 3aT\underline{j} - 4aT\underline{i}$



$\Sigma \underline{M}_O = \underline{r} \times \underline{R} + \underline{M}$

$3aT\underline{j} - 4aT\underline{i} = (y\underline{j} + z\underline{k}) \times (T\underline{i} + T\underline{j}) + M\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

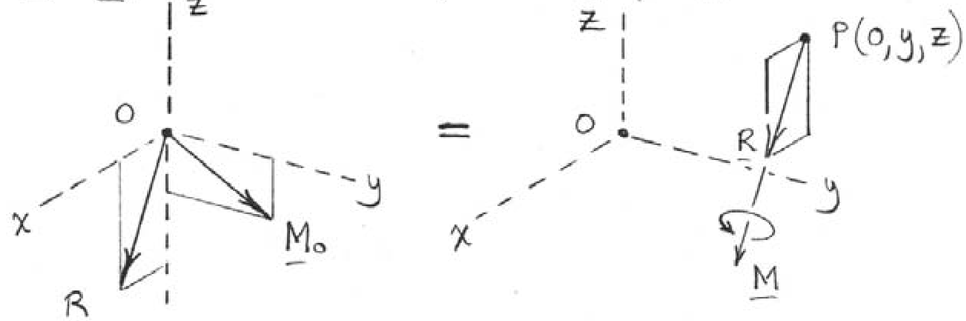
$$\Rightarrow \begin{cases} -4aT = -zT + \frac{M}{\sqrt{2}} \\ 3aT = zT + \frac{M}{\sqrt{2}} \\ 0 = -yT \end{cases} \qquad \text{So} \quad \begin{cases} \underline{y = 0} \\ \underline{z = \frac{7}{2}a} \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$$

So $\underline{M} = -\frac{\sqrt{2}}{2}aT\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$= \underline{\frac{-aT}{2}(\underline{i} + \underline{j})}$ (a negative wrench)

$\boxed{2/157}$ At O: $\begin{cases} \underline{R} = \Sigma \underline{F} = \underline{F\underline{i} - 3F\underline{k}} = \sqrt{10}\,F\left(\frac{1}{\sqrt{10}}\underline{i} - \frac{3}{\sqrt{10}}\underline{k}\right) \\ \underline{M}_o = F(2a)\underline{j} - F(a)\underline{k} = Fa(2\underline{j} - \underline{k}) \end{cases}$

$\underline{R} \cdot \underline{M}_o = 3F^2 a \neq 0$, so $\underline{R} \not\parallel \underline{M}_o$.



$\underline{M}_o = \underline{M} + \underline{r}_{OP} \times \underline{R}$

$Fa(2\underline{j} - \underline{k}) = M\left(\frac{1}{\sqrt{10}}\underline{i} - \frac{3}{\sqrt{10}}\underline{k}\right) + (y\underline{j} + z\underline{k}) \times (F\underline{i} - 3F\underline{k})$

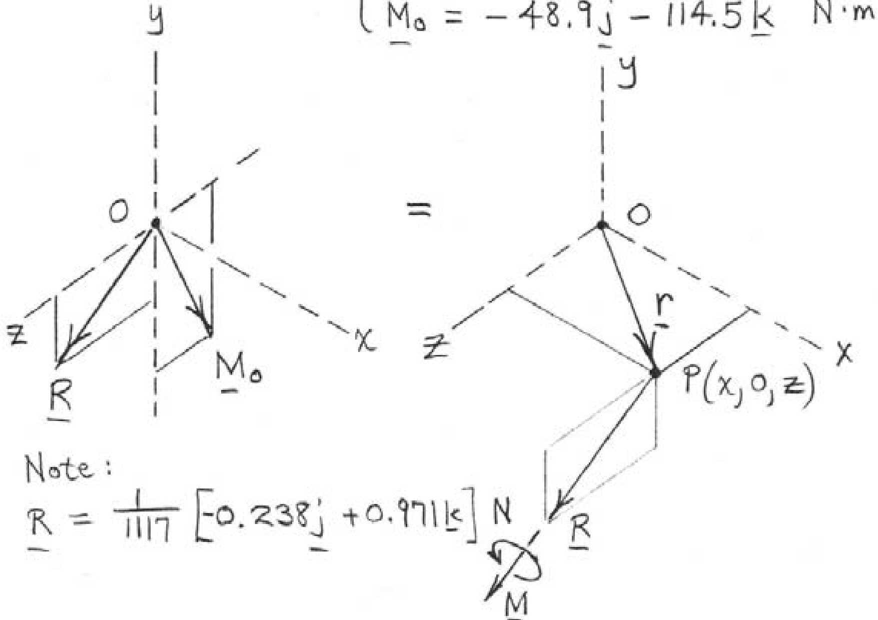
Equate coefficients:

$\left.\begin{array}{l} \underline{i}:\ 0 = -3Fy + \frac{M}{\sqrt{10}} \\ \underline{j}:\ 2Fa = Fz \\ \underline{k}:\ -Fa = -Fy - \frac{3M}{\sqrt{10}} \end{array}\right\}$ Solution: $\underline{y = \frac{a}{10}}$, $\underline{z = 2a}$, $M = 3\sqrt{10}\,aF/10 = 0.949Fa$

$\underline{M} = \frac{3\sqrt{10}\,aF}{10}\left(\frac{1}{\sqrt{10}}\underline{i} - \frac{3}{\sqrt{10}}\underline{k}\right) = \underline{\frac{3aF}{10}(\underline{i} - 3\underline{k})}$

2/158 From Prob. 2/149, the force-couple system at O is

$$\begin{cases} \underline{R} = -266\underline{j} + 1085\underline{k} \text{ N} \\ \underline{M}_O = -48.9\underline{j} - 114.5\underline{k} \text{ N·m} \end{cases}$$



Note:
$\underline{R} = \frac{1}{1117}\left[-0.238\underline{j} + 0.971\underline{k}\right]$ N

$$\underline{M}_O = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + z\underline{k}) \times (-266\underline{j} + 1085\underline{k}) + M(-0.238\underline{j} + 0.971\underline{k})$$

Equate coefficients:

$$\left.\begin{aligned} \underline{i}&: \; 0 = 266z \\ \underline{j}&: \; -48.9 = -1085x - 0.238M \\ \underline{k}&: \; -114.5 = -266x + 0.971M \end{aligned}\right\}$$

Solution:
$x = 0.0669$ m (66.9 mm)
$z = 0$
$M = -99.6$ N·m

▸2/159 $\underline{R} = \Sigma \underline{F} = 100\underline{i} + 100\underline{j}$ N

Direction cosines $l = 1/\sqrt{2}$, $m = 1/\sqrt{2}$, $n = 0$

Let $P = P(x, 0, z)$

$\underline{M}_P = 100z\underline{i} + 100(0.4-x)\underline{k} + 100(0.4-z)\underline{j}$
$- 100(0.3)\underline{k} - 20\underline{j}$
$= 100z\underline{i} + 100(0.2-z)\underline{j} + 100(0.1-x)\underline{k}$ N·m

Let $M = |\underline{M}_P|$. Equate direction cosines of $\underline{R}$ & $\underline{M}_P$ to obtain

$$\frac{100z}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.2-z)}{M} = \frac{1}{\sqrt{2}} \quad \frac{100(0.1-x)}{M} = 0$$

Solution : $\begin{cases} x = 0.1 \text{ m} \\ z = 0.1 \text{ m} \\ M = 10\sqrt{2} \text{ N}\cdot\text{m} \end{cases}$

$\underline{M} = 10\sqrt{2}\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right) = 10\underline{i} + 10\underline{j}$ N·m

▶ 2/160 $\underline{R} = \Sigma \underline{F} = 400(-\cos 24^\circ \underline{j} + \sin 24^\circ \underline{i}) + 800(-\cos 24^\circ \underline{j} - \sin 24^\circ \underline{i})$

$\underline{R} = -162.7\underline{i} - 1096\underline{j}$ N , $R = \sqrt{R_x^2 + R_y^2} = 1108$ N

Dir. cosines : $l = -0.1468$, $m = -0.9892$, $n = 0$

$\underline{M}_A = [(-400(0.2) - 800(0.6))\cos 24^\circ]\underline{i} + [(-400 + 800)(0.2)\cos 24^\circ - 200]\underline{k} + [800(0.6) - 400(0.2)]\sin 24^\circ \underline{j} = -512\underline{i} + 162.7\underline{j} - 126.9\underline{k}$ N·m

$\underline{R} \cdot \underline{M}_A = -95\,100$ N²·m $\neq 0 \Rightarrow R \not\parallel M_A$

Now move system to $P(x, 0, z)$ & form wrench

$\underline{R} = -162.7\underline{i} - 1096\underline{j}$ N

$\underline{M}_P = \underline{M}_A + \underline{r}_{PA} \times \underline{R}$, where $\underline{r}_{PA} = -x\underline{i} - z\underline{k}$

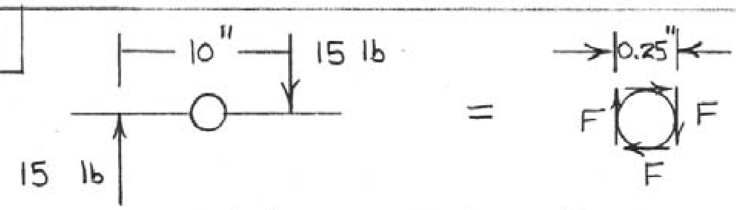
$= (-512 - 1096z)\underline{i} + (162.7z + 162.7)\underline{j} + (-126.9 + 1096x)\underline{k}$

Let $M = |\underline{M}_P|$ & equate dir. cosines of $\underline{R}$ and $\underline{M}_P$:

$\frac{-512 - 1096z}{M} = -0.1468$; $\frac{162.7z + 162.7}{M} = -0.9892$; $\frac{-126.9 + 1096x}{M} = 0$

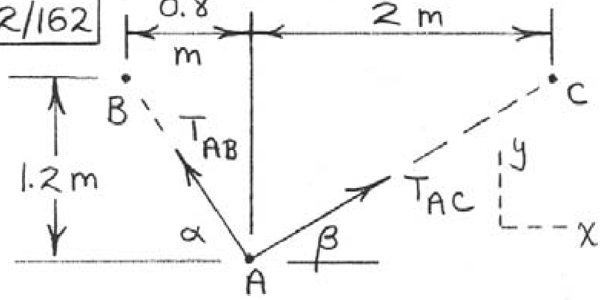
Solve to obtain $\begin{cases} M = -85.8 \text{ N·m} \\ x = 0.1158 \text{ m} \\ z = -0.478 \text{ m} \end{cases}$

2/161



$$M = Fd = 15(10) = 2F(0.25), \quad \underline{F = 300 \text{ lb}}$$

2/162



$$\alpha = \tan^{-1}\left(\frac{1.2}{0.8}\right) = 56.3°$$

$$\beta = \tan^{-1}\left(\frac{1.2}{2}\right) = 31.0°$$

$$\underline{T}_{AB} = T_{AB}\,\underline{n}_{AB} = 0.858(60)(9.81)\left[-\cos 56.3°\,\underline{i} + \sin 56.3°\,\underline{j}\right]$$

$$= \underline{-280\,\underline{i} + 420\,\underline{j}\ \text{N}}$$

$$\underline{T}_{AC} = T_{AC}\,\underline{n}_{AC} = 0.555(60)(9.81)\left[\cos 31.0°\,\underline{i} + \sin 31.0°\,\underline{j}\right]$$

$$= \underline{280\,\underline{i} + 168.1\,\underline{j}\quad \text{N}}$$

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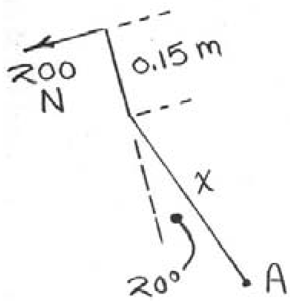
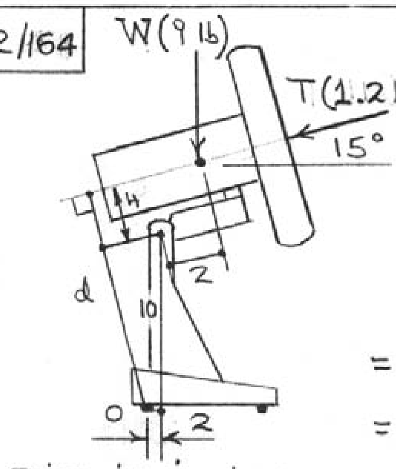


$M_A = Fd: \quad 80 = 200(0.15 + x \cos 20°)$

$x = 0.266$ m or 266 mm

2/164



$M_o = Fd$

$= 1.2(4 + 10\cos 15° - 2\sin 15°)$

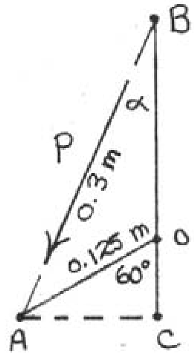
$= 15.77$ lb-in. (↶)

$M_{o_W} = W d_W$

$= 9(2 + 2\cos 15° - 4\sin 15°)$

$= 26.1$ lb-in. (↷)

2/165



$AC = 0.125 \sin 60° = 0.1083 \text{ m}$

$\alpha = \sin^{-1} \dfrac{0.1083}{0.300} = 21.2°$

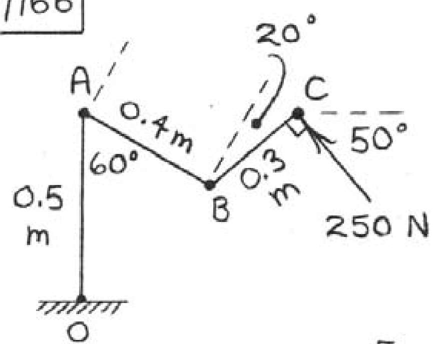
$BC = 0.300 \cos \alpha = 0.280 \text{ m}$

$BO = 0.280 - 0.125 \cos 60° = 0.217 \text{ m}$

$\curvearrowleft + \; M_O = 720 = P \sin\alpha\,(BO) = P \sin 21.2° (0.217)$

$\underline{P = 9.18 \text{ kN}}$

2/166



$$\circlearrowleft + \; M_O = 250\cos 50^\circ\left[0.5 - 0.4\cos 60^\circ + 0.3\sin 40^\circ\right]$$

$$+ 250\sin 50^\circ\left[0.4\sin 60^\circ + 0.3\cos 40^\circ\right]$$

$$= \underline{189.6 \text{ N}\cdot\text{m} \quad \text{CCW}}$$

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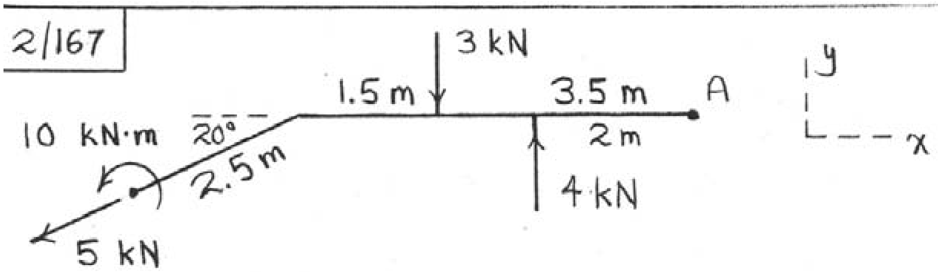


$$R_x = \Sigma F_x = -5\cos 20^\circ = -4.70 \text{ kN}$$

$$R_y = \Sigma F_y = -5\sin 20^\circ + 4 - 3 = -0.710 \text{ kN}$$

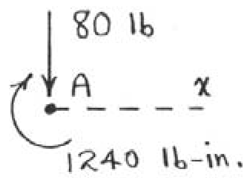
$$R = \sqrt{R_x^2 + R_y^2} = \underline{4.75 \text{ kN}}$$

$$\circlearrowleft + \Sigma M_A = 10 + 5\sin 20^\circ (5) + 3(3.5) - 4(2)$$

$$= \underline{21.1 \text{ kN}\cdot\text{m CCW}}$$

2/168 At A: $R = \Sigma F = 200 + 180 - 300 = \underline{80 \text{ lb } (\downarrow)}$

$\circlearrowleft M_A = 200(8) + 180(28) - 300(18) = \underline{1240 \text{ lb-in.}}$



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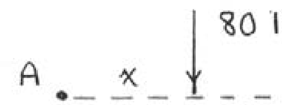
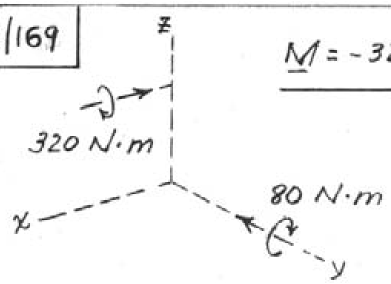


$80x = 1240$, $\underline{x = 15.5 \text{ in.}}$

2/169



$$\underline{M} = -320\underline{i} - 80\underline{j} \quad N\cdot m$$

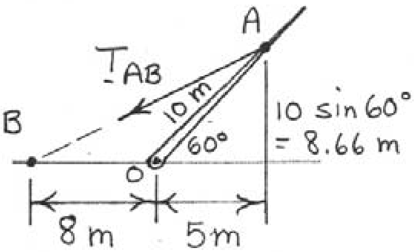
$$\cos\theta_x = \frac{M_x}{|\underline{M}|} = \frac{-320}{\sqrt{320^2 + 80^2}} = \underline{-0.970}$$

2/170 $\underline{P} = P\left(\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right)$; $\underline{r}_{AB} = b(-\underline{i} + \underline{j} + \underline{k})$

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{P}$ to obtain

$$\underline{M}_A = \frac{Pb}{5}\left(-3\underline{i} + 4\underline{j} - 7\underline{k}\right)$$

2/171



$\overline{AB}^2 = 8^2 + 10^2 - 2(8)(10)\cos 120°$

$\overline{AB} = 15.62$ m

(a) $\underline{T}_{AB} = 3\left[-\frac{13}{15.62}\cos 35°\underline{i} - \frac{13}{15.62}\sin 35°\underline{j} - \frac{8.66}{15.62}\underline{k}\right]$

$= \underline{-2.05\underline{i} - 1.432\underline{j} - 1.663\underline{k} \text{ kN}}$

(b) Carry out $\underline{r}_{OB} \times \underline{T}_{AB}$, where $\underline{r}_{OB} = 8(-\cos 35°\underline{i} - \sin 35°\underline{j})$ m to obtain

$\underline{\underline{M}_O = 7.63\underline{i} - 10.90\underline{j} \text{ kN}\cdot\text{m}}$

$\therefore$ $\underline{M_{O_x} = 7.63 \text{ kN}\cdot\text{m}, \ M_{O_y} = -10.90 \text{ kN}\cdot\text{m}, \ M_{O_z} = 0}$

(c) $T_{AO} = \underline{T}_{AB} \cdot \underline{n}_{AO}$

With $\underline{n}_{AO} = -\cos 60°\cos 35°\underline{i} - \cos 60°\sin 35°\underline{j} - \sin 60°\underline{k}$,

we obtain $\underline{T_{AO} = 2.69 \text{ kN}}$

2/172 Coordinates of A: $(x_A, y_A, z_A) = (0, r, 0)$

Coordinates of B: $(x_B, y_B, z_B) = (h, r\cos\theta, r\sin\theta)$

So $\underline{r}_{AB} = h\underline{i} + (r\cos\theta - r)\underline{j} + r\sin\theta\,\underline{k}$

and $\underline{F} = F\underline{n}_{AB} = F\left[\dfrac{h\underline{i} + r(\cos\theta - 1)\underline{j} + r\sin\theta\,\underline{k}}{\sqrt{h^2 + [r(\cos\theta - 1)]^2 + [r\sin\theta]^2}}\right]$

$$= F\left[\frac{h\underline{i} + r(\cos\theta - 1)\underline{j} + r\sin\theta\,\underline{k}}{\sqrt{h^2 + 2r^2(1 - \cos\theta)}}\right]$$

2/173 $\underline{M}_{100} = -100\,(0.200)\,\underline{i} = -20\underline{i}$ N·m

$\underline{M}_{80} = 80\,(0.180\cos 20^\circ)\,(-\underline{j}\sin 30^\circ - \underline{k}\cos 30^\circ)$

$= -6.77\underline{j} - 11.72\underline{k}$ N·m

$\underline{M}_{120} = -120\,(0.300\cos 45^\circ)\underline{k} = -25.5\,\underline{k}$ N·m

So $\underline{\underline{M} = -20\underline{i} - 6.77\underline{j} - 37.2\underline{k}\ \text{N·m}}$

2/174 $\underline{R} = 800\left[-\sin 30^\circ \cos 20^\circ \underline{i} + \sin 30^\circ \sin 20^\circ \underline{j} + \cos 30^\circ \underline{k}\right]$

$$= \underline{-376\,\underline{i} + 136.8\,\underline{j} + 693\,\underline{k} \quad \text{N}}$$

$\underline{M}_O = \underline{r}_{OB} \times \underline{F}$

$\underline{r}_{OB} = \left[300 \sin 20^\circ \underline{i} + 300 \cos 20^\circ \underline{j} + 250 \underline{k}\right]$ mm

$\underline{\underline{M}_O = 161.1\underline{i} - 165.1\underline{j} + 120\,\underline{k} \quad \text{N}\cdot\text{m}}$

2/175 $\underline{R} = \Sigma \underline{F} = 500 \cos 45^\circ (\underline{i}) + 400 \underline{j} - (600 + 500 \sin 45^\circ) \underline{k}$

$$= 354 \underline{i} + 400 \underline{j} - 954 \underline{k} \quad \text{lb}$$

$$R = \sqrt{\overline{354}^2 + \overline{400}^2 + \overline{954}^2} = \underline{1093 \text{ lb}}$$

$$\underline{M} = [500 \cos 45^\circ (3) - 600(3) - 400(10)] \underline{i}$$
$$+ [500 \cos 45^\circ (6) + 500 \sin 45^\circ (7) + 600(6)] \underline{j}$$
$$+ [500 \sin 45^\circ (3) + 400(3)] \underline{k}$$
$$= -4739 \underline{i} + 8196 \underline{j} + 2261 \underline{k} \quad \text{lb-ft}$$

$$M = \sqrt{\overline{4739}^2 + \overline{8196}^2 + \overline{2261}^2} = \underline{9730 \text{ lb-ft}}$$

*2/176

$\Sigma F_x = 0: -360 - 240\sin\theta + T\sin 30° + 400\cos 30° = 0$ (1)

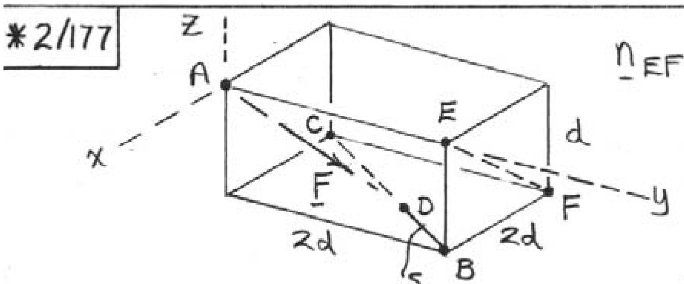
$\Sigma F_y = 600: 240\cos\theta + T\cos 30° + 400\sin 30° = 600$ (2)

Numerical solution of Eqs. (1) & (2):

$\underline{\theta = 21.7°, \quad T = 204 \text{ lb}}$

(We could eliminate T between Eqs. (1) & (2), but the resulting equation is still transcendental.)

*2/177



$$\underline{n}_{EF} = \frac{-2d\underline{i} - d\underline{k}}{\sqrt{5}\,d} = \frac{-2\underline{i} - \underline{k}}{\sqrt{5}}$$

$$\underline{AD} = \underline{AB} + \underline{BD} = 2d\underline{j} - d\underline{k} + s\left[\frac{-2d\underline{i} - 2d\underline{j}}{\sqrt{8}\,d}\right]$$

$$= -\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}$$

$$|\underline{AD}| = \sqrt{\frac{s^2}{2} + \left(2d - \frac{s}{\sqrt{2}}\right)^2 + d^2} = \sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}$$

$$\underline{F} = F\frac{\underline{AD}}{|\underline{AD}|} = F\,\frac{-\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}}{\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

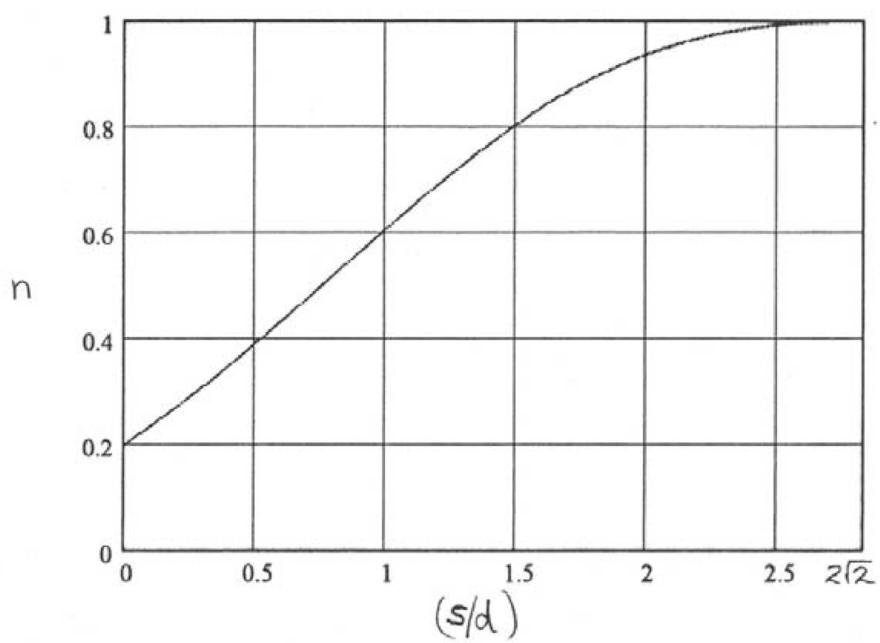
Carry out $\underline{F} \cdot \underline{n}_{EF}$ to obtain the projection

$$\underline{F} \cdot \underline{n}_{EF} = \frac{F\left(s\sqrt{2} + d\right)}{\sqrt{5}\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

The nondimensionalized fraction n of the magnitude F projected is then

$$n = \frac{\underline{F} \cdot \underline{n}_{EF}}{F} = \frac{\sqrt{2}\,\frac{s}{d} + 1}{\sqrt{5}\,\sqrt{\left(\frac{s}{d}\right)^2 + 5 - 2\sqrt{2}\,\frac{s}{d}}}$$

We let $\frac{s}{d}$ vary from 0 to $2\sqrt{2}$ as D moves from B to C. Resulting plot:



*2/178 $\underline{T} = T\underline{n}_{AB}$

$$\underline{T} = T\left[\frac{(d+40\cos\beta)\underline{i} + 40(1-\sin\beta)\underline{j}}{\sqrt{(d+40\cos\beta)^2 + 40^2(1-\sin\beta)^2}}\right]$$

$\left(\beta = \theta + \frac{\pi}{4}\right)$

$\underline{r}_{OB} = (d\underline{i} + 40\underline{j})$

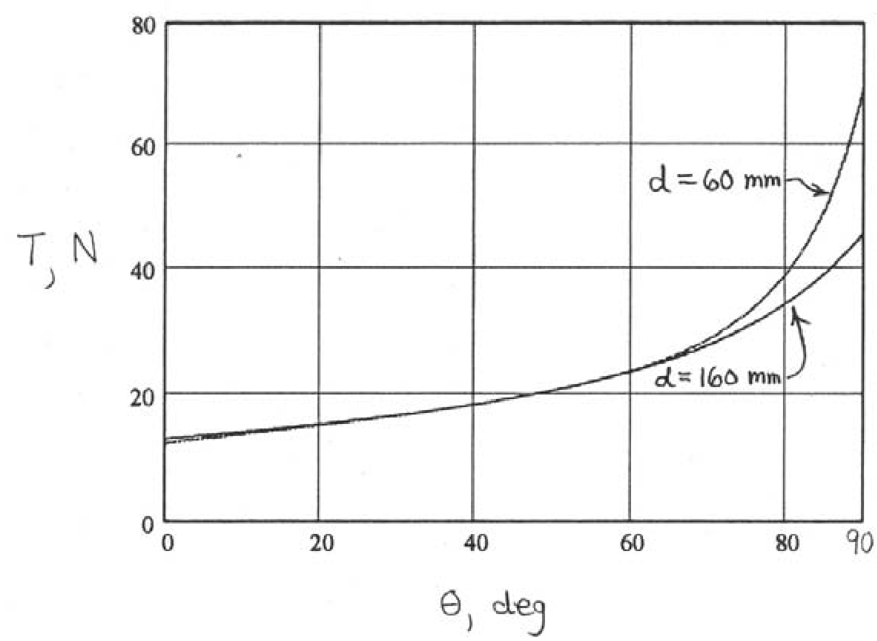
$\Sigma \underline{M}_O = \underline{0}: \quad \underline{r}_{OB} \times \underline{T} + K\beta\underline{k} = \underline{0}$

Carry out the cross product, consider the z-component, and solve for T to obtain

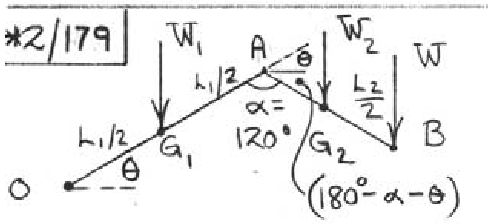
$$T = \frac{12.5\left(\theta+\frac{\pi}{4}\right)\sqrt{d^2 + 80d\cos\left(\theta+\frac{\pi}{4}\right) - 3200\sin\left(\theta+\frac{\pi}{4}\right) + 3200}}{\left[d\sin\left(\theta+\frac{\pi}{4}\right) + 40\cos\left(\theta+\frac{\pi}{4}\right)\right]}$$

(T in N, θ in radians)

Plot of T versus θ for d = 60 mm and for d = 160 mm:



*2/179



$$\begin{cases} W_1 = 120 \text{ lb} & L_1 = 3 \text{ ft} \\ W_2 = 50 \text{ lb} & L_2 = 2 \text{ ft} \\ W = 300 \text{ lb} & \alpha = 120° \end{cases}$$

$$+\circlearrowleft M_O = W_1 \frac{L_1}{2}\cos\theta + W_2\left(L_1\cos\theta + \frac{L_2}{2}\cos\left(180° - \alpha - \theta\right)\right)$$
$$+ W\left(L_1\cos\theta + L_2\cos\left(180° - \alpha - \theta\right)\right)$$

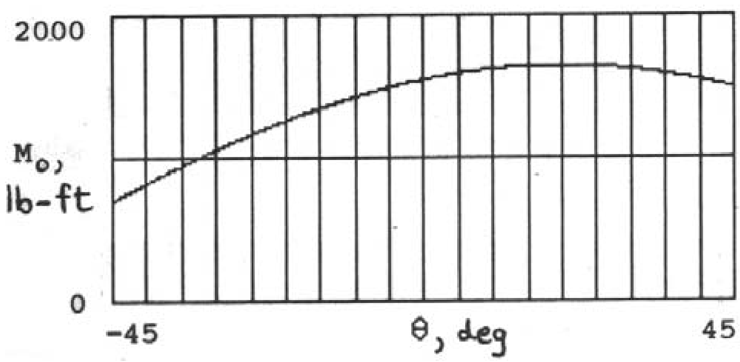
With the above numbers:

$$\underline{M_O = 1230\cos\theta + 650\cos\left(60° - \theta\right)} \quad (\text{in lb-ft})$$

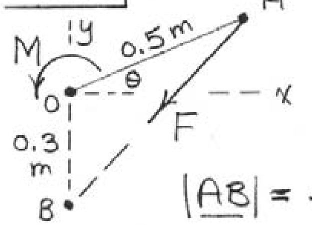
(see plot below)

For $(M_O)_{max}$: $\frac{dM_O}{d\theta} = -1230\sin\theta + 650\sin\left(60° - \theta\right) = 0$

Numerical solution: $\underline{\theta = 19.90°}$, $\underline{(M_O)_{max} = 1654 \text{ lb-ft}}$



*2/180



$$\underline{F} = F\underline{n}_{AB} = F\frac{\underline{AB}}{|\underline{AB}|}$$

$$\underline{AB} = -0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j} \text{ m}$$

$$|\underline{AB}| = \sqrt{(0.5\cos\theta)^2 + (0.5\sin\theta + 0.3)^2}$$

$$= \sqrt{0.34 + 0.3\sin\theta} \text{ m}$$

$$F = k\delta = 600\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right] \text{ N}$$

So

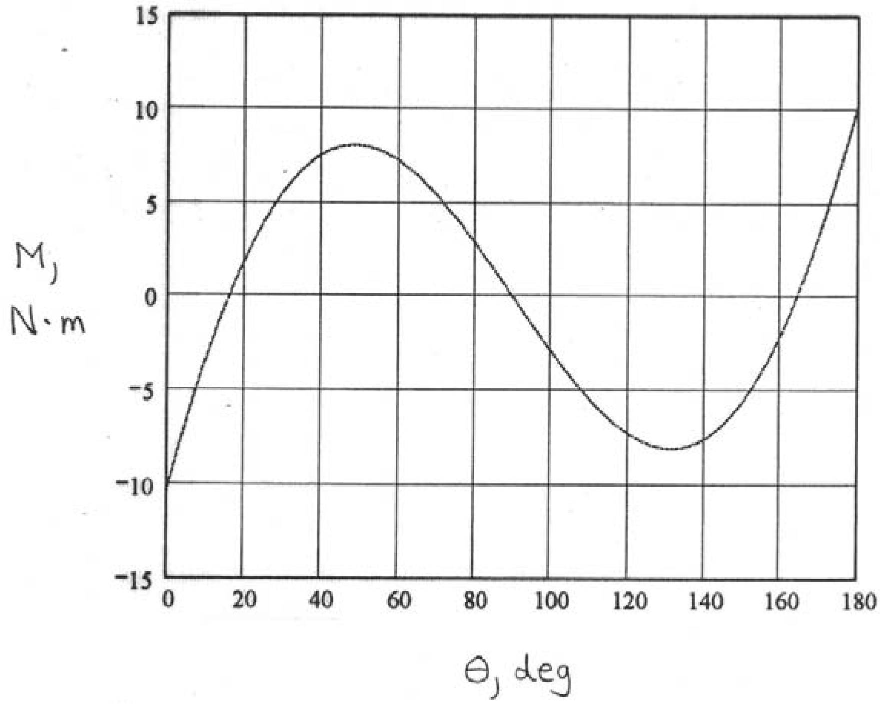
$$\underline{F} = \frac{600\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]}{\sqrt{0.34 + 0.3\sin\theta}}\left[-0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j}\right]$$

Now form $\underline{r}_{OB} \times \underline{F}$, where $\underline{r}_{OB} = -0.3\underline{j}$ m, to obtain $-\dfrac{90\cos\theta\left(\sqrt{0.34+0.3\sin\theta} - 0.65\right)}{\sqrt{0.34 + 0.3\sin\theta}}\underline{k}$

With the above moment plus $M\underline{k}$ summing to zero, we obtain the scalar

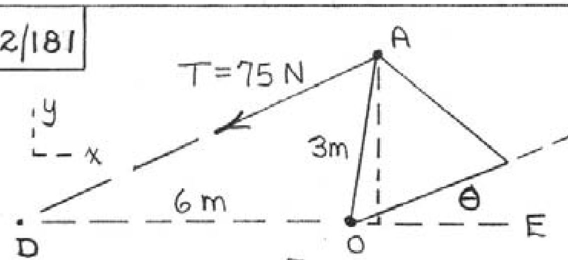
$$M = \frac{90\cos\theta\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]}{\sqrt{0.34 + 0.3\sin\theta}}$$

Plot:



(Note: $M(0) = -10.33$ N·m
$M(180°) = 10.33$ N·m)

*2/181



Angle $AOE = \theta + 60°$

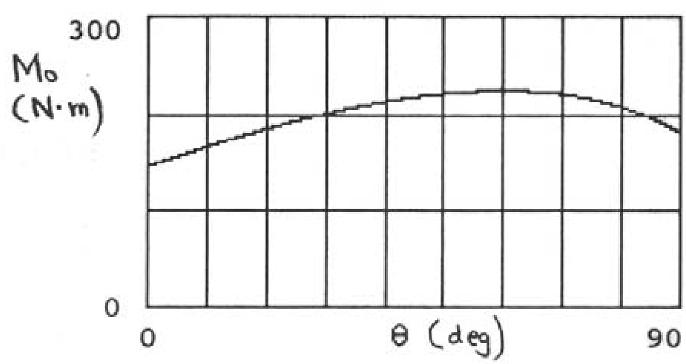
Use $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$

$\underline{r}_{OD} = -6\underline{i}$ m

$$\underline{T} = T\underline{n}_{AD} = 75 \left[\frac{-(6 + 3\cos(\theta+60°))\underline{i} - 3\sin(\theta+60°)\underline{j}}{\sqrt{[6+3\cos(\theta+60°)]^2 + [3\sin(\theta+60°)]^2}} \right] \text{N}$$

$$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = \underline{\frac{1350 \sin(\theta+60°)\ \underline{k}}{\sqrt{45 + 36\cos(\theta+60°)}}} \ \text{N}\cdot\text{m}$$

M_O is a max @ $\underline{\theta = 60°}$: $\underline{M_O = 225 \text{ N}\cdot\text{m}}$



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$$\underline{T} = T\underline{n}_{AB} = T\,\frac{\underline{AB}}{|\underline{AB}|}$$

$$= 120\left[\frac{0.4\underline{i} + 0.8(1-\sin\theta)\underline{j} - 0.8\cos\theta\,\underline{k}}{\sqrt{0.4^2 + 0.8^2(1-\sin\theta)^2 + 0.8^2\cos^2\theta}}\right]$$

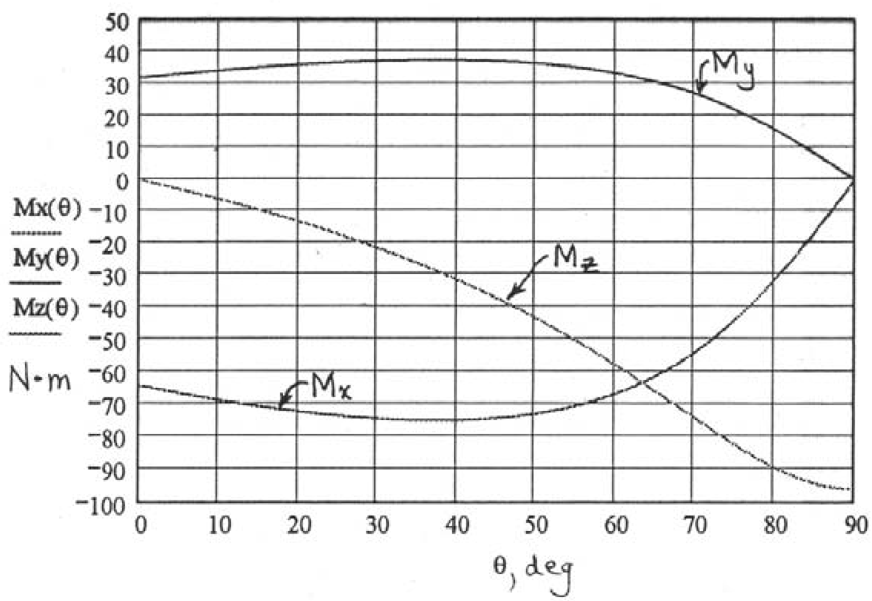
$$= \frac{48\underline{i} + 96(1-\sin\theta)\underline{j} - 96\cos\theta\,\underline{k}}{\sqrt{1.44 - 1.28\sin\theta}} \quad \text{N}$$

$$\underline{r}_{OB} = 0.4\underline{i} + 0.8\underline{j} \text{ m}$$

Carry out $\underline{M}_o = \underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{M}_o = \frac{-76.8\cos\theta\,\underline{i} + 38.4\cos\theta\,\underline{j} - 38.4\sin\theta\,\underline{k}}{\sqrt{1.44 - 1.28\sin\theta}}$$

The $\underline{i}$, $\underline{j}$, & $\underline{k}$ – components of $\underline{M}_o$ are shown in the following plot:



*2/183 $\underline{BC} = -1.2\cos\theta\,\underline{i} + 1.2(1-\sin\theta)\,\underline{j} - 2.4\,\underline{k}$ ft

$$|\underline{BC}| = \sqrt{(1.2\cos\theta)^2 + 1.2^2(1-\sin\theta)^2 + 2.4^2}$$

$$= \sqrt{8.64 - 2.88\sin\theta} \text{ ft}$$

$$F = k\delta = 180\left(\sqrt{8.64 - 2.88\sin\theta} - 2.4\right)$$

$$\underline{F} = F\underline{n}_{BC} = F\frac{\underline{BC}}{|\underline{BC}|}$$

$$= \frac{180\left[\sqrt{8.64 - 2.88\sin\theta} - 2.4\right]}{\sqrt{8.64 - 2.88\sin\theta}}\left[-1.2\cos\theta\,\underline{i} + 1.2(1-\sin\theta)\,\underline{j} - 2.4\,\underline{k}\right]$$

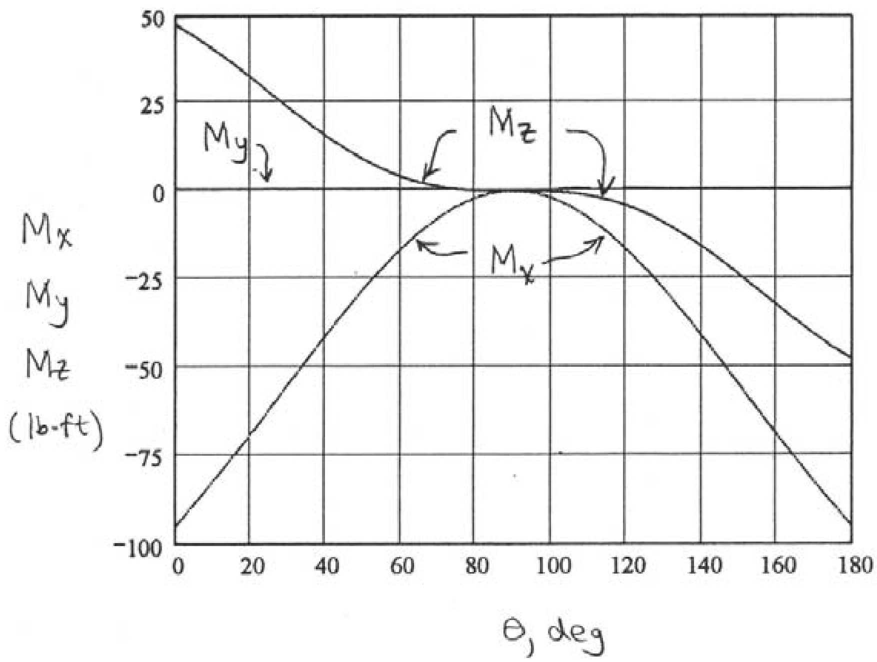
lb

Now, carry out $\underline{M}_O = \underline{r}_{OC} \times \underline{F}$, where $\underline{r}_{OC} = 1.2\underline{j}$ ft, to obtain

$$\underline{M}_O = \frac{\sqrt{8.64 - 2.88\sin\theta} - 2.4}{\sqrt{8.64 - 2.88\sin\theta}}\left[-518\,\underline{i} + 259\cos\theta\,\underline{k}\right]$$

lb-ft

Plots of the moment components:



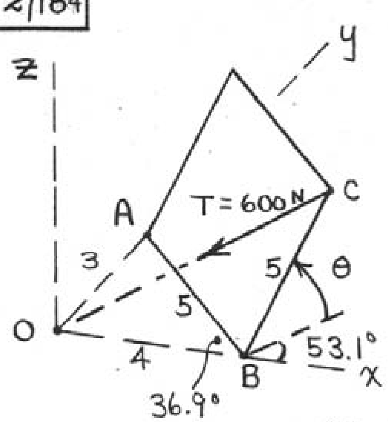
Maximum absolute values:

For M_x : 95.1 lb-ft @ $\theta = 0$ & $\theta = \pi$

For M_z : 47.6 lb-ft @ $\theta = 0$ & $\theta = \pi$

($M_y = 0$ for all θ)

*2/184



$C = C(4 + 5\cos 53.1° \cos\theta,\ 5\sin 53.1° \cos\theta,\ 5\sin\theta)$ m

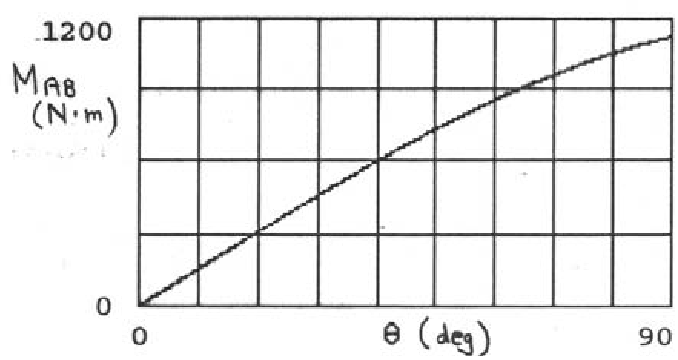
$\underline{n}_{AB} = 0.8\underline{i} - 0.6\underline{j}$

$$\underline{T} = T\underline{n}_{CO} = 600\left[\frac{-(4+3\cos\theta)\underline{i} - (4\cos\theta)\underline{j} - (5\sin\theta)\underline{k}}{\sqrt{(4+3\cos\theta)^2 + (4\cos\theta)^2 + (5\sin\theta)^2}}\right]$$

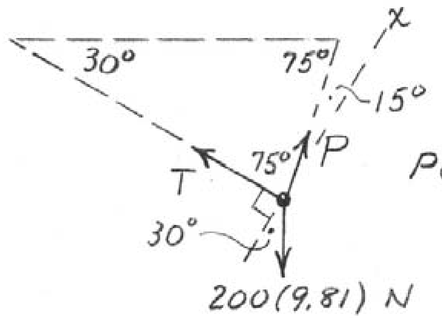
$$= \frac{600\left[-(4+3\cos\theta)\underline{i} - (4\cos\theta)\underline{j} - (5\sin\theta)\underline{k}\right]}{\sqrt{41 + 24\cos\theta}} \text{ N}$$

$$\underline{M}_B = \underline{r}_{OB} \times \underline{T} = \frac{600}{\sqrt{41+24\cos\theta}}\left(-20\sin\theta\,\underline{j} + 16\cos\theta\,\underline{k}\right) \text{ N}\cdot\text{m}$$

Now, $M_{AB} = \underline{M}_B \cdot \underline{n}_{AB} = \dfrac{7200\sin\theta}{\sqrt{41 + 24\cos\theta}}$ N·m



3/1

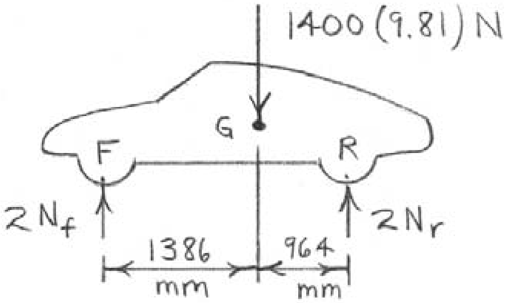


$\Sigma F_x = 0$

$P \cos 15° - 200(9.81) \cos 30° = 0$

$\underline{P = 1759 \text{ N}}$

3/2



$$+\uparrow \Sigma F = 0: \quad 2N_f + 2N_r - 1400(9.81) = 0$$

$$\circlearrowleft + \Sigma M_F = 0: \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

Assumes G midway between left and right wheels.

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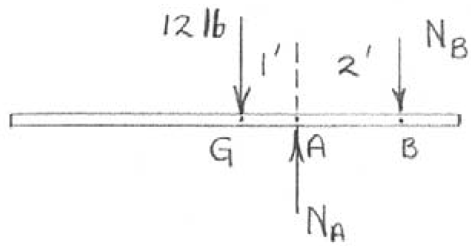

$\circlearrowleft + \; \Sigma M_B = 0 : \quad 12(3) - N_A(2) = 0$

$\underline{N_A = 18 \text{ lb}}$

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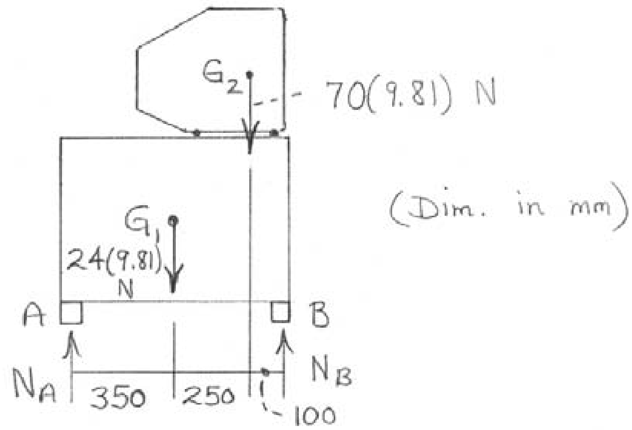


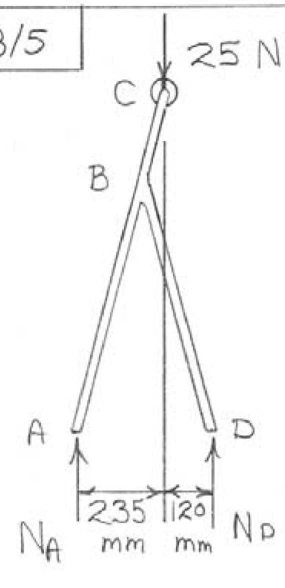
$$\circlearrowleft+ \Sigma M_A = 0: -24(9.81)350 - 70(9.81)(600) + N_B(700) = 0$$

$$\underline{N_B = 706 \text{ N}}$$

$$+\uparrow \Sigma F = 0: \quad N_A + 706 - (70 + 24)(9.81) = 0$$

$$\underline{N_A = 216 \text{ N}}$$

3/5



$\circlearrowleft \Sigma M_A = 0$:

$N_D(355) - 25(235) = 0$

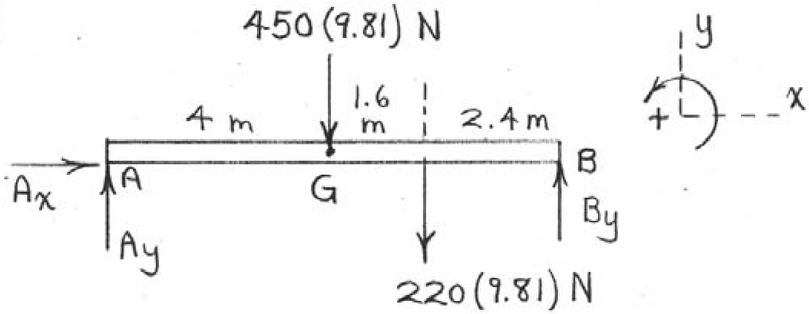
$\underline{N_D = 16.55 \text{ N}}$

$+\uparrow \Sigma F = 0$:

$N_A + 16.55 - 25 = 0$

$\underline{N_A = 8.45 \text{ N}}$

3/6



From $\Sigma F_x = 0$, $A_x = 0$

$\Sigma M_A = 0: \; -450(9.81)4 - 220(9.81)(5.6)$

$+ B_y(8) = 0$, $\underline{B_y = 3720 \text{ N}}$

$\Sigma F_y = 0: \; A_y - 450(9.81) - 220(9.81) + 3720 = 0$

$\underline{A_y = 2850 \text{ N}}$

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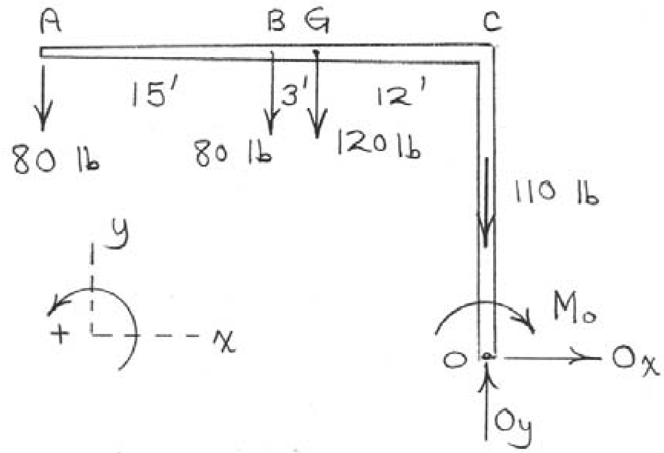


$\Sigma F_x = 0: \quad \underline{O_x = 0}$

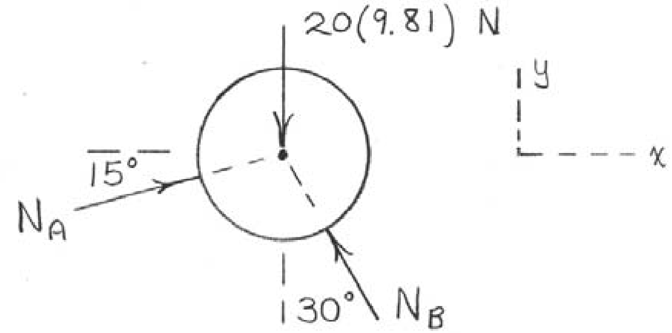
$\Sigma F_y = 0: \quad O_y - 80 - 80 - 120 - 110 = 0$

$\underline{O_y = 390 \text{ lb}}$

$\Sigma M_O = 0: \quad 80(30) + 80(15) + 120(12) - M_O = 0$

$\underline{M_O = 5040 \text{ lb-ft CW}}$

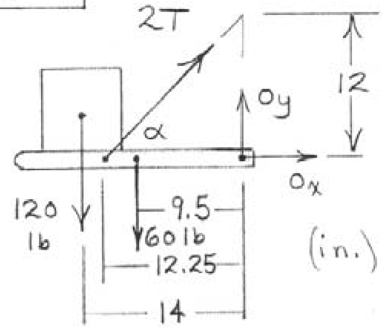
3/8



$$\begin{cases} \Sigma F_x = 0: \quad N_A \cos 15° - N_B \sin 30° = 0 & (1) \\ \Sigma F_y = 0 \quad N_A \sin 15° + N_B \cos 30° - 20(9.81) = 0 & (2) \end{cases}$$

Solution : $\begin{cases} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{cases}$

3/9

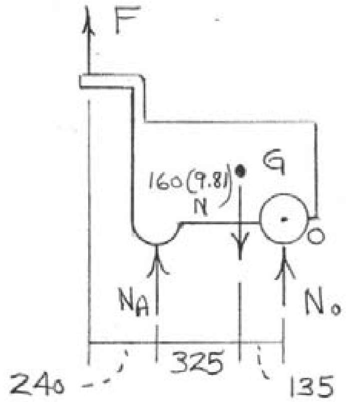


$\alpha = \tan^{-1} \frac{12}{12.25} = 44.4°$

$\circlearrowleft + \sum M_O = 0: \quad 120(14) + 60(9.5) - 2T \sin 44.4°(12.25) = 0$

$\underline{T = 131.2 \text{ lb}}$

3/10



With $F = 0$:

$\circlearrowleft \Sigma M_o = 0$: $160(9.81)(135) - N_A(460) = 0$

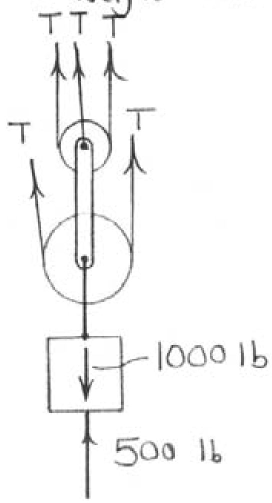
$N_A = 461$ N

For $\frac{1}{2} N_A$:

$\circlearrowleft \Sigma M_o = 0$: $160(9.81)(135) - \frac{461}{2}(460) - F(700) = 0$

$\underline{F = 151.4 \text{ N}}$

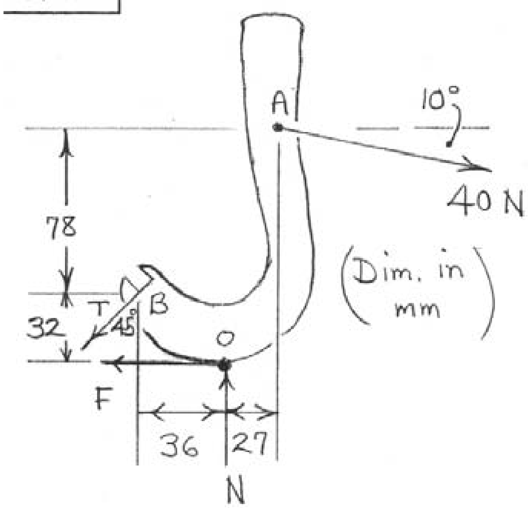
3/11 FBD of 1000-lb weight and lower pair of pulleys:



$\uparrow \Sigma F = 0: \quad 5T + 500 - 1000 = 0, \quad \underline{T = 100 \text{ lb}}$

(We assume that the nonverticality of some of the cables is negligible.)

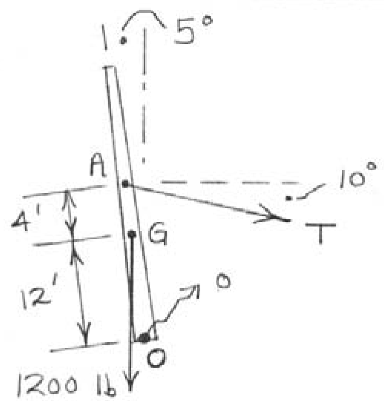
3/12



$$\circlearrowleft + \Sigma M_O = 0: \ -40\cos 10°(110) - 40\sin 10°(27) + T\cos 45°(32) + T\sin 45°(36) = 0$$

$$\underline{T = 94.0 \text{ N}}$$

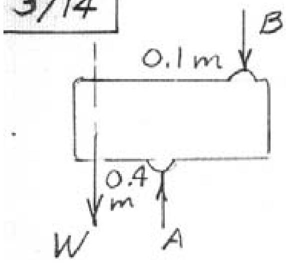
3/13



$$\circlearrowleft + \; \Sigma M_O = 0: \quad 1200(12 \sin 5°) - T \cos 15°(16) = 0$$

$$\underline{T = 81.2 \text{ lb}}$$

3/14



$W = 300(9.81) = 2943$ N

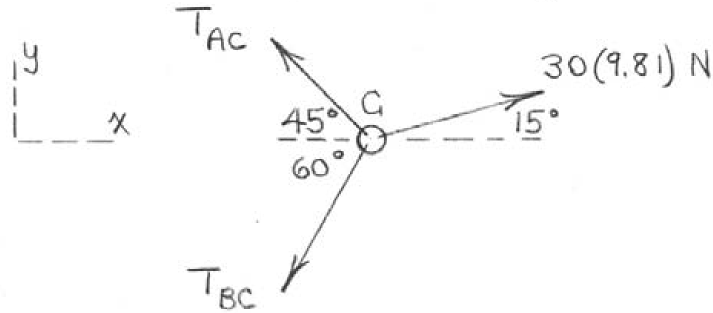
$\Sigma M_A = 0$; $2943(0.4) - B(0.6) = 0$

$\underline{B = 1962 \text{ N}}$ or $\underline{1.962 \text{ kN}}$

$\Sigma F = 0$; $A = 2943 + 1962$

$\underline{= 4910 \text{ N}}$ or $\underline{4.91 \text{ kN}}$

3/15 FBD of junction ring C:

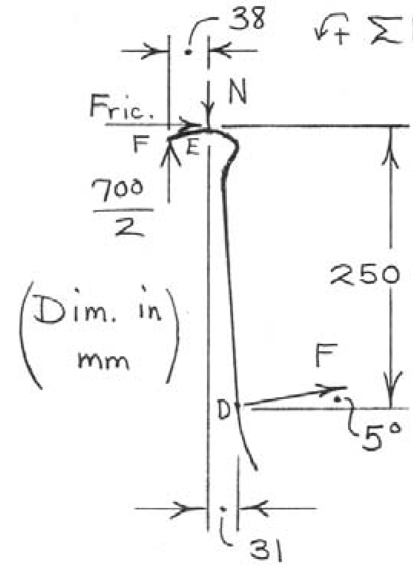


$$\begin{cases} \Sigma F_x = 0: -T_{AC}\cos 45° - T_{BC}\cos 60° + 30(9.81)\cos 15° = 0 \\ \Sigma F_y = 0: T_{AC}\sin 45° - T_{BC}\sin 60° + 30(9.81)\sin 15° = 0 \end{cases}$$

Solve simultaneously to obtain

$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

3/16 Consider the right prybar:

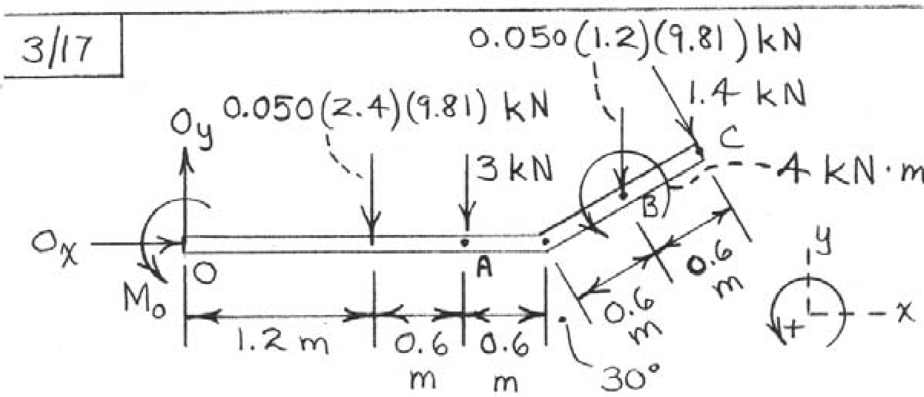


$\circlearrowleft + \Sigma M_E = 0: \; -\frac{700}{2}(38) +$

$F\cos 5°(250) + F\sin 5°(31) = 0$

$\underline{F = 52.8 \text{ N}}$

3/17



$\Sigma F_x = 0: \quad O_x + 1.4 \sin 30° = 0$

$\underline{O_x = -0.7 \text{ kN}}$

$\Sigma F_y = 0: \quad O_y - 0.050(2.4)(9.81) - 3 - 1.4 \cos 30°$

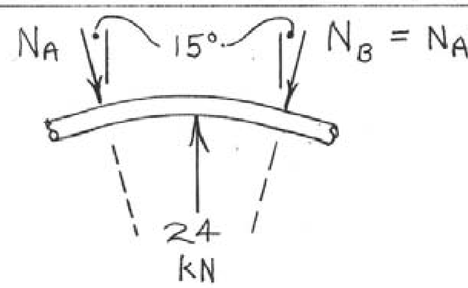
$- 0.050(1.2)(9.81) = 0, \quad \underline{O_y = 5.98 \text{ kN}}$

$\Sigma M_O = 0: \quad M_O - 0.050(2.4)(9.81)(1.2) - 3(1.8)$

$- 0.050(1.2)(9.81)(2.4 + 0.6 \cos 30°) + 4$

$- 1.4(2.4 \cos 30° + 1.2) = 0, \quad \underline{M_O = 9.12 \text{ kN·m}}$

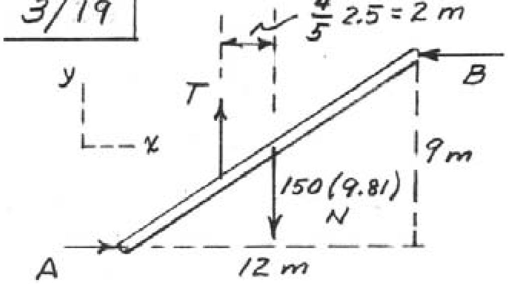
3/18



$\uparrow \sum F = 0$: $24 - 2N_A \cos 15° = 0$

$N_A = N_B = 12.42$ kN

3/19



$\Sigma F_y = 0;\quad T - 150(9.81) = 0$

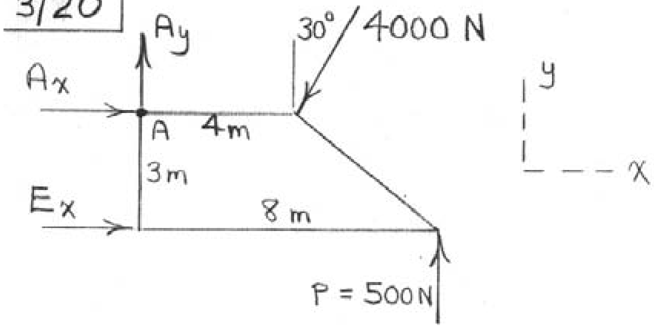
$T = 1472$ N

$\Sigma F_x = 0;\quad A = B$

$\Sigma M = 0;\quad 1472(2) - A(9) = 0$

$\underline{A = B = 327 \text{ N}}$

3/20



$\Sigma F_x = 0: \; A_x + E_x - 4000 \sin 30° = 0$

$\Sigma F_y = 0: \; A_y - 4000 \cos 30° + 500 = 0$

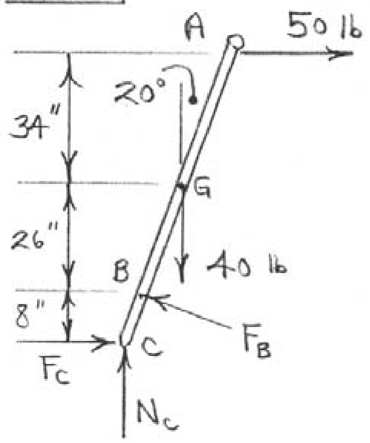
$\Sigma M_A = 0: \; E_x(3) + 500(8) - 4000 \cos 30° (4) = 0$

$\Rightarrow \underline{A_x = -1290 \text{ N}}, \; \underline{A_y = 2960 \text{ N}}, \; \underline{E_x = 3290 \text{ N}}$

For maximum P: $E_x = 0$ and $\Sigma M_A = 0$:

$P(8) - 4000 \cos 30° (4) = 0, \; \underline{P = 1732 \text{ N}}$

3/21



(a) Including 40-lb weight:

$$\circlearrowright + \Sigma M_C = 0: \quad 50(68) + 40\,(34 \tan 20°) - F_B \frac{8}{\cos 20°} = 0$$

$$F_B = 458 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F = 0: \quad F_C - 458 \cos 20° + 50 = 0$$

$$\underline{F_C = 380 \text{ lb}}$$

(b) Exclude 40-lb weight:

$$\circlearrowright + \Sigma M_C = 0: \quad 50(68) - F_B \frac{8}{\cos 20°} = 0$$

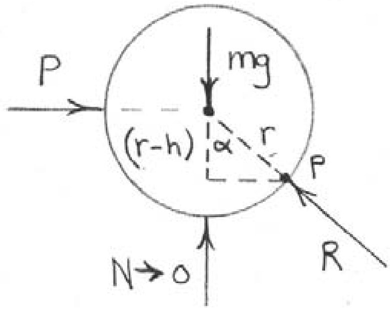
$$F_B = 399 \text{ lb}$$

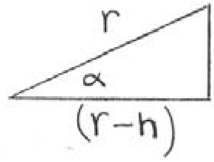
$$\overset{+}{\rightarrow} \Sigma F = 0: \quad F_C - 399 \cos 20° + 50 = 0$$

$$\underline{F_C = 325 \text{ lb}}$$

3/22

$$\cos\alpha = \frac{r-h}{r}$$





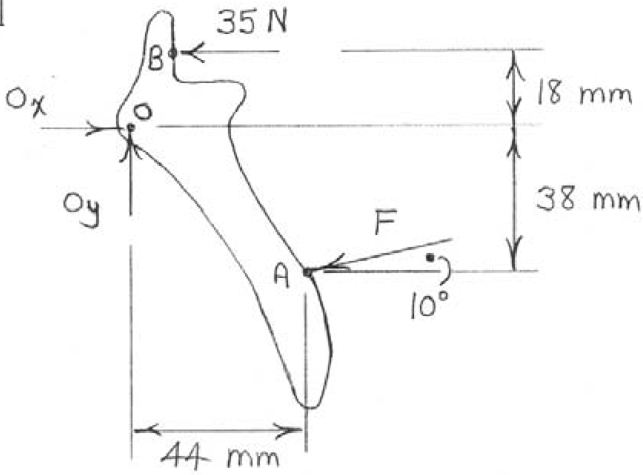
$$\sqrt{r^2-(r-h)^2} = \sqrt{2rh-h^2}$$

$$\sin\alpha = \sqrt{2rh-h^2}\,/\,r$$

$$+\circlearrowleft \Sigma M_p = 0: \quad P(r-h) - mg\,r\sin\alpha = 0$$

$$\Rightarrow \quad \underline{P = \frac{mg\sqrt{2rh-h^2}}{r-h}}$$

3/23



$\circlearrowleft + \Sigma M_O = 0: \quad 35(18) - (F\cos 10°)(38) - (F\sin 10°)(44) = 0$

$\underline{F = 13.98 \text{ N}}$

$\Sigma F_x = 0: \quad O_x - 35 - 13.98 \cos 10° = 0$

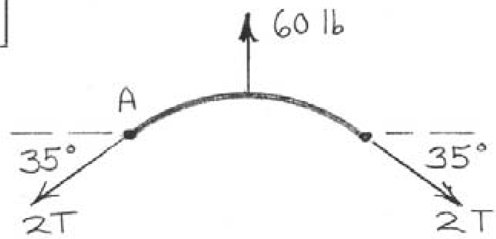
$O_x = 48.8 \text{ N}$

$\Sigma F_y = 0: \quad O_y - 13.98 \sin 10° = 0$

$O_y = 2.43 \text{ N}$

$O = \sqrt{48.8^2 + 2.43^2} = \underline{48.8 \text{ N}}$

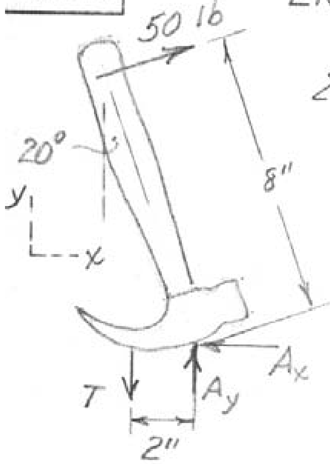
3/24



$+\uparrow \Sigma F = 0: \quad 60 - 4T \sin 35° = 0$

$\underline{T = 26.2 \text{ lb}}$

3/25



$\Sigma M_A = 0;\ 50(8) - 2T = 0,\ \underline{T = 200 \text{ lb}}$

$\Sigma F_x = 0;\ 50 \cos 20° - A_x = 0$

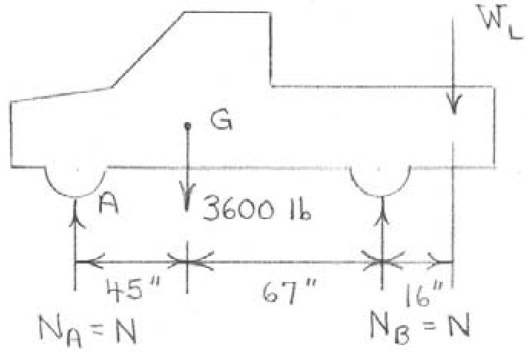
$A_x = 46.98 \text{ lb}$

$\Sigma F_y = 0;\ A_y + 50 \sin 20° - 200 = 0$

$A_y = 182.9 \text{ lb}$

$A = \sqrt{(46.98)^2 + (182.9)^2} = \underline{188.8 \text{ lb}}$

3/26



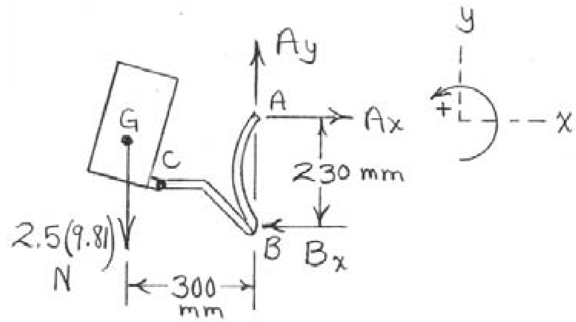
$\circlearrowleft + \; \sum M_A = 0: \; 3600(45) - N(112) + W_L(128) = 0$

$+\uparrow \sum F = 0: \; 2N - 3600 - W_L = 0$

Solve to obtain $N = 2075$ lb

$\underline{W_L = 550 \text{ lb}}$

3/27 Entire unit:



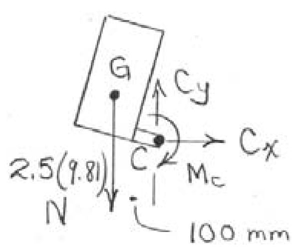
$$\Sigma M_A = 0 : \; 2.5(9.81)(300) - B_x(230) = 0$$

$$\underline{B_x = 32.0 \text{ N}}$$

$$\Sigma F_x = 0 : \quad A_x - 32.0 = 0, \quad \underline{A_x = 32.0 \text{ N}}$$

$$\Sigma F_y = 0 : \quad A_y - 2.5(9.81) = 0, \quad \underline{A_y = 24.5 \text{ N}}$$

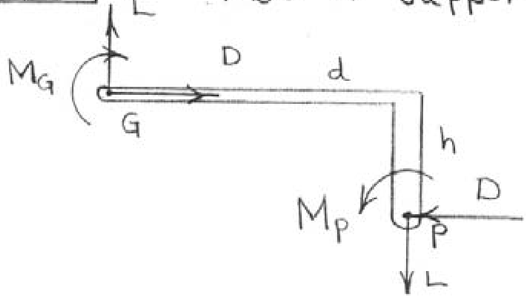
Fixture only:



$$\Sigma M_c = 0 : \; 2.5(9.81)(100) - M_c = 0$$

$$\underline{M_c = 2.45 \text{ N}\cdot\text{m CW}}$$

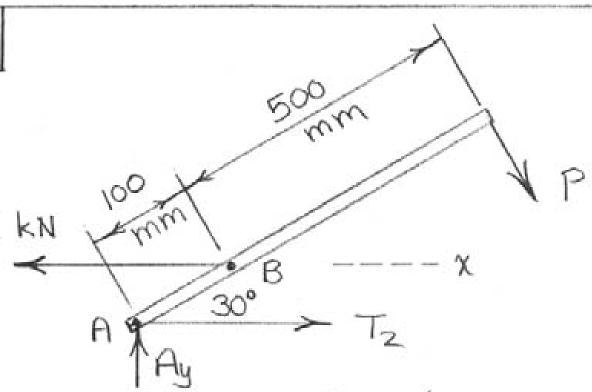
3/28 FBD of support & model:



$\circlearrowleft + \Sigma M_P = 0$: $M_P - M_G - Ld - Dh = 0$

$$\underline{M_G = M_P - Ld - Dh}$$

3/29



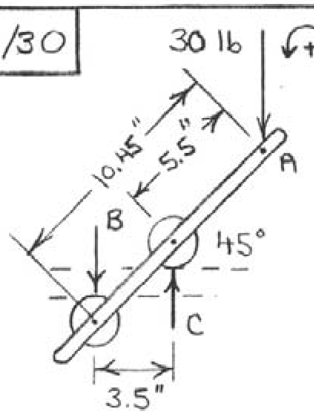
$\curvearrowleft\!+\ \Sigma M_A = 0:\ P(500+100) - 2(100 \sin 30°) = 0$

$P = 0.1667$ kN or $\underline{P = 166.7 \text{ N}}$

$\Sigma F_x = 0: \quad 0.1667 \sin 30° + T_2 - 2 = 0$

$\underline{T_2 = 1.917 \text{ kN}}$

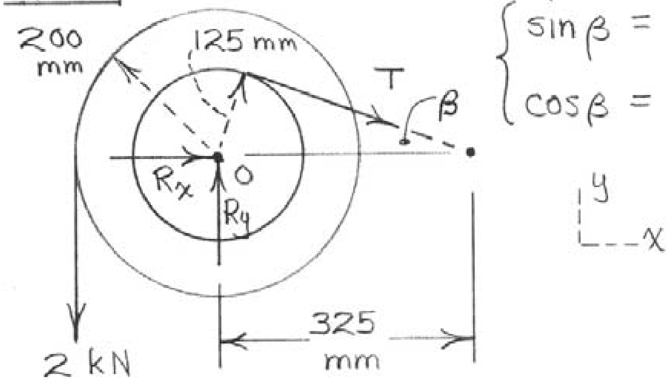
3/30



$\curvearrowleft + \Sigma M_C = 0: \; B(3.5) - (30 \cos 45°)(5.5) = 0, \quad \underline{B = 33.3 \text{ lb}}$

$\Sigma M_B = 0: \; C(3.5) - (30 \cos 45°)(10.45) = 0, \quad \underline{C = 63.3 \text{ lb}}$

3/31



$$\begin{cases} \sin\beta = \dfrac{125}{325} = \dfrac{5}{13} \\ \cos\beta = \dfrac{12}{13} \end{cases}$$

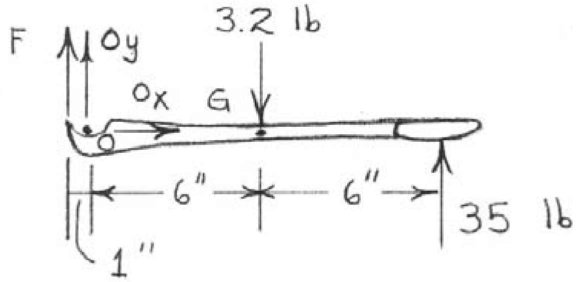
$$\circlearrowleft \Sigma M_O = 0: \quad 2(200) - T(125) = 0, \quad T = 3.2 \text{ kN}$$

$$\Sigma F_x = 0: \quad 3.2\left(\frac{12}{13}\right) + R_x = 0, \quad R_x = -2.95 \text{ kN}$$

$$\Sigma F_y = 0: \quad R_y - 2 - 3.2\left(\frac{5}{13}\right) = 0, \quad R_y = 3.23 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.95^2 + 3.23^2} = \underline{4.38 \text{ kN}}$$

3/32

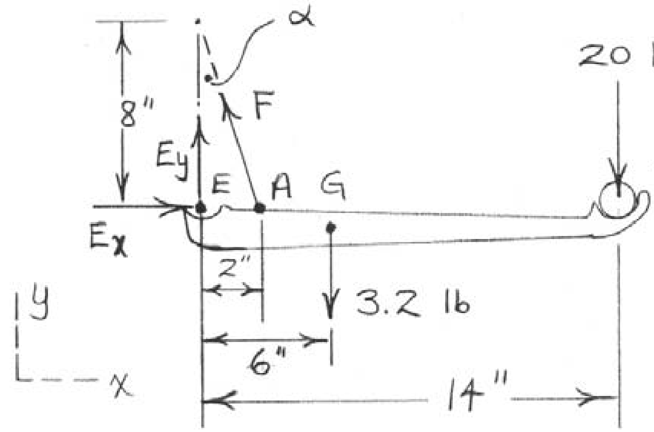


$$\circlearrowleft\!+ \; \Sigma M_O = 0: \; -F(1) - 3.2(6) + 35(12) = 0$$

$$\underline{F = 401 \text{ lb}}$$

3/33

$\alpha = \tan^{-1}\frac{2}{8} = 14.04°$



Assume that A & E are on a horizontal line

$\curvearrowleft + \sum M_E = 0: \quad F\cos 14.04°(2) - 3.2(6) - 20(14) = 0$

$\underline{F = 154.2 \text{ lb}}$

$\sum F_x = 0: -154.2(\sin 14.04°) + E_x = 0$

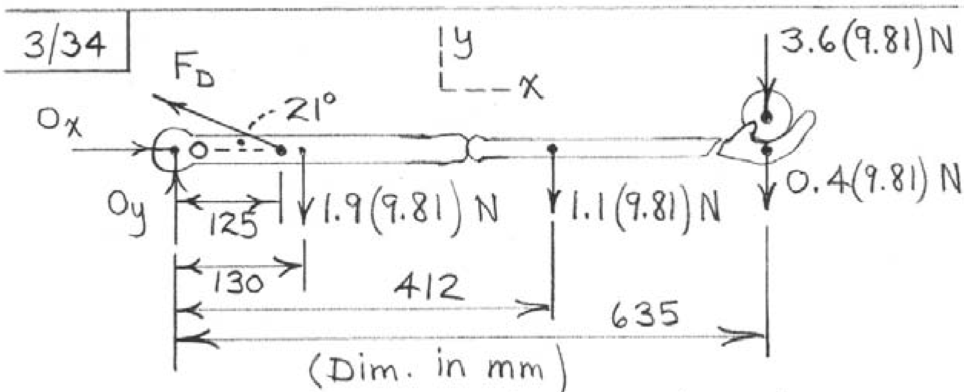
$E_x = 37.4 \text{ lb}$

$\sum F_y = 0: \quad 154.2\cos 14.04° - 3.2 - 20 + E_y = 0$

$E_y = -126.4 \text{ lb}$

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{37.4^2 + 126.4^2} = \underline{131.8 \text{ lb}}$

3/34



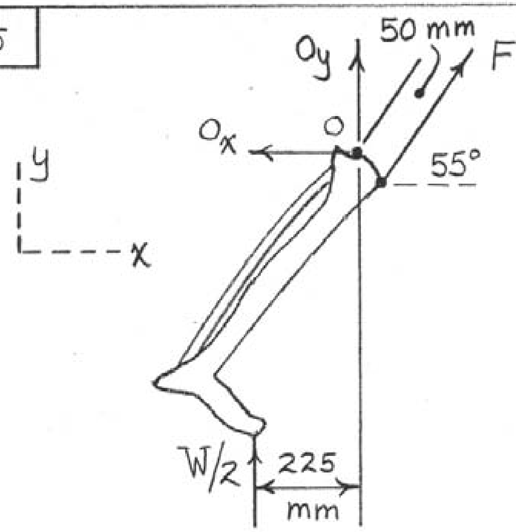
$\curvearrowleft + \sum M_O = 0: \; F_D \sin 21°(125) - 1.9(9.81)(130)$
$- 1.1(9.81)(412) - (3.6+0.4)(9.81)(635) = 0$

$\underline{F_D = 710 \text{ N}}$

$\xrightarrow{+} \sum F_x = 0: \; O_x - 710 \cos 21° = 0, \; \underline{O_x = 662 \text{ N}}$

$+\uparrow \sum F_y = 0: \; O_y + 710 \sin 21° - (1.9 + 1.1 + 3.6 + 0.4)9.81$
$= 0, \; \underline{O_y = -185.6 \text{ N}}$

3/35



$\curvearrowleft + \Sigma M_O = 0: \; F(50) - \frac{W}{2}(225) = 0, \quad \underline{F = 2.25\,W}$

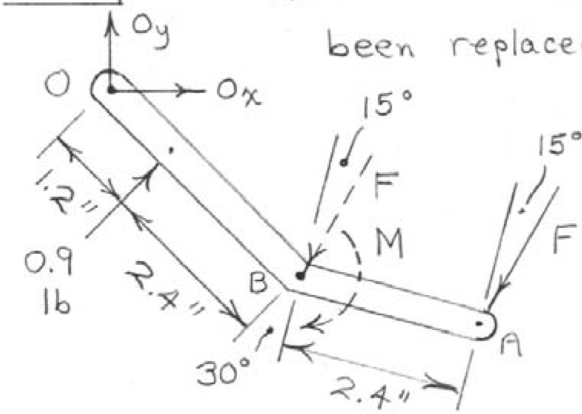
$\Sigma F_x = 0: \; -O_x + 2.25\,W\cos 55° = 0, \; O_x = 1.291\,W$

$\Sigma F_y = 0: \; \frac{W}{2} + O_y + 2.25\,W\sin 55° = 0$

$O_y = -2.34\,W$

$O = \sqrt{O_x^2 + O_y^2} = \sqrt{(1.291\,W)^2 + (2.34\,W)^2} = \underline{2.67\,W}$

3/36



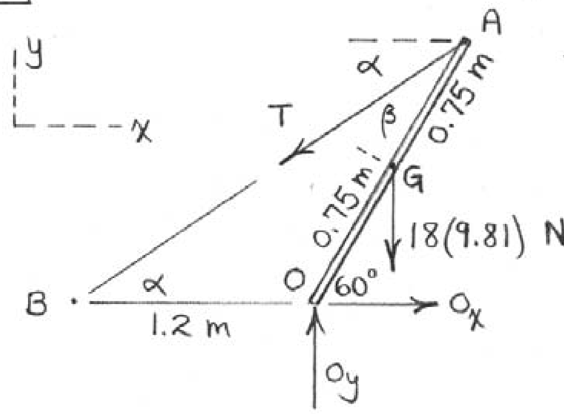
Note that the force F has been replaced by a force-couple system at B (dashed), where $M = F(2.4 \cos 15°)$

$$\circlearrowleft \Sigma M_O = 0: \quad 0.9(1.2) - F(2.4 \cos 15°) - F(3.6 \cos 15°) = 0$$

$$\underline{F = 0.1863 \text{ lb}}$$

3/37

$$\alpha = \tan^{-1}\left[\frac{1.5\sin 60^\circ}{1.5\cos 60^\circ + 1.2}\right] = 33.7^\circ$$



$$\beta = 90^\circ - \alpha - 30^\circ = 26.3^\circ$$

$\circlearrowleft +\ \Sigma M_O = 0$: $T\sin 33.7^\circ(1.2) - 18(9.81)(0.75)\cos 60^\circ = 0$

$\underline{T = 99.5\ \text{N}}$

$\Sigma F_x = 0$: $-99.5\cos 33.7^\circ + O_x = 0$

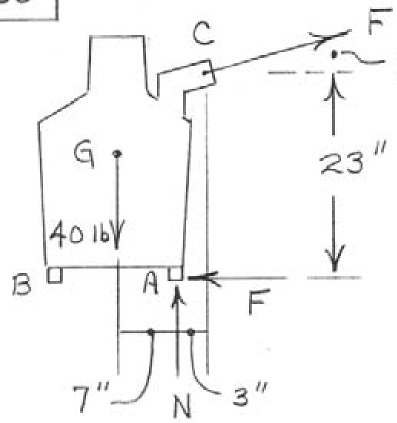
$O_x = 82.8$ N

$\Sigma F_y = 0$: $-99.5\sin 33.7^\circ - 18(9.81) + O_y = 0$

$O_y = 232$ N

So $\underline{O = 246\ \text{N @ } 70.3^\circ \text{ CCW from } +x\text{-axis}}$

3/38

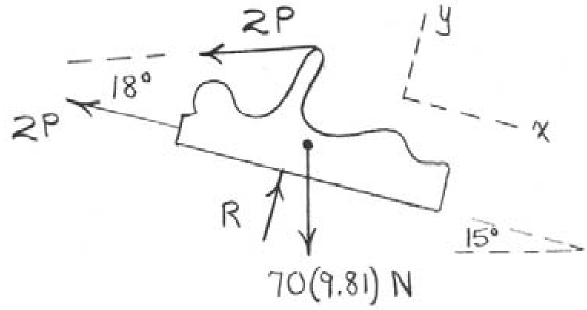


For impending tip, $N_B \to 0$.

$$\circlearrowleft^{+} \sum M_A = 0: \quad 40(7) - F\cos 15°(23) + F\sin 15°(3) = 0$$

$$\underline{F = 13.06 \text{ lb}}$$

3/39



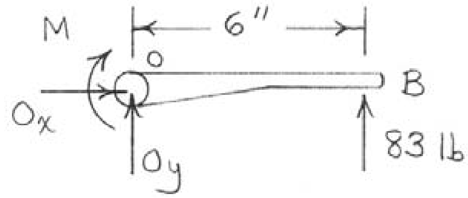
$\Sigma F_x = 0: \ 70(9.81)\sin 15° - 2P - 2P\cos 18° = 0$

$\underline{P = 45.5 \text{ N}}$

$\Sigma F_y = 0: \ R - 70(9.81)\cos 15° - 2(45.5)\sin 18° = 0$

$\underline{R = 691 \text{ N}}$

3/40 Arm OB:



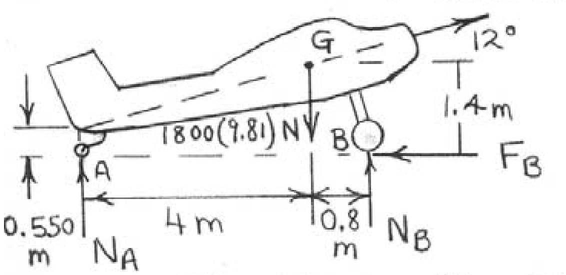
$\circlearrowleft + \sum M_O = 0: \quad 83(6) - M = 0$

$M = 498$ lb-in. or $\underline{41.5 \text{ lb-ft}}$

$F \cos 20° (15) = M = 498$

$\underline{F = 35.3 \text{ lb}}$

3/41



Engine off : $T = 0$, $F_B = 0$

$$\begin{cases} \Sigma M_A = 0: & 1800(9.81)4 - N_B(4.8) = 0 \qquad N_B = 14\,720 \text{ N} \\ \Sigma F_y = 0: & N_A + 14\,720 - 1800(9.81) = 0, \qquad N_A = 2940 \text{ N} \end{cases}$$

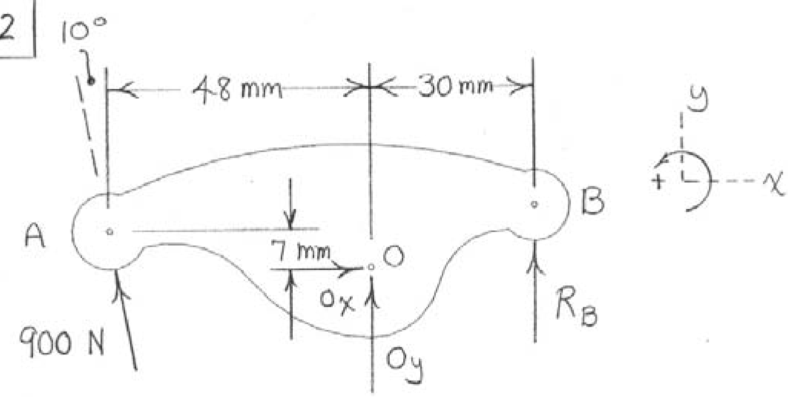
$$\Sigma M_A = 0: \quad 1800(9.81)4 - N_B'(4.8) + 3000\cos 12°(0.550) = 0$$

$$N_B' = 15{,}050 \text{ N}$$

$$\Sigma F_y = 0: \quad N_A' + 15{,}050 - 1800(9.81) + 3000\sin 12° = 0, \quad N_A' = 1983 \text{ N}$$

$$n_A = \frac{N_A' - N_A}{N_A}(100) = \underline{-32.6\%}, \quad n_B = \frac{N_B' - N_B}{N_B} = \underline{2.28\%}$$

3/42



$$\Sigma M_O = 0: -900 \cos 10°(48) + 900 \sin 10°(7)$$
$$+ R_B(30) = 0, \quad \underline{R_B = 1382 \text{ N}}$$

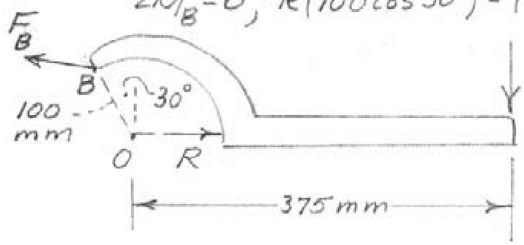
$$\Sigma F_x = 0: \quad O_x - 900 \sin 10° = 0, \quad O_x = 156.3 \text{ N}$$

$$\Sigma F_y = 0: \quad 900 \cos 10° + 1382 + O_y = 0$$
$$O_y = -2270 \text{ N}$$

$$O = \sqrt{O_x^2 + O_y^2} = \underline{2270 \text{ N}}$$

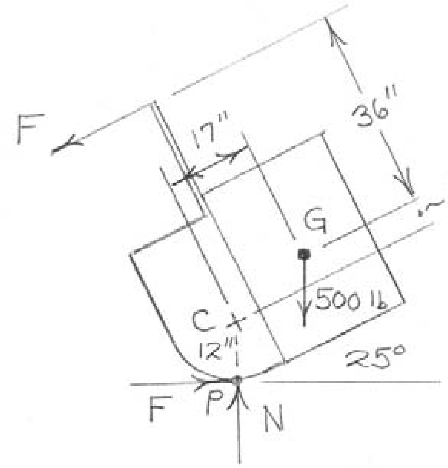
3/43 $M = Fd$; $80 = P(0.375)$, $P = 213$ N

$\Sigma M_B = 0$; $R(100 \cos 30°) - (375 + 100 \sin 30°)(213) = 0$



$\underline{R = 1047 \text{ N}}$

3/44

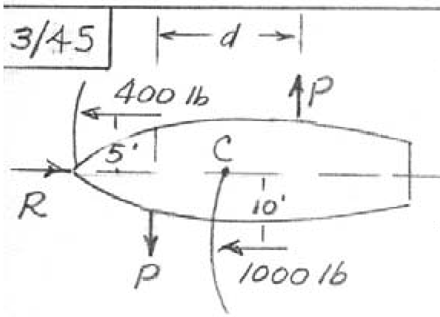


$\circlearrowleft + \Sigma M_P = 0: \; F(36+6+12\cos 25°) -$

$500[17\cos 25°] + 500[6\sin 25°] = 0$

$\underline{F = 121.7 \text{ lb}}$

3/45

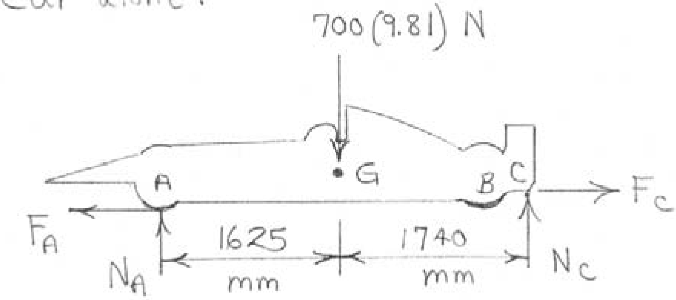


$\Sigma M_C = 0;\ Pd + 400(5)$
$\quad -1000(10) = 0$

Resultant of lateral forces is couple

$M = Pd = \underline{8000\ lb\text{-}ft}$

3/46 Car alone:

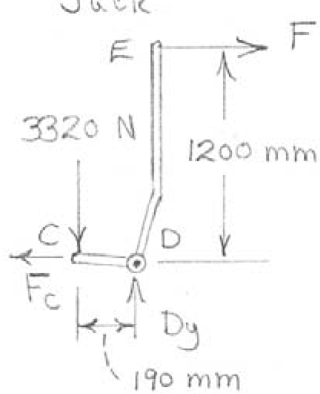


$$\curvearrowleft\!+\ \Sigma M_A = 0:\ -700(9.81)(1625) + N_C(1625+1740) = 0$$

$$N_C = 3320 \text{ N}$$

(assumes moment of F_C about A is negligible)

Jack

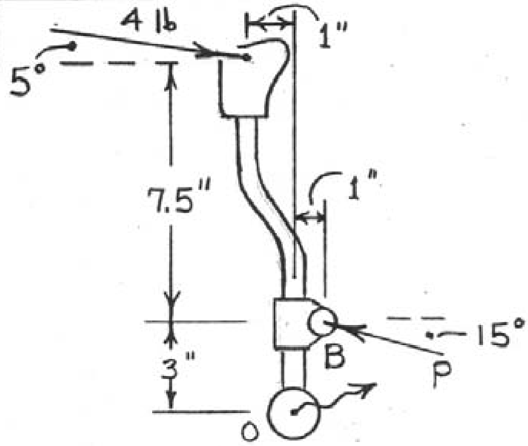


$$\curvearrowleft\!+\ \Sigma M_D = 0:$$

$$3320(190) - F(1200) = 0$$

$$\underline{F = 525 \text{ N}}$$

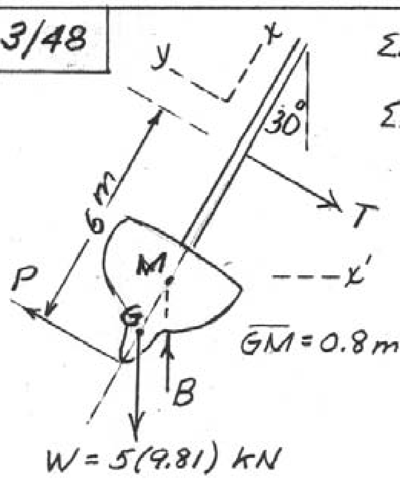
3/47



$$\curvearrowleft + \Sigma M_O = 0: -4\cos 5°(10.5) + 4\sin 5°(1)$$
$$+ P\cos 15°(3) + P\sin 15°(1) = 0$$

$$\underline{P = 13.14 \text{ lb}}$$

3/48



$\Sigma F_x = 0;\quad B = 5(9.81) = 49.05$ kN

$\Sigma F_{x'} = 0;\quad P = T$

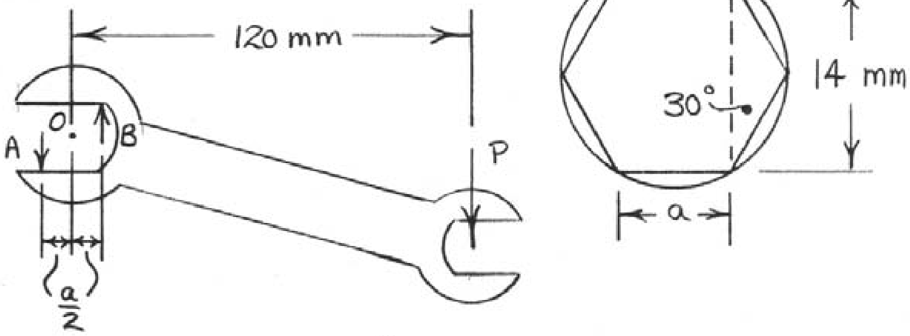
Two couples, so

$\Sigma M = 0;\quad 6T - 5(9.81)(0.8)\sin 30^\circ = 0$

$$T = \frac{5(9.81)(0.8)(0.5)}{6}$$

$$= \underline{3.27 \text{ kN}}$$

3/49



$2a\cos 30^\circ = 14, \quad \frac{a}{2} = 4.04 \text{ mm}$

$\circlearrowleft + \Sigma M_O = 0: \; 0.120P - 24 = 0, \quad \underline{P = 200 \text{ N}}$

(for wrench and bolt)

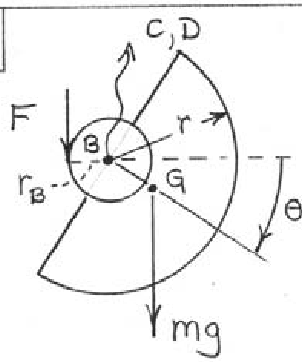
For wrench alone,

$\circlearrowleft + \Sigma M_A = 0: \; 200(0.120 + 0.00404) - B(2 \cdot 0.00404)$

$= 0, \quad \underline{B = 3070 \text{ N}}$

$+\uparrow \Sigma F = 0: \; -A + 3070 - 200 = 0, \quad \underline{A = 2870 \text{ N}}$

3/50



$\overline{BG} = \bar{r} = \frac{3}{8} r$ (from Table D/4)

FBD is of hemisphere, shaft, and gear B.

$\curvearrowleft + \Sigma M_B = 0: \quad F r_B - mg\left(\frac{3}{8} r \cos\theta\right)$

$$F = \frac{3}{8} mg \frac{r}{r_B} \cos\theta$$

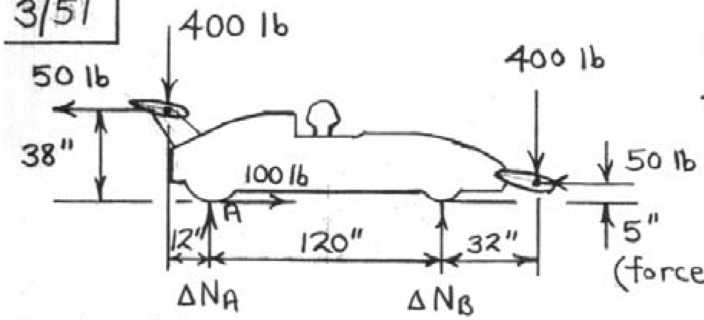
Gear A:

$\curvearrowleft + \Sigma M_B = 0: \quad F r_A - M = 0$

$$M = \left(\frac{3}{8} mg \frac{r}{r_B} \cos\theta\right) r_A$$

$$= \underline{\frac{3mgr}{8} \frac{r_A}{r_B} \cos\theta}$$

3/51



Without airfoils, $N_A = N_B = 850$ lb

(force <u>changes</u> only)

With airfoils,

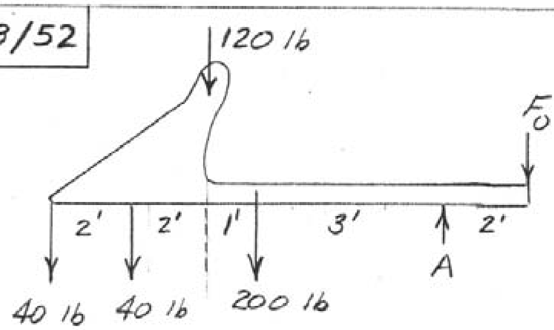
$$+\uparrow \Sigma F_y = 0 : \Delta N_A + \Delta N_B - 2(400) = 0$$

$$\circlearrowleft + \Sigma M_A = 0 : \; 50(38) + 400(12) + \Delta N_B(120) + 50(5) - 400(152) = 0$$

$$\left.\begin{aligned} \Delta N_A &= 351 \text{ lb} \\ \Delta N_B &= 449 \text{ lb} \end{aligned}\right\} \Rightarrow \quad \begin{aligned} N_A &= 850 + 351 = 1201 \text{ lb} \quad (48.0\%) \\ N_B &= 850 + 449 = 1299 \text{ lb} \quad (52.0\%) \end{aligned}$$

Note that a 100-lb propulsive force has been added (at A) to maintain equilibrium.

3/52



$\Sigma M_A = 0;$

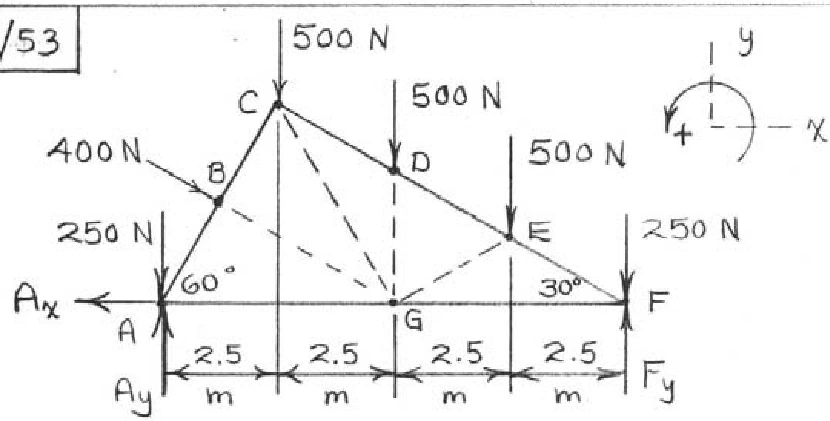
$40(8) + 40(6)$

$+ 120(4) + 200(3)$

$- 2F_O = 0$

$F_O = 820 \text{ lb}$

3/53



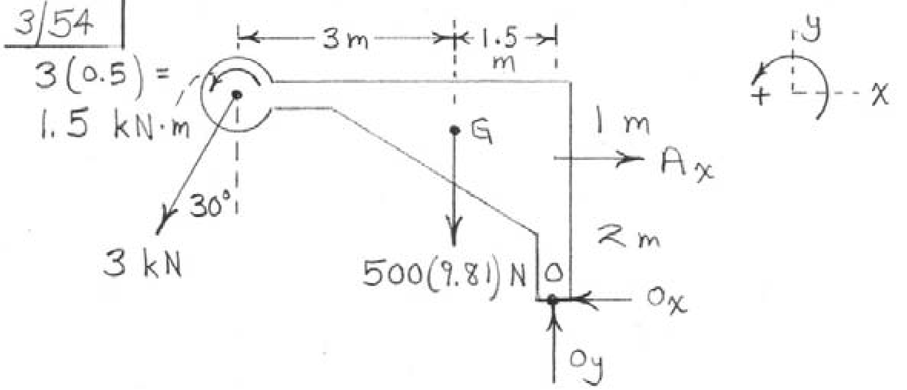
$$\Sigma F_x = 0: \ -A_x + 400\cos 30° = 0, \quad \underline{A_x = 346\text{ N}}$$

$$\Sigma M_A = 0: \ 400\left(\frac{10}{4}\right) + 500(2.5) + 500(5) + 500(7.5) + 250(10) - 10F_y = 0$$

$$\underline{F_y = 1100\text{ N}}$$

$$\Sigma F_y = 0: -250 - 400\sin 30° - 500(3) - 250 + 1100 + A_y = 0, \quad \underline{A_y = 1100\text{ N}}$$

3/54



$\Sigma M_o = 0: \ -2A + 500(9.81)(1.5) + 1500$

$+ 3000\cos 30°(4.5) + 3000 \sin 30°(3) = 0$

$A = 12\,520 \text{ N}$

$\Sigma F_x = 0: \ -3000 \sin 30° + 12\,520 - O_x = 0$

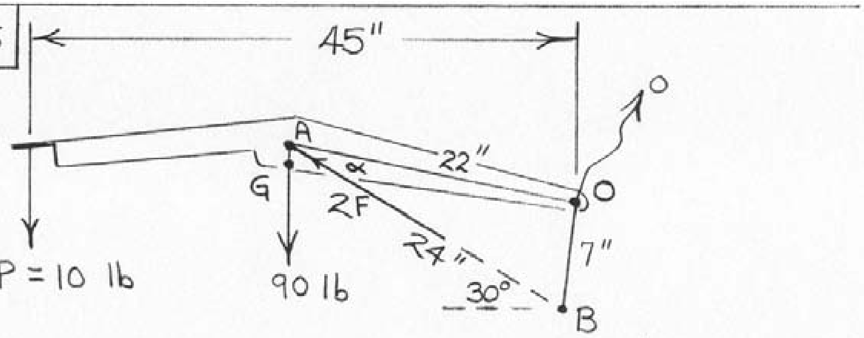
$O_x = 11\,020 \text{ N}$

$\Sigma F_y = 0: \ -3000\cos 30° - 500(9.81) + O_y = 0$

$O_y = 7500 \text{ N}$

$O = \sqrt{11020^2 + 7500^2} = 13\,340 \text{ N}$ or $\underline{13.34 \text{ kN}}$

3/55



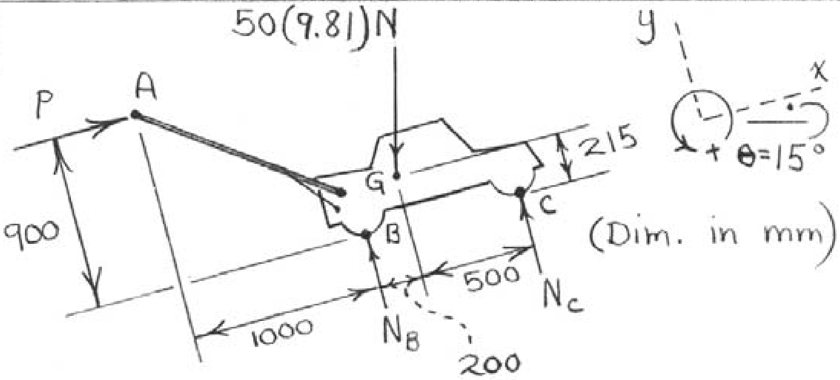
Law of cosines: $7^2 = 22^2 + 24^2 - 2(22)(24)\cos\alpha$

$\alpha = 16.79°$

$\curvearrowleft + \sum M_O = 0: \quad 10(45) - 2F(22\sin\alpha) + 90(22\cos(30° - \alpha))$

$= 0, \qquad \underline{\underline{F = 187.1 \text{ lb}}}$

3/56



$\Sigma F_x = 0: \quad P - 50(9.81)\sin 15° = 0 \qquad (1)$

$\Sigma F_y = 0: \quad N_B + N_C - 50(9.81)\cos 15° = 0 \qquad (2)$

$\Sigma M_C = 0: \quad -P(900) - N_B(700) + 50(9.81)[500 \cos 15° + 215 \sin 15°] = 0 \qquad (3)$

Solution to Eqs. (1)-(3):

$P = 127.0$ N

$N_B = 214$ N

$N_C = 260$ N

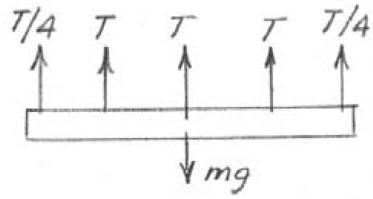
With $\theta = P = 0$:

$P = 0$

$N_B = 350$ N

$N_C = 140.1$ N

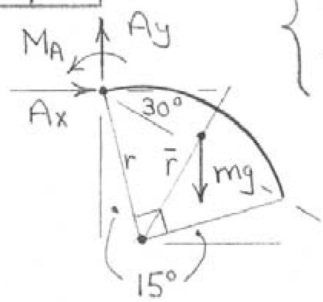
3/57



$$\Sigma F = 0;\ \left(3 + \frac{1}{4} + \frac{1}{4}\right)T - mg = 0$$

$$\underline{T = \frac{2}{7} mg}$$

3/58



From Table D/3, $\bar{r} = \dfrac{r \sin \alpha}{\alpha}$

For $\alpha = \pi/4$, $\bar{r} = 2\sqrt{2}\, r/\pi$

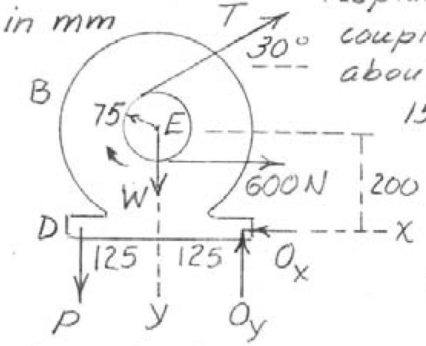
$\circlearrowleft + \sum M_A = 0$:

$M_A - mg\left(r \sin 15° + \bar{r} \cos 60°\right) = 0$

$\underline{M_A = 0.709\ mgr}$

3/59 Torque $M = 100\ N\cdot m = (600-T)(0.225)$, $T = 155.6\ N$

Dimensions in mm



$W = 100(9.81)\ N$

Replace T by force at E and a couple, so that moment of T about D in CW sense becomes

$$155.6(0.2\cos 30° - 0.125 \sin 30°) + 155.6(0.075) = 28.89\ N\cdot m$$

$\Sigma M_D = 0$; $28.89 + 981(0.125) + 600(0.2 - 0.075) - 0.250\,O_y = 0$

$O_y = 906\ N$

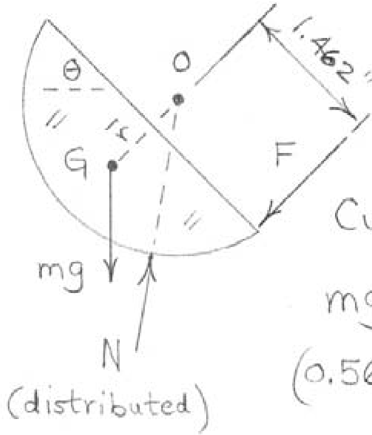
$\Sigma F_x = 0$; $155.6 \cos 30° + 600 - O_x = 0$

$O_x = 735\ N$

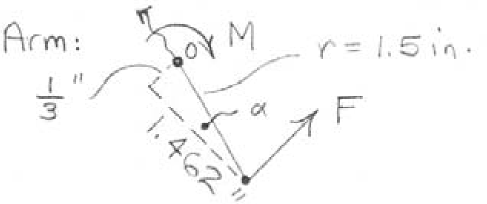
$$R = \sqrt{O_x^2 + O_y^2} = \sqrt{(735)^2 + (906)^2} = 1167\ N \text{ or } \underline{1.167\ kN}$$

3/60

Cube:



Arm:



$$\alpha = \sin^{-1}\frac{1/3}{1.5} = 12.84°$$

Cube: $\curvearrowleft \Sigma M_O = 0$:

$$mg\bar{r}\sin\theta - F(1.462) = 0$$

$$(0.56)(0.55 \cdot 1.5)\sin\theta - F(1.462) = 0$$

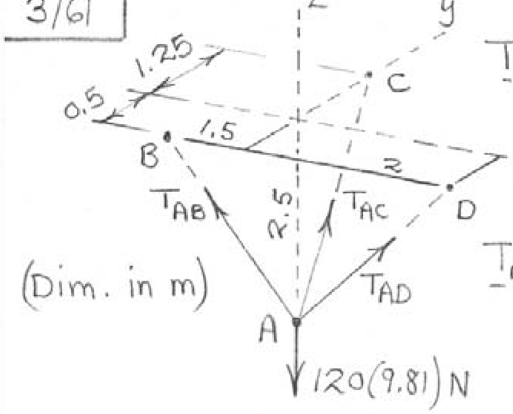
$$F = 0.316\sin\theta \quad (\text{in lb})$$

Arm: $\curvearrowleft \Sigma M_O = 0$:

$$-M + 0.316\sin\theta\,(1.462) = 0$$

$$\underline{M = 0.462\sin\theta \quad \text{lb-in.}}$$

3/61



$$\mathbf{T}_{AB} = T_{AB}\frac{-1.5\mathbf{i} - 0.5\mathbf{j} + 2.5\mathbf{k}}{\sqrt{1.5^2 + 0.5^2 + 2.5^2}} = T_{AB}(-0.507\mathbf{i} - 0.1690\mathbf{j} + 0.845\mathbf{k})$$

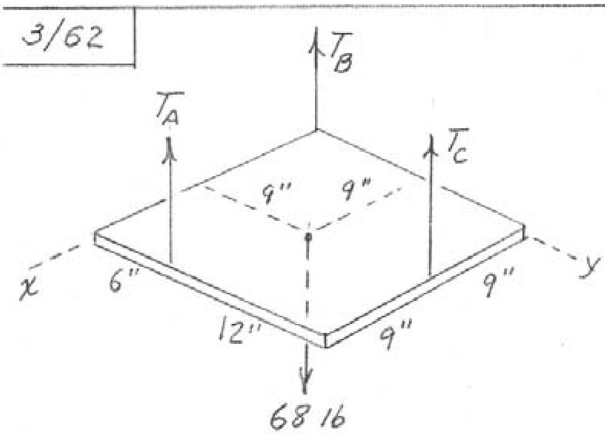
$$\mathbf{T}_{AC} = T_{AC}\frac{1.25\mathbf{j} + 2.5\mathbf{k}}{\sqrt{1.25^2 + 2.5^2}} = T_{AC}(0.447\mathbf{j} + 0.894\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD}\frac{2\mathbf{i} - 0.5\mathbf{j} + 2.5\mathbf{k}}{\sqrt{2^2 + 0.5^2 + 2.5^2}} = T_{AD}(0.617\mathbf{i} - 0.1543\mathbf{j} + 0.772\mathbf{k})$$

$$\begin{cases} \Sigma F_x = 0: -0.507T_{AB} + 0.617T_{AD} = 0 \\ \Sigma F_y = 0: -0.1690T_{AB} + 0.447T_{AC} - 0.1543T_{AD} = 0 \\ \Sigma F_z = 0: \ 0.845T_{AB} + 0.894T_{AC} + 0.772T_{AD} - 120(9.81) = 0 \end{cases}$$

Solution :
$$\begin{cases} T_{AB} = 569 \text{ N} \\ T_{AC} = 376 \text{ N} \\ T_{AD} = 467 \text{ N} \end{cases}$$

3/62



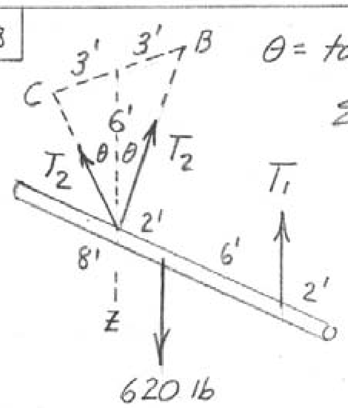
$\Sigma M_x = 0$; $6T_A + 18T_C - 68(9) = 0$, $T_A + 3T_C = 102$

$\Sigma M_y = 0$; $-18T_A - 9T_C + 68(9) = 0$, $2T_A + T_C = 68$

Solve & get $\underline{T_A = 20.4 \text{ lb}}$, $\underline{T_C = 27.2 \text{ lb}}$.

$\Sigma F_z = 0$; $20.4 + T_B + 27.2 - 68 = 0$, $\underline{T_B = 20.4 \text{ lb}}$

3/63



$\theta = \tan^{-1} 3/6 = 26.57°$

$\Sigma M_{CB} = 0;\quad 8T_1 - 620(2) = 0$

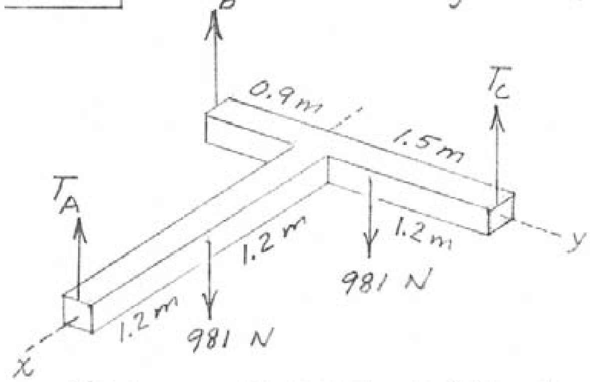
$\underline{T_1 = 155 \text{ lb}}$

$\Sigma F_z = 0;\quad 620 - 155 - 2T_2 \cos\theta = 0$

$T_2 = \dfrac{465}{2 \cos 26.57°} = \underline{260 \text{ lb}}$

3/64

$W = mg = 100(9.81) = 981\ N$



$\Sigma M_y = 0;\ 981(1.2) - 2.4T_A = 0;\quad \underline{T_A = 490\ N}$

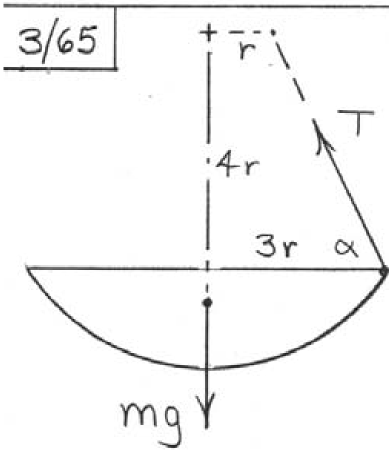
$\Sigma M_x = 0;\ 1.5T_C - 0.9T_B - 981(0.3) = 0$

$\Sigma F_z = 0;\ 491 + T_B + T_C - 2(981) = 0,\ T_B + T_C = 1472\ N$

so $1.5T_C - 0.9(1472 - T_C) = 294.3,\quad \underline{T_C = 674\ N}$

$\underline{T_B = 797\ N}$

3/65



T (one of three)

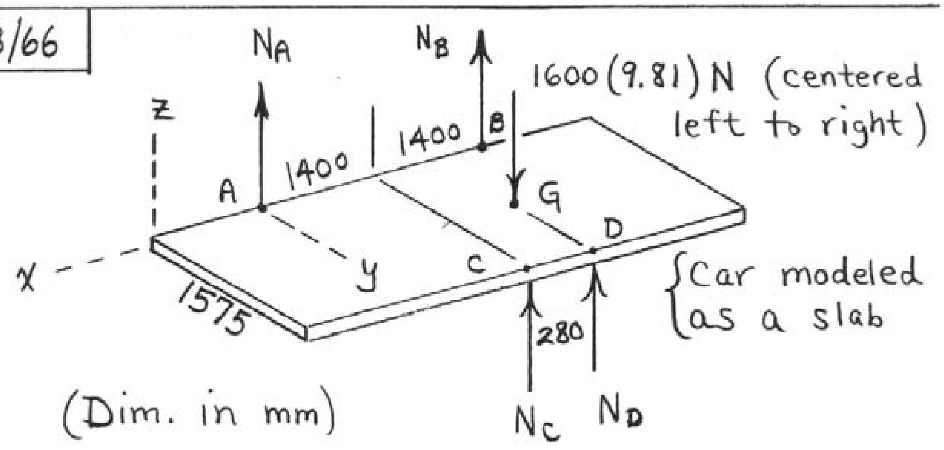
$$\alpha = \tan^{-1}\frac{4r}{2r} = 63.4^\circ$$

$+\uparrow \Sigma F = 0:$

$$3(T\sin\alpha) - mg = 0$$

$$\underline{T = 0.373mg}$$

3/66



Jacking at C ($N_D = 0$):

$\Sigma M_x = 0: \ -1600(9.81)\left(\frac{1575}{2}\right) + N_C\,(1575) = 0$

$\underline{N_C = 7850 \text{ N}}$

$\Sigma M_y = 0: -1600(9.81)(1680) + N_B(2800) + N_C(1400) = 0$

$\Sigma F_z = 0: \ N_A + N_B + N_C - 1600(9.81) = 0$

$\Rightarrow \underline{N_A = 2350 \text{ N}}, \quad \underline{N_B = 5490 \text{ N}}$

Jacking at D ($N_C = 0$): $\quad \underline{N_D = 7850 \text{ N}}$ {Same as for N_C

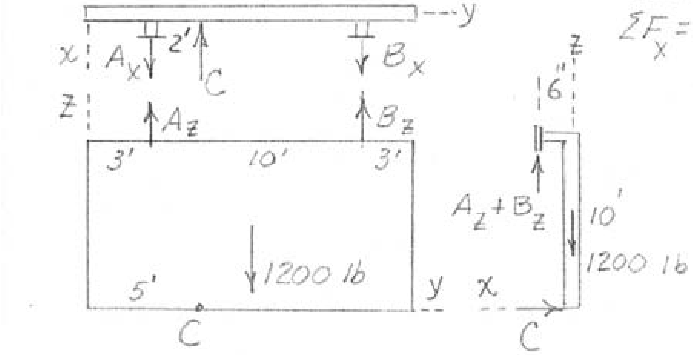
$\Sigma M_y = 0: \ -1600(9.81)(1680) + N_B(2800) + N_D(1680) = 0$

$\Sigma F_z = 0: \ N_A + N_B + N_D - 1600(9.81) = 0$

$\Rightarrow \underline{N_A = 3140 \text{ N}}, \quad \underline{N_B = 4710 \text{ N}}$

3/67 x-z; $\Sigma M_{AB}=0$; $10C-1200(6/12)=0$, $C=60$ lb

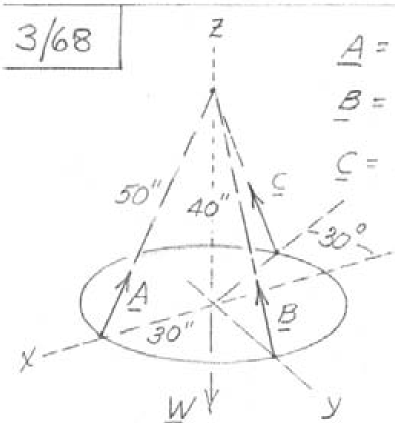
x-y; $\Sigma M_A=0$; $2(60)-10B_x=0$; $\underline{B_x=12\text{ lb}}$



$\Sigma F_x=0$; $A_x+12-60=0$

$\underline{A_x=48\text{ lb}}$

3/68



$\underline{A} = \frac{A}{5}(-3\underline{i} + 4\underline{k})$

$\underline{B} = \frac{B}{5}(-3\underline{j} + 4\underline{k})$

$\underline{C} = \frac{C}{5}(3\underline{i}\cos 30° + 3\underline{j}\sin 30° + 4\underline{k})$

$\underline{W} = -600\underline{k}$ lb

$\Sigma\underline{F} = 0;\quad \frac{1}{5}(-3A + \frac{3\sqrt{3}}{2}C)\underline{i} + \frac{1}{5}(-3B + \frac{3}{2}C)\underline{j}$

$+ \frac{1}{5}(4A + 4B + 4C - 5[600])\underline{k} = \underline{0}$

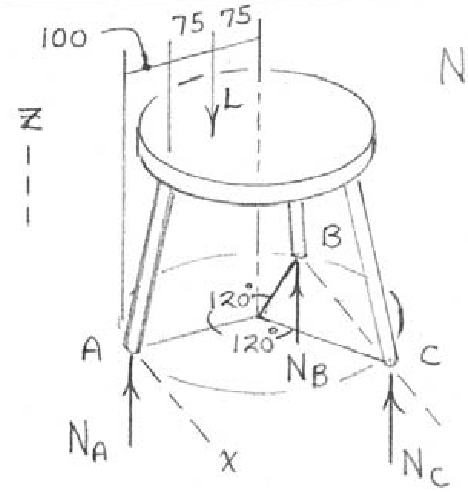
Thus $A = \frac{\sqrt{3}}{2}C$, $B = \frac{C}{2}$, $4(A+B+C) = 3000$

$4(\frac{\sqrt{3}}{2} + \frac{1}{2} + 1)C = 3000$, $C = \frac{3000}{2(3+\sqrt{3})} = \underline{317 \text{ lb}}$

$B = \underline{158 \text{ lb}}$

$A = \underline{275 \text{ lb}}$

3/69



Note: x-axis is $\parallel$ BC.

$N_B = N_C = N$, by symmetry.

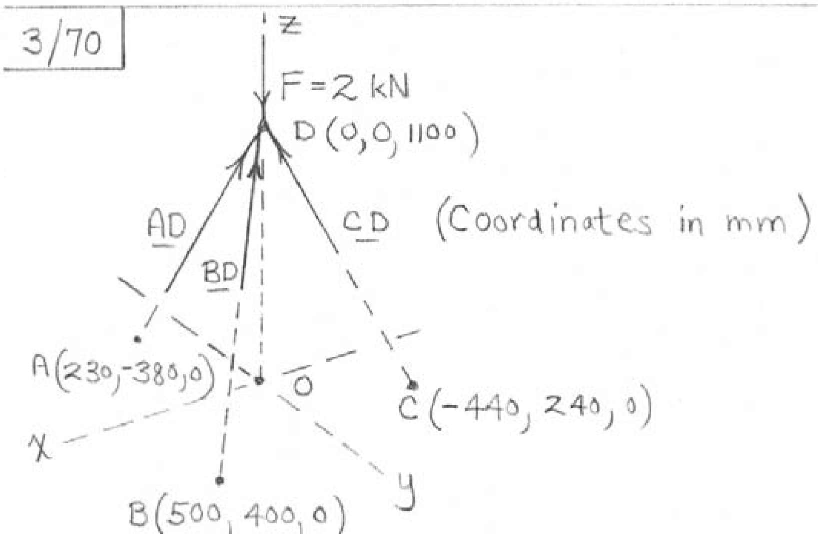
$$\Sigma M_x = 0: \quad -L(175) + 2N(250 + 250\cos 60°) = 0$$

$$\underline{N = 0.233L = N_B = N_C}$$

$$\Sigma F_z = 0: \quad N_A + 2(0.233L) - L = 0$$

$$\underline{N_A = 0.533L}$$

3/70



$$\underline{AD} = AD\,\frac{-230\underline{i} + 380\underline{j} + 1100\underline{k}}{\sqrt{230^2 + 380^2 + 1100^2}} = AD(-0.1939\underline{i} + 0.320\underline{j} + 0.927\underline{k})$$

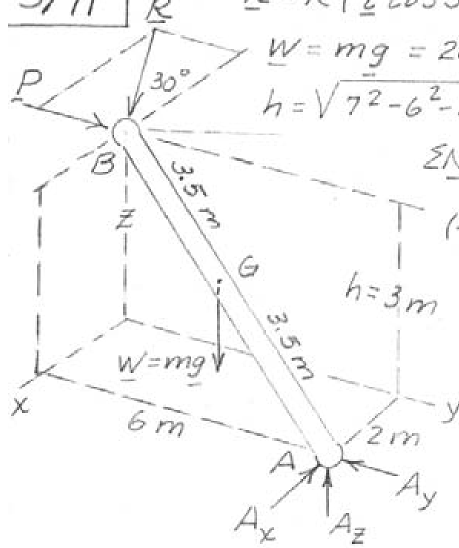
$$\underline{BD} = BD\,\frac{-500\underline{i} - 400\underline{j} + 1100\underline{k}}{\sqrt{500^2 + 400^2 + 1100^2}} = BD(-0.393\underline{i} - 0.314\underline{j} + 0.864\underline{k})$$

$$\underline{CD} = CD\,\frac{440\underline{i} - 240\underline{j} + 1100\underline{k}}{\sqrt{440^2 + 240^2 + 1100^2}} = CD(0.364\underline{i} - 0.1985\underline{j} + 0.910\underline{k})$$

From $\Sigma \underline{F} = \underline{0}$:
$$\begin{cases} -0.1939AD - 0.393BD + 0.364CD = 0 \\ 0.320AD - 0.314BD - 0.1985CD = 0 \\ 0.927AD + 0.864BD + 0.910CD - 2 = 0 \end{cases}$$

Solution : $\underline{AD = 0.925\text{ kN}}$, $\underline{BD = 0.376\text{ kN}}$, $\underline{CD = 0.898\text{ kN}}$
(all compression)

3/71



$\underline{R} = R(\underline{i}\cos 30^\circ + \underline{j}\sin 30^\circ) = \frac{R}{2}(\sqrt{3}\,\underline{i} + \underline{j})$

$\underline{W} = mg = 200(9.81)(-\underline{k}) = -1962\,\underline{k}$ N

$h = \sqrt{7^2 - 6^2 - 2^2} = 3$ m

$\Sigma \underline{M}_A = \underline{0}$; $\underline{r}_{AG} \times \underline{W} + \underline{r}_{AB} \times (\underline{P} + \underline{R}) = \underline{0}$

$$(-1\underline{i} - 3\underline{j} + 1.5\underline{k}) \times (-1962\underline{k})$$

$$+ (-2\underline{i} - 6\underline{j} + 3\underline{k}) \times \left(P\underline{j} + \frac{R\sqrt{3}}{2}\underline{i} + \frac{R}{2}\underline{j}\right) = \underline{0}$$

Simplify & get

$$(5886 - 3P - 3R/2)\,\underline{i}$$

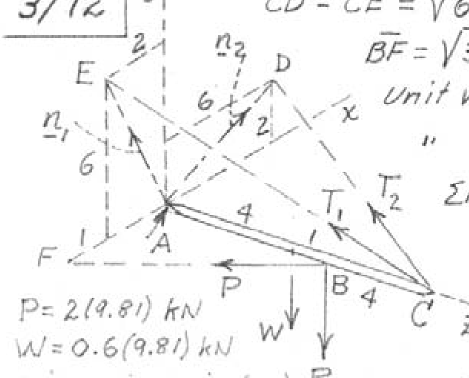
$$+ (-1962 + 3\sqrt{3}R/2)\,\underline{j}$$

$$+ (-2P + R + 3\sqrt{3}R)\,\underline{k} = 0$$

$R = \frac{2(1962)}{3\sqrt{3}} = \underline{755 \text{ N}}$

$3P = 5886 - \frac{3}{2}755$, $P = \underline{1584 \text{ N}}$

3/72



$\overline{CD} = \overline{CE} = \sqrt{6^2+2^2+9^2} = 11$ m

$\overline{BF} = \sqrt{3^2+5^2} = 5.83$ m

Unit vector $\underline{n}_1 = (-\underline{i}+3\underline{j})/\sqrt{10}$

" " $\underline{n}_2 = (3\underline{i}+\underline{j})/\sqrt{10}$

$\Sigma M_{AD} = 0;$

$$9\underline{k} \times \frac{T_1}{11}(-2\underline{i}+6\underline{j}-9\underline{k})\cdot(3\underline{i}+\underline{j})/\sqrt{10}$$

$$+5\underline{k} \times \frac{2(9.81)}{5.83}(-3\underline{i}-5\underline{k})\cdot(3\underline{i}+\underline{j})/\sqrt{10}$$

$$+5\underline{k} \times 2(9.81)(-\underline{j})\cdot(3\underline{i}+\underline{j})/\sqrt{10}$$

$$+4.5\underline{k} \times 0.6(9.81)(-\underline{j})\cdot(3\underline{i}+\underline{j})/\sqrt{10} = 0$$

$P = 2(9.81)$ kN

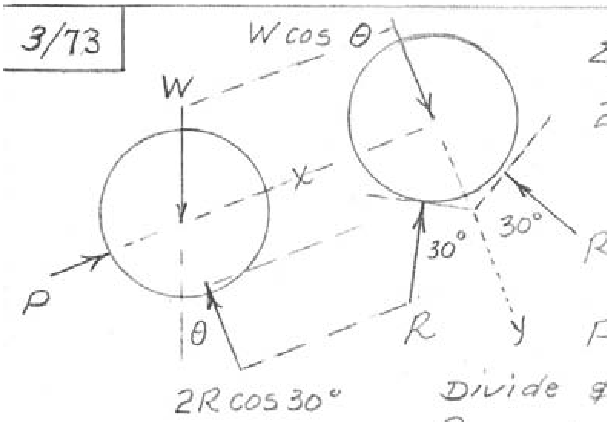
$W = 0.6(9.81)$ kN

Dimensions in (m)

Simplify & get $\frac{180T_1}{11\sqrt{10}} + \frac{294.3}{5.83\sqrt{10}} = \frac{373.8}{\sqrt{10}}$, $\underline{T_1 = 19.76 \text{ kN}}$

(Similarly, if T_2 is desired, $\Sigma M_{AE} = 0$ will give $T_2 = 16.86$ kN)

3/73



$\Sigma F_y = 0;$

$2R\cos 30^\circ - W\cos\theta = 0$

$R\sqrt{3} = W\cos\theta$

$\Sigma F_x = 0;$

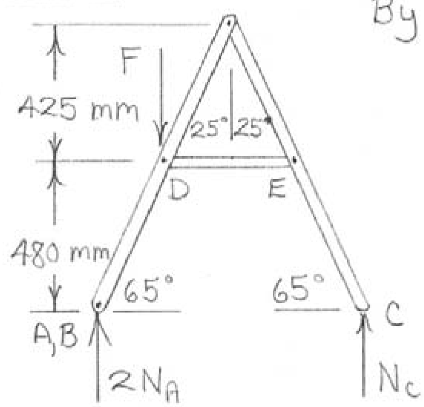
$P - W\sin\theta = 0$

Divide & get

$$\frac{P}{R\sqrt{3}} = \tan\theta$$

So with $P = R$, $\theta = \tan^{-1} 1/\sqrt{3} = \underline{30^\circ}$

3/74



By inspection, $N_A = N_B$.

$\circlearrowleft + \sum M_C = 0: \; -2N_A(2 \cdot 905 \tan 25°)$

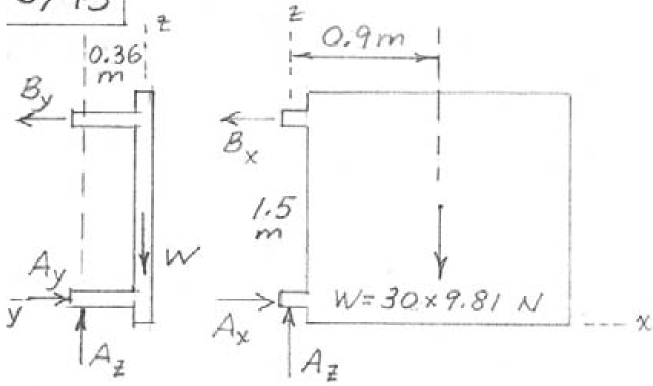
$+F(905 \tan 25° + 425 \tan 25°) = 0$

$\underline{N_A = 0.367F = N_B}$

$+\uparrow \sum F = 0: \; 2(0.367F) + N_C - F = 0$

$\underline{N_C = 0.265F}$

3/75

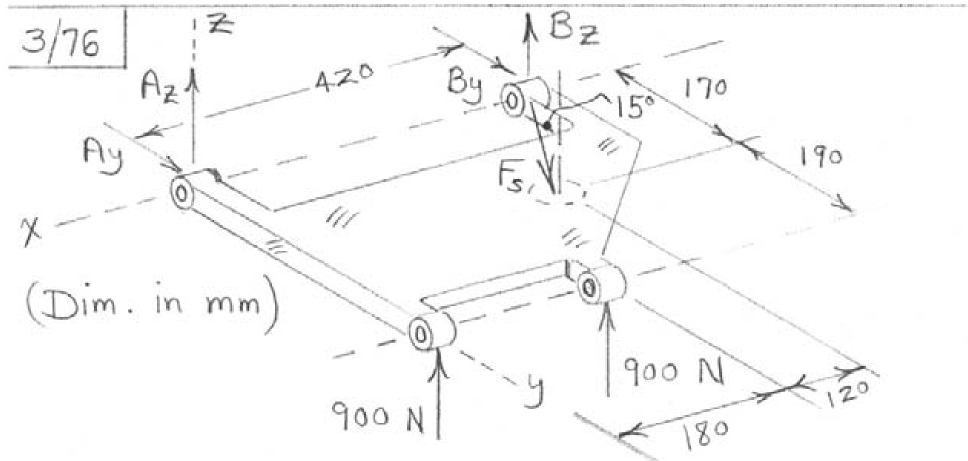


x-z; $\Sigma M_A = 0$; $1.5 B_x - 0.9(30)(9.81) = 0$, $B_x = 176.6$ N

y-z; $\Sigma M_A = 0$; $1.5 B_y - 0.36(30)(9.81) = 0$, $B_y = 70.6$ N

$$B = \sqrt{176.6^2 + 70.6^2} = \underline{190.2 \text{ N}}$$

3/76



$\Sigma M_x = 0: \ 2(900)(360) - F_S \cos 15°(170) = 0, \ \underline{F_S = 3950 \text{ N}}$

$\Sigma M_y = 0: \ B_z(420) + 900(180) - 3950 \cos 15°(300) = 0$

$B_z = 2340 \text{ N}$

$\Sigma M_z = 0: -B_y(420) - 3950 \sin 15°(300) = 0, \ B_y = -730 \text{ N}$

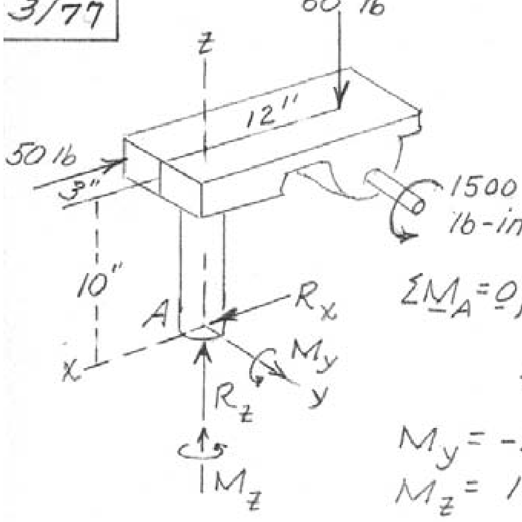
$\Sigma F_y = 0: \ A_y - 730 + 3950 \sin 15° = 0, \quad A_y = -292 \text{ N}$

$\Sigma F_z = 0: \ A_z + 2340 - 3950 \cos 15° + 1800 = 0, \ A_z = -325 \text{ N}$

$F_A = \sqrt{A_y^2 + A_z^2} = \underline{437 \text{ N}}$

$F_B = \sqrt{B_y^2 + B_z^2} = \underline{2450 \text{ N}}$

3/77



$\Sigma \underline{M}_A = \underline{0}$; $[-60(12) - 50(10) + 1500 + M_y]\underline{j} + [-50(3) + M_z]\underline{k} = \underline{0}$

$M_y = -280$ lb-in.

$M_z = 150$ lb-in

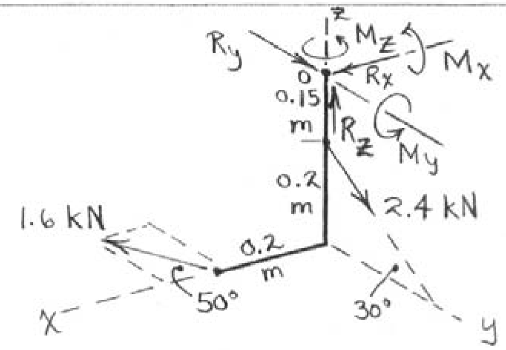
$\underline{M} = -280\underline{j} + 150\underline{k}$ lb-in.

$\Sigma \underline{F} = \underline{0}$; $(R_x - 50)\underline{i} + (R_z - 60)\underline{k} = \underline{0}$

$R_x = 50$ lb, $R_z = 60$ lb

$\underline{R} = 50\underline{i} + 60\underline{k}$ lb

3/78



$\Sigma F_x = 0: \; R_x + 1.6 \cos 50°, \qquad R_x = -1.028 \text{ kN}$

$\Sigma F_y = 0: \; R_y + 2.4 \cos 30° - 1.6 \sin 50° = 0, \quad R_y = -0.853 \text{ kN}$

$\Sigma F_z = 0: \; R_z - 2.4 \sin 30° = 0, \qquad R_z = 1.2 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{1.796 \text{ kN}}$

$\Sigma M_{o_x} = 0: \; M_x + 2.4 \cos 30° (0.15) - 1.6 \sin 50° (0.35) = 0$

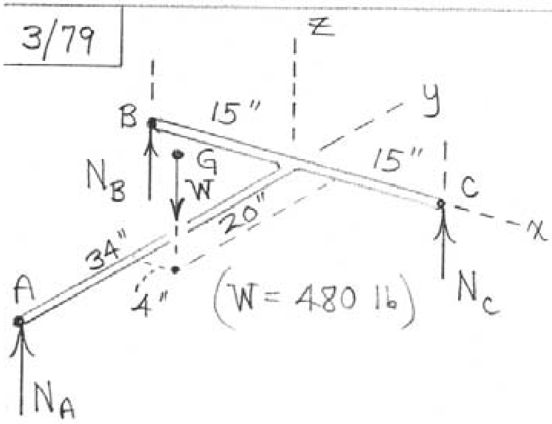
$M_x = 0.1172 \text{ kN·m}$

$\Sigma M_{oy} = 0: \; M_y - 1.6 \cos 50° (0.35) = 0, \quad M_y = 0.360 \text{ kN·m}$

$\Sigma M_{oz} = 0: \; M_z - 1.6 \sin 50° (0.2) = 0, \quad M_z = 0.245 \text{ kN·m}$

$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{0.451 \text{ kN·m}}$

3/79



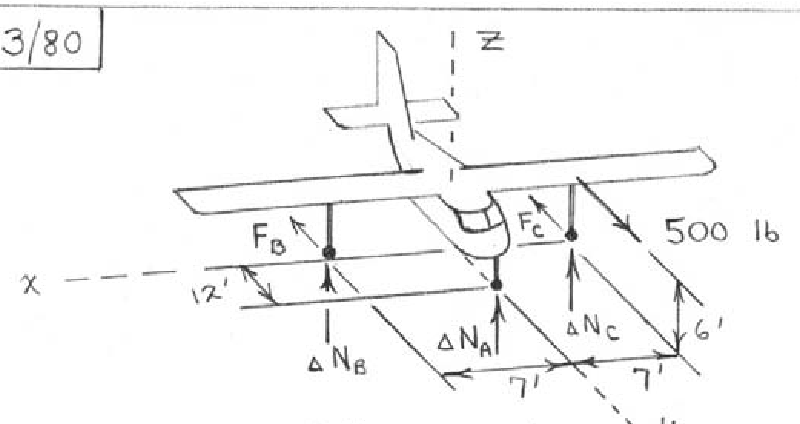
$$\Sigma F_z = 0: \quad N_A + N_B + N_C - 480 = 0$$

$$\Sigma M_x = 0: \quad 480(20) - N_A(34) = 0$$

$$\Sigma M_y = 0: \quad N_B(15) - N_C(15) + 480(4) = 0$$

Solution: $\begin{cases} N_A = 282 \text{ lb} \\ N_B = 34.8 \text{ lb} \\ N_C = 162.8 \text{ lb} \end{cases}$

3/80

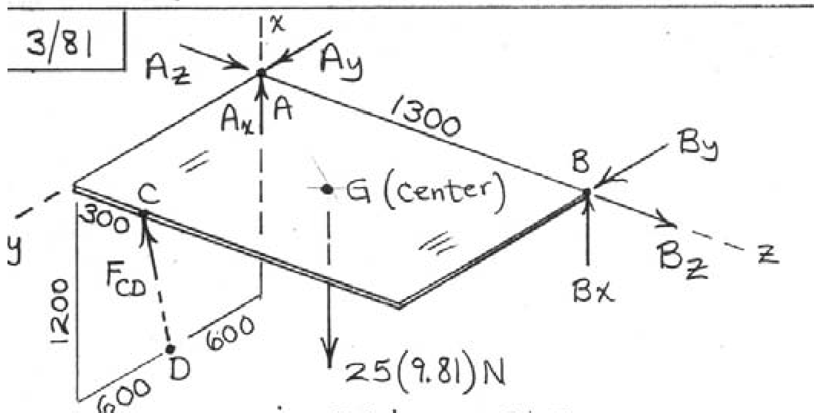


$\Sigma M_x = 0: \quad \Delta N_A(12) - 500(6) = 0, \qquad \underline{\Delta N_A = 250 \text{ lb}}$

$$\left.\begin{array}{ll} \Sigma F_z = 0: & \Delta N_A + \Delta N_B + \Delta N_C = 0 \\ \Sigma M_y = 0: & \Delta N_C(7) - \Delta N_B(7) = 0 \end{array}\right\} \quad \underline{\Delta N_B = \Delta N_C = -125 \text{ lb}}$$

More information would be required to determine F_B and F_C. x-components of friction at B and C are possible.

3/81



$$\underline{F}_{CD} = F\left[\frac{1.2\underline{i} + 0.6\underline{j} + 0.3\underline{k}}{\sqrt{1.2^2+0.6^2+0.3^2}}\right] = F\left(0.873\underline{i} + 0.436\underline{j} + 0.218\underline{k}\right)$$

$\Sigma M_z = 0: \; 25(9.81)(0.6) - 0.873F(1.2) = 0$

$\underline{F = 140.5 \text{ N}}$

$\Sigma M_y = 0: \; B_x(1.3) - 25(9.81)(0.65) + 0.873F(0.3) = 0$

$B_x = 94.3 \text{ N}$

$\Sigma F_x = 0: \; A_x + 94.3 + 0.873F - 25(9.81) = 0$

$A_x = 28.3 \text{ N}$

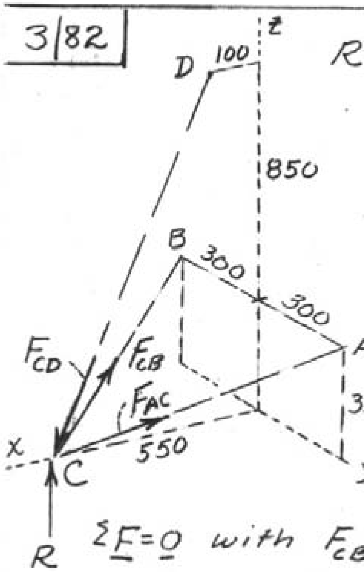
$\Sigma M_x = 0: \; -B_y(1.3) + 0.218F(0.6) = 0$

$B_y = 14.15 \text{ N}$

$\Sigma F_y = 0: \; A_y + 14.15 + 0.436F = 0, \quad A_y = -75.5 \text{ N}$

$A_n = \sqrt{75.5^2 + 28.3^2} = \underline{80.6 \text{ N}}, \quad B_n = \sqrt{94.3^2 + 14.15^2} = \underline{95.4 \text{ N}}$

3/82



$R = \frac{1}{3}mg$; From Table C2 for Mars $g = 3.73\ m/s^2$

$R = \frac{1}{3}600(3.73) = 746\ N$

$\overline{AC} = \sqrt{(550)^2 + (300)^2 + (350)^2} = 718\ mm$

$\overline{CD} = \sqrt{(450)^2 + (1200)^2} = 1282\ mm$

$\underline{F}_{AC} = \frac{F_{AC}}{718}(-550\underline{i} + 300\underline{j} + 350\underline{k})$

$\underline{F}_{CB} = \frac{F_{CB}}{718}(-550\underline{i} - 300\underline{j} + 350\underline{k})$

$\underline{F}_{CD} = \frac{F_{CD}}{1282}(450\underline{i} - 1200\underline{k})$

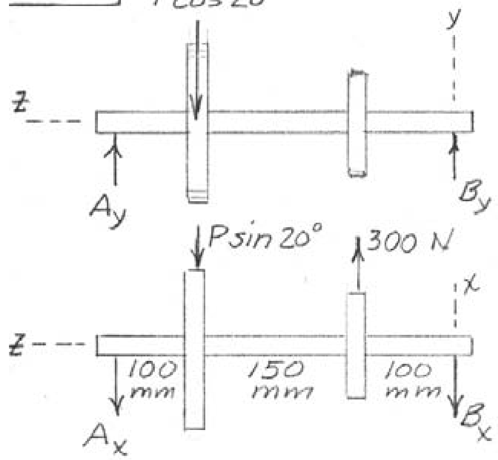
$\Sigma\underline{F} = \underline{0}$ with $F_{CB} = F_{AC}$ gives upon collecting terms

$$\left(-\frac{2F_{AC}}{718}550 + \frac{F_{CD}}{1282}450\right)\underline{i} + \left(746 + \frac{2F_{AC}}{718}350 - \frac{F_{CD}}{1282}1200\right)\underline{k} = \underline{0}$$

Equate coefficients to zero & solve simultaneously to get $F_{CD} = \underline{1046\ N}$ compression

$F_{AC} = F_{CB} = \underline{240\ N}$ tension

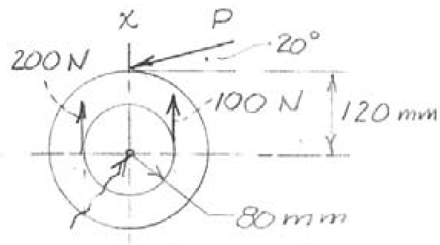
3/83



$\Sigma M_z = 0;$

$(200-100)0.080$
$\quad - 0.120\, P\cos 20° = 0$

$\underline{P = 70.9 \text{ N}}$



y-z; $\Sigma M_A = 0$; $0.35 B_y - 70.9 \cos 20°(0.1) = 0$, $B_y = 19.05$ N

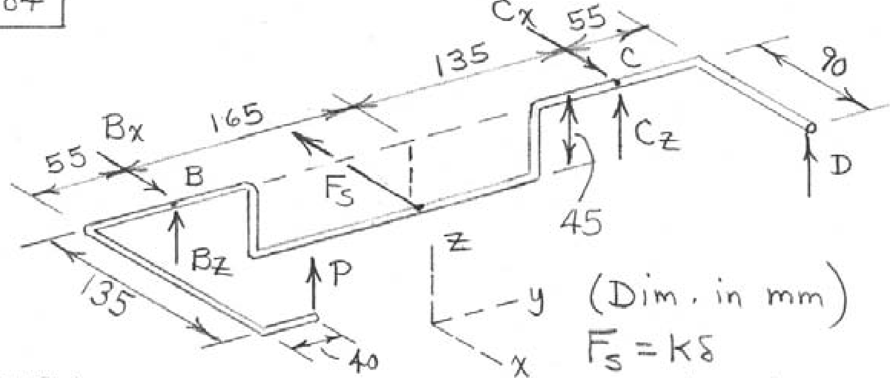
$\Sigma F_y = 0$; $A_y + 19.05 - 70.9 \cos 20° = 0$, $A_y = 47.6$ N

x-z; $\Sigma M_A = 0$; $300(0.25) - 70.9 \sin 20°(0.1) - 0.35 B_x = 0$
$B_x = 207$ N

$\Sigma F_x = 0$; $A_x + 70.9 \sin 20° + 207 - 300 = 0$; $A_x = 68.4$ N

$A = \sqrt{68.4^2 + 47.6^2} = \underline{83.3 \text{ N}}$, $B = \sqrt{207^2 + 19.05^2} = \underline{208 \text{ N}}$

3/84



$F_s = k\delta$

$= 900(0.060) = 54\ N$

D=0 :

$\Sigma M_{BC} = 0: \ -P(135) + 54(45) = 0, \quad \underline{P_{min} = 18\ N}$

$\Sigma M_{Bx} = 0: \ -18(15) + C_z(300) = 0, \quad C_z = 0.9\ N$

$\Sigma M_{Bz} = 0: \ 54(165) - C_x(300) = 0, \quad C_x = 29.7\ N$

$\Sigma F_x = 0: \ 29.7 + B_x - 54 = 0, \quad B_x = 24.3\ N$

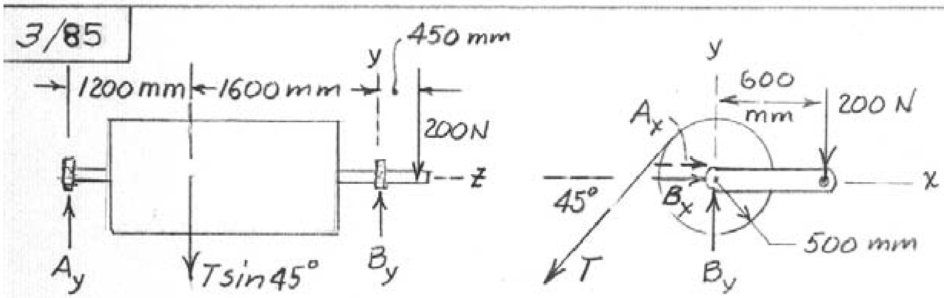
$\Sigma F_z = 0: \ 0.9 + B_z + 18 = 0, \quad B_z = -18.9\ N$

$B = \sqrt{B_x^2 + B_z^2} = \underline{30.8\ N}, \ C = \sqrt{C_x^2 + C_z^2} = \underline{29.7\ N}$

If $P = P_{min}/2 = 18/2 = 9\ N, \ (D \neq 0)$:

$\Sigma M_{BC} = 0: \ -9(135) + 54(45) - D(90) = 0, \ \underline{D = 13.5\ N}$

3/85



$\Sigma M_z = 0;\ 0.500T - 0.600(200) = 0, \quad T = 240\ \text{N}$

$\Sigma M_x = 0;\ 2.800A_y + 0.450(200) - 240(0.707)(1.600) = 0$

$A_y = 64.8\ \text{N}$

$\Sigma F_y = 0;\ 64.8 - 240(0.707) + B_y - 200 = 0, \quad B_y = 305\ \text{N}$

$\Sigma M_y = 0;\ 240(0.707)(1.600) - 2.800A_x = 0, \quad A_x = 97.0\ \text{N}$

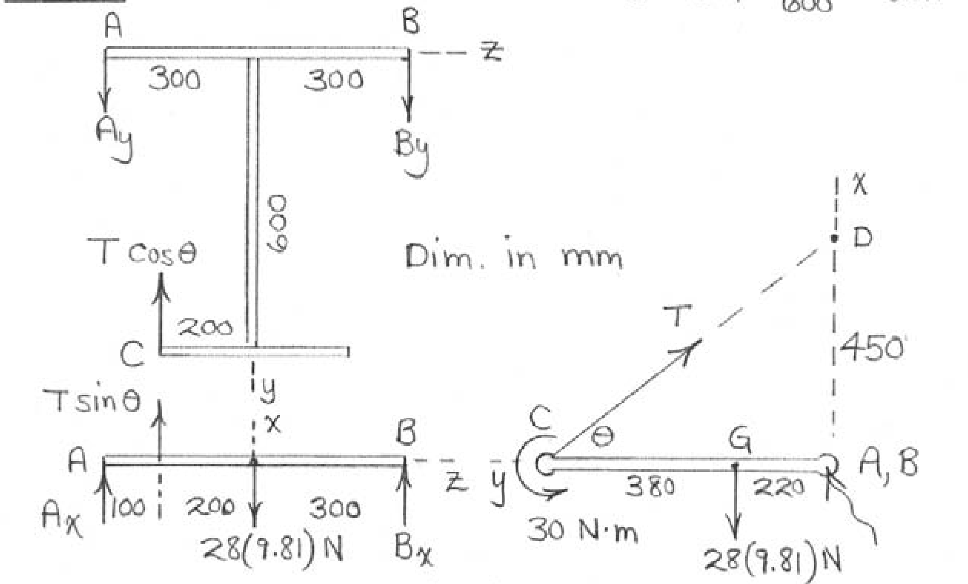
$\Sigma F_x = 0;\ 97.0 + B_x - 240(0.707) = 0, \quad B_x = 72.7\ \text{N}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(97.0)^2 + (64.8)^2} = \underline{116.7\ \text{N}}$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(72.7)^2 + (305)^2} = \underline{313\ \text{N}}$

3/86

$$\theta = \tan^{-1}\frac{450}{600} = 36.9^\circ$$



$(x\text{-}y)\ \Sigma M_z = 0:\ 28(9.81)(0.220) - T\sin 36.9^\circ(0.600) + 30 = 0, \quad T = 251\text{ N}$

$(x\text{-}z)\ \Sigma M_B = 0:\ 28(9.81)(0.300) - 251\sin 36.9^\circ(0.500) - 0.600A_x = 0, \quad A_x = 11.74\text{ N}$

$\Sigma F_x = 0:\ 11.74 + 251\sin 36.9^\circ - 28(9.81) + B_x = 0, \quad B_x = 112.2\text{ N}$

$(y\text{-}z)\ \Sigma M_B = 0:\ 251\cos 36.9^\circ(0.500) - 0.6A_y = 0, \quad A_y = 167.5\text{ N}$

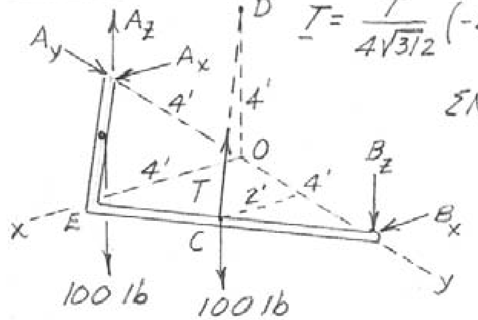
$\Sigma F_y = 0:\ 167.5 + B_y - 251\cos 36.9^\circ = 0, \quad B_y = 33.5\text{ N}$

$$A = \sqrt{11.74^2 + 167.5^2} = \underline{167.9\text{ N}}$$

$$B = \sqrt{112.2^2 + 33.5^2} = \underline{117.1\text{ N}}$$

Couple may be applied at any place on rigid body with the same external effect.

3/87



$\overline{CD} = \sqrt{(4/\sqrt{2})^2 + 4^2} = 4\sqrt{3/2}$ ft

$\underline{T} = \dfrac{T}{4\sqrt{3/2}}(-2\underline{i} - 2\underline{j} + 4\underline{k}) = \dfrac{T}{\sqrt{6}}(-\underline{i} - \underline{j} + 2\underline{k})$

$\Sigma M_{AB} = 0;\ 200(2) - T_z(2) = 0$

$T_z = \dfrac{2T}{\sqrt{6}} = 200,\ T = 100\sqrt{6}$ lb

$\underline{T = 245 \text{ lb}}$

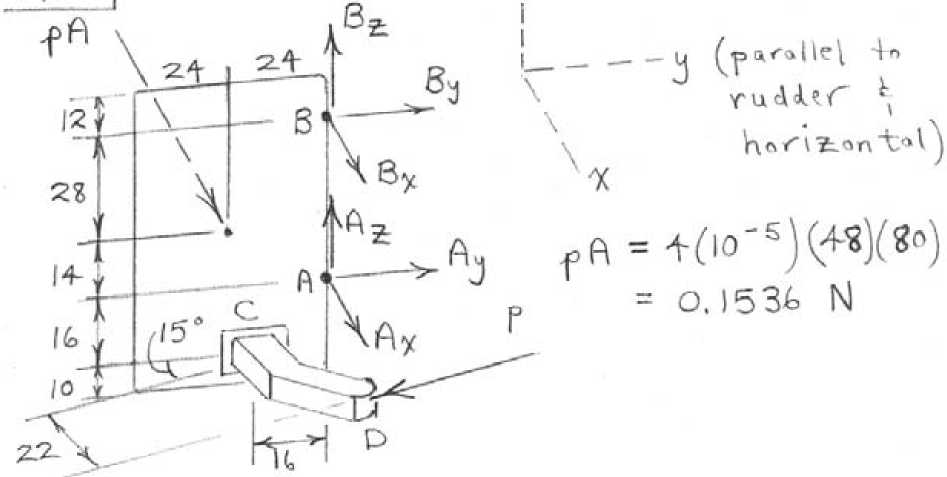
$(\Sigma M_B)_x = 0;\ 100(2) + 100(6) - 200(2) - 8A_z = 0,\ A_z = 50$ lb

$\Sigma F_y = 0;\ A_y - \dfrac{100\sqrt{6}}{\sqrt{6}} = 0,\ A_y = 100$ lb

$(\Sigma M_B)_z = 0;\ 8A_x - \dfrac{100\sqrt{6}}{\sqrt{6}}(2) - \dfrac{100\sqrt{6}}{\sqrt{6}}(2) = 0,\ A_x = 50$ lb

$A = \sqrt{\overline{50}^2 + \overline{100}^2 + \overline{50}^2} = \underline{122.5 \text{ lb}}$

3/88 (Dim. in mm)



$pA = 4(10^{-5})(48)(80)$
$= 0.1536\ \text{N}$

$\sum M_{AB} = 0: -P(22 - 16 \sin 15°) + 0.1536(24) = 0$

$\underline{P = 0.206\ \text{N}}$

$\sum M_{B_x} = 0: \quad A_y(42) - 0.206 \cos 15°(58) = 0$

$\underline{A_y = 0.275\ \text{N}}$

$\sum M_{A_x} = 0: \quad -B_y(42) - 0.206 \cos 15°(16) = 0$

$\underline{B_y = -0.0760\ \text{N}}$

3/89 From Prob. 2/105,

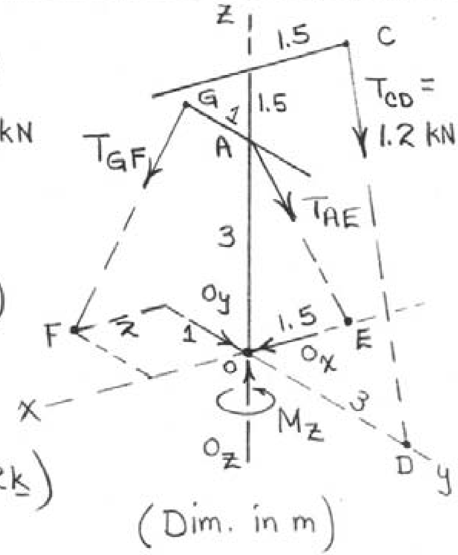
$\underline{T}_{CD} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}$ kN

$\underline{T}_{AE} = T_{AE}\dfrac{-1.5\underline{i} - 3\underline{k}}{\sqrt{1.5^2 + 3^2}}$

$= T_{AE}\left(-0.447\underline{i} - 0.894\underline{k}\right)$

$\underline{T}_{GF} = T_{GF}\dfrac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$

$= T_{GF}\left(0.555\underline{i} - 0.832\underline{k}\right)$



$\Sigma \underline{M}_O = \underline{0}: \quad \underline{OC} \times \underline{T}_{CD} + \underline{OA} \times \underline{T}_{AE} + \underline{OG} \times \underline{T}_{GF} + M_z\underline{k} = \underline{0}$

$(-1.5\underline{i} + 4.5\underline{k}) \times (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) + 3\underline{k} \times$

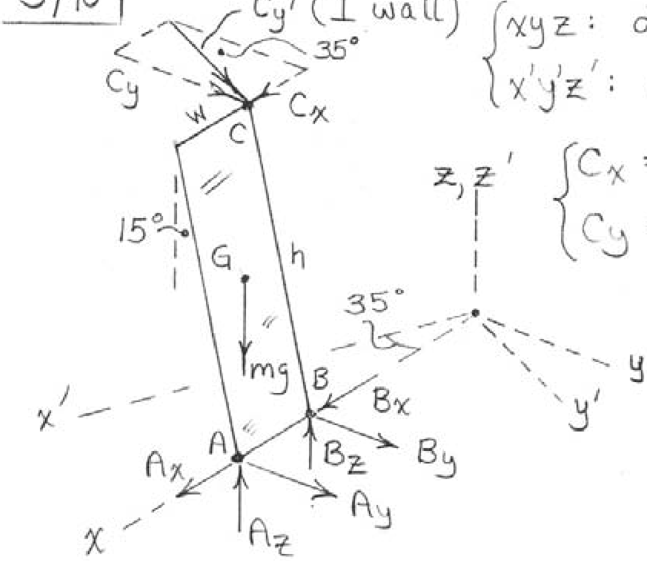
$T_{AE}(-0.447\underline{i} - 0.894\underline{k}) + (-\underline{j} + 3\underline{k}) \times T_{GF}(0.555\underline{i} - 0.832\underline{k})$

$+ M_z\underline{k} = \underline{0} \Rightarrow \begin{cases} 0.832T_{GF} - 2.89 = 0 \\ 1.664\,T_{GF} - 1.342T_{AE} = 0 \\ 0.555\,T_{GF} - 0.962 + M_z = 0 \end{cases}$

Solve to obtain $\underline{T_{GF} = 3.47 \text{ kN}, \quad T_{AE} = 4.30 \text{ kN}},$

$M_z = -0.962 \text{ kN}\cdot\text{m}$

3/90



$\begin{cases} xyz: \text{ door-based} \\ x'y'z': \text{ wall-based} \end{cases}$

$\begin{cases} C_x = C_{y'} \sin 35° \\ C_y = C_{y'} \cos 35° \end{cases}$

$\Sigma M_x = 0: \quad mg \frac{h}{2} \sin 15° - C_y h \cos 15° = 0, \quad C_y = 0.1340\, mg$

$C_{y'} = \frac{C_y}{\cos 35°} = \underline{0.1636\, mg}, \quad C_x = C_y \tan 35° = 0.0938\, mg$

$\Sigma M_{B_z} = 0: \quad C_x h \sin 15° + A_y w = 0, \quad \underline{A_y = -0.0243\, mg \frac{h}{w}}$

$\Sigma M_{B_y} = 0: \quad mg \frac{w}{2} + C_x h \cos 15° - A_z w = 0$

$\underline{A_z = mg\left(\frac{1}{2} + 0.0906 \frac{h}{w}\right)}$

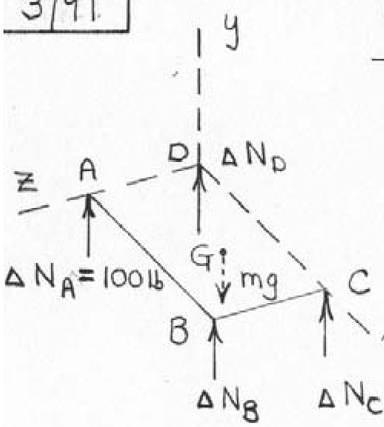
$\Sigma M_{A_z} = 0: \quad -B_y w - C_y w + C_x h \sin 15° = 0$

$\underline{B_y = mg\left(0.0243 \frac{h}{w} - 0.1340\right)}$

$\Sigma M_{A_y} = 0: \quad B_z w - mg \frac{h}{2} + C_x h \cos 15° = 0$

$\underline{B_z = mg\left(\frac{1}{2} - 0.0906 \frac{h}{w}\right)}$

3/91



Location of G does not change. Thus,

$\underline{\Delta N_D = -100 \text{ lb}}$ (preserves total rear-axle loading)

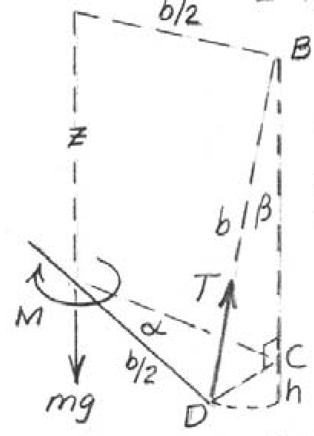
$\underline{\Delta N_B = -100 \text{ lb}}$ (preserves total right-side loading)

$\underline{\Delta N_C = 100 \text{ lb}}$ (preserves total normal force; preserves total front-axle loading)

(Note: The results for ΔN_B & ΔN_C hold only if the track (distance between tire centers) at the front is equal to that at the rear.)

3/92 $\quad \Sigma F_z = 0;\ 2T\cos\beta - mg = 0 \quad \text{- - - - - - - - - - (1)}$

$\Sigma M_z = 0;\ 2T\sin\beta\cos\frac{\alpha}{2}\left(\frac{b}{2}\right) - M = 0 \quad \text{- - - (2)}$



$\overline{CD} = 2\,\frac{b}{2}\sin\frac{\alpha}{2} = b\sin\beta,\ \beta = \frac{\alpha}{2} \quad \text{- - (3)}$

Divide (2) by (1) & substitute (3) & get

$$\frac{2T\frac{b}{2}\sin\beta\cos\beta}{2T\cos\beta} = \frac{M}{mg},\quad \sin\beta = \frac{2M}{bmg}$$

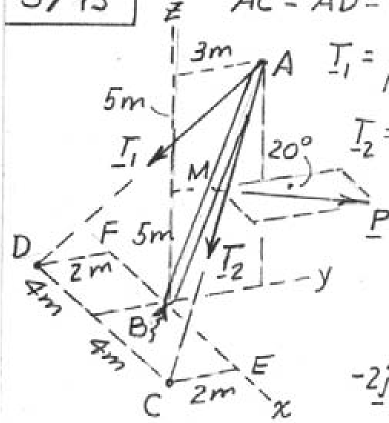
Thus $\cos\beta = \sqrt{1-\left(\frac{2M}{bmg}\right)^2}$

& $h = b(1-\cos\beta)$

so $\overline{h = b\left(1-\sqrt{1-\left(\frac{2M}{bmg}\right)^2}\right)}$

For $h \to b,\ \cos\beta \to 0,\ \sin\beta \to \pi/2$ & $\underline{M \to \frac{bmg}{2}}$

3/93



$\overline{AC} = \overline{AD} = \sqrt{4^2 + 5^2 + 10^2} = 11.87\ m$; $|\underline{P}| = 20$ kN

$\underline{T}_1 = \frac{T_1}{11.87}(-4\underline{i} - 5\underline{j} - 10\underline{k})$; $\sin 20° = 0.3420$

$\underline{T}_2 = \frac{T_2}{11.87}(4\underline{i} - 5\underline{j} - 10\underline{k})$ $\cos 20° = 0.9397$

$\Sigma \underline{M}_x = 0$; $\underline{r}_{EC} \times \underline{T}_2 \cdot \underline{i} + \underline{r}_{FD} \times \underline{T}_1 \cdot \underline{i}$

$+ \underline{r}_{BM} \times \underline{P} \cdot \underline{i} = 0$

$-2\underline{j} \times \frac{T_2}{11.87}(4\underline{i} - 5\underline{j} - 10\underline{k}) \cdot \underline{i}$

$-2\underline{j} \times \frac{T_1}{11.87}(-4\underline{i} - 5\underline{j} - 10\underline{k}) \cdot \underline{i} + (1.5\underline{j} + 5\underline{k}) \times 20(0.342\underline{i}$
$+ 0.9397\underline{j}) \cdot \underline{i} = 0$

Simplify & get $T_1 + T_2 = 55.79$ - - - - (1)

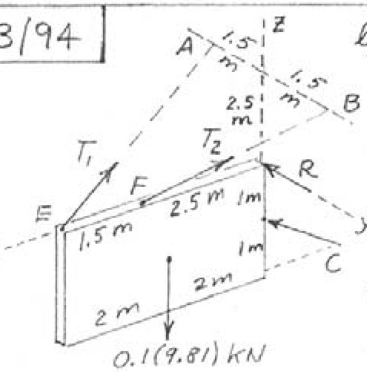
$\Sigma \underline{M}_z = 0$; $\underline{r}_{BA} \times (\underline{T}_1 + \underline{T}_2) \cdot \underline{k} + \underline{r}_{BM} \times \underline{P} \cdot \underline{k} = 0$

$(3\underline{j} + 10\underline{k}) \times \frac{1}{11.87}([-4T_1 + 4T_2]\underline{i} + [-5T_1 - 5T_2]\underline{j} - [10T_1 + 10T_2]\underline{k}) \cdot \underline{k}$

Simplify to $T_1 - T_2 = 10.14$ - - - - (2) $-20(0.3420)(1.5) = 0$

Solve (1) & (2) & get $\underline{T_1 = 33.0\text{ kN}}$, $T_2 = \underline{22.8\text{ kN}}$

3/94



$\ell_1 = \overline{AE} = \sqrt{4^2 + 1.5^2 + 2.5^2} = 4.95\ m$

$\ell_2 = \overline{BF} = \sqrt{2.5^2 + 1.5^2 + 2.5^2} = 3.84\ m$

$\underline{T}_1 = \frac{T_1}{\ell_1}(-4\underline{i} - 1.5\underline{j} + 2.5\underline{k})$

$\underline{T}_2 = \frac{T_2}{\ell_2}(-2.5\underline{i} + 1.5\underline{j} + 2.5\underline{k})$

Note: $\underline{T}_1$, $\underline{T}_2$, $\underline{R}$, and weight all pass through x-axis, so $\underline{C}$ must also. Thus $C_y = 0$

$\Sigma M_{AB} = 0;\ C_x(3.5) - 9.81(2) = 0,\quad C_x = 0.561\ kN$

$\Sigma M_z = 0;\ 4\underline{i} \times \frac{T_1}{\ell_1}(-1.5\underline{j}) + 2.5\underline{i} \times \frac{T_2}{\ell_2}(1.5\underline{j}) = \underline{0},\quad 8T_1/\ell_1 = 5T_2/\ell_2$

$\Sigma F_x = 0;\ -\frac{T_1}{\ell_1}(4) - \frac{T_2}{\ell_2}(2.5) + 0.561 = 0,\quad 8T_1/\ell_1 + 5T_2/\ell_2 = 1.121\ kN$

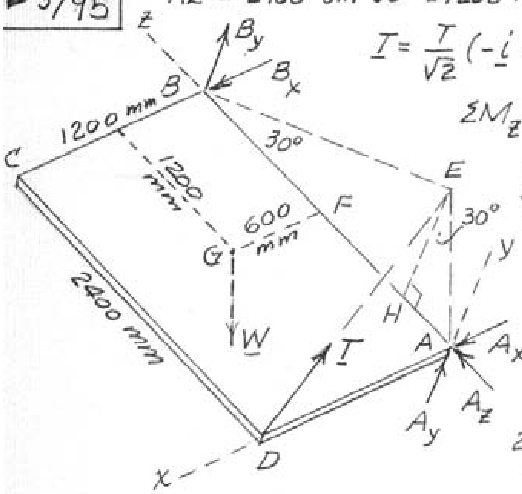
solve & get $T_1 = 1.121\,\ell_1/16 = \underline{0.347\ kN}$, $T_2 = \frac{1.121}{10}\ell_2 = \underline{0.431\ kN}$

$\Sigma F_y = 0;\ \frac{1.121}{16}\ell_1\frac{1.5}{\ell_1} - \frac{1.121}{10}\ell_2\frac{1.5}{\ell_2} + R = 0,\quad \underline{R = 0.0631\ kN}$

$\Sigma F_z = 0;\ \frac{1.121}{16}\ell_1\frac{2.5}{\ell_1} + \frac{1.121}{10}\ell_2\frac{2.5}{\ell_2} + C_z - 0.981 = 0,\quad C_z = 0.526\ kN$

Thus $C = \sqrt{(0.561)^2 + (0.526)^2} = \underline{0.768\ kN}$

▶ 3/95 $\overline{AE} = 2400 \sin 30^\circ = 1200$ mm, $\overline{AH} = 600$ mm, $\overline{HE} = 1200 \cos 30^\circ$
$= 600\sqrt{3}$ mm



$\underline{T} = \frac{T}{\sqrt{2}}\left(-\underline{i} + \frac{\sqrt{3}}{2}\underline{j} + \frac{1}{2}\underline{k}\right)$

$\Sigma M_z = 0;\ T_y \overline{AD} - W_y \overline{GF} = 0$

$\frac{T\sqrt{3}}{2\sqrt{2}}(1200) = 40(9.81)\frac{\sqrt{3}}{2}(600)$

$T = 20(9.81)\sqrt{2} = \underline{277\ N}$

$\Sigma M_x = 0;\ -B_y \overline{AB} + W_y \overline{FA} = 0$

$2400 B_y = 40(9.81)\frac{\sqrt{3}}{2}(1200)$

$B_y = 98.1\sqrt{3} = 169.9$ N

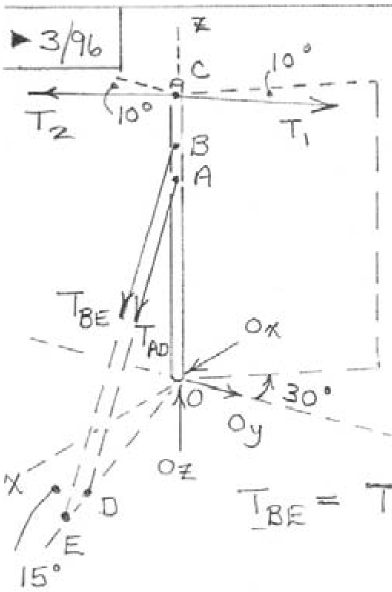
$\Sigma M_y = 0;\ B_x \overline{AB} + W_z \overline{GF} - T_z \overline{AD} = 0$

$2400 B_x + 40(9.81)0.5(600) - \frac{277}{2\sqrt{2}}(1200) = 0$

$B_x = 0$

$B = B_y = \underline{169.9\ N}$ ($B_x = 0$ can be obtained by inspection by noting $\Sigma M_{AE} = 0$ eliminates all terms except $B_x \overline{BE}$ so $B_x = 0$)

▶ 3/96



$$\begin{cases} \overline{OA} = 9\text{ m}, \quad \overline{OB} = 11\text{ m}, \quad \overline{OC} = 13\text{ m} \\ \overline{OD} = 8\text{ m}, \quad \overline{OE} = 10\text{ m} \end{cases}$$

$$\underline{T}_1 = 950[-\sin 30° \cos 10° \underline{i} + \cos 30° \cos 10° \underline{j} - \sin 10° \underline{k}]$$

$$\underline{T}_2 = 950[-\cos 10° \underline{j} - \sin 10° \underline{k}]$$

(in N)

$$\underline{T}_{BE} = T\left[\frac{10\cos 15° \underline{i} + 10 \sin 15° \underline{j} - 11\underline{k}}{\sqrt{221}}\right]$$

$$\underline{T}_{AD} = T\left[\frac{8\cos 15° \underline{i} + 8 \sin 15° \underline{j} - 9\underline{k}}{\sqrt{145}}\right]$$

$\Sigma \underline{M}_o = \underline{0}$: $\quad \underline{r}_{oc} \times (\underline{T}_1 + \underline{T}_2) + \underline{r}_{OB} \times \underline{T}_{BE} + \underline{r}_{OA} \times \underline{T}_{AD} = \underline{0}$

Set $\underline{r}_{oc} = 13\underline{k}$ m, $\underline{r}_{OB} = 11\underline{k}$ m, & $\underline{r}_{OA} = 9\underline{k}$ m, carry out cross products, and set either the $\underline{i}$- component or the $\underline{j}$- component to zero to obtain $\underline{T = 471 \text{ N}}$.

$\Sigma F_x = 0$: $\quad -950 \sin 30° \cos 10° + 471 \dfrac{10 \cos 15°}{\sqrt{221}} + 471 \dfrac{8 \cos 15°}{\sqrt{145}} + O_x = 0$

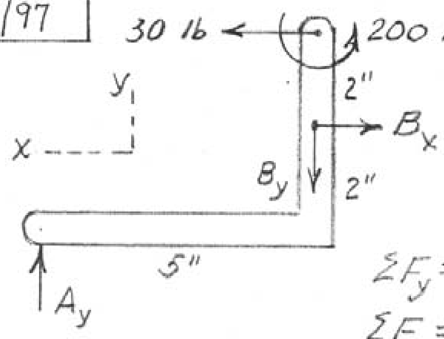
$$O_x = -140.0 \text{ N}$$

$\Sigma F_y = 0$: $\quad 950 \cos 30° \cos 10° - 950 \cos 10° + 471 \dfrac{10 \sin 15°}{\sqrt{221}} + 471 \dfrac{8 \sin 15°}{\sqrt{145}} + O_y = 0$

$$O_y = -37.5 \text{ N}$$

Then $O = \sqrt{O_x^2 + O_y^2} = \underline{144.9 \text{ N}}$

3/97



$\Sigma M_B = 0;$

$200 + 30(2) - 5A_y = 0$

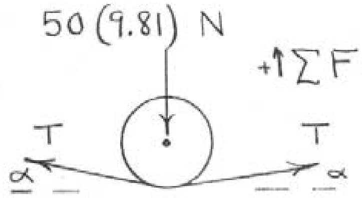
$A_y = 52 \text{ lb}$

$\Sigma F_y = 0; \quad B_y = 52 \text{ lb}$

$\Sigma F_x = 0; \quad B_x = 30 \text{ lb}$

$B = \sqrt{(30)^2 + (52)^2} = \underline{60.0 \text{ lb}}$

3/98 Isolate wheel of unicycle:



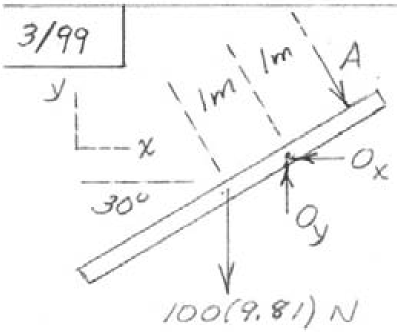
$$\alpha = \tan^{-1}\left(\frac{0.075}{9}\right) = 0.477^\circ$$

$$+\uparrow \Sigma F = 0: \quad 2T\sin\alpha - 50(9.81) = 0$$

$$T = 29\,400 \text{ N}$$

or $\underline{T = 29.4 \text{ kN}}$

3/99



$\Sigma M_O = 0;\ A(1) - (981\cos 30°)1 = 0$

$A = 850\text{ N}$

$\Sigma F_x = 0;\ O_x - 850\sin 30° = 0$

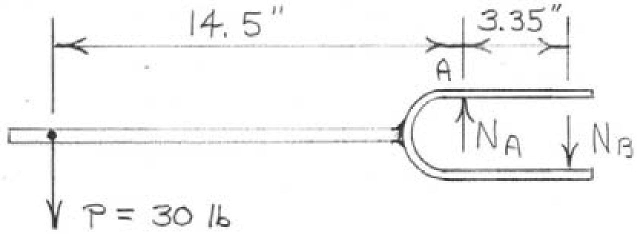
$O_x = 425\text{ N}$

$\Sigma F_y = 0;\ O_y - 850\cos 30° - 981 = 0$

$O_y = 1717\text{ N}$

$O = \sqrt{\overline{425}^2 + \overline{1717}^2} = \underline{1769\text{ N}}$

3/100



$\circlearrowleft \Sigma M_A = 0: \quad 30(14.5) - N_B(3.35) = 0$

$N_B = 129.9 \text{ lb}$

$\uparrow + \Sigma F = 0: \quad -30 + N_A - 129.9 = 0$

$N_A = 159.9 \text{ lb}$

So the forces applied to the stud are

$$\begin{cases} N_A = 159.9 \text{ lb} \downarrow \\ N_B = 129.9 \text{ lb} \uparrow \end{cases}$$

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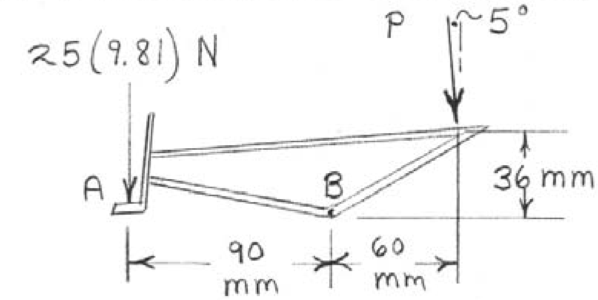
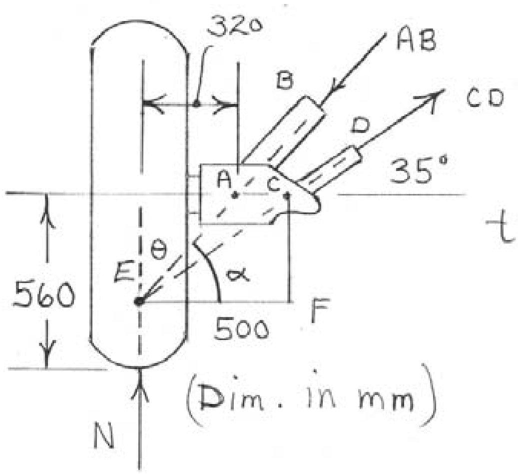


$$\circlearrowleft + \sum M_B = 0: \quad 25(9.81)(90) - P\cos 5°(60) - P\sin 5°(36) = 0$$

$$\underline{P = 351\ \text{N}}$$

Assumptions: The weight of the panel acts at the 90-mm dimension as shown above; panel does not slip.

3/102 The three forces must be concurrent at point E.



$\overline{CF} = 500 \tan 35°$

$= 350 \text{ mm}$

$\tan\theta = \dfrac{320}{350}$

$\theta = 42.4°$

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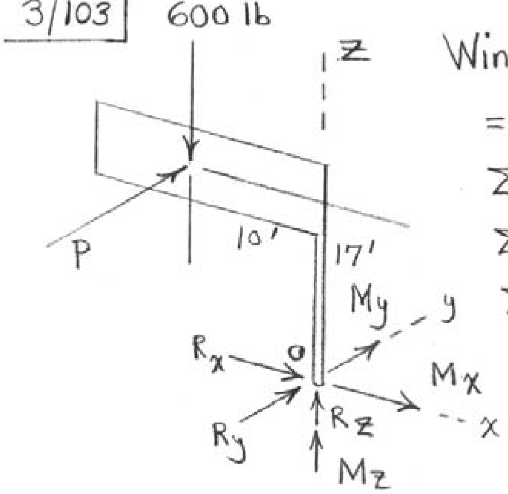

Wind force $P = pA$

$= 17.5(6)(12) = 1260 \text{ lb}$

$\Sigma F_x = 0 \Rightarrow R_x = 0$

$\Sigma F_y = 0 \Rightarrow R_y = -1260 \text{ lb}$

$\Sigma F_z = 0 \Rightarrow R_z = 600 \text{ lb}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$= \underline{1396 \text{ lb}}$

$\Sigma \underline{M}_O = \underline{0}: \quad \underline{M} + (-10\underline{i} + 17\underline{k}) \times (1260\underline{j} - 600\underline{k}) = \underline{0}$

$\Rightarrow \underline{M} = 21{,}400\,\underline{i} + 6000\,\underline{j} + 12{,}600\,\underline{k} \text{ lb-ft}$

$\underline{M = 25{,}600 \text{ lb-ft}}$

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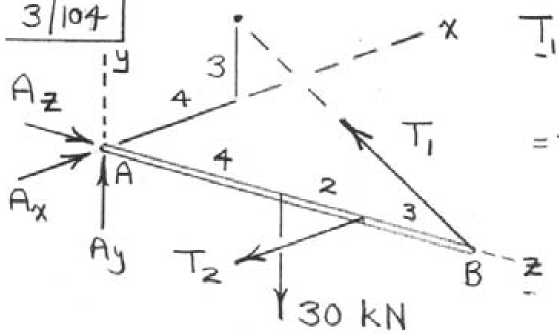

$$\underline{T}_1 = T_1\left[\frac{4\underline{i} + 3\underline{j} - 9\underline{k}}{\sqrt{4^2+3^2+9^2}}\right]$$

$$= T_1\left(0.389\underline{i} + 0.291\underline{j} - 0.874\underline{k}\right)$$

$$\underline{T}_2 = -T_2\,\underline{i}$$

$\Sigma M_x = 0: \; 30(4) - 0.291\,T_1(9) = 0, \quad \underline{T_1 = 45.8 \text{ kN}}$

$\Sigma M_y = 0: \; -T_2(6) + 0.389\,T_1(9) = 0, \quad \underline{T_2 = 26.7 \text{ kN}}$

$\Sigma \underline{F} = \underline{0} : \; \underline{A} + 45.8\left(0.389\underline{i} + 0.291\underline{j} - 0.874\underline{k}\right)$

$-26.7\underline{i} - 30\underline{j} = 0$

$\underline{A} = 8.89\underline{i} + 16.67\underline{j} + 40.0\underline{k} \quad \text{kN}$

$\underline{A = 44.2 \text{ kN}}$

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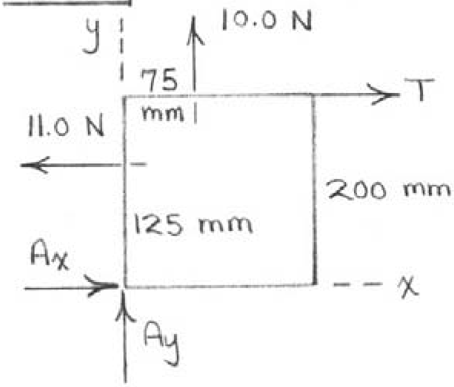

Isolate entire system of plate, pulleys, & tape.

$\circlearrowleft^{+} \sum M_A = 0: \quad T(200) - 10.0(75) - 11.0(125) = 0$

$\underline{T = 10.62 \text{ N}}$

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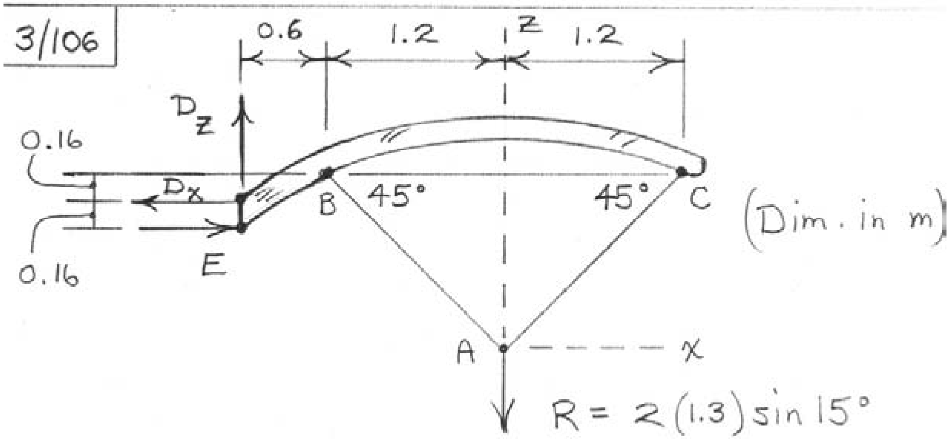

$R = 2(1.3)\sin 15°$
$= 0.673$ kN

$\Sigma F_z = 0: \ D_z - 0.673 = 0, \quad D_z = 0.673$ kN

$\circlearrowleft + \Sigma M_E = 0: \ 0.16 D_x - 0.673(1.8) = 0$

$D_x = 7.57$ kN

$D = \sqrt{0.673^2 + 7.57^2} = \underline{7.60 \text{ kN}}$

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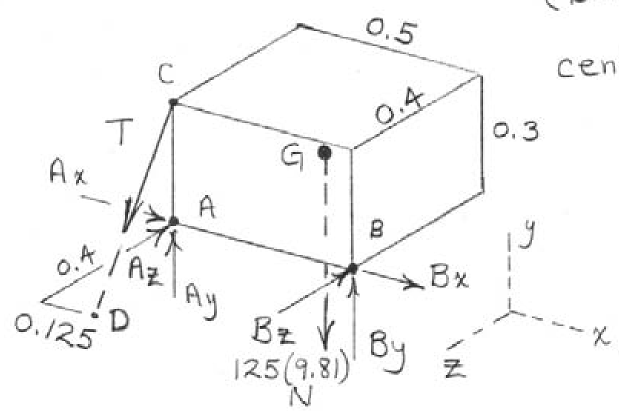

$$\underline{T} = T\,\underline{n}_{CD} = T\,\frac{\overline{CD}}{\overline{CD}}$$

$$= T\left[\frac{0.125\,\underline{i} - 0.3\,\underline{j} + 0.4\,\underline{k}}{\sqrt{0.125^2 + 0.3^2 + 0.4^2}}\right]$$

$$= T\left(0.243\,\underline{i} - 0.582\,\underline{j} + 0.776\,\underline{k}\right)$$

$$\Sigma M_{A_x} = 0: \; 0.776T(0.3) - 125(9.81)(0.2) = 0$$

$$\underline{T = 1053 \text{ N}}$$

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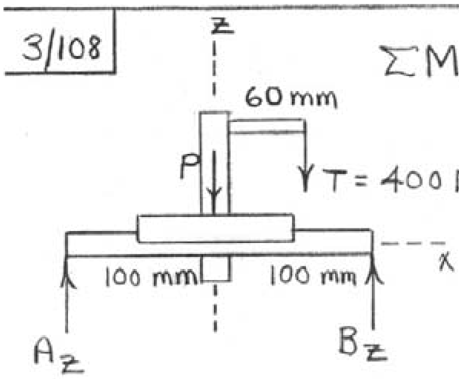

$\Sigma M_x = 0$: $P(200) - 400(120\cos 30^\circ) = 0$

$P = 208$ N

$\Sigma M_{A_y} = 0$: $208(100)$

$+ 400(160) - B_z(200) = 0$

$B_z = 424$ N

$\Sigma F_z = 0$: $A_z + 424 - 208 - 400 = 0$, $A_z = 183.9$ N

Because $A_y = B_y = 0$, $\underline{A = A_z = 183.9 \text{ N}}$, $\underline{B = B_z = 424 \text{ N}}$

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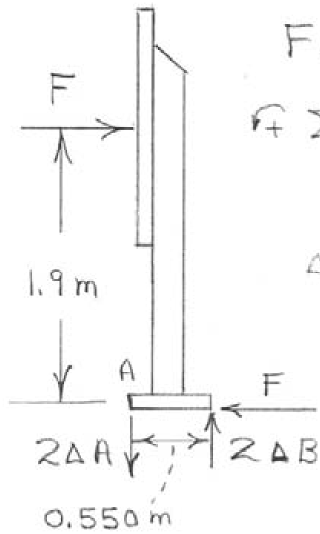


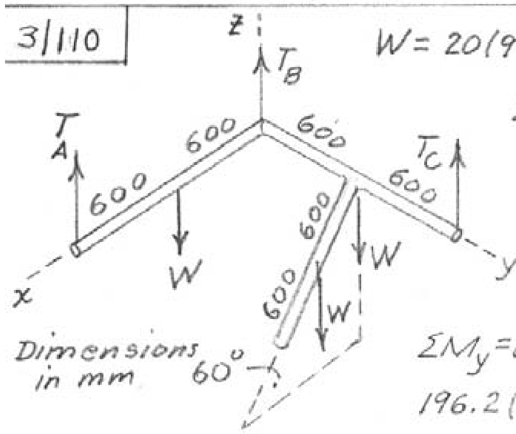
$F = pA = 175(1.6 \cdot 3) = 840$ N

From $+\!\uparrow \Sigma F = 0$, $\Delta A = \Delta B$

$\circlearrowleft\!+ \; \Sigma M_A = 0: \; -840(1.9) + 2\Delta B(0.550) = 0$

$\Delta B = \Delta A = \underline{1451 \text{ N}}$

3/110



$W = 20(9.81) = 196.2\ N$

$\Sigma M_x = 0;$

$1200 T_C - 2(196.2)600 = 0$

$\underline{T_C = 196.2\ N}$

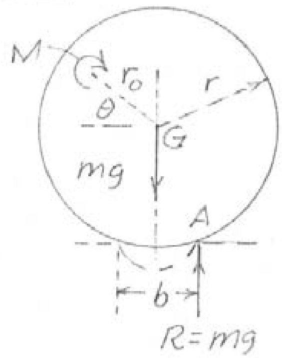
$\Sigma M_y = 0;$

$196.2(600 + 600 \cos 60°) - 1200 T_A = 0$

$\underline{T_A = 147.2\ N}$

$\Sigma F_z = 0;\ T_B + 147.2 + 196.2 - 3(196.2) = 0,\quad \underline{T_B = 245\ N}$

3/111



$\Sigma M_A = 0;\ M - mg\frac{b}{2} = 0$

$\underline{M = \frac{1}{2} mgb}$

independent of r_o, r_i & θ

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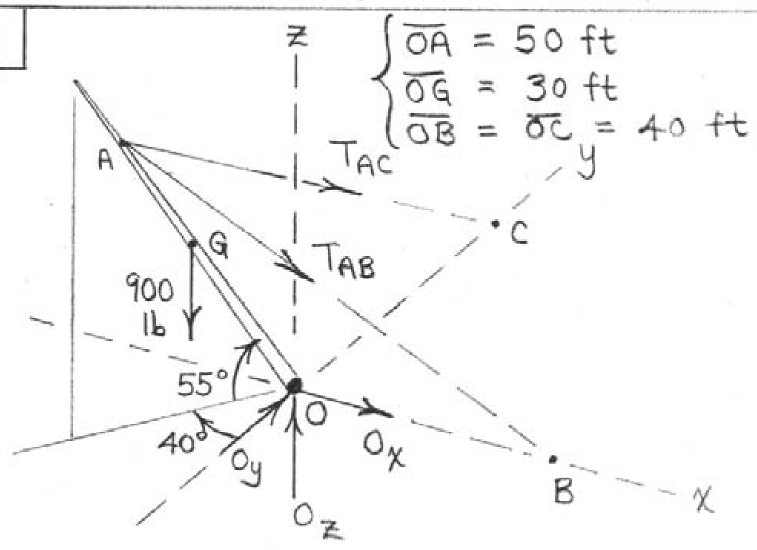

Coordinates of A: $50(-\cos 55° \sin 40°, -\cos 55° \cos 40°, \sin 55°) = (-18.43, -22.0, 41.0)$ ft

Coordinates of G: $30(-\cos 55° \sin 40°, -\cos 55° \cos 40°, \sin 55°) = (-11.06, -13.18, 24.6)$ ft

$$\underline{T}_{AB} = T_{AB}\left[\frac{(18.43+40)\underline{i} + 22.0\underline{j} - 41.0\underline{k}}{\sqrt{(18.43+40)^2 + 22.0^2 + 41.0^2}}\right]$$

$$= T_{AB}\left[0.783\underline{i} + 0.294\underline{j} - 0.549\underline{k}\right]$$

$$T_{AC} = T_{AC}\left[\frac{18.43\underline{i} + (22.0+40)\underline{j} - 41.0\underline{k}}{\sqrt{18.43^2 + (22.0+40)^2 + 41.0^2}}\right]$$

$$= T_{AC}\left[0.241\underline{i} + 0.810\underline{j} - 0.535\underline{k}\right]$$

$$\Sigma F_x = 0: \quad 0.783T_{AB} + 0.241T_{AC} + O_x = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad 0.294T_{AB} + 0.810T_{AC} + O_y = 0 \qquad (2)$$

$$\Sigma F_z = 0: \quad -0.549T_{AB} - 0.535T_{AC} + O_z - 900 = 0 \qquad (3)$$

$$\Sigma M_{BC} = 0: \quad \Sigma \underline{M}_B \cdot \underline{n}_{BC} = 0:$$

$$\left\{-40\underline{i} \times O_z\underline{k} + \left[(-40-11.06)\underline{i} - 13.18\underline{j}\right] \times \left[-900\underline{k}\right]\right\} \cdot \left(-\frac{\sqrt{2}}{2}\underline{i} + \frac{\sqrt{2}}{2}\underline{j}\right) = 0$$

or $$(40\,O_z - 46{,}000)\frac{\sqrt{2}}{2} - 11{,}860\frac{\sqrt{2}}{2} = 0 \qquad (4)$$

$$\Sigma M_{O_y} = 0: \quad 0.549\,T_{AB}(40) - 900(11.06) = 0 \qquad (5)$$

Solve Eqs. (1)-(5) in reverse order to obtain

$$\begin{cases} O_x = -489 \text{ lb} & T_{AB} = 454 \text{ lb} \\ O_y = -582 \text{ lb} & T_{AC} = 554 \text{ lb} \\ O_z = 1445 \text{ lb} & \end{cases}$$

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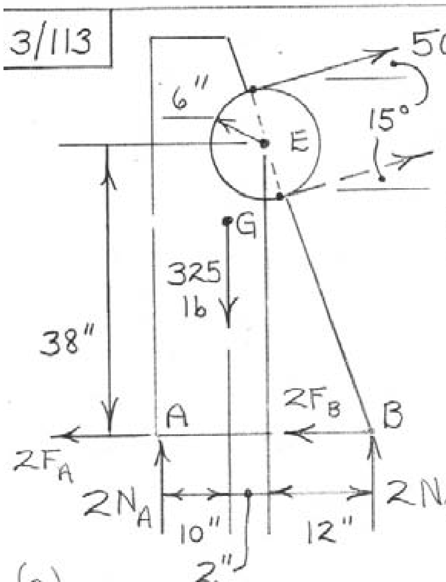

Because of symmetry, $N_A = N_D$ & $N_B = N_C$.
For (a), replace 50-lb force by a 50-lb force & 300 lb-in. CW couple at E (CCW for (b)).

(a)

$\circlearrowleft \Sigma M_A = 0: \; -325(10) + 2N_B(24) - 50\cos 15°(38)$
$+ 50 \sin 15° (12) - 300 = 0, \; \underline{N_B = N_C = 109.0 \text{ lb}}$

$+\uparrow \Sigma F = 0: \; 2(109.0) + 2N_A - 325 + 50 \sin 15° = 0$

$\underline{N_A = N_D = 47.1 \text{ lb}}$

(b) Reverse the sign on the 300 lb-in. couple & obtain
$$\begin{cases} N_B = N_C = 96.5 \text{ lb} \\ \underline{N_A = N_D = 59.6 \text{ lb}} \end{cases}$$

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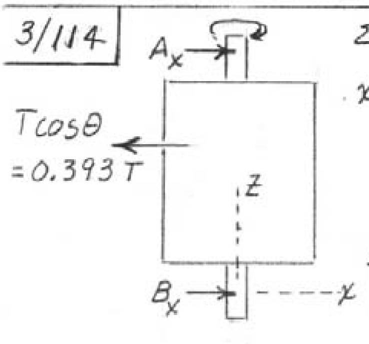


$\Sigma M_z = 0;\ 120 - 0.150T = 0,\ T = 800\text{ N}$

$x\text{-}z;\ \Sigma M_B = 0;\ 0.393(800)(0.360) - 0.700A_x = 0;\ A_x = 161.6\text{ N}$

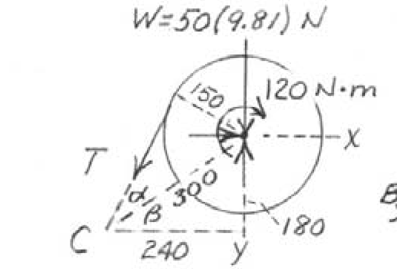
$\Sigma F_x = 0;\ B_x + 161.6 - 0.393(800) = 0$

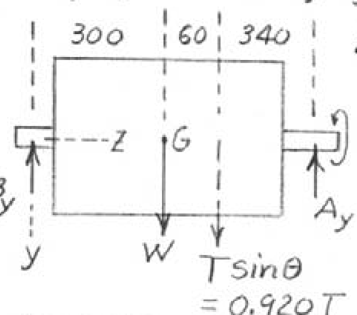
$B_x = 152.6\text{ N}$

$y\text{-}z;\ \Sigma M_B = 0;\ 0.7A_y - 0.920(800)(0.360) - 50(9.81)(0.300) = 0;\ A_y = 588.6\text{ N}$

$\Sigma F_y = 0;\ W + T\sin\theta - A_y - B_y = 0,$

$B_y = 637.6\text{ N}$





$\beta = \tan^{-1}\dfrac{180}{240} = 36.9°$

$\alpha = \sin^{-1}\dfrac{150}{300} = 30°$

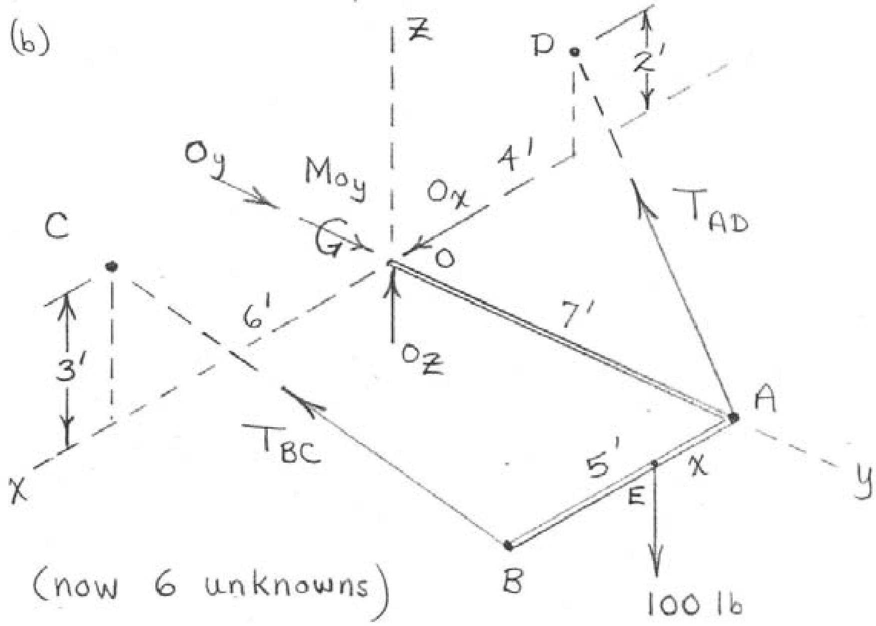
$\theta = \alpha + \beta = 66.9°$

$A = \sqrt{(161.6)^2 + (588.6)^2} = \underline{610\text{ N}}$

$B = \sqrt{(152.6)^2 + (637.6)^2} = \underline{656\text{ N}}$

▸3/115 (a) There are 5 unknown constraint forces. The bar is free to rotate about a line which passes through point O and through which the lines of action of both tension forces pass.

(b)



(now 6 unknowns)

$$\underline{T}_{AD} = T_{AD}\left[\frac{-4\underline{i} - 7\underline{j} + 2\underline{k}}{\sqrt{69}}\right]$$

$$\underline{T}_{BC} = T_{BC}\left[\frac{\underline{i} - 7\underline{j} + 3\underline{k}}{\sqrt{59}}\right]$$

$$\begin{cases} \Sigma F_x = 0: \ O_x - \frac{4}{\sqrt{69}} T_{AD} + \frac{1}{\sqrt{59}} T_{BC} = 0 & (1) \\ \Sigma F_y = 0: \ O_y - \frac{7}{\sqrt{69}} T_{AD} - \frac{7}{\sqrt{59}} T_{BC} = 0 & (2) \\ \Sigma F_z = 0: \ O_z + \frac{2}{\sqrt{69}} T_{AD} + \frac{3}{\sqrt{59}} T_{BC} - 100 = 0 & (3) \\ \Sigma M_{O_x} = 0: \ 7\left(\frac{2}{\sqrt{69}} T_{AD}\right) + 7\left(\frac{3}{\sqrt{59}} T_{BC}\right) - 7(100) = 0 & (4) \\ \Sigma M_{O_y} = 0: \ -5\left(\frac{3}{\sqrt{59}} T_{BC}\right) + M_{O_y} + 100x = 0 & (5) \\ \Sigma M_{O_z} = 0: \ 7\left(\frac{4}{\sqrt{69}} T_{AD}\right) - 7\left(\frac{1}{\sqrt{59}} T_{BC}\right) - 5\left(\frac{7}{\sqrt{59}} T_{BC}\right) = 0 & (6) \end{cases}$$

Solve Eqs. (1)–(6) over $0.5 \le x \le 4.5$ ft and discover that three of the requested quantities are constant:

$$\begin{cases} O_x = 83.3 \text{ lb} \\ O_y = 292 \text{ lb} \\ O_z = 0 \end{cases}$$

$\underline{T_{AD} = 208 \text{ lb} = \text{constant}}$

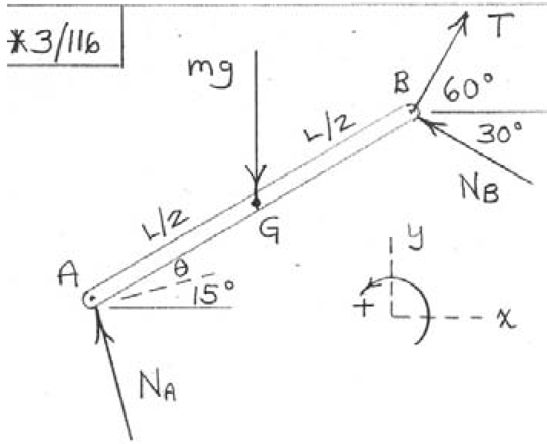
$\underline{T_{BC} = 128.0 \text{ lb} = \text{constant}}$

$\underline{O = \sqrt{O_x^2 + O_y^2 + O_z^2} = 303 \text{ lb} = \text{constant}}$

and $\underline{M_{O_y} = -100x + 250}$ (in lb-ft if x in ft)

(Note that O_y could have been obtained from $\Sigma M_{CD} = 0$ & O_z from $\Sigma M_{AB} = 0$)

*3/116

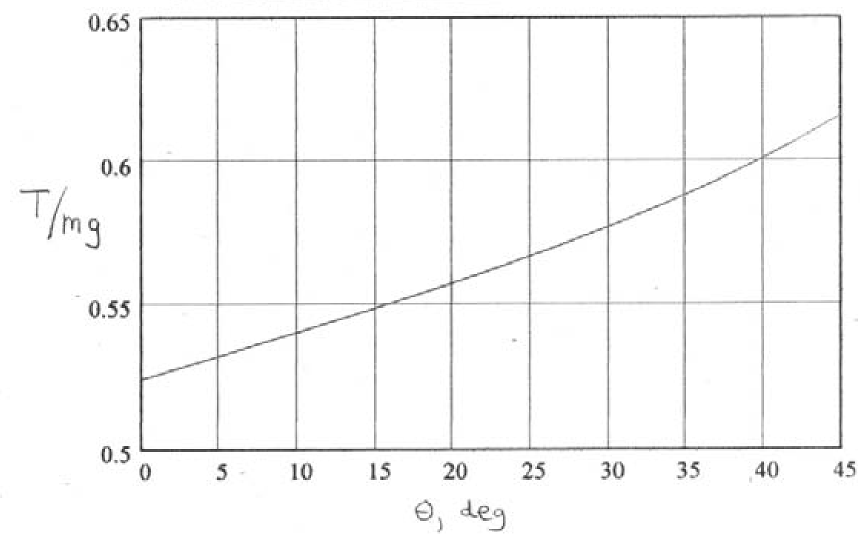


$$\Sigma F_x = 0: -N_A \sin 15° - N_B \cos 30° + T \sin 30° = 0$$

$$\Sigma F_y = 0: \quad N_A \cos 15° + N_B \sin 30° + T \cos 30° - mg = 0$$

$$\Sigma M_B = 0: \quad mg \frac{L}{2} \cos(\theta + 15°) - N_A \cos\theta\,(L) = 0$$

Solving, $T = mg \dfrac{\frac{\sqrt{3}}{2} \cos\theta - \frac{\sqrt{2}}{4} \cos(\theta + 15°)}{\cos\theta}$



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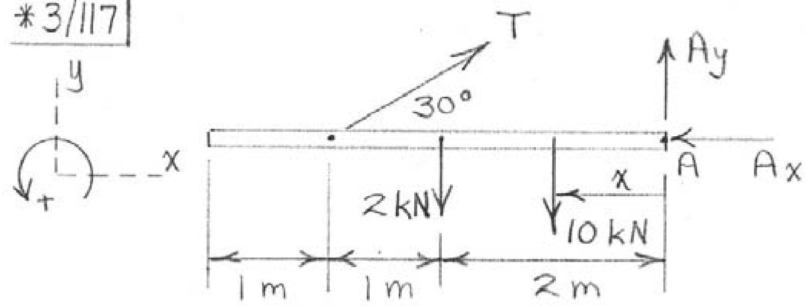


(Weight of beam = $200(10)/1000 = 2$ kN)

$\Sigma M_A = 0: \quad 10x + 2(2) - T\sin 30°(3) = 0$

$$T = \frac{2}{3}(10x + 4) \quad (\text{in kN})$$

$\Sigma F_x = 0: \quad T\cos 30° - A_x = 0$

$$A_x = \frac{1}{\sqrt{3}}(10x + 4)$$

$\Sigma F_y = 0: \quad T\sin 30° - 2 - 10 + A_y = 0$

$$A_y = \frac{1}{3}(-10x + 32)$$

$R = \{A_x^2 + A_y^2\}^{1/2} = \frac{1}{3}\{400x^2 - 400x + 1072\}^{1/2}$

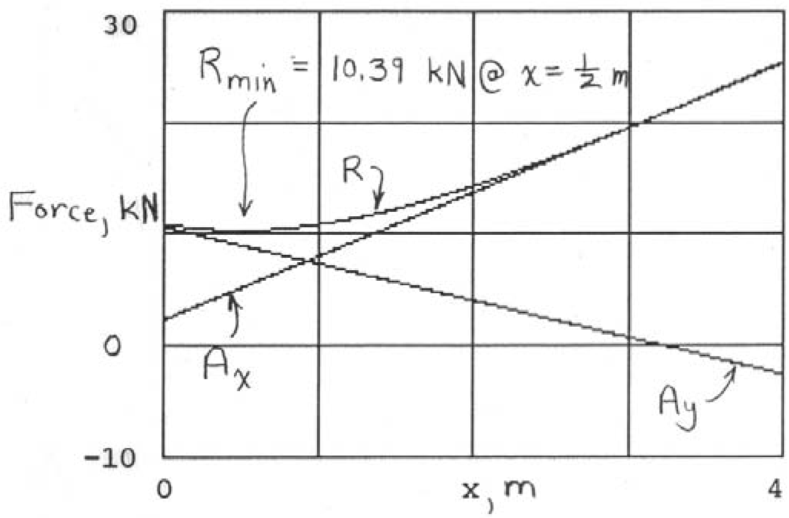
Set $\frac{dR^2}{dx} = 0$: $\quad 800x - 400 = 0$

$$\underline{x = \tfrac{1}{2} \text{ m}}$$

$$R_{min} = \frac{1}{3}\left\{400\left(\tfrac{1}{2}\right)^2 - 400\left(\tfrac{1}{2}\right) + 1072\right\}^{1/2}$$

$$= \underline{10.39 \text{ kN}} \qquad (b)$$

Plot of A_x, A_y, and R:



(a)

(c)

$\underline{R_{max} = 24.3 \text{ kN @ } x = 3.8 \text{ m}}$ is the value of R which must be used for the design of the pin at A.

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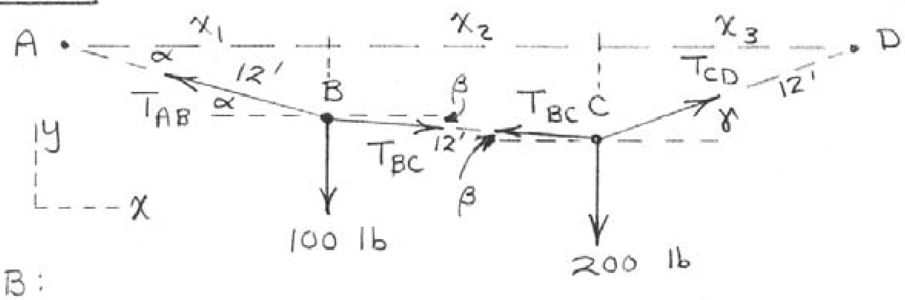

B:

$$\Sigma F_x = 0: \quad -T_{AB}\cos\alpha + T_{BC}\cos\beta = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad T_{AB}\sin\alpha - T_{BC}\sin\beta - 100 = 0 \qquad (2)$$

C:

$$\Sigma F_x = 0: \quad -T_{BC}\cos\beta + T_{CD}\cos\gamma = 0 \qquad (3)$$

$$\Sigma F_y = 0: \quad T_{BC}\sin\beta + T_{CD}\sin\gamma - 200 = 0 \qquad (4)$$

$$\cos\alpha = \frac{x_1}{12}, \quad \cos\beta = \frac{x_2}{12}, \quad \cos\gamma = \frac{x_3}{12}$$

So $\quad 12\cos\alpha + 12\cos\beta + 12\cos\gamma = 35 \qquad (5)$

$\quad \sin\alpha + \sin\beta = \sin\gamma$ (from figure) (6)

Solve numerically:

$$\begin{cases} \alpha = 14.44^\circ \\ \beta = 3.57^\circ \\ \gamma = 18.16^\circ \end{cases} \qquad \begin{aligned} T_{AB} &= 529 \text{ lb} \\ T_{BC} &= 513 \text{ lb} \\ T_{CD} &= 539 \text{ lb} \end{aligned}$$

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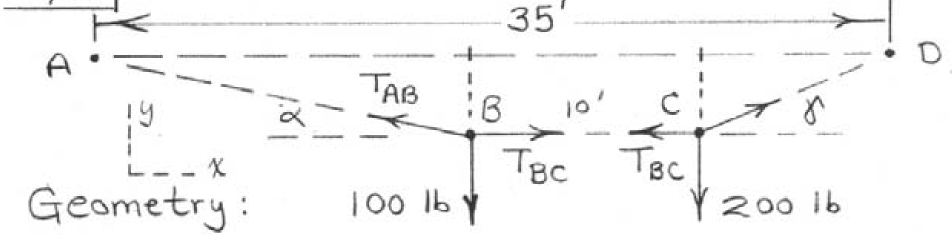


Geometry:

$$\overline{AB} + \overline{BC} + \overline{CD} = 36 \text{ ft} \quad (1)$$

$$\overline{AB}\cos\alpha + \overline{BC} + \overline{CD}\cos\gamma = 35 \text{ ft} \quad (2)$$

$$\overline{AB}\sin\alpha = \overline{CD}\sin\gamma \quad (3)$$

Equilibrium:

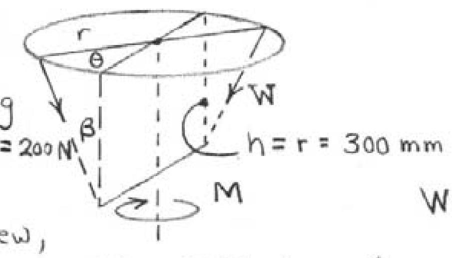
(B) $\begin{cases} \Sigma F_x = 0: \; -T_{AB}\cos\alpha + T_{BC} = 0 & (4) \\ \Sigma F_y = 0: \; T_{AB}\sin\alpha - 100 = 0 & (5) \end{cases}$

(C) $\begin{cases} \Sigma F_x = 0: \; -T_{BC} + T_{CD}\cos\gamma = 0 & (6) \\ \Sigma F_y = 0: \; T_{CD}\sin\gamma - 200 = 0 & (7) \end{cases}$

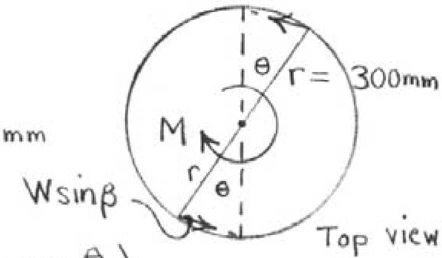
With $\overline{BC}$ set to 10 ft, solve 7 equations in 7 unknowns & obtain

$\overline{AB} = 17.01$ ft	$\alpha = 11.47°$	$T_{AB} = 503$ lb
$\overline{CD} = 8.99$ ft	$\gamma = 22.1°$	$T_{BC} = 493$ lb
		$T_{CD} = 532$ lb

*3/120



$W = mg$
$= 20(10) = 200\text{ N}$

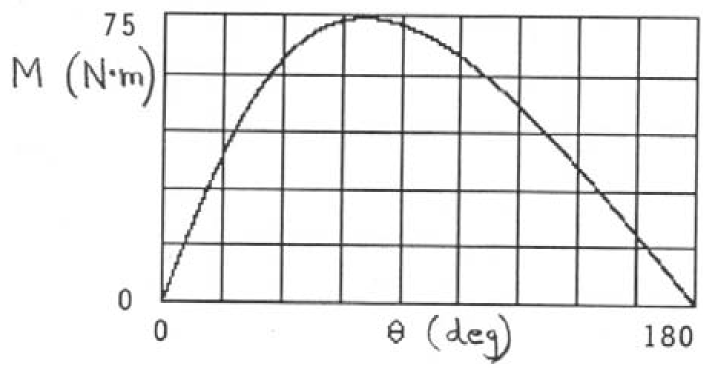


Top view,

$\Sigma M = 0: \quad M = 2W \sin\beta \left(r \cos\frac{\theta}{2}\right)$

where $\quad \sin\beta = 2r\sin\frac{\theta}{2} \Big/ \sqrt{\left(2r\sin\frac{\theta}{2}\right)^2 + h^2}$

With $h = r$: $\quad M = 2Wr \dfrac{\sin\theta}{\sqrt{4\sin^2\frac{\theta}{2}+1}} = 120\dfrac{\sin\theta}{\sqrt{4\sin^2\frac{\theta}{2}+1}}\ \text{N·m}$



$M_{max} = 74.2\ \text{N·m} \ @\ \theta = 67.5°$

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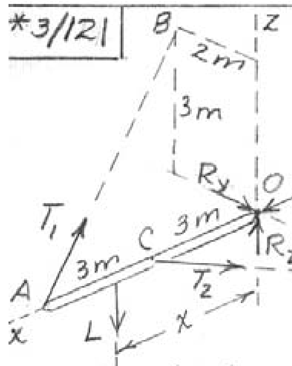


$\overline{AB} = \sqrt{6^2+2^2+3^2} = 7\ m$, $\overline{CD} = \sqrt{3^2+4^2} = 5\ m$

$\underline{T}_1 = \frac{T_1}{7}(-6\underline{i}-2\underline{j}+3\underline{k})$, $\underline{T}_2 = \frac{T_2}{5}(-3\underline{i}+4\underline{j})$, $\underline{L} = -L\underline{k}$

$\underline{r}_1 = 6\underline{i}$, $\underline{r}_2 = 3\underline{i}$, $\underline{r} = x\underline{i}$

$\Sigma \underline{M}_O = \underline{0}: \underline{r}_1 \times \underline{T}_1 + \underline{r}_2 \times \underline{T}_2 + \underline{r} \times \underline{L} = \underline{0}$

$6\underline{i} \times \frac{T_1}{7}(-6\underline{i}-2\underline{j}+3\underline{k})$
$+ 3\underline{i} \times \frac{T_2}{5}(-3\underline{i}+4\underline{j}) + x\underline{i} \times (-L\underline{k}) = 0$

Expand: $\frac{6}{7}T_1(-2\underline{k}-3\underline{j}) + \frac{3}{5}T_2(4\underline{k}) + Lx\underline{j} = \underline{0}$

$-\frac{12}{7}T_1 + \frac{12}{5}T_2 = 0$, $-\frac{18}{7}T_1 + Lx = 0$, so $T_1 = \frac{7}{18}Lx$, $T_2 = \frac{5}{18}Lx$

$\Sigma F_x = 0$; $R_x - \frac{3}{5}T_2 - \frac{6}{7}T_1 = 0$, $R_x = \frac{3}{5}\frac{5}{18}Lx + \frac{6}{7}\frac{7}{18}Lx = \frac{1}{2}Lx$

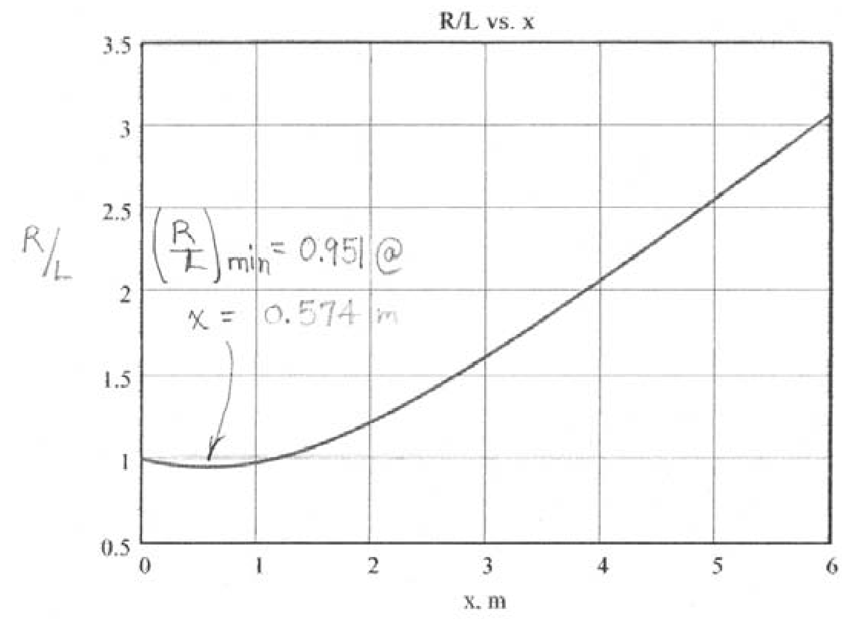
$\Sigma F_y = 0$; $R_y + \frac{4}{5}T_2 - \frac{2}{7}T_1 = 0$, $R_y = -\frac{4}{5}\frac{5}{18}Lx + \frac{2}{7}\frac{7}{18}Lx = -\frac{1}{9}Lx$

$\Sigma F_z = 0$; $R_z - L + \frac{3}{7}T_1 = 0$, $R_z = L - \frac{3}{7}\frac{7}{18}Lx = L(1-\frac{1}{6}x)$

$R^2 = L^2\left(\frac{x^2}{4} + \frac{x^2}{81} + 1 - \frac{x}{3} + \frac{x^2}{36}\right)$

$R/L = \sqrt{(47x^2/162) - x/3 + 1}$

$\frac{dR^2/L^2}{dx} = \frac{47x}{81} - \frac{1}{3} = 0$ for min.; $x = 0.574\ m$



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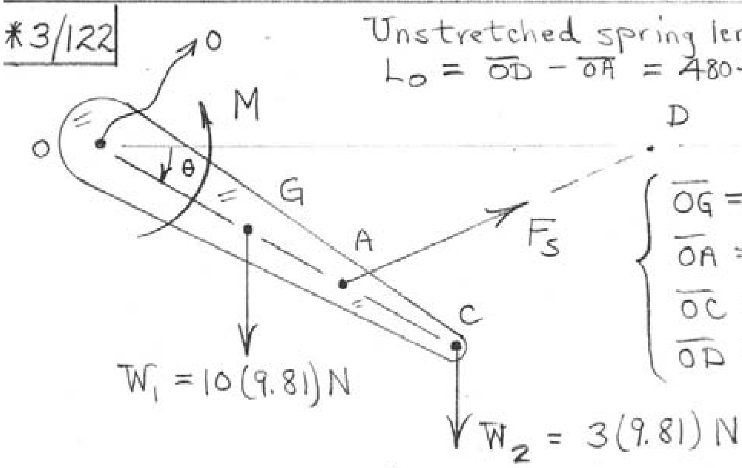
Unstretched spring length is
$L_o = \overline{OD} - \overline{OA} = 480 - 250 = 230$ mm



$\overline{OG} = 150$ mm
$\overline{OA} = 250$ mm
$\overline{OC} = 360$ mm
$\overline{OD} = 480$ mm

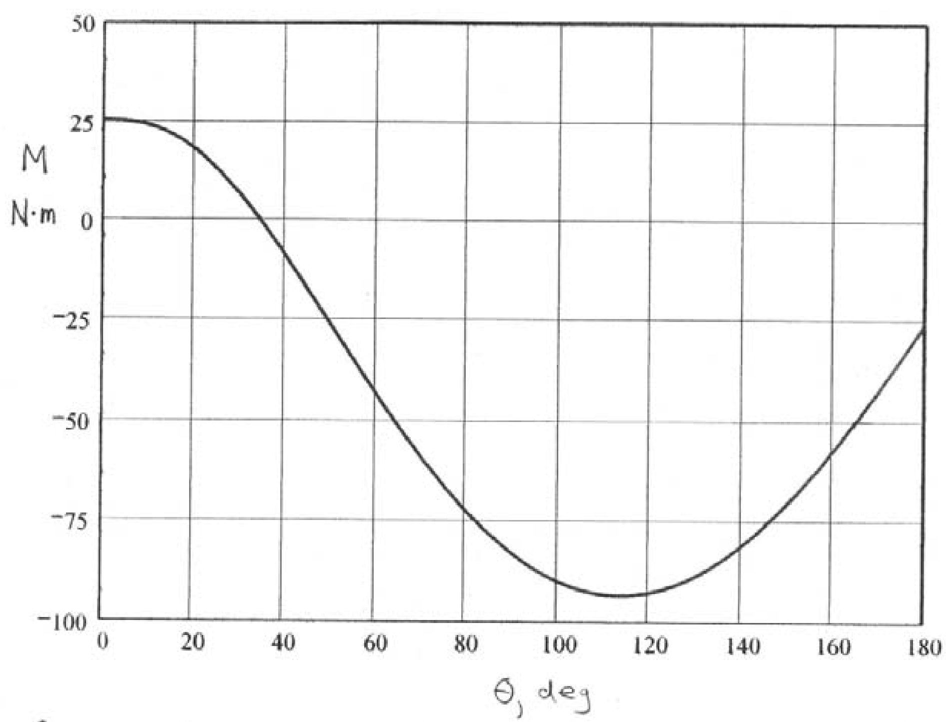
$$\underline{AD} = (\overline{OD} - \overline{OA}\cos\theta)\underline{i} + \overline{OA}\sin\theta\,\underline{j}$$

$$\overline{AD} = \left\{\left[\overline{OD} - \overline{OA}\cos\theta\right]^2 + \left[\overline{OA}\sin\theta\right]^2\right\}^{1/2}$$

$$\underline{F_s} = k\left(\overline{AD} - L_o\right)\frac{\underline{AD}}{\overline{AD}} \; ; \; \underline{r}_{OA} = \overline{OA}\left(\cos\theta\,\underline{i} - \sin\theta\,\underline{j}\right)$$

$$\Sigma M_O = 0: \; M - \left(W_1\overline{OG} + W_2\overline{OC}\right)\cos\theta + \left(\underline{r}_{OA} \times \underline{F_s}\right)_z = 0$$

Carry out algebra, solve for M as a function of θ, & plot to obtain

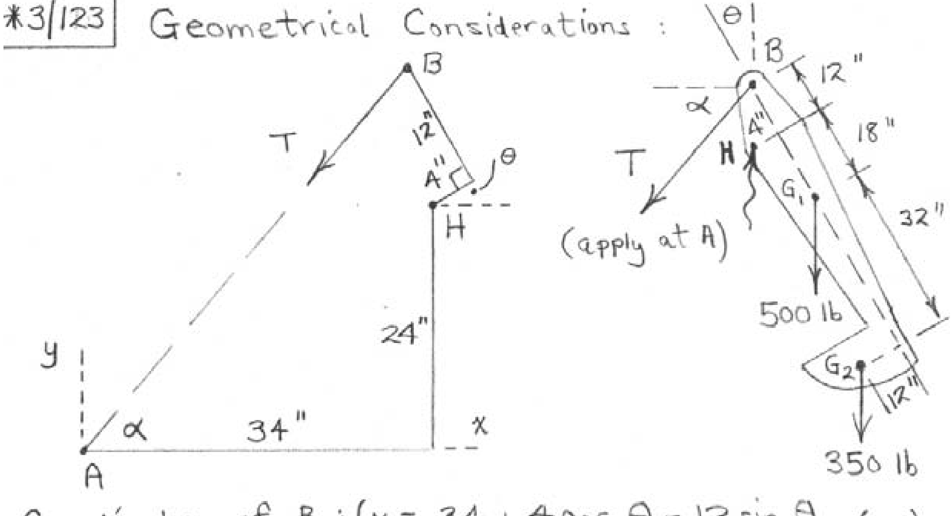


$M = 0$ @ $\theta = 34.6°$

$M_{min} = -93.4$ N·m @ $\theta = 113.9°$

$M_{max} = 25.3$ N·m @ $\theta = 0$

*3/123 Geometrical Considerations :



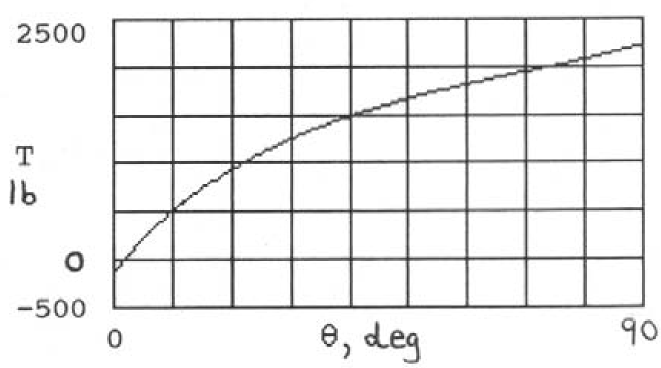
Coordinates of B : $\begin{cases} x = 34 + 4\cos\theta - 12\sin\theta \\ y = 24 + 4\sin\theta + 12\cos\theta \end{cases}$ (in.) (1)

$\therefore\ \alpha = \tan^{-1}\dfrac{y}{x}$ (aims $\underline{T}$) (2)

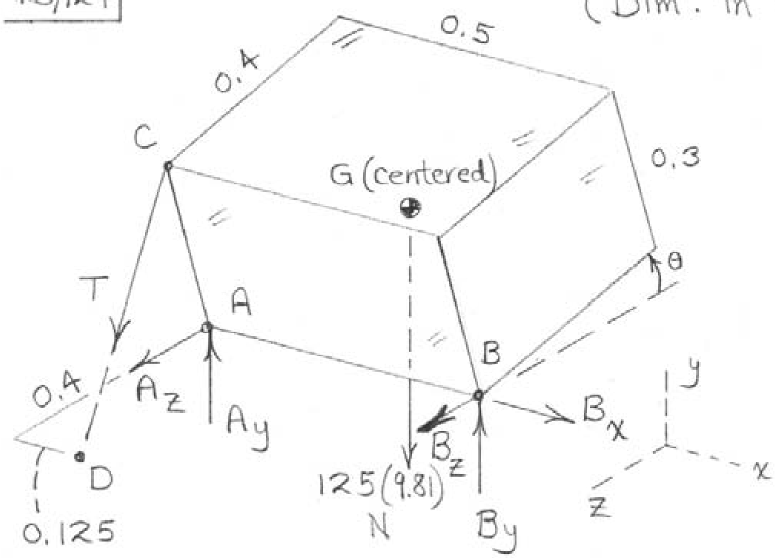
$\circlearrowleft \Sigma M_H = 0 : -(T\cos\alpha)(24) + (T\sin\alpha)(34) - 500(18\sin\theta + 4\cos\theta) - 350(4\cos\theta + (18+32)\sin\theta - 12\cos\theta) = 0$

$\Rightarrow\ T = \dfrac{26{,}500\sin\theta - 800\cos\theta}{34\sin\alpha - 24\cos\alpha}$ (lb; positive is tension in AB)

Solve (1), (2), & (3) & plot: (Note T=0 @ θ= 1.729°)



*3/124 (Dim. in m)



$$\underline{CD} = 0.125\underline{i} - 0.3\cos\theta\,\underline{j} + (0.4 - 0.3\sin\theta)\,\underline{k}$$

Then $\underline{T} = T\underline{n}_{CD} = T\,\dfrac{\underline{CD}}{\overline{CD}}$, or

$$\underline{T} = T\left[\frac{0.125\underline{i} - 0.3\cos\theta\,\underline{j} + (0.4 - 0.3\sin\theta)\,\underline{k}}{\sqrt{0.266 - 0.24\sin\theta}}\right]$$

$$\Sigma F_x = 0: \quad \frac{0.125\,T}{\sqrt{0.266 - 0.24\sin\theta}} + B_x = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad \frac{-0.3\,T\cos\theta}{\sqrt{0.266 - 0.24\sin\theta}} + A_y + B_y - 125(9.81) = 0 \qquad (2)$$

$$\Sigma F_z = 0: \quad \frac{(0.4 - 0.3\sin\theta)T}{\sqrt{0.266 - 0.24\sin\theta}} + A_z + B_z = 0 \qquad (3)$$

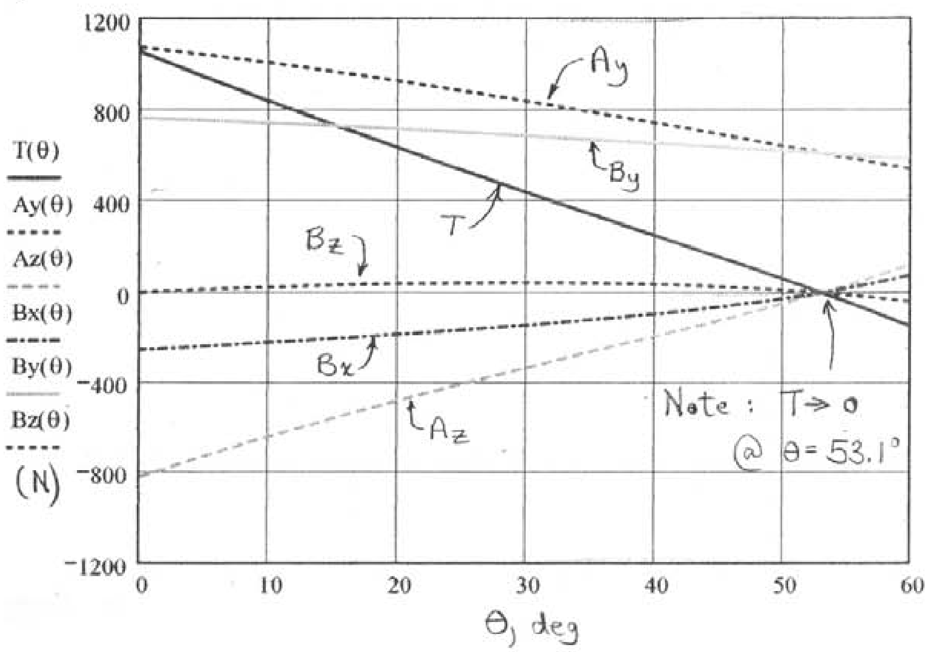
$$\Sigma M_{D_x} = 0: \quad (A_y + B_y)(0.4) - 125(9.81)(0.4 + 0.2\cos\theta - 0.15\sin\theta) = 0 \qquad (4)$$

$$\Sigma M_{D_y} = 0: \quad -B_x(0.4) + A_z(0.125) - B_z(0.375) = 0 \qquad (5)$$

$$\Sigma M_{D_z} = 0: \quad -A_y(0.125) + B_y(0.375) - 125(9.81)(0.125) = 0 \qquad (6)$$

Computer Solution:

$$\begin{cases} T = -12.77 \dfrac{-4\cos\theta + 3\sin\theta}{\cos\theta}\sqrt{425 - 384\sin\theta} \\ A_y = 613 + 460\cos\theta - 345\sin\theta \\ A_z = -12.77 \dfrac{64\cos\theta - 48\sin\theta - 36\sin\theta\cos\theta + 27\cos^2\theta}{\cos\theta} \\ B_x = 63.9 \dfrac{-4\cos\theta + 3\sin\theta}{\cos\theta} \\ B_y = 613 + 153.3\cos\theta - 115.0\sin\theta \\ B_z = -38.3 \dfrac{-4\cos\theta + 3\sin\theta}{\cos\theta}\sin\theta \end{cases}$$



*3/125 With reference to the FBD and solution to Prob. 3/96, the various tension vectors are $\underline{T}_2 = 1000\left(0\underline{i} - \cos 10^\circ \underline{j} - \sin 10^\circ \underline{k}\right)$ N

$$\underline{T}_1 = T_1\left(-\sin 30^\circ \cos 10^\circ \underline{i} + \cos 30^\circ \cos 10^\circ \underline{j} - \sin 10^\circ \underline{k}\right)$$

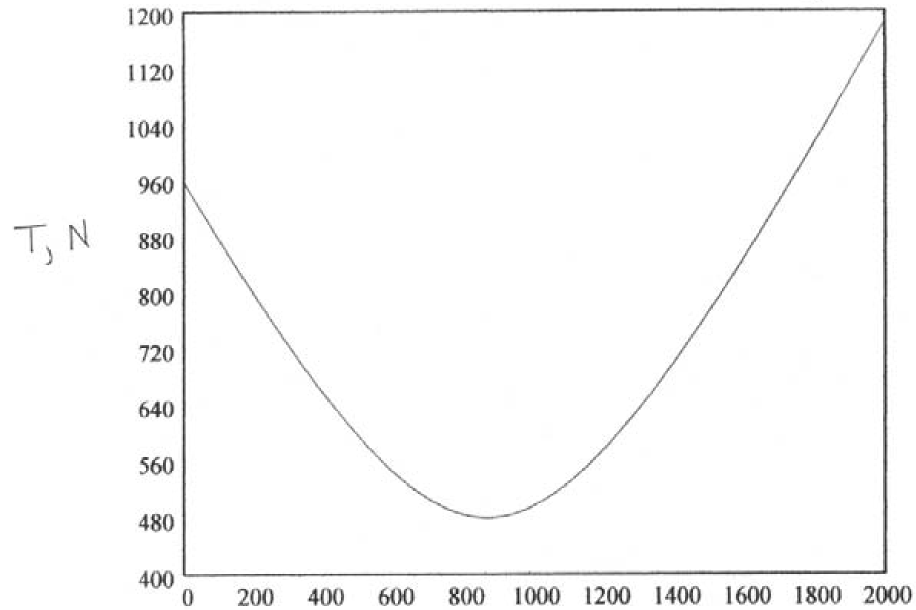
$$\underline{T}_{BE} = \frac{T\left(10\cos\theta\,\underline{i} + 10\sin\theta\,\underline{j} - 11\underline{k}\right)}{\sqrt{(10\cos\theta)^2 + (10\sin\theta)^2 + 11^2}}$$

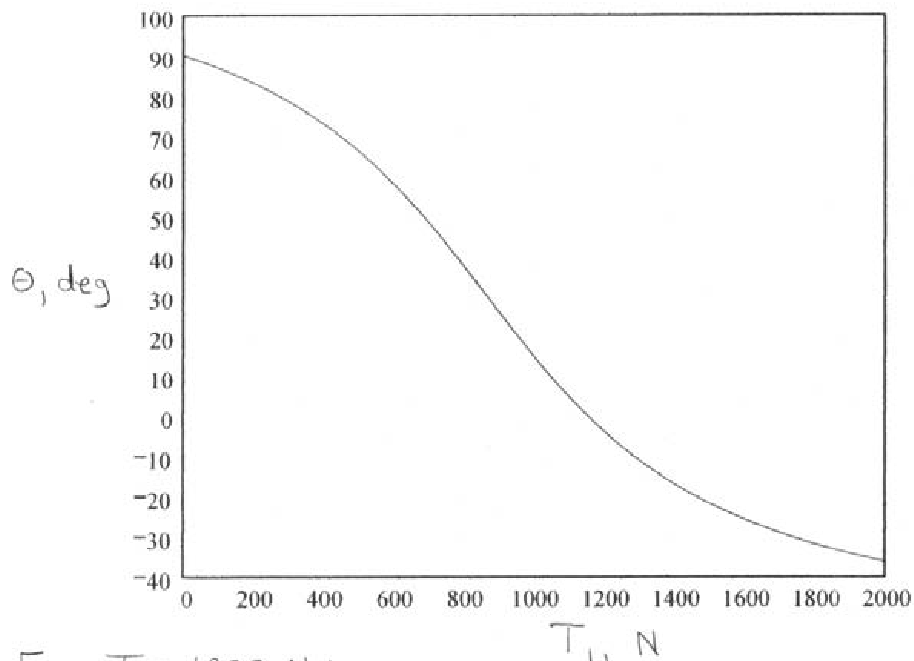
$$\underline{T}_{AD} = \frac{T\left(8\cos\theta\,\underline{i} + 8\sin\theta\,\underline{j} - 9\underline{k}\right)}{\sqrt{(8\cos\theta)^2 + (8\sin\theta)^2 + 9^2}}$$

Needed position vectors are $\underline{r}_{OC} = 13\underline{k}$ m, $\underline{r}_{OB} = 11\underline{k}$ m, & $\underline{r}_{OA} = 9\underline{k}$ m.

$$\Sigma \underline{M}_O = \underline{0}: \quad \underline{r}_{OC} \times (\underline{T}_1 + \underline{T}_2) + \underline{r}_{OB} \times \underline{T}_{BE} + \underline{r}_{OA} \times \underline{T}_{AD} = \underline{0}$$

Carry out cross products, collect terms, & set the $\underline{i}$- & $\underline{j}$-components to zero, solving for T and θ for each value of T_1. Resulting plots:



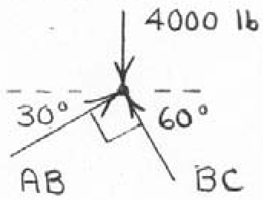


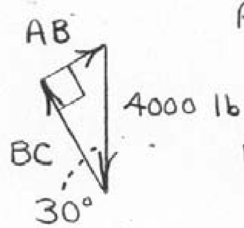
For $T_1 = 1000$ N:

$T = 495$ N, $\theta = 15°$

4/1

Joint B:





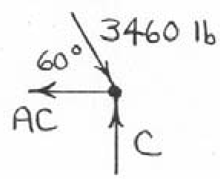
$AB = 4000\left(\frac{1}{2}\right)$

$= \underline{2000 \text{ lb } C}$

$BC = 4000 \sqrt{3}/2$

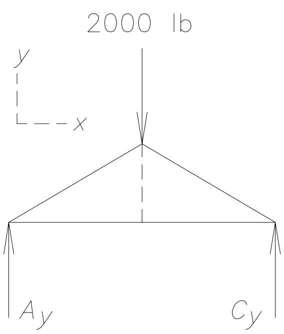
$= \underline{3460 \text{ lb } C}$

Joint C:



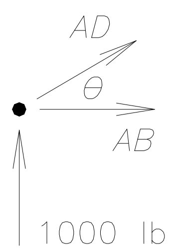
$\xrightarrow{+} \Sigma F_x = 0: \ -AC + 3460 \cos 60° = 0$

$\underline{AC = 1732 \text{ lb } T}$



By symmetry, $A_y = C_y = 1000$ lb

Joint A:



$$\theta = \tan^{-1}\frac{7}{12} = 30.3°$$

$$\begin{cases} \Sigma F_y = 0: \; AD \sin 30.3° + 1000 = 0, \; \underline{AD = -1985 \text{ lb } (C)} \\ \Sigma F_x = 0: \; AD \cos 30.3° + AB = 0, \; \underline{AB = 1714 \text{ lb } T} \end{cases}$$

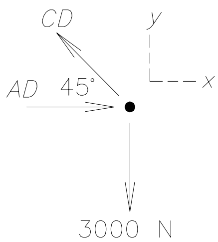
By symmetry: $CD = AC = 1985$ lb C
$BC = AB = 1714$ lb T

By inspection of joint B, BD is a zero-force member.

4/3

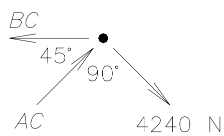
We can begin at joint D without finding the external reactions.

Joint *D*:



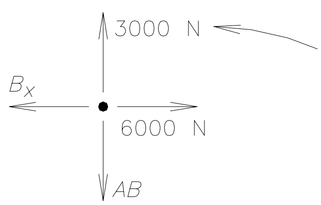
$$\begin{cases} \Sigma F_y = 0: \ CD \sin 45° - 3000 = 0, \ \underline{CD = 4240 \text{ N } T} \\ \Sigma F_x = 0: \ AD - CD \cos 45° = 0, \ \underline{AD = 3000 \text{ N } C} \end{cases}$$

Joint *C*:



$$\begin{cases} \Sigma F_y = 0: \ AC \sin 45° - 4240 \sin 45° = 0, \ \underline{AC = 4240 \text{ N } C} \\ \Sigma F_x = 0: \ -BC + 2(4240 \cos 45°) = 0, \ \underline{BC = 6000 \text{ N } T} \end{cases}$$

Joint *B*:

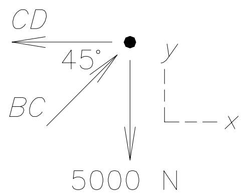


From equilibrium of entire truss

From $\Sigma F_y = 0$, $\underline{AB = 3000 \text{ N } T}$

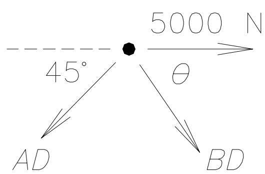
We can begin at joint C without finding the external reactions.

Joint C:



$$\begin{cases} \Sigma F_y = 0: \; BC \sin 45° - 5000 = 0, \; \underline{BC = 7070 \text{ N } C} \\ \Sigma F_x = 0: \; 7070 \cos 45° - CD = 0, \; \underline{CD = 5000 \text{ N } T} \end{cases}$$

Joint D:



$$\overline{DB}^2 = 5^2 + 6^2 - 2(5)(6)\cos 45°, \; \overline{DB} = 4.31 \text{ m}$$

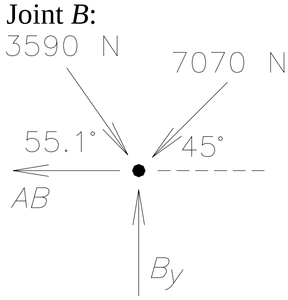
$$\frac{\sin\theta}{5} = \frac{\sin 45°}{4.31} \Rightarrow \theta = 55.1°$$

$$\begin{cases} \Sigma F_x = 0: \; 5000 + BD \cos 55.1° - AD \cos 45° = 0 \\ \Sigma F_y = 0: \; -AD \sin 45° - BD \sin 55.1° = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BD = -3590 \text{ N or } 3590 \text{ N } C}$

$\underline{AD = 4170 \text{ N } T}$

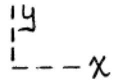
Joint B:



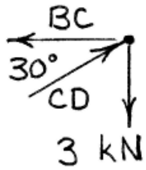
$$\Sigma F_x = 0: \; 3590 \cos 55.1° - 7070 \cos 45° - AB = 0$$

$\underline{AB = -2950 \text{ N or } 2950 \text{ N } C}$

4/5

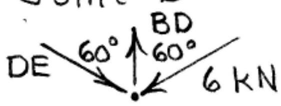


Joint C :



$$\begin{cases} \Sigma F_y = 0: \; CD\left(\frac{1}{2}\right) - 3 = 0, & \underline{CD = 6 \text{ kN } C} \\ \Sigma F_x = 0: \; -BC + 6\frac{\sqrt{3}}{2} = 0, & \underline{BC = 5.20 \text{ kN } T} \end{cases}$$

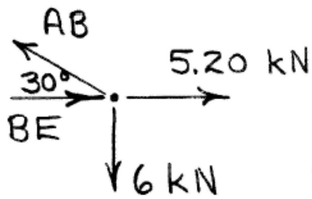
Joint D :



$$\begin{cases} \Sigma F_x = 0 \Rightarrow \; \underline{DE = 6 \text{ kN } C} \\ \Sigma F_y = 0: \; BD - 2(6)\left(\frac{1}{2}\right) = 0 \end{cases}$$

$$\underline{BD = 6 \text{ kN } T}$$

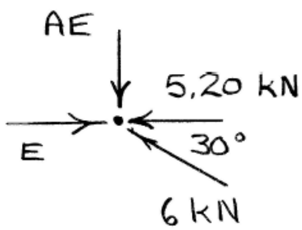
Joint B :



$$\begin{cases} \Sigma F_y = 0: \; AB\left(\frac{1}{2}\right) - 6 = 0, & \underline{AB = 12 \text{ kN } T} \\ \Sigma F_x = 0: \; BE - 12\frac{\sqrt{3}}{2} + 5.20 = 0 \end{cases}$$

$$\underline{BE = 5.20 \text{ kN } C}$$

Joint E :

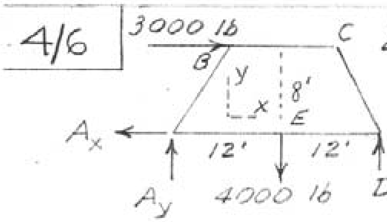


$$\Sigma F_y = 0: \; 6\left(\frac{1}{2}\right) - AE = 0$$

$$\underline{AE = 3 \text{ kN } C}$$

(Joint A checks after external reactions are determined from the truss as a whole.)

4/6



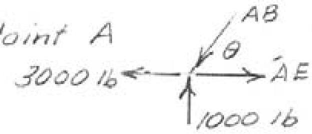
$\Sigma M_A = 0;\ 4000(12) + 3000(8) - 24D = 0$

$D = 3000$ lb

$\Sigma F_x = 0;\ A_x = 3000$ lb

$\Sigma F_y = 0;\ A_y = 4000 - 3000 = 1000$ lb

Joint A

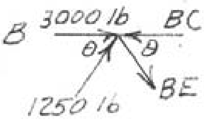


$\Sigma F_y = 0;\ AB\left(\frac{4}{5}\right) - 1000 = 0,\ \underline{AB = 1250 \text{ lb } C}$

$\Sigma F_x = 0;\ AE - 1250\left(\frac{3}{5}\right) - 3000 = 0$

$\underline{AE = 3750 \text{ lb } T}$

Joint B

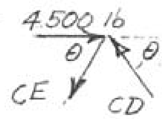


$\Sigma F_y = 0;\ 1250\left(\frac{4}{5}\right) - BE\left(\frac{4}{5}\right) = 0,\ \underline{BE = 1250 \text{ lb } T}$

$\Sigma F_x = 0;\ BC - 3000 - 1250\left(\frac{3}{5}\right) - 1250\left(\frac{3}{5}\right) = 0$

$\underline{BC = 4500 \text{ lb } C}$

Joint C

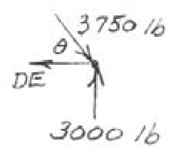


$\Sigma F_y = 0;\ CE \sin\theta - CD \sin\theta = 0,\ CE = CD$

$\Sigma F_x = 0;\ 2CE(3/5) - 4500 = 0,\ \underline{CE = 3750 \text{ lb } T}$

$\underline{CD = 3750 \text{ lb } C}$

Joint D



$\Sigma F_x = 0;\ DE - 3750\left(\frac{3}{5}\right) = 0$

$\underline{DE = 2250 \text{ lb } T}$

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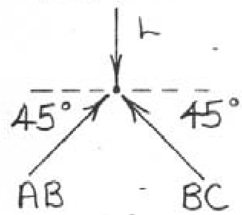
From $\Sigma F_x = 0$, $AB = BC$

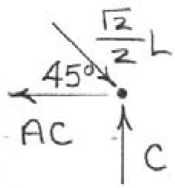
Joint B:



$\Sigma F_y = 0$: $2AB\frac{\sqrt{2}}{2} - L = 0$

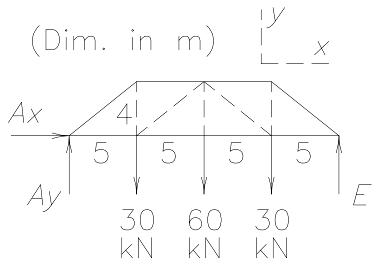
$$AB = \frac{\sqrt{2}}{2}L = BC$$

Joint C:



$\Sigma F_x = 0$: $\frac{\sqrt{2}}{2}L\left(\frac{\sqrt{2}}{2}\right) - AC = 0$

$$\underline{AC = \frac{L}{2}\ T}$$

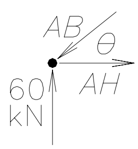


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60 \text{ kN}$ by

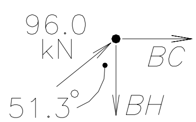
$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$



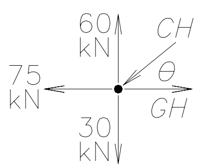
$$\begin{cases} \Sigma F_y = 0: 60 - AB\sin\theta = 0, \ \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0\cos\theta, \ \underline{AH = 75 \text{ kN } T} \end{cases}$$

Joint B:



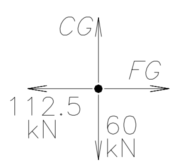
$$\begin{cases} \Sigma F_x = 0: BC + 96.0\sin 51.3° = 0, \ \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0\cos 51.3° = 0, \ \underline{BH = 60 \text{ kN } T} \end{cases}$$

Joint H:



$$\begin{cases} \Sigma F_y = 0: -CH\sin\theta + 30 = 0, \ \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0\cos\theta + GH - 75 = 0, \ \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

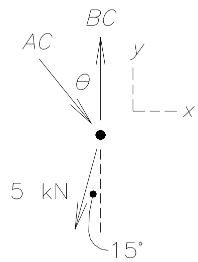
Joint G:



$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$

By symmetry:
FG = 112.5 kN T, CF = 48.0 kN C
CD = 75 kN C, DF = 60 kN T
$\underline{EF = 75 \text{ kN } T, DE = 96.0 \text{ kN } C}$

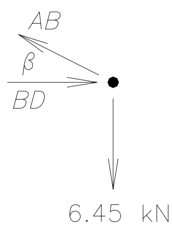
Joint C:



$$\theta = \tan^{-1}\frac{4}{5} = 38.7°$$

$$\begin{cases} \Sigma F_x = 0: \ -5\sin 15° + AC\sin 38.7° = 0, \ \underline{AC = 2.07 \text{ kN } C} \\ \Sigma F_y = 0: \ BC - 5\cos 15° - 2.07\cos 38.7° = 0, \ \underline{BC = 6.45 \text{ kN } T} \end{cases}$$

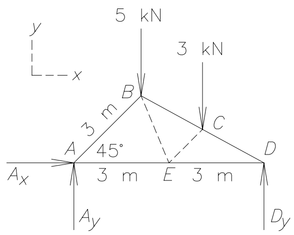
Joint B:



$$\beta = \tan^{-1}\frac{2}{4} = 26.6°$$

$$\begin{cases} \Sigma F_y = 0: \ AB\sin 26.6° - 6.45 = 0, \ \underline{AB = 14.42 \text{ kN } T} \\ \Sigma F_x = 0: \ -14.42\cos 26.6° + BD = 0, \ \underline{BD = 12.89 \text{ kN } C} \end{cases}$$

4/10

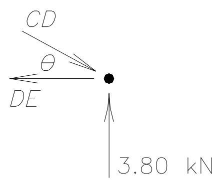


As a whole:

Note: $\overline{CE} = 1.5$ m by similar triangles

$$\Sigma M_A = 0:\ 5(3\cos 45°) + 3(3 + 1.5\cos 45°) - 6D_y = 0$$
$$D_y = 3.80 \text{ kN}$$

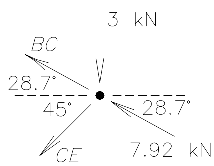
Joint D:



$$\theta = \tan^{-1}\frac{3\sin 45°}{6 - 3\cos 45°} = 28.7°$$

$$\begin{cases} \Sigma F_y = 0:\ 3.80 - CD\sin 28.7° = 0,\ CD = 7.92 \text{ kN } C \\ \Sigma F_x = 0:\ 7.92\cos 28.7° - DE = 0,\ DE = 6.94 \text{ kN } T \end{cases}$$

Joint C:

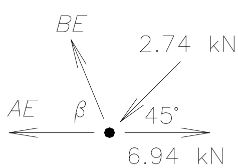


$$\begin{cases} \Sigma F_x = 0:\ -BC\cos 28.7° - CE\cos 45° - 7.92\cos 28.7° = 0 \\ \Sigma F_y = 0:\ BC\sin 28.7° - CE\sin 45° + 7.92\sin 28.7° - 3 = 0 \end{cases}$$

Solve simultaneously to obtain: $BC = -5.70 \text{ kN } (C)$

$\underline{CE = -2.74 \text{ kN } (C)}$

Joint E:

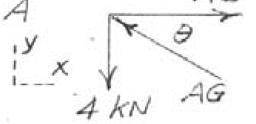


$$\beta = \frac{180° - 45°}{2} = 67.5°$$

$$\Sigma F_y = 0:\ BE\sin 67.5° - 2.74\sin 45° = 0$$

$\underline{BE = 2.10 \text{ kN } T}$

4/11 Joint A



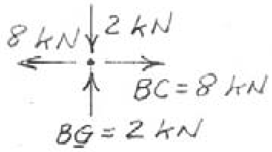
$\theta = \tan^{-1} 1/2 = 26.57°$

$\sin\theta = 1/\sqrt{5}, \quad \cos\theta = 2/\sqrt{5}$

$\Sigma F_y = 0;\ AG/\sqrt{5} - 4 = 0$

$AG = 4\sqrt{5}$ kN C

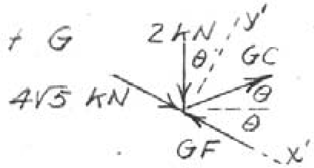
Joint B



$\Sigma F_x = 0;\ AB - 4\sqrt{5}(2/\sqrt{5}) = 0$

$\therefore AB = 8$ kN T

Joint G



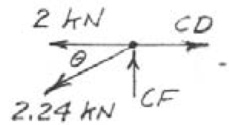
$\Sigma F_{y'} = 0;\ 2\left(\frac{2}{\sqrt{5}}\right) - CG \sin 2\theta = 0$

$CG = \frac{4}{\sqrt{5}} \cdot \frac{1}{0.8} = \underline{2.24 \text{ kN T}}$

$\Sigma F_{x'} = 0;\ 2.24 \cos 2\theta + 2/\sqrt{5} + 4\sqrt{5} - GF = 0$

$GF = 5\sqrt{5} = 11.18$ kN C

Joint C



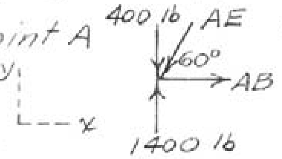
$\Sigma F_y = 0;\ CF - 2.24 \sin\theta = 0$

$\underline{CF = 1.00 \text{ kN C}}$

4/12 Total weight of truss = 7(400) = 2800 lb

By symmetry, reactions at A & C are 1400 lb

Joint A



$\Sigma F_y = 0;\ AE \cos 30° + 400 - 1400 = 0$

$\underline{AE = 2000/\sqrt{3} \text{ lb } C}$

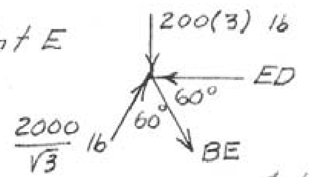
$\Sigma F_x = 0;\ AB - \frac{2000}{\sqrt{3}} \cos 60° = 0$

$\underline{AB = 1000/\sqrt{3} \text{ lb } T}$

By symmetry $\underline{BC = 1000/\sqrt{3} \text{ lb } T}$

$\underline{CD = 2000/\sqrt{3} \text{ lb } C}$

Joint E



$\Sigma F_y = 0;\ BE \sin 60° - \frac{2000}{\sqrt{3}} \sin 60° + 600 = 0$

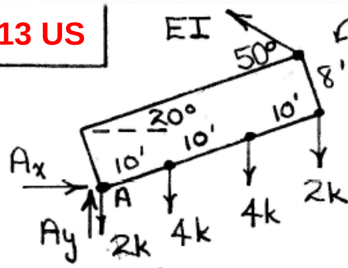
$\underline{BE = 800/\sqrt{3} \text{ lb } T}$

& by symmetry $\underline{BD = 800/\sqrt{3} \text{ lb } T}$

$\Sigma F_x = 0;\ ED - \frac{2000}{\sqrt{3}} \sin 30° - \frac{800}{\sqrt{3}} \sin 30° = 0$

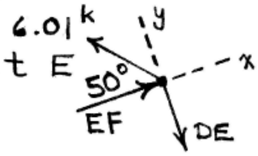
$\underline{ED = 1400/\sqrt{3} \text{ lb } C}$

4/13 US



$$\circlearrowleft \sum M_A = 0: -4\cos 20^\circ\left(10+20+\tfrac{30}{2}\right) + EI\cos 50^\circ(8) + EI\sin 50^\circ(30) = 0, \quad EI = 6.01 \text{ kips}$$

Joint E



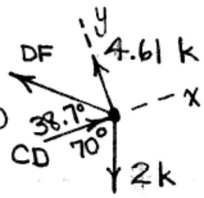
$\sum F_x = 0: EF - 6.01\cos 50^\circ = 0$

<u>EF = 3.87 kips C</u>

$\sum F_y = 0: -DE + 6.01\sin 50^\circ = 0$

<u>DE = 4.61 kips T</u>

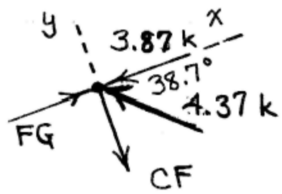
Joint D



$\sum F_y = 0: 4.61 - 2\sin 70^\circ + DF\sin 38.7^\circ = 0$, <u>DF = −4.37 kips (C)</u>

$\sum F_x = 0: 4.37\cos 38.7^\circ - 2\cos 70^\circ + CD = 0$, <u>CD = −2.73 kips (T)</u>

Joint F



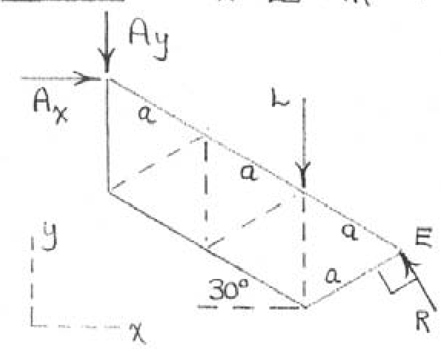
$\sum F_x = 0: -3.87 - 4.37\cos 38.7^\circ + FG = 0$

<u>FG = 7.28 kips C</u>

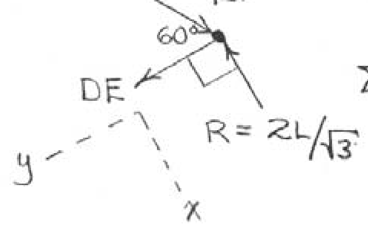
4/14

$$\overset{+}{\circlearrowleft} \sum M_A = 0: \quad L(2a\cos 30^\circ) - R(3a\sin 30^\circ) = 0$$

$$R = 2L/\sqrt{3}$$



Joint E:



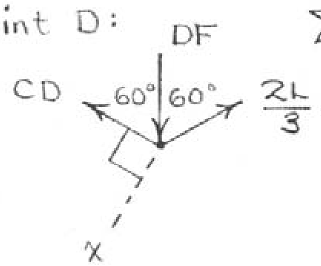
$$\sum F_x = 0: \quad EF\sin 60^\circ - 2L/\sqrt{3} = 0$$

$$\underline{EF = 4L/3 \quad C}$$

$$\sum F_y = 0: \quad DE - \frac{4L}{3}\cos 60^\circ = 0$$

$$\underline{DE = 2L/3 \quad T}$$

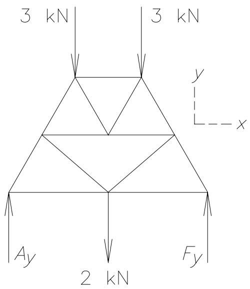
Joint D:



$$\sum F_x = 0: \quad DF\cos 30^\circ - \frac{2L}{3}\cos 30^\circ = 0$$

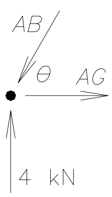
$$\underline{DF = 2L/3 \quad C}$$

4/15



By symmetry, $A_y = F_y = 4$ kN

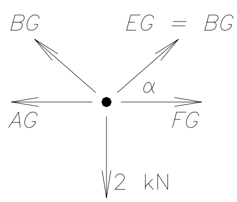
Joint *A*:



$$\theta = \cos^{-1}\frac{1}{2} = 60°$$

$$\begin{cases} \Sigma F_y = 0: \ 4\text{ kN} - AB\sin 60° = 0, \ AB = 4.62\text{ kN } C \\ \Sigma F_x = 0: \ AG - 4.62\cos 60° = 0, \ AG = 2.31\text{ kN } T \end{cases}$$

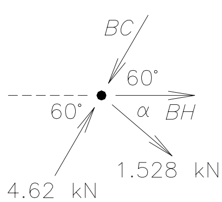
Joint *G*:



$$\alpha = \tan^{-1}\frac{2\sin 60°}{2} = 40.9°$$

$$\Sigma F_y = 0: \ 2BG\sin 40.9° - 2 = 0, \ \underline{BG = 1.528\text{ kN } T}$$

Joint *B*:



$$\Sigma F_y = 0: \ 4.62\sin 60° - BC\sin 60° - 1.528\sin 40.9° = 0$$

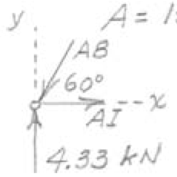
$$\underline{BC = 3.46\text{ kN } C}$$

4/16 Truss as a whole

$\Sigma M_F = 0;\ 2(a + \frac{a}{2}) + 4(2a + \frac{a}{2}) - A(3a) = 0$

$A = 13/3 = 4.33$ kN

Joint A



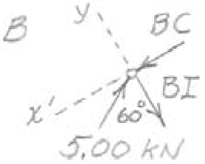
$\Sigma F_y = 0;\ AB \sin 60° - 4.33 = 0$

$AB = 5.00$ kN C

$\Sigma F_x = 0;\ AI - 5 \cos 60° = 0$

$AI = 2.50$ kN T

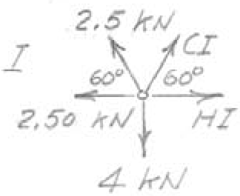
Joint B



$\Sigma F_{y'} = 0;\ 5.00 \cos 60° - BI = 0$

$\underline{BI = 2.50 \text{ kN } T}$

Joint I



$\Sigma F_y = 0;\ (CI + 2.5)\sin 60° - 4 = 0$

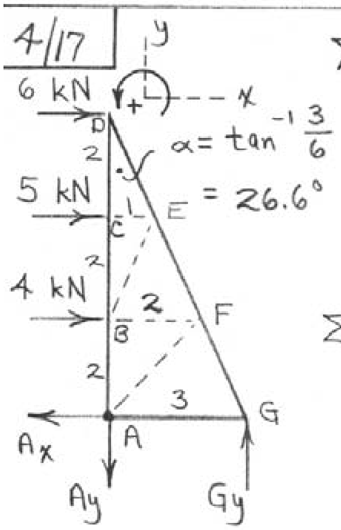
$\underline{CI = 2.12 \text{ kN } T}$

$\Sigma F_x = 0;\ HI + 2.12 \cos 60°$

$- 2.50 - 2.5 \cos 60° = 0$

$\underline{HI = 2.69 \text{ kN } T}$

4/17



$\Sigma M_A = 0: \; -6(6) - 5(4) - 4(2) + G_y(3) = 0$

$G_y = 21.3 \text{ kN}$

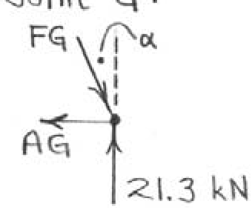
$\Sigma F_y = 0: \; -A_y + G_y = 0$

$A_y = 21.3 \text{ kN}$

$\Sigma F_x = 0: \; 6 + 5 + 4 - A_x = 0$

$A_x = 15 \text{ kN}$

Joint G:



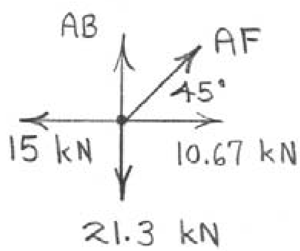
$\Sigma F_y = 0: \; 21.3 - FG \cos\alpha = 0$

$FG = 23.9 \text{ kN C}$

$\Sigma F_x = 0: \; 23.9 \sin\alpha - AG = 0$

$AG = 10.67 \text{ kN T}$

Joint A:



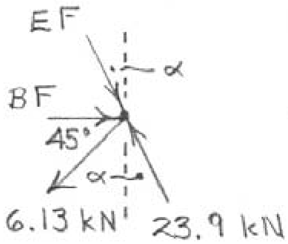
$\Sigma F_x = 0: \; 10.67 - 15 + AF \cos 45° = 0$

$AF = 6.13 \text{ kN T}$

$\Sigma F_y = 0: \; AB + 6.13 \sin 45° - 21.3 = 0$

$AB = 17 \text{ kN T}$

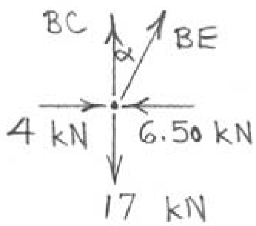
Joint F:



$\Sigma F_y = 0: -EF\cos\alpha + 23.9\cos\alpha$
$- 6.13\sin 45° = 0, \quad EF = 19.01 \text{ kN C}$

$\Sigma F_x = 0: 19.01\sin\alpha + BF - 6.13\cos 45°$
$- 23.9\sin\alpha = 0, \quad \underline{BF = 6.50 \text{ kN C}}$

Joint B:



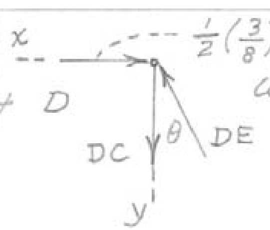
$\Sigma F_x = 0: BE\sin\alpha + 4 - 6.50 = 0$

$\underline{BE = 5.59 \text{ kN T}}$

From inspection of joint C, $\underline{CE = 5 \text{ kN C}}$

4/18

Joint D



$\frac{1}{2}\left(\frac{3}{8}\right)800 = 150\ lb$; $\theta = \tan^{-1}\frac{1}{2} = 26.57°$

$\cos\theta = 2/\sqrt{5}$, $\sin\theta = 1/\sqrt{5}$

$\Sigma F_x = 0$; $DE\left(\frac{1}{\sqrt{5}}\right) - 150 = 0$

$DE = 335\ lb\ C$

$\Sigma F_y = 0$; $DC = 335(2/\sqrt{5}) = 300\ lb\ T$

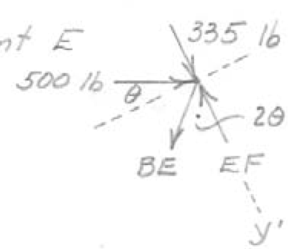
Joint C

$CE = 500\ lb$

$\frac{5}{8}(800) = 500\ lb$

$BC = 300\ lb\ T$

Joint E



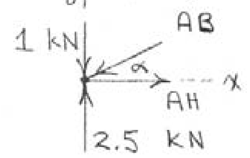
$\Sigma F_{x'} = 0$; $500\left(\frac{2}{\sqrt{5}}\right) - BE\ \sin 2\theta = 0$

$\sin 2\theta = 0.8$

$BE = \frac{1000}{\sqrt{5}}\frac{1}{0.8} = \underline{559\ lb\ T}$

4/19 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

A:



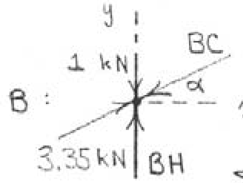
$\Sigma F_y = 0$: $2.5 - 1 - AB \sin\alpha = 0$

$\underline{AB = 3.35 \text{ kN } C}$

$\Sigma F_x = 0$: $-3.35 \cos\alpha + AH = 0$

$\underline{AH = 3 \text{ kN } T}$

B:



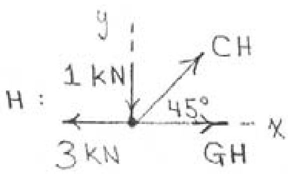
$\Sigma F_x = 0$: $3.35 \cos\alpha - BC \cos\alpha = 0$

$\underline{BC = 3.35 \text{ kN } C}$

$\Sigma F_y = 0$: $-1 + (3.35 - 3.35)\sin\alpha + BH = 0$

$\underline{BH = 1 \text{ kN } C}$

H:



$\Sigma F_y = 0$: $-1 + CH \sin 45° = 0$

$\underline{CH = 1.414 \text{ kN } T}$

$\Sigma F_x = 0$: $-3 + 1.41 \cos 45° + GH = 0$

$\underline{GH = 2 \text{ kN } T}$

By inspection of joint G and $\Sigma F_y = 0$, $\underline{CG = 0}$.

By symmetry,

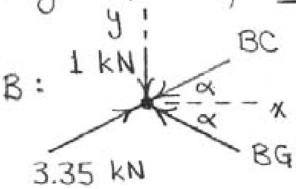
$$\left\{\begin{array}{l} DE = AB = 3.35 \text{ kN } C \\ CD = BC = 3.35 \text{ kN } C \\ EF = AH = 3 \text{ kN } T \\ DF = BH = 1 \text{ kN } C \\ CF = CH = 1.414 \text{ kN } T \\ \underline{FG = GH = 2 \text{ kN } T} \end{array}\right.$$

4/20 By symmetry, $A = E = 2.5 \text{ kN}$; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

Joint A analysis same as Prob. 4/19: $\begin{cases} AB = 3.35 \text{ kN } C \\ AH = 3.00 \text{ kN } T \end{cases}$

By inspection, $\underline{BH = 0}$ and $\underline{GH = AH.}$

B:

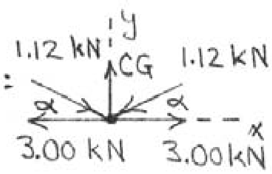


$\Sigma F_y = 0: -1 + 3.35 \sin\alpha + BG \sin\alpha - BC \sin\alpha = 0$

$\Sigma F_x = 0: 3.35 \cos\alpha - BC \cos\alpha - BG \cos\alpha = 0$

$\Rightarrow \underline{BC = 2.24 \text{ kN } C, \quad BG = 1.118 \text{ kN } C}$

G:



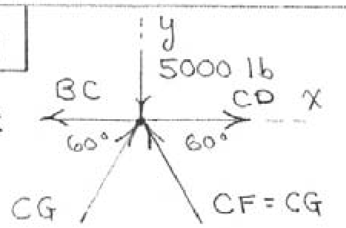
$\Sigma F_y = 0: CG - 2(1.12)\sin\alpha = 0$

$\underline{CG = 1.00 \text{ kN } T}$

By symmetry, $\begin{cases} DE = AB = 3.35 \text{ kN } C \\ CD = BC = 2.24 \text{ kN } C \\ EF = AH = 3.00 \text{ kN } T \\ DF = BH = 0 \\ FG = GH = 3.00 \text{ kN } T \\ DG = BG = 1.118 \text{ kN } C \end{cases}$

4/21

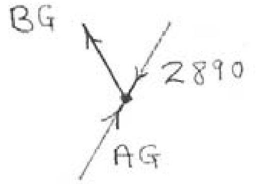
C :



Arguing symmetry,

$\sum F_y = 0: \; 2CG \sin 60° - 5000 = 0$, $\underline{CG = 2890 \text{ lb } C}$

G :



By inspection,

$\underline{AG = 2890 \text{ lb } C}$

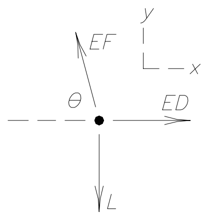
$\underline{BG = 0}$

From joint B : $\underline{AB = 0, \; BC = 0}$.

(Right truss symmetric to left one.)

4/22

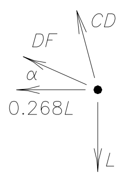
Joint E:



$$\theta = \frac{180° - 30°}{2} = 75°$$

$$\begin{cases} \Sigma F_y = 0: \ EF \sin 75° - L = 0, \ EF = 1.035L \ T \\ \Sigma F_x = 0: \ -1.035L \cos 75° + ED = 0, \ ED = 0.268L \ T \end{cases}$$

Joint D:



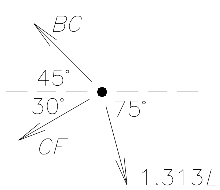
$$\overline{DF}^2 = R^2 + 4R^2 - 2(R)(2R)\cos 30°, \ \overline{DF} = 1.239R$$

$$\frac{\sin 30°}{1.239R} = \frac{\sin \alpha}{R}, \ \alpha = 23.8°$$

$$\begin{cases} \Sigma F_x = 0: \ -0.268L - DF \cos 23.8° - CD \cos 75° = 0 \\ \Sigma F_y = 0: \ -L + DF \sin 23.8° + CD \sin 75° = 0 \end{cases}$$

Solve simultaneously to obtain: $CD = 1.313L \ T$, $DF = 0.664L \ C$

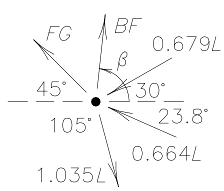
Joint C:



$$\begin{cases} \Sigma F_x = 0: \ -BC \cos 45° - CF \cos 30° + 1.313L \cos 75° = 0 \\ \Sigma F_y = 0: \ BC \sin 45° - CF \sin 30° - 1.313L \sin 75° = 0 \end{cases}$$

Solve simultaneously to obtain: $CF = -0.674L \ (C)$

Joint F:

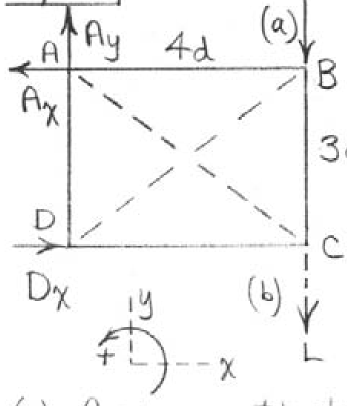


$$\beta = 30° + 23.8° + 30° = 83.8°$$

$$\begin{cases} \Sigma F_x = 0: \ -FG \cos 45° + BF \cos \beta + EF \cos 75° + CF \cos 30° + DF \cos \alpha = 0 \\ \Sigma F_y = 0: \ FG \sin 45° + BF \sin \beta - EF \sin 75° + CF \sin 30° - DF \sin \alpha = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BF = 1.814L \ T}$

4/23



$\Sigma F_y = 0: \; A_y - L = 0, \quad A_y = L$

$\Sigma M_A = 0: \; D_x(3d) - L(4d) = 0$

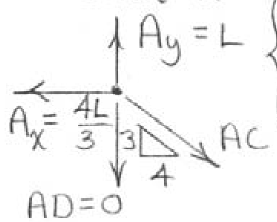
$D_x = \dfrac{4L}{3}$

$\Sigma F_x = 0: \; A_x = D_x = \dfrac{4L}{3}$

(Reactions hold for (a) & (b).)

(a) Assume that BD goes slack. From an inspection of joint B, $\underline{AB = 0}$ and $\underline{BC = L\ C}$. Similarly, from joint D, $\underline{AD = 0}$ and $\underline{CD = \frac{4L}{3}\ C}$.

Joint A:



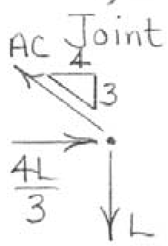
$\Sigma F_y = 0: \; L - \frac{3}{5}AC = 0, \quad \underline{AC = \frac{5L}{3}\ T}$

$\Sigma F_x = 0: \; -\frac{4L}{3} + \frac{5L}{3} \cdot \frac{4}{5} = 0 \ \checkmark$

Because AC is in tension, assumption is valid.

(b) Assume that BD goes slack. From joint B, $\underline{AB = BC = 0}$. From joint D, $\underline{AD = 0}$ & $\underline{CD = \frac{4L}{3}\ C}$.

Joint C:



$\Sigma F_y = 0: \; AC\left(\frac{3}{5}\right) - L = 0, \quad \underline{AC = \frac{5L}{3}\ T}$

$\Sigma F_x = 0: \; \frac{4L}{3} - \frac{5L}{3}\left(\frac{4}{5}\right) = 0 \ \checkmark$

4/24 m = no. of two-force members

j = no. of joints

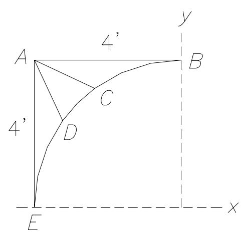
(a) $[m+3=13] > [2j=12]$ so redundant members.
Remove one member connecting B, C, D, and E.

(b) $[m+3=12] = [2j=12]$ so sufficient no. of members, but redundancy in external supports. Place A or F on roller.

(c) $[m+3=9] > [2j=8]$ so redundant members. Supports are also redundant. Remove AE or BE. Supports are then OK.

(d) $[m+3=12] = [2j=12]$ so sufficient no. of members, but redundancy in external supports. Place B on roller or remove member CD.

4/25



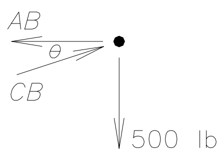
Location of C: $x_c = -\sqrt{4^2 - {y_c}^2}$

$$\overline{AC}^2 = (4 + x_c)^2 + (4 - y_c)^2$$

$$x_c = -(4 - AC\cos 25°),\ y_c = 4 - AC\sin 25°$$

Solve to obtain $\overline{AC} = 1.815$ ft

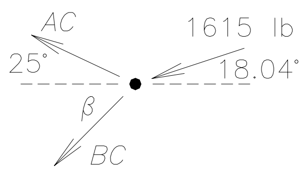
Joint B:

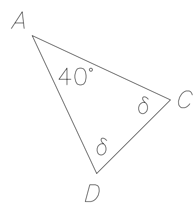


$$\theta = \tan^{-1}\frac{1.815\sin 25°}{4 - 1.815\cos 25°} = 18.04°$$

$$\begin{cases} \Sigma F_y = 0:\ CB\sin 18.04° - 500 = 0,\ CB = 1615 \text{ lb } C \\ \Sigma F_x = 0:\ 1615\cos 18.04° - AB = 0,\ AB = 1535 \text{ lb } T \end{cases}$$

Joint C:





$$\delta = \frac{180° - 40°}{2} = 70°,\ \beta = \delta - 25° = 45°$$

$$\begin{cases} \Sigma F_y = 0:\ AC\sin 25° - CD\sin 45° - 1615\sin 18.04° = 0 \\ \Sigma F_x = 0:\ -AC\cos 25° - CD\cos 45° - 1615\cos 18.04° = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{AC = -779 \text{ lb or } 779 \text{ lb } C}$

By symmetry across a line from the origin to point A, $\underline{AD = AC = 779 \text{ lb } C}$

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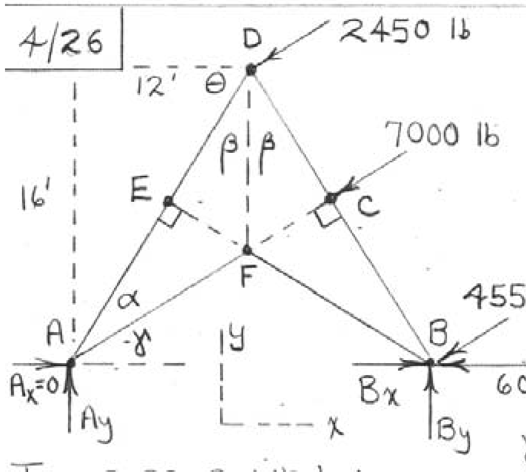


Geometry:

$\theta = \tan^{-1}\frac{16}{12} = 53.1^\circ$

$\beta = 90 - \theta = 36.9^\circ$

$\alpha = 90 - 2\beta = 16.26^\circ$

$\overline{CD} = 20 \sin\alpha = 5.60 \text{ ft}$

$\overline{CB} = 20 - \overline{CD} = 14.40 \text{ ft}$

$\gamma = 90 - (90 - \theta) - \alpha = 36.9^\circ$

Truss as a whole:

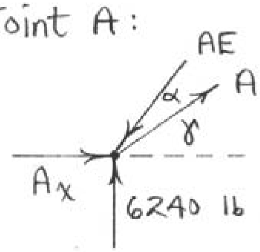
$\circlearrowleft \Sigma M_B = 0: \; 7000(14.4) + 2450(20) - 24A_y = 0$

$A_y = 6240 \text{ lb}$

$\Sigma F_y = 0: \; B_y + 6240 - (2450 + 7000 + 4550)\sin 36.9^\circ = 0, \quad B_y = 2160 \text{ lb}$

$\Sigma F_x = 0: \; B_x - (2450 + 7000 + 4550)\cos 36.9^\circ - 6000 = 0, \quad B_x = 17{,}200 \text{ lb}$

Joint A:



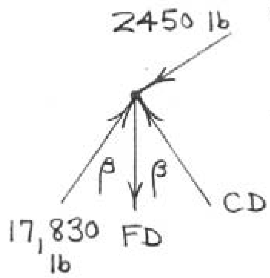
$\Sigma F_x = 0: -AE \cos 53.1° + AF \cos 36.9° = 0$

$\Sigma F_y = 0: 6240 - AE \sin 53.1° + AF \sin 36.9° = 0$

$\Rightarrow AF = 13{,}380$ lb T, $AE = 17{,}830$ lb C

From joint E, $ED = AE = 17{,}830$ lb C

Joint D:



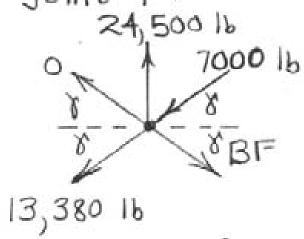
$\Sigma F_x = 0: 17{,}830 \sin 36.9° - CD \sin 36.9°$

$- 2450 \cos 36.9° = 0$, $CD = 14{,}570$ lb C

$\Sigma F_y = 0: (14{,}570 + 17{,}830) \cos 36.9° - FD$

$- 2450 \sin 36.9° = 0$, $FD = 24{,}500$ lb T

Joint F:



$\Sigma F_x = 0: BF \cos 36.9° - (13{,}380 +$

$7000) \cos 36.9° = 0$, $BF = 20{,}400$ lb T

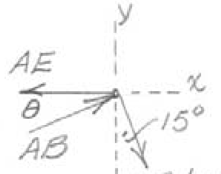
(Joint B checks.)

The maximum force occurs in member FD:

$FD = 24{,}500$ lb T

▶4/27

$\theta = \tan^{-1}\frac{1}{2} = 26.6°$

Joint A 

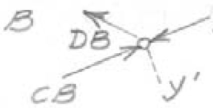
$\Sigma F_y = 0;\ AB \sin 26.6° - 1.8 \cos 15° = 0$

$\underline{AB = 3.89 \text{ kN } C}$

$\Sigma F_x = 0;\ AE = 1.8 \sin 15° + 3.89 \cos\theta$

$AE = 3.94 \text{ kN } T$

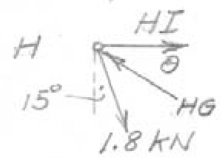
Joint E gives $EB = 0$

Joint B 

$\Sigma F_{y'} = 0$ so $\underline{DB = 0}$

$CB = 3.89 \text{ kN } C$

without diagonals, FD would lengthen & CJ shorten, so FD is tension & $CJ = 0$

Joint H 

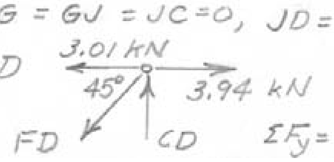
$\Sigma F_y = 0;\ HG \sin 26.6° - 1.8 \cos 15° = 0$

$HG = 3.89 \text{ kN } C$

$\Sigma F_x = 0;\ HI + 1.8 \sin 15° - 3.89 \cos 26.6° = 0$

$HI = 3.01 \text{ kN } T$

with $IG = GJ = JC = 0$, $JD = HI$

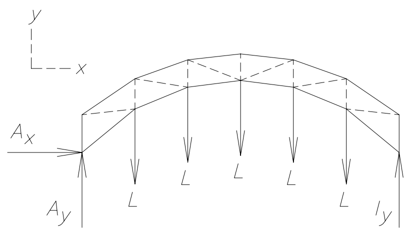
Joint D 

$\Sigma F_x = 0;\ FD \cos 45° + 3.01 = 3.94$

$FD = 1.318 \text{ kN } T$

$\Sigma F_y = 0;\ CD = 1.318 \sin 45° = \underline{0.932 \text{ kN } C}$

▶4/28



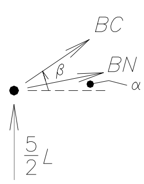
By symmetry, $A_y = I_y = 5/2L$, $A_x = 0$
By inspection of point A , AN = 0

Coordinate origin is at the center of the two concentric arcs.

Location of N: $x^2 + y^2 = 16^2$
For x_N = -8 m: $y_N^2 = -8^2 + 16^2$
y_N = 13.86 m N = (-8, 13.86) m

Location of A: $y_A^2 = -12^2 + 16^2$, y_A = 10.58 m, A = (-12, 10.58) m
Location of B: $y_B^2 = -12^2 + 18^2 =$, y_B = 13.42 m, B = (-12, 13.41) m
Location of C: $y_C^2 = -8^2 + 18^2 =$, y_C = 16.12 m, C = (-8, 16.12) m
Location of D: $y_D^2 = -4^2 + 18^2 =$, y_D = 17.55 m, C = (-4, 17.55) m
Location of M: $y_M^2 = -4^2 + 16^2 =$, y_M = 15.49 m, C = (-4, 15.49) m

Joint B:



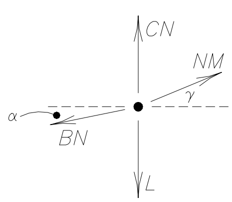
$$\alpha = \tan^{-1}\frac{13.86-13.42}{-8+12} = 6.28°$$

$$\beta = \tan^{-1}\frac{16.12-13.42}{-8+12} = 34.1°$$

$$\begin{cases} \Sigma F_x = 0: \ BC\cos 34.1° + BN\cos 6.28° = 0 \\ \Sigma F_y = 0: \ (5/2)L + BC\sin 34.1° + BN\sin 6.28° = 0 \end{cases}$$

Solve simultaneously to obtain: $BN = 4.44L\ T$, $BC = -5.32L\ (C)$

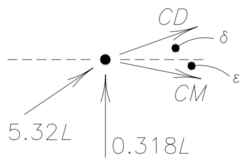
Joint N:



$$\gamma = \tan^{-1}\frac{15.49-13.86}{-4+8} = 22.2°$$

$$\begin{cases} \Sigma F_x = 0: & -4.44L\cos 6.28° + NM\cos 22.2° = 0, \quad NM = 4.76L\ T \\ \Sigma F_y = 0: & -4.44L\sin 6.28° + 4.76\sin 22.2° + CN - L = 0, \quad CN = 0.318L\ C \end{cases}$$

Joint C:



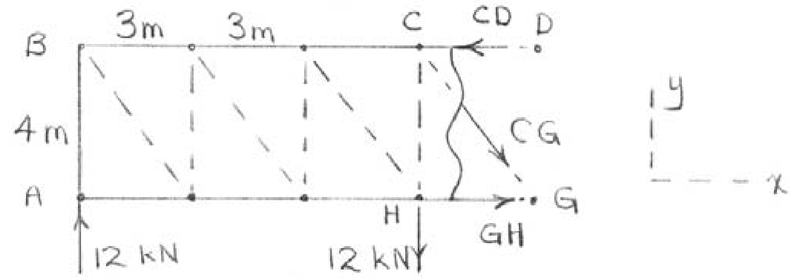
$$\delta = \tan^{-1}\frac{17.55-16.12}{-4+8} = 19.61°$$

$$\varepsilon = \tan^{-1}\frac{16.12-15.49}{8-4} = 8.99°$$

$$\begin{cases} \Sigma F_x = 0: & 5.32L\cos 34.1° + CD\cos 19.61° + CM\cos 8.99° = 0 \\ \Sigma F_y = 0: & 5.32L\sin 34.1° + 0.318L + CD\sin 19.61° - CM\sin 8.99° = 0 \end{cases}$$

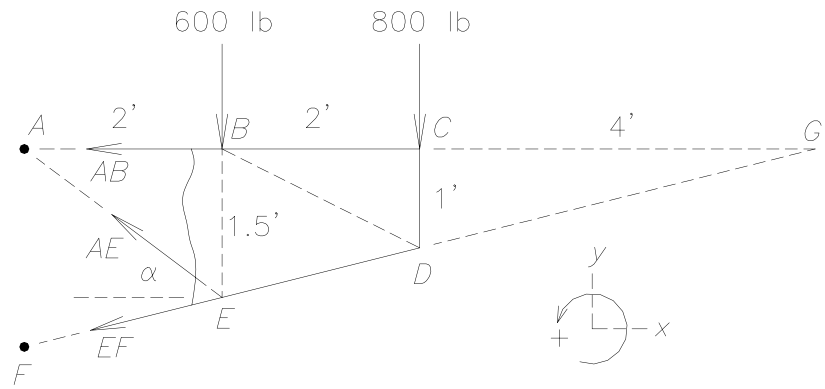
Solve simultaneously to obtain: $\underline{CM = 3.41L\ T}$

4/29 From truss as a whole, $A_y = F_y = 12$ kN and $A_x = 0$. Left section:



By inspection of $\Sigma F_y = 0$, $\underline{CG = 0}$.

$\circlearrowleft \Sigma M_C = 0: \; GH(4) - 12(9) = 0, \quad \underline{GH = 27 \text{ kN T}}$



$$\alpha = \tan^{-1}\frac{1.5}{2} = 36.9^\circ$$

$$\Sigma M_G = 0: \ 800(4) + 600(6) - AE\cos 36.9^\circ(1.5) - AE\sin 36.9^\circ(6) = 0$$

$$\underline{AE = 1417 \text{ lb } T}$$

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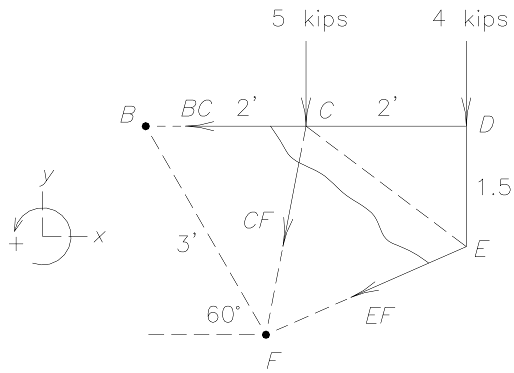
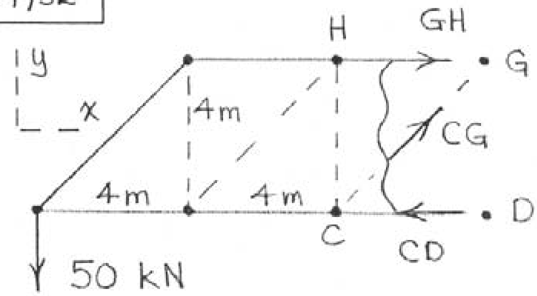


$$\Sigma M_F = 0: \ BC(3\sin 60°) - 5(0.5) - 4(2.5) = 0, \ \underline{BC = 4.81 \text{ kips } T}$$

4/32

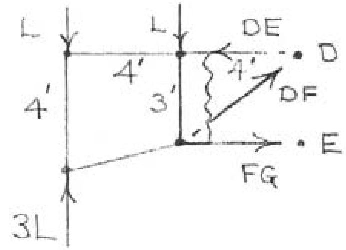


$\Sigma F_y = 0: \quad CG \sin 45° - 50 = 0, \quad \underline{CG = 70.7 \text{ kN } T}$

$\Sigma M_C = 0: \quad GH(4) - 50(8) = 0, \quad \underline{GH = 100 \text{ kN } T}$

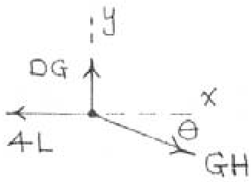
All members except EF are statically determinate, so above solution is unaffected by the redundant support.

4/33 From entire truss, $A = B = 3L$.



$$\curvearrowleft + \sum M_D = 0: \; FG(3) + L(4+8) - 3L(8) = 0, \quad FG = 4L \; T$$

Joint G:



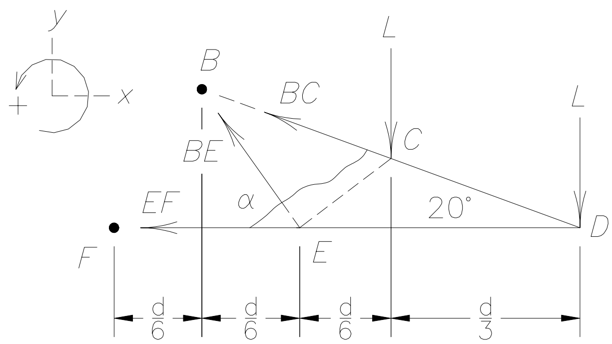
$$\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.04^\circ$$

$$\sum F_x = 0: \; -4L + GH \cos\theta = 0$$

$$GH = 4.12L \; T$$

$$\sum F_y = 0: \; DG - 4.12 \sin\theta = 0$$

$$\underline{DG = L \; T}$$



$$\alpha = \tan^{-1}\frac{\frac{4}{6}d\tan 20°}{\frac{d}{6}} = 55.5°$$

$$\Sigma M_D = 0: \; L\left(\frac{d}{3}\right) - BE\left(\frac{d}{2}\right)(\sin 55.5°) = 0$$

$$\underline{BE = 0.809L \; T}$$

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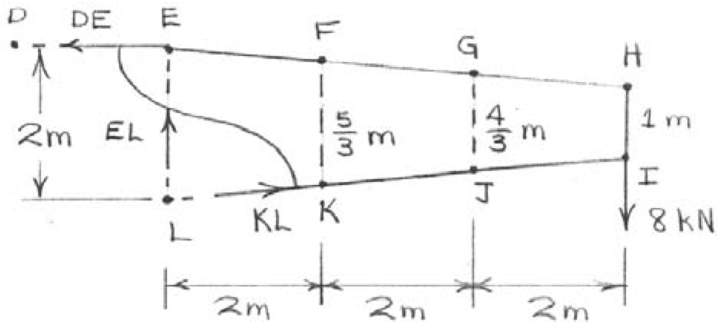
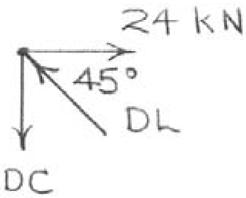


$\Sigma M_L = 0: \; DE(2) - 8(6) = 0, \quad \underline{DE = 24 \text{ kN } T}$

Joint D:



$\overset{+}{\rightarrow} \Sigma F_x = 0: \; 24 - DL \cos 45° = 0$

$\underline{DL = 33.9 \text{ kN } C}$

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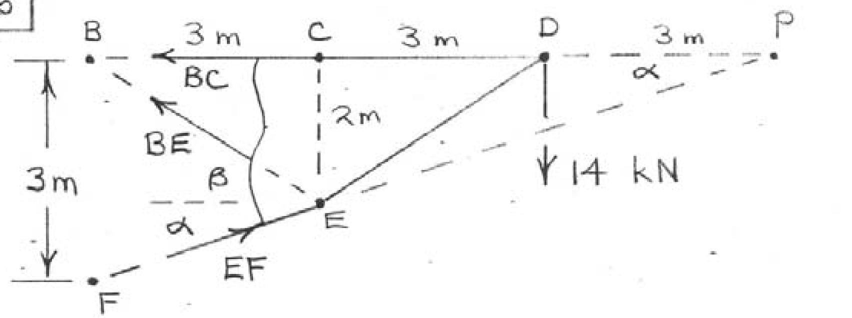

$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43°$, $\beta = \tan^{-1}\frac{2}{3} = 33.7°$

$\circlearrowleft{+}\ \Sigma M_E = 0:\ BC(2) - 14(3) = 0$, $\underline{BC = 21 \text{ kN T}}$

$\circlearrowleft{+}\ \Sigma M_P = 0: -BE\sin\beta(9) + 14(3) = 0$, $\underline{BE = 8.41 \text{ kN T}}$

$\circlearrowleft{+}\ \Sigma M_B = 0:\ EF\cos\alpha(3) - 14(6) = 0$, $\underline{EF = 29.5 \text{ kN C}}$

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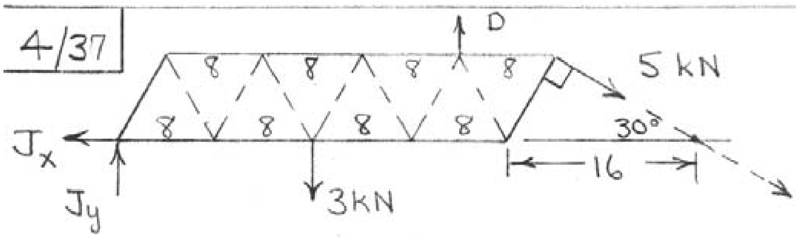
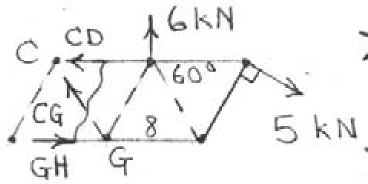


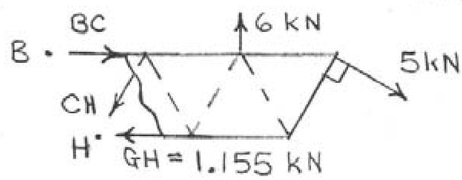
$$\Sigma M_J = 0: \quad -3(16) + D(28) - (5\sin 30°)(48) = 0$$
$$D = 6 \text{ kN}$$



$$\Sigma F_y = 0: \quad CG\sin 60° + 6 - 5\sin 30° = 0$$
$$\underline{CG = -4.041 \text{ kN} \quad C}$$

$$\Sigma M_G = 0: \quad CD(8\sin 60°) + 6(4) - (5\sin 30°)(24) = 0, \quad \underline{CD = 5.20 \text{ kN} \quad T}$$

$$\Sigma M_C = 0: \quad GH(8\sin 60°) + 6(8) - (5\sin 30°)(16) = 0$$
$$GH = -1.155 \text{ kN} \quad T$$



$$\Sigma M_H = 0: \quad 6(12) - (5\sin 30°)(32) - BC\ 8\sin 60° = 0$$
$$\underline{BC = -1.155 \text{ kN} \quad T}$$

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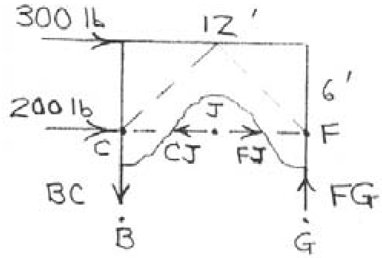


$$\circlearrowleft + \sum M_C = 0: \; -300(6) + FG(12) = 0$$

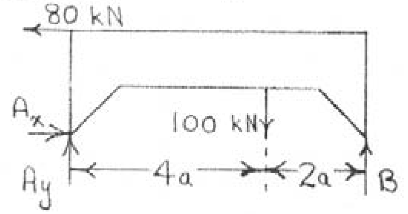
$\underline{FG = 150 \text{ lb } C}$

$$\sum M_F = 0: \; -300(6) + BC(12) = 0$$

$\underline{BC = 150 \text{ lb } T}$

4/39 $\circlearrowleft \Sigma M_A = 0: \quad B(6a) - 100(4a) + 80(2a) = 0$

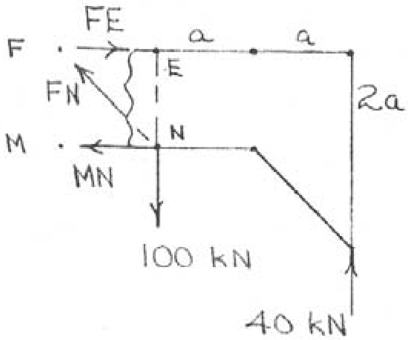
$B = 40 \text{ kN}$



For section,

$\Sigma F_y = 0: \quad \frac{FN}{\sqrt{2}} + 40 - 100 = 0$

$\underline{FN = 84.8 \text{ kN T}}$



$\Sigma M_E = 0: \quad 40(2a) - 84.8 \frac{a}{\sqrt{2}} - MN(a) = 0, \quad \underline{MN = 20 \text{ kN T}}$

For section through GF & LM, $\Sigma F_y = 0$ gives $\underline{GM = 84.8 \text{ kN T}}$.

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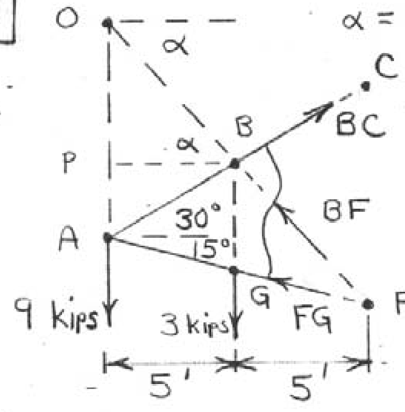

$$\alpha = \tan^{-1}\left[\frac{5\tan 30^\circ + 2(5\tan 15^\circ)}{5}\right] = 48.1^\circ$$

$$\overline{AO} = \overline{AP} + \overline{PO} = \overline{AP} + \overline{PB}\tan\alpha = 5\tan 30^\circ + 5\tan\alpha = 8.45'$$

$$\Sigma M_A = 0: (BF\cos\alpha)\,\overline{AO} - 3(5) = 0$$

$$\underline{BF = 2.66 \text{ kips } C}$$

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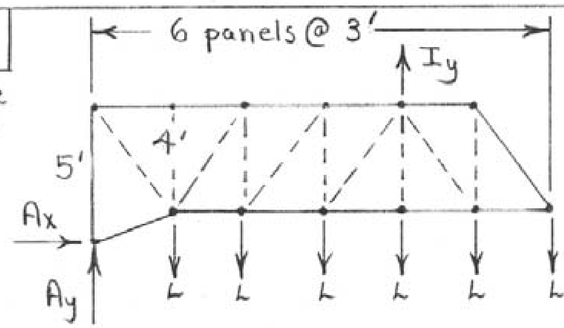
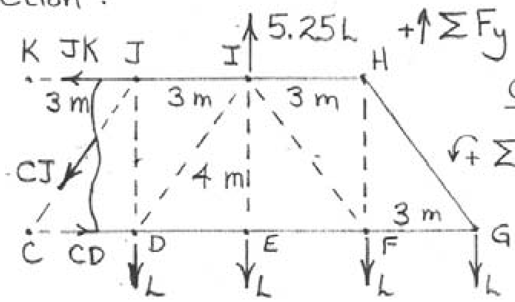
Entire truss:



$$\circlearrowleft + \Sigma M_A = 0: \quad I_y(12) - L(3+6+9+12+15+18) = 0$$

$$I_y = 5.25L$$

Section:



$$+\uparrow \Sigma F_y = 0: \quad 5.25L - 4L - CJ\left(\tfrac{4}{5}\right) = 0$$

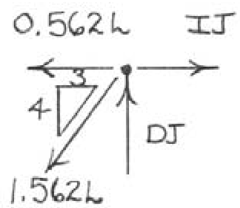
$$\underline{CJ = 1.562L \ T}$$

$$\circlearrowleft + \Sigma M_J = 0: \quad CD(4) + 5.25L(3) - L(3+6+9) = 0$$

$$\underline{CD = 0.562L \ C}$$

From $\overset{+}{\rightarrow} \Sigma F_x = 0$, $JK = 0.562L \ T$.

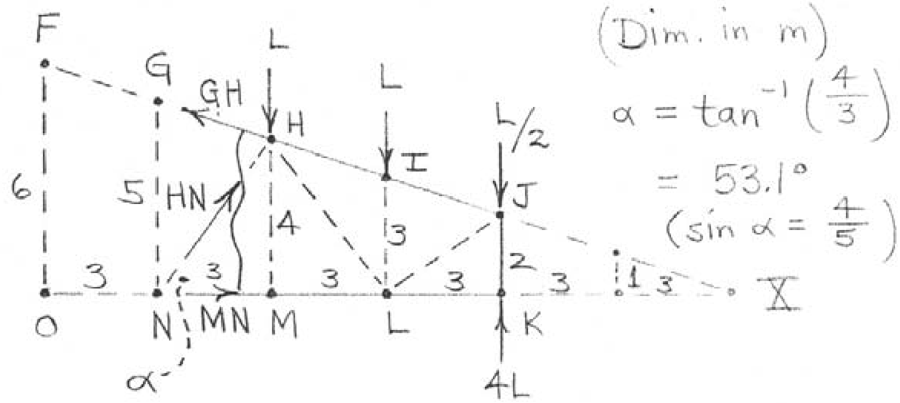
Joint J:



$$+\uparrow \Sigma F_y = 0: \quad DJ - 1.562L\left(\tfrac{4}{5}\right) = 0$$

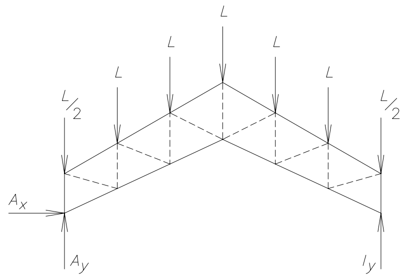
$$\underline{DJ = 1.250L \ C}$$

4/42 From the truss as a whole, the external reactions at A and K are 4L (up).



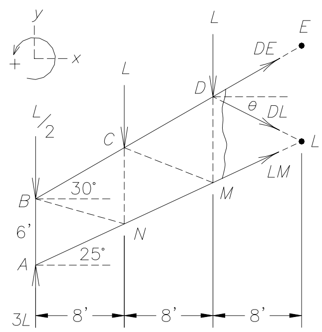
$$\curvearrowleft{+}\ \Sigma M_X = 0: \ \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12)$$

$$-HN\left(\frac{4}{5}\right)(15) = 0, \quad \underline{HN = 0}$$



By symmetry, $A_y = I_y = 3L$

$\Sigma F_x = 0: \ A_x = 0$



$$\theta = \tan^{-1}\frac{6+16\tan 30° - 24\tan 25°}{8} = 26.8°$$

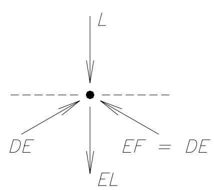
$\Sigma M_L = 0: \ L(8) + L(16) + \frac{L}{2}(24) - 3L(24) - DE(24\tan 30° - 24\tan 25° + 6)\sin 60° = 0$

$DE = -4.80L$ or $\underline{4.80L\ C}$

$$\begin{cases} \Sigma F_x = 0: \ -4.80L\cos 30° + DL\cos 26.8° + LM\cos 25° = 0 \\ \Sigma F_y = 0: \ -4.80L\sin 30° - DL\sin 26.8° + LM\sin 25° + 3L - 2L - \frac{L}{2} \end{cases}$$

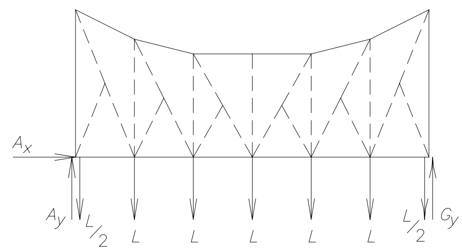
Solve simultaneously to obtain: $\underline{DL = 0.0446L\ T}$, $\underline{LM = 4.54L\ T}$

Joint E:



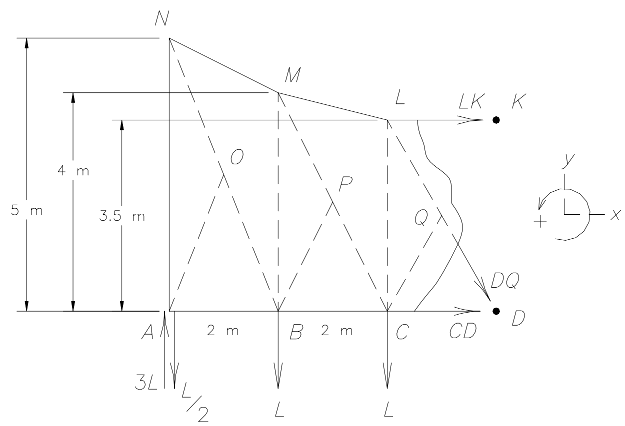
$\Sigma F_y = 0: \ 2(4.80L\sin 30°) - L - EL = 0$

$\underline{EL = 3.80L\ T}$



By symmetry, $A_y = G_y = 3L$

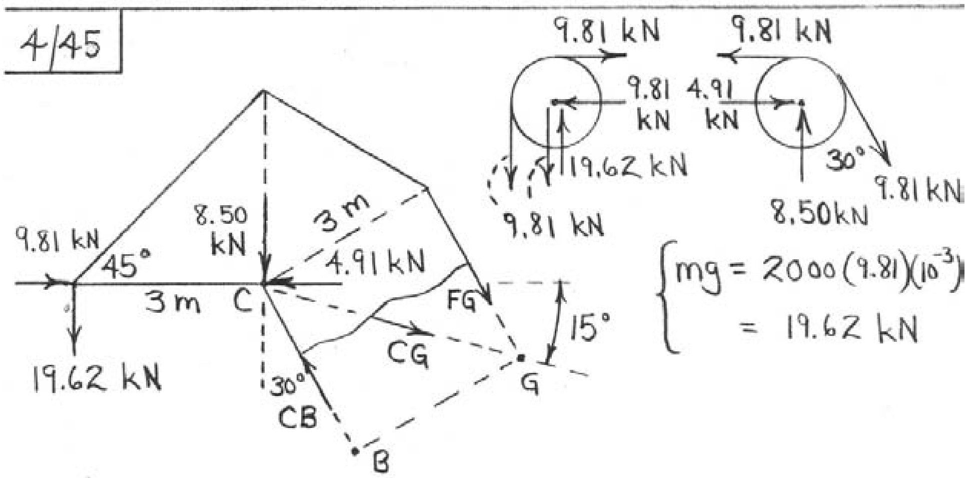
$\Sigma F_x = 0: \; A_x = 0$



$$\Sigma F_y = 0: \; 3L - 2L - \frac{L}{2} - DQ\sin\left(\tan^{-1}\frac{3.5}{2}\right) = 0, \; \underline{DQ = 0.576L \; T}$$

By inspection: $\underline{CQ = 0}$

4/45



$$\begin{cases} mg = 2000(9.81)(10^{-3}) \\ \quad = 19.62 \text{ kN} \end{cases}$$

$\curvearrowleft + \Sigma M_C = 0: \quad 19.62(3) - FG(3) = 0, \quad \underline{FG = 19.62 \text{ kN } T}$

$\curvearrowleft + \Sigma M_G = 0: \quad 19.62(3 + 3\sqrt{2}\cos 15°) - (9.81 - 4.91)(3\sqrt{2}\sin 15°)$

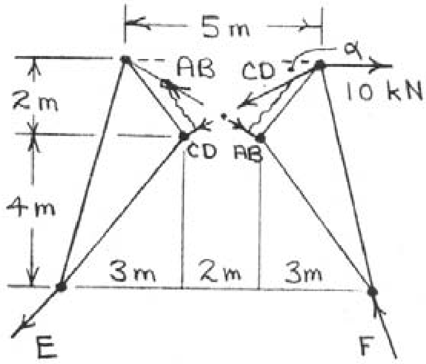
$\quad + 8.50(3\sqrt{2}\cos 15°) - CB(3) = 0$

$\underline{CB = 56.2 \text{ kN } C}$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad CG\cos 15° + 19.62\sin 30° - 56.2\sin 30°$

$\quad + 9.81 - 4.91 = 0, \quad \underline{CG = 13.87 \text{ kN } T}$

4/46 $\quad \alpha = \tan^{-1}\left(\frac{2}{3.5}\right) = 29.7°$



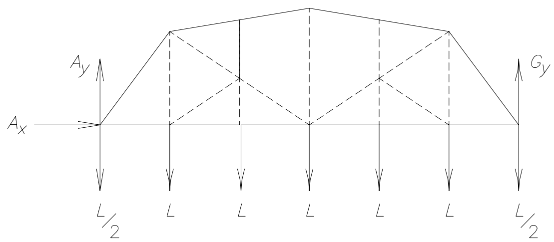
I. $\Sigma M_E = 0: \ CD(4\cos\alpha) - CD(3\sin\alpha) - AB(6\cos\alpha) - AB(1.5\sin\alpha) = 0$

$CD = 3.00\ AB$

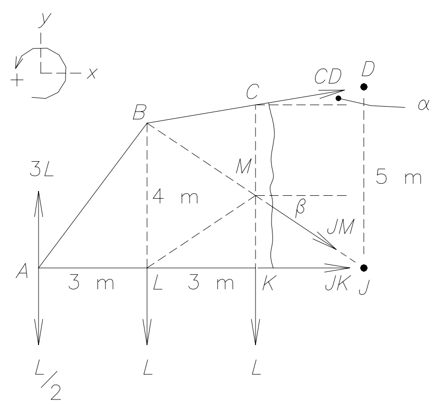
II. $\Sigma M_F = 0: \quad 10(6) + AB(4\cos\alpha) - AB(3\sin\alpha) - CD(6\cos\alpha) - CD(1.5\sin\alpha) = 0$

$60 + 1.985\ AB - 5.954\ CD = 0$

Solving simultaneously, $\quad \underline{AB = 3.78 \text{ kN } C}$



By symmetry, $A_y = G_y = 3L$ $\qquad\qquad$ $\Sigma F_x = 0: \; A_x = 0$



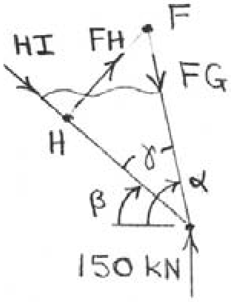
$$\alpha = \tan^{-1}\frac{1}{6} = 9.46°, \; \beta = \tan^{-1}\frac{4}{6} = 33.7°$$

$$\Sigma M_J = 0: \; \frac{L}{2}(9) + L(6) + L(3) - 3L(9) - CD(5)\left[\sin\left(90° - 9.46°\right)\right] = 0$$

$$CD = -2.74L\ (C)$$

$$\Sigma F_y = 0: \; 3L + CD\sin 9.46° - \frac{L}{2} - 2L - JM\sin 33.7° = 0, \; \underline{JM = 0.0901L\ T}$$

4/48 By symmetry, $A = G = 150$ kN



$\curvearrowleft \Sigma M_F = 0: \ 150(4) + HI\,(7.902) = 0$

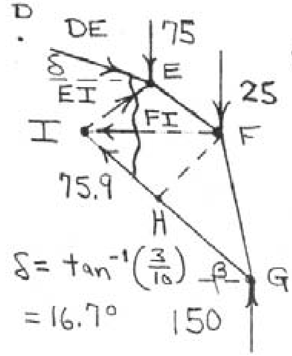
$\underline{HI = -75.9 \text{ kN } T}$

(Note: $\alpha = \tan^{-1}\left(\frac{14}{4}\right) = 74.1°$

$\beta = \tan^{-1}\left(\frac{14}{16}\right) = 41.2°$

$\gamma = \alpha - \beta = 32.9°$

Then $d_\perp = FG \sin\gamma = \sqrt{14^2 + 4^2}\,\sin\gamma = 7.902$ m)



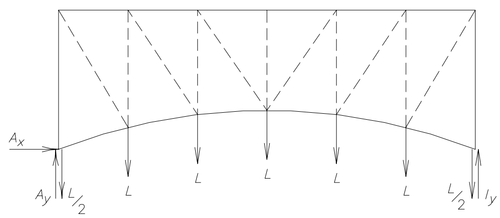
$\delta = \tan^{-1}\left(\frac{3}{10}\right) = 16.7°$

$\curvearrowleft^{+} \Sigma M_E = 0: -25(6) + 150(10)$
$- FI\,(4) - (75.9 \sin\beta)(6) - (75.9 \cos\beta)(4)$
$= 0, \quad \underline{FI = 205 \text{ kN } T}$

$\Sigma M_I = 0: \ -75(6) - 25(12) + 150(16)$
$-(DE \cos\delta)(4) - (DE \sin\delta)(6) = 0$

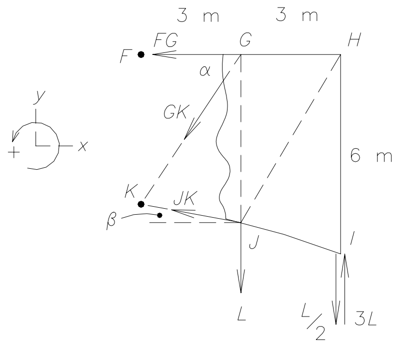
$\underline{DE = 297 \text{ kN } C}$

$\Sigma F_y = 0: \ -75 - 25 + 150 - 297 \sin\delta + 75.9 \sin\beta$
$+ EI \frac{4}{\sqrt{52}} = 0, \quad \underline{EI = -26.4 \text{ kN } T}$



By symmetry, $A_y = H_y = 3L$

$\Sigma F_x = 0: \ A_x = 0$



Origin at center of arc

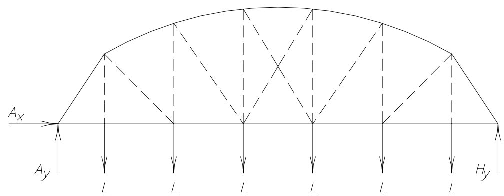
Location of I: $y_I^2 = 25^2 - 9^2$, $B = (9, 23.3)$ m
Location of J: $y_J^2 = 25^2 - 6^2$, $J = (6, 24.3)$ m
Location of G: $y_G = y_I + 6$, $G = (6, 29.3)$ m
Location of K: $y_K^2 = 25^2 - 3^2$, $K = (3, 24.8)$ m

$$\alpha = \tan^{-1}\frac{29.3 - 24.8}{3} = 56.3°, \ \beta = \tan^{-1}\frac{24.8 - 24.3}{3} = 10.39°$$

$$\Sigma M_K = 0: \ 3L(6) - \frac{L}{2}(6) + FG(y_G - y_K) - L(3) = 0, \ FG = -2.66L \ (C)$$

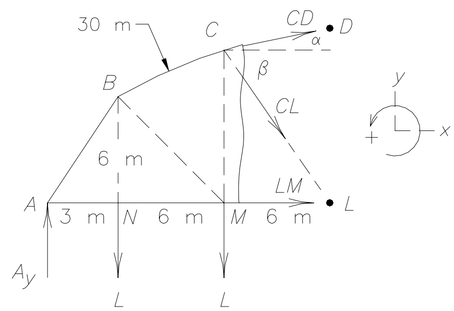
$$\begin{cases} \Sigma F_x = 0: \ 2.66L - GK\cos 56.3° - JK\cos 10.39° = 0 \\ \Sigma F_y = 0: \ \dfrac{3}{2}L + JK\sin 10.39° - GK\sin 56.3° \end{cases}$$

Solve simultaneously to obtain: $\underline{GK = 2.13L\ T}$



By symmetry, $A_y = H_y = 3L$

$\Sigma F_x = 0: \ A_x = 0$



Origin at center of arc

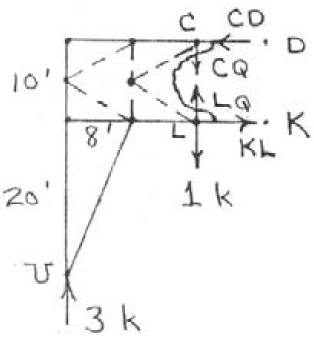
Location of B: $y_B^2 = 30^2 - (-15)^2$, $B = (-15, 26.0)$ m
Location of A: $y_A = y_B - 6$, $A = (-18, 20.0)$ m
Location of C: $y_C^2 = 30^2 - (-9)^2$, $C = (-9, 28.6)$ m
Location of M: $y_M = y_A$, $M = (-9, 20.0)$ m
Location of D: $y_D^2 = 30^2 - (-3)^2$, $D = (-3, 29.8)$ m
Location of L: $y_L = y_A$, $L = (-3, 20.0)$ m

$$\alpha = \tan^{-1}\frac{29.8 - 28.6}{6} = 11.60°, \ \beta = \tan^{-1}\frac{28.6 - 20.0}{6} = 55.2°$$

$$\Sigma M_L = 0: \ -3L(15) + L(6) + L(12) - CD(29.8 - 20.0)\sin(90° - 11.60°) = 0$$
$$CD = -2.79L \text{ or } CD = 2.79L \ C$$

$$\Sigma F_y = 0: \ 3L - L - L - 2.79L\sin 11.60° - CL\sin 55.2° = 0, \ \underline{CL = 0.534L \ T}$$

▸4/51 From truss as a whole, $\left\{ \begin{array}{l} U = 3 \text{ kips} \\ V = 4 \text{ kips} \end{array} \right.$



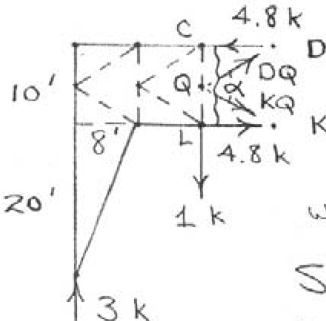
$\circlearrowleft + \Sigma M_C = 0: \; KL(10) - 3(16) = 0$

$KL = 4.8 \text{ kips } T$

$\Sigma M_L = 0: \; CD(10) - 3(16) = 0$

$CD = 4.8 \text{ kips } C$

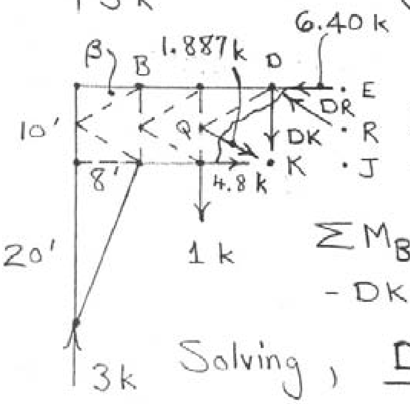
(From a similar right-hand section, $DE = 6.40$ kips C.)



$\Sigma M_D = 0: \; -3(24) + 1(8) + KQ\left(\sqrt{8^2+5^2}\, \sin\alpha\right) + 4.8(10) = 0,$

where $\alpha = 180 - 2\tan^{-1}\left(\frac{8}{5}\right) = 64.0°$

Solving, $KQ = 1.887 \text{ kips } T$



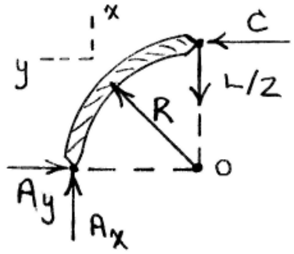
$\beta = \tan^{-1}\left(\frac{5}{8}\right) = 32.0°$

$\Sigma F_x = 0: \; -6.40 + 1.887\cos\beta + 4.8 - DR\cos\beta = 0, \; DR = 0$

$\Sigma M_B = 0: \; -3(8) - 1(8) + 4.8(10) - DK(16) = 0$

Solving, $\underline{DK = 1 \text{ kip } T}$

► 4/52 By symmetry, the force which the right half exerts on the left half at C is horizontal:

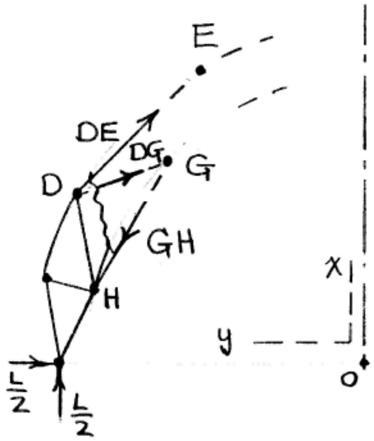


$$\Sigma M_A = 0: \; CR - \frac{L}{2}R = 0$$

$$C = L/2$$

$$\Sigma F_y = 0: \; -A_y + \frac{L}{2} = 0, \; A_y = \frac{L}{2}$$

$$\Sigma F_x = 0: \; A_x - \frac{L}{2} = 0, \; A_x = \frac{L}{2}$$



$$\underline{r}_{OD} + \underline{r}_{DE} = \underline{r}_{OE}$$

$$\therefore \underline{r}_{DE} = \underline{r}_{OE} - \underline{r}_{OD}$$

$$= 1.1R(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$- 1.1R(\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$$

$$= R(0.403\underline{i} - 0.403\underline{j})$$

So force $\underline{DE} = DE \dfrac{\underline{r}_{DE}}{r_{DE}}$

$$= DE(0.707\underline{i} - 0.707\underline{j})$$

Similarly, force $\underline{GH} = GH(-0.866\underline{i} + 0.500\underline{j})$

force $\underline{DG} = DG(0.264\underline{i} - 0.965\underline{j})$

$$\Sigma F_x = 0: \; \frac{L}{2} + 0.707\,DE - 0.866\,GH + 0.264\,DG = 0 \quad (1)$$

$$\Sigma F_y = 0: \; -\frac{L}{2} - 0.707\,DE + 0.500\,GH - 0.965\,DG = 0 \quad (2)$$

$$\Sigma M_O = 0: \; -\frac{L}{2}R\underline{k} + \underline{r}_{OD} \times (\underline{DE} + \underline{DG})$$

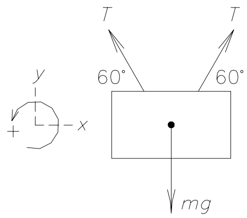
$$+ \underline{r}_{OH} \times \underline{GH},$$

where $\underline{r}_{OH} = 0.9R(\cos 75^\circ \underline{i} + \sin 75^\circ \underline{j})$

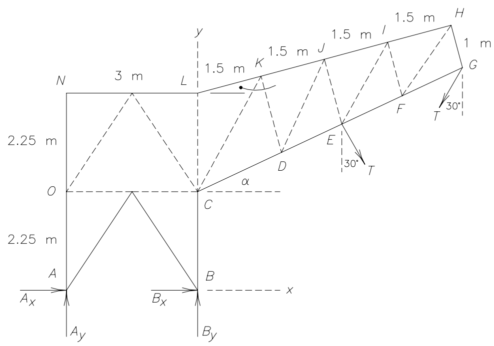
Carrying out the cross products and collecting terms: $-1.063\,DE + 0.869\,GH - 0.782\,DG = \frac{L}{2}$ (3)

Simultaneous solution of Eqs. (1)–(3):

DE = 0.839 L T, GH = 1.090 L C, $\underline{\text{DG} = -0.569\text{ L C}}$



$\Sigma F_y = 0: \; 2T \sin 60° - 1000(9.81) = 0, T = 5660 \text{ N}$



From $\Sigma F_x = 0$, $A_x + B_x = 0$

$C = (0, 2.25)$ m
$L = (0, 4.5)$ m
$H = (6\cos 15°, 4.5 + 6\sin 15°) = (5.80, 6.05)$ m
$G = (6\cos 15° + 1\cos 75°, 4.5 + 6\sin 15° - 1\sin 75°) = (6.05, 5.09)$ m

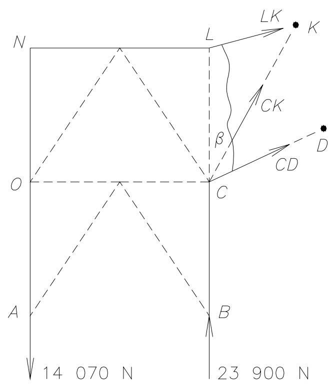
$$\alpha = \tan^{-1}\frac{5.09 - 2.25}{6.05} = 25.1°$$

$$FG = EF = DE = \frac{1.5}{\cos(25.1° - 15°)} = 1.524 \text{ m}$$

$E = (6\cos 15° + 1\cos 75° - 2(1.524\cos 25.1°), 4.5 + 6\sin 15° - 1\sin 75° - 2(1.524\sin 25.1°)) = (3.30, 3.79)$ m

$\Sigma M_B = 0: \; -A_y(3) - T\sin 60°(3.30) - T\cos 60°(3.79) - T\sin 60°(6.05) + T\cos 60°(5.09) = 0$
$A_y = -14\,070 \text{ N}$

$\Sigma F_y = 0: \; -14\,070 + B_y - 1000(9.81) = 0, B_y = 23\,900 \text{ N}$



$$\beta = \tan^{-1}\frac{1.5\cos 15°}{2.25 + 1.5\sin 15°} = 28.8°$$

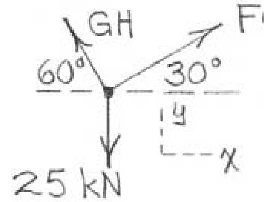
$$\Sigma M_C = 0: \; 14\,070(3) - LK(2.25)(\sin 75°) = 0, \; LK = 19\,420 \text{ N}$$

$$\begin{cases} \Sigma F_x = 0: \; 19\,420\cos 15° + \text{CK}\sin 28.8° + CD\cos 25.1° = 0 \\ \Sigma F_y = 0: \; -14\,070 + 23\,900 + 19\,420\sin 15° + CK\cos 28.8° + CD\sin 25.1° = 0 \end{cases}$$

Solve simultaneously to obtain: $CK = -9290$ N or $\underline{CK = 9290 \text{ N } C}$

▶ 4/54 $FI = 0$, by inspection of joint I.

Joint G:



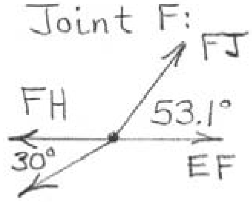
$$\begin{cases} \Sigma F_x = 0: -GH\left(\frac{1}{2}\right) + FG\left(\frac{\sqrt{3}}{2}\right) = 0 \\ \Sigma F_y = 0: GH\left(\frac{\sqrt{3}}{2}\right) + FG\left(\frac{1}{2}\right) - 25 = 0 \end{cases}$$

1st eq.: $GH = \sqrt{3}\, FG$

2nd eq.: $\sqrt{3}\, FG\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2} FG = 25$

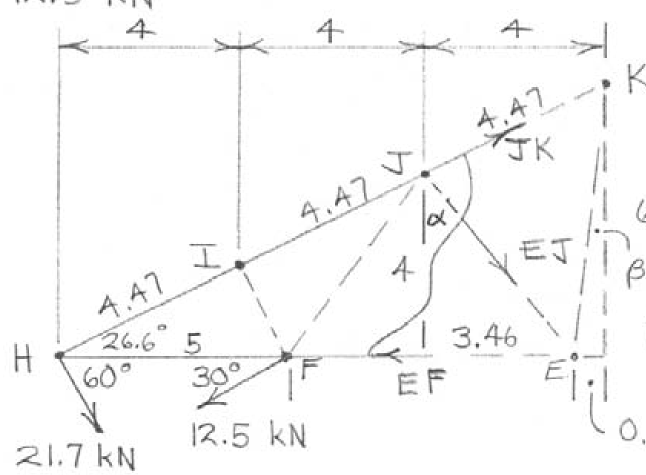
$\Rightarrow FG = 12.5$ kN T, $GH = 21.7$ kN T

Joint F:



$\Sigma F_y = 0: FJ(\sin 53.1°) - 12.5 \sin 30° = 0$

$FJ = 7.81$ kN T



$\alpha = \tan^{-1} \frac{3.46}{4} = 40.9°$

$\beta = \tan^{-1} \frac{0.54}{6} = 5.14°$

$\circlearrowleft + \Sigma M_H = 0: -12.5\left(\frac{1}{2}\right)(5) - EJ\left[\cos 40.9^\circ (8)\right.$
$\left. + \sin 40.9^\circ (4)\right] = 0, \quad EJ = -3.61 \text{ kN}$

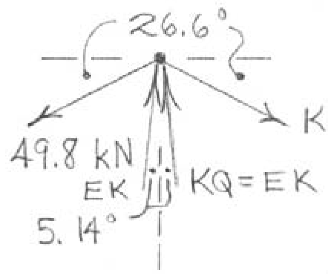
So $\underline{EJ = 3.61 \text{ kN } C}$

$\Sigma F_y = 0: \; JK \sin 26.6^\circ + 3.61 \cos 40.9^\circ - 12.5\left(\frac{1}{2}\right)$
$-21.7\left(\frac{\sqrt{3}}{2}\right) = 0, \quad JK = 49.8 \text{ kN } T$

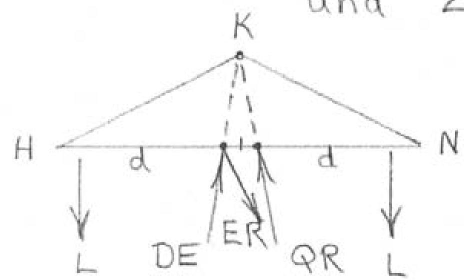
Joint K, using symmetry:

$\Sigma F_y = 0: -2(49.8) \sin 26.6^\circ +$
$2 EK \cos 5.14^\circ = 0,$

$\underline{EK = 22.4 \text{ kN } C}$



By symmetry of loads L
and $\Sigma M_K = 0$, $\underline{ER = 0}$



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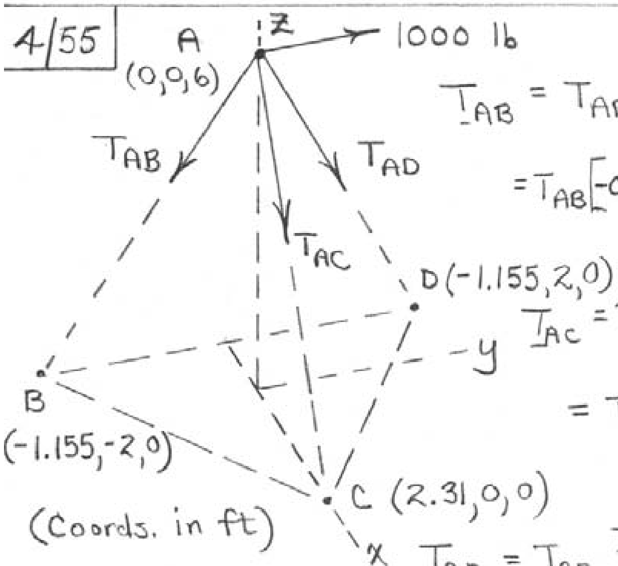


$$\underline{T}_{AB} = T_{AB}\frac{-1.155\underline{i} - 2\underline{j} - 6\underline{k}}{\sqrt{1.155^2 + 2^2 + 6^2}} = T_{AB}\left[-0.1796\underline{i} - 0.311\underline{j} - 0.933\underline{k}\right]$$

$$\underline{T}_{AC} = T_{AC}\frac{2.31\underline{i} - 6\underline{k}}{\sqrt{2.31^2 + 6^2}} = T_{AC}\left[0.359\underline{i} - 0.933\underline{k}\right]$$

$$\underline{T}_{AD} = T_{AD}\frac{-1.155\underline{i} + 2\underline{j} - 6\underline{k}}{\sqrt{1.155^2 + 2^2 + 6^2}} = T_{AD}\left[-0.1796\underline{i} + 0.311\underline{j} - 0.933\underline{k}\right]$$

$\Sigma F_x = 0: \; -0.1796\,T_{AB} + 0.359\,T_{AC} - 0.1796\,T_{AD} = 0 \quad (1)$

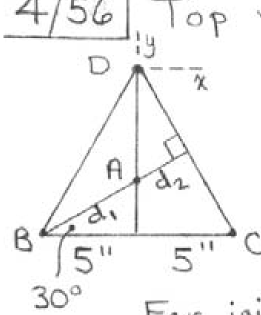
$\Sigma F_y = 0: \; 1000 - 0.311\,T_{AB} + 0.311\,T_{AD} = 0 \quad (2)$

$\Sigma F_z = 0: \; -0.933\,T_{AB} - 0.933\,T_{AC} - 0.933\,T_{AD} = 0 \quad (3)$

Solve Eqs. (1)–(3):

$$\begin{cases} T_{AB} = 1607 \text{ lb} \quad (T) \\ T_{AC} = 0 \\ T_{AD} = -1607 \text{ lb} \quad (C) \end{cases}$$

4/56 Top view of base:



$\cos 30° = \dfrac{d_1 + d_2}{10}$, $d_1 + d_2 = 8.66''$

$\cos 30° = \dfrac{5}{d_1}$, $d_1 = 5.77''$

$d_2 = 8.66 - 5.77 = 2.89''$

For joint A, assuming symmetry:

$$\mathbf{F}_{BA} = P\left[\frac{5\mathbf{i} + 2.89\mathbf{j} + 16\mathbf{k}}{(5^2 + 2.89^2 + 16^2)^{1/2}}\right] = P(0.294\,\mathbf{i} + 0.1697\,\mathbf{j} + 0.941\,\mathbf{k})$$

$\mathbf{F}_{CA} = P(-0.294\,\mathbf{i} + 0.170\,\mathbf{j} + 0.941\,\mathbf{k})$, $\mathbf{F}_{DA} = P(-0.339\,\mathbf{j} + 0.941\,\mathbf{k})$

$\Sigma F_z = 0$ at A: $3(0.941\,P) - 800 = 0$, $P = 283$ lb

For joint C, assuming symmetry:

$\mathbf{F}_{BC} = -Q\mathbf{i}$, $\mathbf{F}_{CD} = Q(-\sin 30°\,\mathbf{i} + \cos 30°\,\mathbf{j})$

Normal $\mathbf{N} = 267\mathbf{k}$ lb

$\Sigma \mathbf{F} = \mathbf{0}$ at C: $\mathbf{N} + \mathbf{F}_{BC} + \mathbf{F}_{CD} + \mathbf{F}_{AC} = \mathbf{0}$

$267\mathbf{k} - Q\mathbf{i} + 283(0.294\,\mathbf{i} - 0.1697\,\mathbf{j} - 0.941\,\mathbf{k})$
$+ Q(-0.5\,\mathbf{i} + 0.866\,\mathbf{j}) = \mathbf{0}$

Solving, $Q = 55.6$ lb $\Rightarrow$ $\underline{BC = BD = CD = 55.6 \text{ lb } T}$

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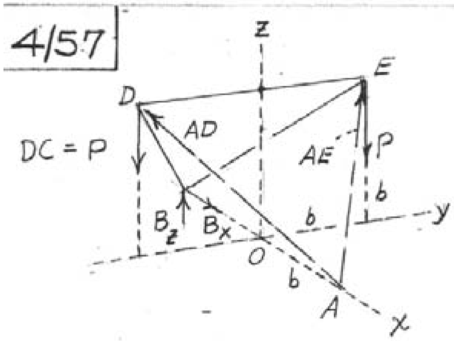

For truss as a whole

$\Sigma M_x = 0$ gives $DC = P$

$\Sigma M_y = 0$ & $\Sigma F_z = 0$ gives $B_z = P$

For unit BDE

$\Sigma M_{DE} = 0$; $B_z b - B_x b = 0$

$\underline{B_x = B_z = P}$

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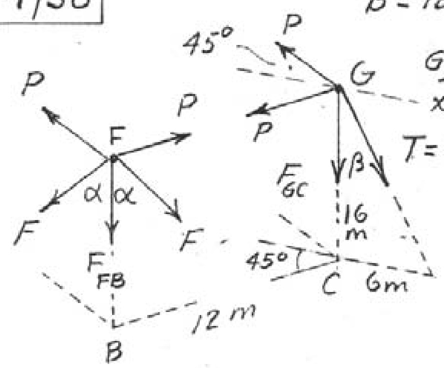


$\beta = \tan^{-1}\frac{6}{16} = 20.6°, \ \sin\beta = 0.351$

$G; \ \Sigma F_x = 0; \ 9(0.351) - 2P(0.707) = 0$

$P = 2.235 \text{ kN}$

$F; \ \sin\alpha = 3/5 = 0.6$

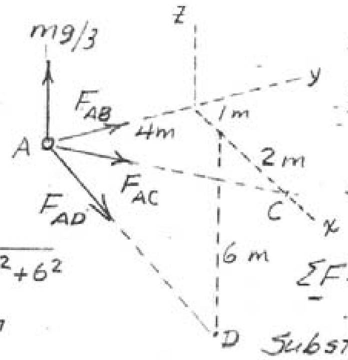
$\Sigma F_{FG} = 0; \ P + F\sin\alpha = 0$

$F = -2.235/0.6 = \underline{-3.72 \text{ kN}}$

4/59 From truss as a whole $\Sigma M_y = 0$ gives tension in vertical wire at C $T_C = \frac{1}{3} mg$

By symmetry & $\Sigma F_z = 0$; $T_A = T_B = \frac{1}{3} mg$

Joint A;



$\underline{F}_{AB} = F_{AB}\underline{j}$

$\underline{F}_{AC} = \frac{F_{AC}}{5}(3\underline{i} + 4\underline{j})$

$\underline{F}_{AD} = \frac{F_{AD}}{\sqrt{53}}(\underline{i} + 4\underline{j} - 6\underline{k})$

$\overline{AD} = \sqrt{4^2 + 1^2 + 6^2} = \sqrt{53}$ m

$\Sigma \underline{F} = \underline{0}$; $\frac{mg}{3}\underline{k} + \underline{F}_{AB} + \underline{F}_{AC} + \underline{F}_{AD} = \underline{0}$

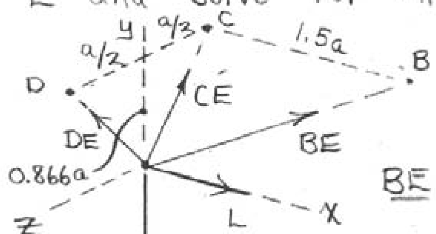
Substitute & collect terms to get

$$\left(\frac{3F_{AC}}{5} + \frac{F_{AD}}{\sqrt{53}}\right)\underline{i} + \left(F_{AB} + \frac{4F_{AC}}{5} + \frac{4F_{AD}}{\sqrt{53}}\right)\underline{j} + \left(\frac{mg}{3} - \frac{6F_{AD}}{\sqrt{53}}\right)\underline{k} = \underline{0}$$

Equate coefficients of $\underline{i}$-, $\underline{j}$-, & $\underline{k}$-terms to zero & get

$F_{AD} = \frac{\sqrt{53}}{18} mg$, $\underline{F_{AC} = -\frac{5}{54} mg}$, $\underline{F_{AB} = -\frac{4}{27} mg}$

4/60 The truss as a whole is statically determinate with six supporting constraints. $j = 6$ & $m = 12$; $3j = m + 6$, so there are sufficient members for stability. C, B, and D are fixed so E is fixed. A and F are also fixed, so the truss is a rigid unit. From an inspection of joint F, $AF = 0$, $\underline{BF = 0}$, $EF = LT$. From an inspection of joint A, $AB = AD = \underline{AE = 0}$. We can now go to joint E and solve for all unknowns there:



$\underline{CE} = CE[0.866\underline{j} - 0.5\underline{k}]$

$\underline{DE} = DE[0.866\underline{j} + 0.5\underline{k}]$

$$\underline{BE} = BE\left[\frac{1.5\underline{i} + 0.866\underline{j} - 0.5\underline{k}}{\sqrt{1.5^2 + 0.866^2 + 0.5^2}}\right]$$

$$= BE[0.832\underline{i} + 0.480\underline{j} - 0.277\underline{k}]$$

$$\left.\begin{array}{l} \Sigma F_x = 0:\ 0.832BE + L = 0 \\ \Sigma F_y = 0:\ 0.480BE + 0.866CE + 0.866DE - L = 0 \\ \Sigma F_z = 0:\ -0.277BE - 0.5CE + 0.5DE = 0 \end{array}\right\}$$

Solution: $\underline{BE = -1.202L\ (C)}$, $\underline{CE = 1.244L\ T}$

$DE = 0.577L\ T$

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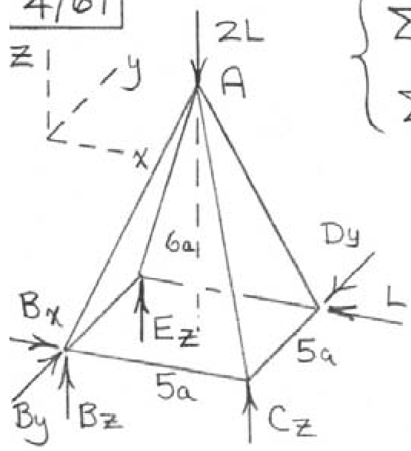
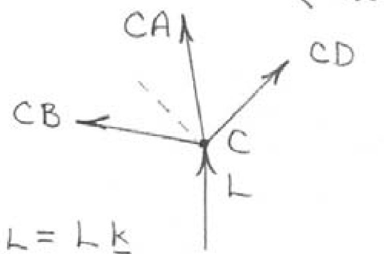


$$\begin{cases} \Sigma M_{BE} = 0 \Rightarrow C_z = L \\ \Sigma M_{B_z} = 0 \Rightarrow D_y = L \end{cases}$$

Joint C : (tensions assumed)



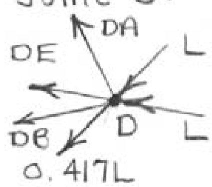
$\underline{CB} = -CB\,\underline{i}$, $\underline{CD} = CD\,\underline{j}$, $\underline{L} = L\,\underline{k}$

$$\underline{CA} = CA\left(\frac{-2.5a\,\underline{i} + 2.5a\,\underline{j} + 6a\,\underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}}\right) = CA(-0.359\,\underline{i} + 0.359\,\underline{j} + 0.862\,\underline{k})$$

$\Sigma \underline{F} = \underline{0}$ yields:

$$\left.\begin{aligned} \underline{i}&: -CB - 0.359\,CA = 0 \\ \underline{j}&: \quad CD + 0.359\,CA = 0 \\ \underline{k}&: \quad L + 0.862\,CA = 0 \end{aligned}\right\} \begin{aligned} CA &= -0.1667L \\ CD &= +0.417L \end{aligned}$$

Joint D:



$\Sigma \underline{F} = \underline{0}$ yields

$$\begin{cases} \underline{i}: -DE - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}: -0.417L - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}: \; 0.862\,DA = 0 \end{cases}$$

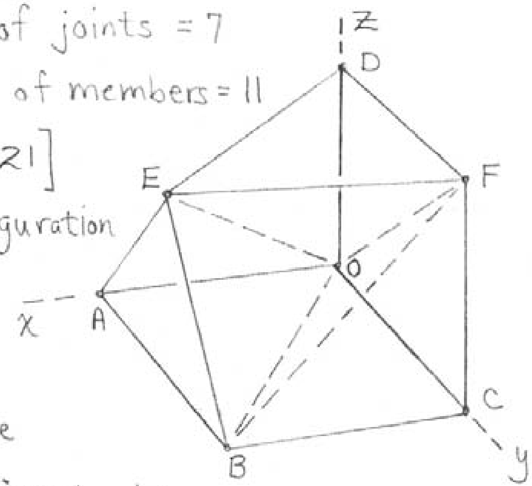
$DA = 0$, $\underline{DB = -2.00L}$

4/62 j = number of joints = 7

m_i = initial number of members = 11

$[m_i + 6 = 17] < [3j = 21]$

So the initial configuration lacks $21 - 17 = 4$ members for internal stabity. A stable configuration is achieved by



(1) Adding OB & OE to produce the rigid tetrahedron ABEO.

(2) Adding OF to produce the rigid tetrahedron ODEF.

(3) Adding BF to produce the rigid tetrahedrons OBCF and OBEF.

With 4 new members, $m = 15$ and $m + 6 = 21$. The number of joints remains $j = 7$; $3j = 21$. So $m + 6 = 3j$; sufficient number of members now present.

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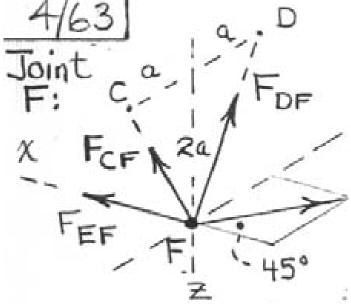
Joint F:



$\underline{L} = \frac{L}{\sqrt{2}}(-\underline{i}+\underline{j})$, $\underline{F}_{EF} = F_{EF}\,\underline{i}$

$\underline{F}_{CF} = \frac{F_{CF}}{\sqrt{5}}(-\underline{j} - 2\underline{k})$

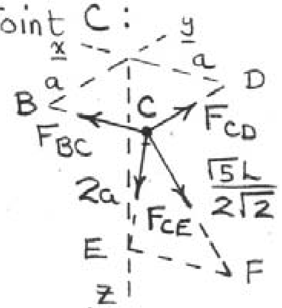
$\underline{F}_{DF} = \frac{F_{DF}}{\sqrt{5}}(\underline{j} - 2\underline{k})$

$\Sigma \underline{F} = \underline{L} + \underline{F}_{EF} + \underline{F}_{CF} + \underline{F}_{DF} = \underline{0}$

$\underline{i}: \; -\frac{L}{\sqrt{2}} + F_{EF} = 0$, $\quad F_{EF} = \frac{L}{\sqrt{2}}$

$$\left.\begin{aligned} \underline{j}: &\; \frac{L}{\sqrt{2}} - \frac{F_{CF}}{\sqrt{5}} + \frac{F_{DF}}{\sqrt{5}} = 0 \\ \underline{k}: &\; -\frac{2}{\sqrt{5}}F_{CF} - \frac{2}{\sqrt{5}}F_{DF} = 0 \end{aligned}\right\} \quad F_{CF} = \frac{\sqrt{5}\,L}{2\sqrt{2}}, \quad F_{DF} = -\frac{\sqrt{5}\,L}{2\sqrt{2}}$$

Joint C:



$\underline{F}_{CD} = F_{CD}\,\underline{j}$, $\underline{F}_{BC} = F_{BC}\,\underline{i}$

$\underline{F}_{CE} = F_{CE}(\underline{i} + \underline{j} + 2\underline{k})/\sqrt{6}$

$\underline{F}_{CF} = F_{CF}(\underline{j} + 2k)/\sqrt{5}$

$= \frac{L}{2\sqrt{2}}(\underline{j} + 2\underline{k})$

$\Sigma F = 0$ yields:

$$\left.\begin{aligned} \underline{i}: &\; 0 + F_{BC} + \frac{F_{CE}}{\sqrt{6}} = 0 \\ \underline{j}: &\; F_{CD} + \frac{F_{CE}}{\sqrt{6}} + \frac{L}{2\sqrt{2}} = 0 \\ \underline{k}: &\; \frac{2F_{CE}}{\sqrt{6}} + \frac{L}{\sqrt{2}} = 0 \end{aligned}\right\} \quad \begin{aligned} &\underline{F_{BC} = \frac{L\sqrt{2}}{4}} \\ &\underline{F_{CD} = 0} \\ &\underline{F_{CE} = -\frac{L\sqrt{3}}{2}} \end{aligned}$$

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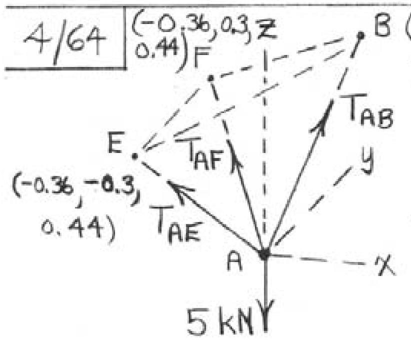


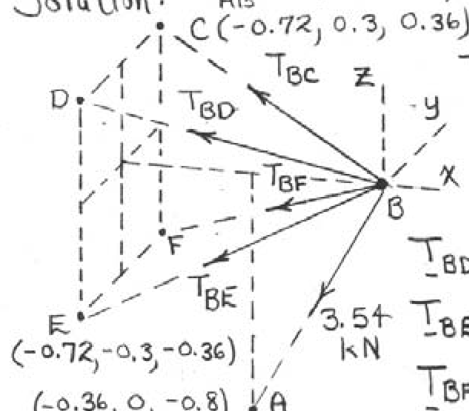
$$\underline{T}_{AB} = T_{AB}\frac{0.36\underline{i} + 0.8\underline{k}}{\sqrt{0.36^2 + 0.8^2}} = T_{AB}\left[0.410\underline{i} + 0.912\underline{k}\right]$$

$$\underline{T}_{AE} = T_{AE}\frac{-0.36\underline{i} - 0.3\underline{j} + 0.44\underline{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AE}\left[-0.560\underline{i} - 0.467\underline{j} + 0.684\underline{k}\right]$$

$$\underline{T}_{AF} = T_{AF}\frac{-0.36\underline{i} + 0.3\underline{j} + 0.44\underline{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AF}\left[-0.560\underline{i} + 0.467\underline{j} + 0.684\underline{k}\right]$$

$$\left.\begin{aligned}
\Sigma F_x = 0:&\ 0.410T_{AB} - 0.560T_{AE} - 0.560T_{AF} = 0\\
\Sigma F_y = 0:&\ -0.467T_{AE} + 0.467T_{AF} = 0\\
\Sigma F_z = 0:&\ 0.912T_{AB} + 0.684T_{AE} + 0.684T_{AF} - 5 = 0
\end{aligned}\right\}$$

Solution: $T_{AB} = 3.54$ kN, $T_{AE} = T_{AF} = 1.296$ kN



$$\underline{T}_{BC} = T_{BC}\frac{-0.72\underline{i} + 0.3\underline{j} + 0.36\underline{k}}{\sqrt{0.72^2 + 0.3^2 + 0.36^2}} = T_{BC}\left[-0.838\underline{i} + 0.349\underline{j} + 0.419\underline{k}\right]$$

$$\underline{T}_{BD} = T_{BD}\left[-0.838\underline{i} - 0.349\underline{j} + 0.419\underline{k}\right]$$

$$\underline{T}_{BE} = T_{BE}\left[-0.838\underline{i} - 0.349\underline{j} - 0.419\underline{k}\right]$$

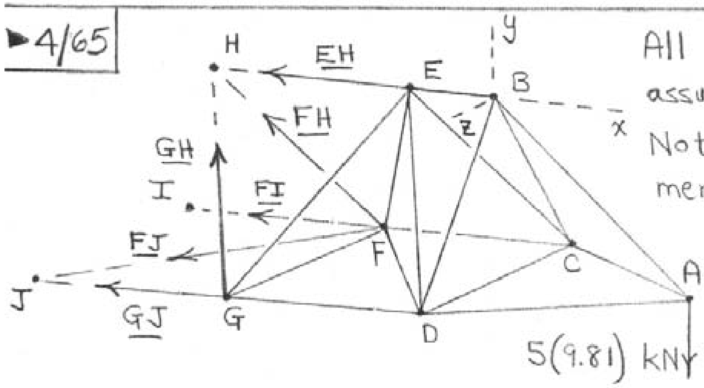
$$\underline{T}_{BF} = T_{BF}\left[-0.838\underline{i} + 0.349\underline{j} - 0.419\underline{k}\right]$$

Note: $T_{BD} = T_{BC}$; $T_{BE} = T_{BF}$ (symmetry)

Set $\Sigma \underline{F} = \underline{0}$ to obtain $T_{BD} = T_{BC} = 1.491$ kN

$$\underline{T_{BE} = T_{BF} = -2.36 \text{ kN (C)}}$$

▶4/65



All members assumed in tension. Note that six members are cut!

$\underline{GJ} = -GJ\underline{i}$, $\underline{FI} = -FI\underline{i}$, $\underline{FJ} = \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k})$

$\sum \underline{M}_H = \underline{0}$: $-49.05(5)\underline{k} + (-2\cos 30°\underline{j} + 2\sin 30°\underline{k})$
$\times(-GJ)\underline{i} + (-2\cos 30°\underline{j} - 2\sin 30°\underline{k}) \times (-FI)\underline{i}$
$+ (\underline{i} - 2\cos 30°\underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k}) = \underline{0}$.

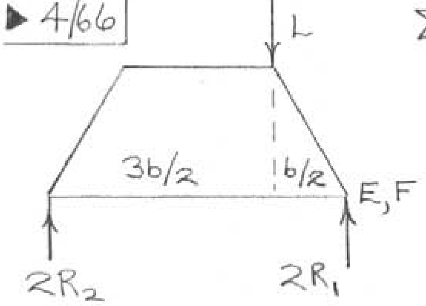
Equating unit vector coefficients to zero:

$-1.225\ FJ = 0 \Rightarrow \underline{FJ = 0}$

$\left.\begin{array}{l} -GJ + FI = 0 \\ -1.732\,GJ - 1.732\,FI = 245 \end{array}\right\}$ $FI = GJ = -70.8$ kN

$\therefore$ $\underline{\text{Force in } GJ = 70.8 \text{ kN } C}$

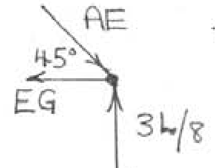
▶ 4/66



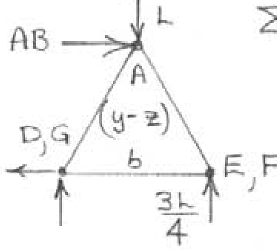
$\Sigma M_{EF} = 0: \ 2R_2(2b) - L\frac{b}{2} = 0$

$R_2 = L/8, \quad R_1 = 3L/8$

Joint E (Vertical plane through AE):



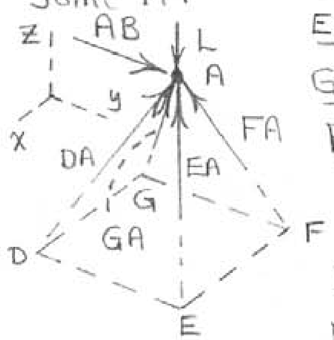
$+\uparrow \Sigma F = 0: \ \frac{3L}{8} - \frac{AE}{\sqrt{2}} = 0, \quad AE = \frac{3L}{4\sqrt{2}}$ C



$\Sigma M_{DG} = 0: \ AB\frac{b}{\sqrt{2}} + L\frac{b}{2} - \frac{3L}{4}b = 0$

$AB = L\sqrt{2}/4$ (Comp)

Joint A:



$\underline{DA} = DA\left(-\frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{EA} = \frac{3L}{4\sqrt{2}}\left(-\frac{1}{2}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{GA} = GA\left(\frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{FA} = \frac{3L}{4\sqrt{2}}\left(\frac{1}{2}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{L} = -L\underline{k}, \quad \underline{AB} = \frac{L\sqrt{2}}{4}\underline{j}$

$\Sigma \underline{F} = \underline{0}: \ \underline{L} + \underline{AB} + \underline{DA} + \underline{EA} + \underline{GA} + \underline{FA} = 0$

Note that DA = GA & EA = FA by symmetry & obtain

$\underline{i}: \ -\frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA + \frac{3L}{8\sqrt{2}} = 0, \quad 0 = 0$ ✓

$\underline{j}: \ \frac{L\sqrt{2}}{4} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} = 0, \quad DA = GA$

$\underline{k}: \ -L + \frac{L}{8} + \frac{3L}{8} + \frac{L}{8} + \frac{3L}{8} = 0, \quad 0 = 0$ ✓

$= \frac{L\sqrt{2}}{8}$ (Comp.)

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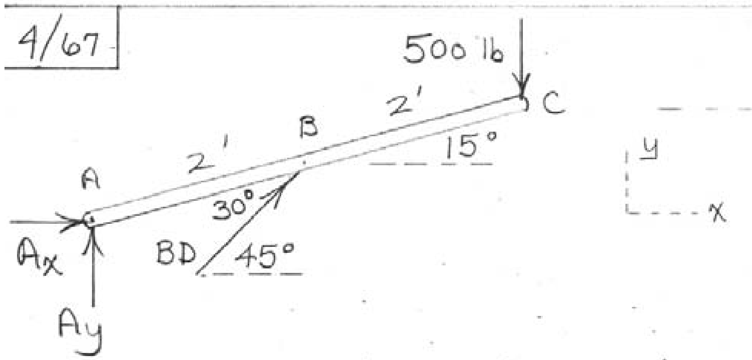

$\curvearrowleft\!+\ \Sigma M_A = 0: \ BD \sin 30°(2) - 500(4\cos 15°) = 0$

$\underline{BD = 1932 \text{ lb}}$

$\underline{\text{Forces at } B \text{ and } D = 1932 \text{ lb}}$

$\Sigma F_x = 0: \ A_x + 1932 \cos 45° = 0, \ A_x = -1366 \text{ lb}$

$\Sigma F_y = 0: \ A_y + 1932 \sin 45° - 500 = 0, \ A_y = -866 \text{ lb}$

$A = \sqrt{1366^2 + 866^2} = \underline{1617 \text{ lb}}$

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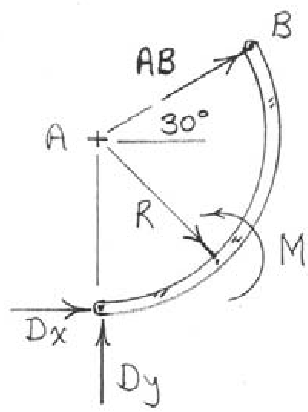

$\curvearrowleft + \Sigma M_D = 0: \ -AB\cos 30°(R) + M = 0$

$$AB = \frac{M}{R\cos 30°} = \frac{M}{R\,\sqrt{3}/2} = \frac{2\sqrt{3}\,M}{3R}$$

Load is a couple, so reactions form a couple:

$$A = D = \frac{2\sqrt{3}\,M}{3R}$$

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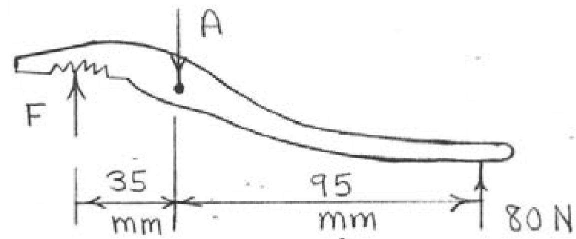
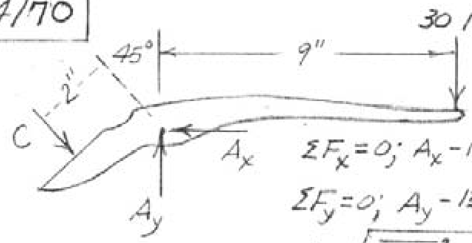


$\circlearrowleft + \sum M_A = 0$: $80(95) - F(35) = 0$, $\underline{F = 217 \text{ N}}$

$+\uparrow \sum F = 0$: $217 - A + 80 = 0$, $\underline{A = 297 \text{ N}}$

4/70



$\Sigma M_A = 0;$

$30(9) - 2C = 0, \quad C = 135 \text{ lb}$

$\Sigma F_x = 0; \; A_x - 135/\sqrt{2} = 0, \quad A_x = 95.4 \text{ lb}$

$\Sigma F_y = 0; \; A_y - 135/\sqrt{2} - 30 = 0, \quad A_y = 125.4 \text{ lb}$

$A = \sqrt{\overline{95.4}^2 + \overline{125.4}^2} = \underline{157.6 \text{ lb}}$

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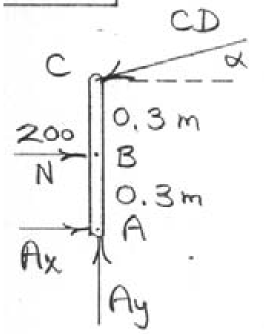
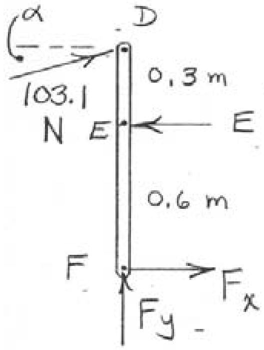


$$\alpha = \tan^{-1}\frac{0.3}{1.2} = 14.04°$$

$$\circlearrowleft + \Sigma M_A = 0: \quad CD\cos\alpha\,(0.6) - 200(0.3) = 0, \quad CD = 103.1 \text{ N}$$



$$\circlearrowleft + \Sigma M_F = 0: -103.1\cos\alpha\,(0.9) + E(0.6) = 0, \quad \underline{E = 150 \text{ N}}$$

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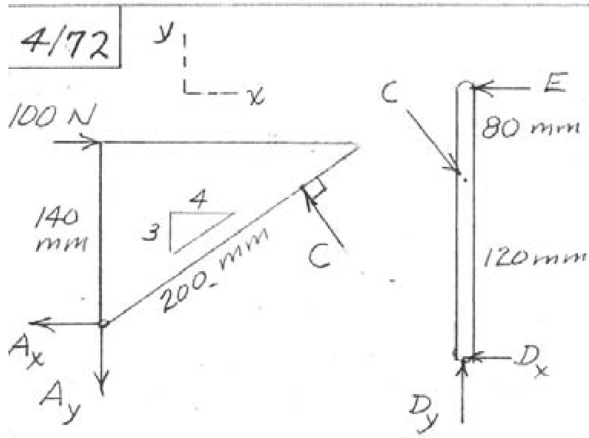

Triangle:

$\Sigma M_A = 0$; $200C - 140(100) = 0$, $C = 70$ N

Bar:

$\Sigma M_D = 0$; $200E - 70\left(\frac{3}{5}\right)120 = 0$; $E = 25.2$ N

$\Sigma F_x = 0$; $D_x + 25.2 - 70(3/5) = 0$, $D_x = 16.8$ N

$\Sigma F_y = 0$; $D_y - 70(4/5) = 0$, $D_y = 56$ N

$D = \sqrt{D_x^2 + D_y^2} = \sqrt{16.8^2 + 56^2} = \underline{58.5 \text{ N}}$

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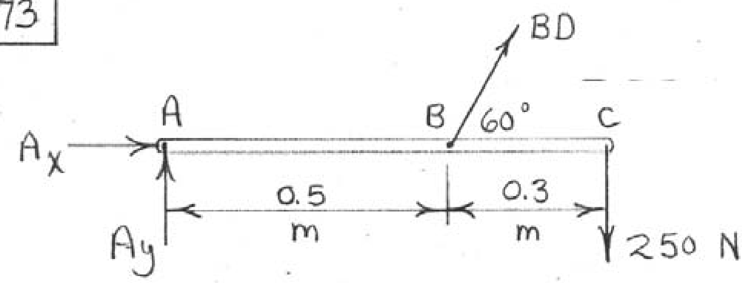
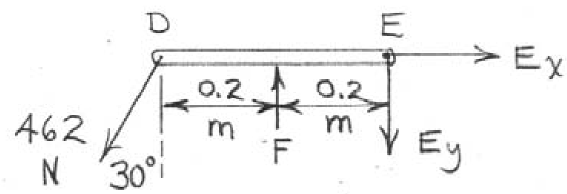


$\circlearrowleft + \ \Sigma M_A = 0: \ (BD \sin 60°)(0.5) - 250(0.8) = 0$

$BD = 462 \text{ N}$



$\circlearrowleft + \ \Sigma M_E = 0: \ 462 \cos 30° (0.4) + F(0.2) = 0$

$\underline{F = 800 \text{ N}}$

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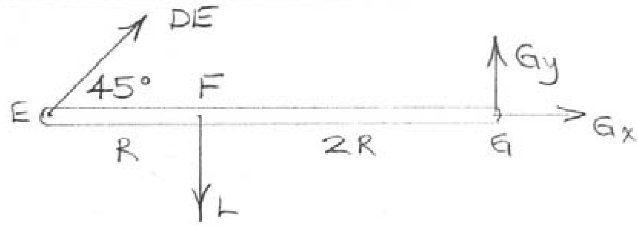
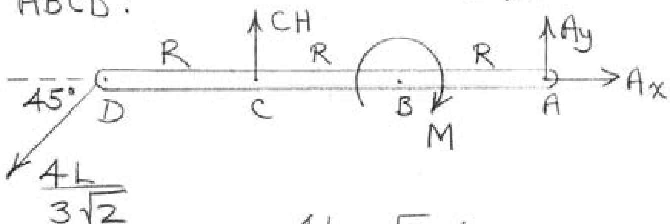
EFG



$$\circlearrowleft+\ \sum M_G = 0: \quad L(2R) - DE\,\frac{\sqrt{2}}{2}(3R) = 0$$

$$DE = \frac{4L}{3\sqrt{2}}$$

ABCD:

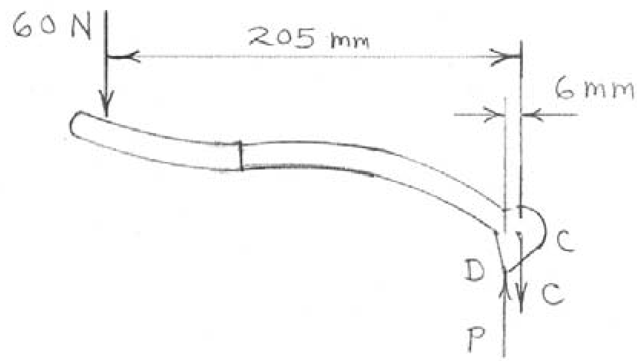


$$\circlearrowleft+\ \sum M_A = 0: \quad \frac{4L}{3\sqrt{2}}\,\frac{\sqrt{2}}{2}(3R) - CH(2R) - M = 0$$

$$CH = L - \frac{M}{2R} \text{ (tension if positive)}$$

$CH = 0$ if $\underline{M = 2RL}$

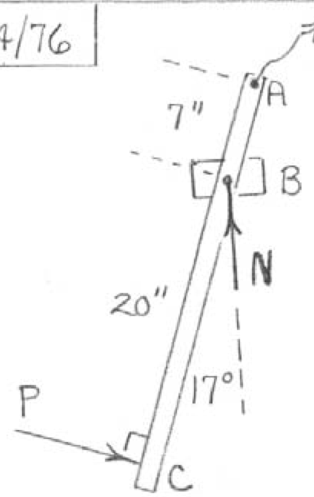
4/75 Upper handle/cam unit:



$\curvearrowleft + \Sigma M_C = 0: \quad 60(205) - P(6) = 0$

$\underline{P = 2050 \text{ N}}$

4/76



$\curvearrowleft + \Sigma M_A = 0:$

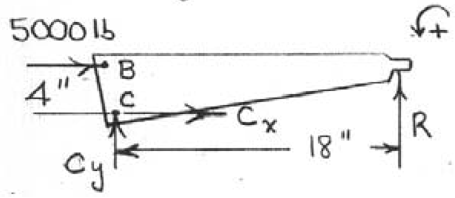
$+P(27) - N \sin 17°(7) = 0$

$\underline{N = 13.19P}$

4/77 Piston force $= (500)(20) = 10{,}000$ lb

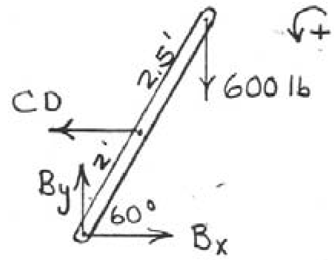
Force in link $AB = 10{,}000/2 = 5000$ lb

Lower jaw:



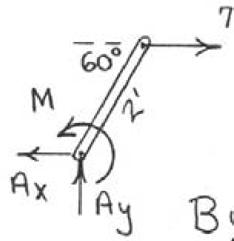
$\circlearrowleft\!+ \ \Sigma M_C = 0: \ R(18) - 5000(4) = 0, \quad \underline{R = 1111 \text{ lb}}$

4/78 Member BE :



$$\circlearrowleft\!+\ \Sigma M_B = 0:\ -600(4.5\cos 60^\circ) + CD(2\sin 60^\circ) = 0,\quad CD = 779\text{ lb}$$

Member AC :



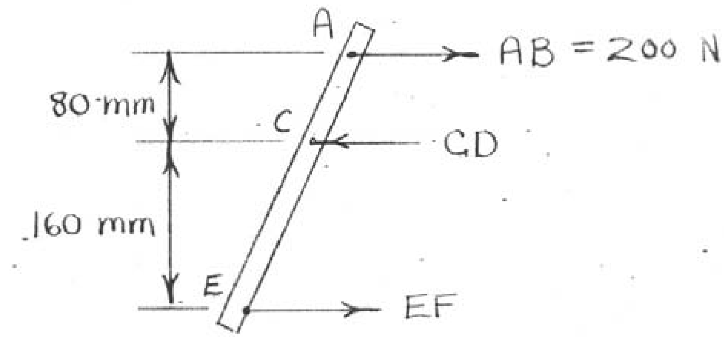
$$\Sigma M_A = 0:\ -779(2\sin 60^\circ) + M = 0$$

$$\underline{M = 1350\text{ lb-ft}\ \circlearrowleft}$$

By inspection, $A_x = 779$ lb, $A_y = 0$

$$\therefore\ A = \sqrt{A_x^2 + A_y^2} = \underline{779\text{ lb}}$$

4/79



$$\circlearrowleft + \sum M_C = 0: \; -200(80) + EF(160) = 0$$

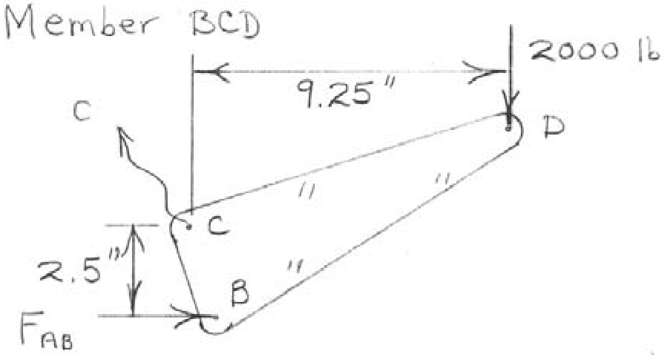
$$\underline{EF = 100 \text{ N } T}$$

$$\overset{+}{\rightarrow} \sum F = 0: \; 200 - CD + 100 = 0$$

$$CD = 300 \text{ N}.$$

So force supported by pin C is $\underline{F = 300 \text{ N}}$

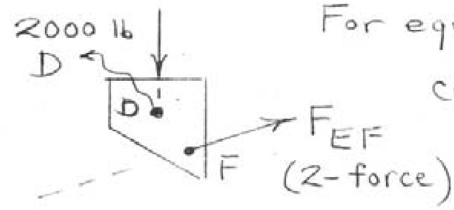
4/80 Member BCD



$\circlearrowleft \sum M_C = 0: \; F_{AB}(2.5) - 2000(9.25) = 0$

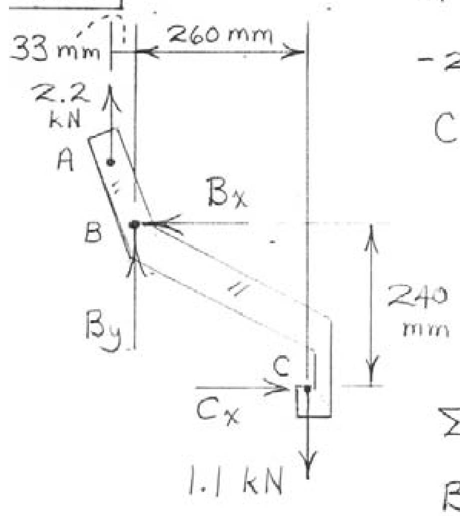
$\underline{F_{AB} = 7400 \text{ lb}}$

The force in member EF is zero, as is disclosed by a FBD of the lifting pad:



For equilibrium, F_{EF} must be concurrent at D, but it cannot be, so it is zero!

4/81



$\circlearrowleft^{+} \Sigma M_B = 0$:

$-2.2(33) + C_x(240) - 1.1(260) = 0$

$C_x = 1.494$ kN

$\Sigma F_x = 0$:

$1.494 - B_x = 0$

$B_x = 1.494$ kN

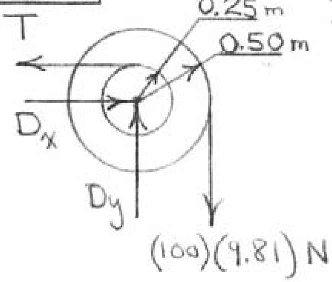
$\Sigma F_y = 0$:

$B_y + 2.2 - 1.1 = 0$

$B_y = -1.1$ kN

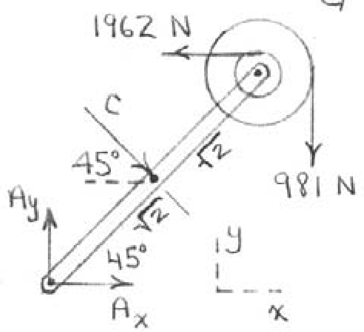
$B = \sqrt{1.494^2 + 1.1^2} = \underline{1.855 \text{ kN}}$

4/82



$\circlearrowleft + \; \Sigma M_D = 0: \quad T(0.25) - 981(0.5) = 0$

$T = 1962 \text{ N}$



$\circlearrowleft + \; \Sigma M_A = 0: \quad -C\sqrt{2}$

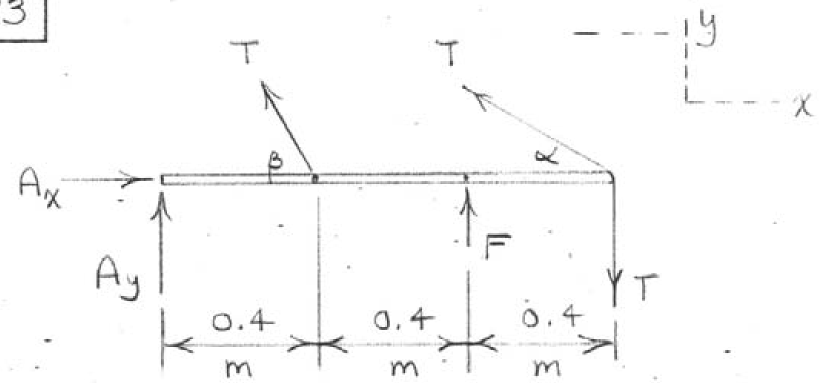
$+ 1962(2 + 0.25) - 981(2 + 0.50) = 0$

$C = 1390 \text{ N}$

$\therefore \quad C_x = 981 \text{ N}$

$C_y = -981 \text{ N}$

4/83



$$\begin{cases} T = 60(9.81) = 589 \text{ N} \\ \alpha = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.6^\circ \\ \beta = \tan^{-1}\left(\frac{0.5}{0.4}\right) = 51.3^\circ \end{cases}$$

$\circlearrowleft + \Sigma M_A = 0: \; T\sin\beta(0.4) + T\sin\alpha(1.2)$

$- T(1.2) + F(0.8) = 0, \quad \underline{F = 314 \text{ N}}$

(Contact at bottom roller)

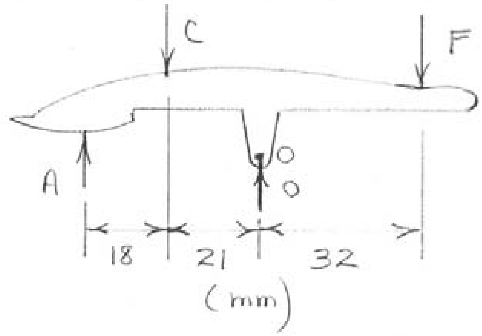
$\Sigma F_x = 0: \; A_x - T\cos\beta - T\cos\alpha = 0, \; A_x = 911 \text{ N}$

$\Sigma F_y = 0: \; A_y + T\sin\beta + T\sin\alpha - T + F = 0$

$A_y = -411 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{999 \text{ N}}$

4/84



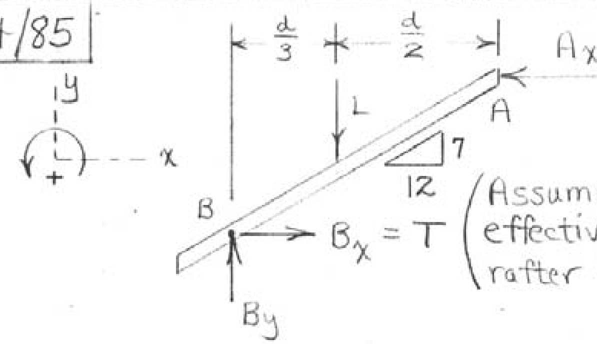
F = 15 N when A = 0 :

$\circlearrowleft + \sum M_O = 0: \; C(21) - 15(32) = 0, \quad C = 22.9 \text{ N}$

When F = 0:

$\circlearrowleft + \sum M_O = 0: \; -A(39) + 22.9(21) = 0, \quad \underline{A = 12.31 \text{ N}}$

4/85



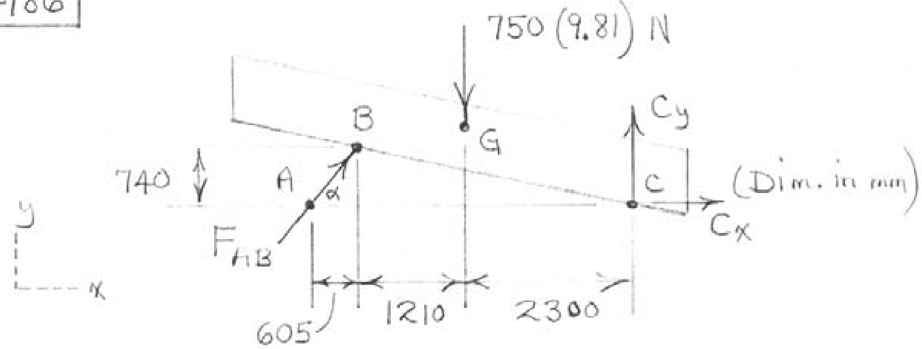
(Assumes cable tension effectively transferred to rafter via proper fastener

$$\Sigma F_x = 0 \Rightarrow T = A_x$$

$$\Sigma M_B = 0 : \quad -L\left(\frac{d}{3}\right) + T\left(\frac{5d}{6} \cdot \frac{7}{12}\right) = 0$$

$$T = \frac{24}{35} L$$

4/86



$\alpha = \tan^{-1}\left(\frac{740}{605}\right) = 50.7°$

$\sum M_C = 0$: $750(9.81)(2300) - F_{AB}\sin\alpha\,(4115) = 0$

$\underline{F_{AB} = 5310 \text{ N}}$

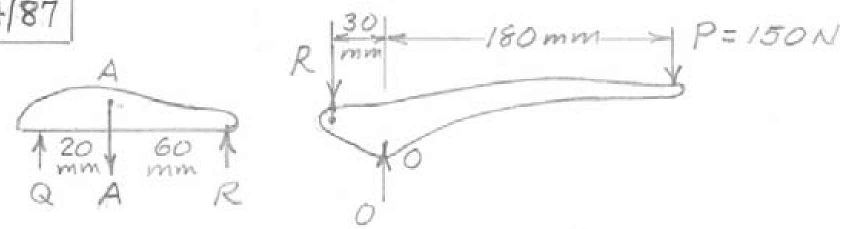
$\sum F_x = 0$: $5310\cos 50.7° + C_x = 0$, $C_x = -3360$ N

$\sum F_y = 0$: $5310\sin 50.7° - 750(9.81) + C_y = 0$

$C_y = 3250$ N

$C = \sqrt{3360^2 + 3250^2} = \underline{4670 \text{ N}}$

4/87

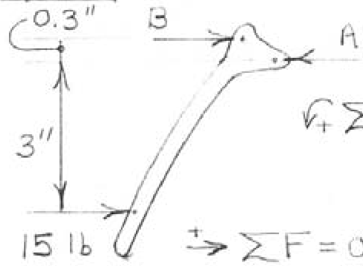


Handle: $\Sigma M_O = 0$; $30R - 180(150) = 0$, $R = 900$ N

Jaw: $\Sigma M_A = 0$; $20Q - 60(900) = 0$, $Q = 2700$ N

or $\underline{Q = 2.7 \text{ kN}}$

4/88 Left handle (with use of symmetry):

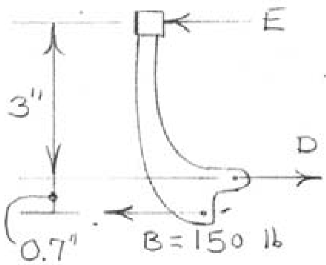


$\circlearrowleft + \Sigma M_A = 0: \; 15(3) - B(0.3) = 0$

$\underline{B = 150 \text{ lb}}$

$\xrightarrow{+} \Sigma F = 0: \; 15 + 150 - A = 0, \quad \underline{A = 165 \text{ lb}}$

Left jaw:



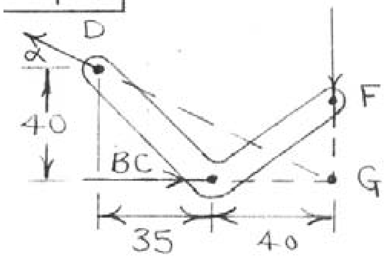
$\circlearrowleft + \Sigma M_D = 0: \; E(3) - 150(0.7) = 0$

$\underline{E = 35 \text{ lb}}$

$\xrightarrow{+} \Sigma F = 0: \; -35 - 150 + D = 0$

$\underline{D = 185 \text{ lb}}$

4/89



DCF is a three-force body; forces intersect at G.

$\alpha = \tan^{-1}\left(\frac{40}{75}\right) = 28.1°$

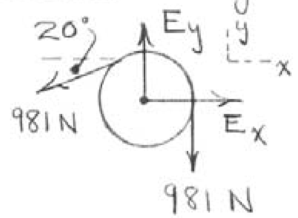
$\Sigma F_y = 0: \ -200 + D \sin\alpha = 0$

$\underline{D = 425 \text{ N}}$

$\Sigma F_x = 0: \ -D\cos\alpha + BC = 0, \ \underline{BC = 375 \text{ N}}$

(BC in compression)

4/90 Pulley:



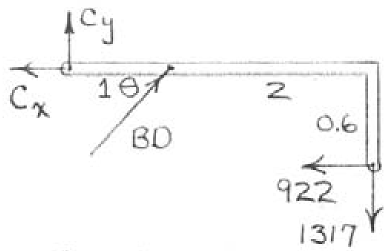
$\Sigma F_x = 0$: $E_x - 981 \cos 20° = 0$

$E_x = 922$ N

$\Sigma F_y = 0$: $E_y - 981 - 981 \sin 20° = 0$

$E_y = 1317$ N

Member CDE:

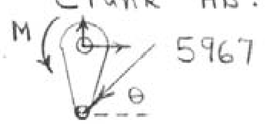


$\theta = \tan^{-1}\left(\frac{1.15}{1}\right) = 48.99°$

$\Sigma M_C = 0$: $(BD \sin\theta)(1) - 922(0.6) - 1317(3) = 0$

$BD = 5967$ N

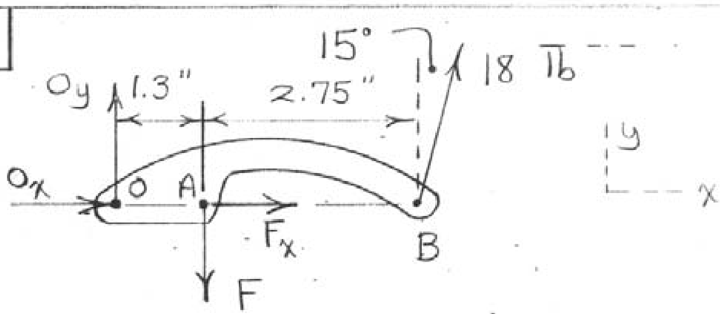
Crank AB:



$\Sigma M_A = 0$: $M - (5967 \cos\theta)(0.5) = 0$

$\underline{M = 1958 \text{ N·m}}$

4/91

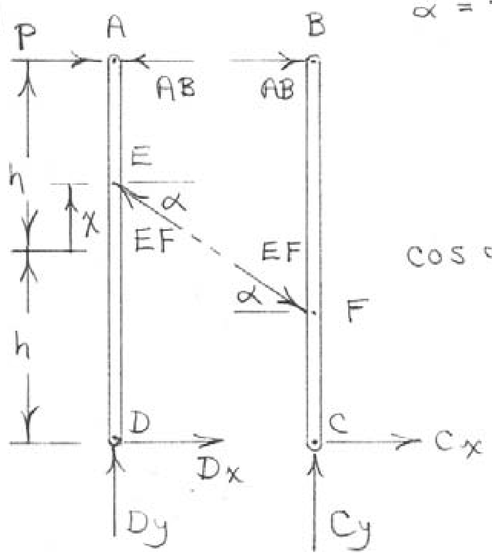


$\circlearrowleft + \Sigma M_O = 0: \; -F(1.3) + 18\cos 15°(2.75 + 1.3) = 0$

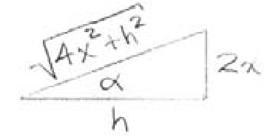
$\underline{F = 54.2 \text{ lb}}$

(Note: Treatment of member OC as a three-force body would yield a constraint relationship between O_x and O_y.).

4/92



$$\alpha = \tan^{-1}\frac{x}{h/2} = \tan^{-1}\frac{2x}{h}$$



$$\cos\alpha = \frac{h}{\sqrt{4x^2+h^2}}$$

Member AED, $\circlearrowleft\!+\ \Sigma M_D = 0$:

$$AB(2h) + EF\cos\alpha\,(h+x) - P(2h) = 0 \qquad (1)$$

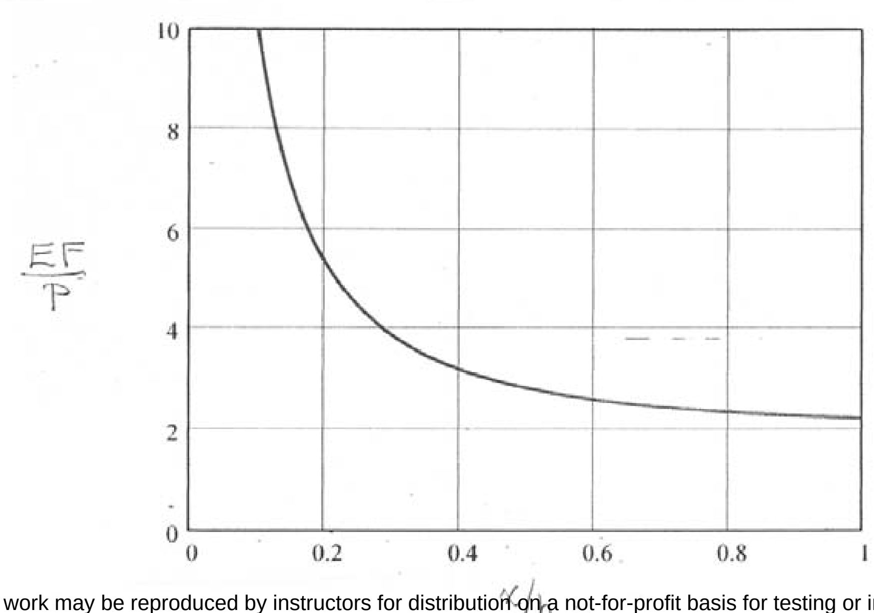
Member BFC, $\circlearrowleft\!+\ \Sigma M_C = 0$:

$$-AB(2h) - EF\cos\alpha\,(h-x) = 0 \qquad (2)$$

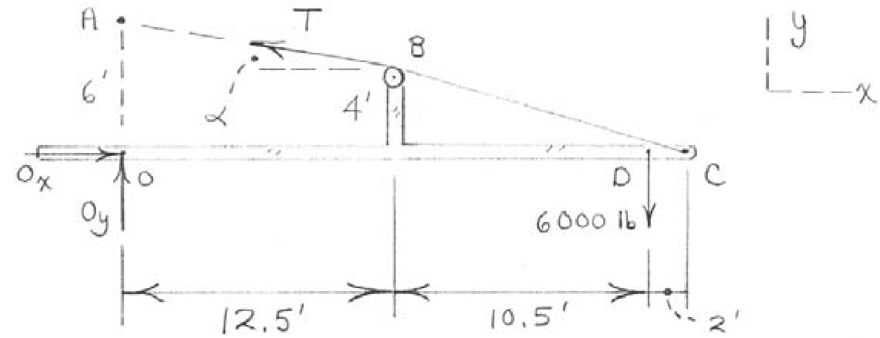
Add (1) & (2), use the above expression for $\cos\alpha$, and solve for EF:

$$EF = P\sqrt{4 + \frac{1}{(x/h)^2}}$$

Undefined @ $x=0$, but structure collapses for $x=0$



4/93 FBD of OBC and portion of cable:



$$\alpha = \tan^{-1}\left(\frac{2}{12.5}\right) = 9.09°$$

$$\circlearrowleft + \Sigma M_O = 0: \quad T\cos\alpha\,(4) + T\sin\alpha\,(12.5) - 6000\,(23) = 0$$

$$\underline{T = 23{,}300 \text{ lb}}$$

$$\Sigma F_x = 0: \quad O_x - 23{,}300\cos\alpha = 0$$

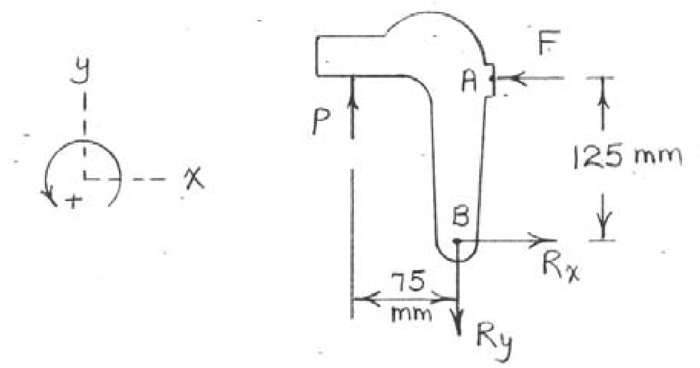
$$O_x = 23{,}000 \text{ lb}$$

$$\Sigma F_y = 0: \quad O_y + 23{,}300\sin\alpha - 6000 = 0$$

$$O_y = 2320 \text{ lb}$$

$$O = \sqrt{23{,}000^2 + 2320^2} = \underline{23{,}100 \text{ lb}}$$

4/94



For $P = 3$ kN :

$\Sigma M_B = 0: \quad 125F - 3(75) = 0, \quad F = 1.8$ kN

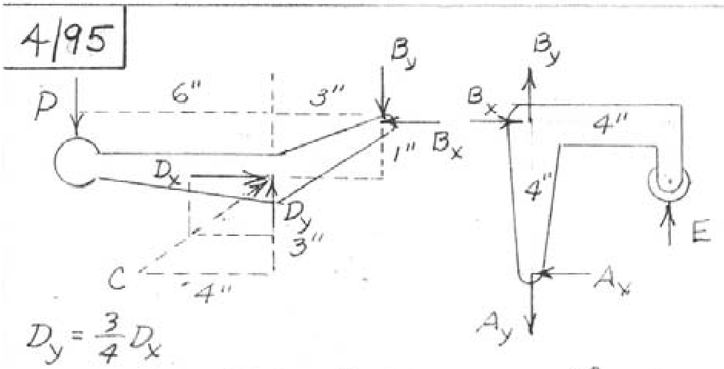
For $F = 2(1.8) = 3.6$ kN, $P = 3(2) = 6$ kN

$\Sigma F_x = 0: \quad R_x - 3.6 = 0, \quad R_x = 3.6$ kN

$\Sigma F_y = 0: \; -R_y + 6 = 0, \quad R_y = 6$ kN

$R = \sqrt{3.6^2 + 6^2} = \underline{7.00 \text{ kN}}$

4/95



$D_y = \frac{3}{4} D_x$

Lever BD; $\Sigma M_B = 0$; $9P + 1D_x - 3(\frac{3}{4}D_x) = 0$, $D_x = \frac{36}{5}P$

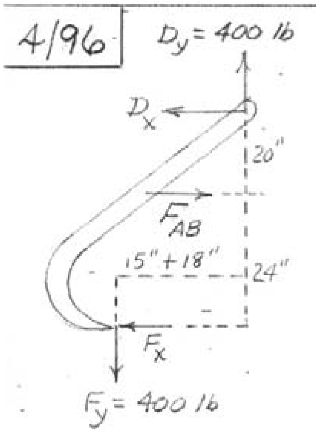
$D_y = \frac{3}{4}D_x = \frac{27}{5}P$

$\Sigma F_x = 0$; $B_x = D_x = \frac{36}{5}P$

$\Sigma F_y = 0$; $B_y + P - \frac{27}{5}P = 0$, $B_y = \frac{22}{5}P$

ABE; $\Sigma M_A = 0$; $4E - 4(\frac{36}{5}P) = 0$, $\underline{E = \frac{36}{5}P = 7.2\,P}$

4/96



From tongs as a whole

$D_y = \frac{1}{2}(800) = 400$ lb $= F_y$

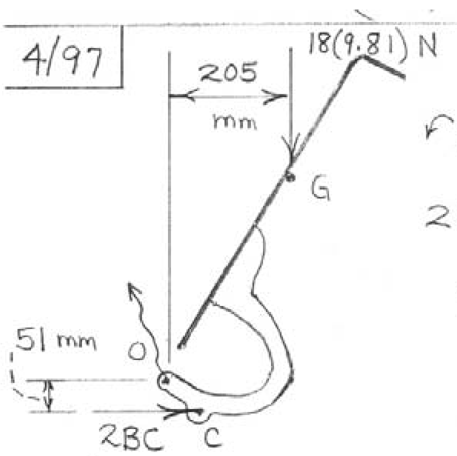
From ED, $D_x = \frac{18}{12} D_y = \frac{18}{12}(400) = 600$ lb

From DF, $\Sigma M_F = 0$;

$F_{AB}(24) - 600(44) - 400(18 + 15) = 0$

$F_{AB} = \underline{1650 \text{ lb}}$ tension

4/97

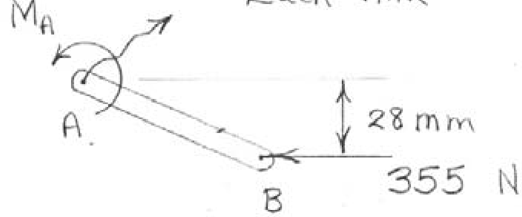


$\circlearrowleft \Sigma M_O = 0:$

$$2BC(51) - 18(9.81)(205) = 0$$

$$BC = 355 \text{ N}$$

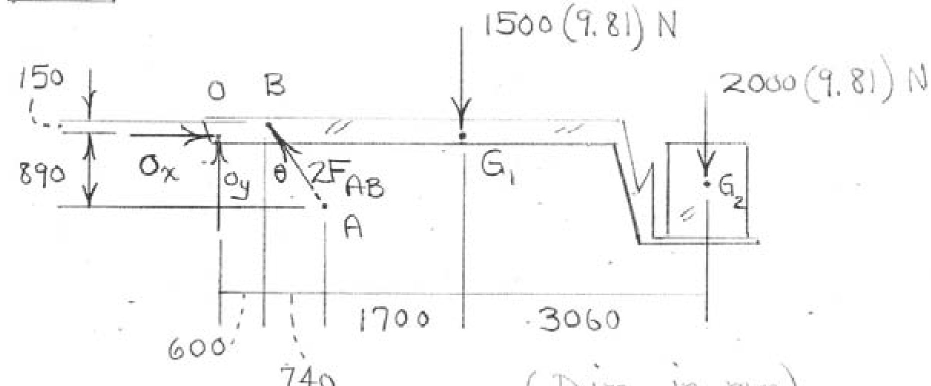
Each link:



$\circlearrowleft + \Sigma M_A = 0: \quad M_A - 355(28) = 0$

$M_A = 9940 \text{ N}\cdot\text{mm}$ or $\underline{9.94 \text{ N}\cdot\text{m} \text{ CCW}}$

4/98



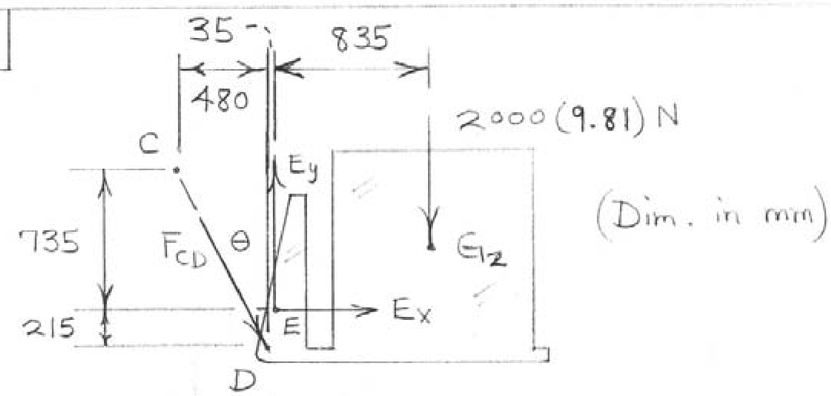
(Dim. in mm)

$$\theta = \tan^{-1}\left(\frac{740}{1040}\right) = 35.4^\circ$$

$$\circlearrowleft \Sigma M_O = 0: 2F_{AB}\cos\theta\,(600) + 2F_{AB}\sin\theta\,(150)$$
$$-1500(9.81)(3040) - 2000(9.81)(6100) = 0$$

$$F_{AB} = 142\,800 \text{ N} \quad \text{or} \quad \underline{142.8 \text{ kN}}$$

4/99



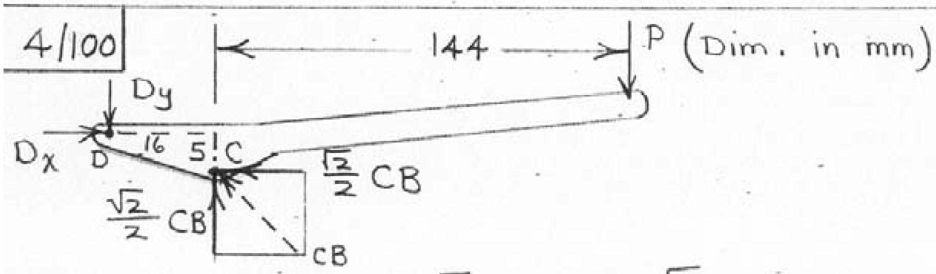
$$\theta = \tan^{-1}\left(\frac{480}{950}\right) = 26.8°$$

$$\Sigma M_E = 0: \; -2000(9.81)(835) + F_{CD}\cos\theta\,(35) + F_{CD}\sin\theta\,(215) = 0$$

$$F_{CD} = 127\,800 \text{ N}$$

or $\underline{127.8 \text{ kN}}$

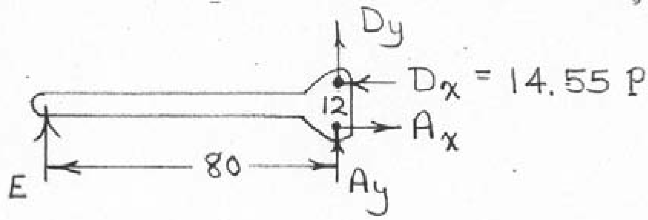
4/100



$\circlearrowleft^{+} \Sigma M_D = 0: \quad P(160) + \frac{\sqrt{2}}{2} CB(5) - \frac{\sqrt{2}}{2} CB(16) = 0$

$CB = 20.6\,P$

$\xrightarrow{+} \Sigma F = 0: \quad D_x - 20.6P\,\frac{\sqrt{2}}{2} = 0, \quad D_x = 14.55P$



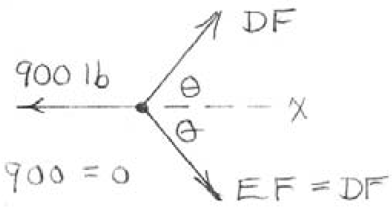
$\circlearrowleft^{+} \Sigma M_A = 0: \quad E(80) - 14.55P(12) = 0$

$\underline{E = 2.18\,P}$

(Note: Mechanical advantage will increase as CB becomes more aligned with CD.)

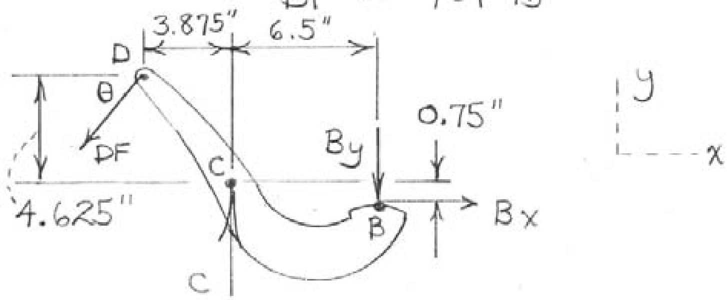
4/101 FBD of joint F :

$$\theta = \tan^{-1}\left(\frac{4.625}{3.875}\right) = 50.0^\circ$$



$$\Sigma F_x = 0: \quad 2DF\cos 50.0^\circ - 900 = 0$$

$$DF = 701 \text{ lb}$$



$$\Sigma F_x = 0: \; -701\cos 50.0^\circ + B_x = 0, \quad B_x = 450 \text{ lb}$$

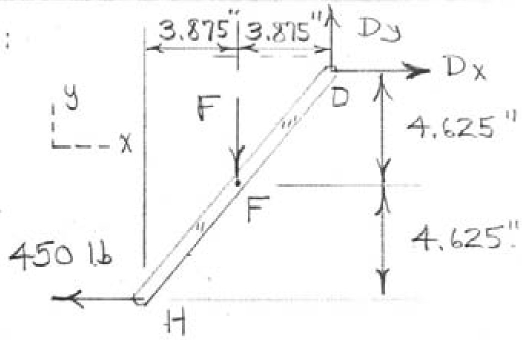
$$\curvearrowleft\!+ \Sigma M_B = 0: \; -C(6.5) + 701\cos 50.0^\circ(4.625 + 0.75)$$
$$+ 701\sin 50.0^\circ(10.375) = 0, \quad \underline{C = 1229 \text{ lb}}$$

$$\Sigma F_y = 0: \; -701\sin 50.0^\circ + 1229 - B_y = 0$$

$$\underline{B_y = 692 \text{ lb} = B_n = A_n}$$

(No horizontal force component at C because of symmetry.)

4/102 Member DFH:



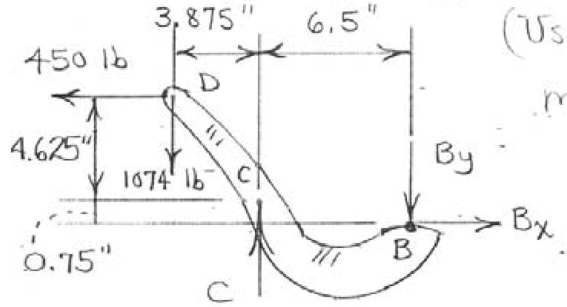
From $\Sigma F_x = 0$,

$D_x = 450 \text{ lb}$

$\circlearrowleft\!+ \Sigma M_D = 0$:

$F(3.875) - 450(9.25) = 0, \quad F = 1074 \text{ lb}$

From $\Sigma F_y = 0$, $\quad D_y = 1074 \text{ lb}$



(Use of symmetry made @ C and F.)

$\circlearrowleft\!+ \Sigma M_B = 0$: $\quad 450(5.375) + 1074(10.375) - C(6.5) = 0$

$\underline{C = 2090 \text{ lb}}$

$\Sigma F_y = 0$: $\quad -1074 + 2090 - B_y = 0$

$\underline{B_y = 1012 \text{ lb}}$

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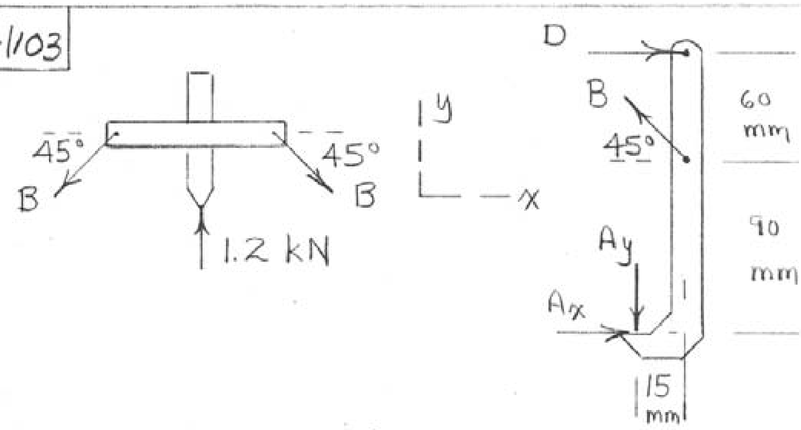


(Upper bar & screw)

$\Sigma F_y = 0: -2B \sin 45^\circ + 1.2 = 0, \quad B = 0.849 \text{ kN}$

(ABD)

$\Sigma M_A = 0: \; 150D - 0.849 \cos 45^\circ (90) - 0.849 \sin 45^\circ (15) = 0$

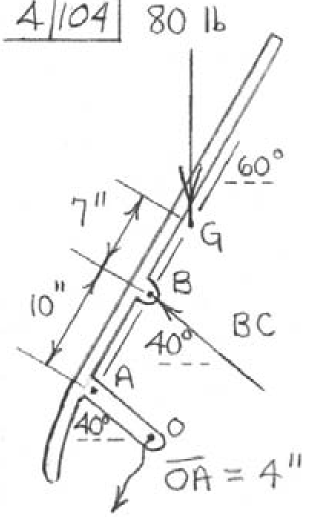
$D = 0.420 \text{ kN}$

$\Sigma F_x = 0: \; A_x - 0.849 \cos 45^\circ + 0.420 = 0, \; A_x = 0.1800 \text{ kN}$

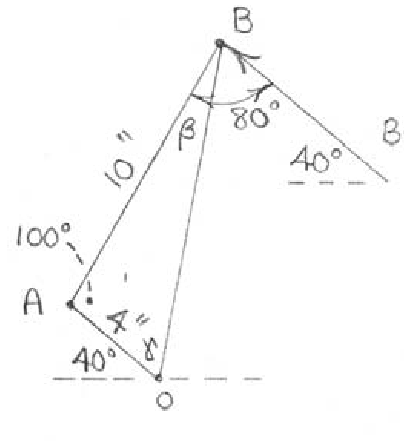
$\Sigma F_y = 0: -A_y + 0.849 \sin 45^\circ = 0, \quad A_y = 0.6 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{0.626 \text{ kN}}$

4/104



Blowup:

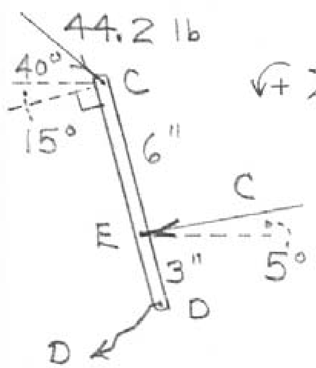


$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10)\cos 100°$, $\overline{OB} = 11.40$ in.

$\frac{\sin\beta}{4''} = \frac{\sin 100°}{11.40''}$, $\beta = 20.2°$

$\curvearrowleft + \sum M_O = 0: \; -80(17\cos 60° - 4\cos 40°)$

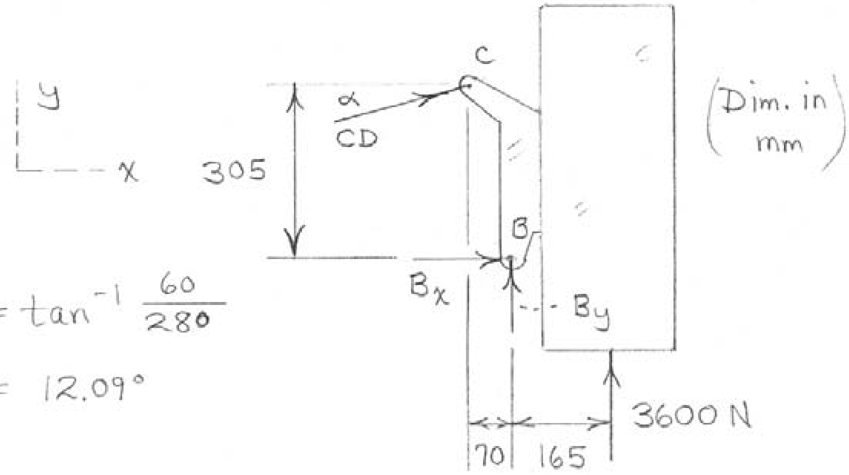
$+ BC\,(11.40\sin(80° - 20.2°)) = 0$, $BC = 44.2$ lb



$\curvearrowleft + \sum M_D = 0: \; -44.2\cos 55°\,(9) + C\cos 10°\,(3) = 0$

$\underline{C = 77.2 \text{ lb}}$

4/105 Wheel unit:



$\alpha = \tan^{-1}\frac{60}{280}$

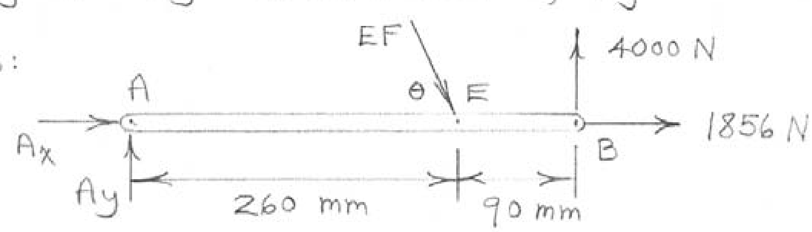
$= 12.09°$

$\curvearrowleft + \Sigma M_B = 0$: $3600(165) - CD\cos\alpha\,(305) - CD\sin\alpha\,(70) = 0$, $CD = 1898$ N

$\Sigma F_x = 0$: $B_x + CD\cos\alpha = 0$, $B_x = -1856$ N

$\Sigma F_y = 0$: $B_y + CD\sin\alpha + 3600 = 0$, $B_y = -4000$ N

AEB:



$\theta = \tan^{-1}\left(\frac{435}{200}\right) = 65.3°$

$\curvearrowleft + \Sigma M_A = 0$: $-EF\sin\theta(260) + 4000(350) = 0$

$EF = 5920$ N

$\Sigma F_x = 0$: $A_x + EF\cos\theta + 1856 = 0$

$A_x = -4330$ N

$\Sigma F_y = 0$: $A_y - EF\sin\theta + 4000 = 0$

$A_y = 1384$ N

So $A = \sqrt{(-4330)^2 + 1384^2} = \underline{4550 \text{ N}}$

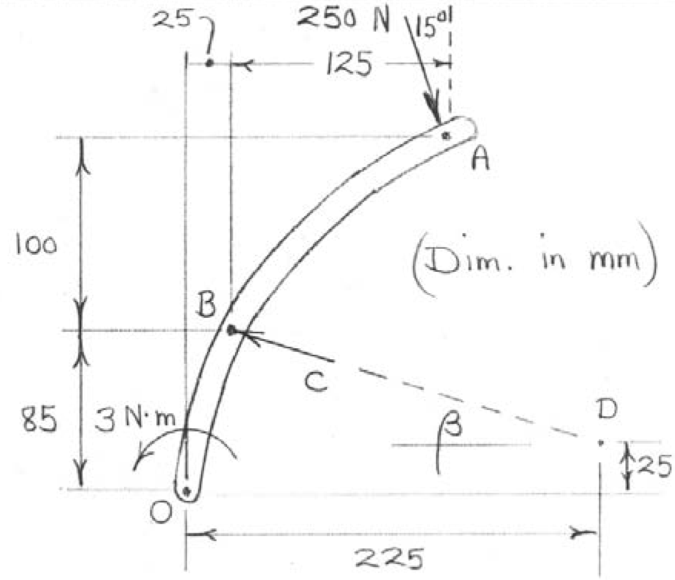
$B = \sqrt{1856^2 + 4000^2} = \underline{4410 \text{ N}}$

$\underline{C = D = 1898 \text{ N}}$, $\underline{E = F = 5920 \text{ N}}$

4/106

$\beta = \tan^{-1}\frac{60}{200}$

$= 16.70°$



$\circlearrowleft + \sum M_O = 0: \ 3000 - 250\cos 15°(150) -$

$250\sin 15°(185) + C\cos\beta(85) + C\sin\beta(25) = 0$

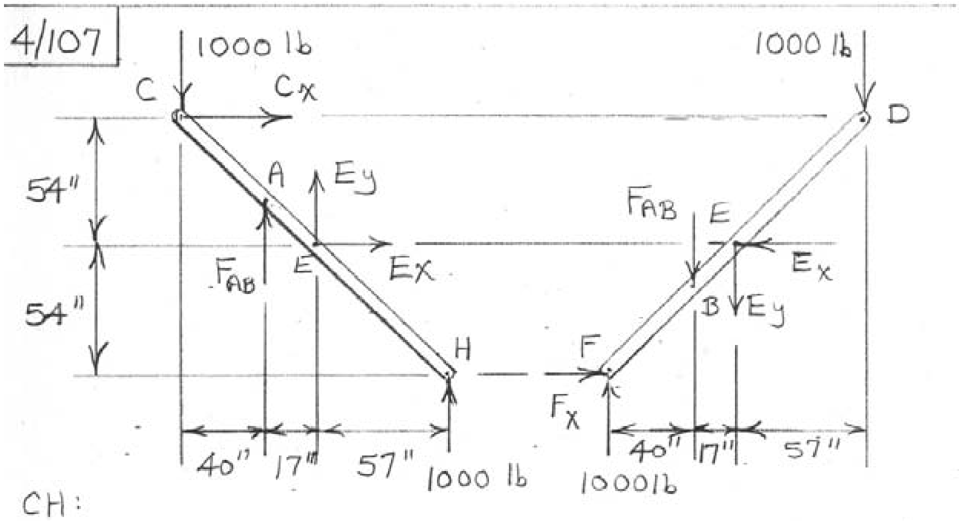
$\underline{C = 510 \text{ N}}$

$C = pA: \quad 510 = p\left(\frac{\pi 45^2}{4}\right)$

$p = 0.321 \frac{\text{N}}{\text{mm}^2}$ or $\underline{321\,000 \text{ Pa}}$

(gauge pressure)

4/107



CH:

$\Sigma F_x = 0: \quad C_x + E_x = 0 \qquad (1)$

$\Sigma F_y = 0: \quad F_{AB} + E_y + 1000 - 1000 = 0 \qquad (2)$

$\circlearrowleft + \Sigma M_C = 0: \quad F_{AB}(40) + E_y(57) + E_x(54) + 1000(114) = 0 \qquad (3)$

DF:

$\Sigma F_x = 0: \quad F_x - E_x = 0 \qquad (4)$

$\Sigma F_y = 0:$ (Same as Eq. (2)).

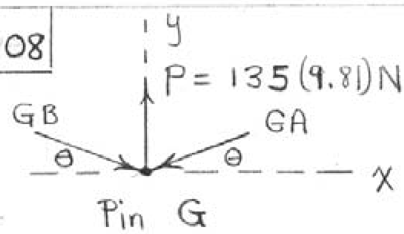
$\circlearrowleft + \Sigma M_F = 0: \quad -F_{AB}(40) - E_y(57) + E_x(54) - 1000(114) = 0 \qquad (5)$

Solution: $\underline{F_{AB} = 6710 \text{ lb}}$

$C_x = F_x = E_x = 0, \quad E_y = -6710 \text{ lb}$

(C_x, F_x evident from overall FBDs !)

4/108

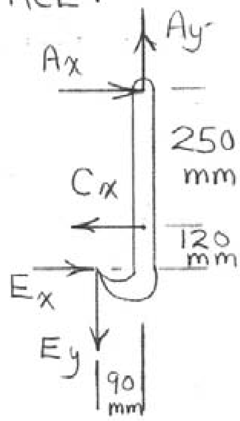


$\theta = \cos^{-1}\frac{340}{350} = 13.73°$

$\sum F_y = 0: \; 135(9.81) - 2\,GA \sin 13.73° = 0$

$GA = GB = 2790$ N

ACE:



$A_x = 2790 \cos 13.73° = 2710$ N

$A_y = 2790 \sin 13.73° = 662$ N

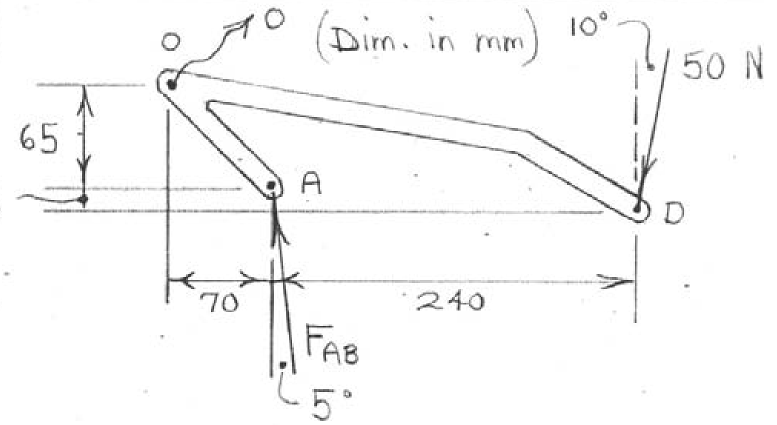
$\sum F_y = 0 \Rightarrow E_y = 662$ N

$\sum M_C = 0: \; 2710(250) - 662(90) - E_x(120) = 0, \quad E_x = 5150$ N

$E = \sqrt{E_x^2 + E_y^2} = 5190$ N

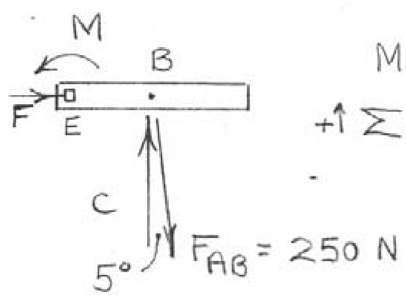
or $\underline{5.19 \text{ kN}}$

4/109



$$\circlearrowleft + \; \Sigma M_O = 0: \quad F_{AB}\left[\cos 5^\circ (70) - \sin 5^\circ (65)\right] - 50\left[\cos 10^\circ (310) + \sin 10^\circ (83)\right] = 0$$

$$F_{AB} = 250 \text{ N}$$



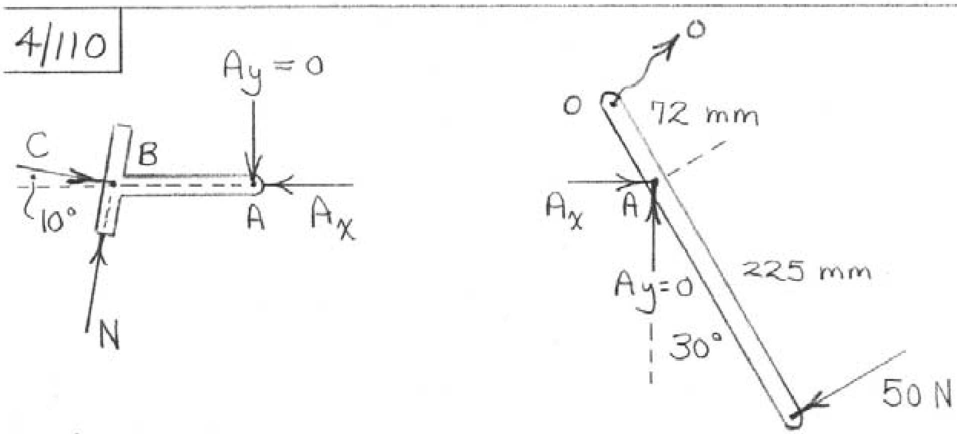
$M = 0$ (from $\Sigma M_B = 0$)

$+\uparrow \Sigma F = 0: \quad C - 250 \cos 5^\circ = 0$

$\underline{C = 249 \text{ N}}$

(a factor $\frac{C}{P} = 4.97$)

4/110



$A_y = 0$ because AB is a three-force body.

Body OA:

$$\circlearrowleft + \Sigma M_O = 0: \quad A_x(.72 \cos 30°) - 50(72+225) = 0$$

$$A_x = 238 \text{ N}$$

Body AB:

$$\xrightarrow{+} \Sigma F = 0: \quad C - 238 \cos 10° = 0$$

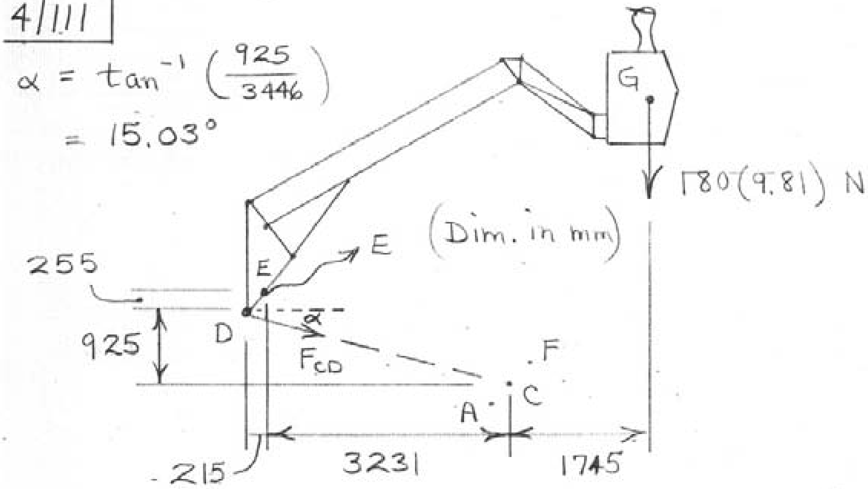
$$\underline{C = 235 \text{ N}}$$

$$\left(\text{a factor } \frac{C}{P} = 4.69\right)$$

4/111

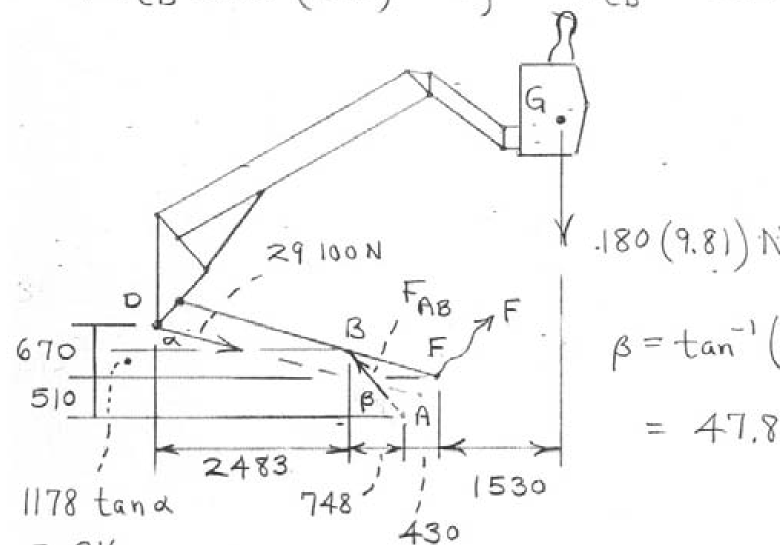
$\alpha = \tan^{-1}\left(\frac{925}{3446}\right)$

$= 15.03°$



$\curvearrowleft + \; \Sigma M_E = 0: -180(9.81)(4976) + F_{CD}\cos\alpha\,(255)$

$+ F_{CD}\sin\alpha\,(215) = 0, \qquad F_{CD} = 29\,100 \text{ N}$



$\beta = \tan^{-1}\left(\frac{510 + 316}{748}\right)$

$= 47.8°$

$1178 \tan\alpha$

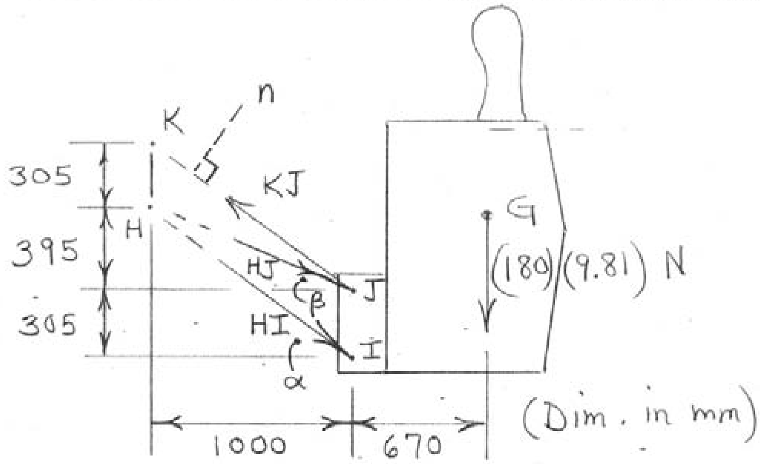
$= 316$

$\curvearrowleft + \; \Sigma M_F = 0: \; -29\,100\cos\alpha\,(670) + 29\,100\sin\alpha\,(3661)$

$- F_{AB}\cos\beta\,(510) - F_{AB}\sin\beta\,(430) - 180(9.81)(1530) = 0$

$\underline{F_{AB} = 9200 \text{ N}}$

4/112

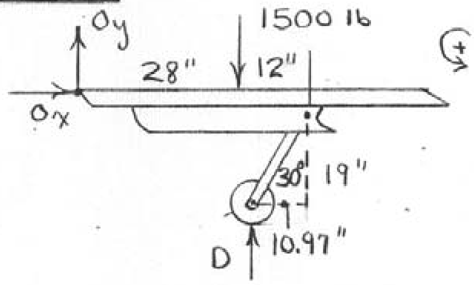


$$\alpha = \tan^{-1}\frac{700}{1000} = 35.0°;\ \beta = \tan^{-1}\frac{395}{1000} = 21.6°$$

$$\Sigma F_n = 0: \quad HJ \sin(\alpha - \beta) - 180(9.81)\cos\alpha = 0$$

$$\underline{HJ = 6220 \text{ N}}$$

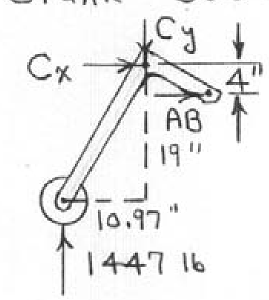
4/113 Entire ramp + mechanism:



$\circlearrowleft\!+\ \sum M_O = 0:\ D(28+12-10.97) - 1500(28) = 0$

$D = 1447$ lb

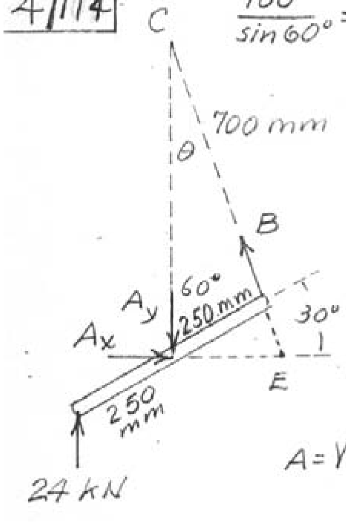
Crank BCD:



$\circlearrowleft\!+\ \sum M_C = 0: -1447(10.97) + AB(4) = 0,\ \underline{AB = 3970 \text{ lb}}$

(Cylinder is in compression)

4/114



$$\frac{700}{\sin 60^\circ} = \frac{250}{\sin\theta}, \quad \theta = \sin^{-1} 0.3093 = 18.02^\circ$$

$$\cos\theta = 0.9510, \quad \tan\theta = 0.3252$$

$$\overline{AC} = 700(0.9510) + 250(0.5) = 791 \text{ mm}$$

$$\Sigma M_C = 0; \quad 24(250)(0.866) - 791 A_x = 0$$

$$A_x = 6.57 \text{ kN}$$

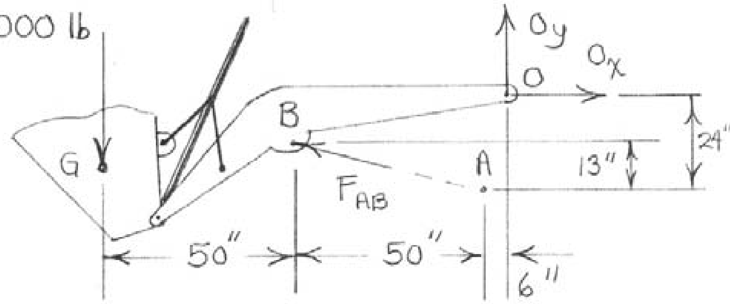
$$\overline{AE} = \overline{AC} \tan\theta = 791(0.3252) = 257 \text{ mm}$$

$$\Sigma M_E = 0; \quad A_y(257) - 24(250 \times .866 + 257) = 0$$

$$A_y = 44.2 \text{ kN}$$

$$A = \sqrt{6.57^2 + 44.2^2} = \underline{44.7 \text{ kN}}$$

4/115



$$\measuredangle OBA = \tan^{-1}\left(\frac{11}{56}\right) + \tan^{-1}\left(\frac{13}{50}\right) = 11.11° + 14.57° = 25.7°$$

$$\overline{OB} = \sqrt{11^2 + 56^2} = 57.1 \text{ in.}$$

$$\circlearrowleft + \Sigma M_O = 0: \quad 4000(106) - F_{AB} \sin 25.7° (57.1) = 0$$

$$\underline{F_{AB} = 17{,}140 \text{ lb}}$$

$$\Sigma F_x = 0: \quad O_x - 17{,}140 \cos 14.57° = 0$$

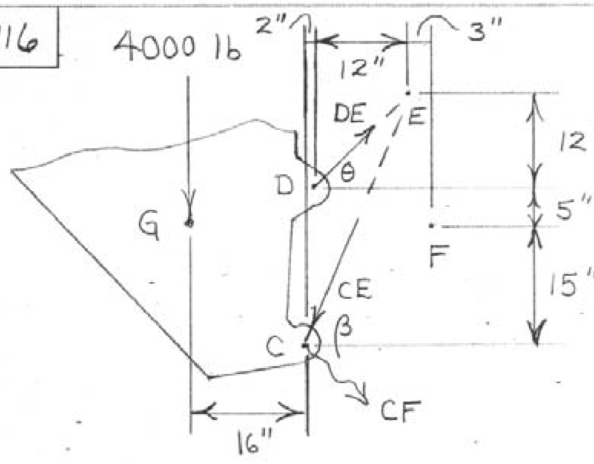
$$O_x = 16{,}590 \text{ lb}$$

$$\Sigma F_y = 0: \quad O_y + 17{,}140 \sin 14.57° - 4000 = 0$$

$$O_y = -313 \text{ lb}$$

$$O = \sqrt{16{,}590^2 + 313^2} = \underline{16{,}590 \text{ lb}}$$

4/116



$\theta = 45°$

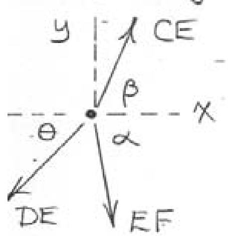
$\beta = \tan^{-1} \frac{32}{14} = 66.4°$

$\alpha = \tan^{-1} \frac{17}{3} = 80.0°$

$\circlearrowleft + \sum M_C = 0: \; 4000(16) - DE\cos 45°(20) + DE \sin 45°(2) = 0$

$DE = 5030 \text{ lb}$

FBD of joint E:



$\sum F_x = 0:$

$-DE\cos\theta + CE\cos\beta + EF\cos\alpha = 0$

$\sum F_y = 0:$

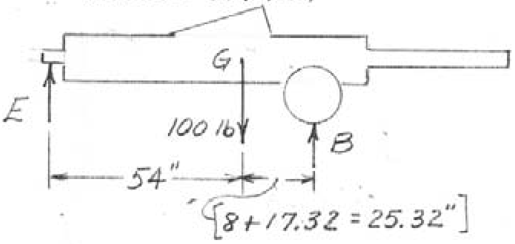
$-DE\sin\theta + CE\sin\beta - EF\sin\alpha = 0$

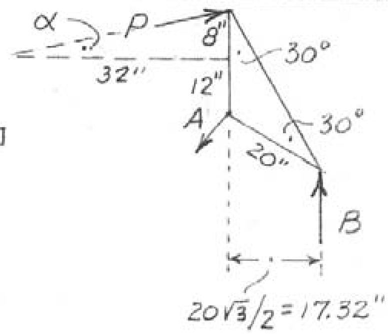
Solve simultaneously to obtain

$$\begin{cases} \underline{CE = 7440 \text{ lb}} \\ EF = 3310 \text{ lb} \end{cases}$$

4/117 $\tan\alpha = \frac{8}{32}$, $\alpha = 14.04°$

$\cos\alpha = 0.9701$





$\Sigma M_E = 0;\ (54 + 25.32)B - 54(100) = 0$

$B = 68.1 \text{ lb}$

$\Sigma M_A = 0;\ 0.9701P(20) - 68.1(17.32) = 0$

$\underline{P = 60.8 \text{ lb}}$

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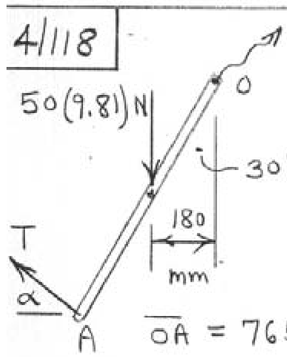
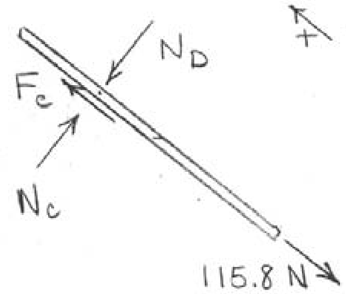


$\overline{OA} = 765$ mm

$$\alpha = \tan^{-1}\left[\frac{765\cos 30° - 410}{570 + 180 - 765\sin 30°}\right] = 34.5°$$

$$\circlearrowleft + \; \Sigma M_O = 0: \; 50(9.81)(180) - T\cos\alpha\,(765\cos 30°) - T\sin\alpha\,(765\sin 30°) = 0, \; T = 115.8 \text{ N}$$

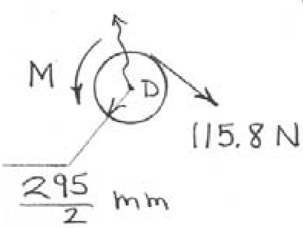
AB :



$$\Sigma F = 0: \; F_c - 115.8 = 0$$

$$F_c = 115.8 \text{ N}$$

Pinion :

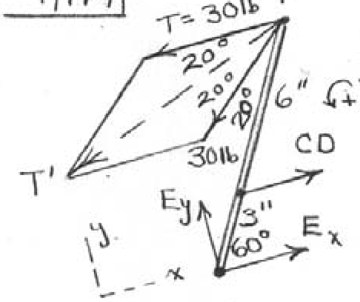


$$\circlearrowleft + \; \Sigma M_D = 0: \; M - 115.8\left(\frac{295}{2}\right) = 0$$

$$M = 17\,080 \text{ N}\cdot\text{mm}$$

or $\underline{17.08 \text{ N}\cdot\text{m}}$

4/119



$T'^2 = 30^2 + 30^2 - 2(30)(30)\cos 140°$

$T' = 56.4$ lb

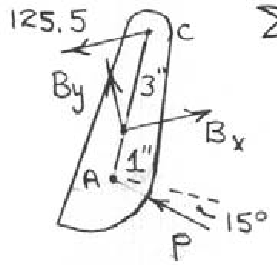
$\circlearrowleft + \Sigma M_E = 0: \; -CD\,(3 \sin 60°) + 56.4(9 \sin 40°) = 0, \quad CD = 125.5$ lb

$\Sigma F_x = 0: \; E_x + 125.5 - 56.4 \cos 20° = 0$

$E_x = -72.6$ lb

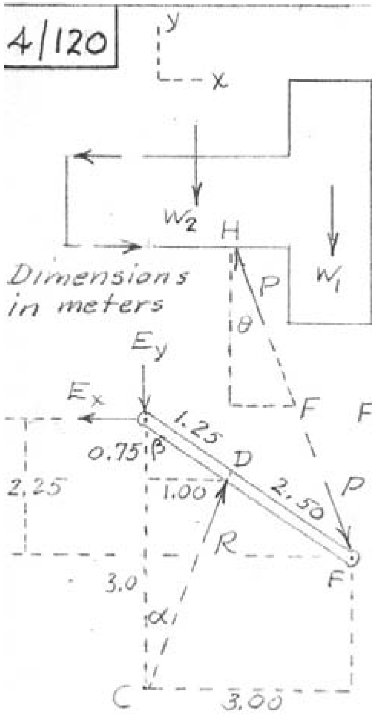
$\Sigma F_y = 0: \; E_y - 56.4 \sin 20° = 0, \quad E_y = 19.28$ lb

$E = \sqrt{E_x^2 + E_y^2} = \underline{75.1 \text{ lb}}$



$\Sigma M_B = 0: \; 125.5\,(3 \sin 60°) - P \cos 15°\,(1) = 0, \quad \underline{P = 338 \text{ lb}}$

4/120



$\sin\theta = \dfrac{(3.00 - 1.75)}{3.25} = \dfrac{5}{13}, \quad \cos\theta = \dfrac{12}{13}$

$\cos\beta = \dfrac{0.75}{1.25} = \dfrac{3}{5}, \quad \sin\beta = \dfrac{4}{5}$

$\tan\alpha = 1/3, \quad \sin\alpha = 1/\sqrt{10}, \quad \cos\alpha = 3/\sqrt{10}$

$W_1 = 1.5(9.81) = 14.72 \text{ kN}$

$W_2 = 2(9.81) = 19.62 \text{ kN}$

$\Sigma F_y = 0; \quad \frac{12}{13}P - 14.72 - 19.62 = 0$

$P = 37.2 \text{ kN}$

For EF; $\Sigma M_E = 0; \quad 37.2\left(\frac{12}{13}\right)(3) - 37.2\left(\frac{5}{13}\right)(2.25) - \frac{R}{\sqrt{10}}(3.0 + 0.75) = 0$

$\underline{R = 59.7 \text{ kN}}$

$\Sigma F_x = 0; \quad E_x - 59.7/\sqrt{10} - 37.2(5/13) = 0$

$E_x = 33.2 \text{ kN}$

$\Sigma F_y = 0; \quad E_y + 37.2(12/13) - 59.7(3/\sqrt{10}) = 0$

$E_y = 22.3 \text{ kN}$

$E = \sqrt{\overline{33.2}^2 + \overline{22.3}^2} = \underline{40.0 \text{ kN}}$

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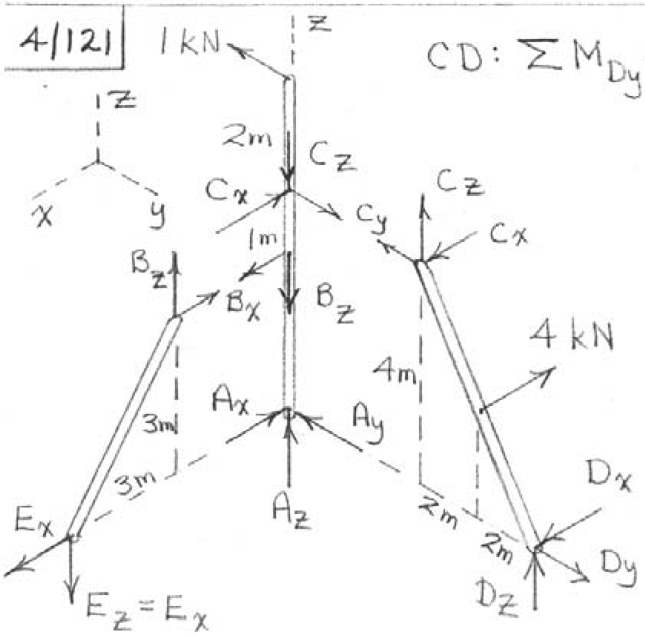


CD: $\Sigma M_{D_y} = 0: \; 4(2) - C_x(4) = 0$

$C_x = 2$ kN

ABC: $\Sigma M_{B_y} = 0: \; 2(1) - A_x(3) = 0, \qquad A_x = 0.667$ kN

$\Sigma F_x = 0: \; B_x - 0.667 - 2 = 0, \qquad B_x = 2.67$ kN

EB: $B_z = E_z = B_x = E_x = 2.67$ kN

ABC: $\Sigma M_{A_x} = 0: \; 1(6) - C_y(4) = 0, \qquad C_y = 1.50$ kN

CD: $\Sigma M_{D_x} = 0: \; C_z = C_y = 1.50$ kN

ABC: $\Sigma F_z = 0: \; A_z - 2.67 - 1.50 = 0, \qquad A_z = 4.17$ kN

$\Sigma F_y = 0: \; A_y + 1 - 1.50 = 0, \qquad A_y = 0.50$ kN

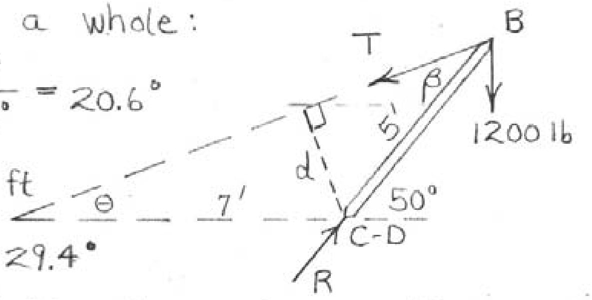
$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \underline{4.25 \text{ kN}}$

4/122 Frame as a whole:

$\theta = \tan^{-1} \dfrac{5 \sin 50°}{7 + 5 \cos 50°} = 20.6°$

$d = 7 \sin 20.6° = 2.46$ ft

$\beta + 20.6° = 50°, \quad \beta = 29.4°$

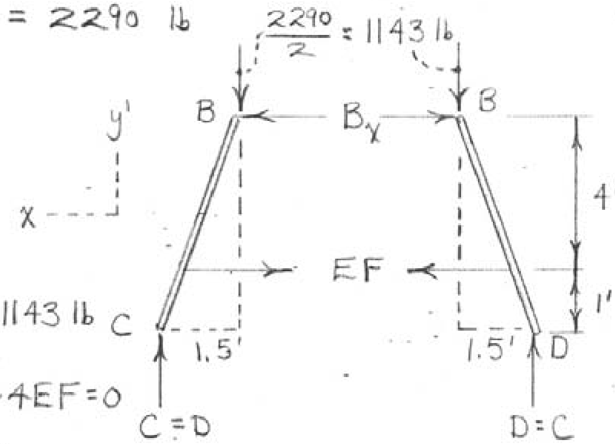


$\Sigma M_x = 0: \ (5 \cos 50°)(1200) - 2.46T = 0, \quad \underline{T = 1569 \text{ lb}}$

$\Sigma F_{(C-D)-B} = 0: \ R - 1200 \cos 40° - 1569 \cos 29.4° = 0$

$R = 2290$ lb

Plane of frame:

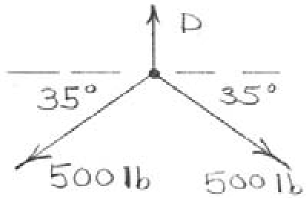


$\Sigma F_{y'} = 0: \ C = D = 1143$ lb

$\Sigma M_B = 0: \ 1143(1.5) - 4EF = 0$

$\underline{EF = 429 \text{ lb}}$

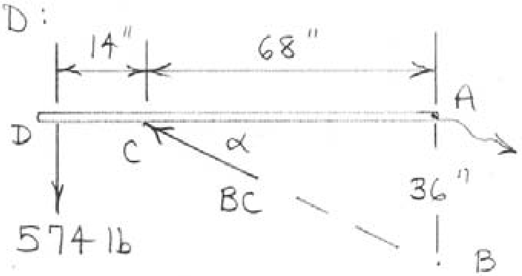
4/123 At D:



$\uparrow \Sigma F = 0: \; D - 2(500)\sin 35° = 0$

$D = 574 \text{ lb}$

ACD:



$\alpha = \tan^{-1}\frac{36}{68}$

$= 27.9°$

$\Sigma M_A = 0: \; 574(82) - BC \sin 27.9° (68) = 0$

$BC = \underline{1478 \text{ lb} = F}$

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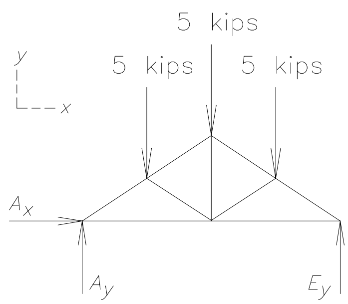
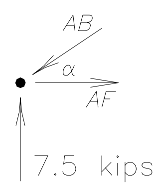


As a whole:

By symmetry, $A_y = E_y = 7.5$ kips

$\Sigma F_x = 0: \ A_x = 0$

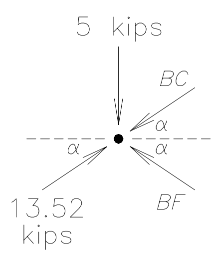
Joint *A*:



$\alpha = \tan^{-1}\dfrac{10}{15} = 33.7°$

$$\begin{cases} \Sigma F_x = 0: \ -13.52\cos 33.7° + AF = 0, \ \underline{AF = 11.25 \text{ kips } T} \\ \Sigma F_y = 0: \ -AB\sin 33.7° + 7.5 = 0, \ \underline{AB = 13.52 \text{ kips } C} \end{cases}$$

Joint *B*:

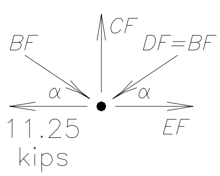


$$\begin{cases} \Sigma F_x = 0: \ (13.52 - BC - BF)\cos\alpha = 0 \\ \Sigma F_y = 0: \ (13.52 - BC = BF)\sin\alpha - 5 = 0 \end{cases}$$

Solve simultaneously to obtain:

$\underline{BC = 9.01 \text{ kips } C}$, $\underline{BF = 4.51 \text{ kips } C}$

Joint *F*:

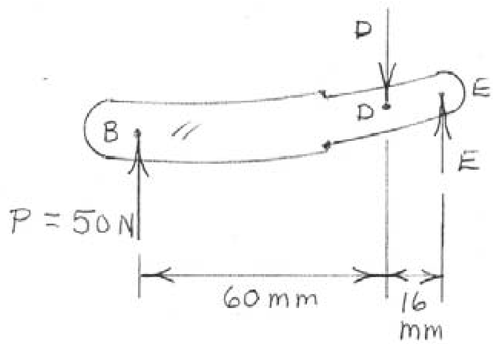


By symmetry, $\underline{EF = AF = 11.25 \text{ kips } T}$
& $\underline{DF = BF = 4.51 \text{ kips } C}$

$\Sigma F_y = 0: \ CF - 2(4.51)\sin 33.7° = 0, \quad \underline{CF = 5 \text{ kips } T}$

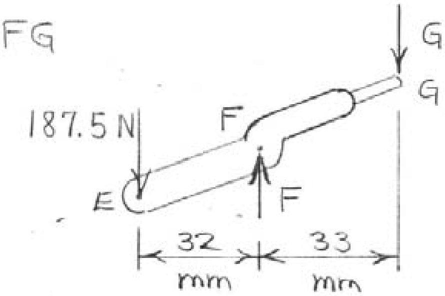
By symmetry, $\underline{CD = 9.01 \text{ kips } C}$, $\underline{DE = 13.52 \text{ kips } C}$, $\underline{DF = 4.51 \text{ kips } C}$

4/125 Handle BDE



$\circlearrowleft + \Sigma M_D = 0: \; -50(60) + E(16) = 0, \quad E = 187.5 \text{ N}$

Jaw EFG



$\circlearrowleft + \Sigma M_F = 0: \; 187.5(32) - G(33) = 0, \quad \underline{G = 181.8 \text{ N}}$

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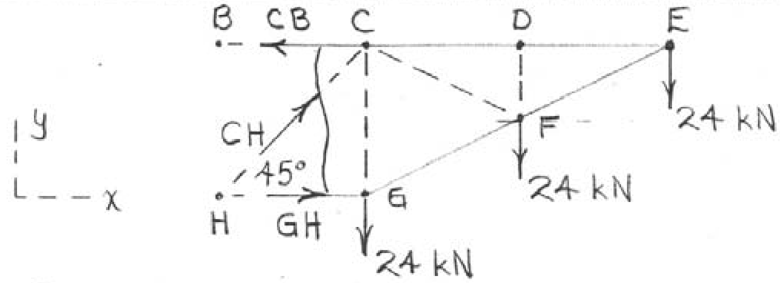
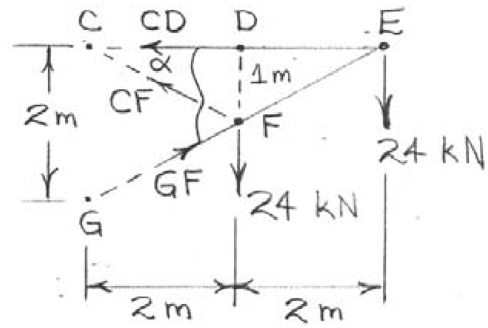


$\Sigma F_y = 0: \; CH \sin 45° - 3(24) = 0$, $\underline{CH = 101.8 \text{ kN C}}$

$\alpha = \tan^{-1}\frac{1}{2} = 26.6°$



$\circlearrowleft + \Sigma M_E = 0: \; 24(2) - (CF \sin 26.6°)(4) = 0$

$\underline{CF = 26.8 \text{ kN T}}$

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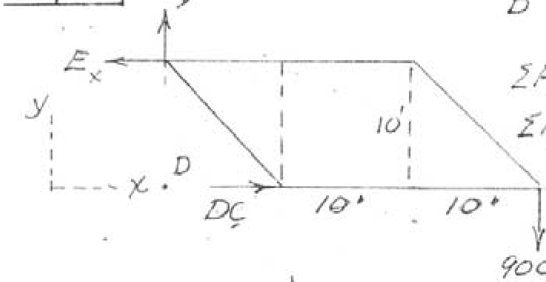


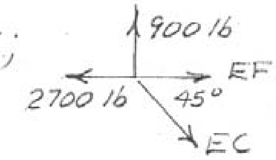
$\Sigma M_D = 0;\ 900(30) - 10E_x = 0$

$E_x = 2700$ lb

$\Sigma F_x = 0;\ DC = 2700$ lb C

$\Sigma F_y = 0;\ E_y = 900$ lb

Joint E;



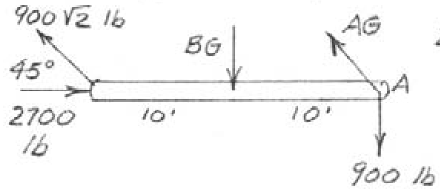
$\Sigma F_y = 0;\ EC/\sqrt{2} - 900 = 0$

$EC = 900\sqrt{2}$ lb

$\Sigma F_x = 0;\ EF + \dfrac{900\sqrt{2}}{\sqrt{2}} - 2700 = 0$

$EF = 1800$ lb T

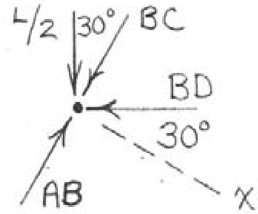
Joint F; $FG = EF$, $FC = 0$



$\Sigma M_A = 0;\ 10BG - \dfrac{900\sqrt{2}}{\sqrt{2}}20 = 0$

$\underline{BG = 1800 \text{ lb C}}$

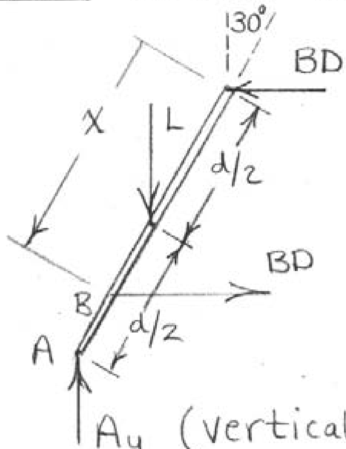
4/128 Joint B :



$\Sigma F_x = 0: \quad \frac{L}{2}\left(\frac{1}{2}\right) - BD\frac{\sqrt{3}}{2} = 0$

$BD = \frac{\sqrt{3}}{6} L\ C$, independent of x.

4/129 Member ABC:



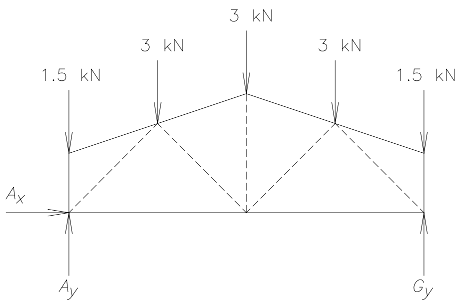
BD (from $\Sigma F_x = 0$ and vertical line of symmetry)

A_y (vertical, from overall frame)

$$\curvearrowleft + \Sigma M_A = 0: \quad BD(d \cos 30°) - BD(d - x)\cos 30° - L\left(\frac{d}{2} \sin 30°\right) = 0$$

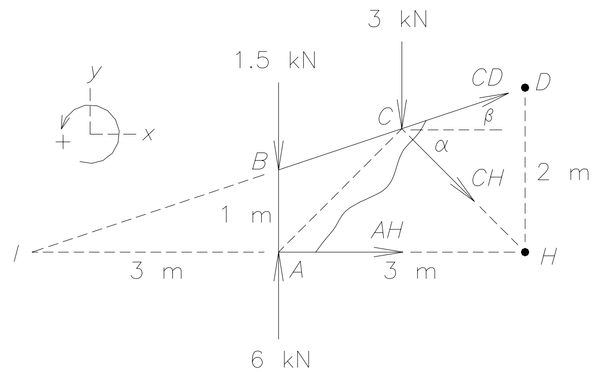
$$\underline{BD = \frac{Ld}{2x} \tan 30° = \frac{0.289 Ld}{x}}$$

(x cannot be zero)



By symmetry, $A_y = G_y = 6$ kN

$\Sigma F_x = 0: \ A_x = 0$



$\alpha = 45°, \ \beta = \tan^{-1}\dfrac{0.5}{1.5} = 18.43°$

$\overline{AC} = \dfrac{3}{\sqrt{2}}$ m

$\Sigma M_I = 0: \ 6(3) - 1.5(3) - 3(4.5) - CH(2\overline{AC}) = 0, \ \underline{CH = 0}$

$\Sigma F_y = 0: \ 6 + CD\sin 18.43° - 1.5 - 3 = 0, \ CD = -4.74 \text{ kN or } \underline{4.74 \text{ kN } C}$

$\Sigma F_x = 0: \ -4.74\cos 18.43° + AH = 0, \ \underline{AH = 4.5 \text{ kN } T}$

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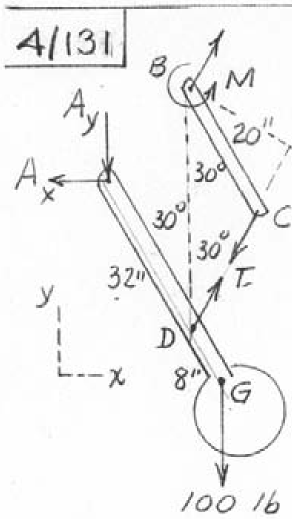

ADG; $\Sigma M_A = 0$; $100(40 \sin 30°)$

$- T(32 \cos 30°) = 0$

$T = 72.2$ lb

BC; $\Sigma M_B = 0$; $72.2(20 \cos 30°) - M = 0$

$M = 1250$ lb-in.

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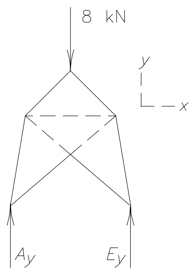
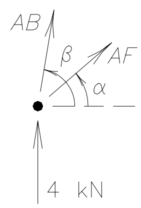


By symmetry, $A_y = E_y = 4$ kN

Joint *A*:

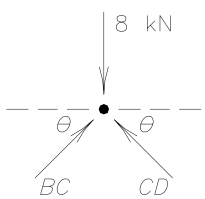


$$\alpha = \tan^{-1}\frac{3}{3.5} = 40.6°,\ \beta = \tan^{-1}\frac{3}{0.5} = 80.5°$$

$$\begin{cases} \Sigma F_x = 0: \ AB\cos 80.5° + AF\cos 40.6° + 4 = 0 \\ \Sigma F_y = 0: \ AB\sin 80.5° + AF\sin 40.6° = 0 \end{cases}$$

Solve to obtain: $AB = -4.73$ kN or 4.73 kN *C*

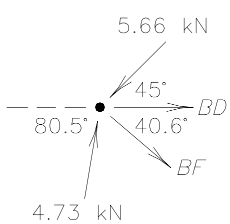
Joint *C*:



By symmetry, *BC* = *CD*

$$\Sigma F_y = 0: \ 2BC\sin 45° - 8 = 0,\ BC = 5.66 \text{ kN } C$$

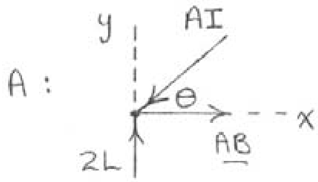
Joint *B*:



$$\Sigma F_y = 0: \ 4.73\sin 80.5° - BF\sin 40.6° - 5.66\sin 45° = 0$$

$$\underline{BF = 1.024 \text{ kN } \ T}$$

4/133 From whole truss, $A = F = 2L$

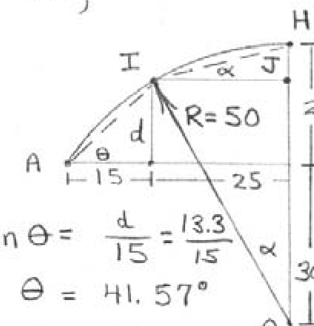


y: $2L - AI \sin\theta$

$AI = 3.01L$ C

x: $-3.01\cos\theta + AB = 0$

$\underline{AB = 2.26L \text{ T}}$



$\sin\alpha = \dfrac{25}{50}$

$\alpha = 30°$

$\tan\alpha = \dfrac{25}{\overline{OJ}}$

$\overline{OJ} = 43.3$

$d = 43.3 - 30$

$= 13.3$ m

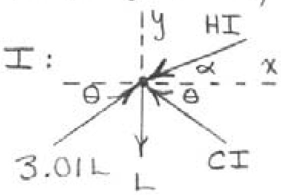
$\tan\theta = \dfrac{d}{15} = \dfrac{13.3}{15}$

$\theta = 41.57°$

$\tan\alpha = \dfrac{50 - 43.3}{25}$

$\alpha = 15°$

From joint B, $\underline{BI = L \text{ T}}$



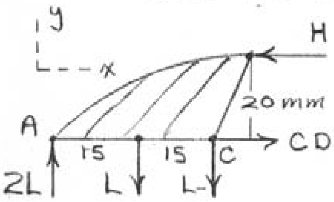
$$\begin{cases} \Sigma F_x = 0: (3.01L - CI)\cos\theta - HI\cos\alpha = 0 \\ \Sigma F_y = 0: (3.01L + CI)\sin\theta - HI\sin\alpha - L = 0 \end{cases}$$

Solve to obtain $\underline{CI = -0.458L \text{ T}}$

$HI = 2.69L$ C

4/134 From whole structure, $A = F = 2L$

Half of structure:

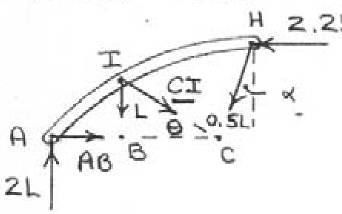


$\circlearrowleft^{+} \sum M_C = 0: \; 20H + 15L - 30(2L) = 0; \quad H = 2.25L$

$\sum F_x = 0 \Rightarrow \; CD = 2.25L \; T$

From joint B, $\underline{BI = L \; T}$

Member AIH:



From previous solution, $\theta = 41.57$, $\alpha = \tan^{-1}\left(\frac{10}{20}\right) = 26.57^\circ$

$\sum F_y = 0: \; 2L - L - CI \sin\theta - 0.5L\cos\alpha = 0, \quad \underline{CI = 0.833L \; T}$

$\sum F_x = 0: \; AB + 0.833L\cos\theta - 0.5L\sin\alpha - 2.25L = 0$

$\underline{AB = 1.850L \; T}$

Problem not solvoble without CH data.

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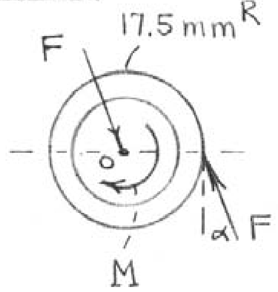
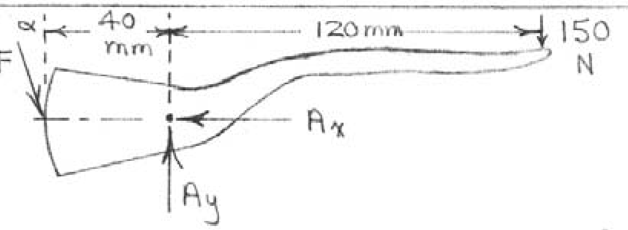
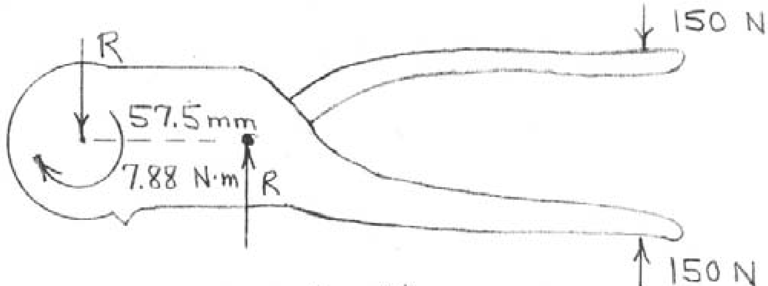




$$\sum M_A = 0:\ 40 F\cos\alpha = 120(150)$$

$$F\cos\alpha = 450\ \text{N}$$

$$\sum M_O = 0:\ 450(0.0175) = \underline{M = 7.88\ \text{N}\cdot\text{m}}$$

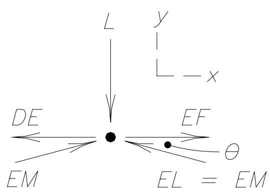


$$\sum M = 0:\ 57.5(10^{-3})R = 7.88$$

$$\underline{R = 137.0\ \text{N}}$$

We can begin at joint E without finding the external reactions.

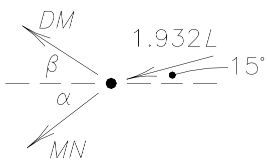
Joint E:



$$\theta = \tan^{-1}\frac{24-24\sin 60^\circ}{24\sin 30^\circ} = 15^\circ$$

$$\begin{cases} \Sigma F_y = 0: \; 2(EM\sin 15^\circ) - L = 0, \; \underline{EM = 1.932\,L\ C} \\ \Sigma F_x = 0: \; 1.932L\cos 15^\circ - DE = 0, \; \underline{DE = 1.366\,L\ C} \end{cases}$$

Joint M:



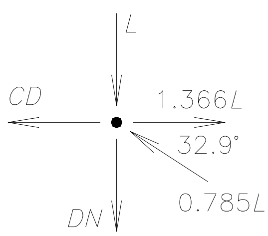
$$\alpha = \tan^{-1}\frac{24\sin 60^\circ - 24\sin 45^\circ}{24\cos 45^\circ - 24\cos 60^\circ} = 37.5^\circ$$

$$\beta = \tan^{-1}\frac{24-24\sin 60^\circ}{24\cos 45^\circ - 24\cos 60^\circ} = 32.9^\circ$$

$$\begin{cases} \Sigma F_x = 0: \; -DM\cos 32.9^\circ - MN\cos 37.5^\circ - 1.932L\cos 15^\circ = 0 \\ \Sigma F_y = 0: \; DM\sin 32.9^\circ - MN\sin 37.5^\circ - 1.932\sin 15^\circ = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{DM = 0.785L\ C}$

Joint D:



$$\Sigma F_y = 0: \; 0.785L\sin 32.9^\circ - L - DN = 0$$

$$\underline{DN = 0.574L\ C}$$

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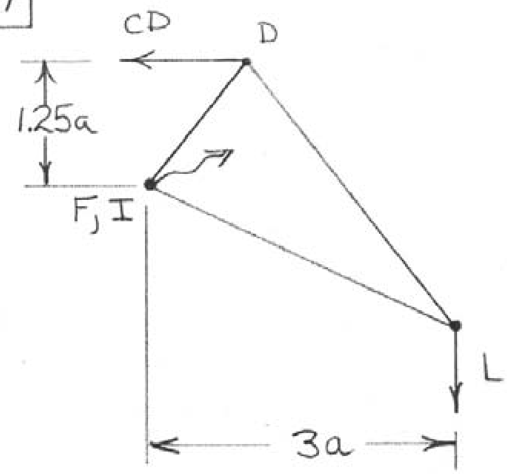

$$\curvearrowleft + \sum M_F = 0: \quad CD(1.25a) - L(3a) = 0$$

$$\underline{CD = 2.4L \ T}$$

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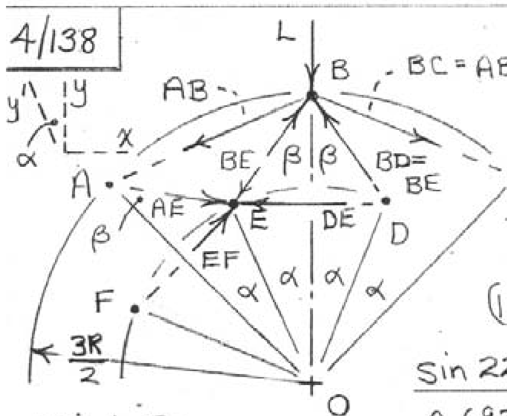


$$\begin{cases} \alpha = 22.5^\circ \\ \angle ABO = 67.5^\circ \end{cases}$$

$$\overline{BE}^2 = R^2 + (1.5R)^2 - 2R \times (1.5R)\cos 22.5^\circ; \quad \overline{BE} = 0.692R$$

$$\frac{\sin 22.5^\circ}{0.692R} = \frac{\sin\beta}{R}, \quad \beta = 33.6^\circ$$

Joint B:

$$\Sigma F_y = 0: \; 2(0.8L)\cos 33.6^\circ - 2\,AB\cos 67.5^\circ - L = 0$$

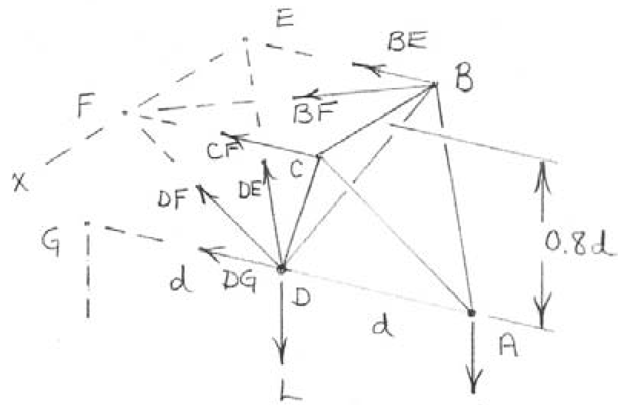
$$\underline{AB = 0.434L \;\; T}$$

For joint E, note that $EF = DE$ & $AE = BE$.

$$\Sigma F_{y'} = 0: \; -2(0.8L)\cos 56.1^\circ + 2DE\cos 67.5^\circ = 0$$

$$\underline{DE = 1.166L \;\; C}.$$

4/139 Section for DG:



$$\Sigma M_x = 0: \quad -DG(0.8d) - L\left(\frac{d}{2}\right) - L\left(\frac{3d}{2}\right) = 0$$

$$\underline{DG = -2.5L \quad (\text{or } 2.5L\ C)}$$

Similarly, $\underline{AD = 0.625L\ C}$

▸ 4/140 Vector expressions for forces at A (treated as tensions) with $F_{AE} = F_{AF} = F_1$, $F_{BE} = F_{BF} = P$, $F_{BD} = F_{BC} = C$, are

$\underline{F}_{AE} = \frac{F_1}{1.552}(-1.2\underline{i} - 0.4\underline{j} + 0.9\underline{k})$, $\underline{F}_{AF} = \frac{F_1}{1.552}(-1.2\underline{i} + 0.4\underline{j} + 0.9\underline{k})$

$\underline{F}_{AB} = \frac{F_{AB}}{1.432}(-0.3\underline{i} + 1.4\underline{k})$, $\underline{F} = 2.2\underline{k}$. For joint A, $\Sigma\underline{F} = 0$ gives $\left[\frac{F_{AB}}{1.432}(-0.3) + \frac{2F_1}{1.552}(-1.2)\right]\underline{i}$

$+\left[2.2 + \frac{F_{AB}}{1.432}(1.4) + \frac{2F_1}{1.552}(0.9)\underline{k}\right] = \underline{0}$

Solve to get $\underline{F_{AB} = -2.681 \text{ kN}}$, $\underline{F_1 = 0.363 \text{ kN}}$

On B: $\underline{F}_{BE} = \frac{P}{1.105}(-0.9\underline{i} - 0.4\underline{j} - 0.5\underline{k})$

$\underline{F}_{BF} = \frac{P}{1.105}(-0.9\underline{i} + 0.4\underline{j} - 0.5\underline{k})$, $\underline{F}_{BD} = \frac{C}{1.105}(-0.9\underline{i} - 0.4\underline{j} + 0.5\underline{k})$

$\underline{F}_{BC} = \frac{C}{1.105}(-0.9\underline{i} + 0.4\underline{j} + 0.5\underline{k})$

For joint B, $\Sigma\underline{F} = \underline{0}$ gives

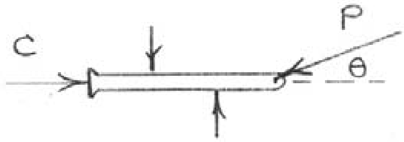
$\left(\frac{-1.8P}{1.105} - \frac{1.8C}{1.105} + 0.3\frac{-2.681}{1.432}\right)\underline{i} + \left(\frac{-P}{1.105} + \frac{C}{1.105} - 1.4\frac{-2.681}{1.432}\right)\underline{k}$

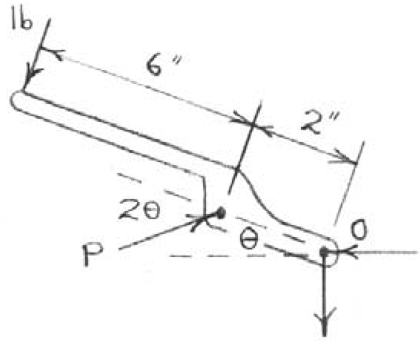
$+ 0\underline{j} = \underline{0}$. Solve to get $P = 1.620$ kN,

$C = -1.275$ kN, $\underline{F_{BE} = P = 1.620 \text{ kN}}$

***4/141**

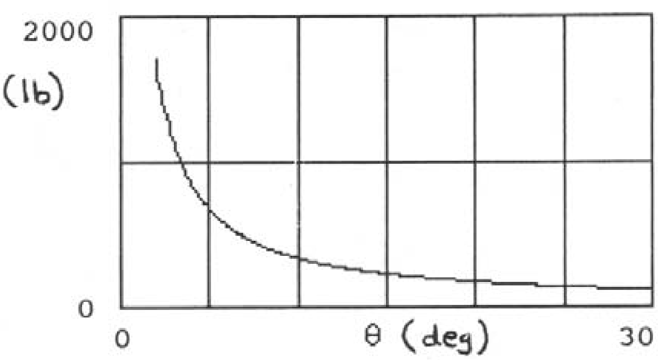
Shaft: $\Sigma F = 0$ gives
$C = P\cos\theta$



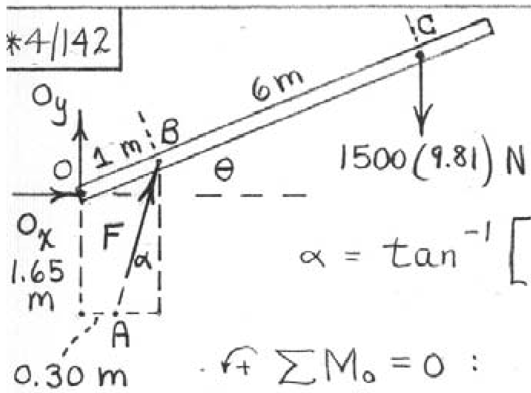


Lever: $\Sigma M_O = 0$: $30(8) - P\sin 2\theta\,(2) = 0$

$P = \dfrac{120}{\sin 2\theta} = \dfrac{60}{\sin\theta\cos\theta}$, $\quad \underline{C = \dfrac{60}{\sin\theta}}$ (lb)



*4/142

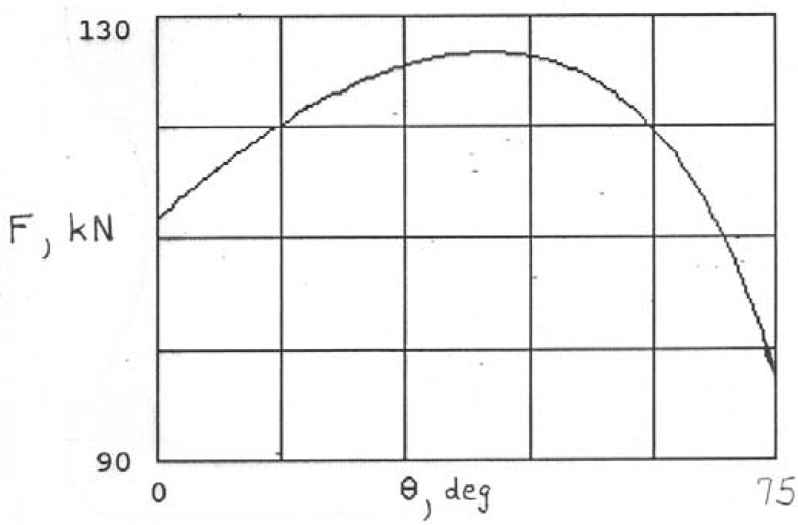


$$\alpha = \tan^{-1}\left[\frac{1\cos\theta - 0.3}{1\sin\theta + 1.65}\right]$$

$\curvearrowleft + \sum M_o = 0$:

$$(F\cos\alpha)(1\cos\theta) - (F\sin\alpha)(1\sin\theta) - 1500(9.81)(7\cos\theta) = 0$$

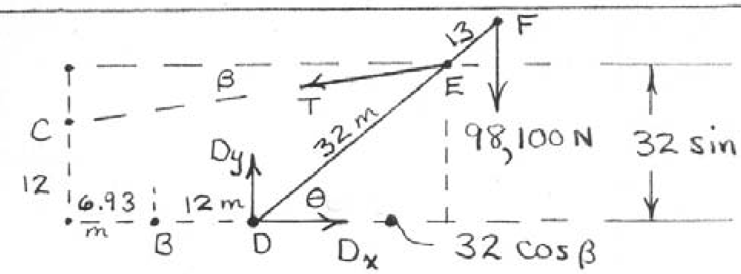
$$F = \frac{103.0\cos\theta}{(\cos\alpha\cos\theta - \sin\alpha\sin\theta)} \quad (\text{in kN})$$



$F_{max} = 126.7$ kN @ $\theta = 39.8°$

*4/143

Boom:

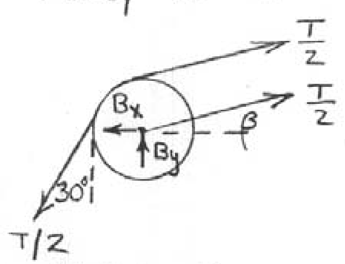


$$\beta = \tan^{-1}\left(\frac{32\sin\theta - 12}{6.93 + 12 + 32\cos\theta}\right) = \tan^{-1}\left(\frac{32\sin\theta - 12}{32\cos\theta + 18.93}\right) \quad (1)$$

$$\circlearrowleft \Sigma M_D = 0: \ (T\cos\beta)(32\sin\theta) - (T\sin\beta)(32\cos\theta) - (98{,}100)(45\cos\theta) = 0$$

$$\text{So} \quad T = \frac{137{,}953\cos\theta}{\cos\beta\sin\theta - \sin\beta\cos\theta} \ (\text{in N}) \quad (2)$$

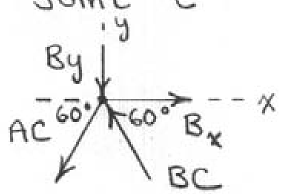
Pulley at C:



Equilibrium yields

$$\begin{cases} B_x = T\cos\beta - \frac{T}{2}\sin 30^\circ \\ B_y = -T\sin\beta + \frac{T}{2}\cos 30^\circ \end{cases} \quad (3)$$

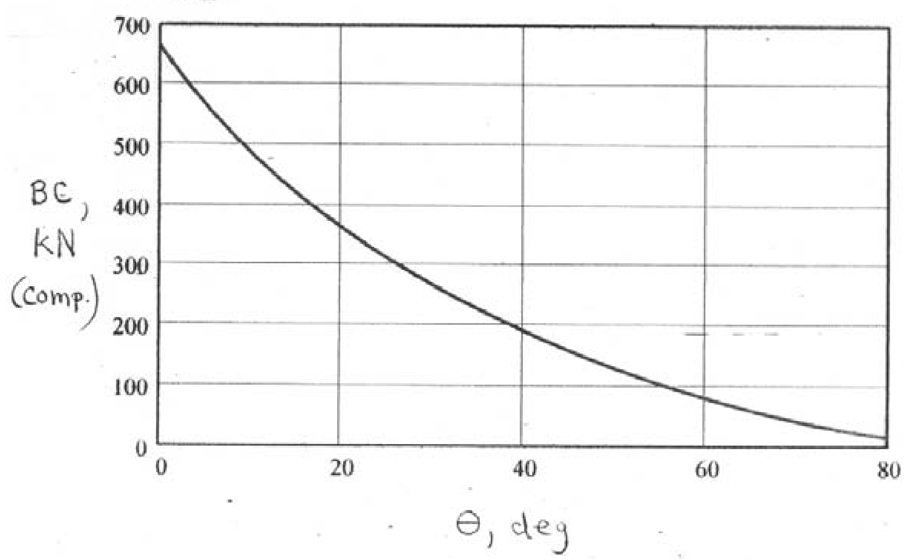
Joint C



Equilibrium yields

$$BC = B_x + 0.5774 B_y \quad (4)$$

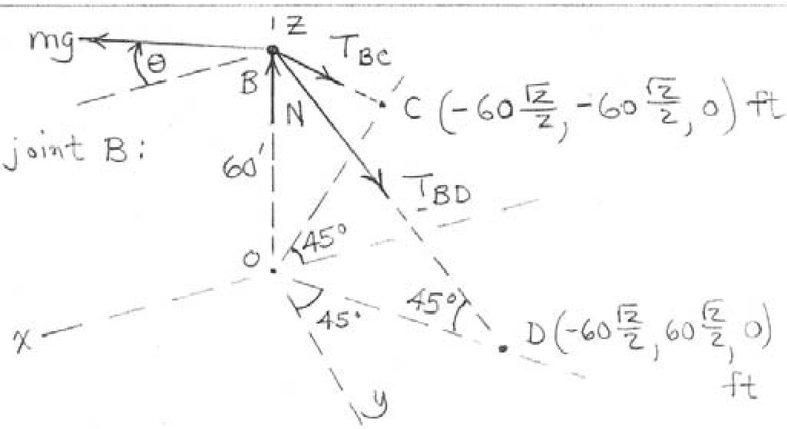
$$(AC = B_x - 0.5774 B_y)$$



BC @ $\theta = 40^\circ$ = 190.5 kN

*4/144

FBD of joint B:



$$\underline{T}_{BC} = T_{BC}\left[\frac{-60\frac{\sqrt{2}}{2}\underline{i} - 60\frac{\sqrt{2}}{2}\underline{j} - 60\underline{k}}{\sqrt{2(60)^2\frac{2}{4} + 60^2}}\right]$$

$$= T_{BC}\left[-0.5\underline{i} - 0.5\underline{j} - 0.707\underline{k}\right]$$

$$\underline{T}_{BD} = T_{BD}\left[-0.5\underline{i} + 0.5\underline{j} - 0.707\underline{k}\right]$$

$$\Sigma F_x = 0: -0.5T_{BC} - 0.5T_{BD} + mg\cos\theta = 0 \quad (1)$$

$$\Sigma F_y = 0: -0.5T_{BC} + 0.5T_{BD} - mg\sin\theta = 0 \quad (2)$$

Solve simultaneously:

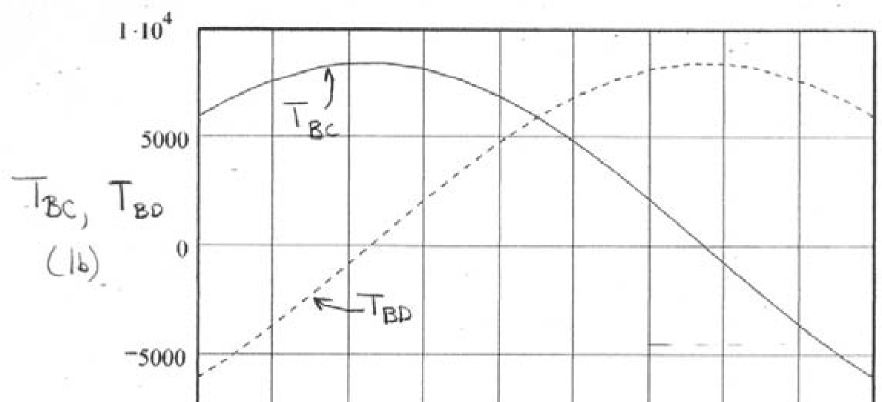
$$T_{BC} = mg(\cos\theta - \sin\theta)$$

$$T_{BD} = mg(\cos\theta + \sin\theta)$$

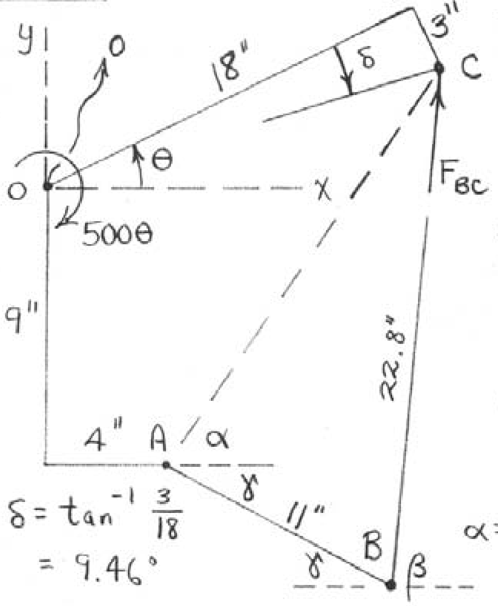
Maximum values

$T_{BC_{max}} = 8490$ lb @ $\theta = -45°$

$T_{BD\,max} = 8490$ lb @ $\theta = 45°$



*4/145 $\overline{OC} = \sqrt{18^2 + 3^2}$; $\overline{AB} = 11''$; $\overline{BC} = \sqrt{14^2 + 17^2}$



$$\overline{AC} = \left\{ \left[18\cos\theta + 3\sin\theta - 4 \right]^2 + \left[18\sin\theta - 3\cos\theta + 9 \right]^2 \right\}^{\frac{1}{2}}$$

$$\measuredangle BAC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2\,\overline{AB}\,\overline{AC}} \right]$$

$$\measuredangle ABC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}} \right]$$

$$\delta = \tan^{-1}\frac{3}{18} = 9.46^\circ$$

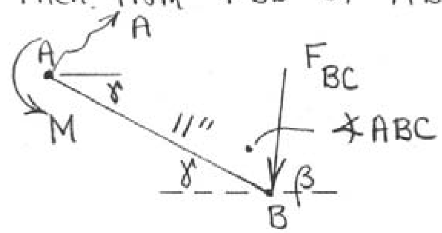
$$\alpha = \tan^{-1}\left[\frac{18\sin\theta - 3\cos\theta + 9}{18\cos\theta + 3\sin\theta - 4} \right]$$

$\gamma = \measuredangle BAC - \alpha$; $\beta = 180^\circ - \gamma - \measuredangle ABC$

From above FBD of door:

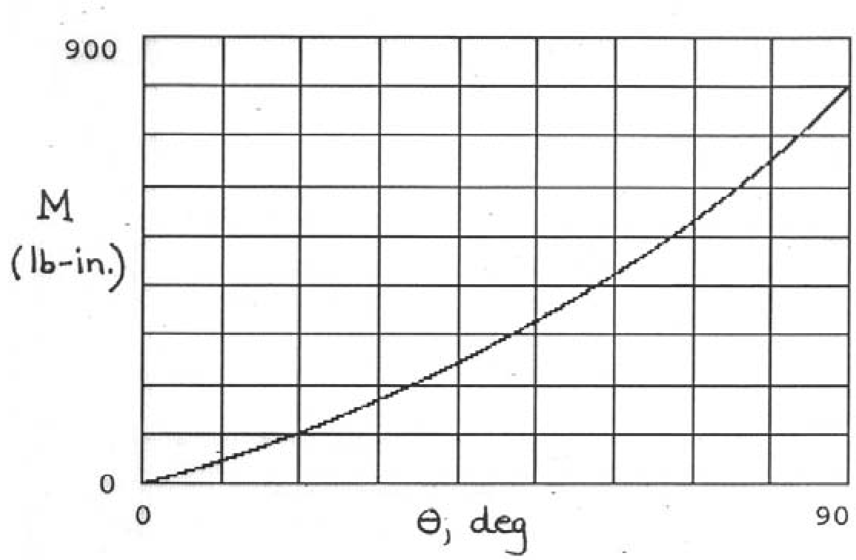
$$\circlearrowleft + \Sigma M_O = 0: \; -500\theta + F_{BC}\left(\overline{OC} \sin(\beta - \theta + \delta) \right) = 0$$

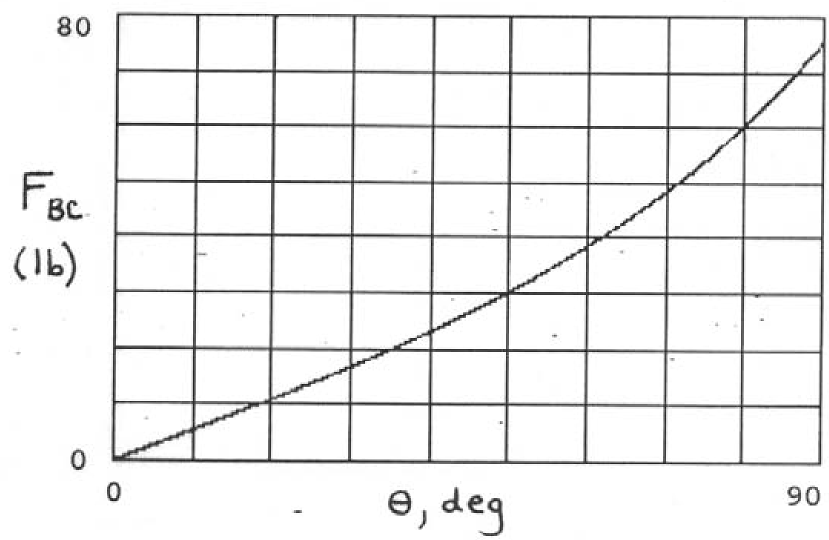
Then from FBD of AB:



$$\circlearrowleft + \Sigma M_A = 0: \; M - F_{BC}\left(11 \sin \measuredangle ABC \right) = 0$$

Solve the above equations (in order) with θ varied from 0° to 90° to obtain the following plots for M and F_{BC}.





At $\theta = 45°$, $M = 285$ lb-in.

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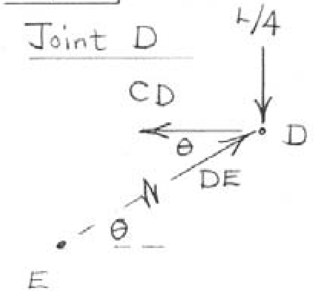
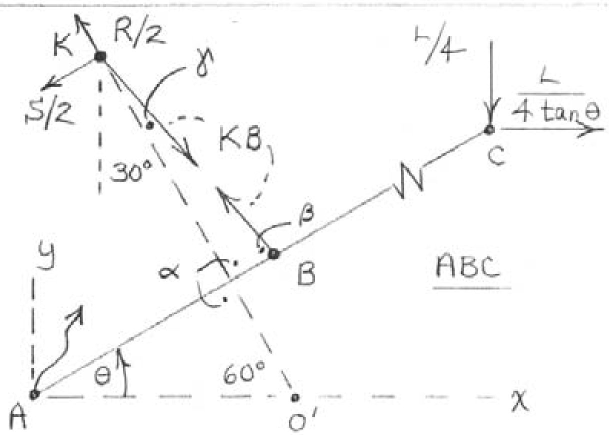
Joint D





From FBD of joint D,

$$\begin{cases} \Sigma F_y = 0: \quad DE \sin\theta - \frac{L}{4} = 0, \quad DE = \dfrac{L}{4\sin\theta} \;\; C \\ \Sigma F_x = 0: \quad \dfrac{L}{4\sin\theta}\cos\theta - CD = 0, \quad CD = \dfrac{L}{4\tan\theta} \;\; T \end{cases}$$

From above right diagrams:

$$\underline{r}_K = \underline{r}_{B/A} + \underline{r}_{K/B} = \overline{AB}\left(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}\right) + \overline{BK}\left(-\sin(\gamma + 30°)\,\underline{i} + \cos(\gamma + 30°)\,\underline{j}\right)$$

Also,

$$\underline{r}_K = \underline{r}_{O'/A} + \underline{r}_{K/O'} = \overline{AO'}\,\underline{i} + \overline{O'K}\left(-\cos 60°\,\underline{i} + \sin 60°\,\underline{j}\right)$$

Equate the two $\underline{r}_K$ expressions, then equate corresponding $\underline{i}$- and $\underline{j}$-coefficients, and then numerically solve for γ and $\overline{O'K}$ as θ varies from 5° to 90°.

Then $\alpha = 180° - 60° - \theta$

and $\beta = 180° - \gamma - \alpha$

From FBD of ABC:

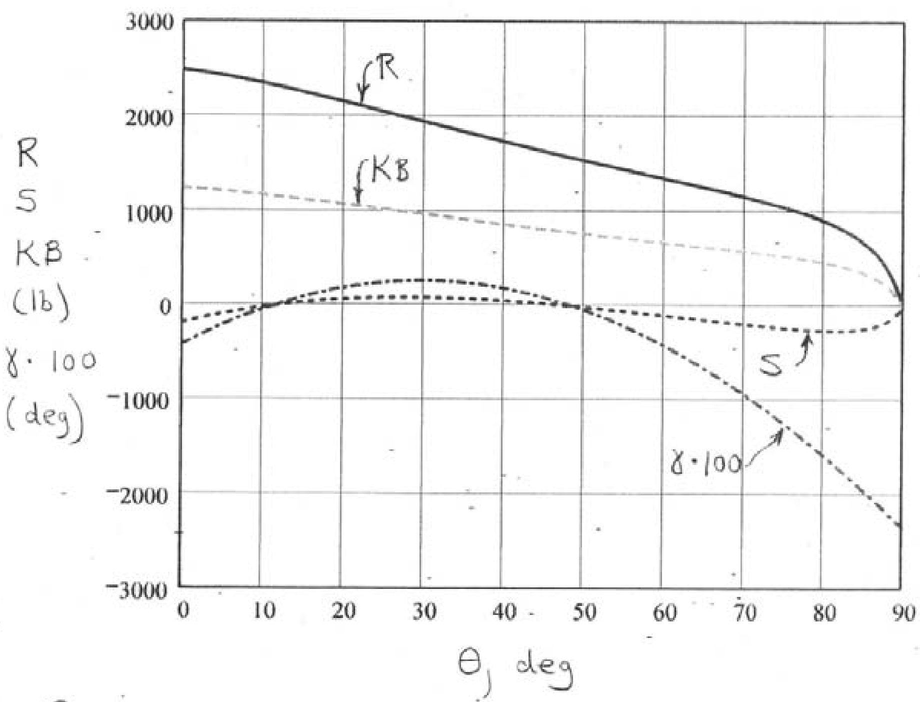
$\curvearrowleft \Sigma M_A = 0: -\frac{L}{4}(\overline{AC}\cos\theta) - \frac{L}{4\tan\theta}(\overline{AC}\sin\theta)$
$+ KB\cos(\beta-\theta)(\overline{AB}\sin\theta) + KB\sin(\beta-\theta)(\overline{AB}\cos\theta) = 0$

Solve this for force KB. From the final FBD,

$\frac{R}{2} - KB\cos\gamma = 0 \Rightarrow$ Gives R

$\frac{S}{2} - KB\sin\gamma = 0 \Rightarrow$ Gives S

For $L = 800$ lb, $\overline{AO'} = 8.75''$, $\overline{AB} = 8''$, $\overline{BK} = 9''$, and $\overline{AC} = 22.5''$, the following plots are constructed:



$R_{max} = 2490$ lb @ $\theta = 0$

$|S|_{max} = 259$ lb @ $\theta = 81.6°$

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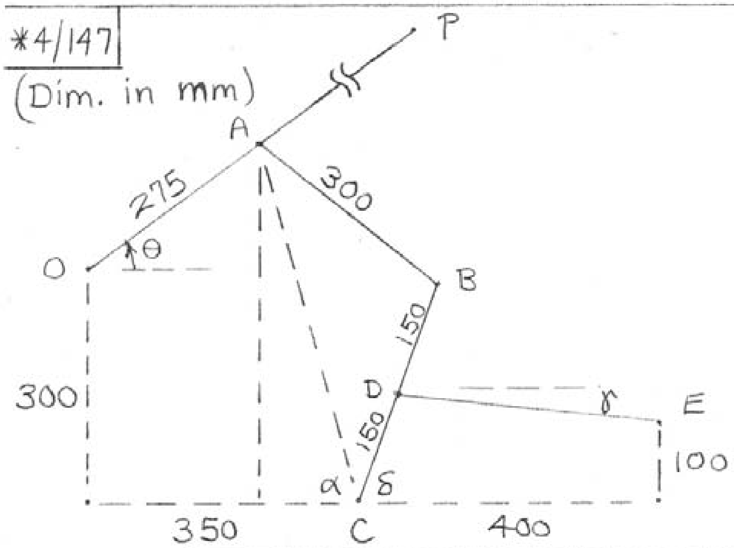
(Dim. in mm)



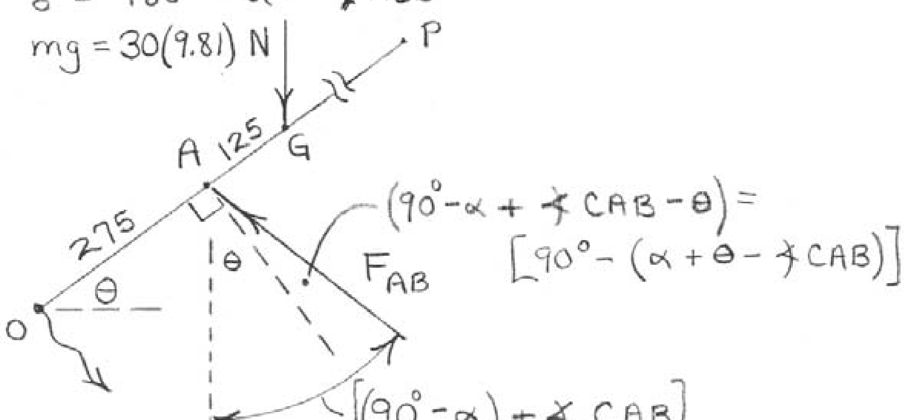
$$\overline{AC} = \sqrt{(350 - 275\cos\theta)^2 + (300 + 275\sin\theta)^2}$$

$$\measuredangle ABC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}}\right]$$

$$\alpha = \tan^{-1}\left[\frac{300 + 275\sin\theta}{350 - 275\cos\theta}\right]$$

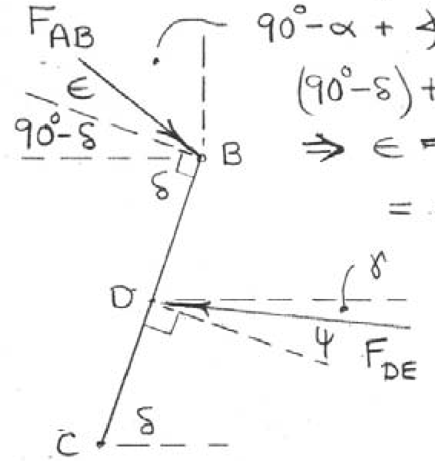
$$\measuredangle CAB = \measuredangle ACB = \frac{180^\circ - \measuredangle ABC}{2}$$

$$\delta = 180^\circ - \alpha - \measuredangle ACB$$



$$\circlearrowleft + \sum M_O = 0: \quad F_{AB} \cos\left[90^\circ - (\alpha + \theta - \measuredangle CAB)\right] \overline{OA} - mg\,\overline{OG} \cos\theta = 0$$

$$F_{AB} = \frac{mg\,\overline{OG} \cos\theta}{\overline{OA} \sin(\alpha + \theta - \measuredangle CAB)}$$



$$(90^\circ - \delta) + (\epsilon) + (90^\circ - \alpha + \measuredangle CAB) = 90^\circ$$

$$\Rightarrow \epsilon = \alpha + \delta - \measuredangle CAB - 90^\circ$$

$$= \alpha + (180^\circ - \alpha - \measuredangle ACB) - \measuredangle CAB - 90^\circ$$

$$\epsilon = 90^\circ - 2\measuredangle ACB$$

Also; $\gamma + \psi = 90^\circ - \delta$

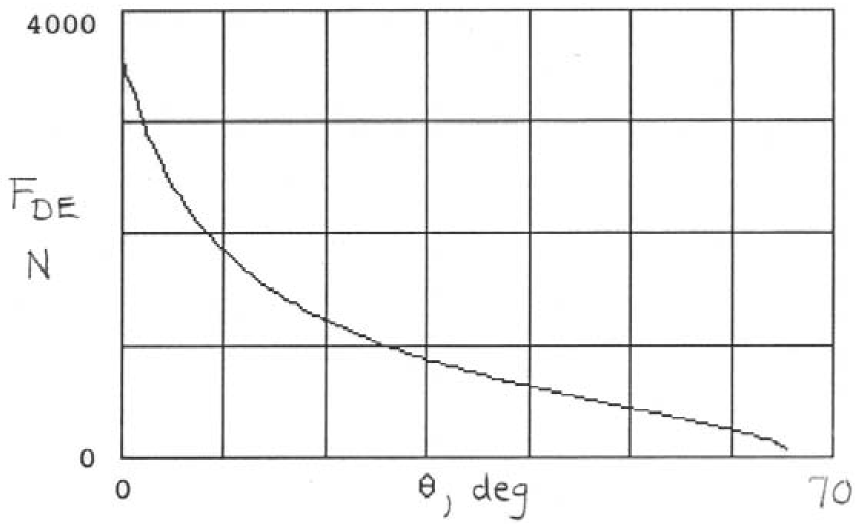
$$\psi = 90^\circ - \delta - \gamma$$

$$\gamma = \tan^{-1}\left[\frac{150 \sin\delta - 100}{400 - 150\cos\delta}\right]$$

$$\circlearrowleft + \sum M_C = 0: \; -F_{AB} \cos\epsilon\,(\overline{CB}) + F_{DE} \cos\psi\,(\overline{CD}) = 0$$

$$F_{DE} = \frac{F_{AB} \cos\epsilon\,(\overline{CB})}{\overline{CD} \cos\psi}$$

To find θ_{max}, set the above expression for $\overline{AC}$ to $\overline{AC} = 600$ mm and solve for $\underline{\theta_{max} = 65.9^\circ}$. Numerically evaluate the above expressions for $0 \le \theta \le \theta_{max}$ to obtain the following plot:



$(F_{DE})_{max} = 3580 \text{ N} @ \theta = 0$

$(F_{DE})_{min} = 0 @ \theta_{max} = 65.9^\circ$ (links AB and BC are collinear and serve as (an unstable !) prop for the door)

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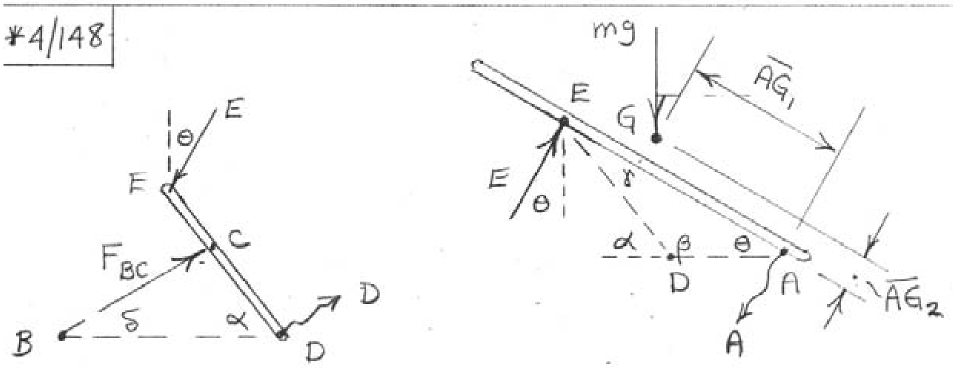


$$\frac{\sin\gamma}{\overline{AD}} = \frac{\sin\theta}{\overline{DE}}, \quad \gamma = \sin^{-1}\left(\frac{\overline{AD}\sin\theta}{\overline{DE}}\right)$$

$$\beta = 180 - (\theta + \gamma), \quad \alpha = 180 - \beta$$

$$\overline{AE} = \sqrt{\overline{DE}^2 + \overline{AD}^2 - 2(\overline{DE})(\overline{AD})\cos\beta}$$

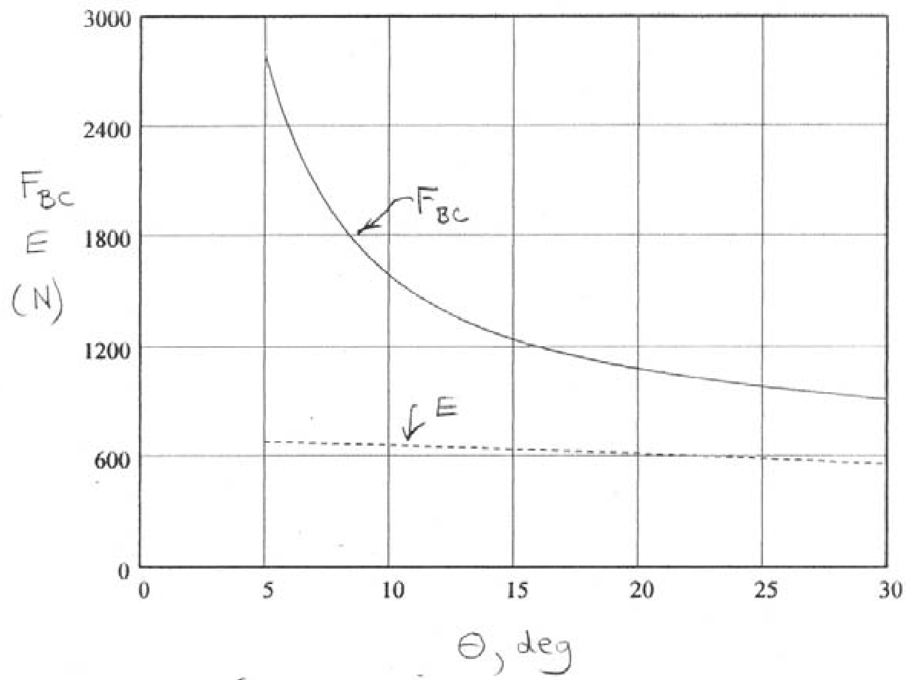
$$\overline{BC} = \sqrt{\overline{BD}^2 + \overline{CD}^2 - 2(\overline{BD})(\overline{CD})\cos\alpha}$$

$$\frac{\sin\alpha}{\overline{BC}} = \frac{\sin\delta}{\overline{CD}}, \quad \delta = \sin^{-1}\left(\frac{\overline{CD}\sin\alpha}{\overline{BC}}\right)$$

Conveyor: $\circlearrowleft\!+\ \Sigma M_A = 0: \ -E(\overline{AE}) + mg(\overline{AG_1})\cos\theta$
$$- mg(\overline{AG_2})\sin\theta = 0 \qquad (1)$$

DE: $\circlearrowleft\!+\ \Sigma M_D = 0$:
$$E\cos\theta\,(\overline{DE}\cos\alpha) + E\sin\theta\,(\overline{DE}\sin\alpha)$$
$$- F_{BC}\sin\delta\,(\overline{BD}) = 0 \qquad (2)$$

Solve Eqs. (1) & (2) for E and F_{BC} as functions of θ for the values $\overline{AD} = 1060$ mm, $\overline{BD} = 1660$ mm, $\overline{DE} = 1945$ mm, $\overline{CD} = 1150$ mm, $\overline{AG_1} = 2130$ mm, $\overline{AG_2} = 500$ mm, and $mg = 100(9.81)$ N to obtain the following plot:

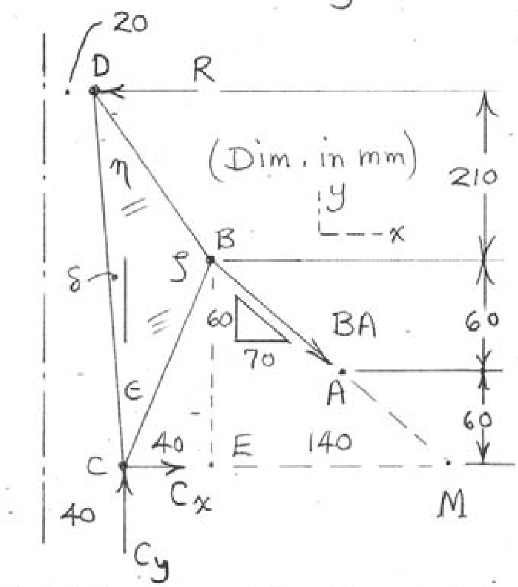


At $\theta = 5°$, $\underline{(F_{BC})_{max} = 2800 \text{ N}}$

*4/149 $\theta = 0$: $C_y = \frac{pA}{2} = \frac{p\pi r^2}{2}$

$= \frac{35(10^6)\pi(0.050)^2}{2}$

$= 137\,400 \text{ N}$



$\delta = \tan^{-1}\frac{20}{330} = 3.47°$

$\overline{CD} = \sqrt{20^2 + 330^2}$

$= 331 \text{ mm}$

Similarly, $\overline{BD} = 218 \text{ mm}$

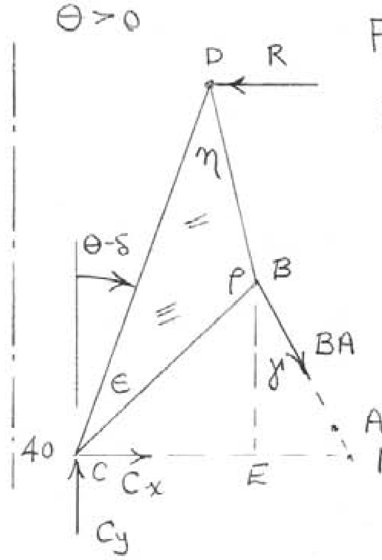
$\overline{BC} = 126.5 \text{ mm}$

$\curvearrowleft\!+ \Sigma M_M = 0$: $R(330) - 137\,400(180) = 0$, $\underline{R = 75\,000 \text{ N}}$

$\Sigma F_y = 0$: $-BA \sin\left(\tan^{-1}\frac{6}{7}\right) + 137\,400 = 0$, $\underline{BA = 211\,000 \text{ N}}$

$\Sigma F_x = 0$: $-75\,000 + 211\,000 \cos\left(\tan^{-1}\frac{6}{7}\right) + C_x = 0$

$\underline{C_x = -85\,400 \text{ N}}$



Find ϵ, ρ, and η:

$$\overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 - 2\,\overline{CD}\,\overline{BC}\cos\epsilon$$

Solve for ϵ, then use

$$\frac{\sin\rho}{\overline{CD}} = \frac{\sin\epsilon}{\overline{BD}} \quad \text{for } \rho$$

$$\frac{\sin\rho}{\overline{CD}} = \frac{\sin\eta}{\overline{BC}} \quad \text{for } \eta$$

$$\overline{CE} = \overline{BC}\sin(\theta-\delta+\epsilon)$$

$$\overline{BE} = \overline{BC}\cos(\theta-\delta+\epsilon)$$

$$110 = \overline{AB}\sin\gamma + \overline{BC}\sin(\theta-\delta+\epsilon) \Rightarrow \text{Solve for } \gamma$$

Then $\overline{EM} = \overline{BE}\tan\gamma$ and $\overline{CM} = \overline{CE} + \overline{EM}$

$$\circlearrowleft + \Sigma M_M = 0: \quad R\left(\overline{CD}\cos(\theta-\delta)\right) - C_y\left(\overline{CM}\right) = 0$$

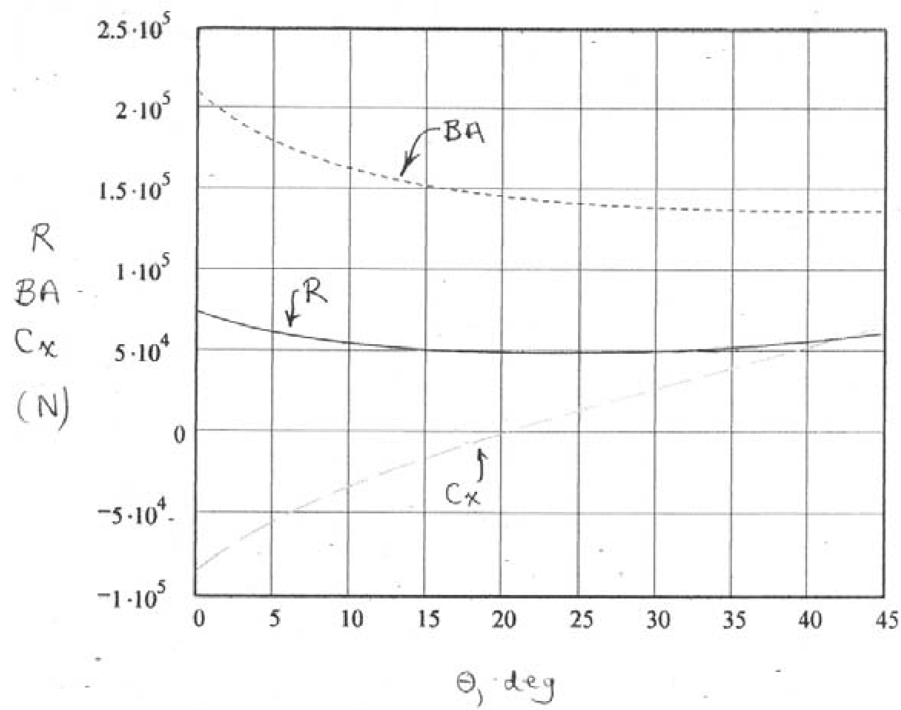
$$\underline{R = \frac{C_y\,\overline{CM}}{\overline{CD}\cos(\theta-\delta)}}$$

$$\Sigma F_y = 0: \quad C_y - BA\cos\gamma = 0, \quad \underline{BA = \frac{C_y}{\cos\gamma}}$$

$$\Sigma F_x = 0: \quad -R + BA\sin\gamma + C_x = 0, \quad \underline{C_x = R - BA\sin\gamma}$$

Solve all of the above in sequential order to obtain the plots below. Note that

$$\underline{R_{min} = 49\,400 \text{ N} \;@\; \theta = 23.2^\circ}$$



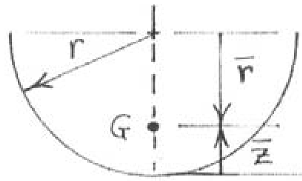
2.5·10^5
2·10^5
1.5·10^5
1·10^5
5·10^4
0
−5·10^4
−1·10^5
R
BA
Cx
(N)
0
5
10
15
20
25
30
35
40
45
θ, deg

5/1 The horizontal coordinate to the centroid is $14 - \frac{1}{3}(14-2) = \underline{10}$

$\boxed{5/2}$ From Sample Problem 5/3 with $r = 8$ and $\alpha = 120° = \frac{2}{3}\pi$:

$$\bar{r} = \frac{2}{3}(8)\,\frac{\sin 120°}{2\pi/3} = \underline{2.21}$$

5/3

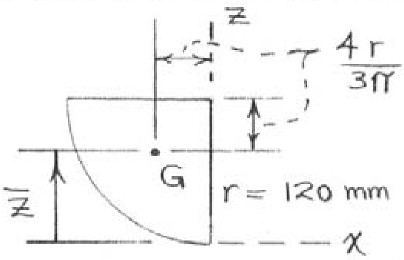


$\bar{r} = \frac{2r}{\pi} = \frac{2(120)}{\pi} = 76.4$ mm

$\bar{z} = r - \bar{r} = 120 - 76.4 = \underline{43.6 \text{ mm}}$

$\underline{\bar{x} = -120 \text{ mm}}$, by inspection

5/4



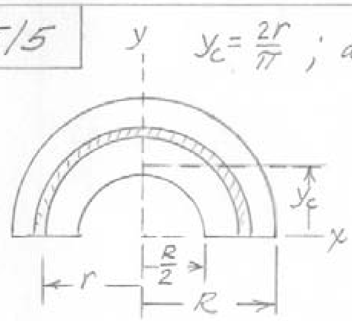
$$\bar{x} = -\frac{4r}{3\pi} = -\frac{4(120)}{3\pi} = \underline{-50.9 \text{ mm}}$$

$\bar{y} = \underline{120 \text{ mm}}$, by inspection

$$\bar{z} = r - \frac{4r}{3\pi} = 120 - 50.9 = \underline{69.1 \text{ mm}}$$

5/5

$y_c = \frac{2r}{\pi}$; $dA = \pi r\,dr$, $A = \pi \int_{R/2}^{R} r\,dr = 3\pi R^2/8$

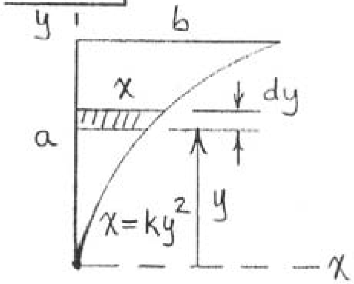


$$\int y_c\,dA = \int_{R/2}^{R} \frac{2r}{\pi}\,\pi r\,dr$$

$$= \frac{2r^3}{3}\Big]_{R/2}^{R} = \frac{7}{12} R^3$$

$\bar{y}A = \int y_c\,dA$; $\bar{y} = \frac{7}{12} R^3 \Big/ \frac{3\pi R^2}{8} = \underline{\frac{14R}{9\pi}}$

5/6



$b = ka^2, \quad k = \frac{b}{a^2}, \quad x = \frac{b}{a^2}y^2$

$A = \int x\,dy = \int_0^a \frac{b}{a^2} y^2 dy$

$= \frac{b}{a^2}\frac{a^3}{3} = \frac{1}{3}ab$

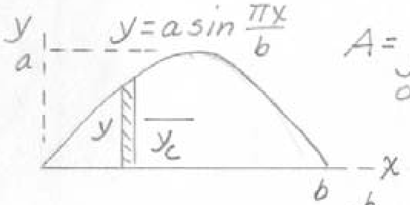
$\int x_c\,dA = \int \frac{x}{2} x\,dy = \int_0^a \frac{b^2 y^4}{2a^4} dy = \frac{ab^2}{10}$

$\bar{x} = \int x_c\,dA/A = \frac{ab^2/10}{ab/3} = \underline{\frac{3}{10}b}$

$\int y_c\,dA = \int y\,x\,dy = \int_0^a y \frac{b}{a^2} y^2 dy = \frac{ba^2}{4}$

$\bar{y} = \int y_c\,dA/A = \frac{ba^2/4}{ba/3} = \underline{\frac{3}{4}a}$

5/7



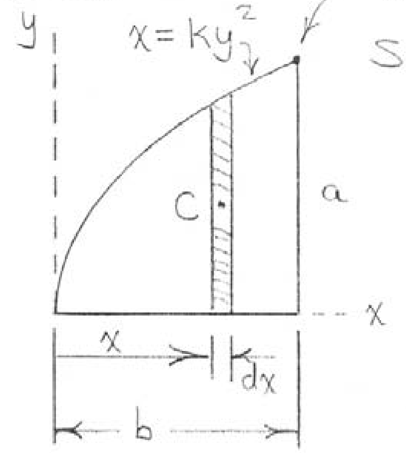
$$dA = y\,dx = a \sin \frac{\pi x}{b}\,dx$$

$$A = \int_0^b a \sin \frac{\pi x}{b}\,dx = -\frac{ab}{\pi} \cos \frac{\pi x}{b}\Big]_0^b = \frac{2ab}{\pi}$$

$$\int y_c\,dA = \int \frac{y}{2}\,dA = \int_0^b \frac{1}{2} a^2 \sin^2 \frac{\pi x}{b}\,dx = \frac{a^2}{2}\frac{b}{\pi}\left(\frac{\pi x}{2b} - \frac{\sin \frac{2\pi x}{b}}{4}\right)\Big|_0^b = a^2 b/4$$

$$\bar{y} = \int y_c\,dA / A = \frac{a^2 b}{4} \Big/ \frac{2ab}{\pi} = \underline{\frac{\pi a}{8}}$$

5/8



$x = ky^2$: $b = ka^2$, $k = \frac{b}{a^2}$

So $x = \frac{b}{a^2} y^2$

$$A = \int dA = \int y\,dx$$

$$= \int_0^b a\sqrt{\frac{x}{b}}\,dx = \frac{a}{\sqrt{b}} \frac{x^{3/2}}{3/2}\Big|_0^b$$

$$= \frac{2}{3}ab$$

$$\int x_c\,dA = \int_0^b \frac{a}{\sqrt{b}} x^{3/2}\,dx = \frac{a}{\sqrt{b}} \frac{x^{5/2}}{5/2}\Big|_0^b = \frac{2}{5}ab^2$$

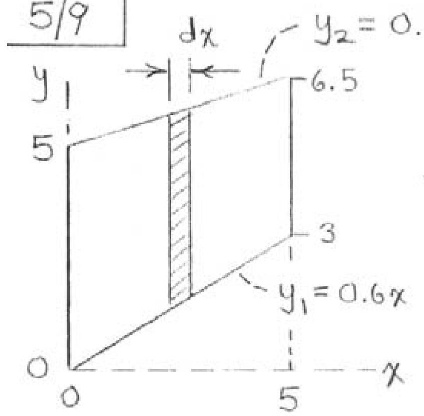
$$\int y_c\,dA = \int \frac{y}{2} y\,dx = \frac{1}{2}\int y^2\,dx$$

$$= \frac{1}{2}\int_0^b \frac{a^2}{b} x\,dx = \frac{1}{2}\frac{a^2}{b}\frac{x^2}{2}\Big|_0^b = \frac{1}{4}a^2 b$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{2}{5}ab^2}{\frac{2}{3}ab} = \underline{\frac{3}{5}b}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{1}{4}a^2 b}{\frac{2}{3}ab} = \underline{\frac{3}{8}a}$$

5/9



$$A = \int_0^5 (y_2 - y_1)\,dx$$

$$= \int_0^5 (0.3x + 5 - 0.6x)\,dx$$

$$= \int_0^5 (5 - 0.3x)\,dx$$

$$= \left(5x - \frac{0.3x^2}{2}\right)_0^5 = 21.25$$

(Note: Trapezoidal area formula could be used)

$$\int x_c\,dA = \int_0^5 x(5 - 0.3x)\,dx = \left(\frac{5}{2}x^2 - 0.1x^3\right)_0^5 = 50$$

$$\int y_c\,dA = \int_0^5 \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \frac{1}{2}\int_0^5 (y_2^2 - y_1^2)\,dx$$

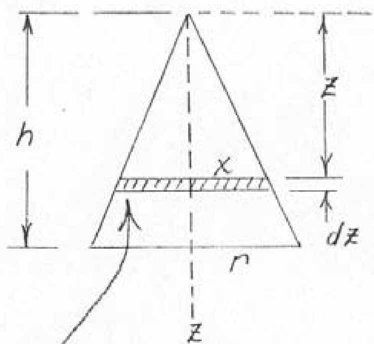
$$= \frac{1}{2}\int_0^5 \left[(0.3x+5)^2 - (0.6x)^2\right]dx = \frac{1}{2}\int_0^5 (25 + 3x - 0.27x^2)\,dx$$

$$= \frac{1}{2}\left[25x + \frac{3x^2}{2} - \frac{0.27x^3}{3}\right]_0^5 = 75.6$$

$$\bar{x} = \int x_c\,dA / A = 50/21.25 = \underline{2.35}$$

$$\bar{y} = \int y_c\,dA / A = 75.6/21.25 = \underline{3.56}$$

5/10 $x = \frac{r}{h} z$, $dV = \pi x^2 dz = \pi \frac{r^2}{h^2} z^2 dz$



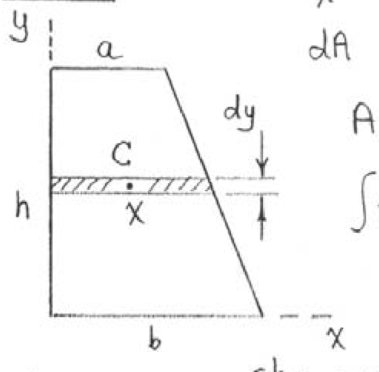
$$V = \pi \frac{r^2}{h^2} \int_0^h z^2 dz = \frac{\pi r^2 h}{3}$$

$$\int z\,dV = \pi \frac{r^2}{h^2} \int_0^h z^3 dz = \frac{\pi r^2 h^2}{4}$$

$$\bar{z} = \int z\,dV / V = \underline{3h/4}$$

(Disk - shaped element viewed edge-on.)

5/11



$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dA = x\,dy = \left[\left(\frac{a-b}{h}\right)y + b\right]dy$$

$$A = \left(\frac{a+b}{2}\right)h = \frac{h}{2}(a+b)$$

$$\int x_c\,dA = \int_0^h \frac{x}{2}\left[\left(\frac{a-b}{h}\right)y + b\right]dy$$

$$= \frac{1}{2}\int_0^h \left[\left(\frac{a-b}{h}\right)y + b\right]^2 dy$$

$$\int x_c\,dA = \frac{1}{2}\int_0^h \left(\frac{a-b}{h}\right)^2 y^2 + 2b\left(\frac{a-b}{h}\right)y + b^2\Big]\,dy$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2\frac{y^3}{3} + 2b\left(\frac{a-b}{h}\right)\frac{y^2}{2} + b^2 y\right]_0^h$$

$$= \frac{h}{6}\left(a^2 + b^2 + ab\right)$$

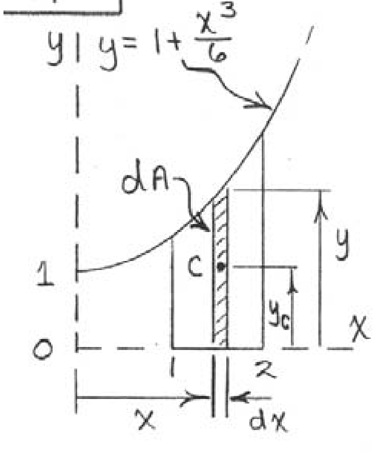
$$\int y_c\,dA = \int_0^h y\left[\left(\frac{a-b}{h}\right)y + b\right]dy = \int_0^h \left[\left(\frac{a-b}{h}\right)y^2 + by\right]dy$$

$$= \left[\left(\frac{a-b}{h}\right)\frac{y^3}{3} + b\frac{y^2}{2}\right]_0^h = \frac{h^2}{3}\left(a + \frac{b}{2}\right)$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{h}{6}\left(a^2+b^2+ab\right)}{\frac{h}{2}(a+b)} = \underline{\frac{a^2+b^2+ab}{3(a+b)}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{h^2}{3}\left(a+\frac{b}{2}\right)}{\frac{h}{2}(a+b)} = \underline{\frac{h(2a+b)}{3(a+b)}}$$

5/12



$$dA = y\,dx = \left(1 + \frac{x^3}{6}\right)dx$$

$$A = \int dA = \int_1^2 \left(1 + \frac{x^3}{6}\right)dx$$

$$= x + \frac{x^4}{24}\Big|_1^2 = \frac{39}{24}$$

$$\int x_c\,dA = \int_1^2 x\left(1 + \frac{x^3}{6}\right)dx$$

$$= \frac{x^2}{2} + \frac{x^5}{30}\Big|_1^2 = \frac{38}{15}$$

$$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \int \frac{y^2}{2}dx = \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{6}\right)^2 dx$$

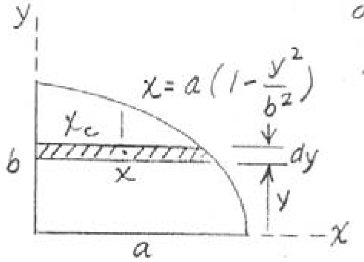
$$= \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{3} + \frac{x^6}{36}\right)dx = \frac{1}{2}\left(x + \frac{x^4}{12} + \frac{x^7}{252}\right)\Big|_1^2$$

$$= \frac{347}{252}$$

So $\bar{x} = \dfrac{\int x_c\,dA}{\int dA} = \dfrac{38/15}{39/24} = \underline{1.559}$

$\bar{y} = \dfrac{\int y_c\,dA}{\int dA} = \dfrac{347/252}{39/24} = \underline{0.847}$

5/13 $dA = x\,dy$, $A = \int_0^b a\left(1 - \frac{y^2}{b^2}\right)dy = a\left[y - \frac{y^3}{3b^2}\right]_0^b = \frac{2}{3}ab$



$$\bar{x}A = \int x_c\,dA = \int_0^b \frac{x}{2}\,x\,dy$$

$$= \frac{1}{2}\int_0^b a^2\left(1 - \frac{2y^2}{b^2} + \frac{y^4}{b^4}\right)dy$$

$$= \frac{a^2}{2}\left[y - \frac{2y^3}{3b^2} + \frac{y^5}{5b^4}\right]_0^b = \frac{4}{15}a^2b$$

$$\bar{x} = \frac{4a^2b/15}{2ab/3} = \underline{\frac{2}{5}a}$$

$$\bar{y}A = \int y\,dA = \int_0^b a\left(y - \frac{y^3}{b^2}\right)dy = a\left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]_0^b = \frac{1}{4}ab^2$$

$$\bar{y} = \frac{ab^2/4}{2ab/3} = \underline{\frac{3}{8}b}$$

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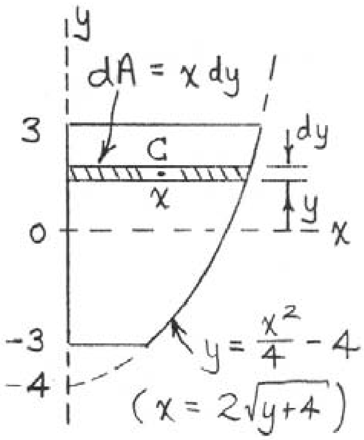


$$A = \int dA = \int x\,dy = \int_{-3}^{3} 2\sqrt{y+4}\,dy$$

$$= 2\,\frac{2}{3}(y+4)^{3/2}\Big|_{-3}^{3} = 23.4$$

$$\int x_c\,dA = \int \frac{x}{2}\,x\,dy = \frac{1}{2}\int x^2 dy$$

$$= \frac{1}{2}\int_{-3}^{3} 4(y+4)\,dy = 2\left(\frac{y^2}{2}+4y\right)_{-3}^{3}$$

$$= 48$$

$$\int y_c\,dA = \int y\,x\,dy = \int_{-3}^{3} y\,2\sqrt{y+4}\,dy$$

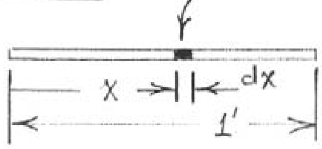
$$= 2\,\frac{2}{15}(3y-8)(y+4)^{3/2}\Big|_{-3}^{3} = 9.47$$

$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{48}{23.4} = \underline{2.05}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{9.47}{23.4} = \underline{0.405}$$

5/15

$dm = \rho\, dx \quad (\rho = \text{mass per unit length})$



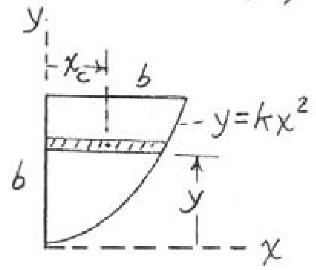
$$m = \int dm = \int \rho\, dx = \int_0^{1'} \rho_0 \left(1 - \frac{x}{2}\right) dx$$

$$= \rho_0 \left[x - \frac{x^2}{4} \right]_0^{1'} = \frac{3}{4}\rho_0$$

$$\int x\, dm = \int_0^{1'} x \rho_0 \left(1 - \frac{x}{2}\right) dx = \rho_0 \left[\frac{x^2}{2} - \frac{x^3}{6} \right]_0^{1'} = \frac{\rho_0}{3}$$

$$\bar{x} = \frac{\int x\, dm}{\int dm} = \frac{\rho_0/3}{3\rho_0/4} = \underline{\frac{4}{9}\ \text{ft}}$$

5/16 $dA = x\,dy$; $k = \frac{b}{b^2} = \frac{1}{b}$ so $x^2 = by$, $x_c = x/2$



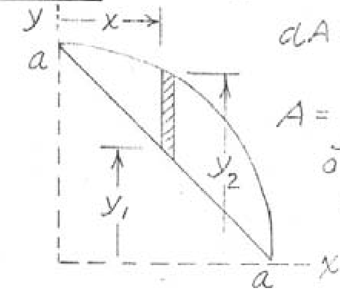
$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{\int_0^b \frac{x}{2}\,x\,dy}{\int_0^b x\,dy}$$

$$= \frac{1}{2}\int_0^b by\,dy \Big/ \int_0^b \sqrt{by}\,dy$$

$$= \frac{1}{4}b^3 \Big/ \frac{2}{3}b^2 = \underline{\frac{3}{8}b}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \int_0^b y\sqrt{by}\,dy \Big/ \frac{2}{3}b^2 = \frac{2}{5}b^3 \Big/ \frac{2}{3}b^2 = \underline{\frac{3}{5}b}$$

5/17 $x^2 + y_2^2 = a^2$; $x + y_1 = a$



$dA = (y_2 - y_1)dx = (\sqrt{a^2 - x^2} - a + x)\,dx$

$$A = \int_0^a (\sqrt{a^2 - x^2} - a + x)\,dx$$

$$= \frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right) - ax + \frac{x^2}{2}\Bigg]_0^a$$

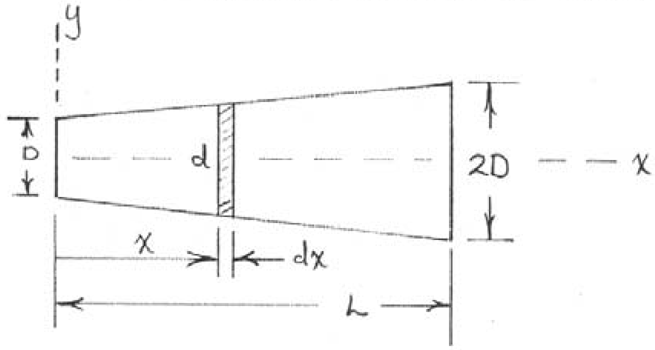
$$= \frac{a^2}{4}(\pi - 2)$$

$$\int x_c\, dA = \int_0^a (x\sqrt{a^2 - x^2} - ax + x^2)\,dx$$

$$= -\frac{1}{3}\sqrt{(a^2 - x^2)^3} - \frac{ax^2}{2} + \frac{x^3}{3}\Bigg]_0^a = \frac{a^3}{6}$$

$$\bar{x} = \int x_c\, dA \Big/ A = \frac{a^3}{6} \Big/ \frac{a^2}{4}(\pi - 2) = \underline{\frac{2a}{3(\pi - 2)}} = \bar{y} \text{ by symm.}$$

5/18



For constant density, $\bar{x} V = \int x \, dV$

Diameter $d = D(1 + x/L)$

So $dV = \frac{\pi d^2}{4} dx = \frac{\pi D^2}{4}\left(1 + \frac{x}{L}\right)^2 dx$

$V = \frac{\pi D^2}{4}\int_0^L \left(1 + \frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[x + \frac{x^2}{L} + \frac{x^3}{3L^2}\right]_0^L$

$= \frac{7}{12}\pi D^2 L$

$\int x \, dV = \frac{\pi D^2}{4}\int_0^L x\left(1 + \frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[\frac{x^2}{2} + \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right]_0^L$

$= \frac{17}{48}\pi D^2 L^2$

$\bar{x} = \frac{\frac{17}{48}\pi D^2 L^2}{\frac{7}{12}\pi D^2 L} = \underline{\frac{17}{28} L}$

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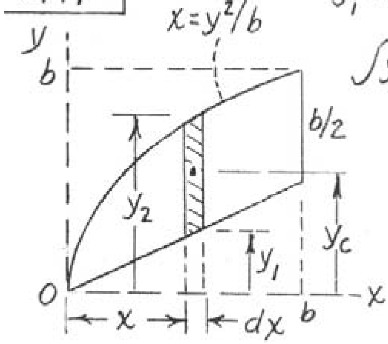


$y_1 = x/2, \quad y_2 = \sqrt{bx}$

$$\int y_c \, dA = \int_0^b \frac{y_1 + y_2}{2} (y_2 - y_1) \, dx$$

$$= \frac{1}{2} \int_0^b (y_2^2 - y_1^2) \, dx$$

$$= \frac{1}{2} \int_0^b \left(bx - \frac{x^2}{4}\right) dx$$

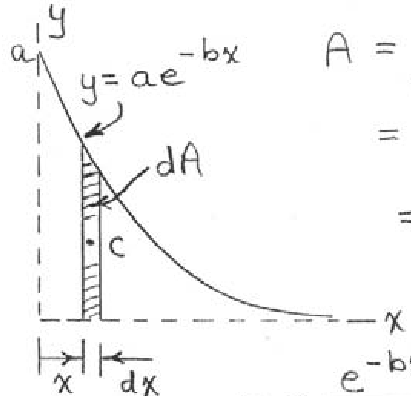
$$= \frac{1}{2} \left[\frac{bx^2}{2} - \frac{x^3}{12} \right]_0^b = \frac{5}{24} b^3$$

$$\int dA = \int_0^b (y_2 - y_1) \, dx = \int_0^b (\sqrt{bx} - x/2) \, dx = \left[\frac{2\sqrt{b}}{3} x^{3/2} - \frac{x^2}{4} \right]_0^b$$

$$= \frac{5}{12} b^2$$

$$\bar{y} = \frac{\int y_c \, dA}{\int dA} = \frac{\frac{5}{24} b^3}{\frac{5}{12} b^2} = \underline{\frac{b}{2}}$$

$\boxed{5/20}$



$dA = y\,dx = ae^{-bx}dx$

$A = \int dA = \int_0^\infty ae^{-bx}dx$

$= -\frac{a}{b}e^{-bx}\Big|_0^\infty = -\frac{a}{b}[0-1]$

$= a/b$

$\int x_c\,dA = a\int_0^\infty xe^{-bx}dx$

$= a\frac{e^{-bx}}{b^2}\left[-bx-1\right]_0^\infty$

$= -\frac{a}{b^2}\left[bxe^{-bx}+e^{-bx}\right]_0^\infty = -\frac{a}{b^2}\left[0+0-(0+1)\right]$

$= a/b^2$

$\int y_c\,dA = \int \frac{y}{2}y\,dx = \int \frac{y^2}{2}dx$

$= \frac{1}{2}\int_0^\infty a^2e^{-2bx}dx = \frac{a^2}{2}\frac{e^{-2bx}}{-2b}\Big|_0^\infty$

$= -\frac{a^2}{4b}[0-1] = \frac{a^2}{4b}$

$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{a/b^2}{a/b} = \underline{\frac{1}{b}}$

$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{a^2/4b}{a/b} = \underline{\frac{a}{4}}$

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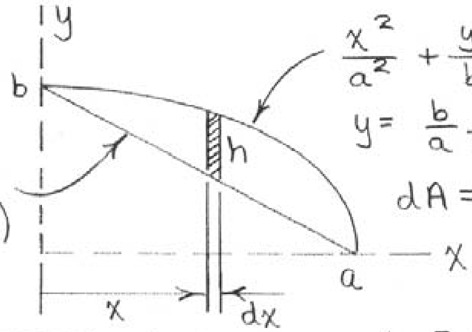


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$dA = h\,dx$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$y = \frac{b}{a}(a - x)$$

$$h = \frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a-x) = \frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right]$$

$$A\bar{x} = \int x\,dA$$

$$A = \int_0^a \frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right]dx$$

$$= \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \left(ax - \frac{x^2}{2}\right)\right]_0^a = \frac{ab}{2}\left[\frac{\pi}{2} - 1\right]$$

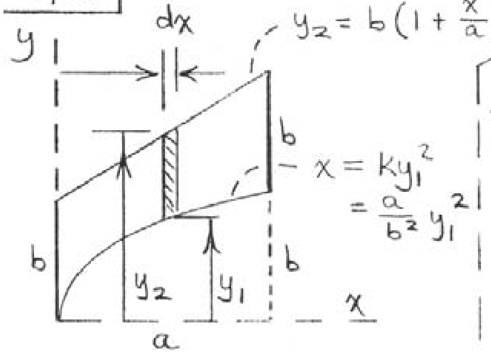
$$\int x\,dA = \int_0^a x\,\frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right]dx$$

$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} - \left(\frac{ax^2}{2} - \frac{x^3}{3}\right)\right]_0^a = \frac{ba^2}{6}$$

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{ba^2/6}{\frac{ab}{2}\left(\frac{\pi}{2}-1\right)} = \underline{\frac{a}{3\left(\frac{\pi}{2}-1\right)}}$$

Similarly, $\underline{\bar{y} = \dfrac{b}{3\left(\frac{\pi}{2}-1\right)}}$

5/22



$$A = \int_0^a (y_2 - y_1)\,dx$$

$$= \int_0^a \left[b\left(1 + \frac{x}{a}\right) - b\left(\frac{x}{a}\right)^{\frac{1}{2}}\right]dx$$

$$= b\left(x + \frac{x^2}{2a}\right) - \frac{b}{\sqrt{a}}\,\frac{2}{3}x^{3/2}\Big|_0^a$$

$$= \frac{5}{6}ab$$

$$\int y_c\,dA = \int_0^a \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$$

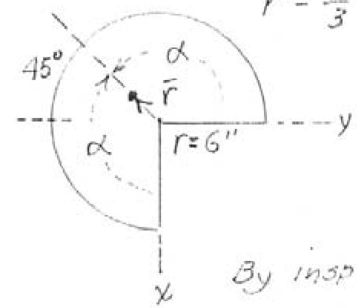
$$= \frac{1}{2}\int_0^a \left[b^2\left(1 + \frac{x}{a}\right)^2 - \frac{b^2}{a}x\right]dx$$

$$= \frac{1}{2}\left[b^2\left(x + \frac{x^2}{a} + \frac{x^3}{3a^2}\right) - \frac{b^2x^2}{2a}\right]\Big|_0^a = \frac{11}{12}ab^2$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{11ab^2/12}{5ab/6} = \underline{\frac{11}{10}b}$$

5/23 From Sample Problem 5/3,

$\bar{r} = \frac{2}{3}\,\frac{r \sin\alpha}{\alpha}$ where $\alpha = 135^\circ$ or $\frac{3}{4}\pi$ rad



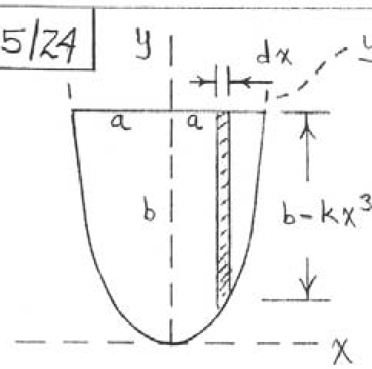
$\bar{r} = \frac{2}{3}(6)\,\frac{\sin 135^\circ}{3\pi/4}$

$= 4\,\frac{4}{3\pi}\,\frac{1}{\sqrt{2}} = \frac{16}{3\pi\sqrt{2}}$ in.

$\bar{x} = \bar{y} = -\bar{r}/\sqrt{2} = -\frac{8}{3\pi}$ in.

By inspection $\bar{z} = 5$ in.

5/24



$y = k|x|^3 : \; b = ka^3, \quad k = b/a^3$

$\therefore \; y = \frac{b}{a^3}|x|^3$

Will treat right half so that absolute value sign may be disregarded.

$$A = 2\int_0^a \left(b - \frac{b}{a^3}x^3\right)dx = 2\left[bx - \frac{b}{4a^3}x^4\right]_0^a = \frac{3}{2}ba$$

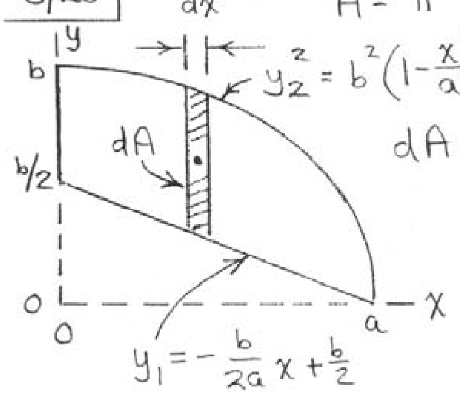
$$\int y_c \, dA = 2\int_0^a \frac{b + \frac{b}{a^3}x^3}{2}\left(b - \frac{b}{a^3}x^3\right)dx$$

$$= \int_0^a \left(b^2 - \frac{b^2}{a^6}x^6\right)dx = b^2\left[x - \frac{x^7}{7a^6}\right]_0^a = \frac{6ab^2}{7}$$

$$\bar{y} = \frac{\int y_c \, dA}{A} = \frac{6ab^2/7}{3ab/2} = \underline{\frac{4}{7}b}$$

$\underline{\bar{x} = 0}$, by inspection.

5/25 $\quad A = \pi\frac{ab}{4} - \frac{1}{2}a\frac{b}{2} = \frac{ab}{4}(\pi - 1)$



$$dA = (y_2 - y_1)\,dx$$
$$= \left[b\sqrt{1-\frac{x^2}{a^2}} - \left(-\frac{b}{2a}x + \frac{b}{2}\right)\right]dx$$
$$= \frac{b}{a}\left[\sqrt{a^2-x^2} + \frac{x}{2} - \frac{a}{2}\right]dx$$

$$\int x_c\,dA = \int_0^a x\,\frac{b}{a}\left[\sqrt{a^2-x^2} + \frac{x}{2} - \frac{a}{2}\right]dx$$
$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} + \frac{x^3}{6} - \frac{ax^2}{4}\right]_0^a = \frac{1}{4}ba^2$$

$$\int y_c\,dA = \int_0^a \left(\frac{y_2+y_1}{2}\right)(y_2-y_1)\,dx = \frac{1}{2}\int_0^a \left(y_2^2 - y_1^2\right)dx$$
$$= \frac{1}{2}\int_0^a \left[b^2\left(1-\frac{x^2}{a^2}\right) - \left(-\frac{b}{2a}x + \frac{b}{2}\right)^2\right]dx$$
$$= \frac{1}{2}\int_0^a \left[b^2\left(1-\frac{x^2}{a^2}\right) - \frac{b^2}{4a^2}x^2 + \frac{b^2}{2a}x - \frac{b^2}{4}\right]dx$$
$$= \frac{1}{2}\left[b^2\left(x - \frac{x^3}{3a^2}\right) - \frac{b^2}{4a^2}\frac{x^3}{3} + \frac{b^2}{2a}\frac{x^2}{2} - \frac{b^2}{4}x\right]_0^a$$
$$= \frac{7}{24}ab^2$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{1}{4}ba^2}{\frac{ab}{4}(\pi-1)} = \underline{\frac{a}{\pi-1}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{7}{24}ab^2}{\frac{ab}{4}(\pi-1)} = \underline{\frac{7b}{6(\pi-1)}}$$

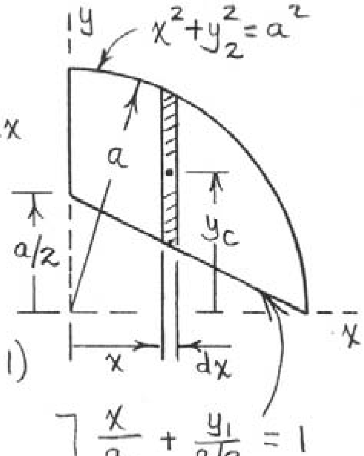
5/26 $\quad \bar{x} = \dfrac{\int x\,dA}{\int dA}$

$dA = (y_2 - y_1)\,dx = \left(\sqrt{a^2-x^2} - \dfrac{a-x}{2}\right)dx$

$A = \displaystyle\int_0^a \left(\sqrt{a^2-x^2} - \frac{a-x}{2}\right)dx$

$= \left[\frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a}\right.\right.$

$\left.\left. - \frac{1}{2}\left(ax - \frac{x^2}{2}\right)\right]_0^a = \frac{a^2}{4}(\pi - 1)$



$\left[\begin{array}{l}\text{Or, by inspection,} \\ A = \frac{1}{4}\pi a^2 - \frac{1}{2}a\,\frac{a}{2} = \frac{a^2}{4}(\pi - 1)\end{array}\right]$

$\displaystyle\int x\,dA = \int_0^a \left[x\sqrt{a^2-x^2} - \frac{1}{2}(ax - x^2)\right]dx$

$= \left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} - \frac{1}{2}\left(\frac{ax^2}{2} - \frac{x^3}{3}\right)\right]_0^a = \frac{a^3}{4}$

So $\bar{x} = \dfrac{a^3/4}{\frac{a^2}{4}(\pi-1)} = \underline{\dfrac{a}{\pi - 1}}$

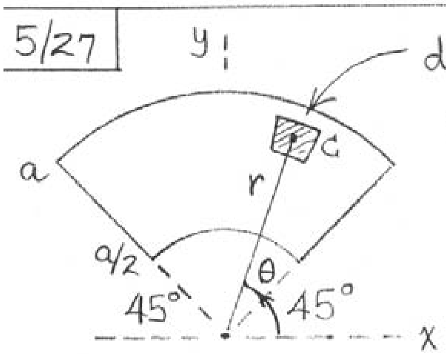
Now, $\bar{y} = \dfrac{\int y_c\,dA}{A}$, where $y_c = \dfrac{y_1 + y_2}{2}$

$\displaystyle\int y_c\,dA = \int \frac{y_1+y_2}{2}(y_2 - y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$

$\displaystyle = \frac{1}{2}\int_0^a \left(a^2 - x^2 - \frac{a^2 - 2ax + x^2}{4}\right)dx = \frac{1}{8}\int_0^a (3a^2 + 2ax - 5x^2)\,dx$

$= \dfrac{7}{24}a^3$. $\quad$ Then $\bar{y} = \dfrac{7a^3/24}{\frac{a^2}{4}(\pi - 1)} = \underline{\dfrac{7a}{6(\pi-1)}}$

5/27



$dA = r\,dr\,d\theta$

$A = \frac{\pi a^2}{4} - \frac{\pi}{4}\left(\frac{a}{2}\right)^2 = \frac{3}{16}\pi a^2$

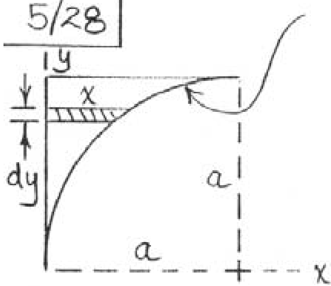
$$\int y_c\,dA = \iint (r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_{\pi/4}^{3\pi/4}\int_{a/2}^{a} \sin\theta\; r^2\,dr\,d\theta = \int_{\pi/4}^{3\pi/4} \sin\theta\,\frac{7}{24}a^3\,d\theta$$

$$= \frac{7}{24}a^3\left(-\cos\theta\right)\Big|_{\pi/4}^{3\pi/4} = \frac{7\sqrt{2}}{24}a^3$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{7\sqrt{2}}{24}a^3}{\frac{3}{16}\pi a^2} = \underline{\frac{14\sqrt{2}}{9\pi}a}$$

5/28



$$(x-a)^2 + y^2 = a^2$$

$$dA = x\,dy = \left(a - \sqrt{a^2-y^2}\right)dy$$

$$\int x_c\,dA = \int \frac{x}{2}\,x\,dy$$

$$= \frac{1}{2}\int_0^a \left(a - \sqrt{a^2-y^2}\right)^2 dy$$

$$\int x_c\,dA = \int_0^a \left(a^2 - a\sqrt{a^2-y^2} - \frac{y^2}{2}\right)dy$$

$$= \left[a^2 y - \frac{a}{2}\left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right) - \frac{y^3}{6}\right]_0^a$$

$$= \left(\frac{5}{6} - \frac{\pi}{4}\right)a^3$$

$$\int y_c\,dA = \int y\,x\,dy = \int_0^a \left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{a^3}{6}$$

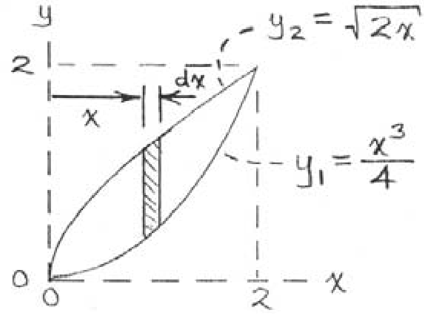
$$A = a^2 - \frac{1}{4}\pi a^2 = a^2\left(1 - \frac{\pi}{4}\right)$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\left(\frac{5}{6} - \frac{\pi}{4}\right)a^3}{\left(1 - \frac{\pi}{4}\right)a^2} = \frac{10-3\pi}{3(4-\pi)}\,a = \underline{0.223a}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{a^3/6}{\left(1-\frac{\pi}{4}\right)a^2} = \frac{2a}{3(4-\pi)} = \underline{0.777a}$$

5/29

$x = y_2^2/2$



$$A = \int dA = \int_0^2 (y_2 - y_1)\,dx = \int_0^2 \left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$
$$= \left(\frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16}\right)_0^2 = 5/3$$

$$\int x_c\, dA = \int_0^2 x\left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$
$$= \left(\frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20}\right)\Big|_0^2 = 8/5$$

$$\int y_c\, dA = \int_0^2 \left(\frac{y_1+y_2}{2}\right)(y_2 - y_1)\,dx = \int_0^2 \frac{1}{2}(y_2^2 - y_1^2)\,dx$$
$$= \frac{1}{2}\int_0^2 \left(2x - \frac{x^6}{16}\right) dx = \frac{1}{2}\left[x^2 - \frac{x^7}{7(16)}\right]_0^2 = 10/7$$

$$\bar{x} = \int x_c\, dA \Big/ A = \frac{8/5}{5/3} = \underline{\frac{24}{25}}$$

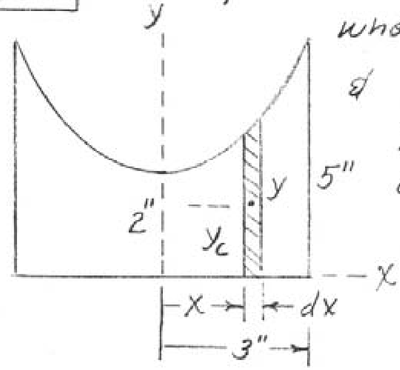
$$\bar{y} = \int y_c\, dA \Big/ A = \frac{10/7}{5/3} = \underline{\frac{6}{7}}$$

5/30

Eq. of curve: $y = 2 + kx^2$

when $x = 3''$, $y = 5''$ so $k = 1/3$

& $y = 2 + x^2/3$



By symmetry use only half of area.

$$A = \int_0^3 y\,dx = \int_0^3 \left(2 + \frac{x^2}{3}\right)dx$$

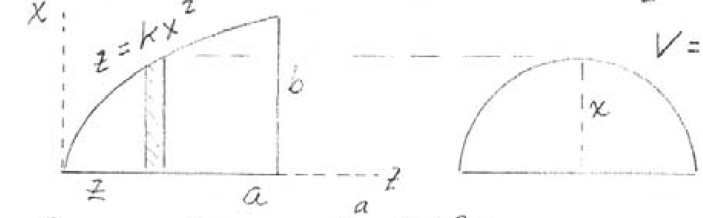
$$= \left[2x + \frac{x^3}{9}\right]_0^3 = 9 \text{ in.}^2$$

$$\bar{y}A = \int y_c\,dA = \int \frac{y}{2}\,dA = \frac{1}{2}\int_0^3 y^2\,dx = \frac{1}{2}\int_0^3 \left(4 + \frac{4}{3}x^2 + \frac{x^4}{9}\right)dx$$

$$= \frac{1}{2}\left[4x + \frac{4x^3}{9} + \frac{x^5}{45}\right]_0^3 = \frac{1}{2}\left[12 + 12 + \frac{27}{5}\right] = 14.70 \text{ in.}^3$$

Thus $\bar{h} = \bar{y} = \frac{14.70}{9} = \underline{1.633 \text{ in.}}$

5/31 $a = kb^2$ so $z = \frac{a}{b^2}x^2$ $\quad dV = \frac{\pi x^2}{2}dz = \frac{\pi}{2}\frac{b^2 z}{a}dz$



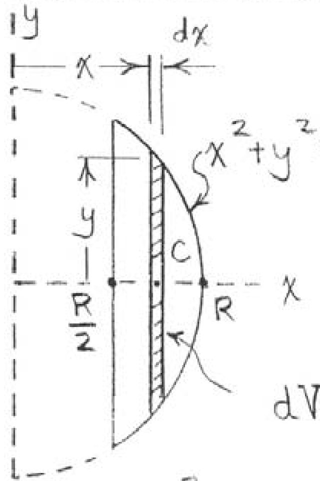
$$V = \frac{\pi b^2}{2a}\int_0^a z\,dz = \frac{\pi b^2}{2a}\frac{a^2}{2} = \frac{\pi}{4}ab^2$$

$$\int z_c\,dV = \int z\,dV = \int_0^a z\,\frac{\pi}{2}\frac{b^2 z}{a}dz$$

$$= \frac{\pi b^2}{2a}\int_0^a z^2\,dz = \frac{\pi b^2}{2a}\frac{a^3}{3} = \frac{\pi b^2 a^2}{6}$$

$$\bar{z} = \int z_c\,dV / V = \frac{\pi b^2 a^2}{6} \Big/ \frac{\pi}{4}ab^2 = \underline{2a/3}$$

5/32



(Note: Shaded element is a circular slice viewed edge-on.

$$dV = \pi y^2 dx = \pi (R^2 - x^2)\,dx$$

$$V = \int dV = \int_{R/2}^{R} \pi (R^2 - x^2)\,dx = \pi \left[R^2 x - \frac{x^3}{3} \right]_{R/2}^{R}$$

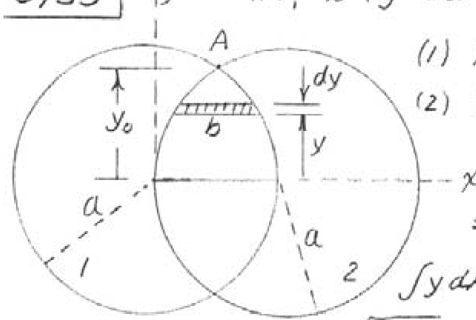
$$= \pi \left[R^3 - \frac{R^3}{3} - \left(\frac{R^3}{2} - \frac{R^3}{24} \right) \right] = \frac{5}{24} \pi R^3$$

$$\int x_c\,dV = \int_{R/2}^{R} x\,\pi (R^2 - x^2)\,dx = \pi \left[\frac{R^2 x^2}{2} - \frac{x^4}{4} \right]_{R/2}^{R}$$

$$= \pi \left[\frac{R^4}{2} - \frac{R^4}{4} - \left(\frac{R^4}{8} - \frac{R^4}{64} \right) \right] = \frac{9}{64} \pi R^4$$

$$\bar{x} = \frac{\int x_c\,dV}{\int dV} = \frac{\frac{9}{64} \pi R^4}{\frac{5}{24} \pi R^3} = \underline{\frac{27}{40} R}$$

5/33 (1), $x^2+y^2=a^2$; (2), $(x-a)^2+y^2=a^2$



(1) $x_1 = +\sqrt{a^2-y^2}$

(2) $x_2 = a - \sqrt{a^2-y^2}$ (Note sign)

$dA = b\,dy = (x_1 - x_2)\,dy$

$= (2\sqrt{a^2-y^2} - a)\,dy$

$\int y\,dA = \int_0^{y_0} (2y\sqrt{a^2-y^2} - ay)\,dy$

For pt A, $x_1 = x_2$ or $\sqrt{a^2-y^2} = a - \sqrt{a^2-y^2}$, $2\sqrt{a^2-y^2} = a$

$y = y_0 = \sqrt{3}a/2$

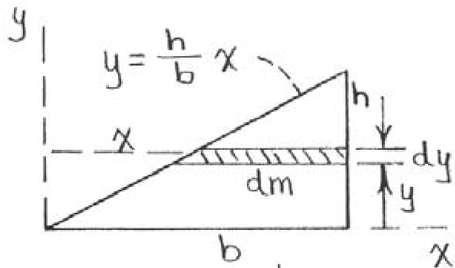
so $\int y\,dA = -\frac{2}{3}\sqrt{(a^2-y^2)^3} - \frac{ay^2}{2}\Big]_0^{y_0} = 5a^3/24 = 0.208a^3$

$$\int dA = \int_0^{y_0} (2\sqrt{a^2-y^2} - a)\,dy = \left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a} - ay\right)_0^{y_0}$$

$$= a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) = 0.614a^2$$

$$\bar{y} = \int y\,dA \Big/ \int dA = \frac{0.208a^3}{0.614a^2} = \underline{0.339\,a}$$

5/34 $dm = \rho dV = \rho dA\, t = t\rho\,(b-x)dy$



$= \left[t_o\left(\frac{y}{h}+1\right)\right]\rho\,(b-x)\,dy$

$= t_o\rho\left(\frac{y}{h}+1\right)\left(b-\frac{b}{h}y\right)dy$

$= t_o\rho b\left(1-\frac{y^2}{h^2}\right)dy$

$$m = \int dm = \int_0^h t_o\rho b\left(1-\frac{y^2}{h^2}\right)dy = t_o\rho b\left[y-\frac{y^3}{3h^2}\right]_0^h$$

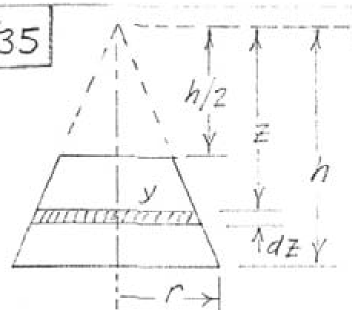
$$= \frac{2}{3}\rho t_o bh$$

$$\int y_c\,dm = \int t_o\rho b\left(1-\frac{y^2}{h^2}\right)y\,dy = t_o\rho b\left[\frac{y^2}{2}-\frac{y^4}{4h^2}\right]_0^h$$

$$= \frac{1}{4}\rho t_o bh^2$$

$$\bar{y} = \frac{\int y_c\,dm}{m} = \frac{\frac{1}{4}\rho t_o bh^2}{\frac{2}{3}\rho t_o bh} = \underline{\frac{3}{8}h}$$

5/35



$dV = \pi y^2 dz$ where $y = \frac{r}{h} z$

$= \pi \frac{r^2}{h^2} z^2 dz$

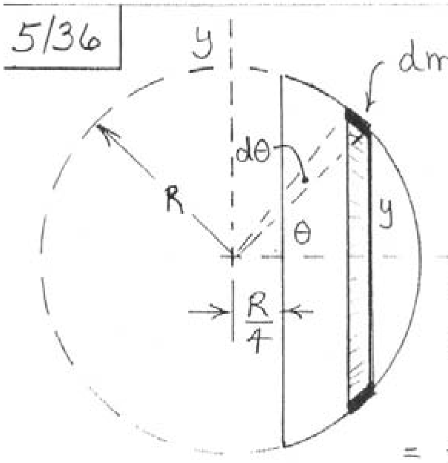
$V = \pi \frac{r^2}{h^2} \int_{h/2}^{h} z^2 dz = \frac{7\pi r^2 h}{24}$

$\int z_c \, dV = \int_{h/2}^{h} z \, \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$

$\bar{z} = \int z_c dV / V = \frac{15}{64} \pi r^2 h^2 \Big/ \frac{7}{24} \pi r^2 h = \frac{45}{56} h$

$\bar{h} = h - \bar{z} = \underline{\frac{11}{56} h}$

5/36

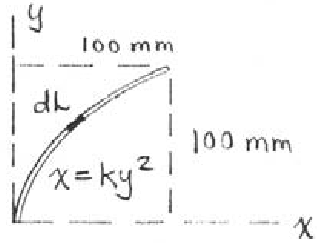


$$dm = \rho\, dA = \rho (R\,d\theta)(2\pi y)$$
$$= 2\pi \rho R (R \sin\theta)\, d\theta$$
$$= 2\pi \rho R^2 \sin\theta\, d\theta$$
$$m = \int dm$$
$$m = \int_0^{\cos^{-1}\frac{1}{4}} 2\pi \rho R^2 \sin\theta\, d\theta$$
$$= 2\pi \rho R^2 (-\cos\theta)\Big|_0^{\cos^{-1}\frac{1}{4}}$$
$$= 1.5 \pi \rho R^2$$

$$\int x_c\, dm = \int (R\cos\theta)(2\pi\rho R^2 \sin\theta\, d\theta)$$
$$= 2\pi\rho R^3 \int \cos\theta \sin\theta\, d\theta = \pi \rho R^3 \int \sin 2\theta\, d\theta$$
$$= \pi \rho R^3 \left(-\tfrac{1}{2}\cos 2\theta\right)\Big|_0^{\cos^{-1}\frac{1}{4}} = \frac{15}{16}\pi\rho R^3$$
$$\bar{x} = \frac{\int x_c\, dm}{\int dm} = \frac{\frac{15}{16}\pi\rho R^3}{1.5\pi\rho R^2} = \underline{\frac{5}{8} R}$$

5/37

$$x = ky^2 = y^2/100\,, \quad \frac{dx}{dy} = \frac{y}{50}$$



$$L = \int dL = \int_0^{100} \sqrt{1 + (dx/dy)^2}\, dy = \int_0^{100} \sqrt{1 + \frac{y^2}{50^2}}\, dy$$

$$= \frac{1}{50}\int_0^{100} \sqrt{50^2 + y^2}\, dy = \frac{1}{50\cdot 2}\left[y\sqrt{50^2+y^2} + 50^2 \ln\left(y + \sqrt{50^2+y^2}\right)\right]_0^{100}$$

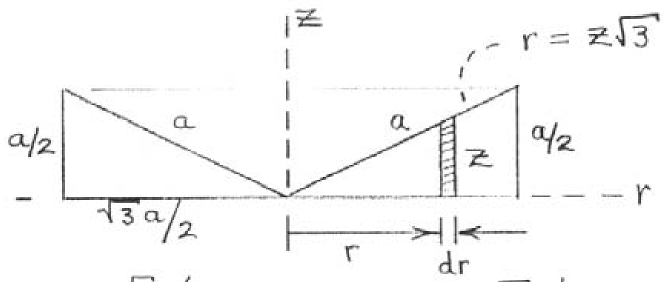
$$= 147.9 \text{ mm}$$

$$\int y_c\, dL = \frac{1}{50}\int_0^{100} y\sqrt{50^2+y^2}\, dy = \frac{1}{50}\,\frac{1}{3}\left(50^2+y^2\right)^{3/2}\Big|_0^{100}$$

$$= 8480 \text{ mm}^2$$

$$\bar{y} = \frac{\int y_c\, dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$

5/38



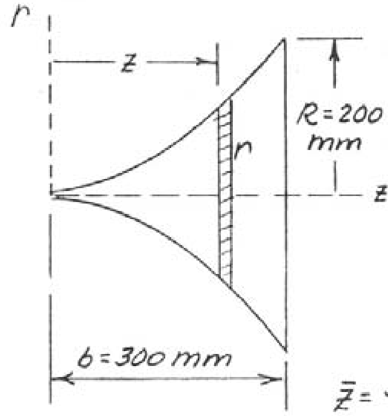
$$V = \int dV = \int_0^{\sqrt{3}a/2} 2\pi r z \, dr = \frac{2\pi}{\sqrt{3}} \int_0^{\sqrt{3}a/2} r^2 dr$$

$$= \frac{2\pi}{\sqrt{3}} \left(\sqrt{3}a/2\right)^3 / 3 = \frac{\pi a^3}{4}$$

$$\int z_c \, dV = \int_0^{\sqrt{3}a/2} \frac{r}{2\sqrt{3}} \frac{2\pi}{\sqrt{3}} r^2 dr = \frac{\pi}{3} \left[\frac{r^4}{4}\right]_0^{\sqrt{3}a/2} = \frac{3\pi a^4}{64}$$

$$\bar{z} = \frac{\int z_c \, dV}{V} = \frac{3\pi a^4/64}{\pi a^3/4} = \underline{\frac{3a}{16}}$$

5/39 $r = kz^3$, $R = kb^3$, so $r = \frac{R}{b^3} z^3$



$$dV = \pi r^2 dz = \pi \frac{R^2}{b^6} z^6 dz$$

$$V = \frac{\pi R^2}{b^6} \int_0^b z^6 dz = \frac{\pi}{7} R^2 b$$

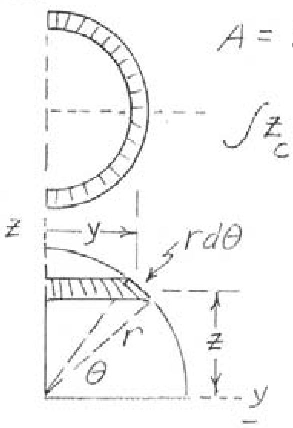
$$\int z_c dV = \int z dV = \frac{\pi R^2}{b^6} \int_0^b z^7 dz$$

$$= \frac{\pi R^2}{8} b^2$$

$$\bar{z} = \int z_c dV / V = \frac{\pi R^2 b^2}{8} \Big/ \frac{\pi}{7} R^2 b$$

$$= 7b/8 = \frac{7(300)}{8} = \underline{263 \text{ mm}}$$

►5/40



$dA = \pi y \, r d\theta = \pi r^2 \cos\theta \, d\theta$

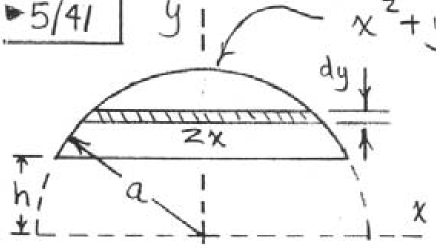
$A = \pi r^2 \int_0^{\pi/2} \cos\theta \, d\theta = \pi r^2 \sin\theta \Big]_0^{\pi/2} = \pi r^2$

$\int z_c \, dA = \int z \, dA = \int_0^{\pi/2} r \sin\theta \, \pi r^2 \cos\theta \, d\theta$

$= \pi r^3 \int_0^{\pi/2} \frac{\sin 2\theta}{2} d\theta = \frac{\pi r^3}{2}$

$\bar{z} = \int z_c \, dA / A = \frac{\pi r^3}{2} \Big/ \pi r^2 = \underline{\frac{r}{2}}$

▶5/41



$x^2+y^2=a^2$, $x=+\sqrt{a^2-y^2}$

$dA = 2x\,dy = 2\sqrt{a^2-y^2}\,dy$

$A = \int dA = \int_h^a 2\sqrt{a^2-y^2}\,dy$

$= 2\left(\frac{1}{2}\right)\left[y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right]_h^a$

$= a^2\left(\frac{\pi}{2} - \sin^{-1}\frac{h}{a}\right) - h\sqrt{a^2-h^2}$

$\int y\,dA = \int_h^a y\,2\sqrt{a^2-y^2}\,dy = 2\left(-\frac{1}{3}\right)\left(a^2-y^2\right)^{3/2}\Big|_h^a$

$= \frac{2}{3}\left(a^2-h^2\right)^{3/2}$

$\bar{y} = \frac{\int y\,dA}{\int dA} = \underline{\frac{\frac{2}{3}\left(a^2-h^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{h}{a}\right) - h\sqrt{a^2-h^2}}}$

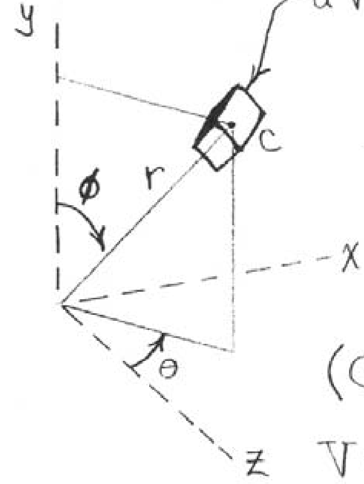
Special cases

$h=0$: $\bar{y} = \frac{\frac{2}{3}a^3}{a^2\frac{\pi}{2}} = \frac{4a}{3\pi}$ (the correct result)

$h=\frac{a}{4}$: $\bar{y} = \frac{\frac{2}{3}\left(a^2-\left(\frac{a}{4}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{4}\right)-\frac{a}{4}\sqrt{a^2-\left(\frac{a}{4}\right)^2}} = \underline{0.562a}$

$h=\frac{a}{2}$: $\bar{y} = \frac{\frac{2}{3}\left(a^2-\left(\frac{a}{2}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{2}\right)-\frac{a}{2}\sqrt{a^2-\left(\frac{a}{2}\right)^2}} = \underline{0.705a}$

▶5/42



$$dV = dr\,(r\,d\phi)(r\sin\phi\,d\theta)$$

$$= r^2 \sin\phi\, d\phi\, d\theta\, dr$$

$$V = \int dV = \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^2 \sin\phi\, d\phi\, d\theta\, dr$$

$$= \frac{7}{12}\pi R^3$$

(Check V by

$$V = \frac{\frac{4}{3}\pi R^3}{2} - \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{2} = \frac{7}{12}\pi R^3)$$

$$\int x_c\, dV = \int (r\sin\phi\sin\theta)(r^2\sin\phi\, d\phi\, d\theta\, dr)$$

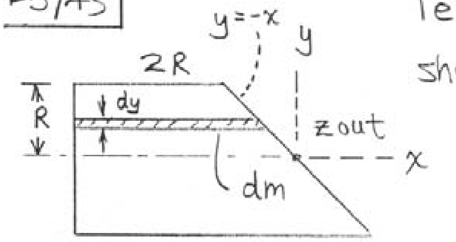
$$= \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^3 \sin^2\phi\, \sin\theta\; d\phi\, d\theta\, dr = \frac{15}{64}\pi R^4$$

$$\bar{x} = \frac{\int x_c\, dV}{V} = \frac{\frac{15}{64}\pi R^4}{\frac{7}{12}\pi R^3} = \underline{\frac{45}{112} R}$$

(Compare to $\bar{x} = \frac{3}{8} R$ for no hole.)

Note: A hemispherical shell of radius r and thickness dr would be a better element.

►5/43



Temporarily use coordinates shown. Element is a thin rectangular plate.

$$dm = \rho\, dV = \rho\,(3R+x)(2z)\,dy$$
$$= 2\rho\,[3R-y]\left[\sqrt{R^2-y^2}\right] dy$$
$$= 6\rho R\sqrt{R^2-y^2}\,dy - 2\rho y\sqrt{R^2-y^2}\,dy$$
$$m = \int dm = \int_{-R}^{R} 6\rho R\sqrt{R^2-y^2}\,dy - \int_{-R}^{R} 2\rho y\sqrt{R^2-y^2}\,dy$$
$$m = 6\rho R\,\tfrac{1}{2}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\frac{y}{R}\right)\Big|_{-R}^{R} - 2\rho\left(-\tfrac{1}{3}\sqrt{(R^2-y^2)^3}\right)\Big|_{-R}^{R}$$
$$= 3\pi\rho R^3$$
$$\int x_c\, dm = \int_{-R}^{R}\left(\frac{x-3R}{2}\right)\left(6\rho R\sqrt{R^2-y^2}\,dy - 2\rho y\sqrt{R^2-y^2}\,dy\right)$$
$$I_1 = -3\rho R\int_{-R}^{R} y\sqrt{R^2-y^2}\,dy = -3\rho R\left(-\tfrac{1}{3}\sqrt{(R^2-y^2)^3}\right)\Big|_{-R}^{R}$$
$$= 0$$
$$I_2 = -9\rho R^2\int_{-R}^{R}\sqrt{R^2-y^2}\,dy$$
$$= -9\rho R^2\cdot\tfrac{1}{2}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\left(\frac{y}{R}\right)\right)\Big|_{-R}^{R}$$
$$= -\tfrac{9}{2}\rho R^2\left(R^2\tfrac{\pi}{2} - \left(-R^2\tfrac{\pi}{2}\right)\right) = -\tfrac{9}{2}\pi\rho R^4$$

$$I_3 = \rho \int_{-R}^{R} y^2 \sqrt{R^2-y^2}\, dy$$

$$= \rho \left(-\frac{y}{4}\sqrt{(R^2-y^2)^3} + \frac{R^2}{8}\left(y\sqrt{R^2-y^2} + R^2 \sin^{-1}\frac{y}{R} \right) \right)_{-R}^{R}$$

$$= \rho \left(\frac{R^2}{8} \cdot \left(R^2 \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) \right) \right) = \frac{1}{8}\pi\rho R^4$$

$$I_4 = -3\rho R \int_{-R}^{R} y \sqrt{(R^2-y^2)}\, dy = 0$$

So total is $\int x_c\, dm = \left(-\frac{9}{2} + \frac{1}{8} \right) \pi \rho R^4$

$$= -\frac{35}{8}\pi\rho R^4$$

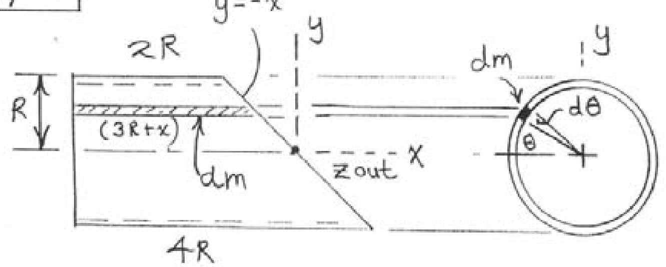
Then $\bar{x} = \dfrac{\int x_c\, dm}{\int dm}$

$$= \frac{-\frac{35}{8}\pi\rho R^4}{3\pi\rho R^3} = -\frac{35}{24}R$$

Relative to left (flat) end, then,

$$\bar{x} = 3R - \frac{35}{24}R = \underline{\frac{37}{24}R} \quad \underline{(1.542R)}$$

▸5/44



Temporarily use coordinates shown. Element is a "stick". ρ is mass per unit area here.

$$dm = \rho(3R+x)R\,d\theta = \rho R(3R-y)\,d\theta$$
$$= \rho R(3R - R\sin\theta)\,d\theta = \rho R^2(3-\sin\theta)\,d\theta$$

$$m = \int dm = \int_0^{2\pi} \rho R^2 (3-\sin\theta)\,d\theta$$
$$= \rho R^2 \Big[3\theta + \cos\theta\Big]_0^{2\pi} = 6\pi\rho R^2$$

$$x_c = -3R + \frac{3R+x}{2} = \frac{x-3R}{2} = -\frac{y+3R}{2}$$
$$= -R\,\frac{\sin\theta+3}{2}$$

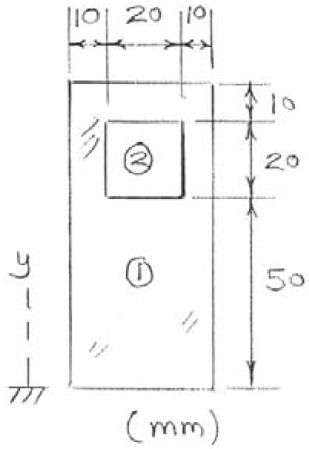
$$\int x_c\,dm = \int_0^{2\pi}\Big[-\frac{R}{2}(\sin\theta+3)\rho R^2(3-\sin\theta)\,d\theta$$
$$= -\frac{\rho R^3}{2}\int_0^{2\pi}(9-\sin^2\theta)\,d\theta$$
$$= -\frac{\rho R^3}{2}\Big[9\theta - \Big(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\Big)\Big]_0^{2\pi} = -\rho R^3\Big(\frac{17\pi}{2}\Big)$$

$$\text{So } \bar{x} = \frac{\int x_c\,dm}{\int dm} = \frac{-\rho R^3(17\pi/2)}{6\pi\rho R^2} = 1.417R$$

Relative to left (flat) end,

$$\bar{x} = 3R - 1.417R = \underline{1.583\,R}$$

5/45

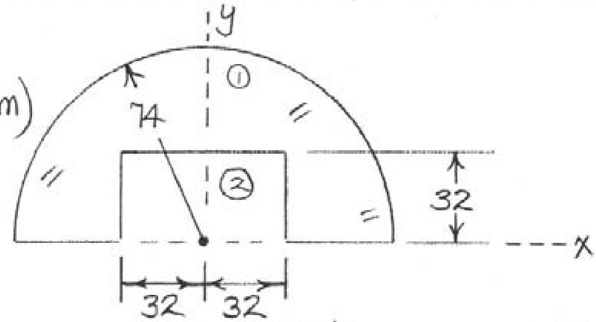


Component	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
1	$80(40) = 3200$	40	128 000
2	$-20(20) = -400$	60	$-24\,000$
	$\Sigma A = 2800$		$\Sigma \bar{y}A = 104\,000$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{104\,000}{2800} = \underline{37.1 \text{ mm}}$$

5/46

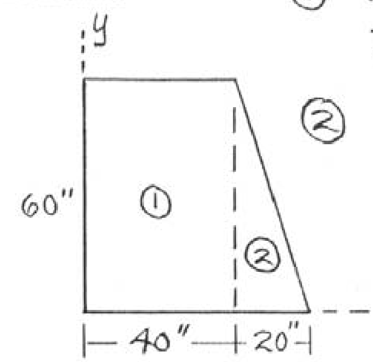
(Dim. in mm)



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\pi \frac{74^2}{2}\left(\frac{4(74)}{3\pi}\right) - 64(32)\left(\frac{32}{2}\right)}{\pi \frac{74^2}{2} - 64(32)}$$

$= \underline{36.2 \text{ mm}}$

5/47



① $A_1 = 40(60) = 2400 \text{ in.}^2$

$\overline{x}_1 = 20 \text{ in.}, \quad \overline{y}_1 = 30 \text{ in.}$

② $A_2 = \frac{1}{2}(20)(60) = 600 \text{ in.}^2$

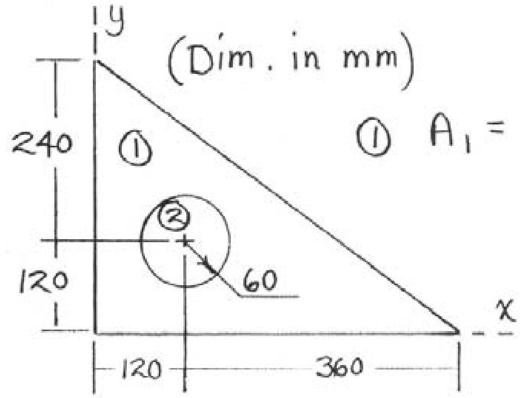
$\overline{x}_2 = 40 + \frac{20}{3} = 46.7 \text{ in.}$

$\overline{y}_2 = \frac{60}{3} = 20 \text{ in.}$

$$\overline{X} = \frac{\Sigma A\overline{x}}{\Sigma A} = \frac{2400(20) + 600(46.7)}{2400 + 600} = \underline{25.3 \text{ in.}}$$

$$\overline{Y} = \frac{\Sigma A\overline{y}}{\Sigma A} = \frac{2400(30) + 600(20)}{2400 + 600} = \underline{28.0 \text{ in.}}$$

5/48



① $A_1 = \frac{1}{2}(480)(360)$

$= 86\,400 \text{ mm}^2$

$\bar{x}_1 = \frac{1}{3}(480) = 160 \text{ mm}$

$\bar{y}_1 = \frac{1}{3}(360) = 120 \text{ mm}$

② $A_2 = -\pi(60)^2 = -11\,310 \text{ mm}^2$

$\bar{x}_2 = \bar{y}_2 = 120 \text{ mm}$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{86\,400(160) - 11\,310(120)}{86\,400 - 11\,310} = \underline{166.0 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{86\,400(120) - 11\,310(120)}{86\,400 - 11\,310} = \underline{120 \text{ mm}}$$

(Because the circular hole is located at the $\bar{y}$ for the triangle, its presence does not alter the $\bar{y}$ result.)

5/49 Triangle: $A = \frac{1}{2}(2h)h = h^2$

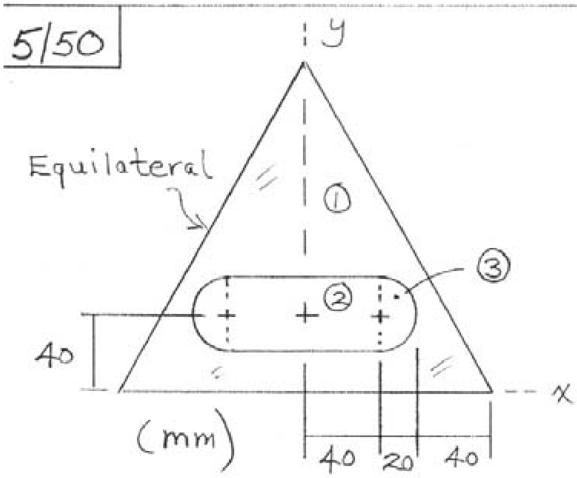
$\bar{y} = h/3$

Semi-circular hole: $A = -\frac{1}{2}\pi(h/2)^2 = -\pi h^2/8$

$\bar{y} = 4(h/2)/3\pi = \frac{2h}{3\pi}$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{h^2(h/3) - (\pi h^2/8)(2h/3\pi)}{h^2 - \pi h^2/8} = \frac{h}{4(1-\pi/8)} = \underline{0.412h}$$

5/50

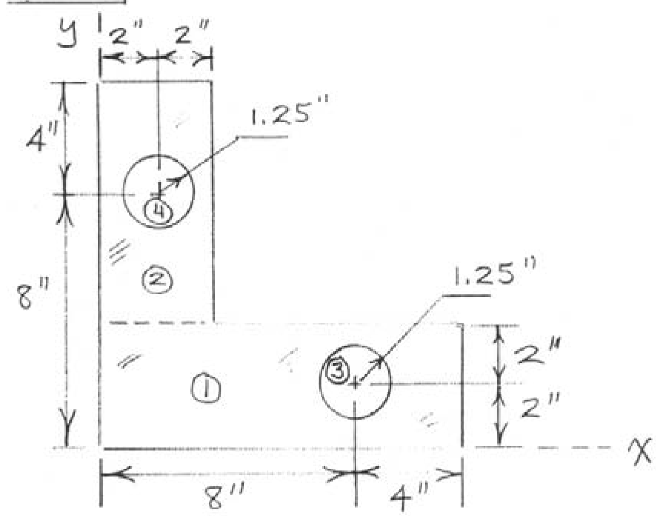


Component	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
Triangle 1	17 320	57.7	10^6
Rectangle 2	− 3200	40	− 128 000
2 semicircles 3	− 1257	40	− 50,300

$\Sigma A = 12\,860$ $\qquad$ $\Sigma \bar{y}A = 822\,000$

$$\overline{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} \quad \frac{822\,000}{12\,860} = 63.9 \text{ mm}$$

5/51



Comp.	A (in.^{2})	$\bar{x}$ (in.)	$\bar{x}A$ (in.^{3})
1	48	6	288
2	32	2	64
3	−4.91	8	−39.3
4	−4.91	2	−9.82

$\Sigma A = 70.2$ $\Sigma \bar{x}A = 303$

$$\bar{X} = \bar{Y} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{303}{70.2} = \underline{4.32 \text{ in.}}$$

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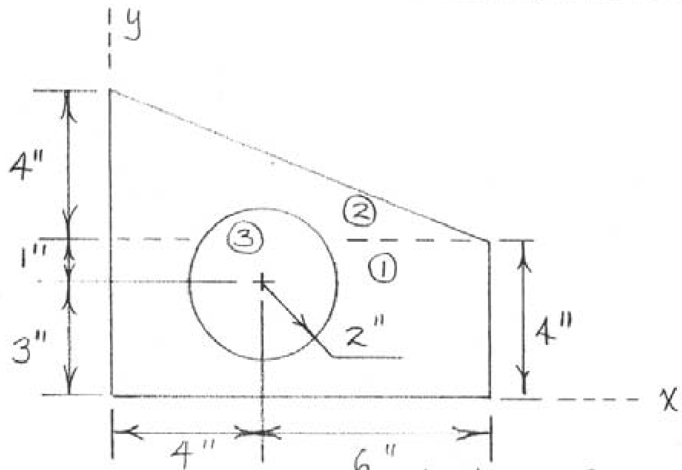

Part	A (in.^{2})	$\bar{x}$ (in.)	$\bar{y}$ (in.)	$A\bar{x}$ (in.^{3})	$A\bar{y}$ (in.^{3})
1	40	5	2	200	80
2	20	$10/3$	$4 + 4/3$	66.7	106.7
3	$-\pi 2^2$	4	3	−50.3	−37.7
Totals	47.4			216	149.0

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{216}{47.4} = \underline{4.56 \text{ in.}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{149.0}{47.4} = \underline{3.14 \text{ in.}}$$

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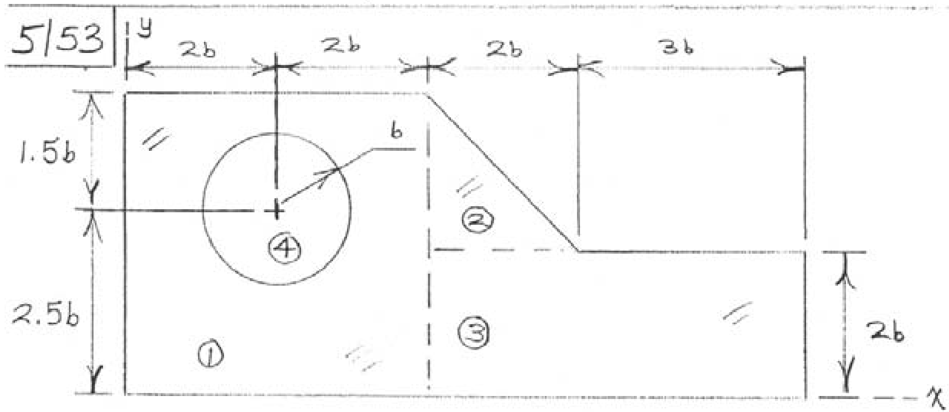


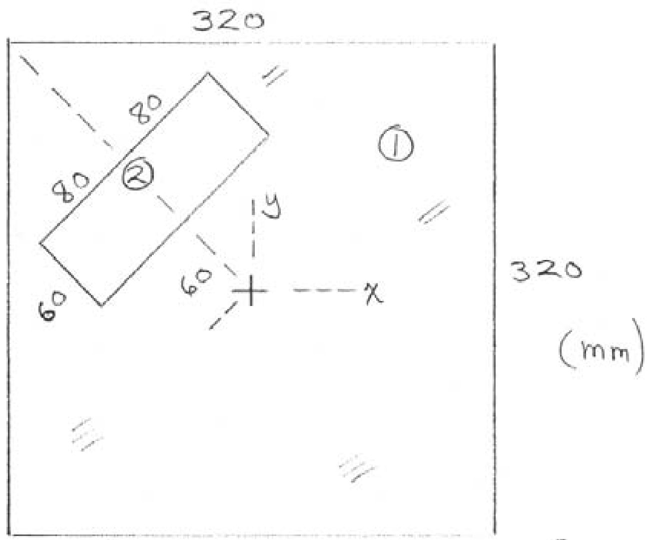
Comp.	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$16b^2$	$2b$	$2b$	$32b^3$	$32b^3$
2	$2b^2$	$\left(4b+\frac{2b}{3}\right)$	$\left(2.5b+\frac{2b}{3}\right)$	$9.33b^3$	$6.33b^3$
3	$10b^2$	$\left(4b+\frac{5b}{2}\right)$	b	$65b^3$	$10b^3$
4	$-\pi b^2$	$2b$	$2.5b$	$-2\pi b^3$	$-2.5\pi b^3$

$\Sigma A = 24.9b^2$ $\qquad$ $\Sigma\bar{x}A = 100.1b^3$ $\qquad$ $\Sigma\bar{y}A = 40.5b^3$

$$\bar{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{100.1b^3}{24.9b^2} = \underline{4.02b}$$

$$\bar{Y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{40.5b^3}{24.9b^2} = \underline{1.628b}$$

5/54



Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}A$ (mm^3)	$\bar{y}A$ (mm^3)
1	$(320)^2$	0	0	0	0
2	$-160(60)$	$-90\frac{\sqrt{2}}{2}$	$90\frac{\sqrt{2}}{2}$	611 000	−611 000

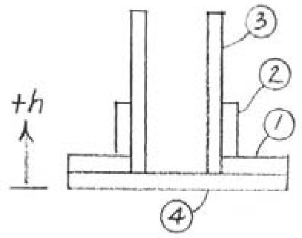
$\Sigma A = 92\,800$ $\qquad$ $\Sigma \bar{x}A = 611\,000$

$\Sigma \bar{y}A = -611\,000$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{611\,000}{92\,800} = \underline{6.58 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{-611\,000}{92\,800} = \underline{-6.58 \text{ mm}}$$

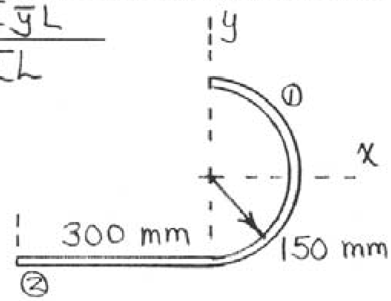
5/55



Part	Size mm	A mm^2	$\bar{h}$ mm	$A\bar{h}$ mm^3
1	10 × 40	800	15	12000
2	10 × 40	800	40	32000
3	10 × 120	2400	70	168 000
4	10 × 160	1600	5	8000
Σ's		5600		220 000

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{220\,000}{5600} = \underline{39.3 \text{ mm}}$$

5/56 $\bar{X} = \dfrac{\Sigma \bar{x} L}{\Sigma L}$, $\bar{Y} = \dfrac{\Sigma \bar{y} L}{\Sigma L}$



$L_1 = \pi r = 150\pi$ mm

$\bar{x}_1 = \dfrac{2r}{\pi} = \dfrac{300}{\pi}$ mm

(from Sample Prob. 5/1)

$\bar{y}_1 = 0$

$L_2 = 300$ mm, $\bar{x}_2 = -150$ mm, $\bar{y}_2 = -150$ mm

So

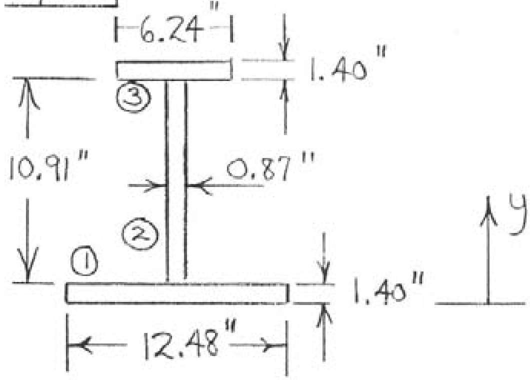
$$\bar{X} = \frac{\frac{300}{\pi}(150\pi) + 300(-150)}{150\pi + 300} = \underline{0}$$

$$\bar{Y} = \frac{\frac{300}{\pi}(0) + 300(-150)}{150\pi + 300} = \underline{-58.3 \text{ mm}}$$

5/57

$$\bar{z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70 \text{ mm}}$$

5/58

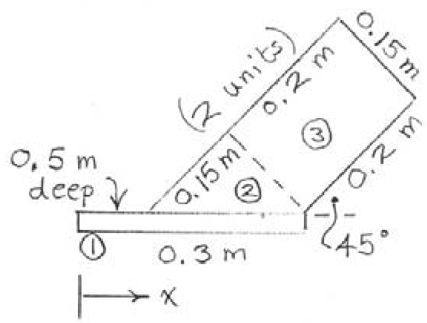


Comp.	A (in.^{2})	$\bar{y}$ (in.)	$A\bar{y}$ (in.^{3})
1	$(12.48)(1.40)$	$\frac{1.40}{2}$	12.23
2	$(10.91)(0.87)$	$1.4+\frac{10.91}{2}$	65.1
3	$(6.24)(1.40)$	$1.4+10.91+\frac{1.4}{2}$	113.7

$\Sigma A = 35.7$ $\Sigma A\bar{y} = 191.0$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{5.35 \text{ in.}}$$

5/59 ρ = mass per unit area

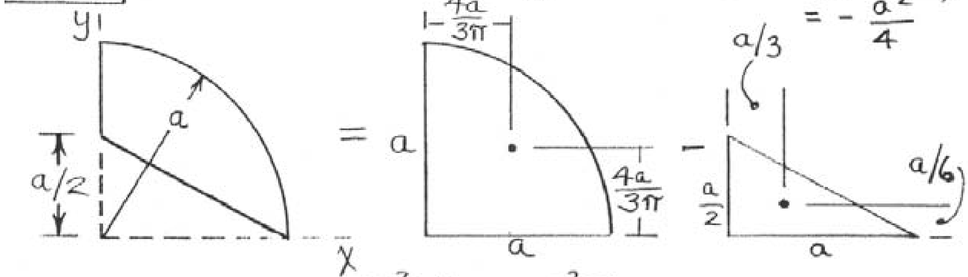


Comp.	m	$\bar{x}$	$\bar{x}m$
①	$(0.5)(0.3)\rho$	0.15	0.0225ρ
②	$2\frac{(0.15)(0.15)}{2}\rho$	$0.3-\left[\frac{2}{3}(0.15)+\frac{1}{3}(0.15)\right]\frac{\sqrt{2}}{2}$	0.00436ρ
③	$2(0.2)(0.15)\rho$	$0.3+\left(\frac{0.2-0.15}{2}\right)\frac{\sqrt{2}}{2}$	0.01906ρ
	$\Sigma m = 0.232\rho$		$\Sigma \bar{x}m = 0.0459\rho$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{0.0459\rho}{0.232\rho} = \underline{0.1975\ m}$$

5/60

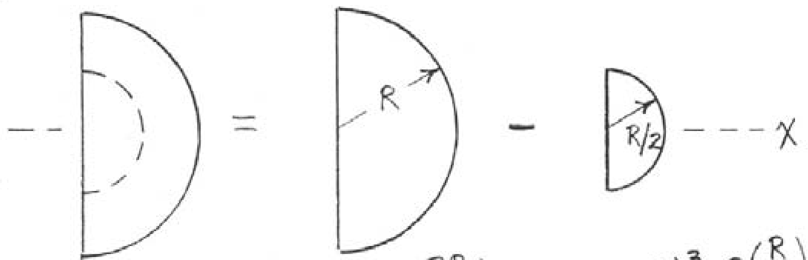
$A = \pi a^2/4$ $\qquad$ $A = -\frac{1}{2}a\left(\frac{a}{2}\right) = -\frac{a^2}{4}$



$$\overline{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\frac{\pi a^2}{4}\frac{4a}{3\pi} - \frac{a^2}{4}\frac{a}{3}}{\frac{\pi a^2}{4} - \frac{a^2}{4}} = \underline{\frac{a}{\pi - 1}}$$

$$\overline{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{\frac{\pi a^2}{4}\frac{4a}{3\pi} - \frac{a^2}{4}\frac{a}{6}}{\frac{\pi a^2}{4} - \frac{a^2}{4}} = \underline{\frac{7}{6}\frac{a}{\pi - 1}}$$

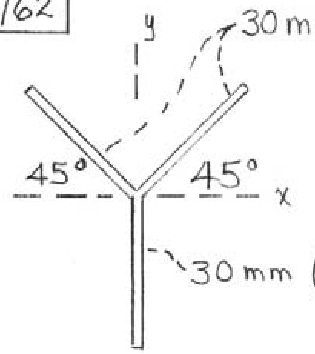
5/61



$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{\frac{2}{3}\pi R^3\left(\frac{3R}{8}\right) - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3 \frac{3\left(\frac{R}{2}\right)}{8}}{\frac{2}{3}\pi R^3 - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$= \underline{\frac{45}{112}R}$$

5/62



$$30\text{ mm}\left(0.3\ \frac{\text{kg}}{\text{m}}\right)\left(\frac{1\text{ m}}{1000\text{ mm}}\right) = 0.009\text{ kg}$$

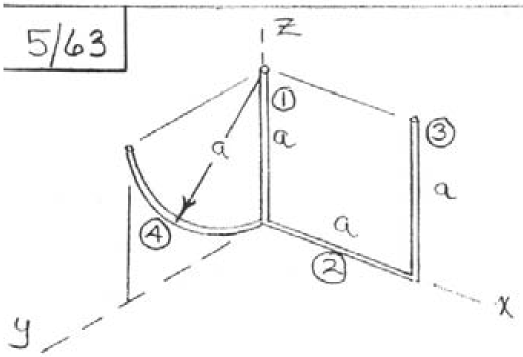
$$30\text{ mm}\left(0.5\ \frac{\text{kg}}{\text{m}}\right)\left(\frac{1\text{ m}}{1000\text{ mm}}\right) = 0.0150\text{ kg}$$

$$\overline{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{2(0.009)(15\sin 45°) - 0.0150(15)}{2(0.009) + 0.0150}$$

$$= \underline{-1.033\text{ mm}}$$

$\underline{\overline{X} = 0}$, by inspection.

5/63



(ρ = mass/length)

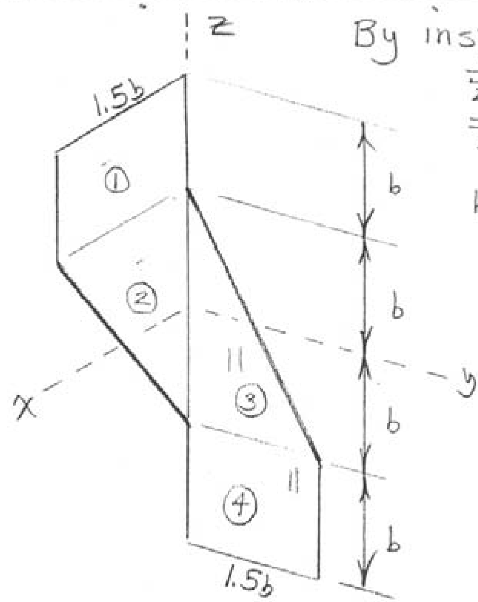
Comp.	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}$	$m\bar{y}$	$m\bar{z}$
1	ρa	0	0	$a/2$	0	0	$\rho a^2/2$
2	ρa	$a/2$	0	0	$\rho a^2/2$	0	0
3	ρa	a	0	$a/2$	ρa^2	0	$\rho a^2/2$
4	$\rho\pi a/2$	0	$-2a/\pi$	$a(1-\frac{2}{\pi})$	0	$-\rho a^2$	$\rho a^2(\frac{\pi}{2}-1)$
Totals	$\rho a(3+\frac{\pi}{2})$				$\frac{3}{2}\rho a^2$	$-\rho a^2$	$\frac{\pi}{2}\rho a^2$

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} = \frac{\frac{3}{2}\rho a^2}{\rho a(3+\frac{\pi}{2})} = \underline{\frac{3a}{6+\pi}}$$

$$\bar{Y} = \frac{\sum m\bar{y}}{\sum m} = \frac{-\rho a^2}{\rho a(3+\pi/2)} = \underline{-\frac{2a}{6+\pi}}$$

$$\bar{Z} = \frac{\sum m\bar{z}}{\sum m} = \frac{\frac{\pi}{2}\rho a^2}{\rho a(3+\pi/2)} = \underline{\frac{\pi a}{6+\pi}}$$

5/64



By inspection,

$\bar{Z} = 0$

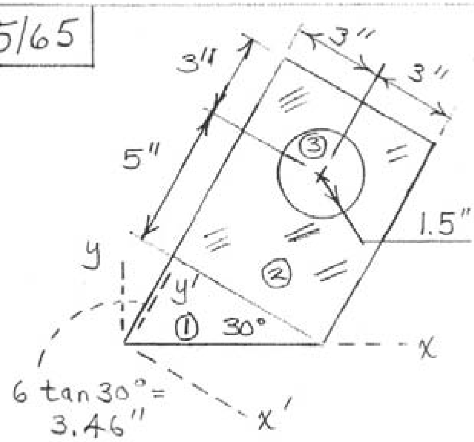
$\bar{X} = \bar{Y}$

Let ρ = area density

Comp.	m	$\bar{x}$	$m\bar{x}$
1	$1.5b^2\rho$	$1.5b/2$	$1.125b^3\rho$
2	$1.5b^2\rho$	$0.5b$	$0.75b^3\rho$
3	$1.5b^2\rho$	0	0
4	$1.5b^2\rho$	0	0
	$\Sigma A = 6b^2$		$\Sigma \bar{x}A = 1.875b^3$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{1.875b^3\rho}{6b^2\rho} = \underline{0.312b} = \bar{Y}$$

5/65



Perform calculations in x'-y' coordinates, then convert to x-y.

Comp.	A	$\bar{x}'$	$\bar{y}'$	$\bar{x}'A$	$\bar{y}'A$
1	$\frac{3.46(6)}{2}$	$6/3$	$3.46\left(\frac{2}{3}\right)$	20.8	24
2	48	3	7.46	144	358
3	$-\pi(1.5)^2$	3	8.46	-21.2	-59.8
	$\Sigma A = 51.3$			143.6	322

$$\bar{X}' = \frac{\Sigma \bar{x}'A}{\Sigma A} = \frac{143.6}{51.3} = 2.80 \text{ in.}$$

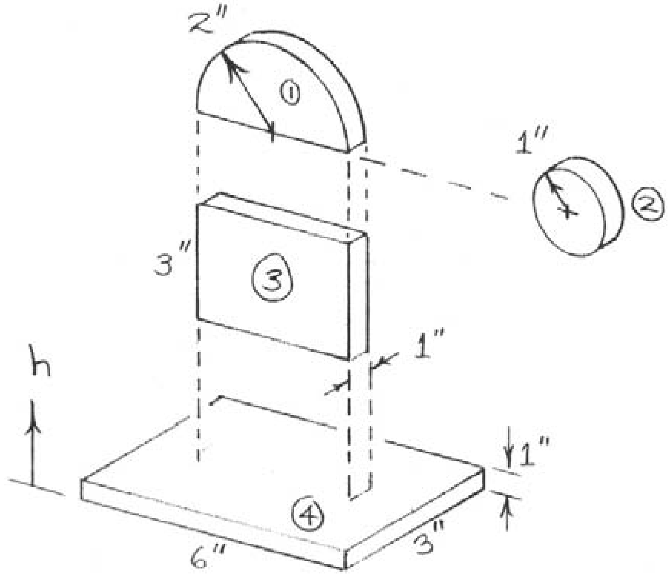
$$\bar{Y}' = \frac{\Sigma \bar{y}'A}{\Sigma A} = \frac{322}{51.3} = 6.28 \text{ in.}$$

Then

$$\bar{X} = 2.80 \cos 30° + 6.28 \sin 30° = \underline{5.56 \text{ in.}}$$

$$\bar{Y} = -2.80 \sin 30° + 6.28 \cos 30° = \underline{4.04 \text{ in.}}$$

5/66



Part	V, in.^{3}	$\bar{h}$, in.	$V\bar{h}$, in.^{4}
①	6.28	4.85	30.5
②	−3.14	4	−12.57
③	12	2.5	30
④	18	0.5	9
Totals	33.1		56.9

$$\bar{H} = \frac{\Sigma V\bar{h}}{\Sigma V} = \frac{56.9}{33.1} = \underline{1.717 \text{ in.}}$$

5/67

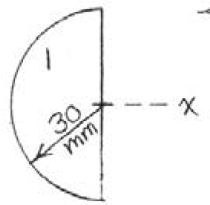
Comp.	V	$\bar{x}$	$\bar{y}$	$\bar{z}$	$V\bar{x}$	$V\bar{y}$	$V\bar{z}$
Rect. Sol.	$8.08(10^6)$	0	0	0	0	0	0
Cyl.	$2.26(10^6)$	185	0	0	$418(10^6)$	0	0
Rod	$17.67(10^3)$	0	175	0	0	$3.09(10^6)$	0
Sphere	$524(10^3)$	0	275	0	0	$144.0(10^6)$	0
Totals	$10.88(10^6)$				$418(10^6)$	$147.1(10^6)$	0

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{418(10^6)}{10.88(10^6)} = \underline{38.5\text{ mm}}$$

$$\bar{Y} = \frac{\Sigma V\bar{y}}{\Sigma V} = \frac{147.1(10^6)}{10.88(10^6)} = \underline{13.52\text{ mm}}$$

$$\underline{\bar{Z} = 0}$$

5/68



Length = 35 mm

$$\left\{\begin{aligned} V_1 &= \frac{\pi}{2}(30)^2(35) = 49\ 500\ \text{mm}^3 \\ \bar{x}_1 &= -\frac{4(30)}{3\pi} = -12.73\ \text{mm} \\ \bar{z}_1 &= 17.5\ \text{mm} \end{aligned}\right.$$

Length = 25 mm

$$V_2 = -\frac{\pi}{2}(20)^2(25) = -15\ 710\ \text{mm}^2$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49\ \text{mm}$$

$$\bar{z}_2 = \frac{1}{2}(10+35) = 22.5\ \text{mm}$$

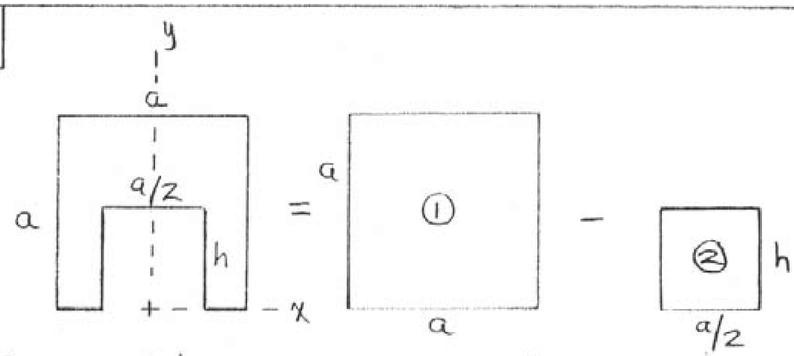
$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{49\ 500(-12.73) - 15\ 710(-8.49)}{49\ 500 - 15\ 710}$$

$$= \underline{-14.71\ \text{mm}}$$

$$\bar{Z}_1 = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{49\ 500\ (17.5) - 15\ 710\ (22.5)}{49\ 500 - 15\ 710}$$

$$= \underline{15.17\ \text{mm}}$$

5/69



$A_1 = a^2$, $\bar{y}_1 = a/2$; $A_2 = -ah/2$, $\bar{y}_2 = h/2$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{a^2\left(\frac{a}{2}\right) - \frac{ah}{2}\left(\frac{h}{2}\right)}{a^2 - \frac{ah}{2}} = \frac{1}{2}\,\frac{a^2 - h^2/2}{a - h/2}$$

$$\frac{d\bar{Y}}{dh} = \frac{1}{2}\,\frac{(a - h/2)(-h) - (a^2 - h^2/2)(-\frac{1}{2})}{(a - h/2)^2}$$

$$= \frac{1}{2}\,\frac{\frac{h^2}{4} - ah + \frac{a^2}{2}}{(a - \frac{h}{2})^2} = 0 \text{ for max } \bar{Y}.$$

So $\frac{1}{4}(h^2 - 4ah + 2a^2) = 0$, $h = a(2 \pm \sqrt{2})$

The plus sign is rejected because h must be less than a.

Hence $h = a(2 - \sqrt{2}) = \underline{0.586a}$

(Note that $\bar{Y} = 0.586a$ for $h = 0.586a$)

5/70 Cube: $\begin{cases} V_1 = 350^3 = 42\,875\,000 \text{ mm}^3 \\ \bar{z}_1 = 175 \text{ mm} \end{cases}$

Hole: $\begin{cases} V_2 = -\pi(100)^2 h \\ \bar{z}_2 = \frac{h}{2} \end{cases}$

$$\bar{Z} = \frac{\Sigma V \bar{z}}{\Sigma V} = \frac{42\,875\,000(175) - \pi(100)^2 h \frac{h}{2}}{42\,875\,000 - \pi(100)^2 h}$$

For the maximum $\bar{Z}$, set $\frac{d\bar{Z}}{dh} = 0$:

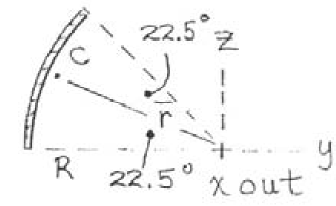
$$\frac{(42\,875\,000 - \pi\, 100^2 h)(-\pi\, 100^2 h) - \left(42\,875\,000(175) - \pi\, 100^2 \frac{h^2}{2}\right) \times (-\pi\, 100^2)}{(\text{denominator})} = 0$$

Set numerator equal to zero to obtain

$$\underline{h = 187.9 \text{ mm}}$$

▸5/71 Detail of "hole" (②): [① = cyl. shell]

$$\bar{r} = \frac{R \sin 22.5^\circ}{\pi/8} = 0.974R$$



$$\begin{cases} \bar{x} = -\frac{L}{8} - \frac{1}{2}\frac{3L}{8} = -\frac{5}{16}L \\ \bar{y} = -\bar{r}\cos 22.5^\circ = -0.900R \\ \bar{z} = \bar{r}\sin 22.5^\circ = 0.373R \end{cases}$$

Comp	m	$\bar{x}$	$\bar{y}$	$\bar{z}$
①	$2\rho\pi RL$	$-\frac{L}{2}$	0	0
②	$-\frac{3}{32}\rho\pi RL$	$-\frac{5L}{16}$	$-0.900R$	$0.373R$

$\Sigma m = 1.906\rho\pi RL$

	$\bar{x}m$	$\bar{y}m$	$\bar{z}m$
①	$-\rho\pi RL^2$	0	0
②	$+0.0293\rho\pi RL^2$	$+0.0844\rho\pi R^2L$	$-0.0350\rho\pi R^2L$

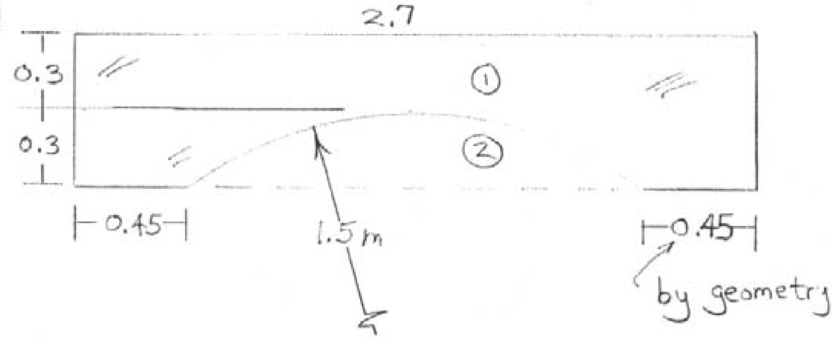
$\Sigma\bar{x}m = -0.971\rho\pi RL^2$ $\quad \Sigma\bar{y}m = 0.0844\rho\pi R^2L$ $\quad \Sigma\bar{z}m = -0.0350\rho\pi R^2L$

$$\bar{X} = \frac{\Sigma\bar{x}m}{\Sigma m} = \frac{-0.971\rho\pi RL^2}{1.906\rho\pi RL} = \underline{-0.509L}$$

$$\bar{Y} = \frac{\Sigma\bar{y}m}{\Sigma m} = \frac{0.0844\rho\pi R^2L}{1.906\rho\pi RL} = \underline{0.0443R}$$

$$\bar{Z} = \frac{\Sigma\bar{z}m}{\Sigma m} = \frac{-0.0350\rho\pi R^2L}{1.906\rho\pi RL} = \underline{-0.01834R}$$

▸5/72



From Prob. 5/41, relative to the base of the current area, we have for the circular portion with $a = 1.5$ m & $h = 1.2$ m:

$$\bar{y} = \frac{\frac{2}{3}\left(1.5^2 - 1.2^2\right)^{3/2}}{1.5^2\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1.2}{1.5}\right)\right] - 1.2\sqrt{1.5^2 - 1.2^2}} - 1.2$$

$$= 0.1211 \text{ m}$$

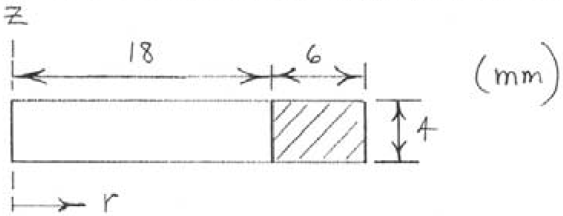
Comp.	A	$\bar{y}$	$\bar{y}A$
1	$(2.7)(0.6)$	0.3	0.486
2	-0.368 *	0.1211	-0.0445
	$\Sigma A = 1.252$		$\Sigma \bar{y} A = 0.441$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.441}{1.252} = \underline{0.353 \text{ m}}$$

* From Prob. 5/41:

$$A = 1.5^2\left(\frac{\pi}{2} - \sin^{-1}\frac{1.2}{1.5}\right) - 1.2\sqrt{1.5^2 - 1.2^2}$$
$$= 0.368 \text{ m}^2$$

5/73



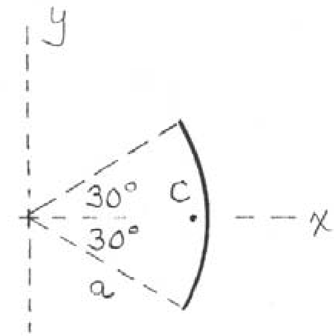
$$A = 2\pi\bar{r}L = 2\pi\left(18+\frac{6}{2}\right)(6+6+4+4)$$

$$= \underline{2640 \text{ mm}^2}$$

$$V = 2\pi\bar{r}A = 2\pi\left(18+\frac{6}{2}\right)(6\cdot 4)$$

$$= \underline{3170 \text{ mm}^3}$$

5/74



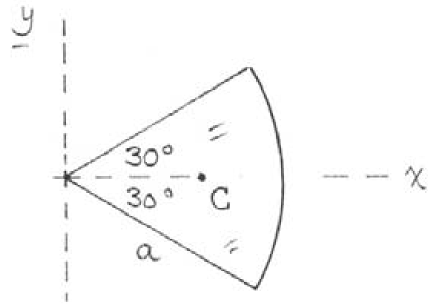
From Table D/3,

$$\bar{x} = \frac{a \sin \pi/6}{\pi/6} = \frac{3a}{\pi}$$

$$A = 2\pi \bar{x} L = 2\pi \left(\frac{3a}{\pi}\right)\left(\frac{\pi}{3} a\right)$$

$$= \underline{2\pi a^2}$$

5/75



From Table D/3:

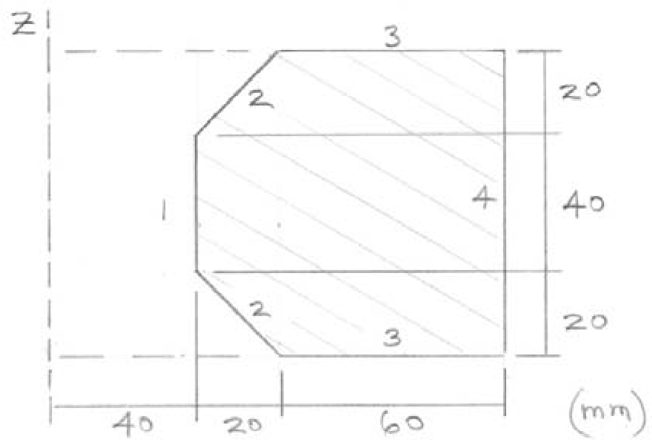
$$\overline{x} = \frac{2}{3}a\,\frac{\sin \pi/6}{\pi/6} = \frac{2a}{3}\,\frac{1}{2}\,\frac{6}{\pi} = \frac{2a}{\pi}$$

$$V = \pi \overline{x} A = \pi\left(\frac{2a}{\pi}\right)\left(\frac{\pi a^2}{6}\right)$$

$$= \underline{\frac{\pi a^3}{3}}$$

$\boxed{5/76}$ $V = \theta \bar{r} A = \pi\left(8 + \frac{2}{3}12\right)\frac{1}{2}(12)(12) = \underline{3620 \text{ mm}^3}$

5/77



$A_1 = \pi \bar{r}_1 L_1 = \pi (40)(40) = 1600\pi \text{ mm}^2$

$A_2 = 2(\pi \bar{r}_2 L_2) = 2\pi (50)(20\sqrt{2}) = 2830\pi \text{ mm}^2$

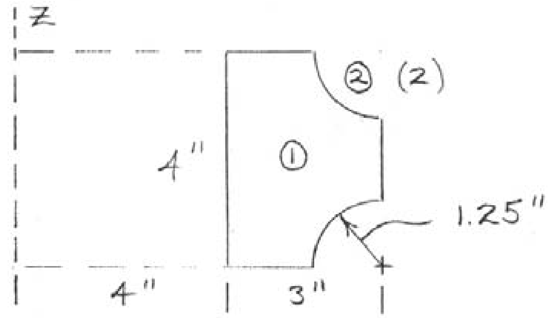
$A_3 = 2(\pi r_3 L_3) = 2\pi (90)(60) = 10\,800\pi \text{ mm}^2$

$A_4 = \pi r_4 L_4 = \pi (120)(80) = 9600\pi \text{ mm}^2$

End faces: $2\{80(60) + 40(20) + 20(20)\} = 12\,000 \text{ mm}^2$

Total: $\underline{90\,000 \text{ mm}^2}$

5/78



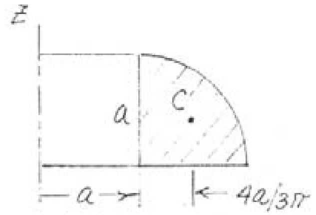
$$\bar{V}_1 = 2\pi \bar{r}_1 A_1 = 2\pi\left(4 + 3/2\right)\left(4\cdot 3\right) = 415 \text{ in.}^3$$

$$V_2 = -2\left(2\pi \bar{r}_2 A_2\right) = -4\pi\left(7 - \frac{4(1.25)}{3\pi}\right)\left(\frac{\pi(1.25)^2}{4}\right)$$

$$= -99.8 \text{ in.}^3$$

$$V = V_1 + V_2 = \underline{315 \text{ in.}^3}$$

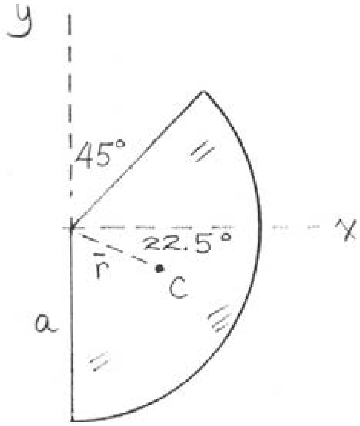
5/79



$$V = \bar{r}\theta A = \left(a + \frac{4a}{3\pi}\right)\frac{\pi}{2}\frac{\pi a^2}{4}$$

$$= \frac{\pi a^3}{24}(3\pi + 4)$$

5/80



From Table D/3, with $\alpha = 67.5°$ (0.375π)

$$\bar{r} = \frac{2}{3}\frac{a\sin(0.375\pi)}{0.375\pi} = 0.523a$$

$$V = 2\pi\bar{x}A = 2\pi(0.523a\cos 22.5°)\left(\pi a^2\,\frac{0.75\pi}{2\pi}\right)$$

$$= \underline{3.58a^3}$$

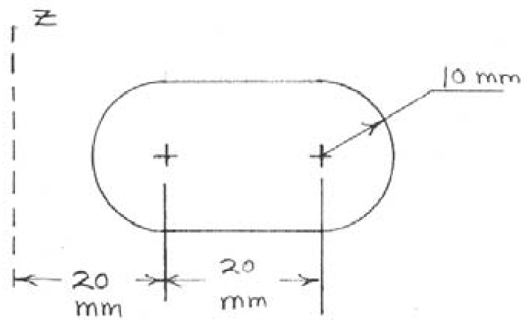
5/81

$$A = 2\pi \bar{r} L + \pi d h$$

$$= 2\pi(8.2)34 + \pi(8)(18) = 2204 \ ft^2$$

No. of gal. for 2 coats $= \frac{2204}{500} \times 2 = \underline{8.82 \ gal}$

5/82



$A = 2\pi \bar{r} L = 2\pi (30)\left[20 + 20 + \pi 10 + \pi 10\right]$

$= \underline{19\ 380\ \text{mm}^2}$

$V = 2\pi \bar{r} A = 2\pi (30)\left[20(20) + \pi (10^2)\right]$

$= \underline{134\ 600\ \text{mm}^3}$

For steel,

$m = \rho V = 7830\ \frac{\text{kg}}{\text{m}^3}\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^3 (134\ 600\ \text{mm}^3)$

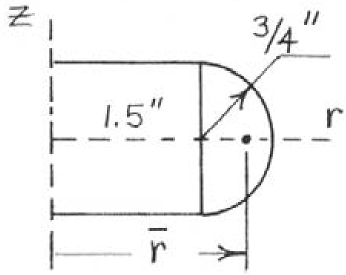
$= \underline{1.054\ \text{kg}}$

5/83

$$\bar{V} = 2\pi \bar{r}_{area} A$$

$$= 2\pi\left(1.5 + \frac{4(0.75)}{3\pi}\right) \frac{\pi(0.75)^2}{2}$$

$$= \underline{10.09 \text{ in.}^3}$$



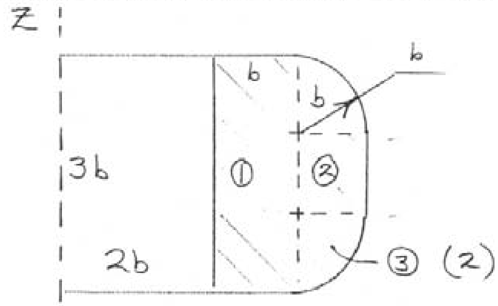
$$A = 2\pi \bar{r}_{arc} L$$

$$= 2\pi\left(1.5 + \frac{2(0.75)}{\pi}\right)(\pi \times 0.75)$$

$$= \underline{29.3 \text{ in.}^2}$$

$$\begin{cases} \bar{r}_{area} = \dfrac{4r}{3\pi} + 1.5 \\ \bar{r}_{arc} = \dfrac{2r}{\pi} + 1.5 \end{cases}$$

5/84



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi\left(2b+\frac{b}{2}\right)(3b^2) = 15\pi b^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi\left(3b+\frac{b}{2}\right)(b^2) = 7\pi b^3$$

$$V_3 = 2(2\pi \bar{r}_3 A_3) = 4\pi\left(3b+\frac{4b}{3\pi}\right)\left(\frac{\pi b^2}{4}\right) = \pi^2 b^3\left(3+\frac{4}{3\pi}\right)$$

$$V = V_1 + V_2 + V_3 = \underline{\pi b^3\left(\frac{70}{3}+3\pi\right)} = \underline{102.9\, b^3}$$

$$A_{1_{\text{inner}}} = 2\pi \bar{r}_{1_i} L_{1_i} = 2\pi(2b)(3b) = 12\pi b^2$$

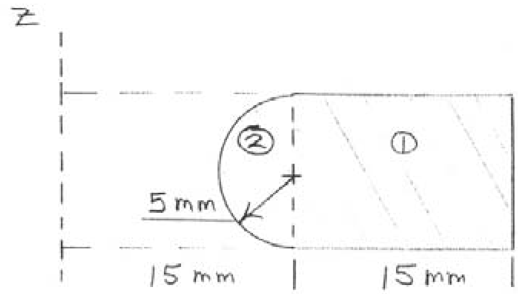
$$A_{1_{\text{top \& bott.}}} = 2\cdot 2\pi \bar{r}_{1_t} L_{1_t} = 4\pi \frac{5b}{2}(b) = 10\pi b^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi(4b)(b) = 8\pi b^2$$

$$A_3 = 2\cdot 2\pi \bar{r}_3 L_3 = 4\pi\left(3b+\frac{2b}{\pi}\right)\left(\frac{\pi b}{2}\right) = 2\pi^2 b^2\left(3+\frac{2}{\pi}\right)$$

$$A = A_1 + A_2 + A_3 = \underline{\pi b^2(34+6\pi)} = \underline{166.0\, b^2}$$

5/85



$$\bar{V}_1 = 2\pi \bar{r}_1 A_1 = 2\pi\left(15 + \frac{15}{2}\right)(15 \cdot 10) = 21\,200 \text{ mm}^3$$

$$\bar{V}_2 = 2\pi \bar{r}_2 A_2 = 2\pi\left(15 - \frac{4 \cdot 5}{3\pi}\right)\left(\frac{\pi 5^2}{2}\right) = 3\,180 \text{ mm}^3$$

$$V = V_1 + V_2 = \underline{24\,400 \text{ mm}^3}$$

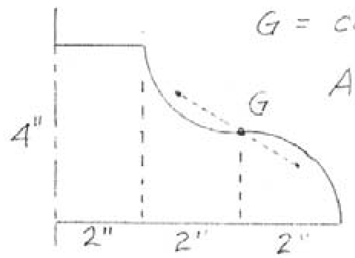
$$A_{1_{outer}} = 2\pi \bar{r}_{1_o} L_{1o} = 2\pi(30)(10) = 1885 \text{ mm}^2$$

$$A_{1_{top \,\&\, bottom}} = 2 \cdot 2\pi r_{1_t} L_{1t} = 4\pi\left(15 + \frac{15}{2}\right)(15) = 4240 \text{ mm}^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi\left(15 - \frac{2 \cdot 5}{\pi}\right)(\pi 5) = 1166 \text{ mm}^2$$

$$A = A_1 + A_2 = \underline{7290 \text{ mm}^2}$$

5/86

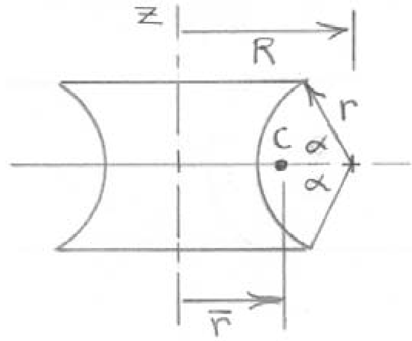


G = centroid by symmetry

$A = 2\pi \bar{r} L = 2\pi(4)\pi(2)$

$= 16\pi^2 = \underline{157.9 \text{ in.}^2}$

5/87



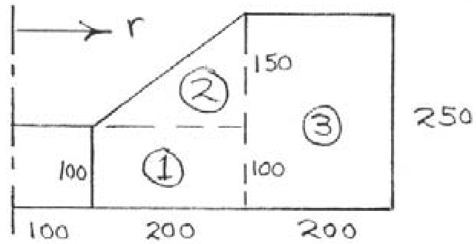
$\bar{r} = R - r\dfrac{\sin\alpha}{\alpha}$ from Sample Problem 5/1

$L = 2r\alpha$

$A = 2\pi\bar{r}L = 2\pi\left(R - r\dfrac{\sin\alpha}{\alpha}\right)(2r\alpha)$

$= \underline{4\pi r\,(R\alpha - r\sin\alpha)}$

5/88



$$V = 2\pi \bar{r} A = 2\pi \sum_{i=1}^{3} \bar{r}_i A_i$$

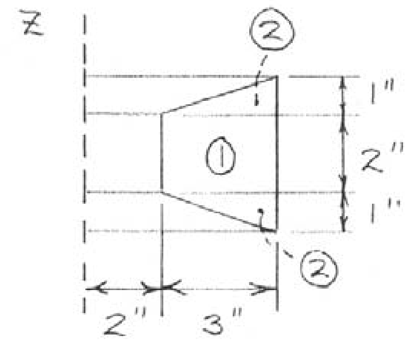
$A_1 = 200(100) = 20(10^3)\text{ mm}^2$, $r_1 = 100 + \frac{200}{2} = 200\text{ mm}$

$A_2 = \frac{200(150)}{2} = 15(10^3)\text{ mm}^2$, $r_2 = 100 + \frac{2}{3}200 = 233\text{ mm}$

$A_3 = 200(250) = 50(10^3)\text{ mm}^2$, $r_3 = 300 + \frac{200}{2} = 400\text{ mm}$

$$V = 2\pi\left[200(20\times10^3) + 233(15\times10^3) + 400(50\times10^3)\right]$$
$$= 172.8(10^6)\text{ mm}^3 \quad \text{or} \quad \underline{V = 0.1728\text{ m}^3}$$

5/89



$$V_1 = \pi \bar{r}_1 A_1 = \pi\left(2+\frac{3}{2}\right)(3\cdot 2) = 66.0 \text{ in.}^3$$

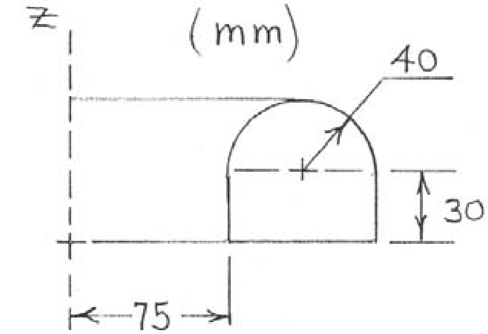
$$V_2 = 2(\pi \bar{r}_2 A_2) = 2\pi\left(5-\frac{3}{3}\right)\left(\frac{1}{2}(3)(1)\right) = 37.7 \text{ in.}^3$$

$$V = V_1 + V_2 = 103.7 \text{ in.}^3$$

$$W = \mu V = \left(168\,\frac{\text{lb}}{\text{ft}^3}\right)\left(103.7\,\text{in.}^3\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= \underline{10.08 \text{ lb}}$$

5/90



$A_1 = \pi \bar{r} L = \pi(75+40)(2\cdot 30 + 80 + \pi 40)$

$= 96\,000 \text{ mm}^2$

End areas $A_2 = 2\left(\frac{\pi}{2}\cdot 40^2 + 80(30)\right)$

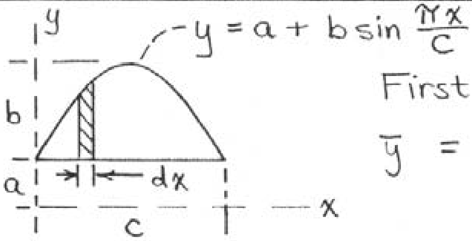
$= 9830 \text{ mm}^2$

Total area $A = A_1 + A_2 = \underline{105\,800 \text{ mm}^2}$

$V = \pi \bar{r} A = \pi(75+40)(30\cdot 80 + \pi 40^2/2)$

$= \underline{1.775(10^6) \text{ mm}^3}$

5/91



First, find $\bar{y}$ by

$$\bar{y} = \int y_c\, dA \Big/ \int dA$$

$$A = \int dA = \int_0^c b\sin\frac{\pi x}{c}\,dx = -\frac{bc}{\pi}\cos\frac{\pi x}{c}\Big|_0^c = \frac{2bc}{\pi}$$

$$\int y_c\, dA = \int_0^c \left[a + \frac{b}{2}\sin\frac{\pi x}{c}\right]\left[b\sin\frac{\pi x}{c}\,dx\right]$$

$$= \int_0^c ab\,\sin\frac{\pi x}{c}\,dx + \int_0^c \frac{b^2}{2}\sin^2\frac{\pi x}{c}\,dx$$

$$= -ab\frac{c}{\pi}\cos\frac{\pi x}{c}\Big|_0^c + \frac{b^2}{2}\left[\frac{x}{2} - \frac{1}{4}\sin\frac{2\pi x}{c}\right]_0^c$$

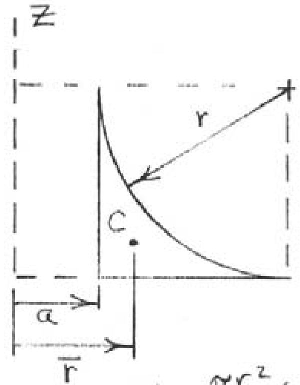
$$= \frac{2abc}{\pi} + \frac{b^2c}{4} = bc\left[\frac{2a}{\pi} + \frac{b}{4}\right]$$

$$\therefore\ \bar{y} = \frac{bc\left[2a/\pi + b/4\right]}{2bc/\pi} = a + \frac{b}{8}\pi$$

$$V = 2\pi\bar{y}A = 2\pi\left(a + \frac{b}{8}\pi\right)\left(2bc/\pi\right)$$

$$= \underline{4bc\left(a + \frac{b\pi}{8}\right)}$$

5/92



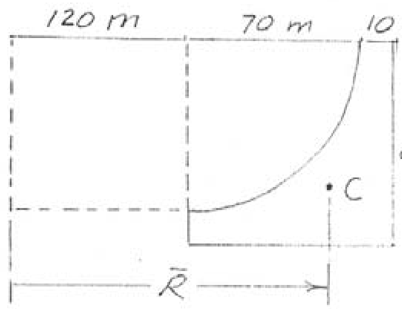
(Refer to Sample Problem 5/3)

$$\bar{r} = \frac{r^2\left(a+\frac{r}{2}\right) - \frac{\pi r^2}{4}\left(a + r - \frac{4r}{3\pi}\right)}{r^2 - \pi r^2/4} = a + \frac{10-3\pi}{3(4-\pi)}r$$

$$V = \theta \bar{r} A = \frac{\pi}{2}\left[a + \frac{10-3\pi}{3(4-\pi)}r\right]\left(1-\frac{\pi}{4}\right)r^2$$

$$= \frac{\pi r^2}{8}\left[(4-\pi)a + \frac{10-3\pi}{3}r\right]$$

5/93



Square: $A = 80^2 = 6400\ m^2$

$\frac{1}{4}$ circle: $A = \frac{1}{4}\pi(70^2)$

$= 3848\ m^2$

Net area $= 2552\ m^2$

$\bar{r}_{square} = 120 + 40 = 160\ m$

$\bar{r}_{\frac{1}{4}circle} = 120 + \frac{4(70)}{3\pi} = 149.7\ m$

$\bar{R} = \frac{\Sigma A\bar{r}}{\Sigma A} = \frac{6400(160) - 3848(149.7)}{2552}$

$= 175.5\ m$

$V = \theta\bar{R}A = \frac{\pi}{3}(175.5)(2552) = 469\ 000\ m^3$

$m = \rho V = 2.40(469\ 000) = \underline{1.126 \times 10^6\ Mg}$

5/94 From the solution to Prob. 5/11,

$$\bar{r} = 8 - \frac{2}{3}\frac{2(1.5)+2}{1.5+2} = 7.05 \text{ m}$$

$$A = \frac{2+1.5}{2}(2) = 3.5 \text{ m}^2$$

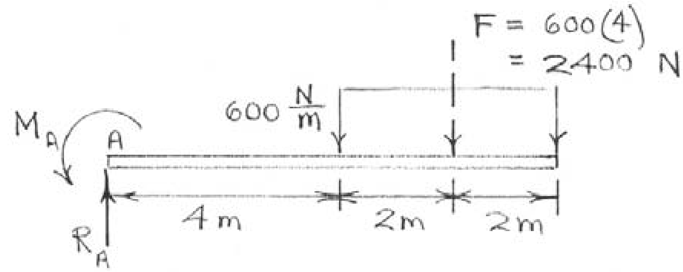
$$\theta = \pi/3$$

So $V = \theta \bar{r} A = \frac{\pi}{3}(7.05)(3.5) = 25.8 \text{ m}^3$

$$W = \rho g V = 2400(9.81)(25.8) = 608(10^3) \text{ N}$$

or $\underline{W = 608 \text{ kN}}$

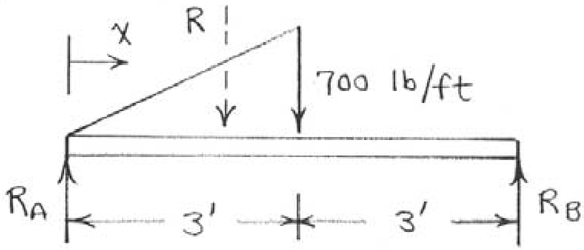
5/95



$+\uparrow \Sigma F = 0 : \; R_A - 2400 = 0, \qquad \underline{R_A = 2.4 \text{ kN}}$

$\circlearrowleft + \Sigma M_A = 0 : \; M_A - 2400(6) = 0, \qquad \underline{M_A = 14.4 \text{ kN}\cdot\text{m}}$

5/96

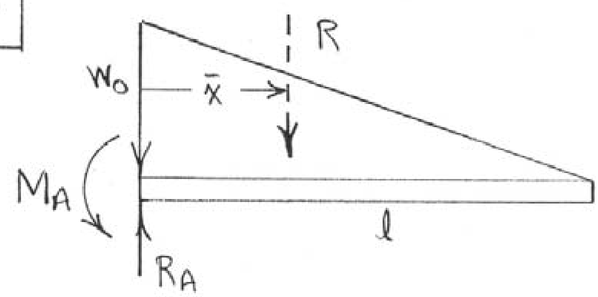


$R = \frac{1}{2}(700)(3) = 1050 \text{ lb}$ @ $\bar{x} = \frac{2}{3}(3) = 2'$

$\circlearrowleft + \sum M_A = 0: \; R_B(6) - 1050(2) = 0, \quad \underline{R_B = 350 \text{ lb}}$

$+\uparrow \sum F = 0: \; R_A + 350 - 1050 = 0, \quad \underline{R_A = 700 \text{ lb}}$

5/97

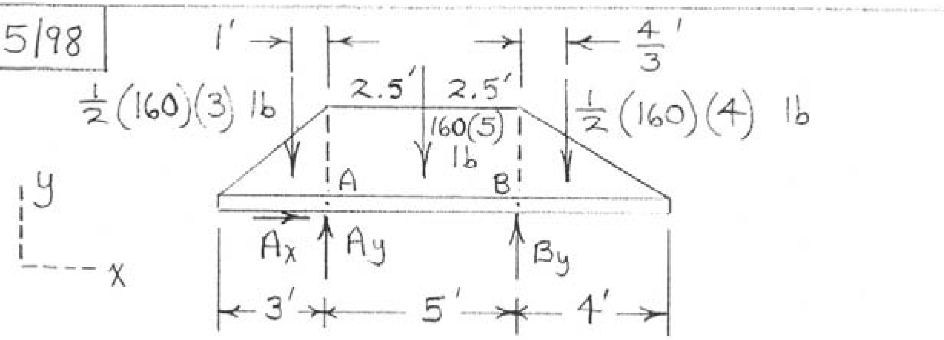


$R = \frac{1}{2} w_0 l$ @ $\bar{x} = \frac{1}{3} l$

$\circlearrowleft + \Sigma M_A = 0$: $M_A - \frac{1}{2} w_0 l \left(l/3 \right) = 0$, $\underline{M_A = \frac{1}{6} w_0 l^2}$

$+\uparrow \Sigma F = 0$: $R_A - \frac{1}{2} w_0 l = 0$, $\underline{R_A = \frac{1}{2} w_0 l}$

5/98



$$\circlearrowleft + \sum M_A = 0: \; 240(1) - 800(2.5) - 320(6.33) + B_y(5) = 0$$

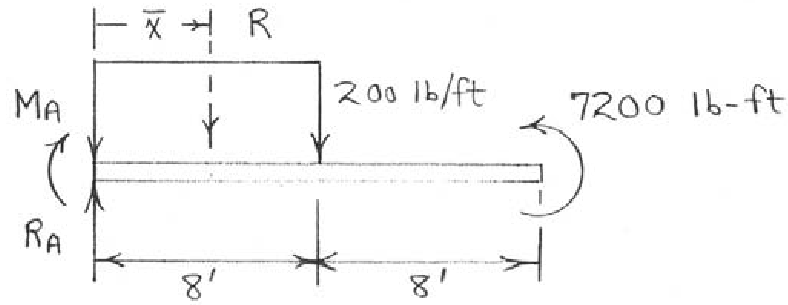
$$\underline{B_y = 757 \text{ lb}}$$

$$\sum F_y = 0: \; A_y + 757 - 240 - 800 - 320 = 0$$

$$\underline{A_y = 603 \text{ lb}}$$

$$\sum F_x = 0 \Rightarrow \underline{A_x = 0}$$

5/99



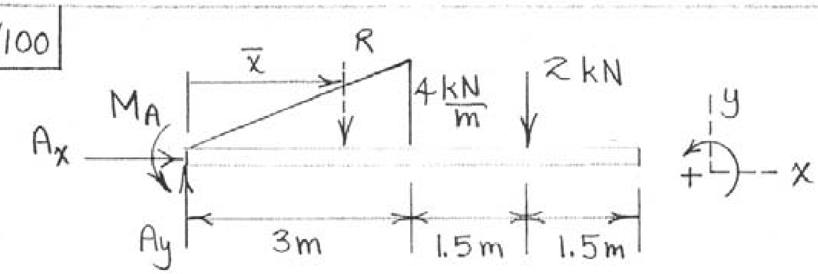
$R = 200(8) = 1600 \text{ lb} \;@\; \bar{x} = 4'$

$\circlearrowleft + \Sigma M_A = 0: \; -M_A - 1600(4) + 7200 = 0$

$\underline{M_A = 800 \text{ lb-ft}}$

$+\uparrow \Sigma F = 0: \; R_A - 1600 = 0, \quad \underline{R_A = 1600 \text{ lb}}$

5/100



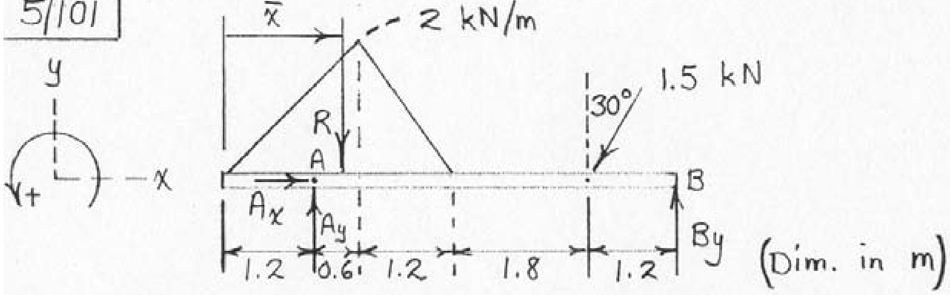
$R = \frac{1}{2}(3)(4) = 6 \text{ kN}$ @ $\bar{x} = \frac{2}{3}(3) = 2 \text{ m}$

$\Sigma M_A = 0: \quad M_A - 6(2) - 2(4.5) = 0, \quad \underline{M_A = 21 \text{ kN}\cdot\text{m}}$

$\Sigma F_y = 0: \quad A_y - 6 - 2 = 0, \quad \underline{A_y = 8 \text{ kN}}$

$\Sigma F_x = 0: \quad \underline{A_x = 0}$

5/101



$$\begin{cases} R = \frac{1}{2}(1.2+0.6+1.2)(2) = 3 \text{ kN} \\ \bar{x} = \dfrac{(1.2+0.6)+(1.2+0.6+1.2)}{3} = 1.6 \text{ m} \end{cases}$$

$\Sigma F_x = 0: \quad A_x - 1.5 \sin 30° = 0, \quad \underline{A_x = 750 \text{ N}}$

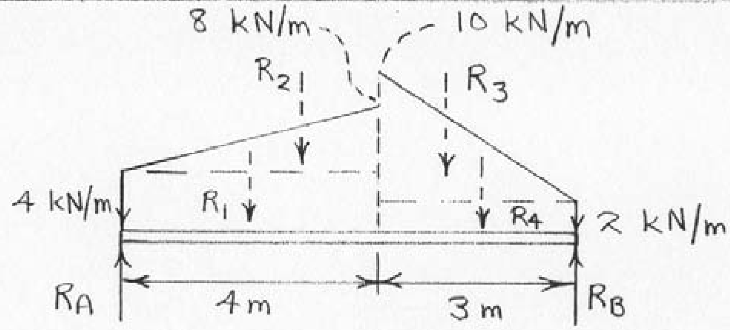
$\Sigma M_A = 0: \quad -3(1.6-1.2) - 1.5\cos 30°(3.6) - B_y(4.8) = 0$

$\underline{B_y = 1.224 \text{ kN}}$

$\Sigma F_y = 0: \quad A_y - 3 - 1.5\cos 30° + 1.224 = 0$

$\underline{A_y = 3.07 \text{ kN}}$

5/102



$R_1 = 4(4) = 16$ kN, $\quad R_2 = \frac{1}{2}(4)(4) = 8$ kN

$R_3 = \frac{1}{2}(8)(3) = 12$ kN, $\quad R_4 = 2(3) = 6$ kN

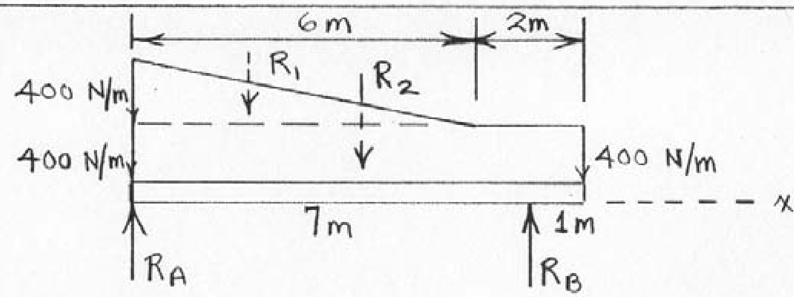
$\circlearrowleft + \sum M_A = 0: \; 7R_B - 16(2) - 8\left(\frac{2}{3}4\right) - 12(4+1)$

$\quad -6(4+1.5) = 0, \quad \underline{R_B = 20.9 \text{ kN}}$

$+\uparrow \sum F = 0: \; R_A + 20.9 - (16+8+12+6) = 0$

$\underline{R_A = 21.1 \text{ kN}}$

5/103



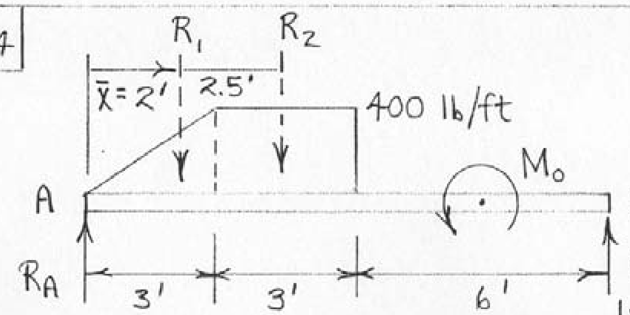
$R_1 = \frac{1}{2}(400)(6) = 1200$ N @ $\bar{x}_1 = \frac{1}{3}(6) = 2$ m

$R_2 = 400(8) = 3200$ N @ $\bar{x}_2 = \frac{1}{2}(8) = 4$ m

$\circlearrowleft + \Sigma M_A = 0$: $R_B(7) - 1200(2) - 3200(4) = 0$, $\underline{R_B = 2170 \text{ N}}$

$+\uparrow \Sigma F = 0$: $R_A - 1200 - 3200 + 2170 = 0$, $\underline{R_A = 2230 \text{ N}}$

5/104



$R_B = 0$ at switch from lower to upper surface

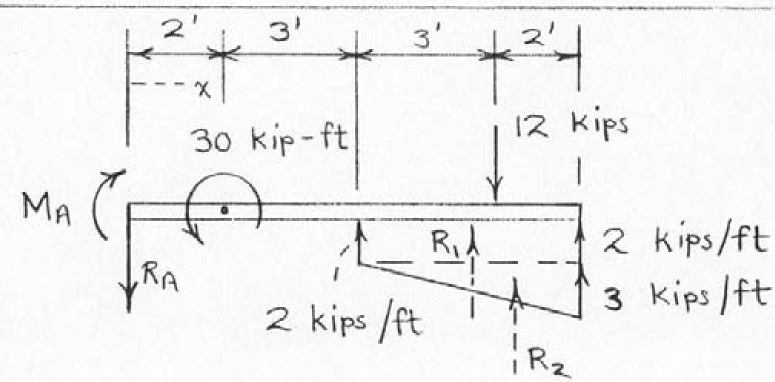
$R_1 = \frac{1}{2}(400)(3) = 600 \text{ lb} \ @ \ \bar{x} = \frac{2}{3}(3) = 2'$

$R_2 = 400(3) = 1200 \text{ lb}$

$\curvearrowleft + \Sigma M_A = 0: \ -600(2) - 1200(4.5) + M_0 = 0$

$\underline{M_o = 6600 \text{ lb-ft}}$

5/105



$R_1 = 2(5) = 10$ kips @ $\bar{x}_1 = 7.5'$

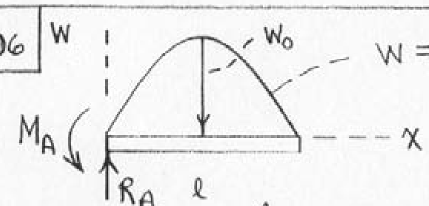
$R_2 = \frac{1}{2}(3)(5) = 7.5$ kips @ $\bar{x}_2 = 8.33'$

$\circlearrowleft + \Sigma M_A = 0: \quad M_A - 30 + 12(8) - 10(7.5) - 7.5(8.33) = 0$

$\underline{M_A = 71.5 \text{ kip-ft}}$

$+\uparrow \Sigma F = 0: \; -R_A + 10 + 7.5 - 12 = 0, \quad \underline{R_A = 5.5 \text{ kips}}$

5/106



$w = w_0 \sin \frac{\pi}{\ell} x$

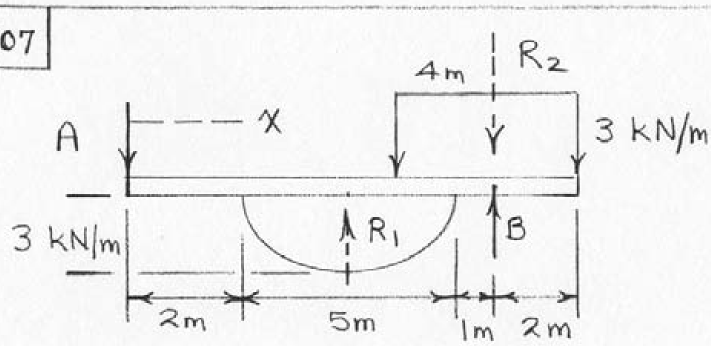
$$R = \int w\,dx = \int_0^{\ell} w_0 \sin \frac{\pi}{\ell} x = -w_0 \frac{\ell}{\pi} \cos \frac{\pi}{\ell} x \Big|_0^{\ell} = \frac{2 w_0 \ell}{\pi}$$

$\bar{x} = \frac{\ell}{2}$, by inspection

$$\curvearrowleft + \Sigma M_A = 0: \quad M_A - \frac{2 w_0 \ell}{\pi}\left(\frac{\ell}{2}\right) = 0, \qquad \underline{M_A = \frac{w_0 \ell^2}{\pi}}$$

$$+\uparrow \Sigma F = 0: \quad R_A - \frac{2 w_0 \ell}{\pi} = 0, \qquad \underline{R_A = \frac{2 w_0 \ell}{\pi}}$$

5/107



$$R_1 = \frac{\pi(5)(3)}{4} = 11.78 \text{ kN} \quad @ \ \bar{x}_1 = 4.5 \text{ m}$$

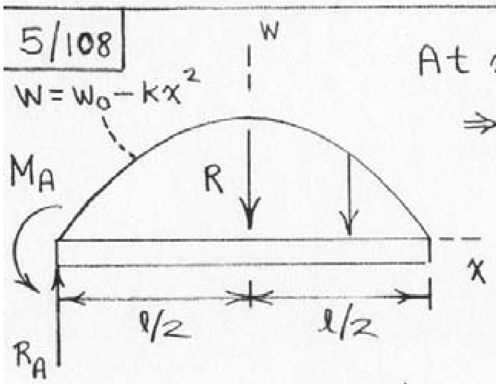
$$R_2 = 4(3) = 12 \text{ kN} \quad @ \ \bar{x}_2 = 8 \text{ m}$$

$$\circlearrowleft + \Sigma M_B = 0: \quad A(8) - 11.78(3.5) = 0, \quad \underline{A = 5.15 \text{ kN}}$$

$$+\uparrow \Sigma F = 0 \quad - 5.15 + 11.78 - 12 + B = 0, \quad \underline{B = 5.37 \text{ kN}}$$

Contact at A is at the upper roller.

5/108



At $x = \frac{l}{2}$, $w = 0 = w_0 - k\left(\frac{l}{2}\right)^2$

$$\Rightarrow k = \frac{4w_0}{l^2}$$

$$w = w_0\left(1 - \frac{4}{l^2}x^2\right)$$

$\bar{x} = 0$, by inspection

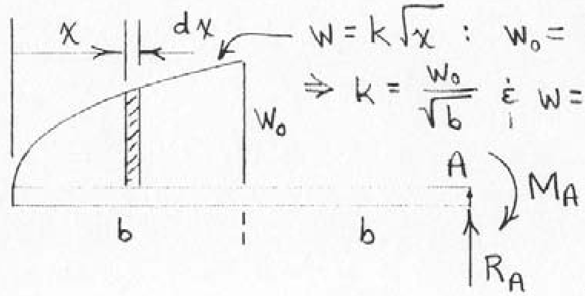
$$R = \int w\,dx = 2\int_0^{l/2} w_0\left(1 - \frac{4}{l^2}x^2\right)dx$$

$$= 2w_0\left[x - \frac{4}{3l^2}x^3\right]_0^{l/2} = \frac{2}{3}w_0 l$$

$+\uparrow\Sigma F = 0: \quad R_A - \frac{2}{3}w_0 l = 0, \quad \underline{R_A = \frac{2}{3}w_0 l}$

$\circlearrowleft + \Sigma M_A = 0: \quad M_A - \frac{2}{3}w_0 l\left(\frac{l}{2}\right) = 0, \quad \underline{M_A = \frac{1}{3}w_0 l^2}$

5/109



$w = k\sqrt{x}$: $w_0 = k\sqrt{b}$

$\Rightarrow k = \frac{w_0}{\sqrt{b}}$ & $w = w_0\sqrt{\frac{x}{b}}$

The force on dx is $dF = w\,dx = w_0\sqrt{\frac{x}{b}}\,dx$

Total load $F = \int dF = \int_0^b w_0\sqrt{\frac{x}{b}}\,dx = \frac{2}{3}w_0 b$

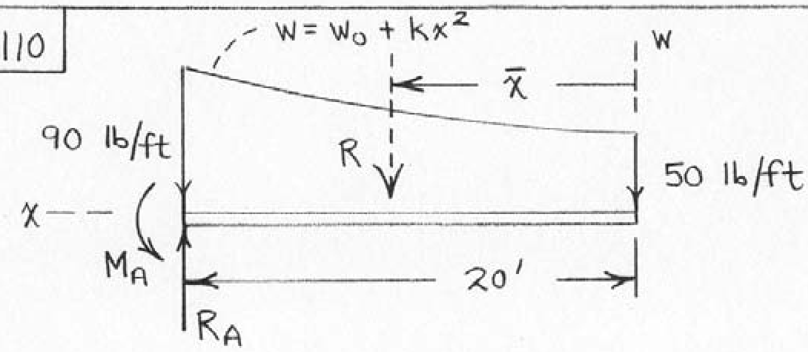
So $\underline{R_A = \frac{2}{3}w_0 b}$

The moment of dF about A is $w\,dx\,(2b-x)$

$$M_A = \int_0^b w\,(2b-x)\,dx = \int_0^b w_0\sqrt{\frac{x}{b}}\,(2b-x)\,dx$$

$$= \frac{w_0}{\sqrt{b}}\int_0^b \left(2b\sqrt{x} - x^{3/2}\right)dx = \underline{\underline{\frac{14}{15}w_0 b^2}}$$

5/110



At $x = 0$, $w = 50 = w_0$

At $x = 20'$, $w = 90 = 50 + k(20)^2$, $k = \frac{1}{10}$ lb/ft^3

So $w = 50 + \frac{x^2}{10}$ (lb/ft)

$$R = \int w\,dx = \int_0^{20}\left(50 + \frac{x^2}{10}\right)dx = \left[50x + \frac{x^3}{30}\right]_0^{20} = 1267 \text{ lb}$$

$$\bar{x} = \frac{\int x\,w\,dx}{R} = \frac{\int x\left(50 + \frac{x^2}{10}\right)dx}{1267} = \frac{\left[\frac{50}{2}x^2 + \frac{x^4}{40}\right]_0^{20}}{1267}$$

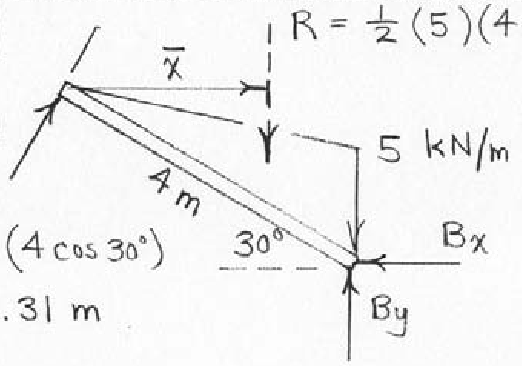
$= 11.05$ ft

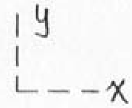
$+\uparrow \Sigma F = 0: \; R_A - 1267 = 0,$ $\qquad \underline{R_A = 1267 \text{ lb}}$

$\circlearrowleft \Sigma M_A = 0: \; M_A - 1267(20 - 11.05) = 0,$ $\underline{M_A = 11.33(10^3) \text{ lb-ft}}$

5/111

$R = \frac{1}{2}(5)(4\cos 30^\circ) = 8.66$ kN





$\bar{x} = \frac{2}{3}(4\cos 30^\circ)$

$= 2.31$ m

$$\left.\begin{array}{l} \sum F_x = 0: \quad A\sin 30^\circ - B_x = 0 \\ \sum F_y = 0: \quad A\cos 30^\circ + B_y - 8.66 = 0 \\ \circlearrowleft^{+} \sum M_B = 0: \quad -A(4) + 8.66\left(\frac{1}{3}4\cos 30^\circ\right) = 0 \end{array}\right\}$$

$\underline{A = 2.5 \text{ kN}}$, $B_x = 1.25$ kN, $B_y = 6.50$ kN

$B = \sqrt{B_x^2 + B_y^2} = \underline{6.61 \text{ kN}}$

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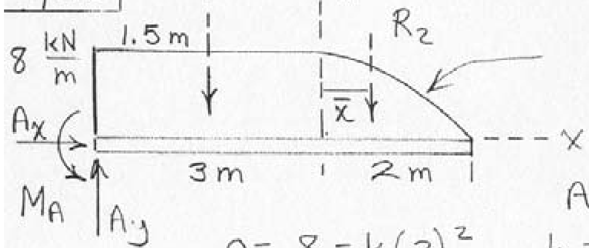


$R_1 = 8(3) = 24$ kN

$w = w_0 - kx^2$

$w_0 = 8$ kN/m

At $x = 2$ m:

$0 = 8 - k(2)^2, \quad k = 2$ kN/m^3

$$R_2 = \int w\,dx = \int_0^2 (8 - 2x^2)\,dx = \left(8x - \frac{2}{3}x^3\right)_0^2$$

$$= 10.67 \text{ kN}$$

$$\bar{x} = \frac{\int x\,w\,dx}{R} = \frac{\int_0^2 (8x - 2x^3)\,dx}{10.67} = \frac{\left(4x^2 - \frac{1}{2}x^4\right)_0^2}{10.67}$$

$$= 0.75 \text{ m}$$

Equilibrium considerations (+ y, x)

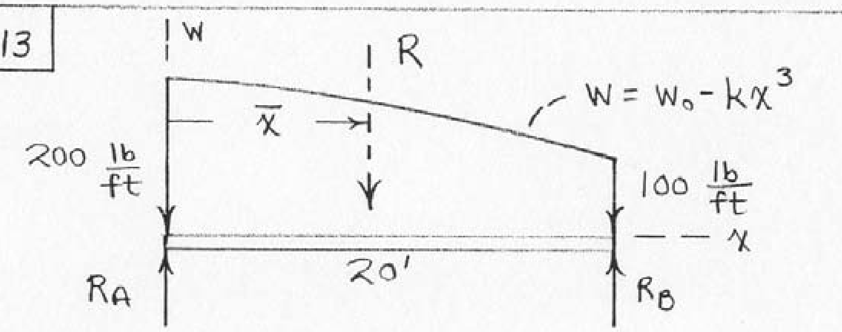
$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0: \quad A_y - 24 - 10.67 = 0, \quad \underline{A_y = 34.7 \text{ kN}}$

$\Sigma M_A = 0: \quad M_A - 24(1.5) - 10.67(3.75) = 0$

$$\underline{M_A = 76 \text{ kN}\cdot\text{m}}$$

5/113



At $x = 0$, $w = w_0 = 200$ lb/ft

At $x = 20'$, $w = 200 - k(20)^3 = 100$, $k = \frac{1}{80}\ \frac{lb}{ft^4}$

So $w = 200 - x^3/80$ (lb/ft)

$$R = \int w\,dx = \int_0^{20}\left(200 - \frac{x^3}{80}\right)dx = \left[200x - \frac{x^4}{320}\right]_0^{20} = 3500 \text{ lb}$$

$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^{20}\left(200 - \frac{x^3}{80}\right)x\,dx}{3500} = \frac{\left[100x^2 - \frac{x^5}{400}\right]_0^{20}}{3500}$$

$$= 9.14'$$

$\circlearrowleft + \sum M_A = 0$: $20R_B - 3500(9.14) = 0$, $\underline{R_B = 1600 \text{ lb}}$

$+\uparrow \sum F = 0$: $R_A + 1600 - 3500 = 0$, $\underline{R_A = 1900 \text{ lb}}$

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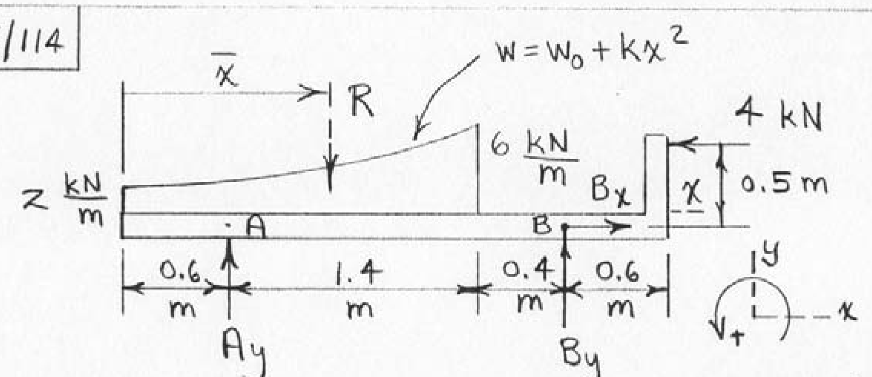

$w = 2 + kx^2$: $6 = 2 + k(2)^2$, $k = 1$ kN/m^3,

$w = 2 + x^2$

$$R = \int w\,dx = \int_0^2 (2 + x^2)\,dx = 2x + \frac{x^3}{3}\Big|_0^2 = 6.67 \text{ kN}$$

$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^2 (2x + x^3)\,dx}{6.67} = \frac{x^2 + \frac{x^4}{4}}{6.67}\Big|_0^2 = 1.2 \text{ m}$$

$\Sigma F_x = 0$: $B_x - 4 = 0$, $\underline{B_x = 4 \text{ kN}}$

$\Sigma M_B = 0$: $4(0.5) + 6.67(2.4 - 1.2) - A_y(1.8) = 0$

$\underline{A_y = 5.56 \text{ kN}}$

$\Sigma F_y = 0$: $5.56 + B_y - 6.67 = 0$, $\underline{B_y = 1.111 \text{ kN}}$

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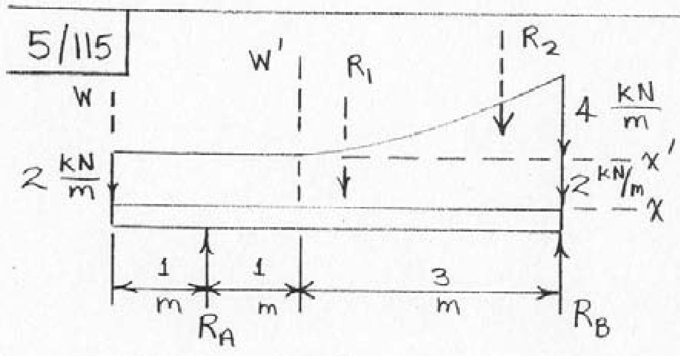


$w' = kx'^2$

At $x' = 3$ m:

$4 = k(3)^2$

$k = \frac{4}{9}$ N/m^3

$w' = \frac{4}{9}x'^2$

$$R_2 = \int_0^3 w'dx' = \int_0^3 \frac{4}{9}x'^2 = \frac{4}{9}\frac{x'^3}{3}\Big|_0^3 = 4 \text{ kN}$$

$$\bar{x}_2' = \int x'w'dx'/R_2 = \frac{1}{4}\int_0^3 \frac{4}{9}x'^3 = \frac{1}{4}\frac{4}{9}\frac{x'^4}{4}\Big|_0^3 = 2.25 \text{ m}$$

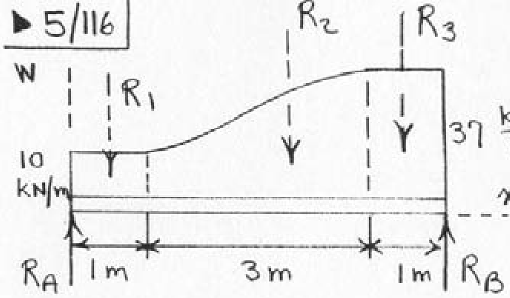
$R_1 = 2(5) = 10$ kN @ $\bar{x}_1 = 2.5$ m | $\bar{x}_2 = 4.25$ m

$\circlearrowleft + \Sigma M_A = 0: -10(1.5) - 4(3.25) + 4R_B = 0$

$\underline{R_B = 7 \text{ kN}}$

$+\uparrow \Sigma F = 0: \; 7 + R_A - 4 - 10 = 0, \quad \underline{R_A = 7 \text{ kN}}$

▶ 5/116



$W = W_0 + k_1 x + k_2 x^2 + k_3 x^3$

$W(1) = 10 = W_0 + k_1 + k_2 + k_3$

$W(4) = 37 = W_0 + k_1(4) + k_2(4)^2 + k_3(4)^3$

$$\frac{dW}{dx} = k_1 + 2k_2 x + 3k_3 x^2 : \begin{cases} 0 = k_1 + 2k_2(1) + 3k_3(1)^2 \\ 0 = k_1 + 2k_2(4) + 3k_3(4)^2 \end{cases}$$

Solve simultaneously to get $W = 21 - 24x + 15x^2 - 2x^3$

$$R_2 = \int W\,dx = \int_1^4 (21 - 24x + 15x^2 - 2x^3)\,dx$$

$$= \left[21x - 12x^2 + 5x^3 - \tfrac{1}{2}x^4\right]_1^4 = 70.5 \text{ kN}$$

$$\bar{x}_2 = \frac{1}{R_2}\int x W\,dx = \frac{1}{70.5}\int_1^4 (21 - 24x + 15x^2 - 2x^3)\,x\,dx$$

$$= \frac{1}{70.5}\left[\frac{21}{2}x^2 - 8x^3 + \frac{15}{4}x^4 - \frac{2}{5}x^5\right]_1^4 = 2.84 \text{ m}$$

$R_1 = 10(1) = 10$ kN @ $\bar{x}_1 = 0.5$ m

$R_3 = 37(1) = 37$ kN @ $\bar{x}_3 = 4.5$ m

$\Sigma M_A = 0: \; 5R_B - 10(0.5) - 70.5(2.84) - 37(4.5) = 0$

$\underline{R_B = 74.4 \text{ kN}}$

$\Sigma F = 0: \; 74.4 - 10 - 70.5 - 37 + R_A = 0, \quad \underline{R_A = 43.1 \text{ kN}}$

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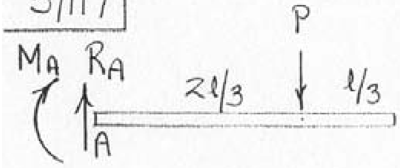
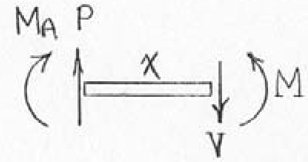


$$+\uparrow \Sigma F = 0 \Rightarrow R_A = P$$

$$\curvearrowleft + \Sigma M_A = 0: \quad -M_A - P\left(\frac{2l}{3}\right) = 0$$

$$M_A = -\frac{2Pl}{3}$$

$0 < x < \frac{2l}{3}$:

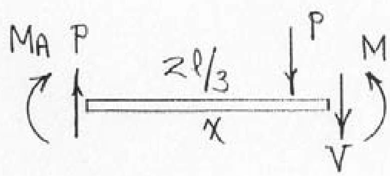


$$+\uparrow \Sigma F = 0 \Rightarrow \quad V = P$$

$$\curvearrowleft + \Sigma M = 0: \quad \frac{2Pl}{3} - Px + M = 0$$

$$M = P\left(x - \frac{2l}{3}\right)$$

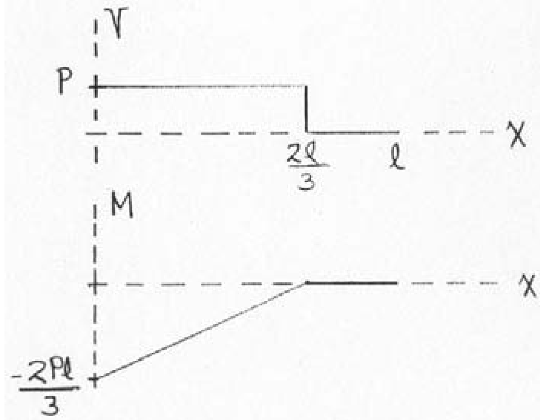
$\frac{2l}{3} < x < l$:



$$+\uparrow \Sigma F = 0: \quad P - P - V = 0, \quad V = 0$$

$$\curvearrowleft + \Sigma M = 0: \quad \frac{2Pl}{3} - P\left(\frac{2l}{3}\right) + M = 0$$

$$M = 0$$



At $x = l/2$:

$$V = P$$

$$M = P\left(\frac{l}{2} - \frac{2l}{3}\right)$$

$$= -\frac{Pl}{6}$$

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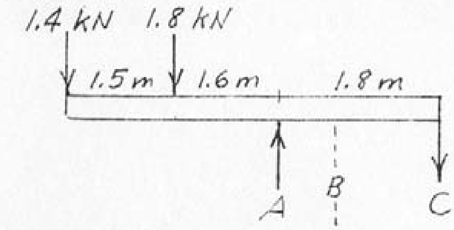


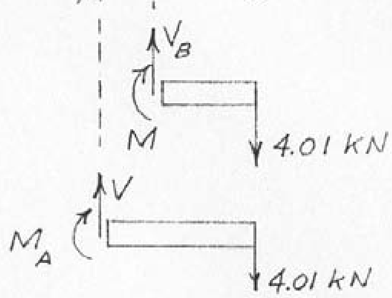
$\Sigma M_A = 0;$

$1.4(3.1) + 1.8(1.6) - 1.8C = 0$

$C = 4.01 \text{ kN}$

$\Sigma F = 0; \ A - 4.01 - 1.4 - 1.8 = 0$

$A = 7.21 \text{ kN}$



$\Sigma F = 0; \ \underline{V_B = 4.01 \text{ kN}}$

$\Sigma M_A = 0; \ M_A + 4.01(1.8) = 0$

$\underline{M_A = -7.22 \text{ kN}\cdot\text{m}}$

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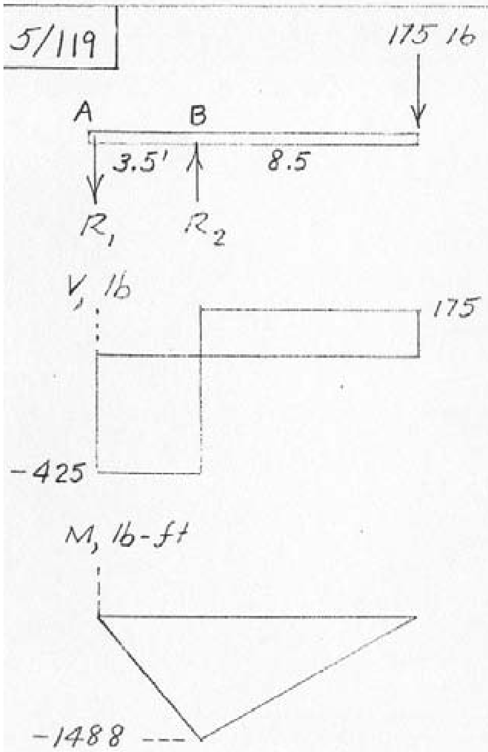

$\Sigma M_{R_1} = 0;\ 12(175) = 3.5 R_2$

$R_2 = 600$ lb

$R_1 = 425$ lb

$M_B = -1488$ lb-ft

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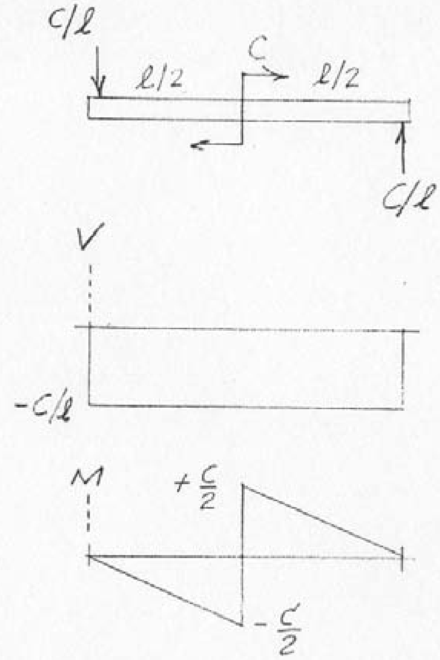

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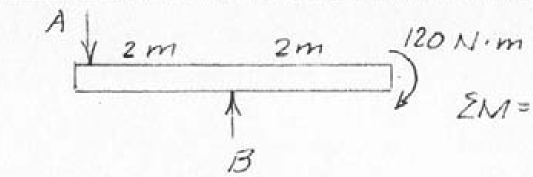
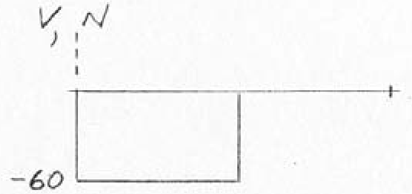
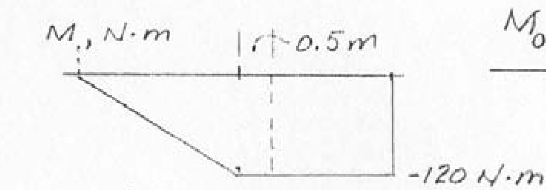


$\Sigma M = 0;\ 2A - 120 = 0$

$A = B = 60\ N$





$M_{0.5\,m} = -120\ N\cdot m$

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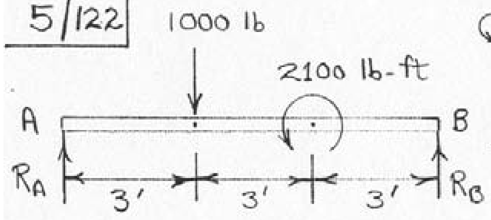
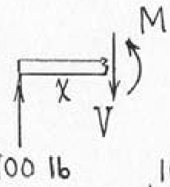


$\circlearrowleft + \Sigma M_A = 0: \; -1000(3) + 2100 + R_B(9) = 0, \quad R_B = 100 \text{ lb}$

$+\uparrow \Sigma F_y = 0: \; R_A - 1000 + 100 = 0$

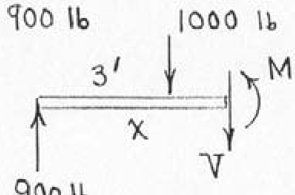
$R_A = 900 \text{ lb}$



$0 < x < 3 \text{ ft}:$

$V = 900 \text{ lb}$

$M = 900x$

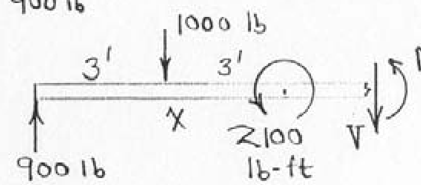


$3 < x < 6 \text{ ft}:$

$+\uparrow \Sigma F = 0: \; 900 - 1000 - V = 0, \quad V = -100 \text{ lb}$

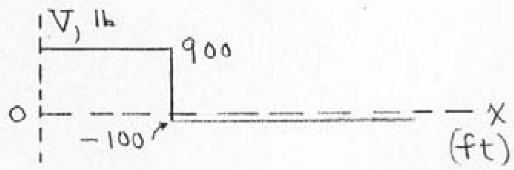
$\circlearrowleft + \Sigma M = 0: \; -900x + 1000(x-3) + M = 0$

$M = 3000 - 100x$



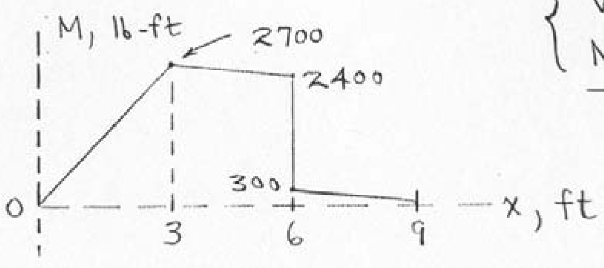
$\Sigma F = 0: \; 900 - 1000 - V = 0, \; V = -100 \text{ lb}$

$\Sigma M = 0: \; -900x + 1000(x-3) + 2100 + M = 0, \quad M = 900 - 100x$



Values at middle:

$\begin{cases} V = -100 \text{ lb} \\ M = 2550 \text{ lb-ft} \end{cases}$



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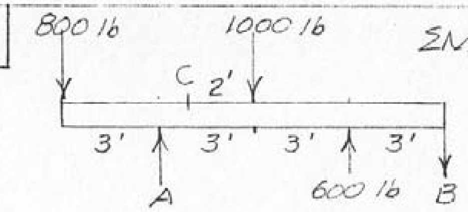
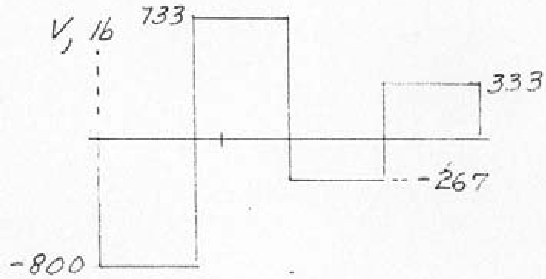
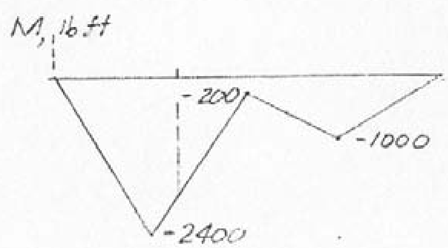


$\Sigma M_A = 0$ gives $B = 333$ lb

$\Sigma M_B = 0$ " $A = 1533$ lb





$$M_C = -\left[2400 - \frac{2400-200}{3}\right]$$

$$= \underline{-1667 \text{ lb-ft}}$$

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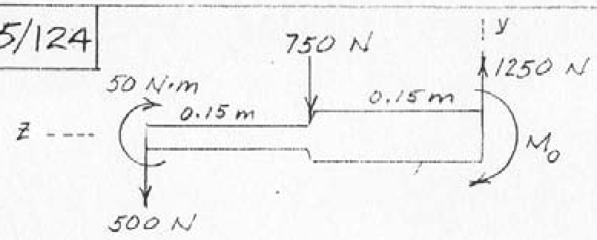
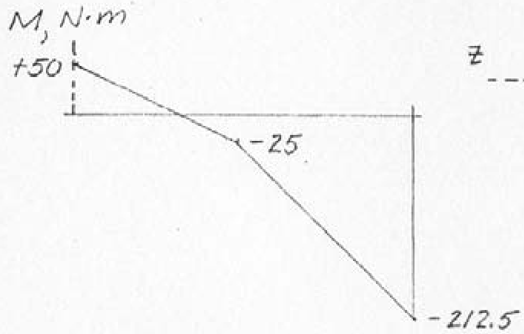
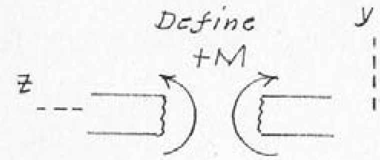


$\Sigma M = 0;\ 50 + M_o - 500(0.30) - 750(0.15) = 0$

$M_o = 212.5\ \text{N·m}$





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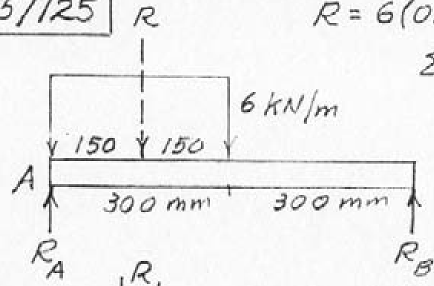
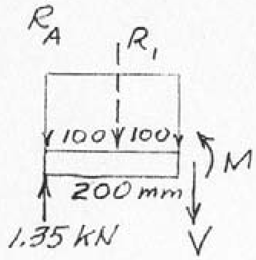


$R = 6(0.3) = 1.8$ kN

$\Sigma M_B = 0;\ 1.8(0.3 + 0.15) - 0.6R_A = 0$

$R_A = 1.35$ kN



$R_1 = 6(0.2) = 1.2$ kN

$\Sigma F = 0;\ 1.35 - 1.2 - V = 0;\ \ \underline{V = 0.15 \text{ kN}}$

$\Sigma M_A = 0;\ M - 1.2(0.1) - 0.15(0.2) = 0$

$\underline{M = 0.15 \text{ kN·m}}$

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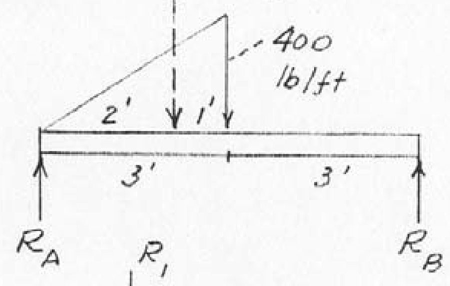
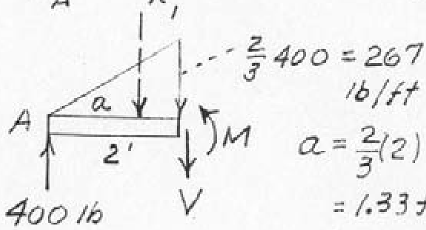
$R = \frac{400}{2}(3) = 600 \text{ lb}$



$\Sigma M_B = 0;\ 600(4) - 6R_A = 0$

$R_A = 400 \text{ lb}$



$R_1 = \frac{267}{2}2 = 267 \text{ lb}$

$\Sigma F = 0;\ 400 - V - 267 = 0$

$\underline{V = 133.3 \text{ lb}}$

$a = \frac{2}{3}(2)$

$= 1.33 \text{ ft}$

$\Sigma M_A = 0;$

$-133.3(2) + M - 267(1.33) = 0$

$\underline{M = 622 \text{ lb-ft}}$

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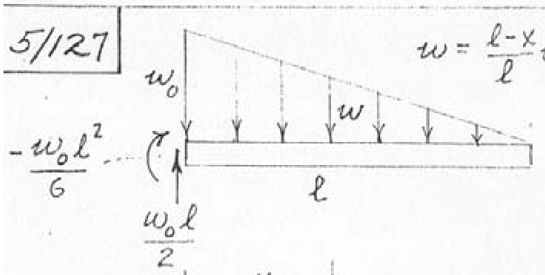
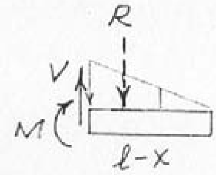


$w = \frac{l-x}{l} w_0$



$R = \frac{w_0}{2}\frac{(l-x)^2}{l}$

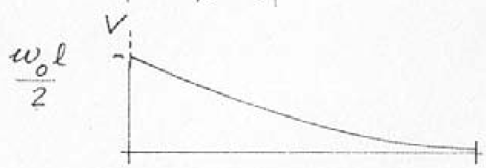
$\Sigma F = 0;\ V = \frac{w_0}{2l}(l-x)^2$

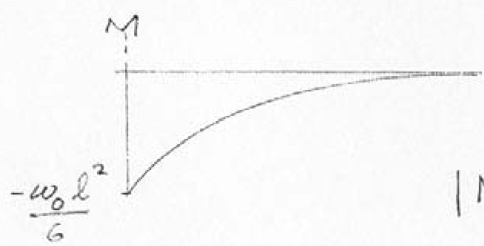
$\Sigma M_V = 0;\ R\frac{l-x}{3} + M = 0$

$M = -\frac{w_0}{2}\frac{(l-x)^2}{l}\frac{l-x}{3}$

$= -\frac{w_0}{6l}(l-x)^3$

$|M|_{max} = \frac{w_0 l^2}{6}$





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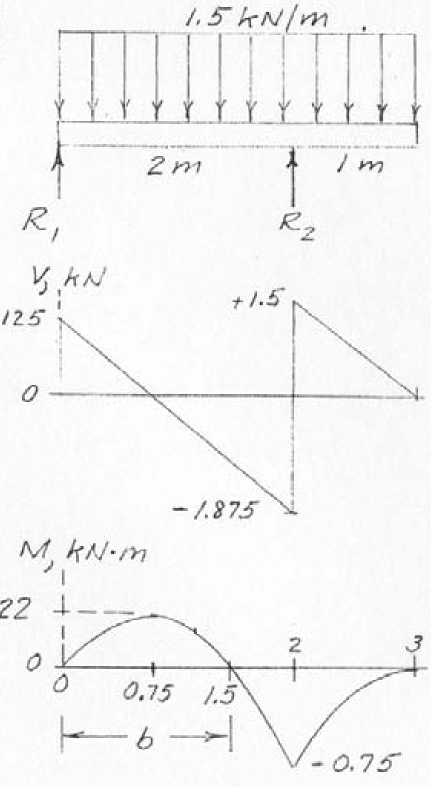


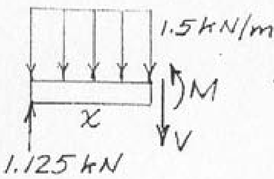
$\Sigma M_{R_1} = 0;$

$1.5(3)(1.5) - 2R_2 = 0$

$R_2 = 3.375 \text{ kN}$

$\Sigma F = 0; \; R_1 + 3.375 - 1.5(3) = 0$

$R_1 = 1.125 \text{ kN}$



$x < 2\text{m}$

$\Sigma F = 0; \; V + 1.5x - 1.125 = 0$

$V = 1.125 - 1.5x$

$\Sigma M_V = 0; \; M + 1.5\dfrac{x^2}{2} - 1.125x = 0$

$M = 1.125x - 0.75x^2$

$b = 1.5 \text{ m}$

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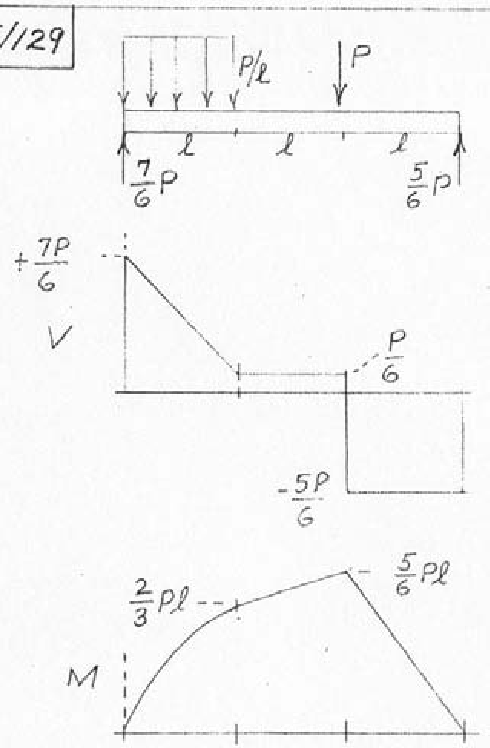

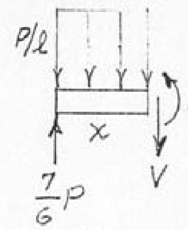

$x < \ell$

$\Sigma F = 0;\ -V - \frac{P}{\ell}x + \frac{7}{6}P = 0$

$V = P\left(\frac{7}{6} - \frac{x}{\ell}\right)$

$\Sigma M_V = 0;\ M + \frac{P}{\ell}x\frac{x}{2} - \frac{7}{6}Px = 0$

$M = \frac{Px}{2}\left(\frac{7}{3} - \frac{x}{\ell}\right)$

$M_{max} = \frac{5}{6}P\ell$

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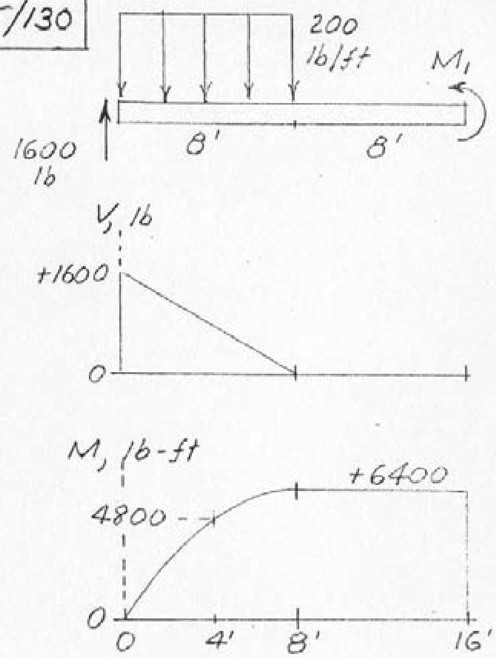
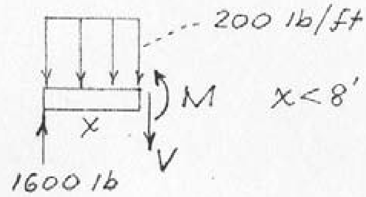


For zero moment on left end,

$M_1 = 200(8)4 = 6400$ lb-ft



$\Sigma F = 0;\ V + 200x - 1600 = 0$

$V = 1600 - 200x$

$\Sigma M_V = 0;\ M + 200\frac{x^2}{2} - 1600x = 0$

$M = 1600x - 100x^2$

$= 100x(16 - x)$

At $x = 4;\ M = 400(12)$

$= \underline{4800 \text{ lb-ft}}$

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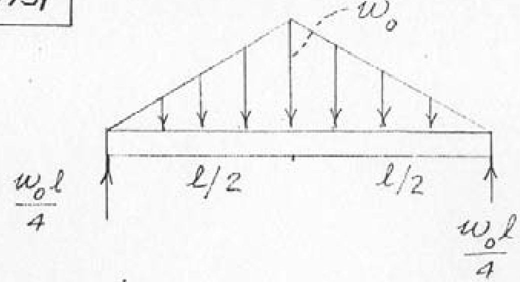
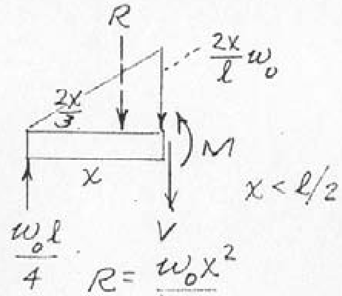
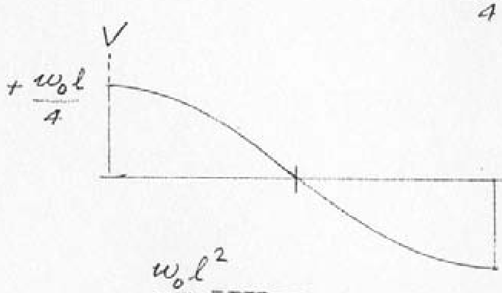




$$R = \frac{w_0 x^2}{\ell}$$

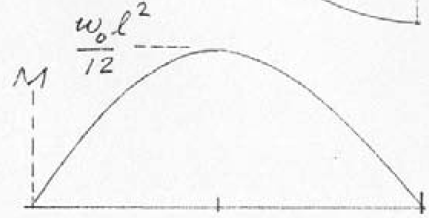


$$\Sigma F = 0;\ V + \frac{w_0 x^2}{\ell} - \frac{w_0 \ell}{4} = 0$$

$$V = w_0 \ell\left(\frac{1}{4} - \frac{x^2}{\ell^2}\right)$$

$$\Sigma M_V = 0;\ M + R\frac{x}{3} - \frac{w_0 \ell}{4} x = 0$$

$$M = w_0 x \ell\left(\frac{1}{4} - \frac{x^2}{3\ell^2}\right)$$



$$\underline{M_{max} = \frac{w_0 \ell^2}{12}}$$

$\boxed{5/132}$ $w = -\frac{dV}{dx} = -\frac{d}{dx}(2200x - 40x^3) = \underline{-2200 + 120x^2}$

$dM = V\,dx;\quad \int_{-1600}^{M} dM = \int_{1}^{3} (2200x - 40x^3)\,dx$

$$M - (-1600) = 2200\,\frac{3^2 - 1^2}{2} - 10(3^4 - 1^4) = 8000$$

$$M = 8000 - 1600 = \underline{6400 \text{ lb-ft}}$$

$\boxed{5/133}$ $M = ky^2$ & $M = Lx$, so $Lx = ky^2$

$Ll = kh^2$

Thus $k = \frac{Ll}{h^2}$ so $y^2 = \frac{Lx}{Ll/h^2}$ so $\underline{y = h\sqrt{x/l}}$

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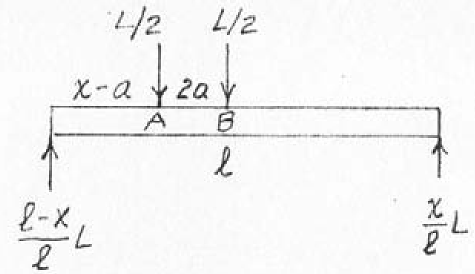

$$M_A = \frac{\ell - x}{\ell} L\,(x-a) = \frac{L}{\ell}\left(-x^2 + [a+\ell]x - a\ell\right)$$

$$\frac{dM_A}{dx} = \frac{L}{\ell}(-2x + a + \ell) = 0 \text{ for max. } M_A\,; \quad \underline{x = \frac{a+\ell}{2}}$$

$$\& \ \underline{M_{A_{max}} = \frac{L}{4\ell}(\ell - a)^2}\ ; \quad M_{B_{x=\frac{a+\ell}{2}}} = \frac{L}{4\ell}(\ell^2 - 2a\ell - 3a^2) < M_A$$

(By symmetry, a second and equal maximum bending moment occurs at $x = \frac{\ell - a}{2}$.)

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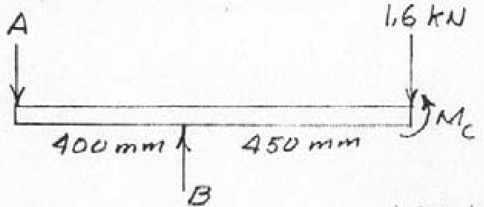


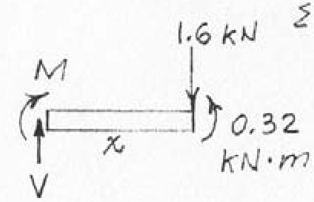
$M_C = 1.6(0.200) = 0.32$ kN·m

$\Sigma M_A = 0$; $0.4B + 0.32 - 0.85(1.6) = 0$

$B = 2.6$ kN

$\Sigma F = 0$; $A + 1.6 - 2.6 = 0$

$A = 1.0$ kN



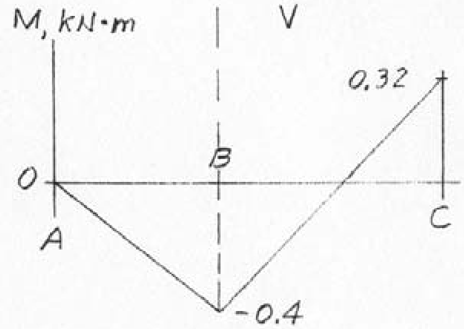
$\Sigma M = 0$

$0.32 - 1.6x - M = 0$

$M = 0.32 - 1.6x$

at B, $x = 0.45$ m

$M_B = -0.40$ kN·m



$M = 0$ when

$x = 0.32/1.6 = 0.2$ m

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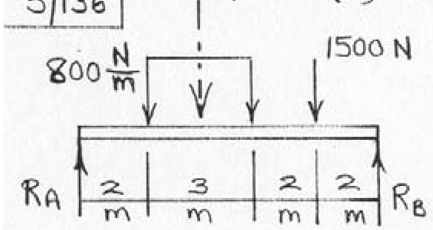
$R = 800(3) = 2400\ \text{N}$



$\sum M_A = 0:\ R_B(9) - 1500(7) - 2400(3.5) = 0,\quad R_B = 2100\ \text{N}$

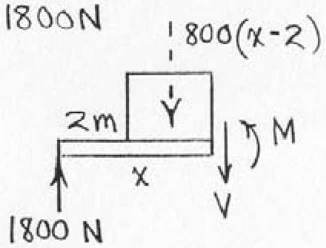
$\sum F = 0:\ R_A - 2400 - 1500 + 2100 = 0$

$R_A = 1800\ \text{N}$

$0 < x < 2\ \text{m}:$

$\sum F = 0 \Rightarrow V = 1800\ \text{N}$

$\sum M = 0 \Rightarrow M = 1800x$



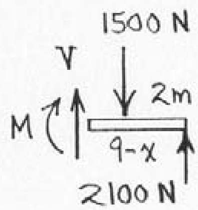
$2 < x < 5\ \text{m}:$

$\sum F = 0:\ 1800 - 800(x-2) - V = 0$

$V = 3400 - 800x$

$\sum M = 0:\ M + 800(x-2)\frac{x-2}{2} - 1800x = 0$

$M = -400x^2 + 3400x - 1600$

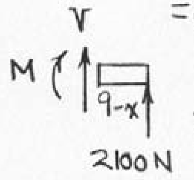


$5 < x < 7:$

$\sum F = 0:\ 2100 - 1500 + V = 0$

$V = -600\ \text{N}$

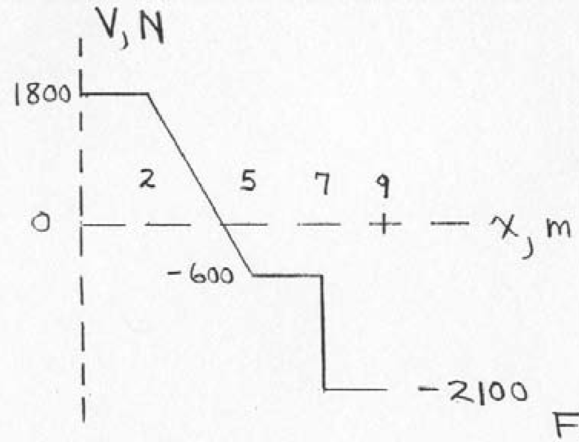
$\sum M = 0:\ -M - 1500(7-x) + 2100(9-x) = 0,\quad M = 8400 - 600x$



$7 < x < 9\ \text{m}:$

$\sum F = 0 \Rightarrow V = -2100\ \text{N}$

$\sum M = 0 \Rightarrow M = 18\,900 - 2100x$



At $x = 6$ m:

$V = \underline{-600 \text{ N}}$

$M = 8400 - 600(6)$

$= \underline{4800 \text{ N·m}}$

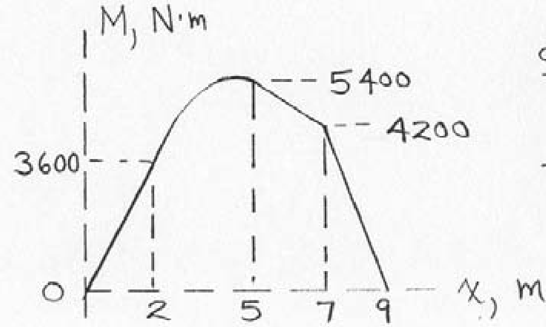
For M_{max},

$$\frac{dM}{dx} = 0$$

$$\frac{d}{dx}\left(-400x^2 + 3400x - 1600\right)$$

$$= -800x + 3400 = 0$$

$$\underline{x = 4.25 \text{ m}}$$



$$M_{max} = -400(4.25)^2 + 3400(4.25) - 1600 = \underline{5620 \text{ N·m}}$$

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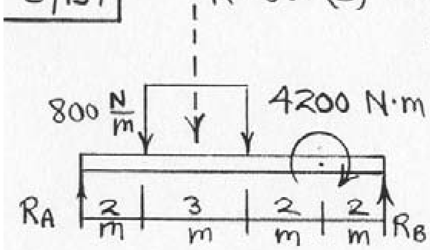
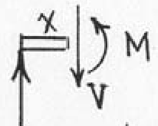
$R = 800(3) = 2400 \text{ N}$



$\Sigma M_A = 0: \; R_B(9) - 2400(3.5) - 4200 = 0, \; R_B = 1400 \text{ N}$

$\Sigma F = 0 \Rightarrow \; R_A = 1000 \text{ N}$

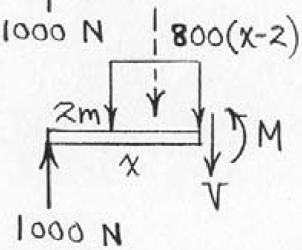
$0 < x < 2 \text{ m}:$



$\Sigma F = 0 \Rightarrow \; V = 1000 \text{ N}$

$\Sigma M = 0 \Rightarrow \; M = 1000x$

$2 < x < 5 \text{ m}:$

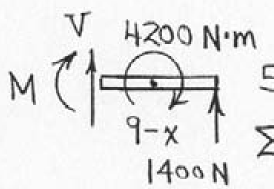


$\Sigma F = 0: \; 1000 - 800(x-2) - V = 0$

$V = 2600 - 800x$

$\Sigma M = 0: \; M + 800(x-2)\frac{x-2}{2} - 1000x = 0$

$M = -400x^2 + 2600x - 1600$

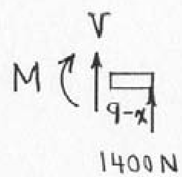


$5 < x < 7 \text{ m}:$

$\Sigma F = 0: \; V + 1400 = 0, \; V = -1400 \text{ N}$

$\Sigma M = 0: \; -M - 4200 + 1400(9-x) = 0$

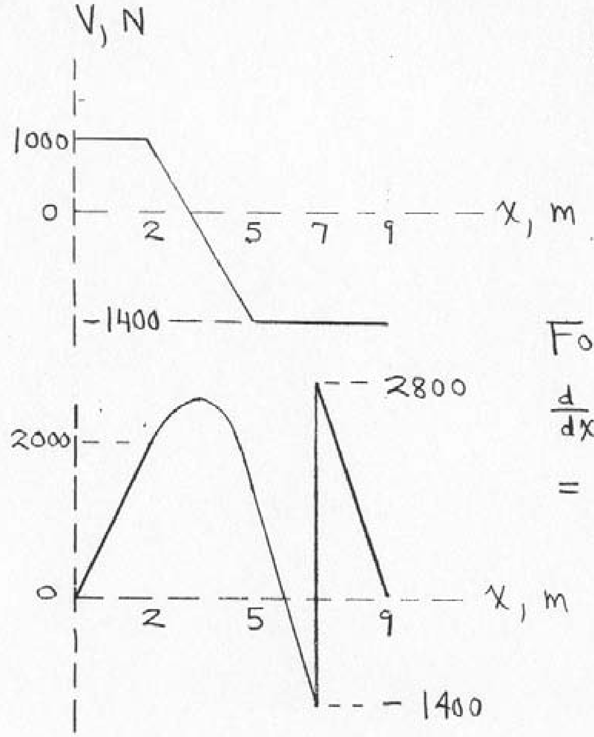
$M = 8400 - 1400x$



$7 < x < 9 \text{ m}:$

$\Sigma F = 0 \Rightarrow \; V = -1400 \text{ N}$

$\Sigma M = 0 \Rightarrow \; M = 12600 - 1400x$



At $x = 6$ m,

$V = -1400$ N

$M = 8400 - 1400(6)$

$= 0$

For M_{max}, $\frac{dM}{dx} = 0$

$\frac{d}{dx}\left(-400x^2 + 2600x - 1600\right)$

$= -800x + 2600 = 0$

$x = 3.25$ m

$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600 = 2625$ N·m

So $M_{max} = 2800$ N·m

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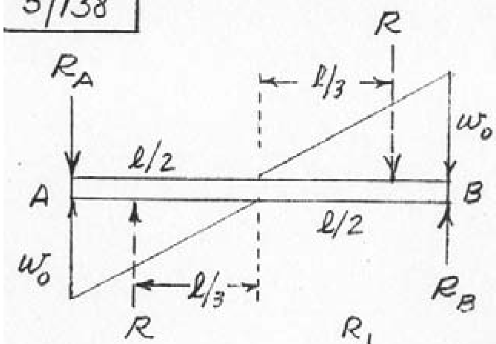
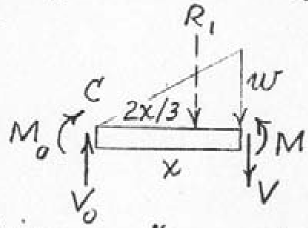


$R = \frac{w_o}{2}\frac{\ell}{2} = w_o\ell/4$

By inspection of the two couples, $R_A = R_B$

$\Sigma M = 0;\ R_A \ell - \frac{w_o \ell}{4}\frac{2\ell}{3} = 0$

$R_A = R_B = \frac{w_o \ell}{6}$



For complete right half of beam $\Sigma F = 0$ gives

$V_o = R - R_B = w_o \ell/12$

$M_o = 0$

For section of length x, $w = \frac{x}{\ell/2} w_o$ & $R_1 = \frac{w}{2} x = w_o x^2/\ell$

$\Sigma F = 0;\ V + w_o x^2/\ell - w_o \ell/12 = 0,\quad \underline{V = w_o \ell\left(\frac{1}{12} - \frac{x^2}{\ell^2}\right)}$

$\Sigma M_C = 0;\ M - \frac{w_o x^2}{\ell}\frac{2x}{3} - w_o \ell\left(\frac{1}{12} - \frac{x^2}{\ell^2}\right)x = 0,\quad \underline{M = \frac{w_o \ell x}{3}\left(\frac{1}{4} - \frac{x^2}{\ell^2}\right)}$

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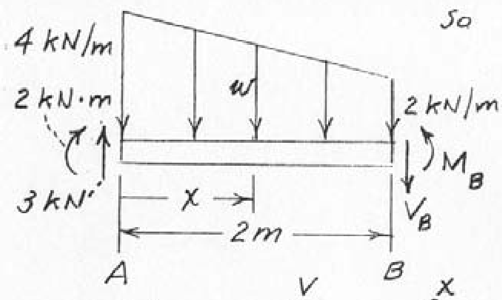
$w = 4 - kx$ where $k = \frac{4-2}{2} = 1 \text{ kN/m}^2$

So $w = 4 - x$ kN/m



$w = -dV/dx$; $\int_3^V dV = -\int_0^x (4-x)\,dx$, $V = 3 - \left[4x - \frac{x^2}{2}\right]_0^x$

$= 3 - 4x + x^2/2$

when $x = 2$ m, $V = \underline{V_B = -3 \text{ kN}}$

$V = \frac{dM}{dx}$; $\int_2^{M_B} dM = \int_0^2 (3 - 4x + \frac{x^2}{2})\,dx$

$M_B - 2 = \left[3x - 2x^2 + \frac{x^3}{6}\right]_0^2 = -\frac{2}{3}$

$M_B = 2 - \frac{2}{3} = \underline{4/3 \text{ kN·m}}$

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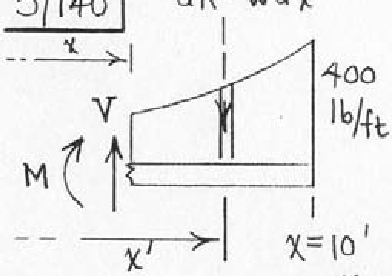

From $w = w_0 + kx^2 = 100 + kx^2$:

$400 = 100 + k(10)^2$

$k = 3 \text{ lb/ft}^3, \quad w = 100 + 3x^2$

$+\uparrow \Sigma F = 0: \quad V - \int_x^{10} w\,dx' = 0$

$V = \int_x^{10} (100 + 3x'^2)\,dx' = 100x' + x'^3 \Big|_x^{10}$

$\underline{V = 2000 - 100x - x^3}$ (in lb if x is in ft)

$\circlearrowleft \Sigma M = 0: \quad -M - \int_x^{10} (x' - x)\,w\,dx' = 0$

$M = -\int_x^{10} (-100x + 100x' + 3x'^3 - 3xx'^2)\,dx'$

$= -\left[-100xx' + 50x'^2 - xx'^3 + \frac{3}{4}x'^4\right]\Big|_x^{10}$

$= \underline{-12{,}500 + 2000x - 50x^2 - \frac{1}{4}x^4}$

(in lb-ft if x is in ft)

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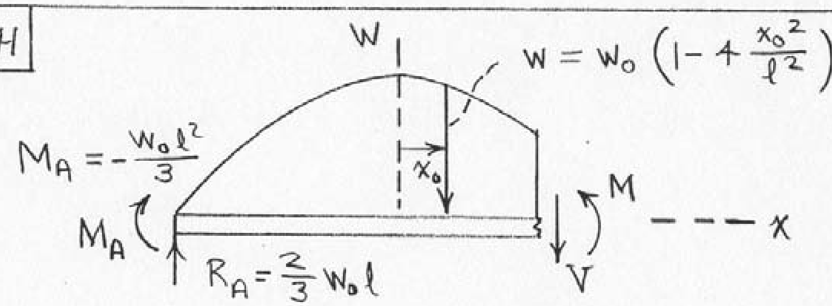


(R_A and M_A from Prob. 5/104)

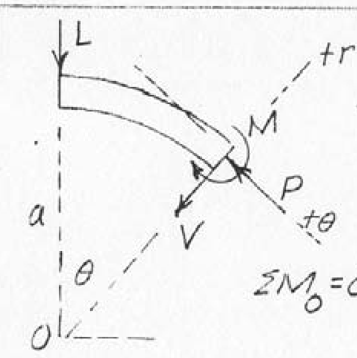
$$+\uparrow \Sigma F = 0: \ \frac{2}{3} w_0 \ell - \int_{-\ell/2}^{x} w_0 \left(1 - 4\frac{x_0^2}{\ell^2}\right) dx_0 - V = 0$$

$$\underline{V = w_0\left(\frac{\ell}{3} - x + \frac{4x^3}{3\ell^2}\right)}$$

$$\circlearrowleft + \Sigma M = 0: \ M + \int_{-\ell/2}^{x} w_0\left(1 - 4\frac{x_0^2}{\ell^2}\right)(x - x_0)\, dx_0$$

$$+ \frac{w_0 \ell^2}{3} - \frac{2}{3} w_0 \ell \left(\frac{\ell}{2} + x\right) = 0, \quad \underline{M = w_0\left(-\frac{\ell^2}{16} + \frac{x\ell}{3} - \frac{x^2}{2} + \frac{x^4}{3\ell^2}\right)}$$

5/142



$\Sigma F_\theta = 0;\ -P + L\sin\theta = 0$

$P = L\sin\theta$

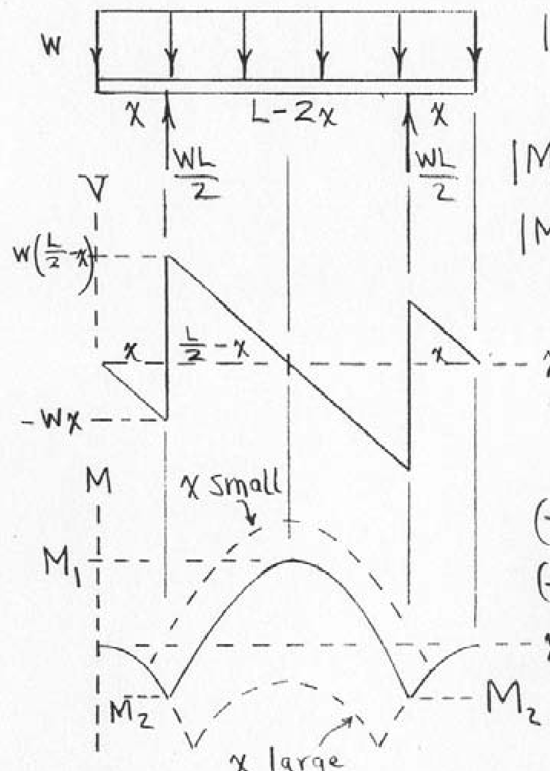
$\Sigma F_r = 0;\ -V - L\cos\theta = 0$

$\underline{V = -L\cos\theta}$

$\Sigma M_O = 0;\ Pa - M = 0$

$M = Pa = \underline{La\sin\theta}$

▶5/143 From areas under shear diagram:



$$|M_2| = \frac{1}{2}wx^2$$

$$|M_1| = \frac{w}{2}\left(\frac{L}{2}-x\right)^2 - \frac{1}{2}wx^2$$

$$= \frac{wL}{2}\left(\frac{L}{4}-x\right)$$

$|M|_{min}$ occurs when $|M_1| = |M_2|$. So

$$\frac{wL}{2}\left(\frac{L}{4}-x\right) = \frac{1}{2}wx^2$$

or $x^2 + Lx - \frac{L^2}{4} = 0$

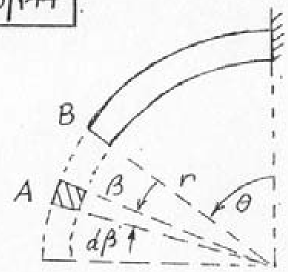
$$\Rightarrow x = \frac{L}{2}\left(-1 \pm \sqrt{2}\right)$$

(−) sign : irrelevant

(+) sign : $x = 0.207L$

$$M_{max} = |M_1| = \frac{1}{2}w\left(0.207L\right)^2 = 0.0214wL^2$$

▶5/144



On element A, $dF = wr\,d\beta$

Torsion about B due to dF is

$$dT = dF(r - r\cos\beta) = wr^2(1-\cos\beta)\,d\beta$$

$$T = wr^2\int_0^{\pi/2-\theta}(1-\cos\beta)\,d\beta$$

$$= wr^2\Big[\beta - \sin\beta\Big]_0^{\frac{\pi}{2}-\theta}$$

$$= wr^2\left[\frac{\pi}{2} - \theta - \sin\left(\frac{\pi}{2}-\theta\right)\right]$$

$$= \underline{wr^2\left[\frac{\pi}{2} - \theta - \cos\theta\right]}$$

Bending moment about B due to dF is

$$dM = dF\,r\sin\beta = wr^2\sin\beta\,d\beta$$

$$M = wr^2\int_0^{\pi/2-\theta}\sin\beta\,d\beta = -wr^2\cos\beta\Big]_0^{\pi/2-\theta}$$

$$= -wr^2\left[\sin\theta - 1\right] = \underline{wr^2(1-\sin\theta)}$$

5/145 Given: $\begin{cases} 2s = 100 \text{ ft}, \quad s = 50 \text{ ft} \\ \mu = 0.00624 \text{ lb/ft} \\ T = 10 \text{ lb} \end{cases}$

$T^2 = \mu^2 s^2 + T_0^2$: $\quad 10^2 = (0.00624 \cdot 50)^2 + T_0^2$

$$T_0 = 9.995 \text{ lb}$$

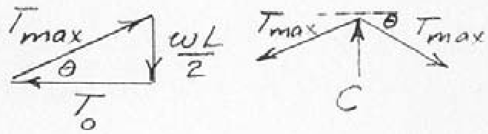
(Eq. 5/22) $T = T_0 + \mu y$: $10 = 9.995 + 0.00624 h$

$$h = 0.780 \text{ ft} \quad \text{or} \quad \underline{h = 9.36 \text{ in.}}$$

5/146 $L = 4200$ ft, $h = 470$ ft, $w = \frac{21{,}300}{2} = 10650$ lb/ft

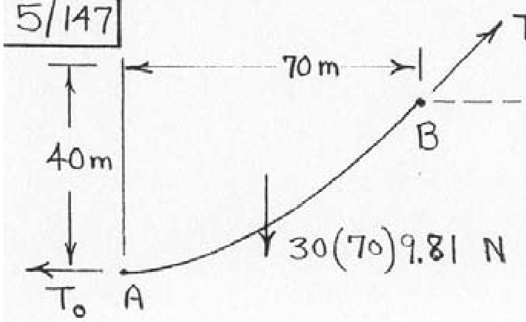
for each cable

$$T_o = \frac{wL^2}{8h} = \frac{10650(4200)^2}{8(470)} = \underline{50.0(10^6) \text{ lb}}$$



$$C = 2T_{max}\sin\theta$$
$$= 2\left(\frac{wL}{2}\right) = wL$$
$$= 10\,650(4200)$$
$$= \underline{44.7(10^6) \text{ lb}}$$

5/147



$$W = 30(9.81) = 294 \ \frac{N}{m}$$

Eq. 5/14: $y = \dfrac{wx^2}{2T_0}$

$$T_0 = \frac{wx^2}{2y} = \frac{294\,(70)^2}{2(40)} = 18.03(10^3)\text{ N or } \underline{18.03 \text{ kN}}$$

$$\tan\theta = \frac{dy}{dx} = \frac{wx}{T_0}.$$

At B, $\theta = \tan^{-1}\left(\dfrac{294(70)}{18\,030}\right) = \underline{48.8°}$

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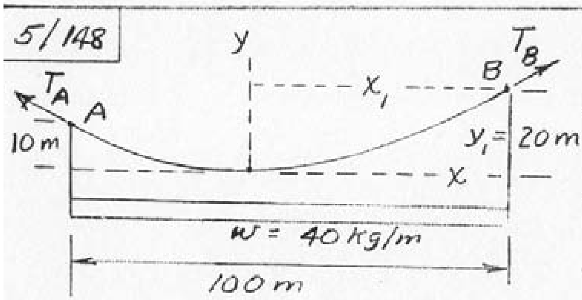

Eq. 5/14 $y = \dfrac{wx^2}{2T_o}$

A: $10 = \dfrac{(40)(9.81)(100-x_1)^2}{2T_o}$

B: $20 = \dfrac{(40)(9.81)x_1^2}{2T_o}$

Combine & get $x_1^2 = 2(100-x_1)^2$, $x_1^2 - 400x_1 + 20000 = 0$

$x_1 = \dfrac{400}{2} \pm \dfrac{1}{2}\sqrt{(16-8)10^4} = 100(2 \pm \sqrt{2}) = 58.6\text{ m (or 341 m)}$

At B; $T_o = \dfrac{40(9.81)(58.6)^2}{2(20)} = 33660\text{ N}$ or $\underline{T_o = 33.7\text{ kN}}$

Also, $T^2 = T_o^2 + w^2x^2$ so

for B, $T_B = \sqrt{(33.7)^2 + [(40)(9.81)(10^{-3})]^2(58.6)^2} = \underline{40.8\text{ kN}}$

for A, $T_A = \sqrt{(33.7)^2 + [(40)(9.81)(10^{-3})]^2(100-58.6)^2} = \underline{37.4\text{ kN}}$

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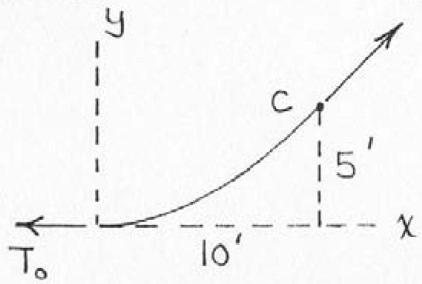

$\mu = 15$ lb/ft

Eq. 5/19 evaluated at point C:

$$5 = \frac{T_0}{\mu}\left[\cosh \frac{10}{T_0/\mu} - 1\right]$$

Numerical solution: $\frac{T_0}{\mu} = 10.74$ ft

Then $T_0 = 10.74(15) = 161.1$ lb

Eq. 5/22 evaluated at C:

$T_c = 161.1 + 15(5) = \underline{236 \text{ lb}}$

Eq. 5/20 : $s = 10.74 \sinh \frac{10}{10.74} = 11.51$ ft

$L = 2s = \underline{23.0 \text{ ft}}$

$\boxed{5/150}$ Eq. 5/22, $T_B = T_0 + \mu y_B$, $T_A = T_0 + \mu y_A$

so $T_B - T_A = \mu(y_B - y_A)$ or $T_B - T_A = \mu h$

Thus $h = \frac{1}{\mu}(T_B - T_A) = \frac{1}{0.12(9.81)}(230 - 110) = \underline{101.9 \text{ m}}$

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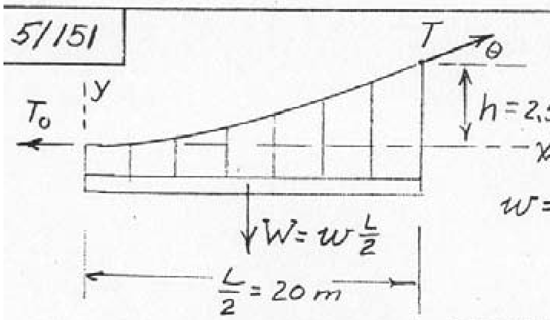
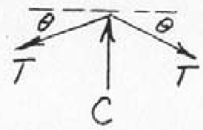




$w = 1.400(9.81) = 13.73 \text{ kN/m}$

From Eq. 5/15b, $T = \frac{wL}{2}\sqrt{1 + (L/4h)^2}$

$$= \frac{13.73(40)}{2}\sqrt{1 + \left[\frac{40}{4(2.5)}\right]^2} = 1133 \text{ kN}$$

$T^2 = W^2 + T_0^2$, $T_0 = \sqrt{(1132)^2 - [(13.73)(20)]^2} = 1099 \text{ kN}$

$\frac{dy}{dx} = \tan\theta = \frac{w(L/2)}{T_0} = \frac{13.73(20)}{1099} = 0.250$, $\theta = 14.04°$

$\Sigma F_y = 0$ at support; $2T\sin\theta - C = 0$

$$C = 2(1133)\sin 14.04° = \underline{549 \text{ kN}}$$

$\boxed{5/152}$ Eq. 5/15 $T = w\sqrt{x^2 + (L^2/8h)^2}$

so $\Delta T = \Delta w\sqrt{x^2 + (L^2/8h)^2}$

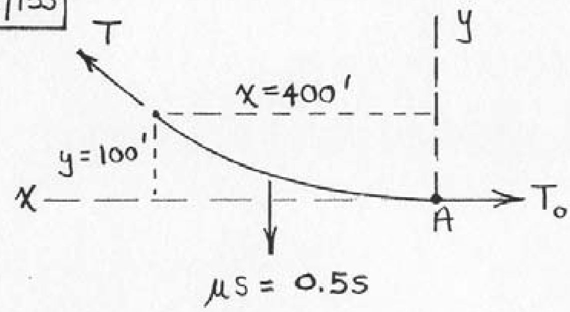
For each cable $480{,}000 = \Delta w\sqrt{(600)^2 + \left(\frac{[2500]^2}{8[500]}\right)^2}$

$= 1674\,\Delta w$

$\Delta w = 286.8$ lb/ft per cable

$w' = 2\Delta w = 2(286.8) = \underline{574 \text{ lb/ft}}$ for both cables

*5/153



Catenary Eq. 5/19 : $y = \frac{T_o}{\mu}\left(\cosh\frac{\mu x}{T_o} - 1\right)$

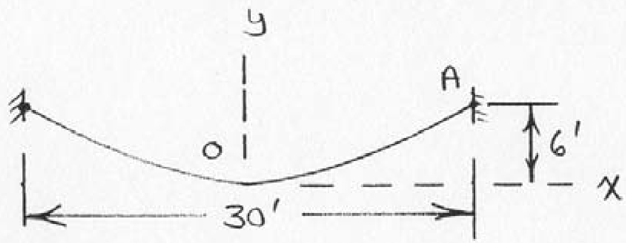
$$100 = \frac{T_o}{0.5}\left(\cosh\frac{0.5(400)}{T_o} - 1\right)$$

Solve numerically to obtain $\underline{T_o = 408 \text{ lb}}$

Parabolic Eq. 5/14 : $y = \frac{wx^2}{2T_o} \cong \frac{\mu x^2}{2T_o}$

So $T_o \cong \frac{\mu x^2}{2y} = \frac{0.5(400)^2}{2(100)} = \underline{400 \text{ lb}}$

*5/154



Eq. 5/19, from O to A : $6 = \frac{T_0}{\mu}\left[\cosh \frac{\mu}{T_0}(15) - 1\right]$

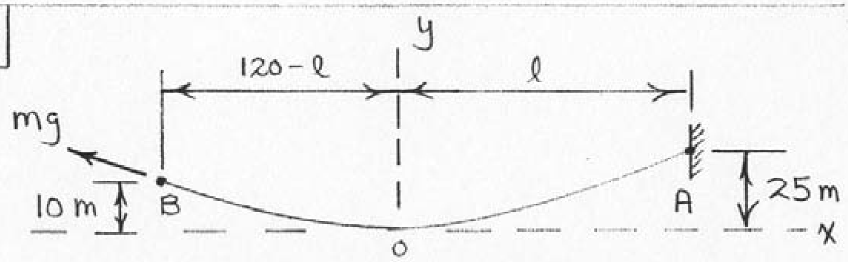
By numerical or graphical means, $\frac{T_0}{\mu} = 19.68\text{m}$

Eq. 5/20: $s = \frac{L}{2} = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$\therefore\ L = 2\left(\frac{T_0}{\mu}\right) \sinh \frac{\mu}{T_0} 15$

$= 2(19.68) \sinh \frac{15}{19.68} = \underline{33.0 \text{ ft}}$

5/155



Eq. 5/14 : $y = \frac{wx^2}{2T_0}$

At A: $25 = \frac{40\ell^2}{2T_0}$, At B: $10 = \frac{40(120-\ell)^2}{2T_0}$

Eliminate T_0 : $0.6\ell^2 - 240\ell + 14\,400 = 0$

$\ell = 73.5$ m (or $\ell = 326$ m)

$T_0 = \frac{4}{5}\ell^2 = \frac{4}{5}(73.5)^2 = 4320$ N

Section O-B : $(mg)^2 = T_0^2 + (wx)^2$

$(9.81m)^2 = (4320)^2 + [40(120-73.5)]^2$

$\underline{m = 480 \text{ kg}}$

*5/156 Please refer to the diagram in the solution to Prob. 5/155. Eq. 5/19:

$$y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$$

$$\left.\begin{aligned} &\text{At } B:\ 10 = \frac{T_0}{40}\left[\cosh\frac{40(120-\ell)}{T_0} - 1\right] \\ &\text{At } A:\ 25 = \frac{T_0}{40}\left[\cosh\frac{40\ell}{T_0} - 1\right] \end{aligned}\right\}$$

Numerical solution: $\begin{cases} T_0 = 4440 \text{ N} \\ \ell = 73.2 \text{ m} \end{cases}$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

At B: $s_B = \frac{4440}{40}\sinh\frac{40(120-73.2)}{4440} = 48.2 \text{ m}$

Equilibrium of section OB:

$(mg)^2 = T_0^2 + (\mu s_B)^2$: $m^2(9.81)^2 = 4440^2 + (40 \cdot 48.2)^2$

$\underline{m = 494 \text{ kg}}$

$\boxed{5/157}$ Eq. 5/19 is $y = \frac{T_0}{\mu}\left(\cosh \frac{\mu x}{T_0} - 1\right)$

So $H - 2 = \frac{300(10^3)}{22(9.81)}\left(\cosh \frac{22(9.81)(250)}{300(10^3)} - 1\right)$

$= 1390\,(\cosh 0.1798 - 1) = 1390\,(1.0162 - 1)$

$= 22.5$ m

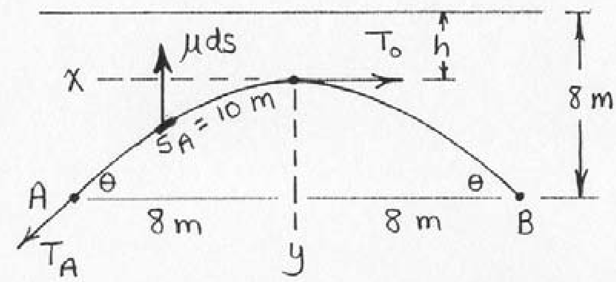
Thus $\underline{H = 24.5 \text{ m}}$

Eq. 5/20, $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} = \frac{300(10^3)}{22(9.81)} \sinh \frac{22(9.81)250}{300(10^3)}$

$= 1390\,(0.1808) = \underline{251 \text{ m}}$

*5/158

$\mu = 560 - 100$
$= 460$ N/m



Eq. 5/20: $s = \frac{T_o}{\mu}\sinh\frac{\mu x}{T_o}$: $10 = \frac{T_o}{460}\sinh\frac{460(8)}{T_o}$

Numerical or graphical solution: $T_o = 3110$ N

Eq. 5/21 : $T = T_o\cosh\frac{\mu x}{T_o} = 3110\cosh\frac{460(8)}{3110}$

$T_A = 5550$ N

Eq. 5/22 : $T = T_o + \mu y$ at A: $5550 = 3110 + 460y$

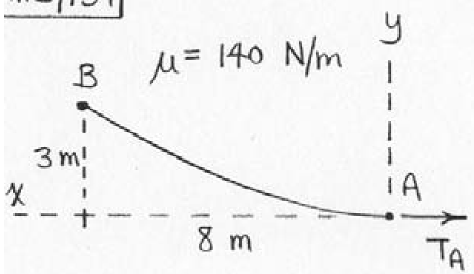
$y = 5.31$ m

Then $h = 8 - 5.31 = \underline{2.69 \text{ m}}$

From Eq. 5/19, $\frac{dy}{dx} = \tan\theta = \sinh\frac{\mu x}{T_o}$

$\tan\theta = \sinh\frac{460(8)}{3110} = \underline{55.9°}$

*5/159



Eq. 5/19: $y = \frac{T_o}{\mu}\left[\cosh \frac{\mu x}{T_o} - 1\right]$

At B: $3 = \frac{T_A}{140}\left[\cosh \frac{140(8)}{T_A} - 1\right]$

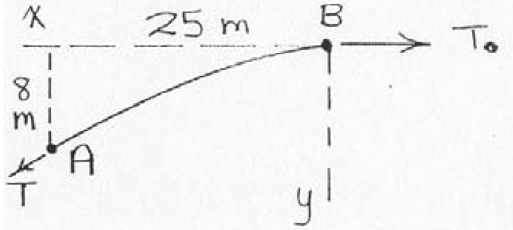
Numerical solution: $\underline{T_A = 1559 \text{ N}}$

Eq. 5/20: $s = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$

At B: $L = \frac{1559}{140} \sinh \frac{140(8)}{1559} = \underline{8.71 \text{ m}}$

*5/160

$\mu = 30$ N/m



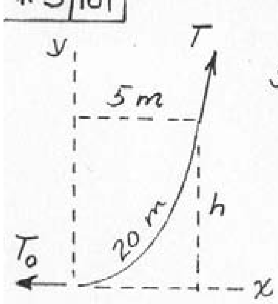
Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$

At A: $8 = \frac{T_0}{\mu}\left[\cosh\frac{25\mu}{T_0} - 1\right]$

Numerical solution: $\frac{T_0}{\mu} = 40.3$ m

$T_0 = 40.3(30) = \underline{1210 \text{ N}}$

*5/161



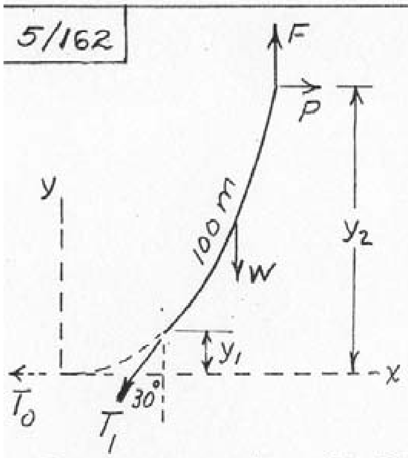
Eq. 5/20, $20 = \frac{T_0}{\mu} \sinh \frac{5\mu}{T_0}$.

Solve by computer or graphically & get $T_0/\mu = 1.532$ m

Eq. 5/19, $y = 1.532\left(\cosh \frac{5}{1.532} - 1\right)$

$h = y = 1.532\,(13.09 - 1) = \underline{18.53 \text{ m}}$

5/162



$W = 0.51(9.81)(100) = 500.3 \text{ N}$

$T_1 = M/r = 400/\frac{0.5}{2} = 1600 \text{ N}$

$\Sigma F_x = 0;\ P - 1600 \sin 30^\circ = 0$

$P = 800 \text{ N} \quad (= T_0)$

$\Sigma F_y = 0;\ F - 500.3 - 1600 \cos 30^\circ = 0$

$F = 1886 \text{ N}$

$T = \sqrt{F^2 + P^2} = \sqrt{(1886)^2 + (800)^2}$

$= 2049 \text{ N}$

From Eq. 5/22, $T = T_0 + \mu y$, $2049 = 800 + 5.003\, y_2$

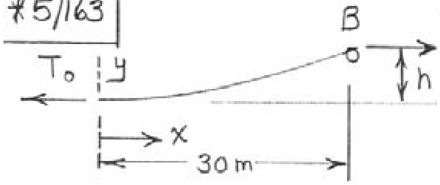
$y_2 = 249.6 \text{ m}$

Also, $1600 = 800 + 5.003\, y_1$

$y_1 = 159.9 \text{ m}$

$H = y_2 - y_1 = \underline{89.7 \text{ m}}$

#5/163



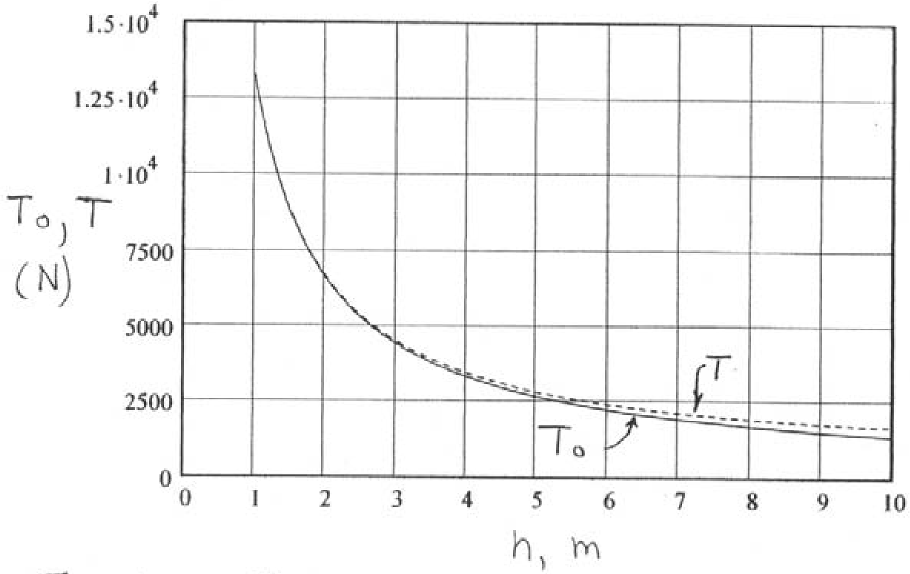
$\mu = \rho g = 3(9.81)$

$\quad = 29.4 \text{ N/m}$

Eq. 5/19 @ B: $h = \dfrac{T_0}{\mu}\left[\cosh\dfrac{\mu x}{T_0} - 1\right]$ (1)

Eq. 5/22 @ B: $T = T_0 + \mu h$ (2)

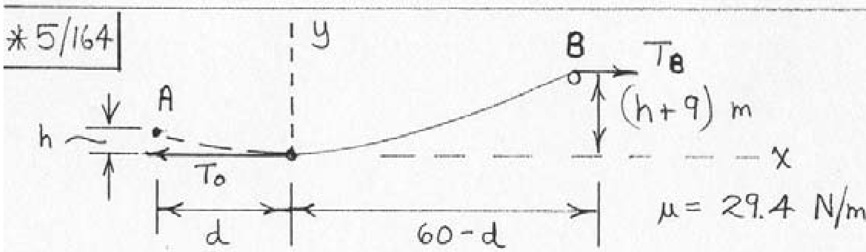
Numerically solve (1), then (2) over $1 \leq h \leq 10$ m to obtain



For $h = 2$ m,

$\underline{T_0 = 6630 \text{ N}}$, $\underline{T = 6690 \text{ N}}$

*5/164



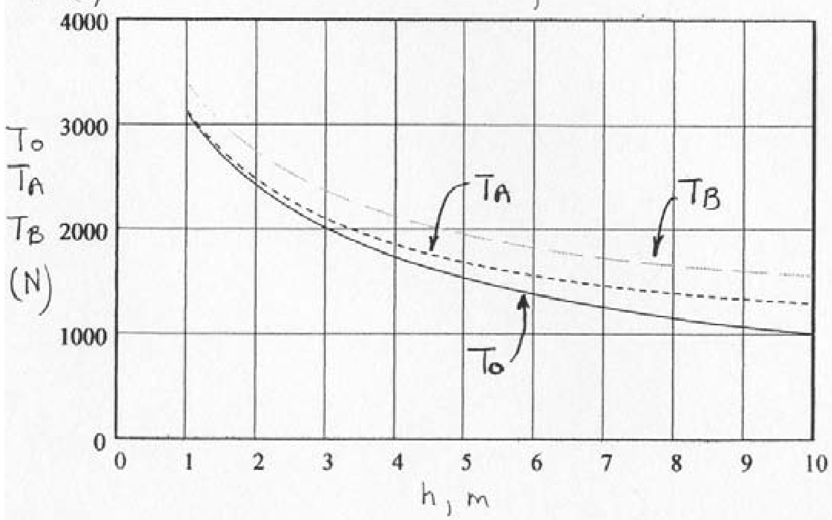
Eq. 5/19 @ B: $h+9 = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu}{T_0}(60-d)\right) - 1\right]$ (1)

Eq. 5/19 @ A: $h = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu}{T_0}d\right) - 1\right]$ (2)

Then Eq. 5/22: $T_A = T_0 + \mu h$ (3)

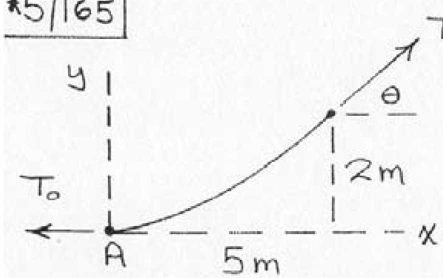
$T_B = T_0 + \mu(h+9)$ (4)

Numerically solve (1) and (2) (for T_0 & d), then (3) & (4) over $1 \leq h \leq 10$ m, to obtain



When $h = 2$ m, $\underline{T_0 = 2410 \text{ N}}$, $\underline{T_A = 2470 \text{ N}}$, $\underline{T_B = 2730 \text{ N}}$

*5/165



$$\begin{cases} \mu_e = 0.75(9.81) = 7.36 \text{ N/m} \\ \mu_f = 1.25(9.81) = 12.26 \text{ N/m} \end{cases}$$

Eq. 5/19: $2 = \frac{T_o}{\mu}\left[\cosh \frac{\mu}{T_o} 5 - 1\right]$

Numerical solution : $\frac{T_o}{\mu} = 6.56$ m

Slope $\frac{dy}{dx} = \sinh \frac{\mu x}{T_o} = \sinh \frac{5}{6.56} = 0.838$

$\theta = \tan^{-1} 0.838 = \underline{40.0°}$ (empty or full)

$T_o = 6.56\,\mu = 6.56(7.36) = 48.3$ N (empty)

$= 6.56\,(12.26) = 80.4$ N (full)

$T = T_o + \mu y = 48.3 + 7.36(2) = \underline{63.0 \text{ N (empty)}}$

$= 80.4 + 12.26(2) = \underline{105.0 \text{ N (full)}}$

*5/166 (a) Use $w = \mu = 1.2(9.81) = 11.77$ N/m

Eq. 5/14 : $y = \frac{wx^2}{2T_0}$ @ A : $2.4 = \frac{11.77(5)^2}{2T_0}$

$T_0 = 61.3$ N

So $y_P = \frac{11.77x^2}{2(61.3)} = 0.096x^2$ (see plots below)

Eq. 5/16 : $s_A = l_A\left[1 + \frac{2}{3}\left(\frac{h_A}{l_A}\right)^2 - \frac{2}{5}\left(\frac{h_A}{l_A}\right)^4 + \cdots\right]$

$= 5\left[1 + \frac{2}{3}\left(\frac{2.4}{5}\right)^2 - \frac{2}{5}\left(\frac{2.4}{5}\right)^4 + \cdots\right] = 5.66$ m

So the required length is $L_P = 2s_A = \underline{11.32 \text{ m}}$

(b) Eq. 5/9 : $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

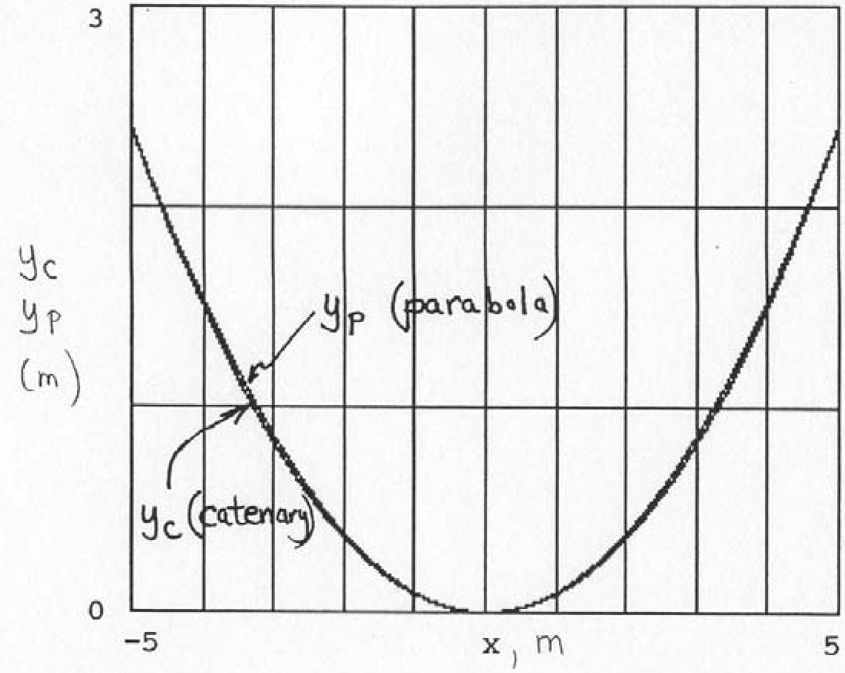
At A : $2.4 = \frac{T_0}{11.77}\left[\cosh\frac{11.77(5)}{T_0} - 1\right]$

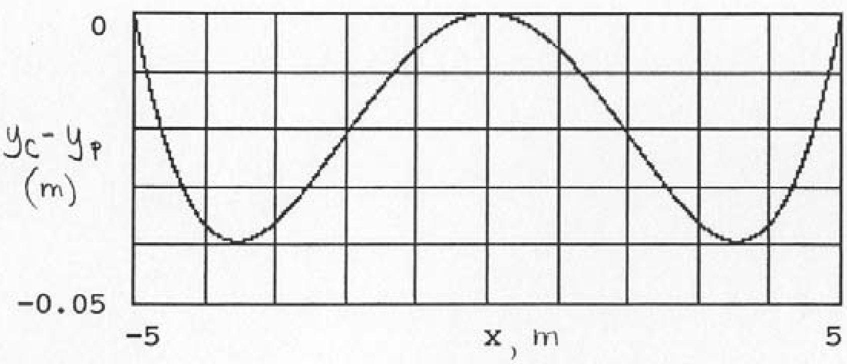
Numerical solution : $T_0 = 65.5$ N

So $y_C = 5.57\left[\cosh(0.1796x) - 1\right]$ (see plots)

Eq. 5/20 : $s_A = \frac{65.5}{11.77}\sinh\frac{11.77(5)}{65.6} = 5.70$ m

The required length is $L_C = 2s_A = \underline{11.40 \text{ m}}$





Even with an expanded vertical scale, y_p and y_c are nearly indistinguishable from each other. Note that y_p is above y_c except at the end points (A and B) and the center. So the fact that $L_c > L_p$ makes sense!

5/167 From Eq. 5/19 with $x = 100$ m, $y = 32$ m

$$32 = \left(\frac{T_o}{\mu}\right)\left[\cosh\left(\frac{100\mu}{T_o}\right) - 1\right]$$

Solve by computer or graphically & get $\frac{T_o}{\mu} = 161.3$ m

From Eq. 5/22, $60(10^3) = T_o + 32\mu$

Solve simultaneously & get $\mu = 310$ N/m

Thus $\rho_{ice} = \frac{\mu}{g} - \rho_{cable} = \frac{310}{9.81} - 18.2 = 13.44$ kg/m of ice

$\boxed{*5/168}$ $\mu = 0.5(9.81) = 4.90$ N/m

Eq. 5/19 @ A : $y_A = \frac{T_0}{\mu}\left[\cosh\frac{\mu x_A}{T_0} - 1\right]$ (1)

Eq. 5/19 @ B : $y_A + 9 = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu}{T_0}(x_A+12)\right) - 1\right]$ (2)

$\frac{dy}{dx} = \sinh\frac{\mu x}{T_0}$ @ A :

$$\tan 15° = \sinh\left[\frac{\mu}{T_0}(x_A)\right] \quad (3)$$

Solve Eqs. (1) - (3) numerically :

$T_0 = 71.5$ N, $x_A = 3.86$ m, $y_A = 0.514$ m

Then

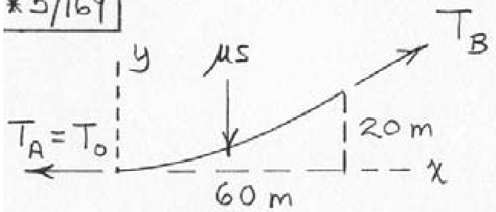
$T_B = T_0 + \mu y_B = 71.5 + 4.90(0.514 + 9)$

$= \underline{118.2 \text{ N}}$

$\tan\theta_B = \sinh\left[\frac{4.90}{71.5}(3.86+12)\right]$

$\underline{\theta_B = 52.8°}$

*5/169



$$\mu = (10+20)9.81$$
$$= 294 \text{ N/m}$$

Eq. 5/19: $y = \dfrac{T_o}{\mu}\left(\cosh \dfrac{\mu x}{T_o} - 1\right)$

Numbers: $20 = \dfrac{T_A}{294}\left(\cosh \dfrac{294(60)}{T_A} - 1\right)$

Numerical solution: $T_A = 27\,400 \text{ N}$

or $\underline{27.4 \text{ kN}}$

Eq. 5/22: $T = T_o + \mu y$

$$T_B = 27\,400 + 294(20) = 33\,300 \text{ N}$$

or $\underline{33.3 \text{ kN}}$

Eq. 5/20: $s = \dfrac{T_o}{\mu} \sinh \dfrac{\mu x}{T_o}$

$$= \frac{27\,400}{294} \sinh \frac{294(60)}{27\,400} = \underline{64.2 \text{ m}}$$

$\boxed{*5/170}$ Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

Numbers and rearranging:

$$600 = \frac{T_0}{3.10}\left(\cosh\frac{3.10(1600)}{T_0} - 1\right) \text{ or } \frac{1860}{T_0} = \cosh\frac{4960}{T_0} - 1$$

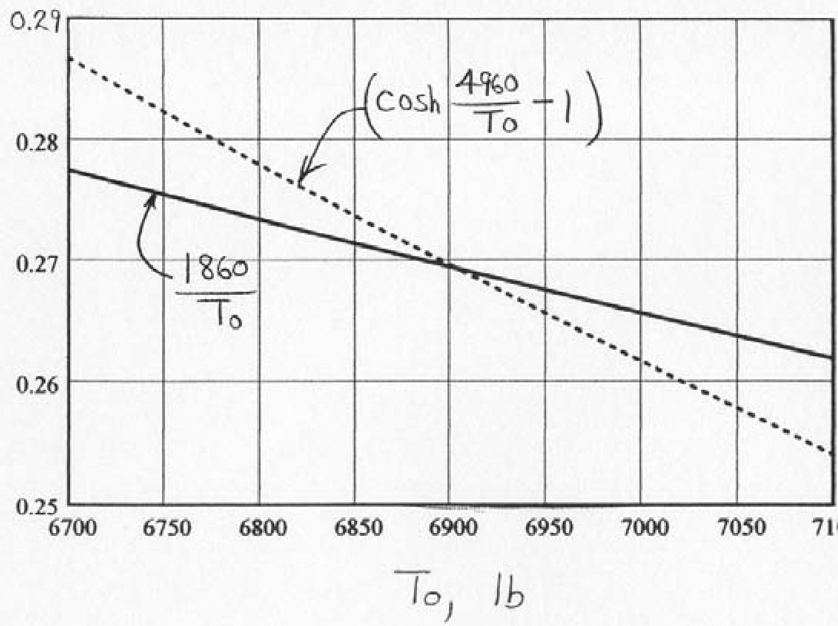
Numerical or graphical (see below) solution:

$\underline{T_0 = 6900 \text{ lb}}$. Then from Eq. 5/22:

$T = T_0 + \mu y = 6900 + 3.10(600) = \underline{8760 \text{ lb}}$

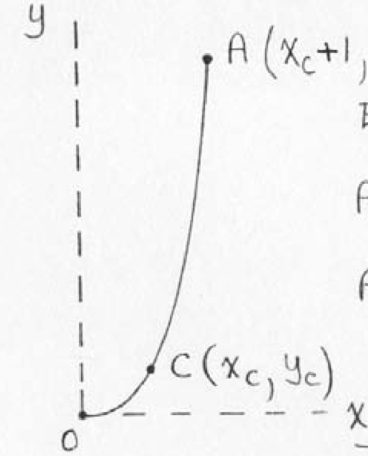
Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

$$= \frac{6900}{3.10}\sinh\frac{3.10(1600)}{6900} = \underline{1741 \text{ ft}}$$



*5/171 Architect's plan : $(T_A)_{arch} = 6(100) = 600$ N

Builder's arrangement :



Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$

At C: $y_c = \frac{T_0}{100}\left[\cosh\frac{100\,x_c}{T_0} - 1\right]$

At B: $y_c + 6 = \frac{T_0}{100}\left[\cosh\frac{100(x_c+1)}{T_0} - 1\right]$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

So $s_A - s_c = 6.1 = \frac{T_0}{100}\left[\sinh\frac{100\,(x_c+1)}{T_0} - \sinh\frac{100\,x_c}{T_0}\right]$

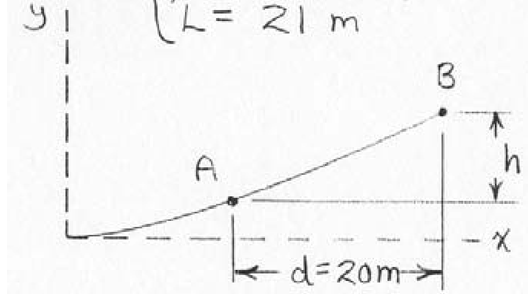
Numerical solution of three equations :

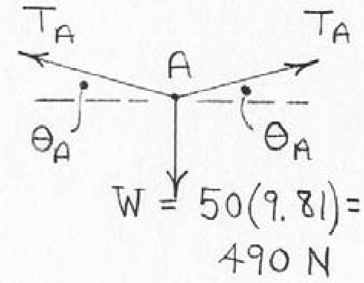
$x_c = 1.071$ m, $y_c = 1.088$ m, $T_0 = 65.5$ N

Eq. 5/22: $T = T_0 + \mu y$, so $T_A = 65.5 + 100\,(1.088 + 6) = 774$ N

Percent increase $n = \frac{774 - 600}{600}(100) = \underline{29.0\,\%}$ (!)

*5/172 $\begin{cases} \mu = 1.2\,(9.81) = 11.77 \text{ N/m} \\ L = 21 \text{ m} \end{cases}$





From FBD of junction ring at A,

$+\uparrow \Sigma F = 0: \quad 2T_A \sin\theta_A - W = 0$

or $\left[T_0 + \mu y_A\right] \sin\left[\tan^{-1}\left(\sinh \frac{\mu x_A}{T_0}\right)\right] - \frac{W}{2} = 0 \quad (1)$

Eq. 5/19 @ A: $\quad y_A = \frac{T_0}{\mu}\left[\cosh \frac{\mu x_A}{T_0} - 1\right] \quad (2)$

Eq. 5/19 @ B: $\quad y_A + h = \frac{T_0}{\mu}\left[\cosh \frac{\mu(x_A + d)}{T_0} - 1\right] \quad (3)$

Eq. 5/20: $\quad s_B - s_A = \frac{T_0}{\mu}\left[\sinh \frac{\mu(x_A+d)}{T_0} - \sinh \frac{\mu x_A}{T_0}\right] = L \quad (4)$

Solution of (1) – (4) with $W = 0$: $h = 5.57$ m

With $W \neq 0$: $h = 6.30$ m

So $\delta = 6.30 - 5.57 = \underline{0.724 \text{ m}}$

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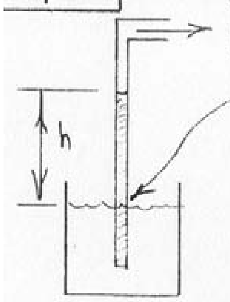

(Assume vacuum pump reduces pressure to zero.)

Pressure here must be atmospheric:

$\rho g h = p_{at}$

$(1000)(9.81)(h) = 1.0133(10^5)$

$\underline{h = 10.33 \text{ m}}$

For mercury: $13\,570(9.81)(h) = 1.0133(10^5)$

$\underline{h = 0.761 \text{ m}}$ (29.97 in.)

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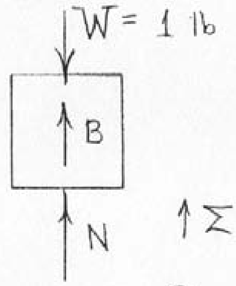


The buoyancy force B is

$$B = \mu_w V = \mu_w \frac{W}{\mu_{ss}}$$

$$= \frac{62.4}{490}(1) = 0.1273 \text{ lb}$$

$\uparrow \Sigma F = 0$: $N + 0.1273 - 1 = 0$, $N = 0.873$ lb

So the force which the weight exerts on the beaker bottom is 0.873 lb (down). The scale reading, however, increases by 1 lb as the weight is added. The difference is due to the increased water pressure on the beaker bottom as the water level rises.

5/175 Force on bottom = weight of water

$= \rho g V = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(0.3\,m)(0.7\,m)(0.4\,m)$

$= \underline{824\ N}$ (down, at center of bottom)

Force on front & back $= p_{av} A_f = \frac{\rho g h}{2} A_f$

$= \frac{1000(9.81)(0.4)}{2}(0.7)(0.4) = \underline{549\ N}$ (outward, at $\frac{2}{3}$ depth)

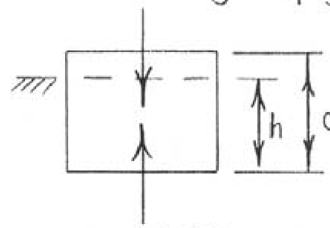
Force on each end glass $= p_{av} A_e = \frac{\rho g h}{2} A_e$

$= \frac{1000(9.81)(0.4)}{2}(0.3)(0.4) = \underline{235\ N}$ (outward, at $\frac{2}{3}$ depth)

(All side forces centered horizontally)

5/176

$W = mg = \rho_1 V g = \rho_1 abcg$



$B = \rho_2 V_{sub} g = \rho_2 abhg$

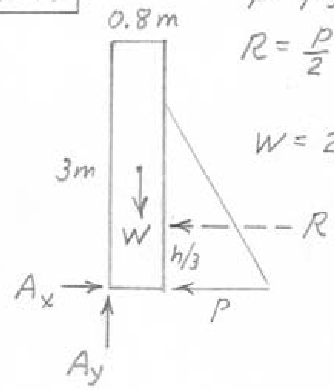
$+\uparrow \Sigma F = 0: \ \rho_2 abhg - \rho_1 abcg = 0, \quad h = \frac{\rho_1}{\rho_2} c$

$r = \frac{h}{c} = \underline{\frac{\rho_1}{\rho_2}}$

Oak in water: $r = \frac{800}{1000} = \underline{0.8}$

Steel in mercury: $r = \frac{7830}{13\,570} = \underline{0.577}$

5/177



$p = \rho g h = 1760(9.81)\,h$

$R = \frac{p}{2} h(1) = 1760(9.81)\frac{h^2}{2} = 8633 h^2$ N

per meter of horizontal width

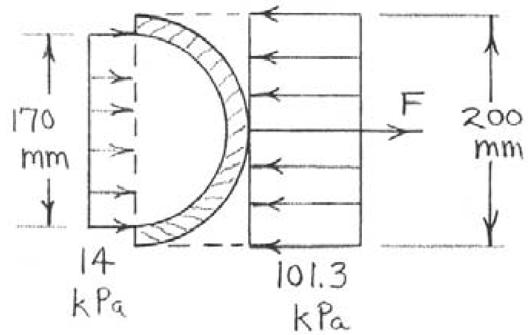
$W = 2400(9.81)(3)(0.8) = 56.5(10^3)$ N

$\Sigma M_A = 0;\quad R\frac{h}{3} - 0.4W = 0$

$8633 h^2 \frac{h}{3} = 0.4(56.5)(10^3)$

$h^3 = 7.85\ ,\ \underline{h = 1.988\text{ m}}$

5/178



$$\xrightarrow{+} \Sigma F = 0: \quad F + 14(10^3)\frac{\pi(0.170)^2}{4} - 101.3(10^3)\frac{\pi(0.200)^2}{4} = 0$$

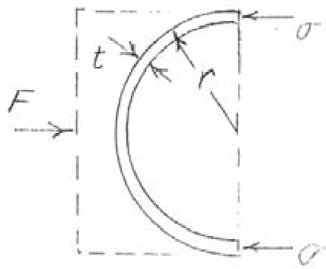
$F = 2860$ N or $\underline{F = 2.86 \text{ kN}}$

5/179

$F = pA = \rho g h A = \rho g h \pi r^2$

$\Sigma F = 0;\ \rho g h \pi r^2 - \sigma(\pi r^2 - \pi[r-t]^2)$

$$\sigma = \frac{\rho g h r}{2t} \frac{1}{1 - t/2r}$$

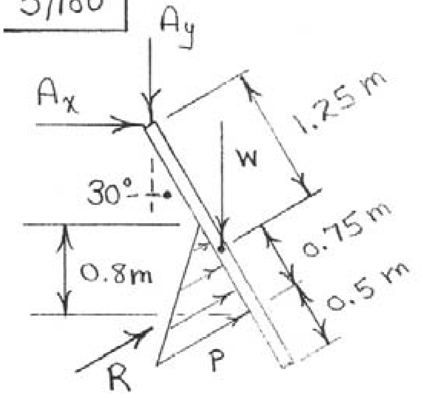


$$= \frac{1.03(10^3)(9.81)\frac{1.500}{2}3(10^3)}{2(0.025)} \frac{1}{1 - \frac{25}{1500}}$$

$$= 454.7(10^6)(1.0169)$$

or $\underline{\sigma = 463 \text{ MPa}}$

5/180



$p = \rho g h = 1000(9.81)(0.8)$

$= 7850$ Pa

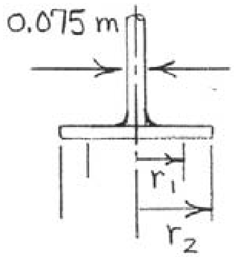
$R = \frac{1}{2}(7850)\left(\frac{0.8}{\cos 30°}\right)$

$= 3620$ N/m

$\curvearrowright + \Sigma M_A = 0: \quad W(1.25 \sin 30°) - 3620\left(2.5 - 0.5 - \frac{1}{3}\frac{0.8}{\cos 30°}\right)$

$= 0; \quad \underline{W = 9810 \text{ N/m}}$

5/181



$\left\{ \begin{array}{l} r_1 = 0.2 \text{ m} \\ r_2 = 0.3 \text{ m} \end{array} \right.$

$p = \rho g h = 1030(9.81)(0.6) = 6060 \text{ Pa}$

$R = pA = 6060\,\pi\left(0.3^2 - \frac{0.075^2}{4}\right) = 1687 \text{ N}$

Pressure supported by seal

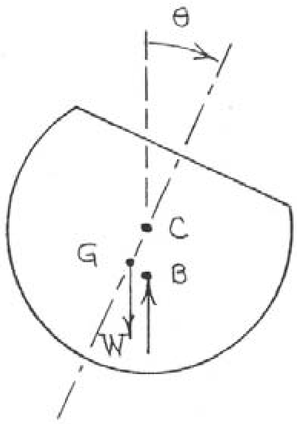
$\sigma = R/\pi(r_2^2 - r_1^2) = \frac{1687}{\pi(0.3^2 - 0.2^2)}$

$= 10\,740 \text{ Pa}$ or $\underline{10.74 \text{ kPa}}$

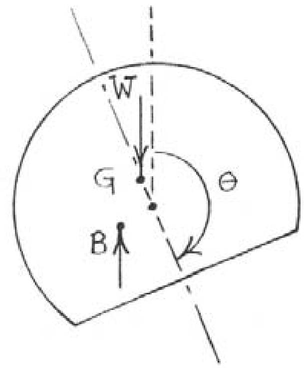
Force to lift plunger

$P = R = 1687 \text{ N}$ or $\underline{1.687 \text{ kN}}$

5/182



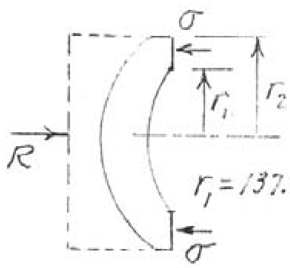
CCW couple tends to make $\theta = 0$



CW couple tends to make $\theta = 180°$

5/183

$p = \rho g h = 1030(9.81)(1000) = 10.104(10^6)$ Pa



$R = pA = 10.104(10^6)\,\pi\,(0.175)^2$

$= 972(10^3)$ N

$r_1 = 137.5$ mm

$r_2 = 175$ mm

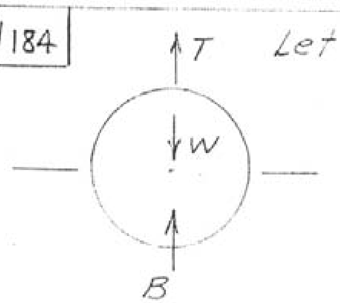
$\sigma = R/A_o$

$= \dfrac{972(10^3)}{\pi\left([0.175]^2 - [0.1375]^2\right)}$

$= 26.4(10^6)$ Pa

or $\underline{\sigma = 26.4 \text{ MPa}}$

5/184



Let γ_c = wt. density of concrete $= 150\ lb/ft^3$

γ_w = wt. density of fresh water $= 62.4\ lb/ft^3$

L = length of cylinder = 6 ft

r = radius of cylinder = 2 ft

For equil., $T = W - B$

$$= \gamma_c \pi r^2 L - \gamma_w \frac{\pi r^2}{2} L$$

$$= \pi r^2 L \left(\gamma_c - \frac{1}{2}\gamma_w\right)$$

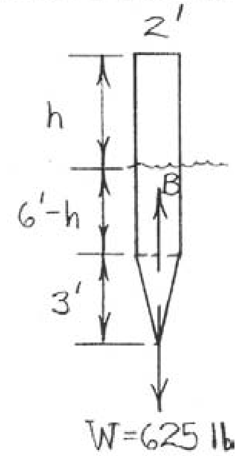
$$= \pi (2^2)(6)\left(150 - \frac{62.4}{2}\right) = \underline{8960\ lb}$$

5/185

$$B = \mu V = 64\left[\pi \frac{2^2}{4}(6-h) + \frac{1}{3}\pi \frac{2^2}{4}(3)\right]$$

$$= 64\pi \frac{2^2}{4}\left[6-h+1\right]$$

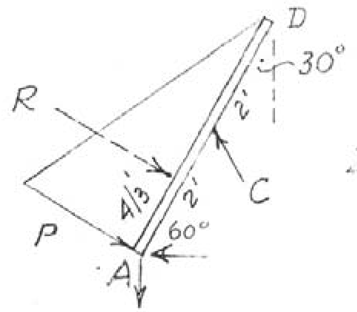
$$= 64\pi\left[7-h\right]$$



$$W = B: \quad 625 = 64\pi\left[7-h\right]$$

$$\underline{h = 3.89 \text{ ft}}$$

5/186



$p = \rho g h = 62.4(4 \cos 30°) = 216 \text{ lb/ft}^2$

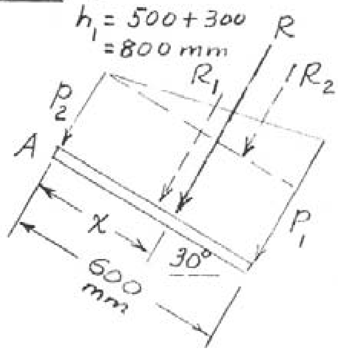
$R = \frac{p}{2}$ Area per panel

$= \frac{216}{2}(4)(2) = 865 \text{ lb}$

$\Sigma M_A = 0;\ 2C \cos 30° - 865(4/3) = 0$

$C = \frac{865(2)}{3 \cos 30°} = \underline{666 \text{ lb}}$

5/187 $h_2 = 500$ mm
$h_1 = 500 + 300 = 800$ mm



$p_2 = \rho g h_2 = 900(9.81)(0.500)$
$= 4.415(10^3)$ Pa

$p_1 = \rho g h_1 = 900(9.81)(0.800)$
$= 7.063(10^3)$ Pa

$R_1 = p_2 A = 4.415(10^3)(0.600)(0.400)$
$= 1059$ N

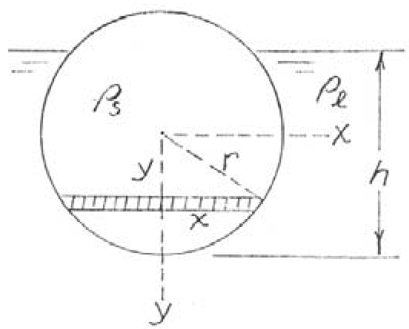
$R_2 = \frac{p_1 - p_2}{2} A$
$= \frac{(7.063 - 4.415)10^3(0.600)(0.400)}{2}$
$= 318$ N

$R = R_1 + R_2 = 1059 + 318 = \underline{1377 \text{ N}}$

$Rx = \Sigma M_A$; $1377x = 1060(300) + 318(400)$

$x = \frac{445\,000}{1377} = \underline{323 \text{ mm}}$

5/188 Buoyancy $B = \rho_\ell g V$, V = submerged volume



$W = mg = \rho_s \frac{4}{3}\pi r^3 g$

$dV = \pi x^2 dy = \pi(r^2 - y^2)dy$

$$B = \rho_\ell g \pi \int_{-(h-r)}^{r} (r^2 - y^2)\,dy$$

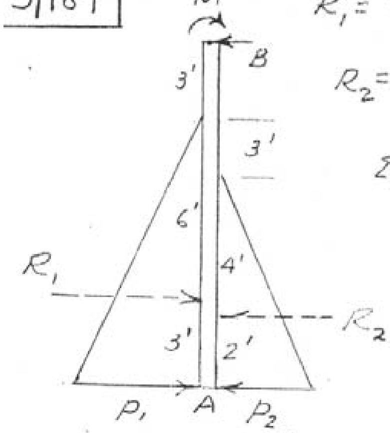
$$= \rho_\ell g \pi \left[r^2 y - \frac{y^3}{3} \right]_{-(h-r)}^{r}$$

$$= \frac{1}{3}\rho_\ell g \pi h^2 (3r - h)$$

Thus with $B = W$, $\rho_s g \frac{4}{3}\pi r^3 = \frac{1}{3}\rho_\ell g \pi h^2 (3r - h)$

Rearrange & get $\underline{\rho_s = \rho_\ell \left(\frac{h}{2r}\right)^2 \left(3 - \frac{h}{r}\right)}$

5/189



Horiz length of gate is 10 ft

$R_1 = \frac{p_1 A_1}{2} = \frac{62.4(9)}{2}(9)(10) = 25{,}270$ lb

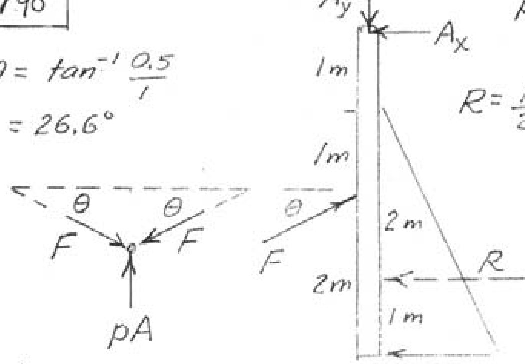
$R_2 = \frac{p_2 A_2}{2} = \frac{64(6)}{2}(6)(10) = 11{,}520$ lb

$\Sigma M_B = 0$; $M + 11{,}520(10) - 25{,}270(9) = 0$

$M = 11.22(10^4)$ lb-ft

5/190

$\theta = \tan^{-1}\frac{0.5}{1}$
$= 26.6°$



$p = \rho g h = 1.0(9.81)(3)$
$= 29.4$ kPa

$R = \frac{p}{2}$ Area $= \frac{29.4}{2}(3)(2)$
$= 88.3$ kN

Gate:

$\Sigma M_A = 0;\ (F\cos 26.6°)2 - 88.3(3) = 0,\ F = 148.1$ kN

Toggle:

$\Sigma F = 0;\ pA - 2F\sin\theta = 0;\ \frac{\pi(0.150)^2}{4}p = 2(148.1)(10^3)\sin 26.6°$

$p = 7.49(10^6)$ Pa or $\underline{p = 7.49\text{ MPa}}$

5/191 Submerged volume V is

$$V = 2(350)(40)(25) + 6\pi\frac{(30)^2}{4}(h-25)$$

$$= 70(10^4) + 0.4241(10^4)(h-25)$$

$$B = \gamma V = 64(70)10^4 + 64(0.4241)(10^4)(h-25)$$

$$= 4480(10^4) + 27.14(10^4)(h-25)$$

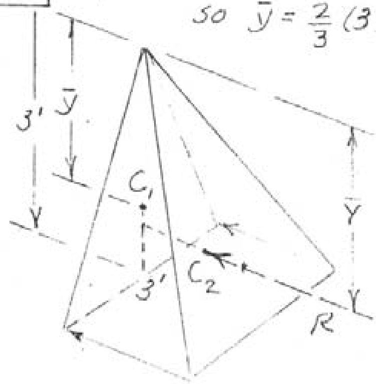
$$W = 26{,}000(2240) = 5824(10^4)\ \text{lb}$$

$$B = W,\quad 27.14(10^4)(h-25) = (5824-4480)10^4$$

$$h - 25 = 49.5,\quad \underline{h = 74.5\ \text{ft}}$$

5/192 C_1 = centroid of triangular area

so $\bar{y} = \frac{2}{3}(3) = 2$ ft



C_2 = centroid of tetrahedron

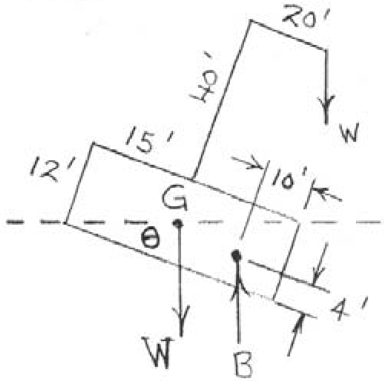
so $H = \bar{Y} = \frac{3}{4}(3) = \underline{2.25 \text{ ft}}$

$R = \rho g \bar{y} A$

$= (62.4)(2)\frac{1}{2}(3)(3)$

$= \underline{562 \text{ lb}}$

5/193



$\theta = \tan^{-1}\frac{12}{30} = 21.8°$

Moment arm of B about G

$(15-10)\cos\theta - (6-4)\sin\theta =$

3.90 ft

Moment arm of w about G

$(40+6)\sin\theta + 20\cos\theta$

$= 35.7$ ft

$B = \rho g V = 64(12)(15)(80) = 922{,}000$ lb

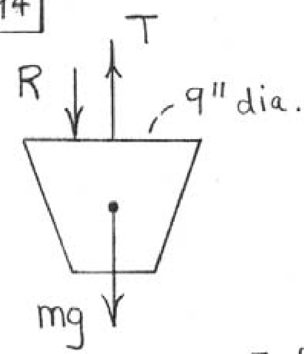
$\Sigma M_G = 0: \; 35.7w - 3.90(922{,}000) = 0$

$\underline{w = 100{,}800 \text{ lb}}$

$W = B - w = 922{,}000 - 100{,}800 = 821{,}000$ lb

or $W = \dfrac{821{,}000}{2240} = \underline{366 \text{ long tons}}$

5/194



$$p = \rho g h = 62.4(20) = 1248 \ \frac{\text{lb}}{\text{ft}^2}$$

$$R = pA = 1248\left[\pi\left(\frac{4.5}{12}\right)^2\right]$$

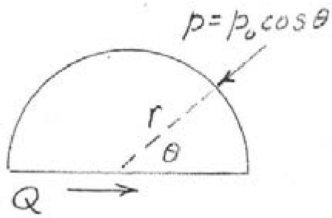
$$= 551 \text{ lb}$$

$$mg = \rho V g = \rho g\left[\frac{7\pi r^2 h}{24}\right] \quad (\text{from Prob. } 5/35)$$

$$= 450\left[\frac{7\pi\ (4.5/12)^2(12/12)}{24}\right] = 58.0 \text{ lb}$$

$$+\uparrow \Sigma F = 0: \quad T - 551 - 58.0 = 0, \quad \underline{T = 609 \text{ lb}}$$

5/195



$$Q = \int_0^{\pi} (p_o \cos\theta)(\cos\theta)\, r\, d\theta$$

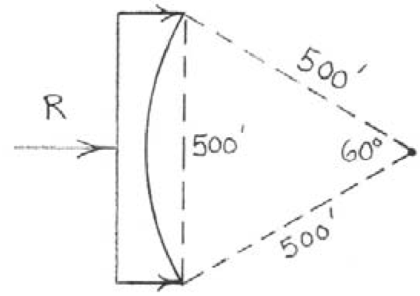
$$= p_o r \int_0^{\pi} \cos^2\theta\, d\theta$$

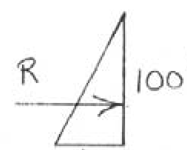
$$= p_o r \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= p_o r \left[\frac{\pi}{2} \right]$$

$$\underline{Q = \tfrac{1}{2}\pi r p_o}$$

5/196

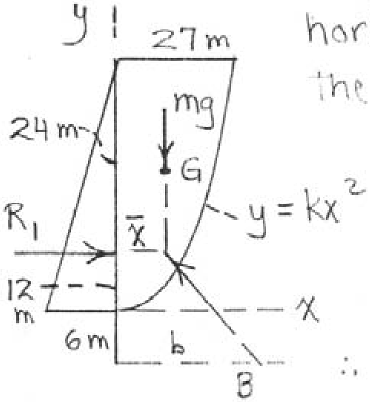




$P_{100} = \mu h = 62.4(100) = 6240 \frac{lb}{ft^2}$

$R = P_{av}\,(\text{Area}) = \frac{6240}{2}(500)(100) = \underline{156.0(10^6)\ lb}$

5/197 Take a vertical section of water of unit horizontal length. Let ρ be the water density in t/m^3.



$y = kx^2$: $36 = k(27)^2$, $k = \frac{4}{81}\ m^{-1}$

$\bar{x} = \dfrac{\int x\,dA}{\int dA}$, $dA = x\,dy = 2\frac{4}{81}x^2 dx$

$\therefore\ \bar{x} = \dfrac{\int_0^{27} \frac{x}{2}\frac{8}{81}x^2 dx}{\int_0^{27}\frac{8}{81}x^2 dx} = 10.12\ m$

$A = \int dA = 648\ m^2$, $mg = 648\rho g$

$R_1 = \frac{1}{2}\,36\rho g\,(36)(1) = 648\rho g$

Resultant of mg & R_1 passes through B, so $\Sigma M_B = 0$. Thus $648\rho g(18) = 648\rho g\,(b - 10.12)$

$\underline{b = 28.1\ m}$

5/198 The gage pressure 12 m below the surface is $p = \rho g h = (1000)(9.81)(12) = 117\ 700\ \text{N/m}^2$.

(a) Cover area $A_{cov} = \pi\left(\frac{0.75}{2}\right)\left(\frac{0.5}{2}\right) = 0.295\ \text{m}^2$

Force on cover $= pA_{cov} = 34\ 700$ N

Seal area $A_s = A_{cov} - \pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) = 0.1325\ \text{m}^2$

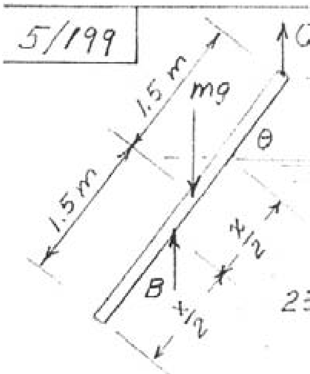
$\sigma A_s = pA_{cov}$, $\sigma = \frac{34\ 700}{0.1325} = 262\ 000\ \frac{\text{N}}{\text{m}^2}$

or $\underline{\sigma = 262\ \text{kPa}}$

(b) $16\,\Delta T = pA_{hole} = 117\ 700\left[\pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right)\right]$

$\underline{\Delta T = 1192\ \text{N}}$

5/199



Let A = cross-sectional area of plank

$mg = 800(3)A(9.81) = 23.54(10^3)A$ N

$B = \rho_w g A x$

$= 1000(9.81)A\left(3 - \frac{1}{\sin\theta}\right)$

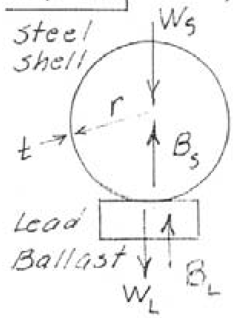
$\Sigma M_O = 0;$

$$23.54(10^3)A(1.5\cos\theta) - 9.81(10^3)A\left(3 - \frac{1}{\sin\theta}\right)\cdot \left[\frac{1}{2}\left(3 - \frac{1}{\sin\theta}\right) + \frac{1}{\sin\theta}\right]\cos\theta = 0$$

Simplify & get $23.54(1.5) = \frac{9.81}{2}\left(9 - \frac{1}{\sin^2\theta}\right)$

& $\sin^2\theta = 0.5556$, $\sin\theta = 0.7454$, $\underline{\theta = 48.2°}$

5/200



For equilibrium $W_S + W_L = B_S + B_L$

ρ_S = density of steel = 7.83 Mg/m^3

ρ_W = " " salt water = 1.03 Mg/m^3

ρ_L = " " lead = 11.37 Mg/m^3

$r = 1.00$ m, $t = 0.035$ m

V_L = volume of lead, m^3

m = mass of lead = $\rho_L V_L$

so $\rho_S g\, 4\pi r^2 t + \rho_L g V_L = \rho_W g \frac{4}{3}\pi (r + \frac{t}{2})^3 + \rho_W g V_L$

$V_L g(\rho_L - \rho_W) = 4\pi r^2 g \left[\frac{r}{3}(1 + \frac{t}{2r})^3 \rho_W - \rho_S t\right]$

$m(1 - \frac{\rho_W}{\rho_L}) = 4\pi r^2 \left[\frac{r}{3}(1 + \frac{t}{2r})^3 \rho_W - \rho_S t\right]$

$m(1 - \frac{1.03}{11.37}) = 4\pi(1)^2 \left[\frac{1}{3}(1 + \frac{0.035}{2})^3 1.03 - 7.83(0.035)\right]$

$0.9094 m = 1.1008$; $m = \underline{1.210 \text{ Mg}}$ (metric tons)

5/201 The pressure at the bottom of the 3-m wall is $p = \rho g h = 2400(9.81)(3) = 70\,600 \text{ N/m}^2$

Each tie controls an area A given by

$$pA = T, \quad A = \frac{T}{p} = \frac{6\,500}{70\,600} = 0.0920 \text{ m}^2$$

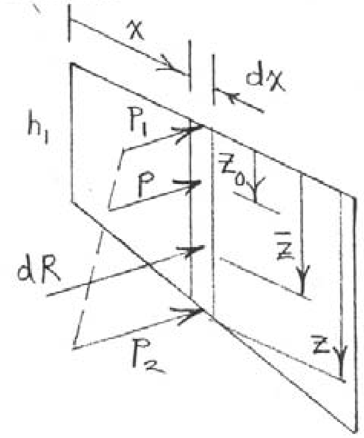
This square area has a side d given by

$$d^2 = A, \quad d = 0.303 \text{ m}$$

Using the pressure at the very bottom of the wall gives us a conservative design; a good figure for d would be $\underline{d = 0.300 \text{ m}}$.

5/202

Method I : Direct integration



ρ = water density , $p = \rho g (b + z_0)$

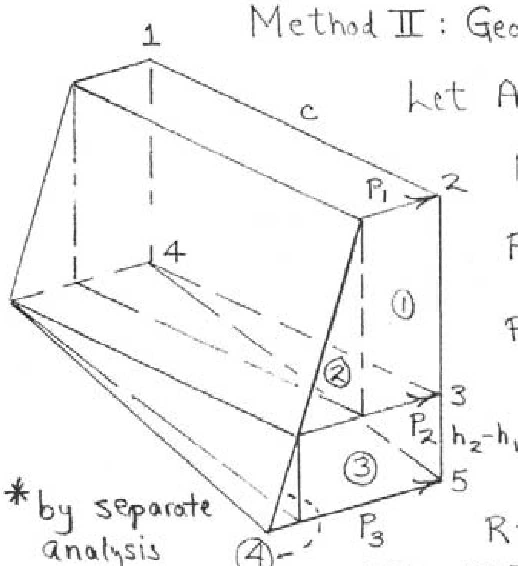
$z = h_1 + kx = f(x)$

(1) Calculate dR for elemental area $z\,dx$ & find $\bar{z}$

(2) Integrate & get $R = \int dR$

(3) $R\bar{z} = \int \bar{z}\, dR$

Method II : Geometry of pressure-area volumes



Let $A_a = 1\text{-}2\text{-}3\text{-}4$, $A_b = 3\text{-}4\text{-}5$

$R_1 = p_1 A_a$, $\bar{z}_1 = h_1/2$

$R_2 = \frac{1}{2}(p_2 - p_1)A_a$, $\bar{z}_2 = \frac{2h_1}{3}$

$R_3 = p_2 A_b$, $\bar{z}_3 = h_1 + \frac{2}{3}(h_2 - h_1)$

$R_4 = \frac{1}{2}(p_3 - p_2)(h_2 - h_1)\frac{c}{3}$

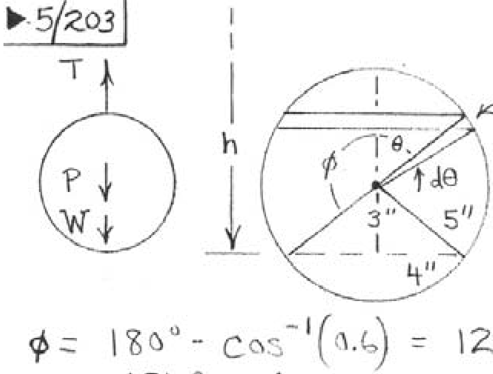
$\bar{z}_4 = \frac{1}{2}(h_2 + h_1)$ *

$R = R_1 + R_2 + R_3 + R_4$

$\bar{z} = (R_1\bar{z}_1 + R_2\bar{z}_2 + R_3\bar{z}_3 + R_4\bar{z}_4)/R$

* by separate analysis

▶5/203



$\Sigma F = 0: \; T - P - W = 0$

Pressure force $P = \int p\, dA \cos\theta$

$\rho g = 62.4/1728 = 0.0361 \text{ lb/in.}^3$

$\phi = 180^\circ - \cos^{-1}(0.6) = 126.9^\circ$

$V = \frac{126.9}{180} \frac{4}{3}\pi(5^3) = 369 \text{ in.}^3, \quad B = 369\rho g \text{ (lb)}$

$p = \rho g(h - 3 - r\cos\theta) = \rho g(h' - r\cos\theta), \quad h' = h - 3$

$$P = \int_0^{\phi} \rho g(h' - r\cos\theta)(2\pi r\sin\theta)\, r\, d\theta \cos\theta$$

$$= 2\pi r^2 \rho g \int_0^{\phi} (h'\sin\theta\cos\theta - r\sin\theta\cos^2\theta)\, d\theta$$

$$= 2\pi r^2 \rho g \left[\int_{\theta=0}^{\theta=\phi} \frac{h'}{4}\sin 2\theta\, d(2\theta) - \int_0^{\phi} r(\sin\theta - \sin^3\theta)\, d\theta \right]$$

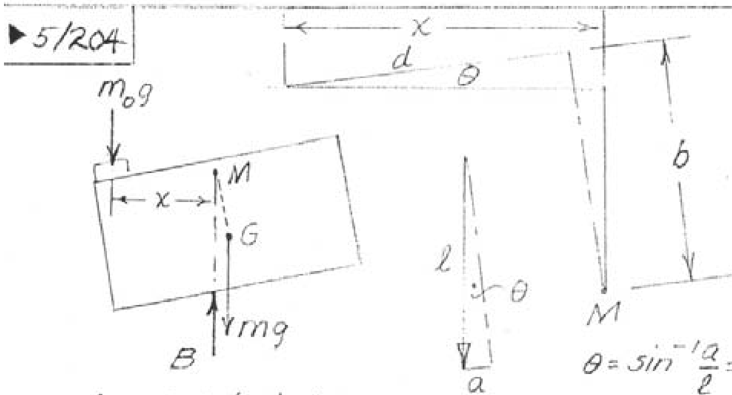
$$= 2\pi r^2 \rho g \left[-\frac{h'}{4}\cos 2\theta - r\left(-\cos\theta + \frac{\cos\theta}{3}(2 + \sin^2\theta)\right)\right]_0^{\phi}$$

$$= 2\pi r^2 \rho g \left[0.32h' - 2.03\right] \quad \text{(lb)}$$

When $T = W$, $P = 0$: $h' = \frac{2.03}{0.32} = h - 3$

$\underline{h = 9.33 \text{ in.}}$

▸5/204



$m_o = 27$ Mg

$m = 12(10^3)$ Mg

$\theta = \sin^{-1}\frac{a}{\ell} = \sin^{-1}\frac{0.2}{6} = 1.91^\circ$

$\sin\theta = 0.0333$, $\cos\theta = 0.9994$

$x = d\cos\theta + b\sin\theta$

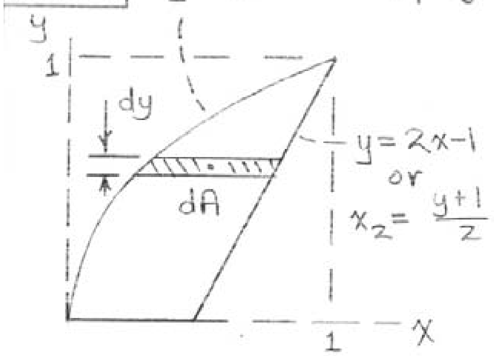
$= 7.8(0.9994) + 1.8(0.0333)$

$= 7.86$ m

$\Sigma M_M = 0$; $mg\,\overline{GM}\sin\theta - m_o g x = 0$

$\overline{GM} = \frac{m_o x}{m\sin\theta} = \frac{27(7.86)}{12(10^3)(0.0333)} = \underline{0.530 \text{ m}}$

5/205 $y = x^{1/3}$ or $x_1 = y^3$



$$dA = (x_2 - x_1)\,dy = \left(\frac{y}{2} + \frac{1}{2} - y^3\right)dy$$

$$A = \int dA = \int_0^1 \left(\frac{y}{2} + \frac{1}{2} - y^3\right)dy = \left(\frac{y^2}{4} + \frac{y}{2} - \frac{y^4}{4}\right)\Big|_0^1 = \frac{1}{2}$$

$$\int y_c\,dA = \int_0^1 \left(\frac{y^2}{2} + \frac{y}{2} - y^4\right)dy = \left(\frac{y^3}{6} + \frac{y^2}{4} - \frac{y^5}{5}\right)\Big|_0^1 = \frac{13}{60}$$

$$\int x_c\,dA = \int_0^1 \left(\frac{x_1 + x_2}{2}\right)(x_2 - x_1)\,dy = \frac{1}{2}\int_0^1 (x_2^2 - x_1^2)\,dy$$

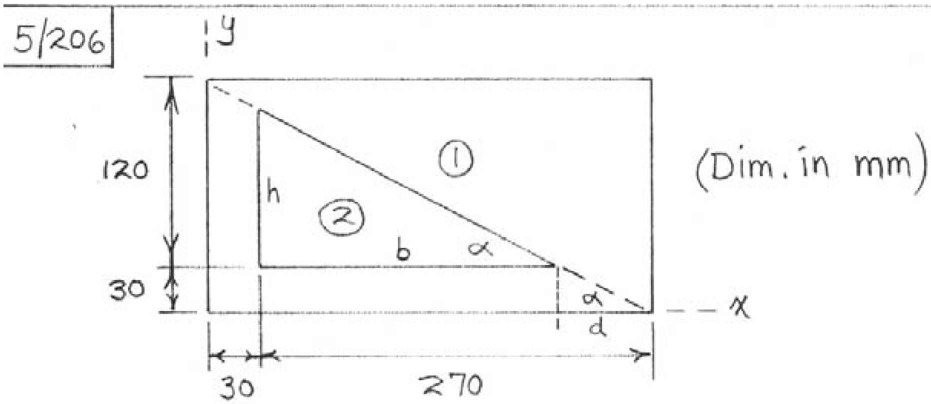
$$= \frac{1}{2}\int_0^1 \left(\frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - y^6\right)dy = \frac{1}{2}\left(\frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^7}{7}\right)\Big|_0^1$$

$$= \frac{37}{168}$$

$$\bar{y} = \int x_c\,dA \Big/ \int dA = \frac{13/60}{1/2} = 13/30$$

$$\bar{x} = \int y_c\,dA \Big/ \int dA = \frac{37/168}{1/2} = 37/84$$

5/206



$\tan\alpha = \frac{150}{300} = \frac{1}{2}; \quad \frac{30}{d} = \tan\alpha = \frac{1}{2}$, so $d = 60$ mm

Then $b = 300 - (30 + 60) = 210$ mm

$\frac{h}{b} = \tan\alpha = \frac{1}{2}, \quad h = \frac{b}{2} = 105$ mm

$A_1 = 150(300) = 45\,000 \text{ mm}^2$

$\bar{x}_1 = 150$ mm, $\bar{y}_1 = 75$ mm

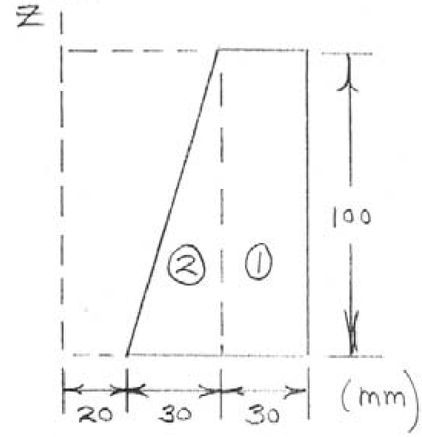
$A_2 = -\frac{1}{2}bh = -\frac{1}{2}(210)(105) = -11\,025 \text{ mm}^2$

$\bar{x}_2 = 30 + \frac{1}{3}(210) = 100$ mm, $\bar{y}_2 = 30 + \frac{1}{3}(105) = 65$ mm

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{45\,000(150) - 11\,025(100)}{45\,000 - 11\,025} = \underline{166.2 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{45\,000(75) - 11\,025(65)}{45\,000 - 11\,025} = \underline{78.2 \text{ mm}}$$

5/207



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi (65)(30 \cdot 100) = 1.225(10^6)\ mm^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi (40) \frac{30 \cdot 100}{2} = 0.377(10^6)\ mm^3$$

$$V = V_1 + V_2 = 1.602(10^6)\ mm^3$$

$$m = \rho V = 7210\ \frac{kg}{m^3}\left[1.602(10^6)\ mm^3 \cdot \left(\frac{1\ m}{1000\ mm}\right)^3\right]$$

$$= \underline{11.55\ kg}$$

$\boxed{5/208}$

For circular arc, $\bar{y} = \dfrac{b \sin 30°}{\pi/6} = 3b/\pi$

$\bar{z} = b$ (See Samp. Prob. 5/1)

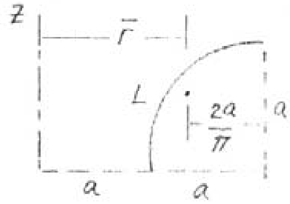
$$\overline{Y} = \frac{\Sigma \bar{y} L}{\Sigma L} = \frac{b(0) + 2b\left(\frac{b}{2}\cos 30°\right) + \frac{\pi}{3}b\left(\frac{3b}{\pi}\right)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.461\,b}$$

$$\overline{Z} = \frac{\Sigma \bar{z} L}{\Sigma L} = \frac{b\left(\frac{b}{2}\right) + 2b(b) + \frac{\pi}{3}b(b)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.876\,b}$$

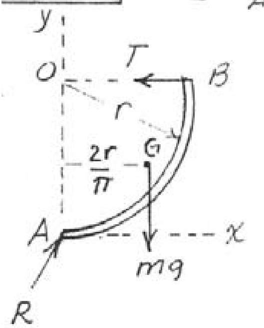
5/209 $L = \pi a/2$; $\bar{r} = 2a - \frac{2a}{\pi} = 2a(1 - \frac{1}{\pi})$



$A = \frac{\pi}{2}\bar{r}L = \frac{\pi}{2}2a(1 - \frac{1}{\pi})\frac{\pi a}{2}$

$= \frac{\pi a^2}{2}(\pi - 1)$

5/210



$\Sigma M_A = 0;\ mg\frac{2r}{\pi} - Tr = 0,\ T = \frac{2mg}{\pi}$

$\Sigma F_x = 0;\ R_x = \frac{2mg}{\pi}$

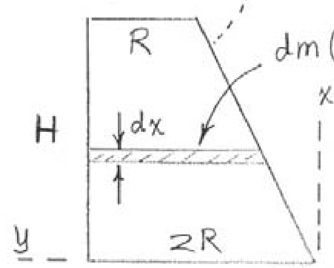
$\Sigma F_y = 0;\ R_y = mg$

$R = \sqrt{R_x^2 + R_y^2} = mg\sqrt{\frac{4}{\pi^2} + 1} = \underline{1.185\,mg}$

5/211

$y = \frac{R}{H}x$

$$dm = \rho\, dV = \rho\pi\left(\frac{2R-y}{2}\right)^2 dx$$
$$= \frac{\pi\rho}{4}\left(4R^2 - 4Ry + y^2\right) dx$$
$$= \frac{\pi\rho}{4}\left[4R^2 - 4R\,\frac{R}{H}x + \left(\frac{R}{H}x\right)^2\right] dx$$
$$= \frac{\pi\rho R^2}{4}\left[4 - 4\frac{x}{H} + \frac{x^2}{H^2}\right] dx$$



$$m = \int dm = \int_0^H \frac{\pi\rho R^2}{4}\left[4 - 4\frac{x}{H} + \frac{x^2}{H^2}\right] dx$$
$$= \frac{7}{12}\pi\rho R^2 H$$

$$\int x_c\, dm = \int_0^H \frac{\pi\rho R^2}{4}\left[4x - 4\frac{x^2}{H} + \frac{x^3}{H^2}\right] dx$$
$$= \frac{11}{48}\pi\rho R^2 H^2$$

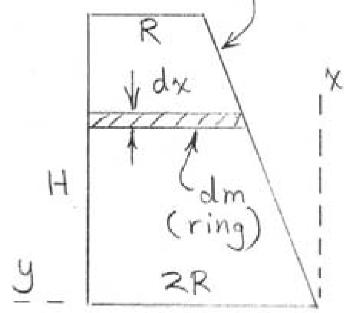
Then $\bar{x} = \dfrac{\int x_c\, dm}{\int dm} = \dfrac{\frac{11}{48}\pi\rho R^2 H^2}{\frac{7}{12}\pi\rho R^2 H}$

$$= \underline{\frac{11}{28}H = \bar{h}}$$

(ρ = mass per unit volume)

5/212

$y = \frac{R}{H}x$



$$dm = \rho\, dA = \rho\left[\frac{2R-y}{2}\cdot 2\pi k\right]$$

$$= \pi\rho\left[2R - \frac{R}{H}x\right]k\,dx$$

$$= \pi\rho R\left[2-\frac{x}{H}\right]k\,dx$$

$$m = \int dm = \int_0^H \pi\rho R\left(2-\frac{x}{H}\right)k\,dx$$

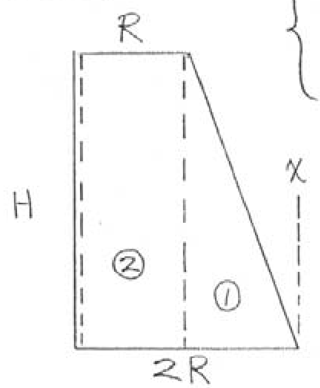
$$= \frac{3}{2}k\pi\rho RH$$

$$\int x_c\,dm = \int_0^H \pi\rho Rk\left(2x-\frac{x^2}{H}\right)dx = \frac{2}{3}k\pi\rho RH^2$$

$$\bar{x} = \frac{\int x_c\,dm}{\int dm} = \frac{\frac{2}{3}k\pi\rho RH^2}{\frac{3}{2}k\pi\rho RH} = \underline{\frac{4}{9}H = \bar{h}}$$

(ρ = mass per unit area, k is a parameter which accounts for the variation of the slant height of the differential elemental ring around the circumference of the body (the same for all elements.))

5/213



① Solid body (Prob. 5/211)
② Cylindrical hole

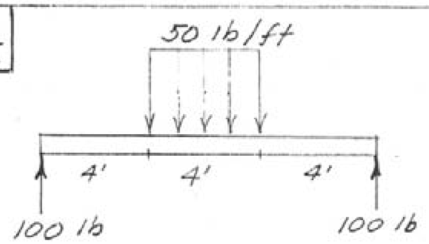
(Refer to the solution for Prob. 5/211)

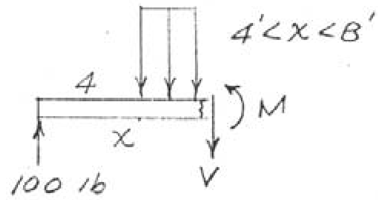
Body	m	$\bar{x}$	$\bar{x}m$
①	$\frac{7}{12}\pi\rho R^2 H$	$\frac{11}{28}H$	$\frac{77}{336}\pi\rho R^2 H^2$
②	$-\pi\rho\left(\frac{R}{2}\right)^2 H$	$\frac{H}{2}$	$-\frac{1}{8}\pi\rho R^2 H^2$

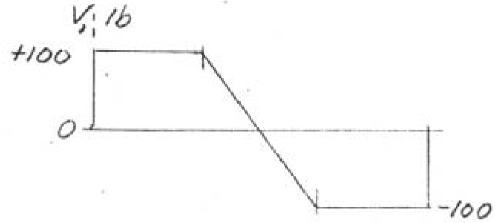
$\Sigma m = \frac{1}{3}\pi\rho R^2 H$ $\qquad \Sigma\bar{x}m = \frac{35}{336}\pi\rho R^2 H^2$

$$\bar{X} = \frac{\Sigma\bar{x}m}{\Sigma m} = \frac{\frac{35}{336}\pi\rho R^2 H^2}{\frac{1}{3}\pi\rho R^2 H} = \underline{\frac{5}{16}H = \bar{h}}$$

5/2.14





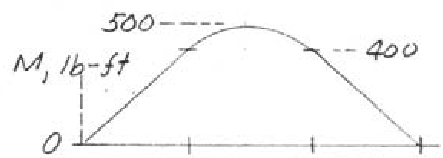


$\Sigma F = 0;\ V + 50(x-4) - 100 = 0$

$V = 50(6-x)$

$\Sigma M_V = 0;\ M + 50\frac{(x-4)^2}{2} - 100x = 0$

$M = 25(-x^2 + 12x - 16)$



Set $\frac{dM}{dx} = 0$ to get

$M_{max} = 500$ lb-ft

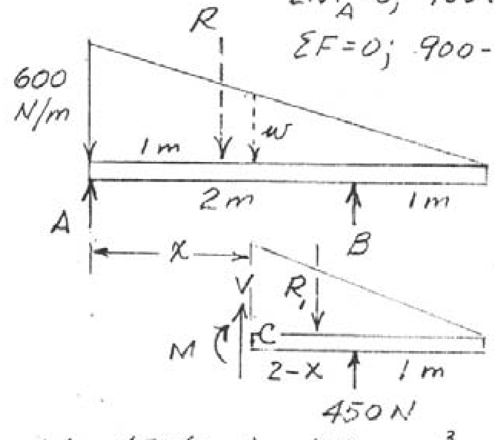
@ $x = 6$ ft

5/215

$R = \frac{600}{2}(3) = 900\ N$;

$\Sigma M_A = 0;\ 900(1) - 2B = 0,\ B = 450\,N$

$\Sigma F = 0;\ 900 - 450 - A = 0,\ A = 450\,N$



$R_1 = \frac{w}{2}(2 - x + 1) = \frac{3-x}{2} w$

where $w = \frac{3-x}{3} 600\ N/m$

So $R_1 = 100(3-x)^2\ N$

$\Sigma M_C = 0;$

$450(2-x) = 100(3-x)^2 \frac{3-x}{3} + M$

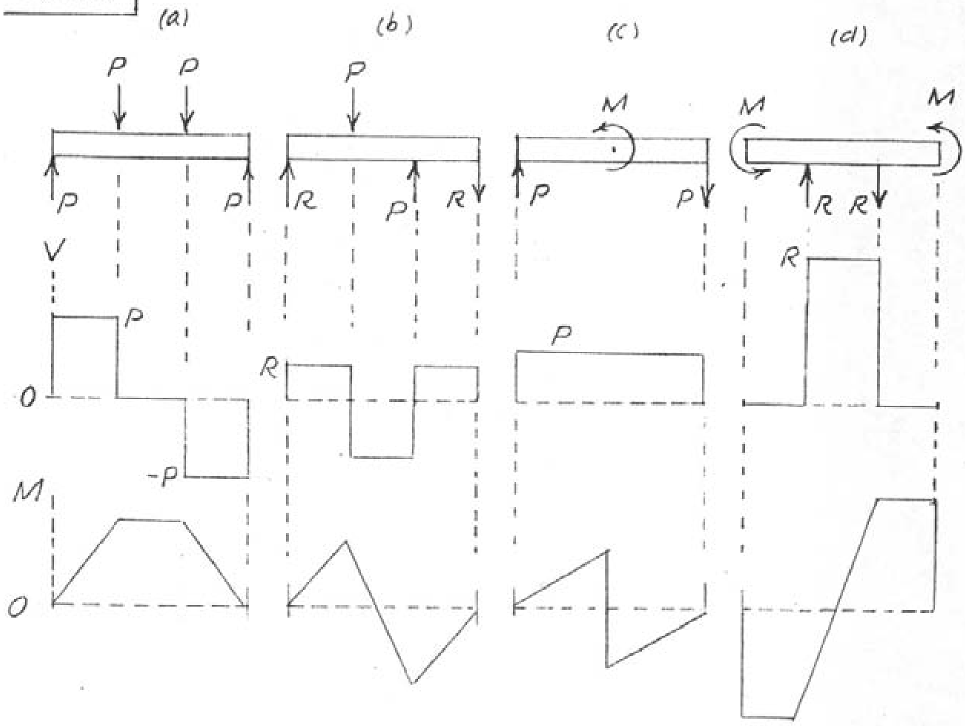
$M = 450(2-x) - \frac{100}{3}(3-x)^3 \qquad 0 \le x \le 2\ m$

$\frac{dM}{dx} = -450 + 100(3-x)^2 = 0$ for max or min

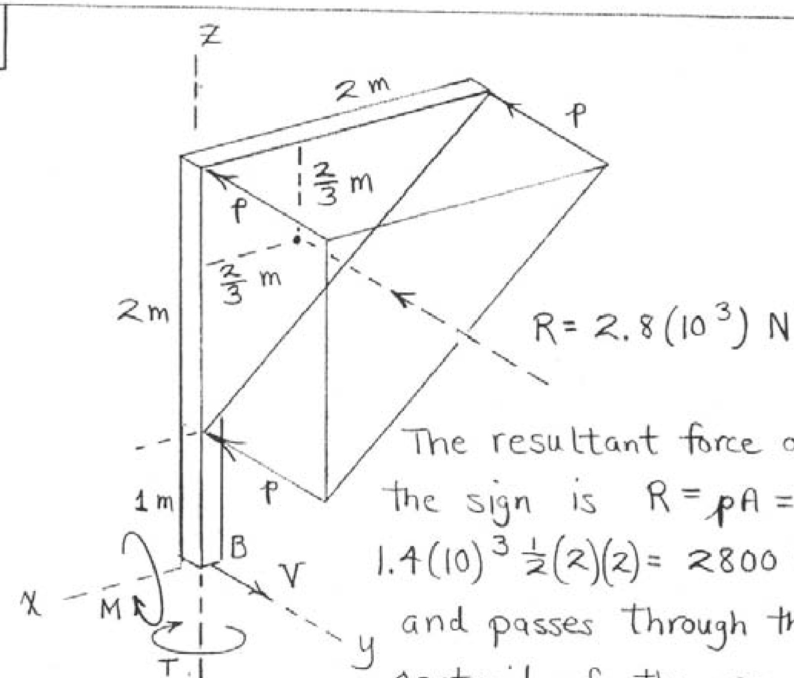
$3 - x = \pm\sqrt{4.5}, \quad x = 3(1 \pm 1/\sqrt{2}) = \underline{0.879\,m}$ (or 5.12)

$M_{max} = 450(2 - 0.879) - \frac{100}{3}(3 - 0.879)^3 = \underline{186.4\ N\cdot m}$

5/216



5/217



The resultant force on the sign is $R = pA = 1.4(10)^3 \frac{1}{2}(2)(2) = 2800$ N and passes through the centroid of the pressure-area prism.

By inspection,
$$\begin{cases} V = \underline{2.8 \text{ kN}} \\ M = 2.8\left(3 - \frac{2}{3}\right) = \underline{6.53 \text{ kN}\cdot\text{m}} \\ T = 2.8\left(\frac{2}{3}\right) = \underline{1.867 \text{ kN}\cdot\text{m}} \end{cases}$$

5/218

$$\bar{X} = 0 = \frac{\pi D^2}{4} r\cos\theta + \frac{\pi d^2}{4} r\cos 30° - \frac{\pi d^2}{4} r, \quad D^2\cos\theta = d^2\left(1-\frac{\sqrt{3}}{2}\right)$$

$$\bar{Y} = 0 = \frac{\pi D^2}{4} r\sin\theta - \frac{\pi d^2}{4} r - \frac{\pi d^2}{4} r\sin 30°$$

$$D^2\sin\theta = d^2\left(1+\frac{1}{2}\right)$$

Divide : $\dfrac{\sin\theta}{\cos\theta} = \dfrac{3/2}{1-\sqrt{3}/2}$, $\underline{\theta = 84.9°}$

$D^2 = \dfrac{3d^2/2}{\sin 84.9°}$, $\underline{D = 1.227\,d}$

5/219 Hole: $V = -9h \quad (\text{in.}^3), \quad \bar{z} = h/2$

Cylinder: $V = \pi 6^2(10) = 1131 \text{ in.}^3, \quad \bar{z} = 5 \text{ in.}$

$$\bar{z} = \frac{\Sigma \bar{z}V}{\Sigma V} = \frac{-9h\left(\frac{h}{2}\right) + 1131(5)}{-9h + 1131}$$

For max. $\bar{z}$, $\dfrac{d\bar{z}}{dh} = 0$

$$\frac{d\bar{z}}{dh} = \frac{(-9h + 1131)(-9h) - \left(-\frac{9h^2}{2} + 5655\right)(-9)}{(-9h + 1131)^2} = 0$$

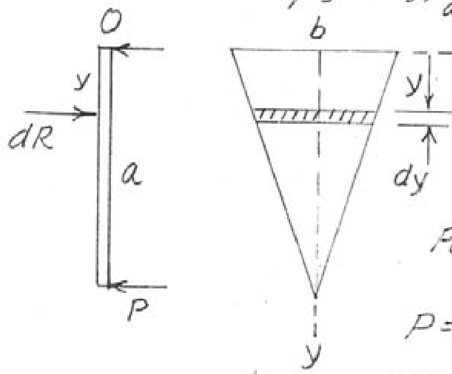
$$\Rightarrow 9h^2 - 1131h - 4.5h^2 + 5655 = 0$$

$$4.5h^2 - 1131h + 5655 = 0$$

$$h = \frac{1131 \pm \sqrt{1131^2 - 4(4.5)(5655)}}{9} = \underline{5.10 \text{ in.}} \text{ or } 246 \text{ in.}$$

5/220 $dA = 2x\,dy = \frac{b}{a}(a-y)\,dy, \quad p = \rho g\,(h+y)$

$$dR = \rho g\,(h+y)\,\frac{b}{a}\,(a-y)\,dy$$



$\Sigma M_O = 0; \quad Pa - \int y\,dR = 0$

$$Pa = \rho g\,\frac{b}{a}\int_0^a y(h+y)(a-y)\,dy$$

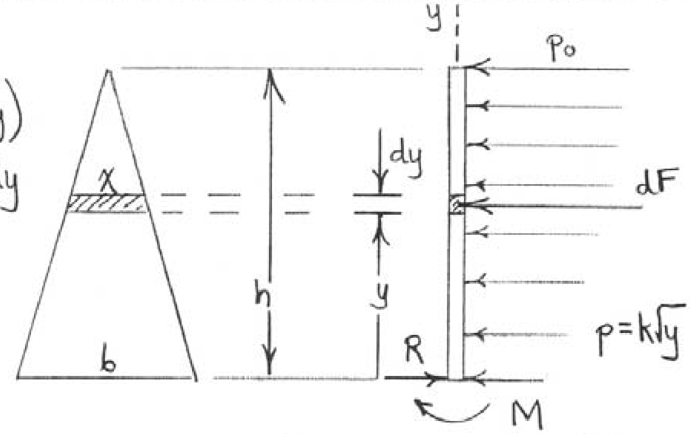
$$Pa = \rho g\,\frac{b}{a}\,\frac{a^3}{6}\left(h+\frac{a}{2}\right)$$

$$\underline{P = \frac{\rho g a b}{6}\left(h+\frac{a}{2}\right)}$$

5/221

$x = \frac{b}{h}(h-y)$

$dA = \frac{b}{h}(h-y)\,dy$



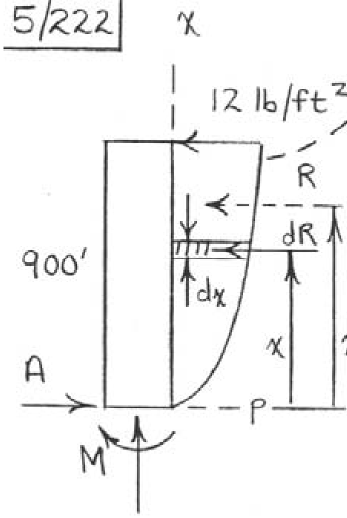
$p = k\sqrt{y}$: $p_o = k\sqrt{h}$, $k = \frac{p_o}{\sqrt{h}}$, so $p = p_o\sqrt{\frac{y}{h}}$

$dF = p\,dA = p_o\sqrt{\frac{y}{h}}\;\frac{b}{h}(h-y)\,dy$

$dM = y\,dF = p_o\frac{b}{h\sqrt{h}}\left(y^{3/2}h - y^{5/2}\right)dy$

$M = \int_0^h dM = \frac{p_o b}{h\sqrt{h}}\left[\frac{y^{5/2}}{5/2}h - \frac{y^{7/2}}{7/2}\right]_0^h = \underline{\frac{4}{35}p_o b h^2}$

5/222



$p = k\sqrt{x}$: $12 = k\sqrt{900}$

$k = 0.4 \frac{\text{lb}}{\text{ft}^{5/2}}$, $p = 0.4\sqrt{x}$

$dR = 200\,p\,dx = 200(0.4\sqrt{x})$

$= 80\sqrt{x}\;dx$

$R = \int dR = \int_0^{900'} 80\sqrt{x}\;dx$

$= 80\,\frac{2}{3}\,x^{3/2}\Big|_0^{900'} = 1.440(10^6)$ lb

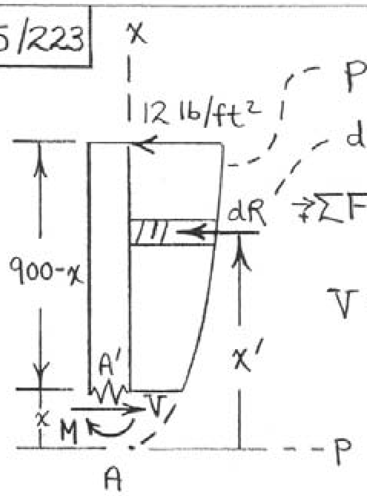
$\int x\,dR = \int_0^{900} 80\,x^{3/2}\,dx = 80\,\frac{2}{5}\,x^{5/2}\Big|_0^{900'}$

$= 7.78(10^8)$ lb-ft

Thus, $A = 1.440(10^6)$ lb

$M = 7.78(10^8)$ lb-ft

▸5/223



$p = 0.4\sqrt{x} \quad (\text{lb/ft}^2)$

$dR = 80\sqrt{x'}\,dx'$

$\rightarrow \Sigma F = 0: \quad V - \int_x^{900'} 80\sqrt{x'}\,dx' = 0$

$V = 80\,\frac{2}{3}\,x'^{3/2}\Big|_x^{900} =$

$\underline{1.440(10^6) - \frac{160}{3}x^{3/2} \ (\text{lb})}$

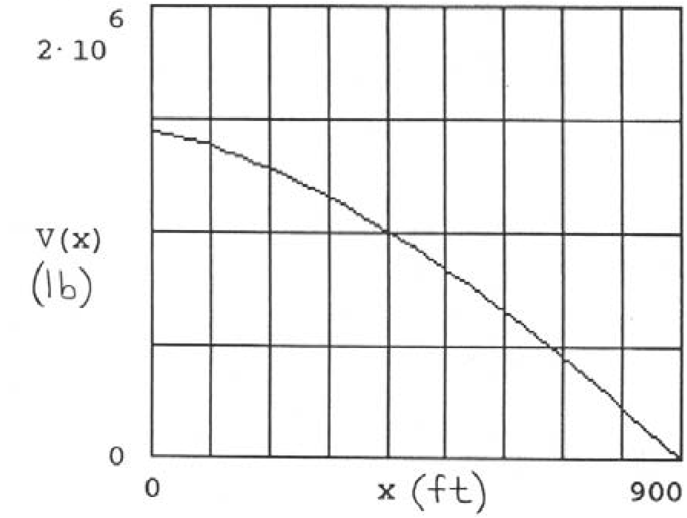
$\underline{V\Big|_{x=450'} = 0.931(10^6) \text{ lb}}$

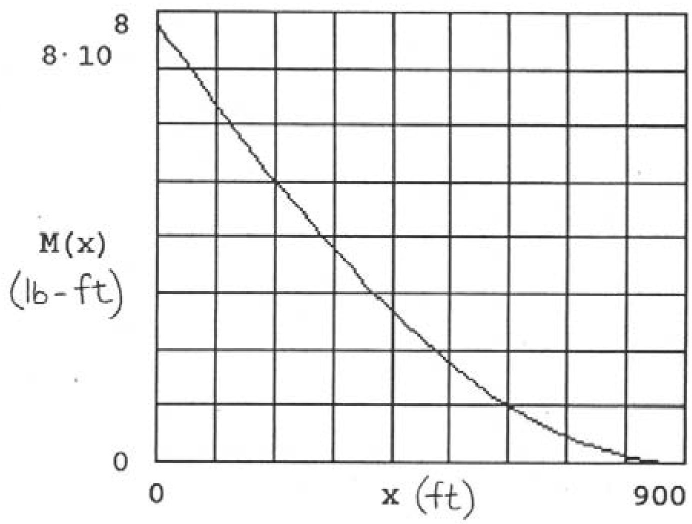
$\Sigma M_{A'} = 0: \quad M - \int_x^{900} 80\sqrt{x'}\,(x' - x)\,dx' = 0$

$M = 80\left\{\frac{2}{5}x'^{5/2} - x\,\frac{2}{3}x'^{3/2}\right\}\Big|_x^{900}$

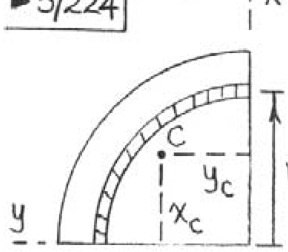
$= \underline{7.78(10^8) - 1.440(10^6)x + \frac{64}{3}x^{5/2}}$

$M\Big|_{x=450'} = \underline{2.21(10^8) \text{ lb-ft}}$

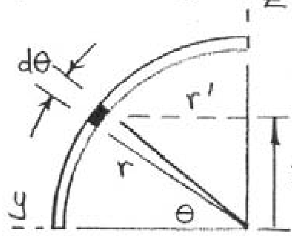

2·10^6
V(x)
(lb)
0
0
x (ft)
900


8·10^8
M(x)
(lb-ft)
0
0
x (ft)
900

▶ 5/224



$(r' = r\cos\theta)$



Element is quarter-circular ring parallel to x-y plane with area

$$dA = \frac{\pi r'}{2} r\,d\theta = \frac{\pi}{2} r^2 \cos\theta\,d\theta$$

Centroid of element is at

$$x_c = y_c = \frac{2r'}{\pi} = \frac{2}{\pi} r\cos\theta \quad \text{(S.P. 5/1)}$$

$$\int x_c\,dA = \int \left(\frac{2}{\pi} r\cos\theta\right)\frac{\pi}{2} r^2 \cos\theta\,d\theta$$

$$= r^3 \int_0^{\pi/2} \cos^2\theta\,d\theta = \frac{\pi r^3}{4}$$

$$\int y_c\,dA = \int x_c\,dA = \frac{\pi r^3}{4}$$

$$\int z_c\,dA = \int (r\sin\theta)\left(\frac{\pi}{2} r^2 \cos\theta\,d\theta\right) = \frac{\pi r^3}{2}\int_0^{\pi/2} \sin\theta\cos\theta\,d\theta$$

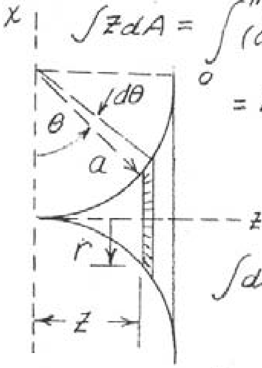
$$= \pi r^3/4$$

$$A = \int \frac{\pi}{2} r^2 \cos\theta\,d\theta = \frac{\pi r^2}{2}\int_0^{\pi/2} \cos\theta\,d\theta = \pi r^2/2$$

From $\bar{x} = \dfrac{\int x_c\,dA}{A}$, etc, we have

$$\bar{x} = \bar{y} = \bar{z} = \frac{\pi r^3/4}{\pi r^2/2} = \underline{r/2}$$

▶ 5/225 $dA = 2\pi r\, a\, d\theta = 2\pi a^2 (1-\cos\theta)\, d\theta$



$$\int z\, dA = \int_0^{\pi/2} (a\sin\theta)(2\pi a^2)(1-\cos\theta)\, d\theta$$

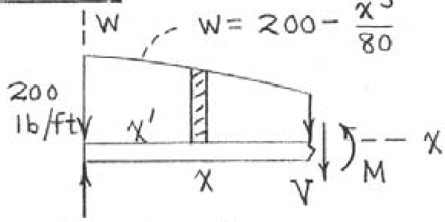
$$= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta\cos\theta)\, d\theta$$

$$= 2\pi a^3 \left(1 - \tfrac{1}{2}\right) = \pi a^3$$

$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1-\cos\theta)\, d\theta = 2\pi a^2 \left(\frac{\pi}{2} - 1\right)$$

$$\bar{z} = \frac{\int z\, dA}{A} = \frac{\pi a^3}{2\pi a^2\left(\frac{\pi}{2}-1\right)} = \underline{\frac{a}{\pi - 2}}$$

*5/226 (w & R_A from Prob. 5/113)

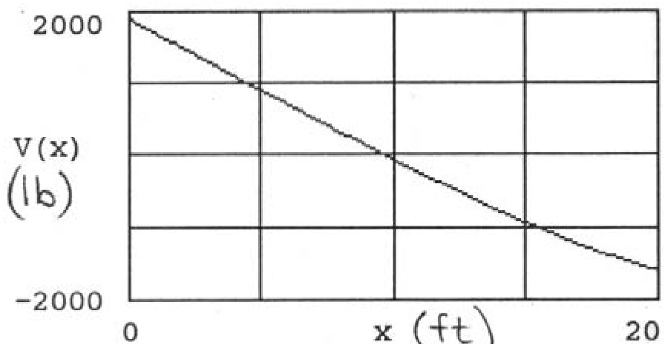


$$+\uparrow\Sigma F = 0: \quad 1900 - V - \int_0^x \left(200 - \frac{x'^3}{80}\right) dx'$$

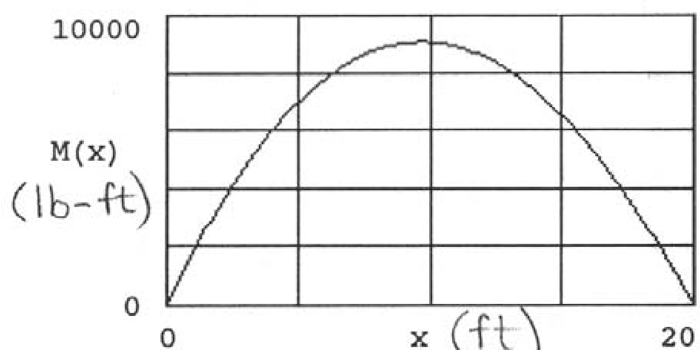
$$\underline{V = 1900 - 200x + \frac{x^4}{320} \quad \text{(lb)}}$$

$$\Sigma M = 0: \quad M + \int_0^x \left(200 - \frac{x'^3}{80}\right)(x - x')\, dx' - 1900x = 0$$

$$\underline{M = 1900x - 100x^2 + \frac{x^5}{1600} \quad \text{(lb-ft)}}$$

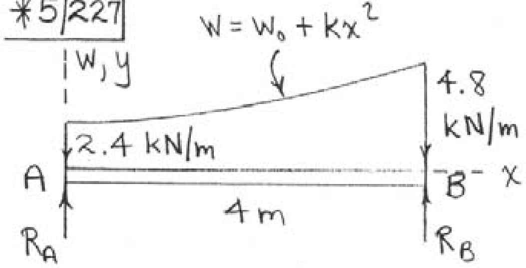


$\underline{V_{max} = 1900 \text{ lb @ } x = 0}$



$\underline{M_{max} = 9080 \text{ lb-ft @ } x = 9.63'}$

*5/227



$w_0 = 2.4$ kN/m

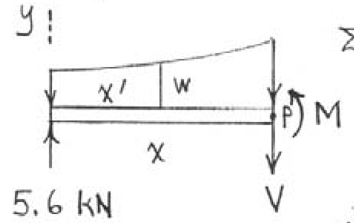
$4.8 = 2.4 + k(4)^2$

$k = 0.15$ kN/m^3

So $w = 2.4 + 0.15x^2$

$\circlearrowleft \Sigma M_A = 0: \int_0^4 (2.4 + 0.15x^2)x\,dx - 4R_B = 0, \quad R_B = 7.2$ kN

$\Sigma F_y = 0: R_A + 7.2 - \int_0^4 (2.4 + 0.15x^2)\,dx = 0, \quad R_A = 5.6$ kN



$\Sigma F_y = 0: 5.6 - \int_0^x (2.4 + 0.15x^2)\,dx - V = 0$

$\underline{V = 5.6 - 2.4x - 0.05x^3}$

$\circlearrowleft \Sigma M_P = 0: M + \int_0^x (2.4 + 0.15x'^2)(x - x')\,dx' - 5.6x = 0$

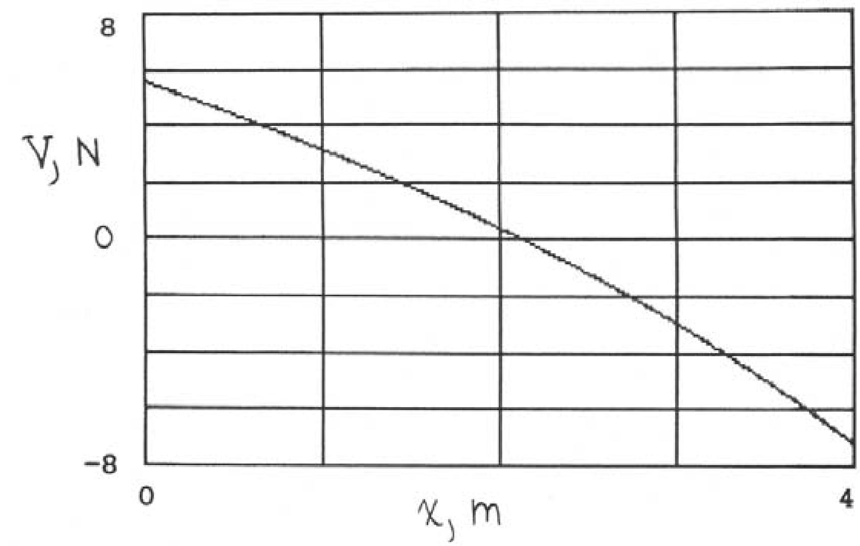
$M = 5.6x - \left[2.4xx' + \frac{0.15}{3}x'^3x - \frac{2.4}{2}x'^2 - \frac{0.15}{4}x'^4\right]_0^x$

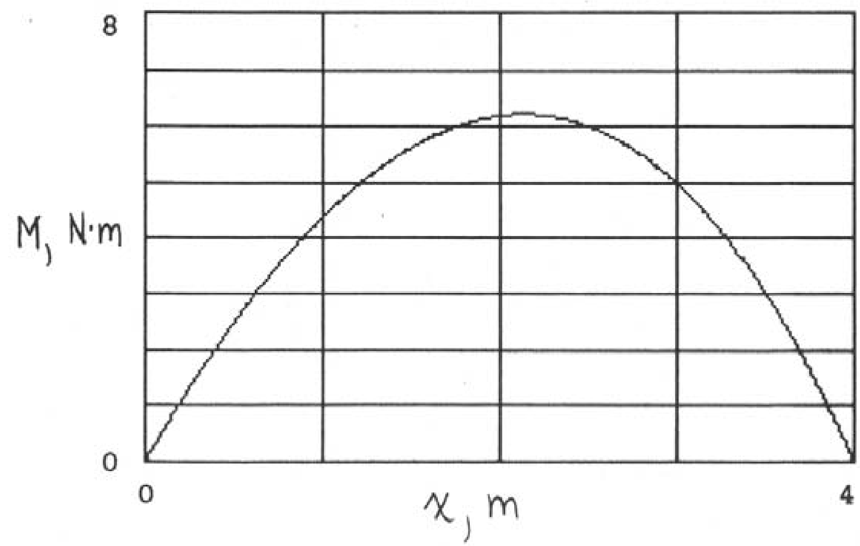
$= 5.6x - 1.2x^2 - 0.0125x^4$

For a maximum, $\frac{dM}{dx} = 5.6 - 2.4x - 0.05x^3 = 0$

Solve numerically to obtain

$\underline{M_{max} = 6.23 \text{ kN·m @ } x = 2.13 \text{ m}}$


8
V, N
0
−8
0
x, m
4

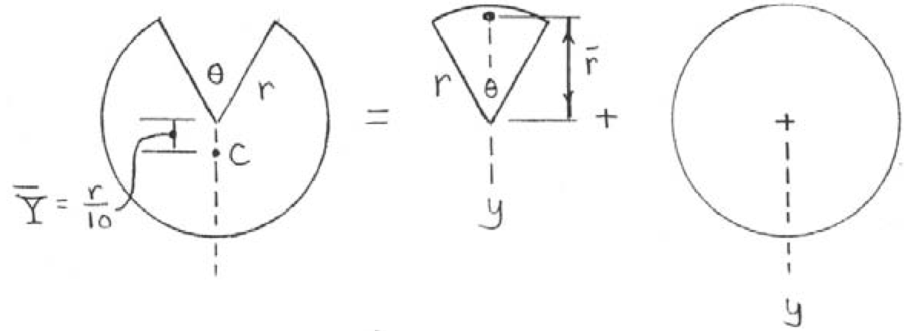

8
M, N·m
0
0
x, m
4

*5/228

Arc = $(2\pi-\theta)r$

Negative arc $r\theta$

Positive arc 2π



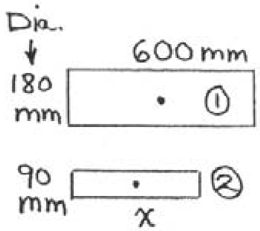
From Sample Problem 5/1, $\bar{r} = \dfrac{r \sin\frac{\theta}{2}}{\theta/2}$

$$\overline{Y} = \frac{\Sigma L\bar{y}}{\Sigma L} : \quad \frac{r}{10} = \frac{(-r\theta)\left(-r\,\frac{\sin\frac{\theta}{2}}{\theta/2}\right) + 2\pi r(0)}{(2\pi-\theta)r}$$

Simplify to $10 \sin\frac{\theta}{2} = \pi - \frac{\theta}{2}$ and solve numerically to obtain $\underline{\theta = 33.1^\circ}$

*5/229

$$V_1 = \frac{\pi\, 180^2}{4}(600) = 15\,270(10^3)\ \text{mm}^3$$



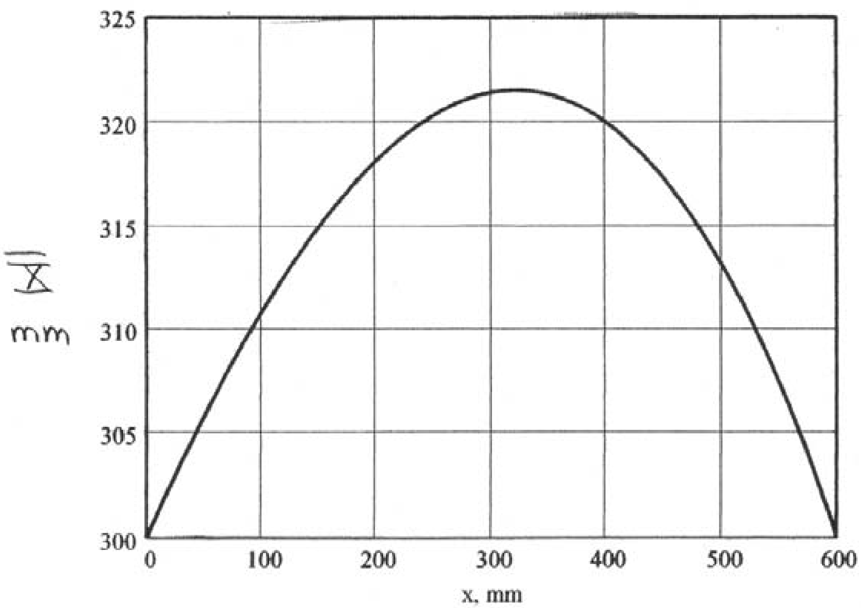
$$\bar{x}_1 = 300\ \text{mm}$$

$$V_2 = -\frac{\pi\, 90^2}{4}x = -6.36(10^3)x$$

$$\bar{x}_2 = -\frac{x}{2}$$

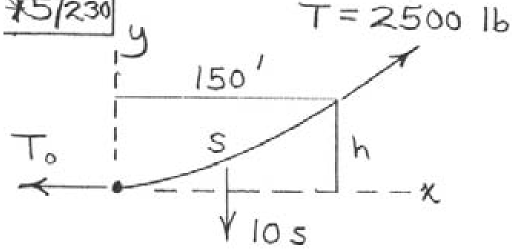
$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{15\,270(10^3)(300) - 6.36(10^3)x\left(\frac{x}{2}\right)}{15\,270(10^3) - 6.36(10^3)x}$$

$$= \frac{4580(10^3) - 3.18x^2}{15\,270 - 6.36x}$$



$$\bar{X}_{max} = 322\ \text{mm} \ @\ x = 322\ \text{mm}$$

*5/230



From Eq. 5/21,

$$\frac{T}{T_o} = \cosh\frac{\mu x}{T_o}$$

$$\frac{2500}{T_o} = \cosh\frac{10(150)}{T_o}$$

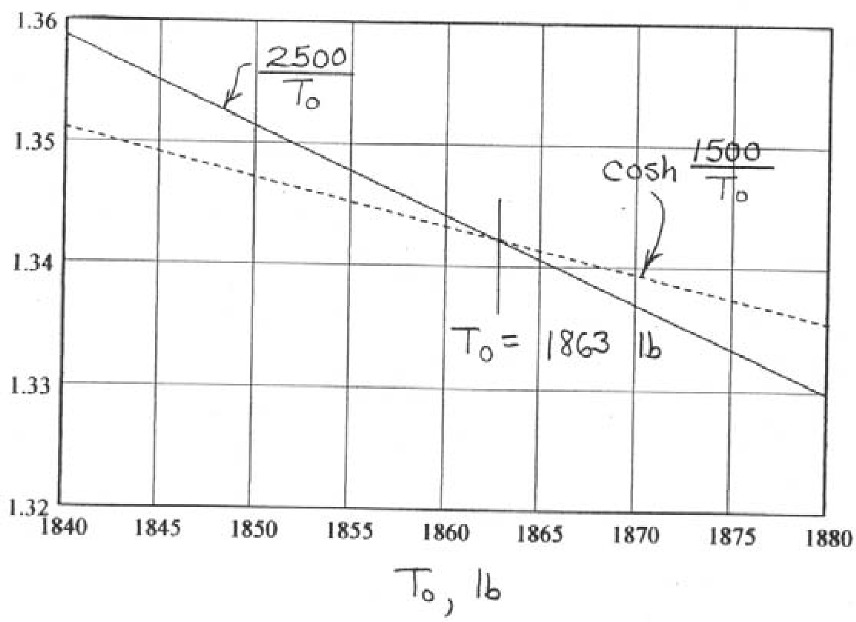
Numerical or graphical (see below) solution :

$T_o = 1863$ lb. Then Eq. 5/22 :

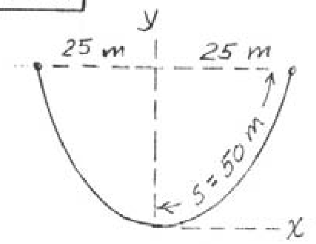
$T = T_o + \mu h$: $2500 = 1863 + 10h$, $\underline{h = 63.7 \text{ ft}}$

5/22: $S = 2s = 2\frac{T_o}{\mu}\sinh\frac{\mu x}{T_o}$

$$= 2\,\frac{1863}{10}\sinh\frac{10(150)}{1863} = \underline{333 \text{ ft}}$$



*5/231



Eq. 5/20 $s = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$

$50 = \frac{T_o}{\mu} \sinh \frac{25\mu}{T_o}$

$\frac{50\mu}{T_o} - \sinh \frac{25\mu}{T_o} = R = 0$

Write and run program for $R = f\left(\frac{\mu}{T_o}\right)$ & find μ/T_o for $R = 0$. Result is $\mu/T_o = 0.0871$

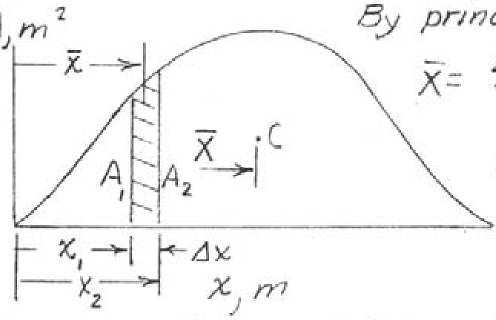
From Eq. 5/19, $y = \frac{T_o}{\mu}\left(\cosh \frac{\mu x}{T_o} - 1\right)$

$= \frac{1}{0.0871}\left(\cosh 0.0871[25] \quad -1\right)$

$h = y = 3.468/0.0871 = \underline{39.8 \text{ m}}$

Result depends only on the geometry of the catenary.

*5/232 V = submerged volume



By principle of moments

$$\bar{X} = \frac{\Sigma \bar{x}\Delta V}{\Delta V}$$

$$\bar{x} = \tfrac{1}{2}(x_1 + x_2)$$

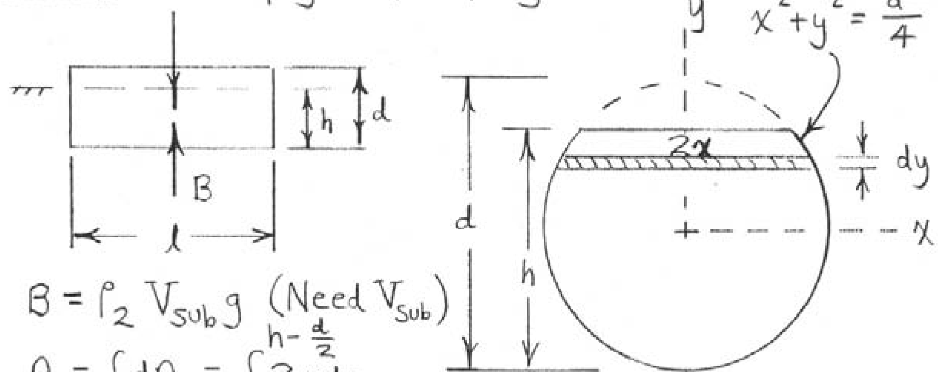
$$\Delta V = \tfrac{1}{2}(A_1 + A_2)(x_2 - x_1)$$

Thus $$\bar{X} = \frac{\Sigma \frac{1}{2}(x_1 + x_2)\frac{1}{2}(A_1 + A_2)(x_2 - x_1)}{\Sigma \frac{1}{2}(A_1 + A_2)(x_2 - x_1)}$$

$$= \frac{\Sigma \frac{1}{2}(A_1 + A_2)(x_2^2 - x_1^2)}{\Sigma (A_1 + A_2)(x_2 - x_1)}$$

Write & carry out program for quotient of sums & get $\underline{\bar{X} = 24\text{ m}}$

*5/233 $W = \rho_1 V g = \rho_1 \pi \frac{d^2}{4} l g$



$B = \rho_2 V_{sub} g$ (Need V_{sub})

$$A = \int dA = \int_{-d/2}^{h-\frac{d}{2}} 2x\,dy$$

$$= 2\int_{-d/2}^{h-d/2} \sqrt{\frac{d^2}{4}-y^2}\,dy = 2\cdot\frac{1}{2}\left[y\sqrt{\frac{d^2}{4}-y^2}+\frac{d^2}{4}\sin^{-1}\frac{2y}{d}\right]_{-\frac{d}{2}}^{h-\frac{d}{2}}$$

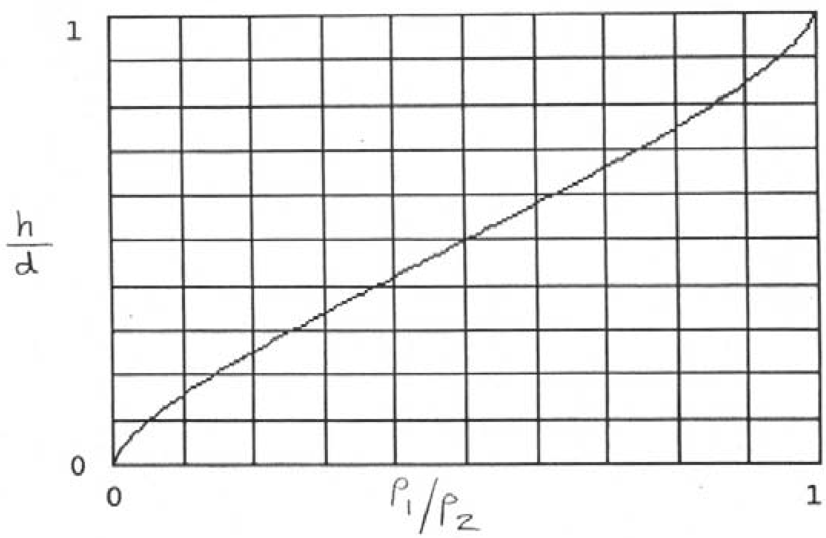
$$= \left(h-\frac{d}{2}\right)\sqrt{hd-h^2}+\frac{d^2}{4}\sin^{-1}\left(\frac{2h-d}{d}\right)+\frac{\pi d^2}{8}$$

So $B = \rho_2 g A l$, A given just above

$\uparrow \Sigma F = 0: \quad \rho_2 g A l - \rho_1 \pi \frac{d^2}{4} l g = 0$

or $\left(h-\frac{d}{2}\right)\sqrt{hd-h^2}+\frac{d^2}{4}\sin^{-1}\left(\frac{2h-d}{d}\right)+\frac{\pi d^2}{8} = \frac{\rho_1}{\rho_2}\pi\frac{d^2}{4}$

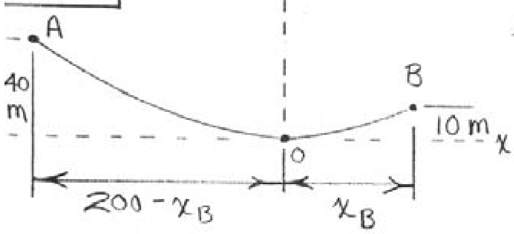
Strategy: Set $d=1$ and numerically solve the above equation for h for values of $\frac{\rho_1}{\rho_2}$ between 0 and 1:



For pine wood and salt water, $\rho_1 = 480 \ \frac{kg}{m^3}$ and $\rho_2 = 1030 \ \frac{kg}{m^3}$. So $\frac{\rho_1}{\rho_2} = \frac{480}{1030} = 0.466$

Numerical solution : $\frac{h}{d} = \underline{0.473} = r$

*5/234



$\mu_{cable} = 20(9.81) = 196.2 \frac{N}{m}$

Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh \frac{x}{T_0/\mu} - 1\right]$

At B: $10 = \frac{T_0}{\mu}\left[\cosh \frac{x_B}{T_0/\mu} - 1\right]$

At A: $40 = \frac{T_0}{\mu}\left[\cosh \frac{200 - x_B}{T_0/\mu} - 1\right]$

Simultaneous numerical solution: $\begin{cases} x_B = 67.1 \text{ m} \\ T_0/\mu = 227 \text{ m} \end{cases}$

$T_A = T_0 + \mu y_A$: $75\,000 = 227\mu + \mu(40)$

$\mu = 281$ N/m

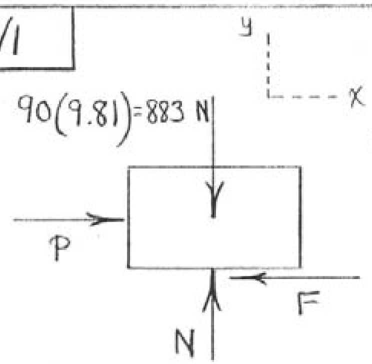
$\mu = \mu_{cable} + \mu_{ice}$: $281 = 196.2 + \mu_{ice}$

$\mu_{ice} = 84.7$ N/m

$\rho = \frac{\mu}{g} = \frac{84.7}{9.81} = \underline{8.63 \text{ kg/m}}$

The configuration does not depend on μ.

6/1



$\Sigma F_y = 0$: $N - 883 = 0$, $N = 883$ N

$F_{max} = \mu_s N = 0.5(883) = 441$ N

$\Sigma F_x = 0$ yields $F = P$ for equilibrium

(a) $P = 300$ N, $\underline{F = 300 \text{ N}} < F_{max}$, OK

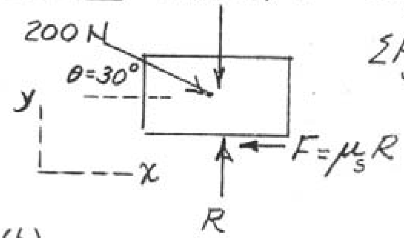
(b) $P = 400$ N, $\underline{F = 400 \text{ N}} < F_{max}$, OK

(c) $P = 500$ N, $F = 500 \text{ N} > F_{max}$, motion

So $F = \mu_k N = 0.4(883) = \underline{353 \text{ N}}$

(all to the left)

6/2



(a) $\Sigma F_x = 0;\ 200\cos 30^\circ - \mu_s R = 0$

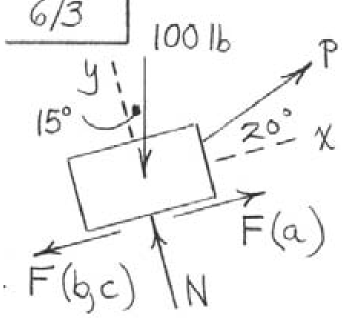
$\Sigma F_y = 0;\ R - 200\sin 30^\circ - 50(9.81) = 0$

$R = 590.5\ \text{N}$

So $\mu_s = \dfrac{200\cos 30^\circ}{590.5} = \underline{0.29}$

(b)
For $\theta = 45^\circ$, $\Sigma F_x = 0$ gives $F = 200\cos 45^\circ = \underline{141.4\ \text{N}}$
which is $< \mu_s R_b$

6/3



(a) $P = 0$

$\Sigma F_y = 0: \ N - 100\cos 15° = 0$

$N = 96.6 \text{ lb}$

Assume equilibrium:

$\Sigma F_x = 0: \ F - 100\sin 15° = 0$

$F = 25.9 \text{ lb}$

$F_{max} = \mu_s N = 0.25(96.6) = 24.1 < F$; assumption invalid and $F = F_k = \mu_k N = 0.2(96.6) = \underline{19.32 \text{ lb}}$ up the incline.

(b) $P = 40$ lb

$\Sigma F_y = 0: \ N - 100\cos 15° + 40\sin 20° = 0, \quad N = 82.9 \text{ lb}$

$\Sigma F_x = 0: \ 40\cos 20° - 100\sin 15° - F = 0, \quad \underline{F = 11.71 \text{ lb}}$

$F_{max} = \mu_s N = 0.25(82.9) = 20.7 \text{ lb}$; assumption OK

(c) $P = 60$ lb

$\Sigma F_y = 0: \ N - 100\cos 15° + 60\sin 20° = 0, \quad N = 76.1 \text{ lb}$

$\Sigma F_x = 0: \ 60\cos 20° - 100\sin 15° - F = 0, \quad F = 30.5 \text{ lb}$

$F_{max} = \mu_s N = 0.25(76.1) = 19.02 \text{ lb} < F$; assumption invalid

$F = \mu_k N = 0.2(76.1) = \underline{15.21 \text{ lb}}$ down the incline

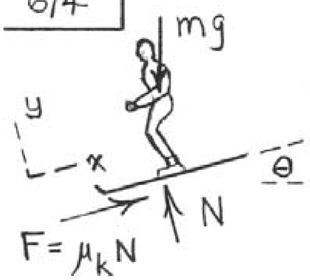
(d) To initiate motion, set $F = \mu_s N = 0.25N$ down the incline:

$\Sigma F_y = 0: \ N - 100\cos 15° + P\sin 20° = 0$

$\Sigma F_x = 0: \ P\cos 20° - 100\sin 15° - 0.25N = 0$

Solve to obtain $\begin{cases} N = 79.9 \text{ lb} \\ \underline{P = 48.8 \text{ lb}} \end{cases}$

6/4



$\Sigma F_x = 0: \; 0.08N - mg\sin\theta = 0$

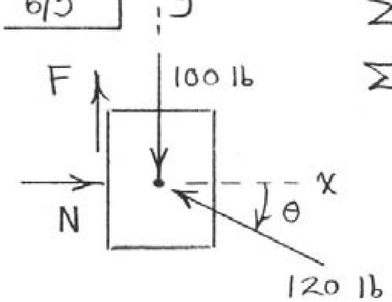
$$N = \frac{mg\sin\theta}{0.08}$$

$\Sigma F_y = 0: \; N - mg\cos\theta = 0$

$$N = mg\cos\theta$$

So $\dfrac{mg\sin\theta}{0.08} = mg\cos\theta$, $\tan\theta = 0.08$, $\underline{\theta = 4.57°}$

6/5



$\Sigma F_x = 0: \quad N - 120\cos\theta = 0$

$\Sigma F_y = 0: \quad F - 100 + 120\sin\theta = 0$

(a) $\theta = 15°$

$N = 115.9 \text{ lb}, \quad F = 68.9 \text{ lb}$

$F_{max} = \mu_s N = 0.50\,(115.9)$

$= 58.0 \text{ lb}$

$F > F_{max}$, so assumption of equilibrium is invalid and $F = F_k = \mu_k N = 0.40\,(115.9)$

$= \underline{46.4 \text{ lb (up)}}$

(b) $\theta = 30°$

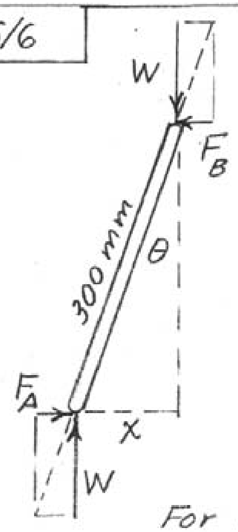
$N = 103.9 \text{ lb}, \quad F = 40 \text{ lb}$

$F_{max} = \mu_s N = 0.50\,(103.9) = 52.0 \text{ lb}$

$F < F_{max}$, so equilibrium assumption OK

& $\underline{F = 40 \text{ lb (up)}}$

6/6



For $x = 75$ mm, $\theta = \sin^{-1}\frac{75}{300} = 14.5°$

Friction angle $\phi = \tan^{-1}\mu_s$

for A is $\phi_A = \tan^{-1}0.40 = 21.8°$

" B " $\phi_B = \tan^{-1}0.30 = 16.7°$

Since $\theta < \phi_A$ & ϕ_B, bar does not slip & $F_A = F_B = W\tan\theta$

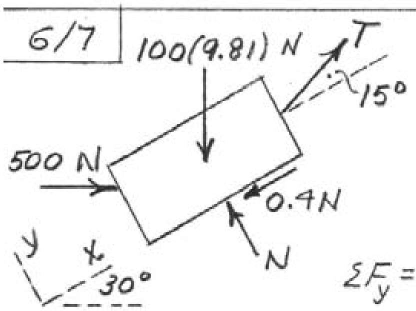
$= 50(9.81)\tan 14.5°$

$= \underline{126.6\text{ N}}$

For increased x bar slips first at B with $\theta = \phi_B = 16.7°$. Thus $x_{max} = 300\sin 16.7°$

$= \underline{86.2\text{ mm}}$

6/7



$\Sigma F_x = 0;$

$T\cos 15° - 981 \sin 30° - 0.4N + 500 \cos 30° = 0$

$0.966T - 0.4N = 57.5$

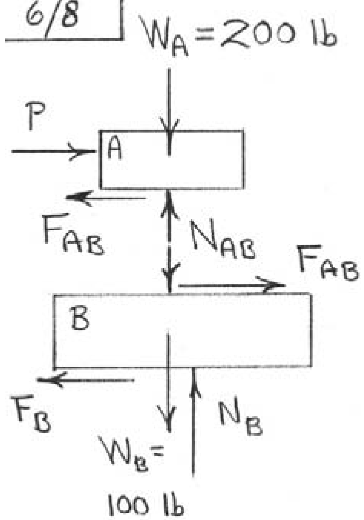
$\Sigma F_y = 0;$ $N - 500 \sin 30° - 981 \cos 30° + T \sin 15° = 0$

$0.259T + N = 1099.6$

Solve simultaneously & get $N = 979$ N

$\underline{T = 465 \text{ N}}$

6/8



From $\uparrow \Sigma F_y = 0$,

$N_{AB} = 200$ lb, $N_B = 300$ lb

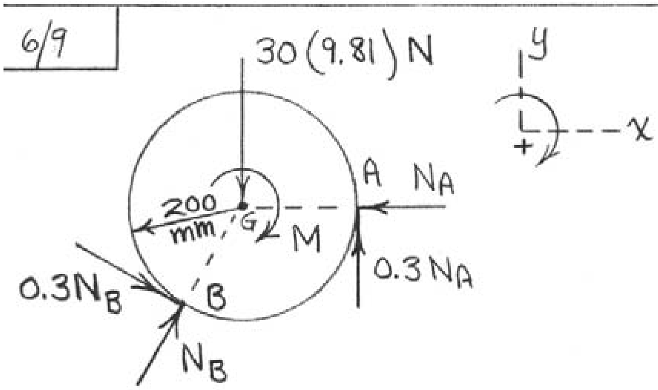
$$\begin{cases} F_{AB_{max}} = \mu_s N_{AB} = 0.5(200) = 100 \text{ lb} \\ F_{B_{max}} = \mu_s N_B = 0.25(300) = 75 \text{ lb} \end{cases}$$

(a) $P = 60$ lb : Both blocks remain stationary, as $\rightarrow \Sigma F_x = 0$ yields $F_{AB} = 60$ lb and $F_B = 60$ lb, both less than the maximum

(b) $P = 80$ lb : No relative motion between A and B, but sliding between B and the supporting surface; motion is to the right.

(c) $P = 120$ lb : A slides to the right relative to B; $F_{AB} = \mu_k N_{AB} = 0.3(200) = 60$ lb $< F_{B_{max}}$) so B remains stationary.

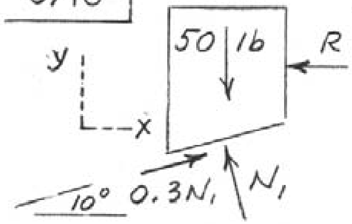
6/9



$$\begin{cases} \Sigma M_G = 0: \quad M - 0.3\,(N_A + N_B)\,0.2 = 0 & (1) \\ \Sigma F_x = 0: \quad N_B \sin 30° + 0.3\,N_B \cos 30° - N_A = 0 & (2) \\ \Sigma F_y = 0: \quad N_B \cos 30° - 0.3\,N_B \sin 30° - 30(9.81) \\ \qquad + 0.3\,N_A = 0 & (3) \end{cases}$$

Solution of Eqs. (1)–(3): $\begin{cases} N_B = 312 \text{ N} \\ N_A = 237 \text{ N} \\ M = 32.9 \text{ N·m} \end{cases}$

6/10

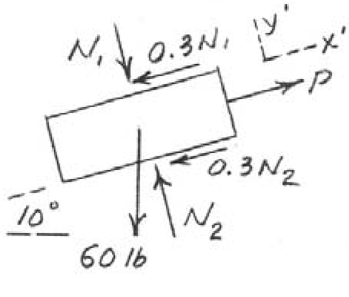


50-lb block: $\Sigma F_y = 0$;

$$N_1 \cos 10^\circ + 0.3 N_1 \sin 10^\circ - 50 = 0$$

$$N_1 (0.985 + 0.0521) = 50$$

$$N_1 = 50/1.0369 = 48.2 \text{ lb}$$



60-lb block;

$\Sigma F_{y'} = 0$; $N_2 - 48.2 - 60 \cos 10^\circ = 0$

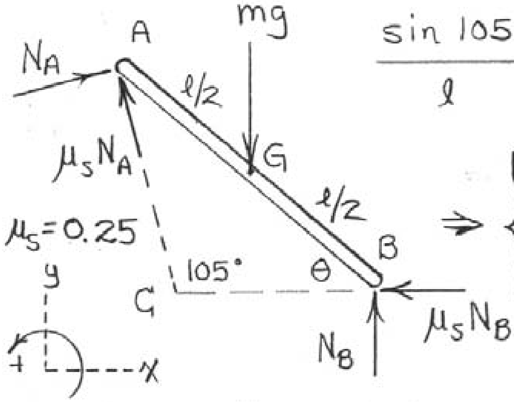
$$N_2 = 107.3 \text{ lb}$$

$\Sigma F_{x'} = 0$; $P - 0.3(48.2 + 107.3) - 60 \sin 10^\circ = 0$

$$P = 46.7 + 10.4$$

$$\underline{P = 57.1 \text{ lb}}$$

6/11



From law of sines:

$$\frac{\sin 105°}{l} = \frac{\sin\theta}{\overline{AC}} = \frac{\sin(75°-\theta)}{\overline{BC}}$$

$$\Rightarrow \begin{cases} \overline{AC} = \dfrac{l \sin\theta}{\sin 105°} \\ \overline{BC} = \dfrac{l\sin(75°-\theta)}{\sin 105°} \end{cases}$$

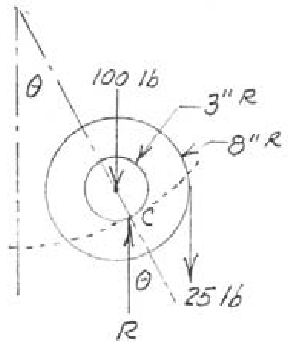
$$\Sigma F_x = 0: \; N_A \cos 15° - 0.25 N_A \sin 15° - 0.25 N_B = 0$$

$$\Sigma F_y = 0: \; N_A \sin 15° + 0.25 N_A \cos 15° + N_B - mg = 0$$

$$\Sigma M_C = 0: \; -N_A\left(\frac{l\sin\theta}{\sin 105°}\right) + N_B\left(\frac{l\sin(75°-\theta)}{\sin 105°}\right) - mg\left[\frac{l\sin(75°-\theta)}{\sin 105°} - \frac{l}{2}\cos\theta\right] = 0$$

Solution: $\begin{cases} N_A = 0.244mg \\ N_B = 0.878mg \\ \theta = 59.9° \end{cases}$

6/12



$\Sigma M_C = 0;\ 100(3\sin\theta) - 25(8 - 3\sin\theta) = 0$

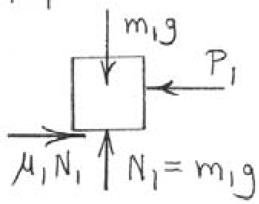
$\sin\theta = 8/15,\ \underline{\theta = 32.2°}$

$\mu_{min} = \tan\theta = \tan 32.2°$

$= \underline{0.630}$

6/13

m_1 alone:

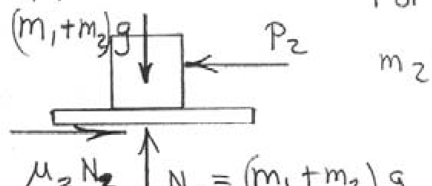


For impending slip between person and board,

$\xrightarrow{+} \Sigma F = 0: \quad \mu_1 m_1 g - P_1 = 0$

$P_1 = \mu_1 m_1 g$

m_1 plus m_2:



For impending slip beneath m_2, $\xrightarrow{+} \Sigma F = 0$:

$-P_2 + \mu_2 (m_1+m_2) g = 0$

$P_2 = \mu_2 (m_1+m_2) g$

For first slipping to occur beneath m_2:

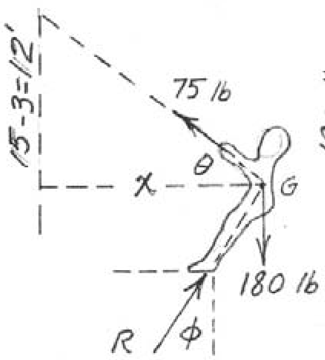
$P_2 < P_1$

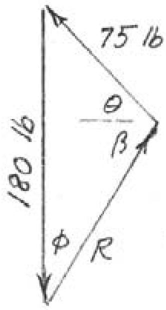
$\mu_2 (m_1 + m_2) g < \mu_1 m_1 g$

$$\underline{\mu_2 < \frac{m_1}{m_1+m_2} \mu_1}$$

Numbers: $\underline{\mu_2} < \frac{80}{80+10}(0.60) = \underline{0.533}$

6/14





$\phi = \tan^{-1} 0.40 = 21.8°$

$\beta = 90 - 21.8 = 68.2°$

Law of sines

$$\frac{180}{\sin(\theta+\beta)} = \frac{75}{\sin 21.8°}$$

$$\theta + \beta = \sin^{-1} \frac{180 \sin 21.8}{75}$$

$$= \sin^{-1} 0.891 = 63.0°$$

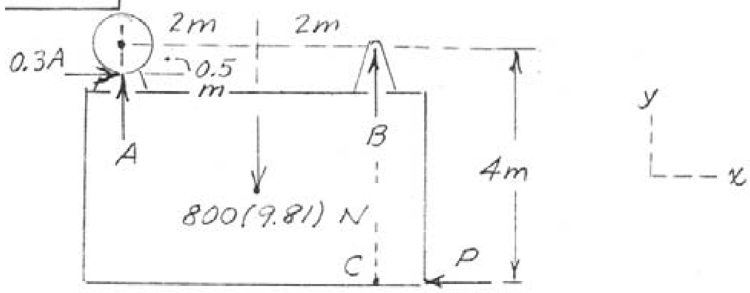
or $117.0°$

$63.0°$ sol. not possible,

so $\theta = 117.0 - 68.2 = 48.8°$

$\frac{12}{x} = \tan 48.8°$, $x = 12/1.14 = \underline{10.52 \text{ ft}}$

6/15

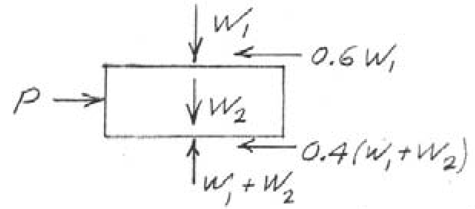


$\Sigma M_C = 0;\quad 800(9.81)2 - 4A - 0.3A(4-0.5) = 0,\ A = 3108\ \text{N}$

$\Sigma F_x = 0;\quad P - 0.3(3108) = 0,\ \underline{P = 932\ \text{N}}$

6/16 There are two possibilities

(a) Middle block moves; bottom one does not

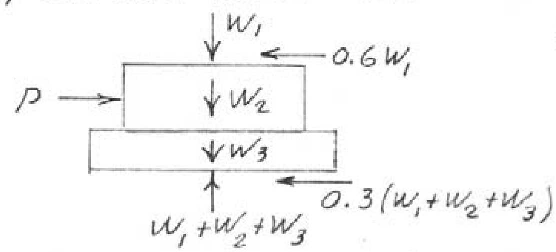


$W_1 = 100(9.81) = 981\ N$

$W_2 = 50(9.81) = 490.5\ N$

$\Sigma F = 0;\quad P = 0.6(981) + 0.4(981 + 490.5) = 1177\ N$

(b) Bottom block moves with middle block

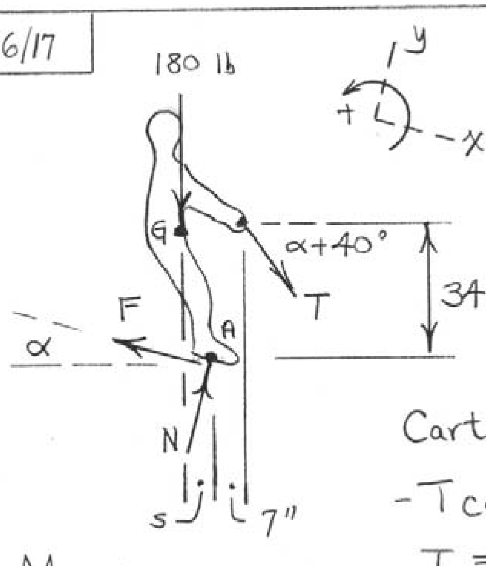


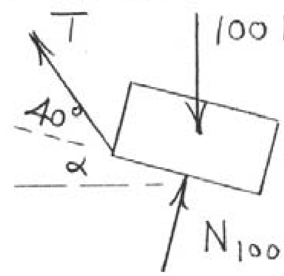
$W_3 = 20(9.81) = 196.2\ N$

$\Sigma F = 0;\ P = 0.6(981) + 0.3(981 + 490.5 + 196.2) = 1089\ N$

$1088 < 1177$ so case (b) occurs & $\underline{P = 1089\ N}$

6/17





$\alpha = \tan^{-1} \frac{3}{10} = 16.70°$

Cart: $\Sigma F_x = 0$:

$-T \cos 40° + 100 \sin \alpha = 0$

$T = 37.5$ lb

Man:

$\Sigma F_x = 0$: $-F + 180 \sin \alpha + 37.5 \cos 40° = 0$

$F = 80.5$ lb

$\Sigma F_y = 0$: $N - 180 \cos \alpha - 37.5 \sin 40° = 0$

$N = 196.5$ lb

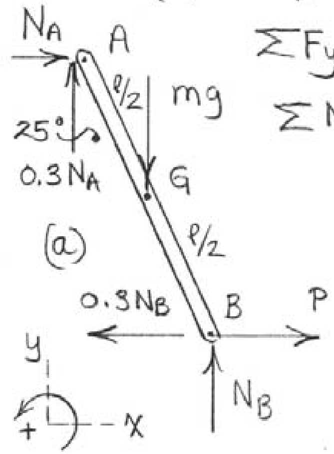
$\mu_s = \frac{F}{N} = \frac{80.5}{196.5} = \underline{0.409}$

$\Sigma M_A = 0$: $180 s - 37.5 \cos(\alpha + 40°)(34)$

$- 37.5 \sin(\alpha + 40°)(7) = 0$

$\underline{s = 5.11 \text{ in.}}$

6/18

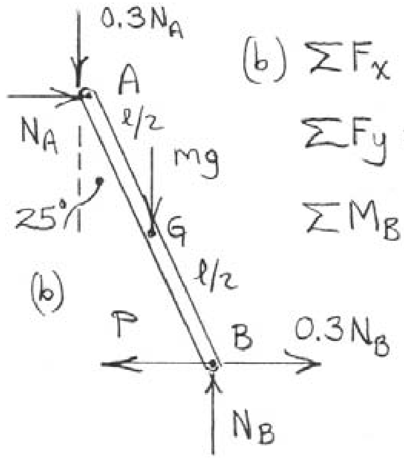


(a) $\Sigma F_x = 0: \; P - 0.3N_B + N_A = 0$

$\Sigma F_y = 0: \; N_B - mg + 0.3N_A = 0$

$\Sigma M_B = 0: \; mg\left(\frac{\ell}{2}\sin 25^\circ\right) - N_A\left(\ell\cos 25^\circ\right) - 0.3N_A\left(\ell\sin 25^\circ\right) = 0$

Solution: $\begin{cases} N_A = 0.205\,mg \\ N_B = 0.939\,mg \\ \underline{P = 0.0771\,mg} \end{cases}$



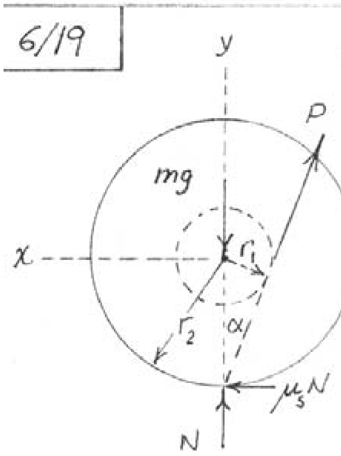
(b) $\Sigma F_x = 0: \; -P + 0.3N_B + N_A = 0$

$\Sigma F_y = 0: \; N_B - mg - 0.3N_A = 0$

$\Sigma M_B = 0: \; mg\left(\frac{\ell}{2}\sin 25^\circ\right) - N_A\left(\ell\cos 25^\circ\right) + 0.3N_A\left(\ell\sin 25^\circ\right) = 0$

Solution: $\begin{cases} N_A = 0.271\,mg \\ N_B = 1.081\,mg \\ \underline{P = 0.595\,mg} \end{cases}$

6/19



Forces must be concurrent for equilibrium so $\alpha = \sin^{-1}\frac{r_1}{r_2}$

$\Sigma F_y = 0$; $P\cos\alpha + N - mg = 0$

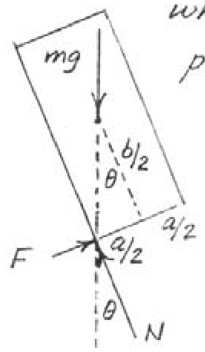
$\Sigma F_x = 0$; $P\sin\alpha - \mu_s N = 0$

Eliminate N & get

$$P = \frac{\mu_s mg}{\sin\alpha + \mu_s\cos\alpha} = \frac{\mu_s mg r_2}{r_1 + \mu_s\sqrt{r_2^2 - r_1^2}}$$

6/20 Block slips if $F = \mu N$ or $mg \sin\theta = \mu mg \cos\theta$

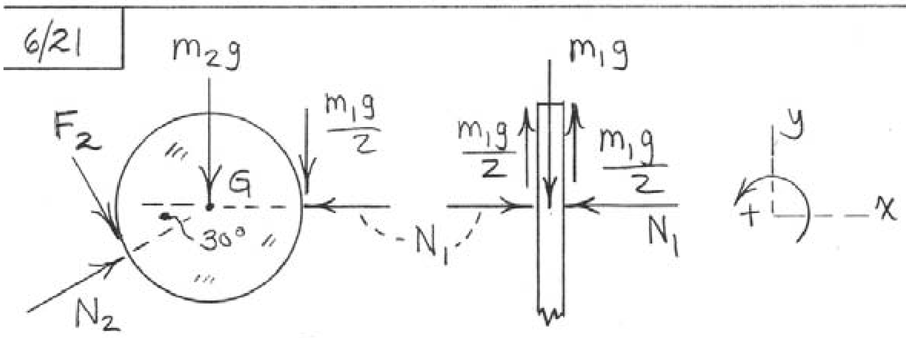
when angle reaches $\theta = \tan^{-1}\mu$



provided $\frac{a}{2} > \frac{b}{2}\tan\theta$ or $\underline{a > \mu b}$

Tips first if $\underline{a < \mu b}$

6/21



Cylinder ② Paper ①

Cylinder:

$$\begin{cases} \Sigma F_x = 0: \quad N_2 \cos 30° + F_2 \sin 30° - N_1 = 0 & (1) \\ \Sigma F_y = 0: \quad N_2 \sin 30° - F_2 \cos 30° - m_2 g - \frac{m_1 g}{2} = 0 & (2) \\ \Sigma M_G = 0: \quad F_2 r - \frac{m_1 g}{2} r = 0 & (3) \end{cases}$$

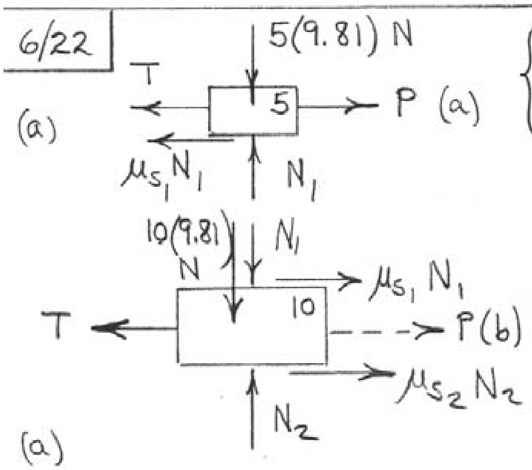
Solve simultaneously to obtain

$$\mu = \frac{m_1 g/2}{N_1} = \frac{m_1}{3.73 m_1 + 3.46 m_2} > \frac{F_2}{N_2}$$

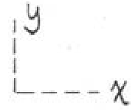
where we have assumed and then verified that slipping occurs first on right side of the cylinder.

For $m_1 >> m_2$, $\underline{\mu = 0.268}$

6/22



$$\begin{cases} \mu_{s_1} = 0.50 \\ \mu_{s_2} = 0.60 \end{cases}$$



From $\Sigma F_y = 0$,

$$\begin{cases} N_1 = 49.0 \text{ N} \\ N_2 = 147.2 \text{ N} \end{cases}$$

$$\Sigma F_x = 0: \begin{cases} P - T - 0.50(49.0) = 0 \\ -T + 0.50(49.0) + 0.60(147.2) = 0 \end{cases}$$

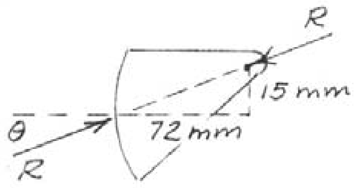
$\underline{T = 112.8 \text{ N}}$, $\underline{P = 137.3 \text{ N}}$

(b) Now P is applied to 10-kg block & we reverse all friction forces above:

$$\Sigma F_x = 0: \begin{cases} -T + 0.50(49.0) = 0 \\ -T - 0.50(49.0) - 0.60(147.2) + P = 0 \end{cases}$$

$\underline{T = 24.5 \text{ N}}$, $\underline{P = 137.3 \text{ N}}$

6/23



Req'd. minimum coeff. of friction is

$\mu_s = \tan\phi = \tan\theta$

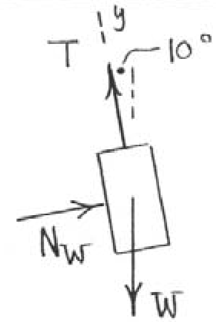
$= \frac{15}{72} = \underline{0.208}$

$\Sigma F = 0$ for rope; $2R\sin\theta = 600$

$R = \frac{300}{15/\sqrt{15^2+72^2}} = \underline{1471\ N}$

6/24 $\Sigma F_y = 0: \; T - W\cos 10° = 0$

$$T = W \cos 10°$$

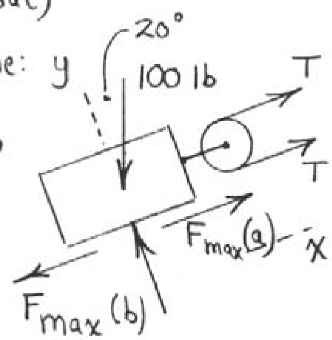


100-lb block:

$\Sigma F_y = 0: \quad N - 100\cos 20° = 0$

$$N = 94.0 \text{ lb} \quad \text{(throughout)}$$

(a) Motion impends down incline:



$\Sigma F_x = 0: \; 2T - 100\sin 20° + F_{max} = 0$

With $F_{max} = \mu_s N = 0.3(94.0)$

$= 28.2$ lb and $T = W\cos 10°$,

$W = 3.05$ lb

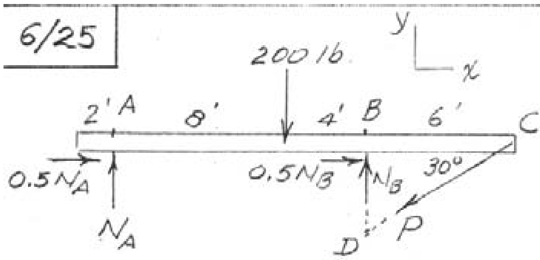
(b) Motion impends up incline:

$\Sigma F_x = 0: \; 2T - 100\sin 20° - F_{max} = 0$

Similarly, $W = 31.7$ lb

Hence the allowable range is $\underline{3.05 \leq W \leq 31.7 \text{ lb}}$

6/25



$\overline{BD} = 6 \tan 30°$
$= 6/\sqrt{3}$

$\Sigma M_C = 0; \quad 10(200) - 18N_A - 6N_B = 0$

$\Sigma M_D = 0; \quad 4(200) - 0.5(N_A + N_B)\frac{6}{\sqrt{3}} - 12N_A = 0$

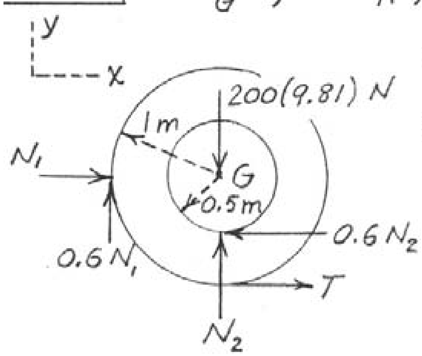
solve simultaneously & get $N_A = 26.08$ lb

$N_B = 255.1$ lb

$\Sigma F_x = 0; \quad 0.866P - 0.5(26.08 + 255.1) = 0$

$P = \underline{162.3 \text{ lb}}$

6/26 $\Sigma M_G = 0;\ 0.6N_1(1) + 0.6N_2(0.5) - T(1) = 0$



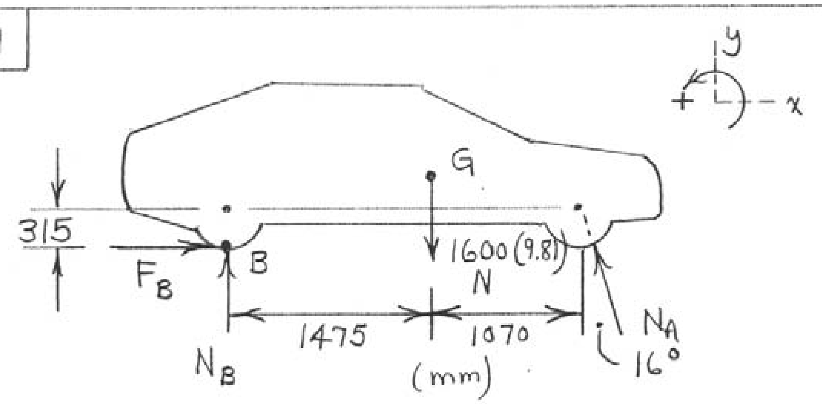
$\Sigma F_x = 0;\ T - 0.6N_2 + N_1 = 0$

$\Sigma F_y = 0;\ N_2 + 0.6N_1 - 1962 = 0$

Combine & get

$N_1 = 331\text{ N},\ N_2 = 1764\text{ N},\ \underline{T = 727\text{ N}}$

6/27

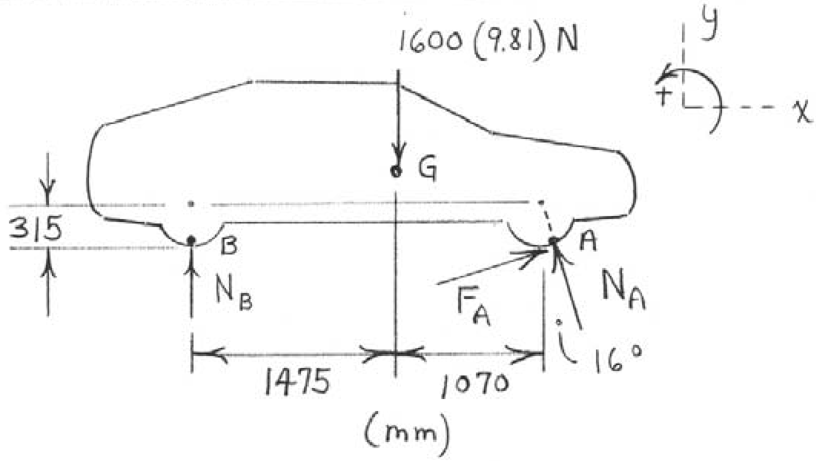


$$\begin{cases} \Sigma F_x = 0: & F_B - N_A \sin 16° = 0 \quad (1) \\ \Sigma F_y = 0: & N_B + N_A \cos 16° - 1600(9.81) = 0 \quad (2) \\ \Sigma M_B = 0: & -1600(9.81)(1475) + N_A \cos 16° (2545) \\ & + N_A \sin 16° (315) = 0 \quad (3) \end{cases}$$

Solution : $\begin{cases} N_A = 9140 \text{ N} \\ N_B = 6910 \text{ N} \\ F_B = 2520 \text{ N} \end{cases}$

$$\mu_s = \frac{F_B}{N_B} = \frac{2520}{6910} = \underline{0.365}$$

6/28



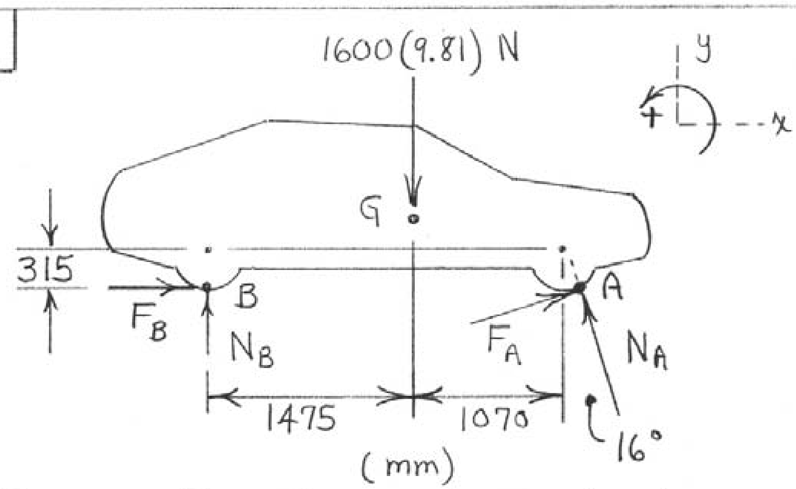
$$\begin{cases} \Sigma F_x = 0: \quad F_A \cos 16° - N_A \sin 16° = 0 & (1) \\ \Sigma F_y = 0: \quad N_B + F_A \sin 16° + N_A \cos 16° - 1600(9.81) = 0 & (2) \\ \Sigma M_A = 0: \quad -N_B\left(2545 + 315 \sin 16°\right) \\ \qquad + 1600\,(9.81)\left(1070 + 315 \sin 16°\right) = 0 & (3) \end{cases}$$

Solution: $\begin{cases} N_A = 8460 \text{ N} \\ N_B = 6900 \text{ N} \\ F_A = 2420 \text{ N} \end{cases}$

$$\mu_s = \frac{F_A}{N_A} = \frac{2420}{8460} = \underline{0.287}$$

Note that the value of μ_s can be determined from Eq. (1) alone.

6/29



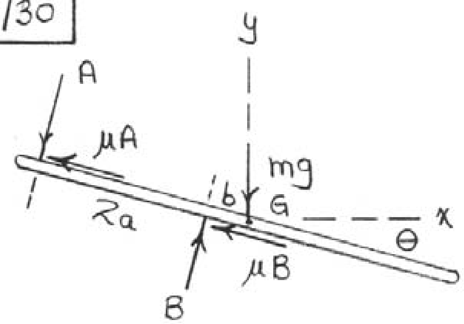
$$\begin{cases} \Sigma F_x = 0: \quad F_B + F_A \cos 16° - N_A \sin 16° = 0 & (1) \\ \Sigma F_y = 0: \quad N_B + F_A \sin 16° + N_A \cos 16° - 1600(9.81) = 0 & (2) \\ \Sigma M_A = 0: \; -N_B(2545 + 315 \sin 16°) & \end{cases}$$

$$+ F_B(315 - 315 \cos 16°) + 1600(9.81)(1070 + 315 \sin 16°) = 0 \quad (3)$$

Also, set $\begin{cases} F_A = \mu_s N_A & (4) \\ F_B = \mu_s N_B & (5) \end{cases}$

Solution: $\begin{cases} F_A = 1378 \text{ N} & F_B = 1087 \text{ N} \\ N_A = 8750 \text{ N} & N_B = 6900 \text{ N} \\ & \underline{\mu_s = 0.1575} \end{cases}$

6/30



$\circlearrowleft + \Sigma M_G = 0: \quad A(2a+b) - Bb = 0, \quad \frac{B}{A} = \frac{2a+b}{b}$

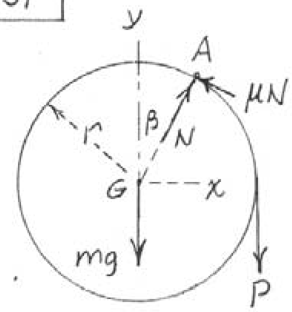
$\Sigma F_x = 0: \quad (B-A)\sin\theta - \mu(A+B)\cos\theta = 0$

$$\tan\theta = \mu\frac{A+B}{B-A} = \mu\frac{1+B/A}{\frac{B}{A}-1}$$

Substitute $\frac{B}{A}$: $\tan\theta = \mu\dfrac{1+(2a+b)/b}{(2a+b)/b - 1}$

or $\underline{\theta = \tan^{-1}\left(\mu\frac{a+b}{a}\right)}$

6/31



$\Sigma F_x = 0; \; N \sin\beta - \mu N \cos\beta = 0$

$\tan\beta = \mu$

$\underline{\beta = \tan^{-1}\mu}$

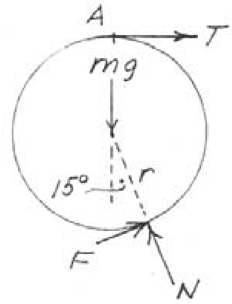
$\Sigma M_A = 0;$

$P(r - r\sin\beta) - mgr\sin\beta = 0$

$P = mg \dfrac{\sin\beta}{1-\sin\beta}$

But $\sin\beta = \dfrac{\mu}{\sqrt{1+\mu^2}}$ so $\underline{P = mg \dfrac{\mu}{\sqrt{1+\mu^2} - \mu}}$

$\boxed{6/32}$



Assume equilibrium, $\Sigma M_A = 0$

$F(r + r\cos 15°) = N r \sin 15°$

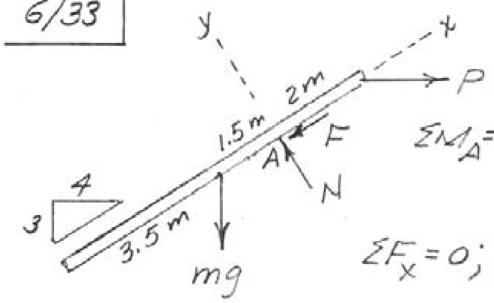
$\frac{F}{N} = \frac{0.2588}{1 + 0.9659} = 0.1317$

But $F_{max} = 0.20N$ $(> 0.1317\,N)$

so $F = 0.1317N$ can be supported

& paper rolls <u>without slipping</u>

6/33



$\Sigma M_A = 0;\ \frac{3}{5}P(2) - \frac{4}{5}mg(1.5) = 0$

$P = mg$

$\Sigma F_x = 0;\ \frac{4}{5}mg - F - \frac{3}{5}mg = 0$

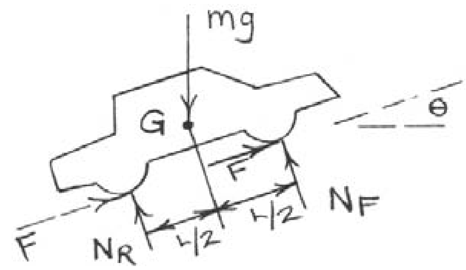
$F = \frac{1}{5}mg$

$\Sigma F_y = 0;\ N - \frac{3}{5}mg - \frac{4}{5}mg = 0,\ N = \frac{7}{5}mg$

$\mu_{s\,(min)} = F/N = \frac{1}{5}/\frac{7}{5} = \frac{1}{7} = 0.14 < 0.30$ so friction is sufficient to maintain equil.

∴ ans. is yes. $F = \frac{1}{5}60(9.81) = \underline{117.7\text{ N}}$

6/34 Consider the FBD below and the equilibrium equation $\Sigma M_G = 0$. The presence of the propulsive friction forces F, whether applied at the front or at the rear, increases the rear normal forces and decreases the front ones. Increased normals mean increased available propulsive friction forces.



Thus the rear-wheel drive car would climb the steeper grade.

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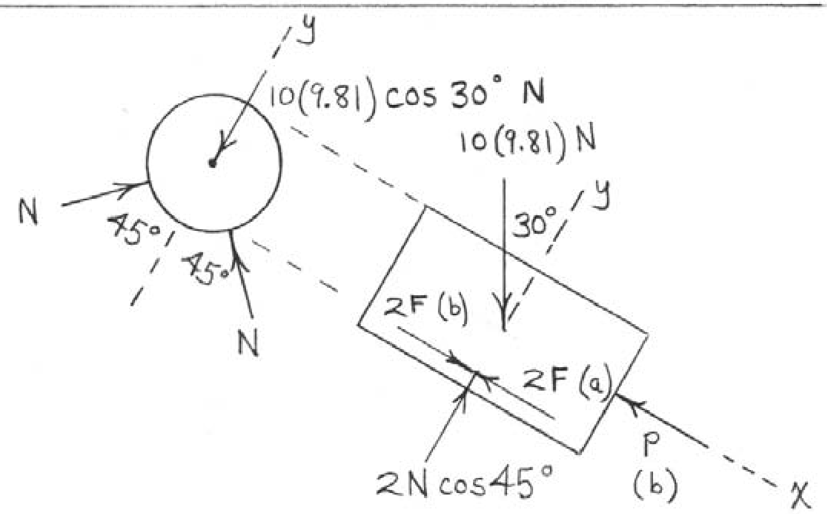

$\Sigma F_y = 0: \; 2N\cos 45° - 10(9.81)\cos 30° = 0, \; N = 60.1$ N

(a) $P = 0$

$\Sigma F_x = 0: -2F + 10(9.81)\sin 30° = 0, \; \underline{F = 24.5 \text{ N}}$

Check: $F_{max} = \mu_s N = 0.5(60.1) = 30.0 \text{ N} > F = 24.5 \text{ N}$

So we indeed have static equilibrium.

(b) $P \neq 0$

$\Sigma F_x = 0: -P + 10(9.81)\sin 30° + 2(0.5 \cdot 60.1) = 0$

$\underline{P = 109.1 \text{ N}}$

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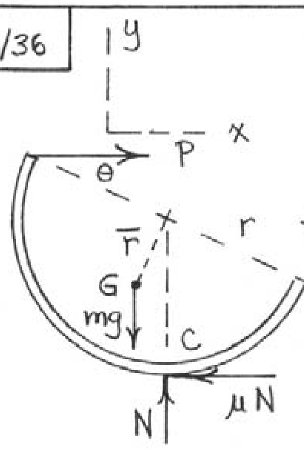


$\Sigma F_y = 0 \Rightarrow N = mg$

$\Sigma F_x = 0 \Rightarrow P = \mu_s N$

$\curvearrowright^{+} \Sigma M_C = 0: \; P(r + r\sin\theta) - mg\bar{r}\sin\theta = 0, \quad \bar{r} = \dfrac{2r}{\pi}$

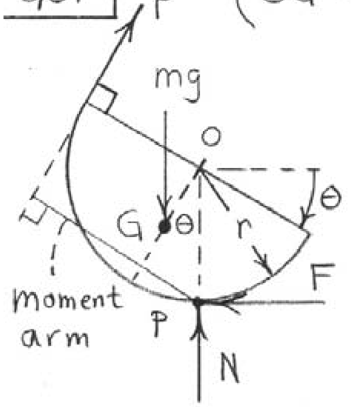
$\therefore \mu_s mg(r + r\sin\theta) = mg\dfrac{2r}{\pi}\sin\theta$

$\sin\theta\left[\dfrac{2}{\pi} - \mu_s\right] = \mu_s, \quad \underline{\theta = \sin^{-1}\left(\dfrac{\pi\mu_s}{2 - \pi\mu_s}\right)}$

$\theta = 90^\circ$ when $\dfrac{\pi\mu_s}{2 - \pi\mu_s} = 1$ or $\underline{\mu_{90^\circ} = \dfrac{1}{\pi} = 0.318}$

6/37

$\left(\overline{OG} = \bar{r} = \frac{4r}{3\pi}\right)$



$$\Sigma F_x = 0: \quad P\sin\theta - F = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad N - mg + P\cos\theta = 0 \qquad (2)$$

$$\Sigma M_P = 0: \quad mg\,\frac{4r}{3\pi}\sin\theta - P(r + r\sin\theta) = 0 \qquad (3)$$

When slipping impends, $F = \mu_s N \qquad (4)$

$(4) \rightarrow (1): \quad P\sin\theta - \mu_s N = 0 \qquad (5)$

$(2): \quad N = mg - P\cos\theta \qquad (6)$

$(6) \rightarrow (5): \quad P\sin\theta - \mu_s(mg - P\cos\theta) = 0$

$$\Rightarrow P = \frac{\mu_s mg}{\sin\theta + \mu_s\cos\theta} \qquad (7)$$

$(7) \rightarrow (3)$ & simplification yields

$$\mu_s = \frac{4\sin^2\theta}{3\pi(1+\sin\theta) - 4\sin\theta\cos\theta}$$

For $\theta = 40°$, $\underline{\mu_s = 0.1223}$

(7) then gives $\underline{P = 0.1661\,mg}$

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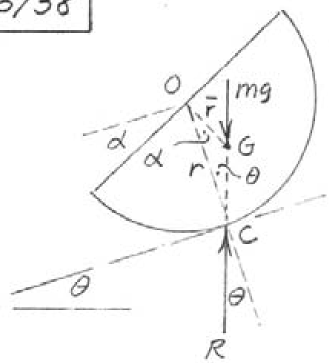
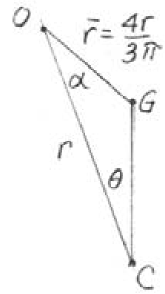




$\bar{r} = \frac{4r}{3\pi}$ $\frac{r}{\sin(\pi - \theta - \alpha)} = \frac{\bar{r}}{\sin\theta}$

$$\frac{r}{\sin(\theta + \alpha)} = \frac{4r/3\pi}{\sin\theta}$$

Max. angle occurs when

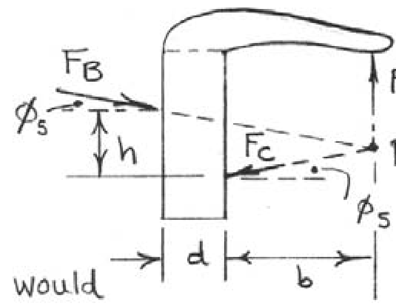
$\theta = \phi = \tan^{-1}\mu = \tan^{-1}0.30 = \underline{16.70°}$

Thus $\sin(\theta + \alpha) = \frac{3\pi}{4}\sin\theta = \frac{3\pi}{4}(0.2873) = 0.6770$

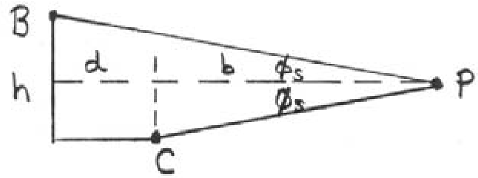
so $\theta + \alpha = 42.6°$ & $\alpha = 42.6 - 16.7 = \underline{25.9°}$

6/39

For equilibrium the three forces must be concurrent at P. For b larger than that shown, friction angles ϕ would be less than $\phi_s = \tan^{-1}\mu_s$ and no slippage is possible.



From geometry,

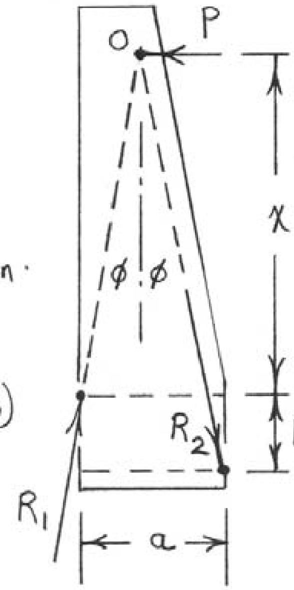


$$\frac{h - b\tan\phi_s}{b+d} = \tan\phi_s$$

$$h - b\mu_s = (b+d)\mu_s \,, \quad \underline{b = \frac{1}{2}\left(\frac{h}{\mu_s} - d\right)}$$

6/40

For equilibrium, the forces must be concurrent at O and $\phi = \tan^{-1}\mu_s$ for impending motion with $x = x_{min}$.

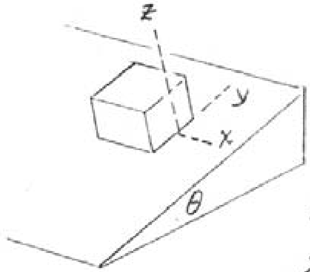


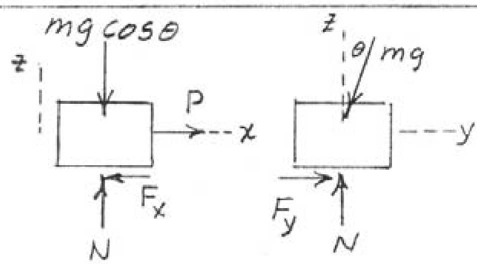
$$a = x\tan\phi + (x+b)\tan\phi$$

$$= x\mu_s + (x+b)\mu_s = \mu_s(2x+b)$$

$$x = \frac{a - b\mu_s}{2\mu_s}$$

6/41





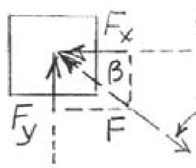
$\Sigma F_y = 0$; $F_y = mg \sin\theta$

$\Sigma F_x = 0$; $F_x = P$; $\Sigma F_z = 0$; $N = mg\cos\theta$

$F = \mu_s N = \sqrt{F_x^2 + F_y^2}$ so $(\mu_s mg\cos\theta)^2 = P^2 + (mg\sin\theta)^2$

so $\underline{P = mg\sqrt{\mu_s^2\cos^2\theta - \sin^2\theta}}$

where $\mu_s > \tan\theta$ otherwise block slips without P



$\beta = \tan^{-1}\frac{F_y}{F_x} = \underline{\tan^{-1}\left(\frac{mg\sin\theta}{P}\right)}$

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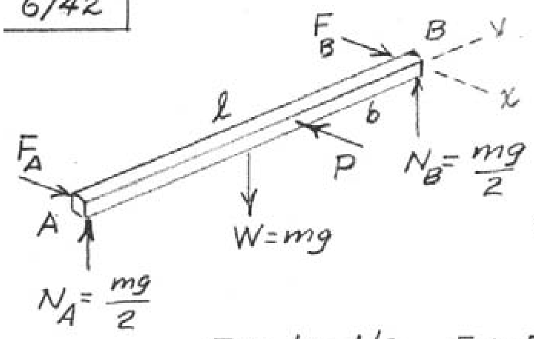


$\Sigma M_{B_z} = 0$

$Pb - F_A \ell = 0$

$F_A = Pb/\ell$

$\Sigma F_x = 0;\ F_B + \frac{Pb}{\ell} - P = 0$

$F_B = P(1 - b/\ell)$

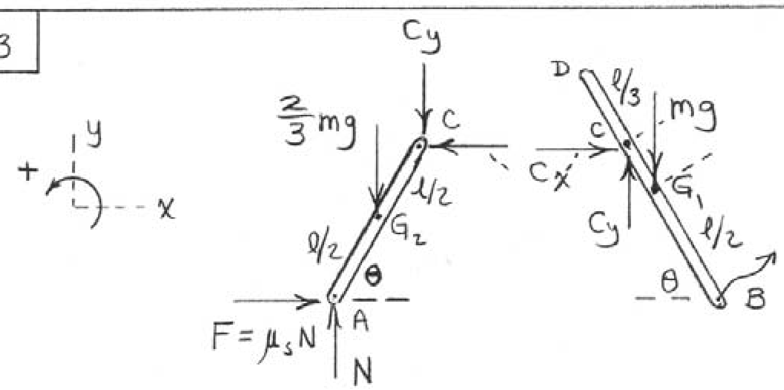
For $b < \ell/2$, $F_B > F_A$ so slips first at B

Hence $F_B = \mu_s N_B = \mu_s mg/2$

Thus $\mu_s mg/2 = P(1 - b/\ell)$ so $\underline{P = \dfrac{\mu_s mg}{2(1 - b/\ell)}}$

& $F_A = \dfrac{\mu_s mg}{2(1 - b/\ell)} \dfrac{b}{\ell} = \underline{\dfrac{\mu_s mg}{2(\frac{\ell}{b} - 1)}}$

6/43



Body BD:

$$\sum M_B = 0: -C_x\left(\frac{2\ell}{3}\sin\theta\right) - C_y\left(\frac{2\ell}{3}\cos\theta\right) + mg\frac{\ell}{2}\cos\theta = 0 \quad (1)$$

Body AC:

$$\sum F_x = 0: \quad \mu_s N - C_x = 0 \quad (2)$$

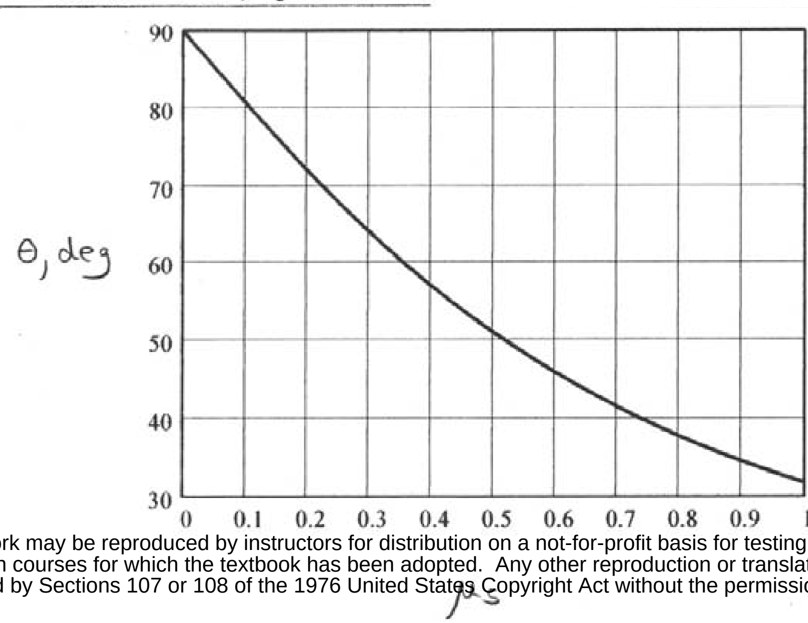
$$\sum F_y = 0: \quad N - \frac{2}{3}mg - C_y = 0 \quad (3)$$

$$\sum M_C = 0: \mu_s N\left(\frac{2\ell}{3}\sin\theta\right) - N\left(\frac{2\ell}{3}\cos\theta\right) + \frac{2}{3}mg\left(\frac{\ell}{3}\cos\theta\right) = 0 \quad (4)$$

Solve Eqs. (1)-(4) for C_x, C_y, N, and θ as functions of μ_s. Solution for θ is

$$\theta = \tan^{-1}\frac{13}{21\mu_s} \text{ (plotted below)}$$

$\theta \to 90°$ as $\mu_s \to 0$
$\theta = 51.1°$ for $\mu_s = 0.50$
$\theta = 31.8°$ for $\mu_s = 1$



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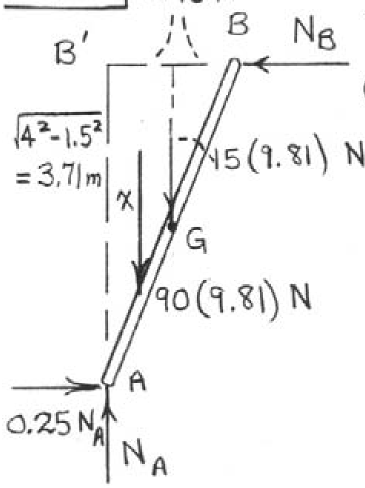


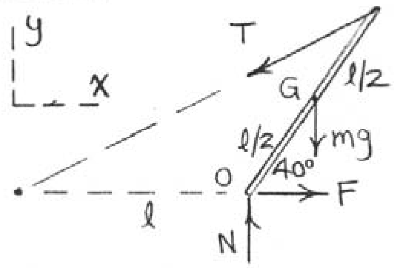
$+\uparrow \Sigma F = 0 \Rightarrow N_A = (90+15)\,9.81 = 1030\text{ N}$

$\circlearrowleft + \Sigma M_{B'} = 0: 0.25\,N_A(3.71) - 90(9.81)x$

$-15(9.81)(0.75) = 0, \quad x = 0.957\text{ m}$

$\dfrac{s}{x} = \dfrac{4}{1.5}, \quad \underline{s = 2.55\text{ m}}$

6/45



$$\underline{T} = T\left\{\frac{(-l - l\cos 40^\circ)\underline{i} - (l\sin 40^\circ)\underline{j}}{\sqrt{(l + l\cos 45^\circ)^2 + (l\sin 45^\circ)^2}}\right\}$$

$$= T\{-0.940\underline{i} - 0.342\underline{j}\}$$

$\circlearrowleft + \Sigma M_o = 0: \quad 0.342T(l) - mg\frac{l}{2}\cos 40^\circ = 0$

$T = 1.120\ mg$

$\Sigma F_x = 0: \quad F - 1.120\ mg(0.940) = 0, \quad F = 1.052\ mg$

$\Sigma F_y = 0: \quad N - 1.120\ mg\ (0.342) - mg = 0$

$N = 1.383\ mg$

$$\mu_s = \frac{F}{N} = \frac{1.052\ mg}{1.383\ mg} = \underline{0.761}$$

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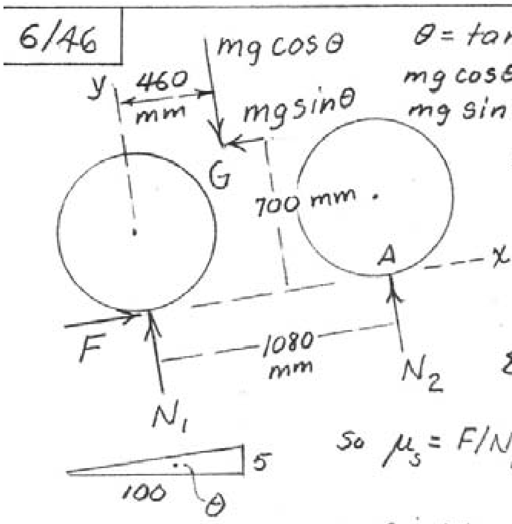


$\theta = \tan^{-1} 5/100 = 2.86^\circ$

$mg\cos\theta = 82(9.81)(0.9998) = 803.4\ N$

$mg\sin\theta = 82(9.81)(0.0499) = 40.2\ N$

$\Sigma M_A = 0;$

$803.4(620) + 40.2(700) - 1080 N_1 = 0$

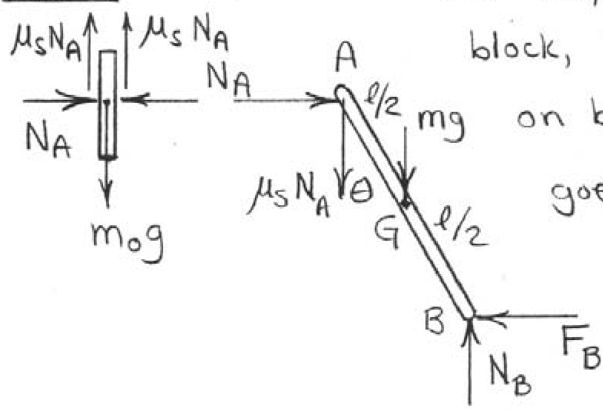
$N_1 = 487.3\ N$

$\Sigma F_x = 0;\ F = 40.2\ N$

So $\mu_s = F/N_1 = 40.2/487.3 = \underline{0.082}$

If μ_s were doubled, friction force remains $\underline{40.2\ N}$

6/47



For impending slip of the block, the friction force on both of its faces goes to the maximum.

(Block) $+\uparrow \Sigma F = 0$: $2\mu_s N_A - m_o g = 0$, $N_A = \dfrac{m_o g}{2\mu_s}$ (1)

(Bar) $\curvearrowleft + \Sigma M_B = 0$: $mg\left(\frac{l}{2}\sin\theta\right) + \mu_s N_A (l\sin\theta) - N_A (l\cos\theta) = 0$ (2)

With Eq. (1), Eq. (2) becomes

$$mg\left(\frac{l}{2}\sin\theta\right) + \mu_s \frac{m_o g}{2\mu_s} l\sin\theta - \frac{m_o g}{2\mu_s} l\cos\theta = 0$$

Solving for θ: $\theta = \tan^{-1}\left[\dfrac{1}{\mu_s\left(1+\frac{m}{m_o}\right)}\right] = \theta_{min}$

(Bar) $\xrightarrow{+} \Sigma F = 0$: $N_A - F_B = 0$

$$F_B = N_A = \frac{m_o g}{2\mu_s}$$

$+\uparrow \Sigma F = 0$: $N_B - mg - \mu_s N_A = 0$

$$N_B = mg + \mu_s \frac{m_o g}{2\mu_s} = \left(m + \frac{m_o}{2}\right)g$$

$$(\mu_s)_B = \frac{F_B}{N_B} = \frac{m_o g/(2\mu_s)}{\left(m+\frac{m_o}{2}\right)g} = \frac{1}{\mu_s\left(1+2\frac{m}{m_o}\right)}$$

Numbers ($\mu_s = 0.5$ throughout)

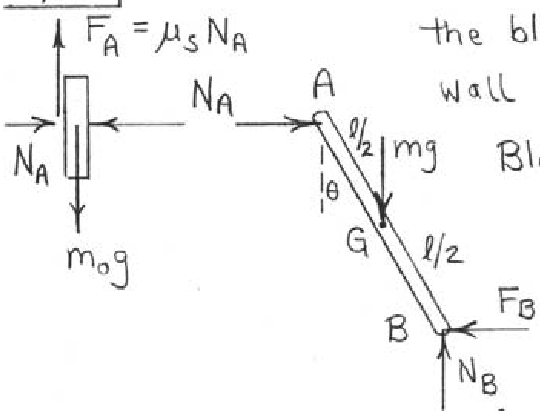
(a) $\frac{m}{m_o} = 0.1$: $\theta_{min} = 61.2°$, $(\mu_s)_B = 1.667$

($\mu_s > 1$ perhaps unusual, but quite possible)

(b) $\frac{m}{m_o} = 1$: $\theta_{min} = 45°$, $(\mu_s)_B = 0.667$

(c) $\frac{m}{m_o} = 10$: $\theta_{min} = 10.30°$, $(\mu_s)_B = 0.0952$

6/48



The friction force F_A between the block m_o and the vertical wall is set to the maximum.

Block: $+\uparrow \Sigma F = 0$:

$$\mu_s N_A = m_o g$$

$$N_A = \frac{m_o g}{\mu_s} \quad (1)$$

Bar: $\circlearrowleft + \Sigma M_B = 0$: $mg\left(\frac{l}{2}\sin\theta\right) - N_A\left(l\cos\theta\right) = 0$

$$N_A = \frac{mg}{2}\tan\theta \quad (2)$$

Combine (1) & (2): $\frac{m_o g}{\mu_s} = \frac{mg}{2}\tan\theta$

$$\Rightarrow \underline{\theta = \tan^{-1}\left(\frac{2}{\mu_s}\frac{m_o}{m}\right) = \theta_{min}}$$

Numbers: $\theta_{min} = \tan^{-1}\left(\frac{2}{0.5}\cdot\frac{1}{10}\right) = \underline{21.8°}$

Check to see that no slippage occurs at B.

(Bar) $\xrightarrow{+} \Sigma F = 0$: $N_A - F_B = 0$

So $F_B = N_A = \frac{mg}{2}\tan\theta$ (from Eq. (2))

$+\uparrow \Sigma F = 0$: $N_B - mg = 0$, $N_B = mg$

$(F_B)_{max} = \mu_s N_B = \mu_s mg$

Numbers: $\begin{cases} F_B = \frac{mg}{2}\tan 21.8° = 0.2mg \\ (F_B)_{max} = 0.5mg \end{cases}$

$F_B < (F_B)_{max}$, so no slippage at B.

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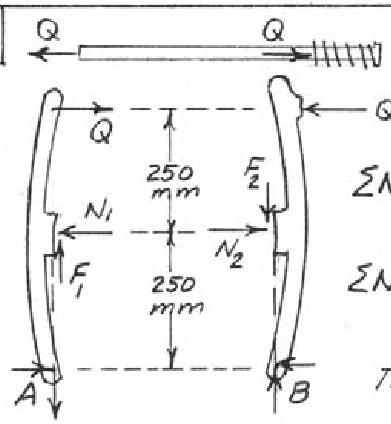


FBD's for $P=0$

$F_1 = 0.2N_1$, $F_2 = 0.2N_2$

$\Sigma M_A = 0$; $0.250N_1 = 0.500Q$

$N_1 = 2Q$

$\Sigma M_B = 0$; $0.250N_2 = 0.500Q$

$N_2 = 2Q$

Thus $N_1 = N_2 = N$

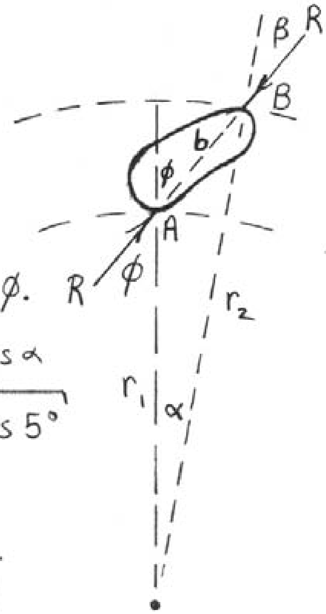
Flywheel: $\Sigma M = 0$; $M = F_1 r + F_2 r$

$100 = 0.2N(0.200) + 0.2N(0.200)$

$N = 1250$ N

So $Q = N/2 = 625$ N ; $Q = k\delta$, $k = \dfrac{625}{0.030} = \underline{20.8(10^3)\ \dfrac{N}{m}}$

▶ 6/50 Given: $\begin{cases} r_1 = \overline{OA} = 5/16'' \\ r_2 = \overline{OB} = 7/16'' \\ \alpha = 5° \end{cases}$



$\phi > \beta$ so slipping would occur first at A. Hence $\mu_{min} = \tan\phi$.

Law of cosines: $b^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\alpha$

$$b = \sqrt{(5/16)^2 + (7/16)^2 - 2\left(\tfrac{5}{16}\right)\left(\tfrac{7}{16}\right)\cos 5°}$$

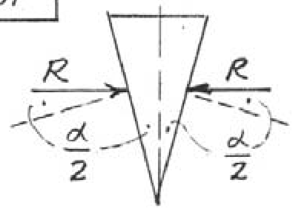
$$= 0.1291 \text{ in.}$$

Law of sines: $\dfrac{b}{\sin\alpha} = \dfrac{r_2}{\sin(\pi - \phi)}$

$$\frac{0.1291}{\sin 5°} = \frac{7/16}{\sin(\pi - \phi)} = \frac{7/16}{\sin\phi}$$

$$\phi = 17.18°,\ \mu_{min} = \tan\phi = \tan 17.18° = \underline{0.309}$$

6/51



Critical α occurs when

$$\frac{\alpha}{2} = \phi = \tan^{-1}\mu$$

so $\alpha = 2 \tan^{-1} 0.20$

$$= 2(11.31°) = \underline{22.6°}$$

6/52 Self-locking condition is

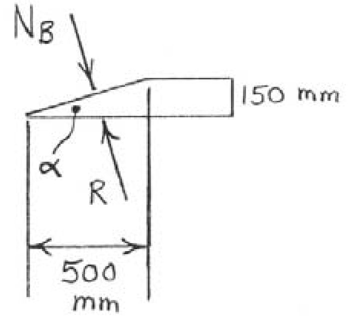
$$\alpha = \phi$$

$$\tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\mu_s$$

$$\mu_s = \frac{L}{2\pi r} = \frac{5}{2\pi(3)}$$

$$= \underline{0.265}$$

6/53 FBD of portable ramp:



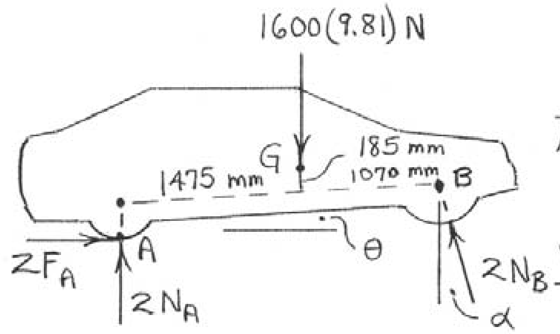
$$\alpha = \tan^{-1}\left(\frac{150}{500}\right) = 16.70^\circ$$

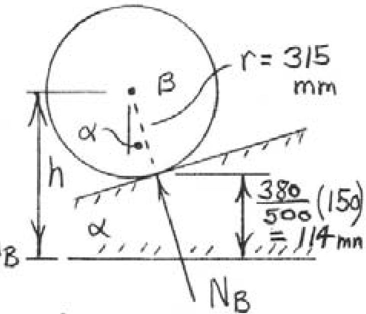
$$\tan\alpha = \frac{F}{N} = \mu_s$$

So $\mu_s = \tan\alpha$

$= \tan 16.70^\circ$

$\underline{\mu_s = 0.3}$





So $\theta = \sin^{-1}\frac{416 - 315}{2545} = 2.27^\circ$ $\left\{ \begin{array}{l} h = 114 + 315\cos\alpha \\ = 416 \text{ mm} \end{array} \right.$

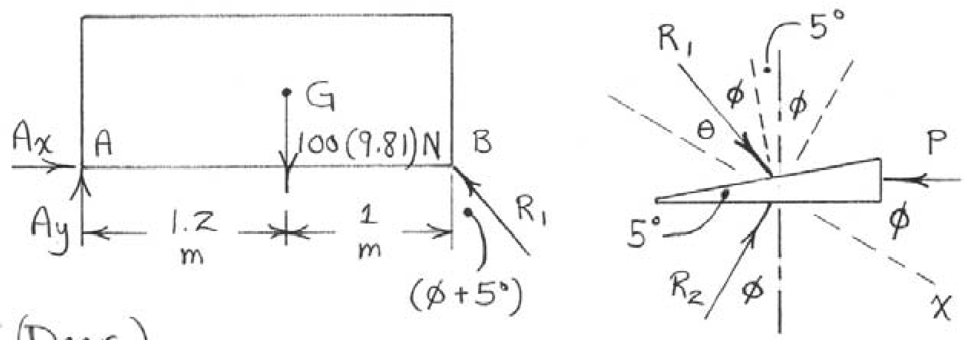
$$\circlearrowleft^{+}\ \Sigma M_A = 0: -1600(9.81)(1475\cos\theta - 185\sin\theta)$$
$$+ 2N_B\cos(\alpha-\theta)(315\sin\theta + 2545) + 2N_B\sin(\alpha-\theta)\,315\cos\theta = 0$$

$$N_B = 4500 \text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad 2F_A - 2N_B\sin\alpha = 0$$

$$\underline{F_A = 1294 \text{ N}}$$

6/54 $\phi = \tan^{-1}(0.60) = 31.0°$



(Door)

$\Sigma M_A = 0: \ R_1 \cos(31.0° + 5°)(2.2) - 100(9.81)(1.2) = 0$

$R_1 = 661 \text{ N}$

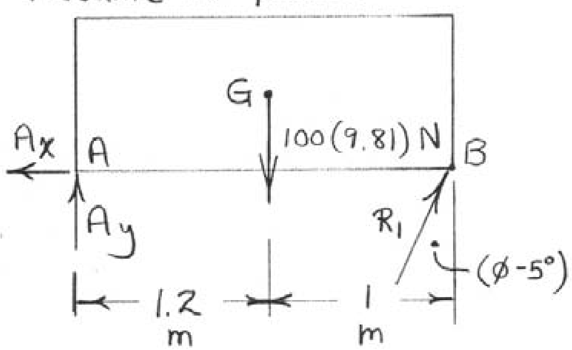
(Wedge) (Note $\theta = 90° - 2\phi - 5° = 23.1°$)

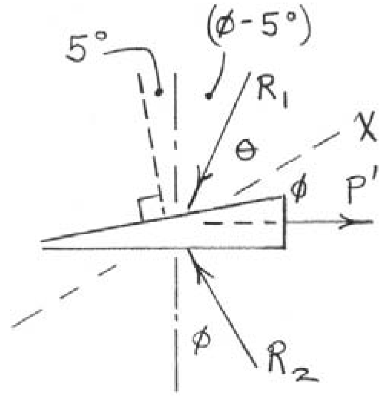
$\Sigma F_x = 0: \ 661 \cos 23.1° - P \cos 31.0° = 0$

$\underline{P = 709 \text{ N}}$

6/55 $\quad \phi = \tan^{-1}(0.60) = 31.0°$

Assume A pinned:





(Door)

$\Sigma M_A = 0: \; R_1 \cos(31.0° - 5°)(2.2) - 100(9.81)(1.2) = 0$

$R_1 = 595$ N 3)

(Wedge) $\quad$ (Note $\theta = 90° - 2\phi + 5° = 33.1°$) 4)

$\Sigma F_x = 0: \; P' \cos 31.0° - 595 \cos 33.1° = 0$

$\underline{P' = 582 \text{ N}}$ 5)

(Door) $\Sigma F_x = 0: \; -A_x + 595 \sin(\phi - 5°) = 0, \; A_x = 261$ N

$\Sigma F_y = 0: \; A_y - 100(9.81) + 595 \cos(\phi - 5°) = 0, \; A_y = 446$ N

$\frac{A_x}{A_y} = \frac{261}{446} = 0.584 < (\mu_s = 0.6)$; assumption OK, A will not slip.

6/56

Helix angle $\alpha = \tan^{-1} \frac{L}{\pi d} = \tan^{-1} \frac{1/12}{\pi (3/8)} = 4.05°$

Friction angle $\phi = \tan^{-1} \mu_s = \tan^{-1}(0.20) = 11.31°$

Tighten : $M = Pr \tan(\alpha + \phi) = 80 \frac{3/8}{2} \tan(4.05° + 11.31°)$

$= \underline{4.12 \text{ lb-in.}}$

Loosen : $M' = Pr \tan(\phi - \alpha) = 80 \frac{3/8}{2} \tan(11.31° - 4.05°)$

$= \underline{1.912 \text{ lb-in.}}$ (in direction opposite to that of M)

Note : $\alpha < \phi$, so screw is self-locking (a good feature for a clamp!)

6/57 $M = Wr \tan(\alpha + \phi)$

where $W = 450$ N, $r = 0.025$ m,

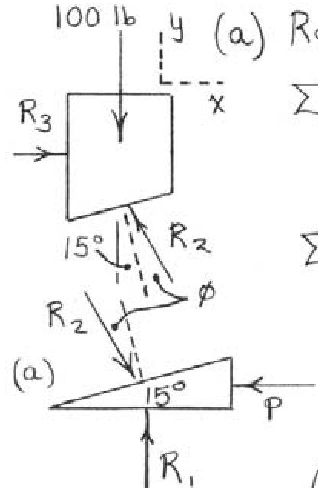
$$\alpha = \tan^{-1}\frac{Lead}{2\pi r} = \tan^{-1}\frac{0.020}{2\pi(0.025/2)} = \tan^{-1}0.255$$
$$= 14.29°$$

$\phi = \tan^{-1}\mu = \tan^{-1}0.20 = 11.31°$; $\phi + \alpha = 25.60°$

So $M = 450\left(\frac{0.025}{2}\right)\tan 25.60° = \underline{2.69 \text{ N·m}}$

6/58 Friction angle $\phi = \tan^{-1}(0.2) = 11.31°$

(a) Rollers under wedge



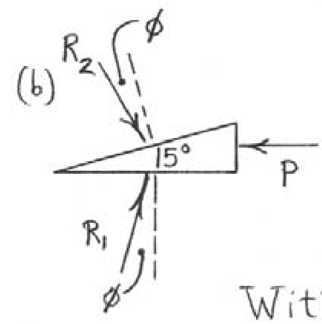
$\Sigma F_y = 0: \ -100 + R_2 \cos(15° + 11.31°) = 0$

$R_2 = 111.6 \text{ lb}$

$\Sigma F_x = 0: \ R_2 \sin(15° + 11.31°) - P = 0$

$\underline{P = 49.4 \text{ lb}}$

(b) Rollers removed



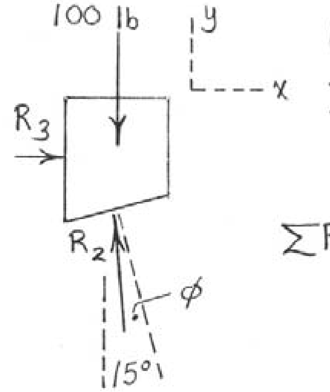
Value of R_2 from 100-lb body is unchanged.

$\Sigma F_x = 0: \ R_2 \sin(15° + 11.31°) - P + R_1 \sin(11.31°) = 0$

With R_1 determined from overall equilibrium as $R_1 = 100/\cos 11.31° = 102.0 \text{ lb}$, we solve for P as

$\underline{P = 69.4 \text{ lb}}$

6/59 Friction angle $\phi = \tan^{-1}(0.2) = 11.31°$



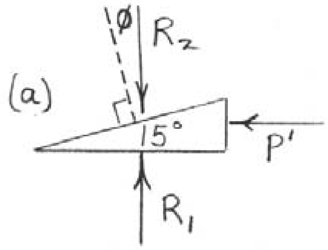
(a) Rollers under wedge

$\Sigma F_y = 0: -100 + R_2 \cos(15° - 11.31°) = 0$

$R_2 = 100.2$ lb

$\Sigma F_x = 0: R_2 \sin(15° - 11.31°) - P' = 0$

$\underline{P' = 6.45 \text{ lb}}$ (to the left)

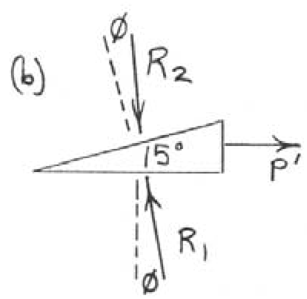


(b) Rollers removed

$R_2 = 100.2$ lb, from (a).

From overall equilibrium,

$R_1 = \frac{100}{\cos 11.31°} = 102.0$ lb



$\Sigma F_x = 0: R_2 \sin(15° - 11.31°) + P'$

$- R_1 \sin(11.31°) = 0$

$\underline{P' = 13.55 \text{ lb}}$ (to the right)

6/60 Helix angle: $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{1/5}{2\pi\left(\frac{1.150}{2}\right)} = 3.17°$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.25) = 14.04°$

(a) To tighten, $M_a = 2Tr\tan(\alpha + \phi)$

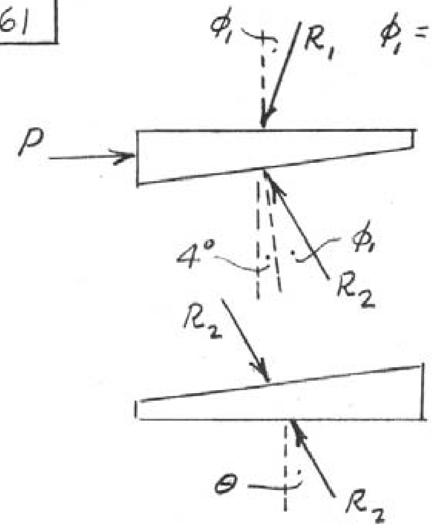
$$M_a = 2(10{,}000)\,\frac{1.150}{2}\tan(3.17° + 14.04°) = \underline{3560 \text{ lb-in.}}$$

(b) To loosen, $M_b = 2Tr\tan(\phi - \alpha)$

$$M_b = 2(10{,}000)\,\frac{1.150}{2}\tan(14.04° - 3.17°) = \underline{2210 \text{ lb-in.}}$$

6/61

$\phi_1 = \tan^{-1} 0.3 = 16.70°$



In order for column to be raised, bottom wedge must not move. Therefore μ_2 must be greater than $\tan\theta$ or

$$(\mu_2)_{min} = \tan(4 + 16.70)$$

$$= \underline{0.378}$$

6/62 For the screw,

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{6}{20\pi} = 5.45°$$

$$\phi = \tan^{-1}(0.25) = 14.04°$$

Eq. 6/3: $M = Wr\tan(\alpha + \phi)$

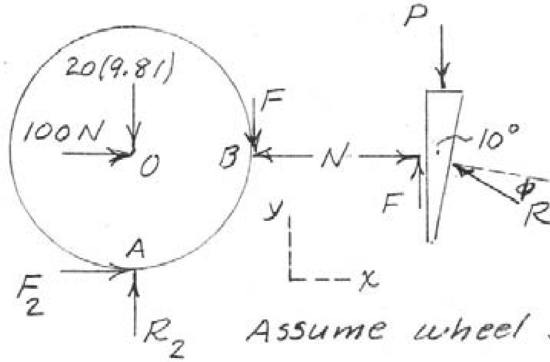
$$24 = P\,\frac{20/2}{1000}\tan(5.45° + 14.04°)$$

$$P = 6780 \text{ N} \quad \text{(to remove collar)}$$

Collar: $\mu p A = P$: $0.30p\,(0.050\pi \cdot 0.060) = 6780$

$$p = 2.40(10^6) \text{ Pa or } \underline{2400 \text{ kPa}}$$

6/63 $\quad \phi = \tan^{-1} 0.30 = 16.70°$



Assume wheel slips at B

wheel: $\Sigma M_O = 0$; $F_2 = F = 0.3N$
$\Sigma F_x = 0$; $N = 100 + F_2$
$\Big\}$ $0.7N = 100$
$N = 142.9\ N$
$F_2 = F = 0.3(142.9)$
$= 42.9$

$\Sigma F_y = 0$; $R_2 = 196.2 + 42.9$
$= 239.1\ N$

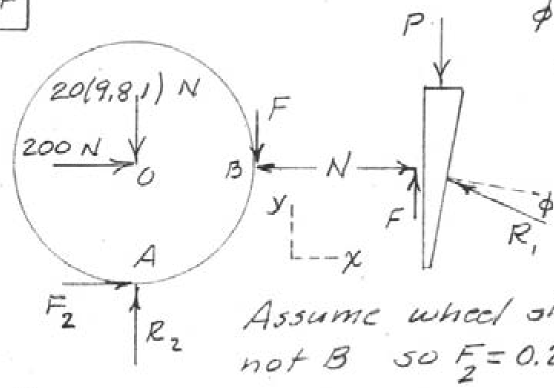
$(F_2 = 42.9) < (0.25 \times 239.1 = 59.8)$ so assumption OK

Wedge: $\Sigma F_x = 0$; $R_1 \cos 26.70° = 142.9$, $R_1 = 159.9\ N$

$\Sigma F_y = 0$; $P = 42.9 + 159.9 \sin 26.70° = \underline{114.7\ N}$

6/64

$\phi = \tan^{-1} 0.30$
$= 16.70°$



Assume wheel slips at A but not B so $F_2 = 0.25R_2$

Wheel: $\Sigma M_O = 0$; $F = F_2 = 0.25R_2$

$\Sigma F_y = 0$; $R_2 = 196.2 + F = 196.2 + 0.25R_2$

Solve & get $R_2 = 262$ N & $F = 0.25(262) = 65.4$ N

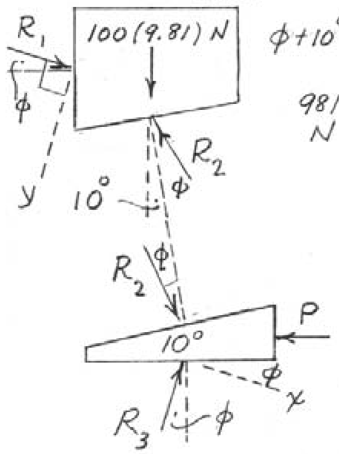
$\Sigma F_x = 0$; $N = 200 + 65.4 = 265.4$ N

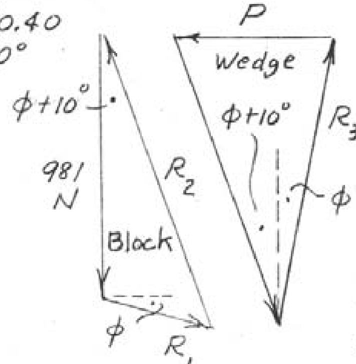
$(F = 65.4) < (0.3 \times 265.4 = 79.6)$ so assumption OK

wedge: $\Sigma F_x = 0$; $R_1 \cos 26.70° = 265.4$, $R_1 = 297$ N

$\Sigma F_y = 0$; $P = 65.4 + 297 \sin 26.70° = \underline{198.8\ N}$

6/65 $\phi = \tan^{-1} 0.40 = 21.80°$





Block: $\Sigma F_y = 0$;

$981 \cos 21.80° = R_2 \cos 53.60°$

$R_2 = 1535$ N

Wedge: $\Sigma F_x = 0$;

$1535 \cos 36.40° = P \cos 21.80°$

$P = 1331$ N

$2\phi + 10° = 53.60°$

$90° - (2\phi + 10°) = 36.40°$

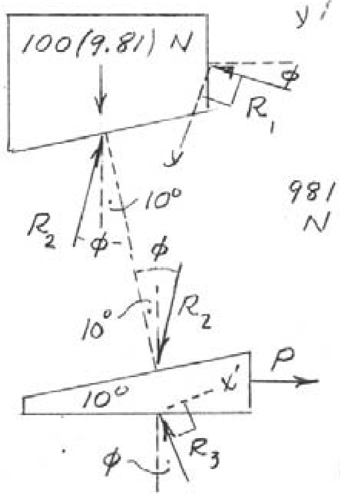
Screw: $\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{10}{2\pi(15)} = 6.06°$; $\phi = \tan^{-1} 0.25 = 14.04°$

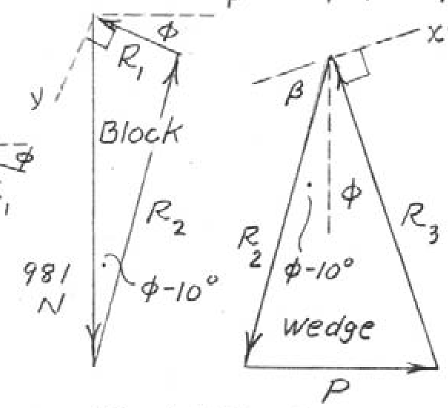
$\phi + \alpha = 20.09°$

$M = Pr \tan(\phi + \alpha) = 1331(0.015) \tan 20.09° = \underline{7.30 \text{ N}\cdot\text{m}}$

6/66 $\phi = \tan^{-1} 0.40 = 21.80°$ $\beta = 90° - (2\phi - 10°) = 56.40°$

$\phi + 10° = 31.80°$





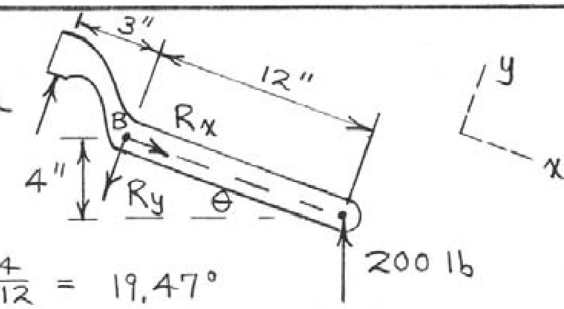
Block: $\Sigma F_y = 0$;
$981 \cos 21.80 = R_2 \cos 10°$
$R_2 = 925$ N

Wedge: $\Sigma F_{x'} = 0$;
$925 \cos 56.40 = P \cos 21.80$
$P = 551$ N

For screw,
$\alpha = 6.06°$ & $\phi = 14.04°$ (Prob. 6/65)

$M' = Pr \tan(\phi + \alpha) = 551(0.015) \tan 20.09° = \underline{3.02 \text{ N}\cdot\text{m}}$
Note (+) sign, $(\phi + \alpha)$

6/67



$\theta = \sin^{-1}\frac{4}{12} = 19.47°$

$\Sigma M_B = 0: \; 200 \cos 19.47° (12) - 3L = 0, \quad L = 754$ lb

Screw: $\begin{cases} \text{helix angle } \alpha = \tan^{-1}\dfrac{1/12}{\pi(1/2)} = 3.04° \\ \text{friction angle } \phi = \tan^{-1}(0.20) = 11.31° \end{cases}$

Tighten screw: $M = Lr\tan(\phi + \alpha)$

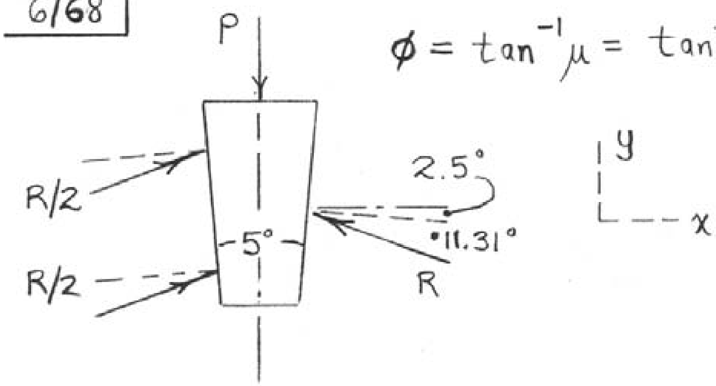
$= 754(0.25)\tan(11.31° + 3.04°) = \underline{48.2 \text{ lb-in.}}$

Loosen screw: $M' = Lr\tan(\phi - \alpha)$

$= 754(0.25)\tan(11.31° - 3.04°) = \underline{27.4 \text{ lb-in.}}$

6/68

$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$



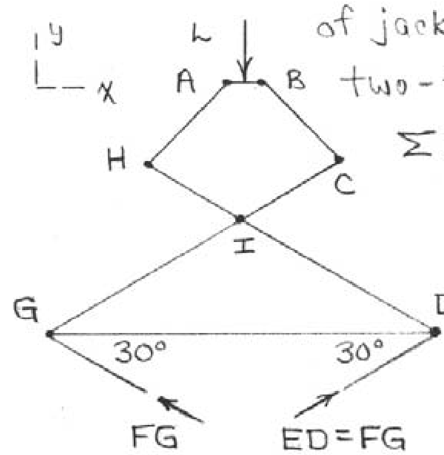
$\Sigma F_x = 0$ for shaft: $R \cos(11.31° + 2.5°) = T = 200$

$R = 206$ lb

$\Sigma F_y = 0$ for wedge: $P = 2R \sin(11.31° + 2.5°)$

$\underline{P = 98.3 \text{ lb}}$

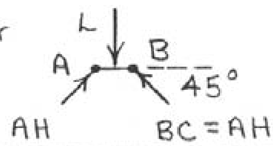
▸6/69 Calculate force in member FG. FBD of jack with FG & DE (both two-force members) cut:



$\Sigma F_y = 0: \; 2FG \sin 30° = L$

$FG = L$

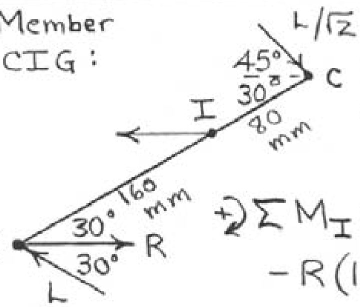
Member AB:



$\Sigma F_y = 0: \; 2AH \sin 45° = L$

$AH = L/\sqrt{2}$

Member CIG:



(Due to symmetry, force at I is horizontal; R is force from rod.)

$\circlearrowleft \Sigma M_I = 0: \; \frac{L}{\sqrt{2}}(80 \sin 75°) + L(160 \sin 60°) - R(160 \sin 30°) = 0, \quad R = 2.42L$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{0.003}{2\pi\,(0.010/2)} = 5.45°$

To raise load: $M = Rr \tan(\alpha + \phi)$

$= [2.42(1000)(9.81)]\left[\frac{0.010}{2}\right] \tan(11.31° + 5.45°)$

$= \underline{35.7 \text{ N·m}}$

To lower load: $M' = Rr \tan(\phi - \alpha)$

$= [2.42(1000)(9.81)]\left[\frac{0.010}{2}\right] \tan(11.31° - 5.45°)$

$= \underline{12.15 \text{ N·m}}$

▶6/70 For equil. of screw (refer to prob. illust.)

$\Sigma F = 0$; $W = \Sigma R_1 \cos(\alpha+\gamma) = \cos(\alpha+\gamma)\,\Sigma R_1$

$\Sigma M = 0$; $M = \Sigma R_1 r \sin(\alpha+\gamma) = r\sin(\alpha+\gamma)\,\Sigma R_1$

combine & set $M = Wr\tan(\alpha+\gamma) = Wr\dfrac{\tan\alpha + \tan\gamma}{1-\tan\alpha\tan\gamma}$

But $\tan\gamma = \dfrac{R\sin\phi}{R\cos\phi\cos\beta/2} = \mu/\cos\beta/2$

& $\tan\beta/2 = \dfrac{L}{2h}\cos\alpha$, $\tan\dfrac{\theta}{2} = \dfrac{L}{2h}$, so $\tan\dfrac{\beta}{2} = \tan\dfrac{\theta}{2}\cos\alpha$

& $\cos\dfrac{\beta}{2} = 1\Big/\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$. Thus $\tan\gamma = \mu\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$

Hence

$$M = Wr\,\frac{\tan\alpha + \mu\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}{1-\mu\tan\alpha\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}$$

where $\tan\alpha = \dfrac{L}{2\pi r}$

6/71

$M = Rr\sin\phi$, $\sin\phi = \dfrac{M}{Rr} = \dfrac{3}{2(40)(9.81)(0.040/2)}$

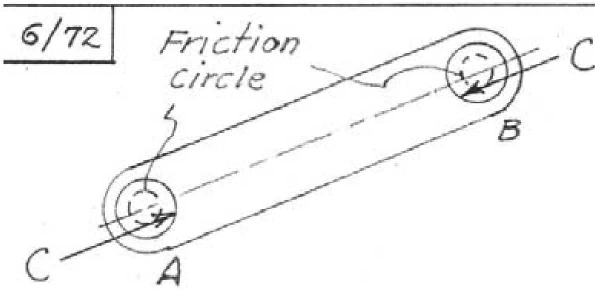
$\phi = 11.02°$

$\mu = \tan\phi = \underline{0.1947}$

$r_f = r\sin\phi = \dfrac{0.040}{2}\sin 11.02° = 0.00382\text{ m}$

or $\underline{r_f = 3.82\text{ mm}}$

6/72

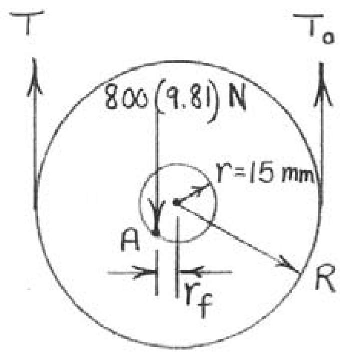


6/73 $M = \frac{2}{3}\mu P R$

A on B: $M = \frac{2}{3}(0.40)(80)\left(\frac{9}{2}\right) = \underline{96 \text{ lb-in.}}$

B on C: $96.0 = \frac{2}{3}\mu(80)\left(\frac{12}{2}\right)$, $\underline{\mu = 0.3}$

6/74



$\phi = \tan^{-1} 0.25 = 14.04°$

$r_f = r \sin\phi = 15 \sin 14.04$

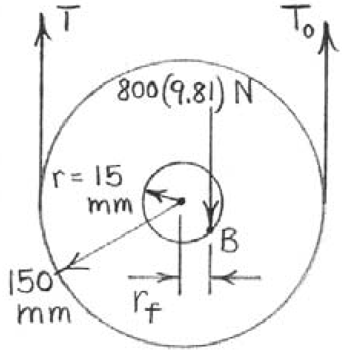
$= 3.64 \text{ mm}$

$+\uparrow \Sigma F = 0: \quad T + T_o - 800(9.81) = 0 \qquad (1)$

$+\circlearrowleft \Sigma M_A = 0: \quad T(150 - 3.64) - T_o(150 + 3.64) = 0 \qquad (2)$

Solve (1) & (2) : $\begin{cases} T = 4020 \text{ N} \\ \underline{T_o = 3830 \text{ N}} \end{cases}$

6/75



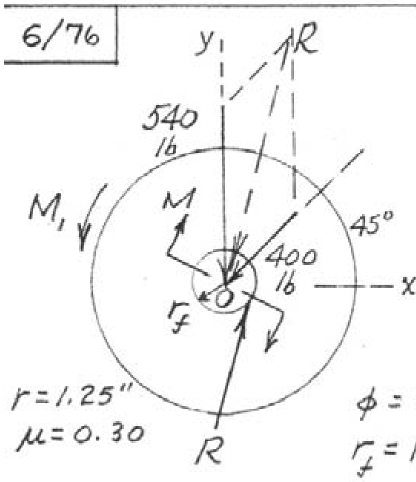
From the solution to Prob. 6/74, $r_f = 3.64$ mm

$$+\uparrow \Sigma F = 0: \quad T + T_0 - 800(9.81) = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_B = 0: \quad T(150 + 3.64) - T_0(150 - 3.64) = 0 \qquad (2)$$

Solve (1) & (2): $\begin{cases} T = 3830 \text{ N} \\ T_0 = 4020 \text{ N} \end{cases}$

6/76



Replace 400-lb force by force and couple M_1

$M_1 = 400(22) = 8800$ lb-in.

$R_x = 400 \cos 45° = 282.8$ lb

$R_y = 540 + 400 \sin 45°$

$= 822.8$ lb

$R = \sqrt{(282.8)^2 + (822.8)^2} = 870.1$ lb

$\phi = \tan^{-1} 0.30 = 16.70°$

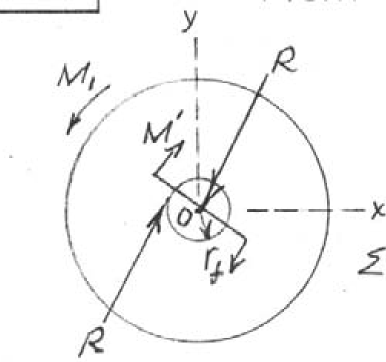
$r_f = r \sin\phi = 1.25 \sin 16.70° = 0.359$ in.

$\Sigma M_O = 0$; $M - 8800 - 870.1(0.359) = 0$

$M = 8800 + 313 = 9113$ lb-in.

or $M = 9113/12 = \underline{759 \text{ lb-ft}}$

6/77



From Prob. 6/76, $R = 870.0$ lb

$\phi = 16.70°$

$r_f = r \sin\phi = 0.359$ in.

$M_1 = 8800$ lb-in.

$\Sigma M_O = 0;\ M' + 870.0(0.359) - 8800 = 0$

$M' = 8800 - 313$

$= 8487$ lb-in.

or $\underline{M' = 707 \text{ lb-ft}}$

6/78 $\mu = 0.80 - kr$: $0.50 = 0.80 - k(3)$, $k = \frac{1}{10}$ in.$^{-1}$

So $\mu = 0.80 - \frac{r}{10}$ (r in in.)

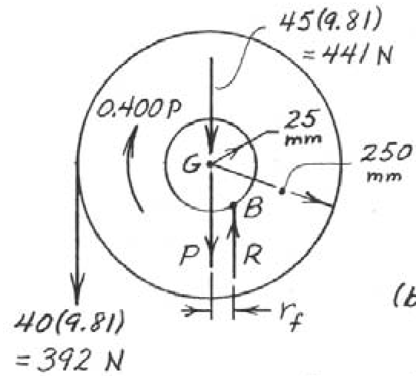
Downward force $R = pA$, $p = \frac{6+10}{\pi(3)^2} = \frac{16}{9\pi}$ lb/in.^{2}

$$M_z = \int \mu p \, dA \times r = \int_0^{2\pi}\int_0^3 \left(0.80 - \frac{r}{10}\right)\frac{16r}{9\pi} \, r \, dr \, d\theta$$

$$= 2\pi \frac{16}{9\pi}\int_0^3 \left(0.80 - \frac{r}{10}\right) r^2 dr = \frac{32}{9}\left[\frac{0.80r^3}{3} - \frac{r^4}{40}\right]_0^3$$

$$= \underline{18.40 \text{ lb-in.}}$$

6/79 FBD of shaft and attached drum, with force P replaced by a force-couple system at G:



(a) $r_f = 0$:

$$\circlearrowleft + \Sigma M_G = 0: \; 392(0.250) - 0.400P = 0, \quad \underline{P = 245 \text{ N}}$$

(b) $\varphi = \tan^{-1} 0.2 = 11.31°$

$$r_f = r \sin\varphi = 25 \sin 11.31° = 4.90 \text{ mm}$$

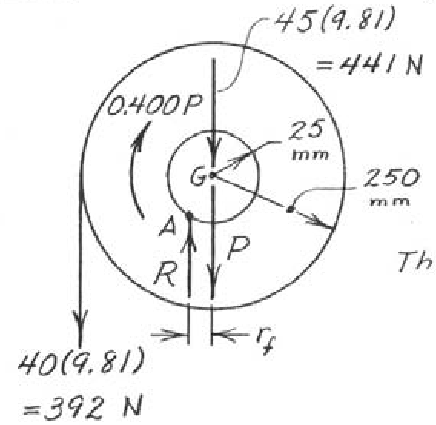
$$\circlearrowleft + \Sigma M_B = 0: \; 392(0.250 + 0.00490) + 441(0.00490) + P(0.00490) - 0.400P = 0$$

$$\underline{P = 259 \text{ N}}$$

(This solution assumes that the bearing reaction can be represented by a single force R as shown above.)

6/80

$r_f = 4.90$ mm (from Prob. 6/79)



$$\circlearrowleft \Sigma M_A = 0: 392(0.250-0.00490) - 0.400P - (441+P)(0.00490) = 0$$

$$\underline{P = 232 \text{ N}}$$

The values are NOT symmetric:

$$\underline{\begin{cases} P_{down} = 232.18 \text{ N} \\ P_{n.f.} = 245.25 \text{ N} \\ P_{up} = 258.64 \text{ N} \end{cases}}$$

6/81 $dM = (\mu p\, dA) r \quad$ where $p = k/r^2$

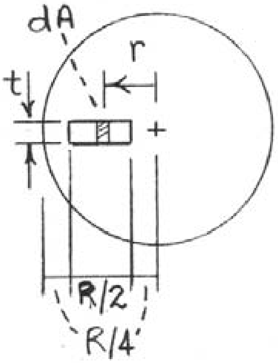
$$M = \int_0^{2\pi}\int_{r_i}^{r_o} \mu p r (r\,dr\,d\theta) = 2\pi\mu k \int_{r_i}^{r_o} dr = 2\pi\mu k (r_o - r_i)$$

Also $L = \int p\,dA = \int_0^{2\pi}\int_{r_i}^{r_o} \frac{k}{r^2} r\,dr\,d\theta = 2\pi k \ln r \Big]_{r_i}^{r_o}$

or $L = 2\pi k \ln \frac{r_o}{r_i}$, $2\pi k = \frac{L}{\ln r_o/r_i}$

Thus $M = \mu L \frac{r_o - r_i}{\ln (r_o/r_i)}$

6/82



(t small)

$$P = pA = p\left(t\,\frac{R}{2}\right)$$

$$M = \int \mu p r\,dA = \mu p \int_{R/4}^{3R/4} r t\,dr$$

$$= \mu p t \left.\frac{r^2}{2}\right|_{R/4}^{3R/4}$$

$$= \frac{\mu p t}{2}\left[\left(\frac{3R}{4}\right)^2 - \left(\frac{R}{4}\right)^2\right]$$

$$= \frac{1}{4}\mu p t R^2$$

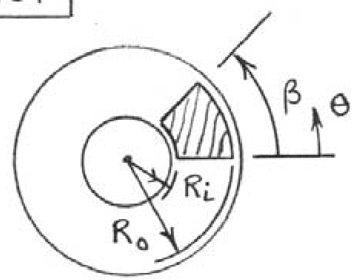
$$= \frac{1}{4}\mu\left(\frac{P}{tR/2}\right)t R^2 = \underline{\frac{1}{2}\mu P R}$$

6/83 For constant pressure Eq. 6/5a gives

$$M = \frac{2}{3}\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{2}{3}(0.35)(1)\frac{(150)^3 - (75)^3}{(150)^2 - (75)^2} = 40.8 \text{ N}\cdot\text{m}$$

For wheel $\Sigma M = 0$; $F(0.3) - 40.8 = 0$, $\underline{F = 136.1 \text{ N}}$

6/84



$$P = pA = p\int_0^\beta \int_{R_i}^{R_o} r\,dr\,d\theta$$

$$= \frac{p}{2}\int_0^\beta (R_o^2 - R_i^2)\,d\theta$$

$$= \frac{p}{2}(R_o^2 - R_i^2)\beta$$

$$M = 2\int \mu p r\,dA = 2\mu p \int_0^\beta \int_{R_i}^{R_o} r^2\,dr\,d\theta$$

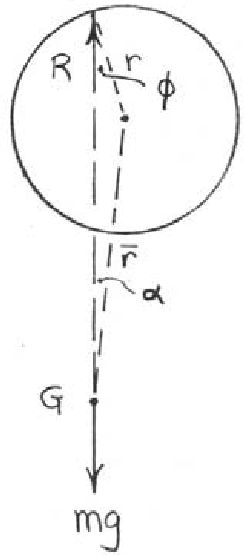
$$= \frac{2\mu p}{3}(R_o^3 - R_i^3)\beta$$

$$= \frac{2\mu}{3}\frac{2P}{(R_o^2 - R_i^2)\beta}(R_o^3 - R_i^3)\beta$$

$$= \underline{\frac{4\mu P}{3}\,\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}}$$

Same form as Eq. 6/5a except for factor of 2 for 2 pads. No β dependence. Pressure variation with θ would not change the moment M.

6/85



$$r \sin \phi = \bar{r} \sin \alpha$$

$$\mu = \tan \phi = \frac{\bar{r} \sin \alpha}{\sqrt{r^2 - \bar{r}^2 \sin^2 \alpha}}$$

or

$$\mu = \frac{1}{\sqrt{\left(\frac{d/2}{\bar{r} \sin \alpha}\right)^2 - 1}}$$

6/86 $\quad p = p_o\left(1 - \frac{r}{2a}\right); \quad dA = 2\pi r\, dr$

$$L = \int p\, dA = \int_0^a p_o\left(1 - \frac{r}{2a}\right) 2\pi r\, dr = 2\pi p_o\left[\frac{r^2}{2} - \frac{r^3}{6a}\right]_0^a$$

$$= \frac{2}{3}\pi p_o a^2 \quad \text{so} \quad p_o = \frac{3L}{2\pi a^2}$$

$$M = \int \mu p r\, dA = \int_0^a \mu p_o\left(r - \frac{r^2}{2a}\right) 2\pi r\, dr = 2\pi\mu p_o\left[\frac{r^3}{3} - \frac{r^4}{8a}\right]_0^a$$

$$= \frac{5}{12}\pi\mu p_o a^3 = \underline{\frac{5}{8}\mu L a}$$

6/87 $L = \frac{1}{4}(960 - 4 \times 40) = 200 \text{ lb}$

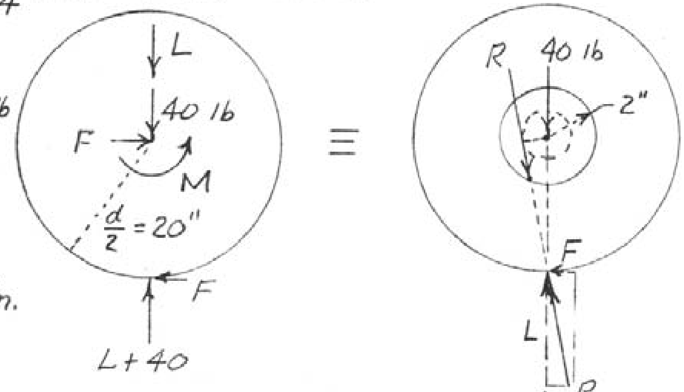
$P = 4F$

$F = \frac{1}{4} 16 = 4 \text{ lb}$

$M = F\frac{d}{2}$

$= 4(20)$

$= 80 \text{ lb-in.}$

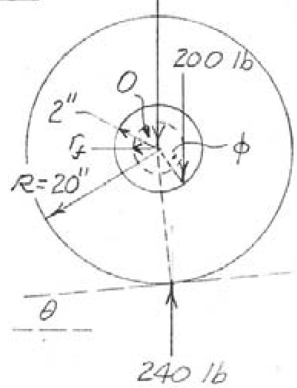


$M = Rr\sin\phi \approx Lr\sin\phi$

$80 \approx 200(2)\sin\phi$

$\phi = \sin^{-1} 0.2 = 11.54°, \quad \mu = \tan\phi = \underline{0.204}$

6/88



$r_f = r \sin\phi = 2 \sin(\tan^{-1} 0.2)$

$= 2 \sin^{-1} 11.31° = 2(0.1961)$

$= 0.3922$ in.

$\Sigma M_O = 0;$

$200 r_f - 240 R \sin\theta = 0$

$\sin\theta = \dfrac{200 \times 0.3922}{240 \times 20} = 0.01634$

$\underline{\theta = 0.936°}$

$\boxed{6/89}$ $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{11}{2\pi\frac{120}{2}} = 1.671°$

$\phi = \tan^{-1} 0.15 = 8.53°$

Screw: (a) Raise : $M_s = Wr\tan(\alpha+\phi)$

$= \frac{1}{2}(10+3)(9.81)\frac{120}{2}\tan(1.671° + 8.53°) = 689 \text{ N·m}$

(b) Lower: $M_s = Wr\tan(\phi-\alpha)$

$= \frac{1}{2}(10+3)(9.81)\frac{120}{2}\tan(8.53° - 1.671°) = 460 \text{ N·m}$

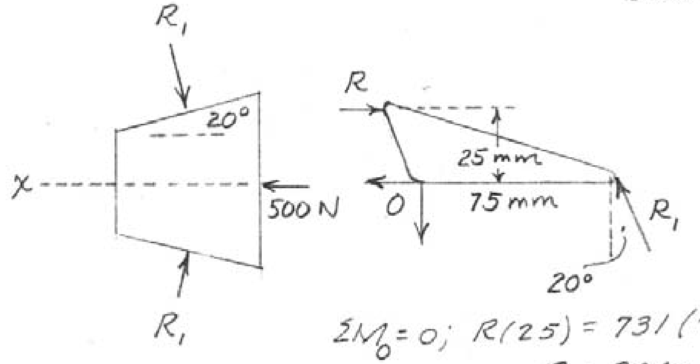
Collar bearing : $M_c = \frac{2}{3}\mu P\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

$= \frac{2}{3}(0.15)\left(\frac{10+3}{2} + 0.9\right)(9.81)\frac{(250/2)^3 - (125/2)^3}{(250/2)^2 - (125/2)^2}$

$= 1059 \text{ N·m}$

Total moment per screw $\begin{cases} \text{(a)} & M = 689 + 1059 = \underline{1747 \text{ N·m}} \\ \text{(b)} & M = 460 + 1059 = \underline{1519 \text{ N·m}} \end{cases}$

6/90 $\Sigma F_x = 0;\ 2R_1 \sin 20° = 500,\ R_1 = \dfrac{500}{2 \sin 20°} = 731\ N$



$\Sigma M_O = 0;\ R(25) = 731\,(75 \cos 20°)$

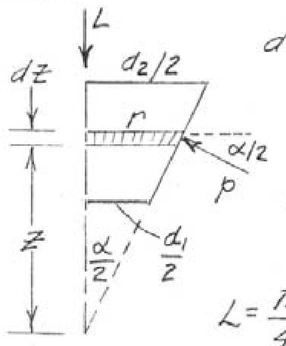
$R = 2060\ N$

For 5 pairs of surfaces with uniform pressure

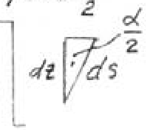
$$M = 5\left(\tfrac{2}{3}\right)\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{10}{3}(0.15)(4120)\frac{\overline{0.15}^3 - \overline{0.05}^3}{\overline{0.15}^2 - \overline{0.05}^2}$$

$\underline{M = 335\ N \cdot m}$

►6/91 $dL = p(2\pi r)\, ds \, \sin\frac{\alpha}{2}$ where $ds = dz/\cos\frac{\alpha}{2}$



$$dL = 2\pi p \sin\frac{\alpha}{2}\left(z \tan\frac{\alpha}{2}\right)\frac{dz}{\cos \alpha/2}$$



$$= 2\pi p \tan^2\frac{\alpha}{2}\, z\, dz$$

$$L = 2\pi p \tan^2\frac{\alpha}{2}\int_{z_1}^{z_2} z\, dz \quad \text{where} \quad z_1 = \frac{d_1/2}{\tan \alpha/2}$$

$$z_2 = \frac{d_2/2}{\tan \alpha/2}$$

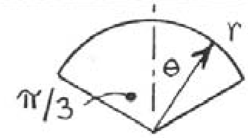
$$L = \frac{\pi p}{4}(d_2^2 - d_1^2)$$

$$M = \int r \mu p \, dA = \mu p \int_{z_1}^{z_2}\left(z \tan\frac{\alpha}{2}\right)\left(2\pi z \tan\frac{\alpha}{2}\right)\frac{dz}{\cos \alpha/2}$$

$$= 2\pi \mu p \frac{\tan^2\frac{\alpha}{2}}{\cos \alpha/2}\int_{z_1}^{z_2} z^2 dz = 2\pi \mu p \frac{1}{24 \sin\frac{\alpha}{2}}(d_2^3 - d_1^3)$$

$$\underline{M = \frac{\mu L}{3\sin\frac{\alpha}{2}}\,\frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}}$$

▶6/92



$$dM = \mu p\, dA\,(r \sin\theta) = \mu p\,(r\,d\theta)(2\pi r \sin\theta)\, r\sin\theta$$

$$= 2\pi \mu r^3 p_0 \cos\theta \sin^2\theta\, d\theta$$

$$M = 2\pi\mu r^3 p_0 \int_0^{\pi/3} \cos\theta \sin^2\theta\, d\theta$$

$$= 2\pi\mu r^3 p_0 \left.\frac{\sin^3\theta}{3}\right|_0^{\pi/3} = \frac{\sqrt{3}}{4}\pi\mu r^3 p_0$$

But $L = \int p\cos\theta\, dA = \int_0^{\pi/3} p_0 \cos^2\theta\,(r\,d\theta)(2\pi r\sin\theta)$

$$= 2\pi r^2 p_0 \int_0^{\pi/3} \cos^2\theta \sin\theta\, d\theta = 2\pi r^2 p_0 \left(-\frac{\cos^3\theta}{3}\right)_0^{\pi/3}$$

$$= \frac{7}{12}\pi r^2 p_0$$

Substitute $\pi r^2 p_0 = \frac{12}{7}L$ to obtain

$$M = \frac{\sqrt{3}}{4}\mu r\left(\frac{12L}{7}\right) = \underline{\frac{3\sqrt{3}}{7}\mu r L}$$

6/93

$$\frac{T_2}{T_1} = e^{\mu\beta} : \quad \frac{mg}{mg/10} = e^{\mu(3\pi)}, \quad \underline{\mu = 0.244}$$

$\boxed{6/94}$ Use $\frac{T_2}{T_1} = e^{\mu\beta}$, where $\beta = \frac{\pi}{2}$

(a) $\frac{P}{W} = e^{0.4(\pi/2)}$, $\underline{P = 1.874W}$

(b) $\frac{W}{P} = e^{0.4(\pi/2)}$, $\underline{P = 0.533W}$

$\boxed{6/95}$ $\frac{T_2}{T_1} = e^{\mu\beta}$, $\frac{T}{200} = e^{0.30(5\pi/2)}$ where $\beta = \frac{5}{4}(2\pi)$ rad

$\frac{T}{200} = e^{2.356} = 10.55$, $T = 10.55(200) = 2110$ N

or $\underline{T = 2.11 \text{ kN}}$

6/96 $T_2 = T_1 e^{\mu\beta}$

$18\,000 = 240 e^{0.30\beta}$, $e^{0.30\beta} = 75.0$

$0.30\beta = 4.32$, $\beta = 14.39$ rad

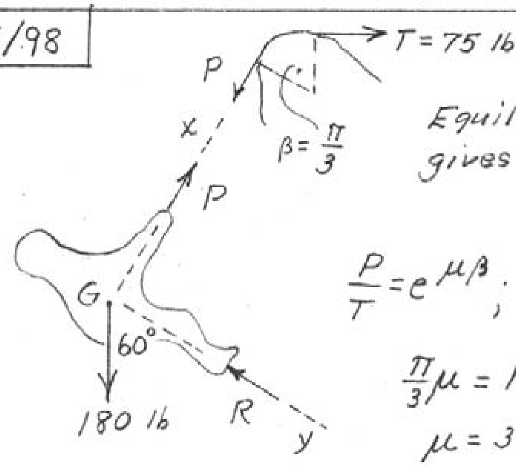
No. of turns $n = \frac{\beta}{2\pi} = \frac{14.39}{2\pi} = \underline{2.29}$ turns

6/97 $T_2 = T_1 e^{\mu\beta}$, $T_2 = 50$ lb, $T_1 = 20$ lb, $\beta = \pi$

So $\frac{50}{20} = e^{\pi\mu}$, $e^{\pi\mu} = 2.5$, $\pi\mu = \ln 2.5$

$\pi\mu = 0.9163$, $\underline{\mu = 0.292}$

6/98



Equil. of climber, $\Sigma F_x = 0$

gives $P = 180 \sin 60°$

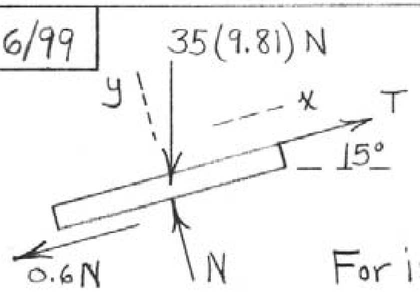
$= 155.9$ lb

$\frac{P}{T} = e^{\mu\beta}$; $\frac{155.9}{75} = e^{\pi\mu/3}$

$\frac{\pi}{3}\mu = \ln 2.078 = 0.732$

$\mu = 3(0.732)/\pi = \underline{0.699}$

6/99



$\Sigma F_y = 0$:

$N - 35(9.81)\cos 15° = 0$

$N = 332$ N

For impending motion up incline:

$-0.6(332) - 35(9.81)\sin 15° + T = 0$, $T = 288$ N

Cord friction : $\dfrac{P}{288} = e^{0.4[15 \frac{\pi}{180}]}$

$\underline{P = 320 \text{ N}}$

6/100 $T_2 = T_1 e^{\mu\beta}$, $\beta = 2$ turns $+ 60°$

$= 2(360°) + 60° = 780°$

or 13.61 rad

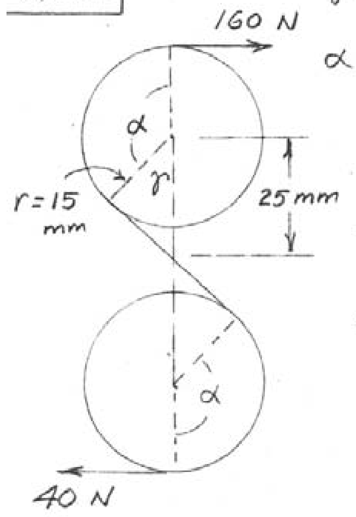
$$T = \left(\frac{2}{16}\right) e^{0.7(13.61)} = \underline{1720 \text{ lb}}$$

$\boxed{6/101}$ $\frac{4}{mg} = e^{\mu\beta}$, $\frac{mg}{1.6} = e^{\mu\beta}$

Thus $\frac{4}{mg} = \frac{mg}{1.6}$, $m^2 g^2 = 4(1.6)$

$m = \frac{\sqrt{4(1.6)}}{9.81} = 0.258$ Mg or $\underline{m = 258 \text{ kg}}$

6/102



$\gamma = \cos^{-1}\frac{15}{25} = 53.13°$

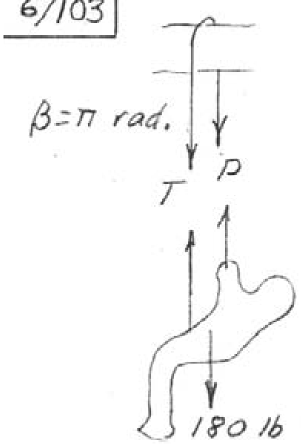
$\alpha = 180 - 53.13 = 126.87°$

$\beta = 2\alpha = \frac{2(126.87)}{180}\pi = 4.429 \text{ rad}$

$\frac{T_2}{T_1} = e^{\mu\beta}$, $\frac{160}{40} = e^{4.429\mu}$

$4.429\mu = 1.3863$, $\underline{\mu = 0.313}$

6/103



$T = Pe^{\mu\beta}$

$= Pe^{0.6\pi} = 6.59\ P$

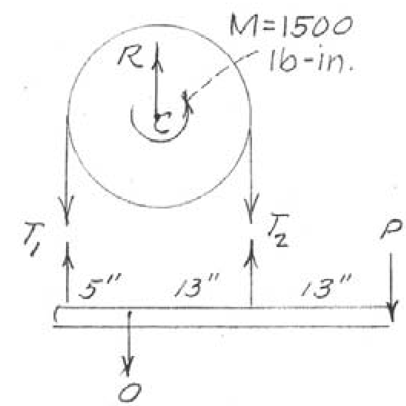
Man; $\Sigma F = 0$; $T + P = 180$

Combine & set

$P(1 + 6.59) = 180$ lb

$P = \dfrac{180}{7.59} = \underline{23.7 \text{ lb}}$

6/104 $\quad T_2/T_1 = e^{\mu\beta} = e^{0.20\pi} = 1.874 \quad$ ----(a)



$\Sigma M_C = 0;\ (T_2 - T_1)\,9 = 1500$

$T_2 - T_1 = 166.7 \quad$ ----(b)

Lever: $\Sigma M_O = 0;\ 26P + 5T_1 - 13T_2 = 0 \quad$ ---(c)

Combine (a) & (b) & get

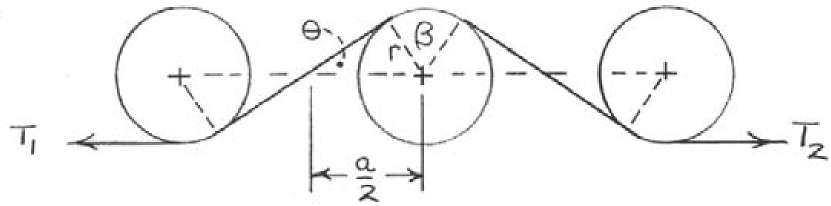
$T_2 = 1.874(T_2 - 166.7),\ T_2 = 357$ lb.

$T_1 = 357 - 166.7 = 190.7$ lb

Substitute in (c) & get

$26P + 5(190.7) - 13(357) = 0, \quad \underline{P = 142.0 \text{ lb}}$

6/105

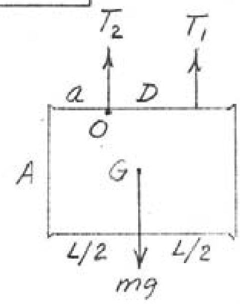


$\theta = \sin^{-1}\frac{r}{a/2}$, $\beta = 2\theta = 2\sin^{-1}\frac{2r}{a}$

$\frac{T_2}{T_1} = e^{\mu\beta}$: $1.15 = e^{0.1\beta}$, $\beta = 1.398$ rad or $80.1°$

Thus $80.1° = 2\sin^{-1}\frac{2(20)}{a}$, $\underline{a = 62.2 \text{ mm}}$

6/106 Slipping impends for rope when $T_2 = T_1 e^{\mu\beta}$ (1)



Equil. of drum

$\Sigma F = 0;\ T_1 + T_2 = mg$ ----------- (2)

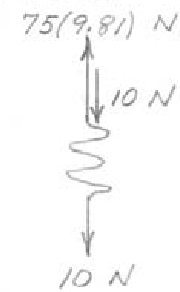
$\Sigma M_0 = 0;\ mg\left(\frac{L}{2} - a\right) = T_1 D$ --------- (3)

(2) & (3) $T_1\left(\frac{L}{2} - a - D\right) = T_2\left(a - \frac{L}{2}\right)$

Combine with (1) & get

$$\frac{\frac{L}{2} - a - D}{a - \frac{L}{2}} = e^{\mu\beta}, \quad \underline{a = \frac{L}{2} - \frac{D}{1 + e^{\mu\pi}}} \quad \text{where } \beta = \pi \text{ rad.}$$

6/107 $L = 75(9.81)$ N, $T_2 = T_1 e^{\mu\beta}$,

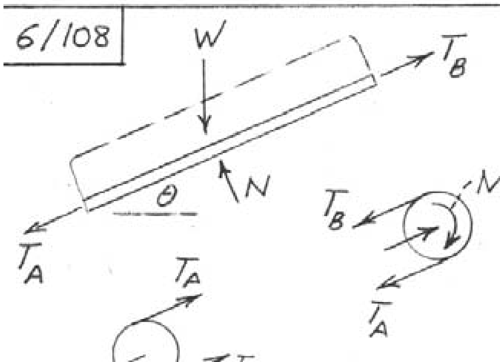


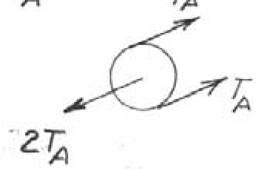
$$75(9.81) - 10 = 10e^{\mu(3+\frac{1}{2})2\pi}$$

$$72.6 = e^{21.99\mu}$$

$$21.99\mu = 4.285, \quad \underline{\mu = 0.1948}$$

6/108





$\theta = \tan^{-1} \frac{20}{50} = 21.8°$

$\sin \theta = 0.3714$

$W \sin \theta = 30(150)(0.3714)$
$= 1671$ lb

$T_B = 1000 + \frac{1671}{2} = 1836$ lb

$T_A = 1000 - \frac{1671}{2} = 164$ lb

$\frac{T_B}{T_A} = e^{\mu \beta}$; $\frac{1836}{164} = e^{\pi \mu} = 11.17$

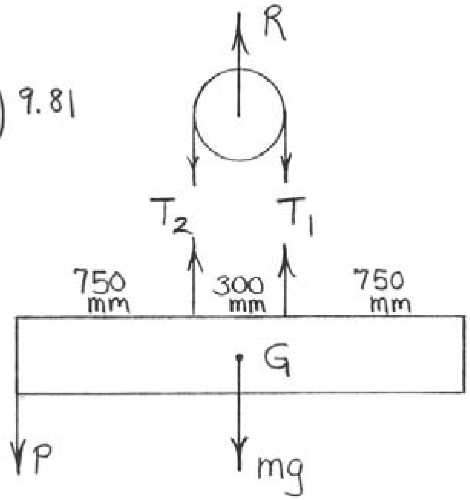
$\pi \mu = \ln 11.17 = 2.413$

$\mu = \frac{2.413}{\pi} = \underline{0.768}$

6/109

$mg = 74(0.75+0.30+0.75)\,9.81$

$= 1307$ N



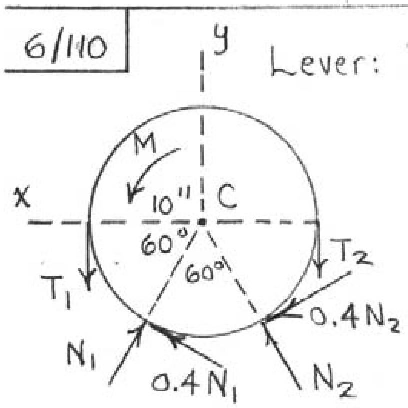
Beam $+\uparrow \Sigma F = 0: \; T_1 + T_2 - P - 1307 = 0 \qquad (1)$

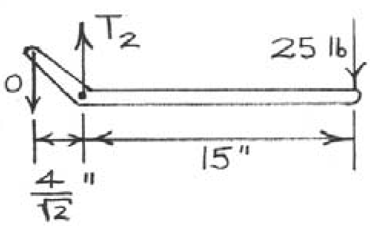
$\circlearrowleft + \Sigma M_G = 0: \; P\left(0.75 + \frac{0.30}{2}\right) + \frac{0.30}{2} T_1 - \frac{0.30}{2} T_2 = 0 \qquad (2)$

Drum: $T_2 = T_1 e^{\mu\beta} = T_1 e^{0.5\pi} \qquad (3)$

Solve Eqs. (1), (2), and (3): $\left\{ \begin{array}{l} T_1 = 252 \text{ N} \\ T_2 = 1215 \text{ N} \\ \underline{P = 160.3 \text{ N}} \end{array} \right.$

6/110





Lever: $\sum M_O = 0: \; \frac{4T_2}{\sqrt{2}} = 25\left(15 + \frac{4}{\sqrt{2}}\right)$

$T_2 = 157.6$ lb

Band: $\frac{T_2}{T_1} = e^{\mu\beta}$

$\frac{157.6}{T_1} = e^{0.3\pi}$, $\quad T_1 = 61.4$ lb

Pipe: $\sum F_y = 0$:

$N_1\left(\frac{\sqrt{3}}{2} + 0.4\left(\frac{1}{2}\right)\right) + N_2\left(\frac{\sqrt{3}}{2} - 0.4\left(\frac{1}{2}\right)\right)$

$- 157.6 - 61.4 = 0 \qquad$ (a)

$\sum F_x = 0: \; N_1\left(\frac{1}{2} - 0.4\frac{\sqrt{3}}{2}\right)$

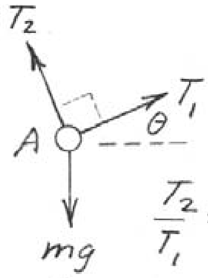
$- N_2\left(\frac{1}{2} + 0.4\frac{\sqrt{3}}{2}\right) = 0 \qquad$ (b)

Solve (a) & (b): $N_1 = 184.5$ lb, $\quad N_2 = 33.5$ lb

$\circlearrowleft + \sum M_C = 0: \; M - 157.6(10) + 61.4(10)$

$- 0.4(184.5 + 33.5)(10) = 0, \qquad \underline{M = 1834 \text{ lb-in.}}$

6/111



Equilibrium of A gives

$T_1 = mg \sin\theta$

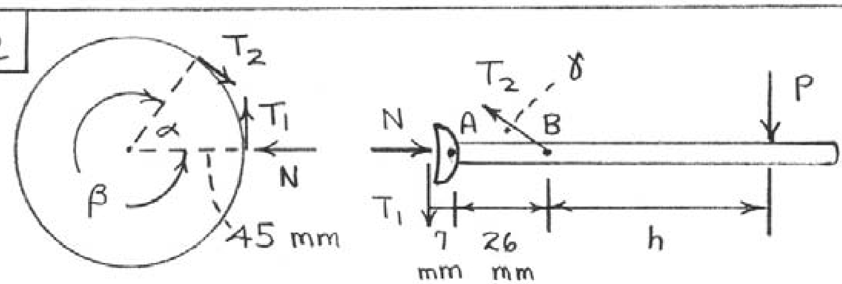
$T_2 = mg \cos\theta$

$$\frac{T_2}{T_1} = e^{\mu\beta}; \quad \frac{mg\cos\theta}{mg\sin\theta} = e^{3\pi\mu/2}$$

For $\theta = 20°$, $\text{ctn}\, 20° = e^{3\pi\mu/2} = 2.747$

or $\mu = \frac{2}{3\pi} \ln 2.747 = \underline{0.214}$

6/112



$$\cos\alpha = \frac{45}{45+7+26}, \quad \alpha = 54.8^\circ, \quad \gamma = 180-90-\alpha = 35.2^\circ$$

$$\beta = 360 - \alpha = 305^\circ \text{ or } 5.33 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.25(5.33)} = 3.79 \qquad \text{(a)}$$

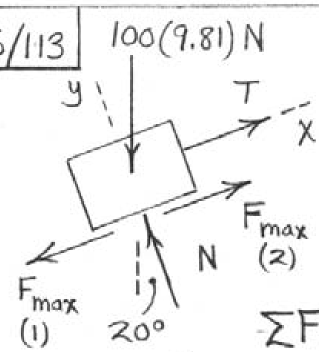
Bar: $\Sigma M_P = 0$: $T_1(h+7+26) - T_2 \sin\gamma\,(h) = 0$

$$\text{or} \quad \frac{T_2}{T_1} = \frac{h+33}{0.577h} \qquad \text{(b)}$$

From (a) & (b), $\underline{h = 27.8 \text{ mm}}$

(For actual wrench, $h \cong 100$ mm)

6/113



$(F_{max} = 0.3N)$

$\Sigma F_y = 0: \; N - 100(9.81)\cos 20° = 0$

$N = 922$ N (throughout)

(a) No cord friction: $T = mg$

(1) Motion impends up incline:

$\Sigma F_x = 0: -0.3(922) - 981 \sin 20° + T = 0$

$T = 612$ N, $m = \frac{612}{9.81} = 62.4$ kg

(2) Motion impends down the incline:

$\Sigma F_x = 0: \; 0.3(922) - 981 \sin 20° + T = 0, \quad T = 59.0$ N

$m = \frac{59.0}{9.81} = 6.01$ kg ; so $\underline{6.01 \text{ kg} \leq m \leq 62.4 \text{ kg}}$

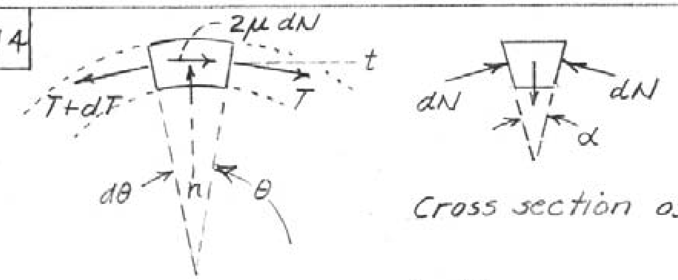
(b) Cord friction: $T \neq mg$, $\beta = 110°\left(\frac{\pi}{180°}\right) = 1.920$ rad

(1) Motion impends up: $\frac{mg}{612} = e^{0.3(1.920)}$, $m = 111.0$ kg

(2) Motion impends down: $\frac{59.0}{mg} = e^{0.3(1.920)}$, $m = 3.38$ kg

So $\underline{3.38 \text{ kg} \leq m \leq 111.0 \text{ kg}}$

6/114



Cross section of belt

$\Sigma F_n = 0;\quad T \sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2} = 2\,dN \sin\frac{\alpha}{2}$

or $\quad T\,d\theta = 2\,dN \sin\frac{\alpha}{2}$

$\Sigma F_t = 0;\quad T\cos\frac{d\theta}{2} + 2\mu\, dN = (T+dT)\cos\frac{d\theta}{2}$

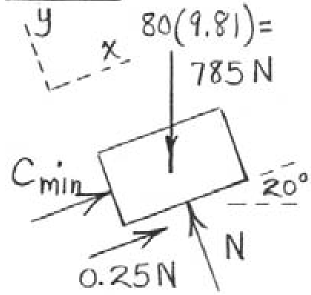
or $\quad 2\mu\, dN = dT$

combine & get $\quad \frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\, d\theta$

$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}} \int_0^{\beta} d\theta,\quad \ln\frac{T_2}{T_1} = \frac{\mu\beta}{\sin\frac{\alpha}{2}},\quad \underline{T_2 = T_1 e^{\mu\beta/\sin\frac{\alpha}{2}}}$

$n = 1/\sin 17.5° = \underline{3.33}$

6/115



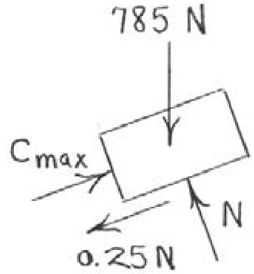
$\Sigma F_y = 0: \quad N - 785 \cos 20° = 0$

$N = 737 \text{ N}$ Throughout

(a) C_{min}

$\Sigma F_x = 0: \; C_{min} + 0.25\,(737)$

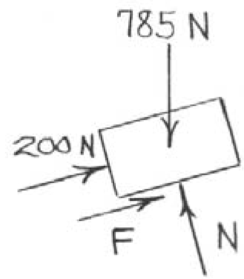
$- 785 \sin 20° = 0, \quad \underline{C_{min} = 84.0 \text{ N}}$



(b) C_{max}

$\Sigma F_x = 0: \; C_{max} - 0.25\,(737)$

$- 785 \sin 20° = 0, \quad C_{max} = \underline{453 \text{ N}}$



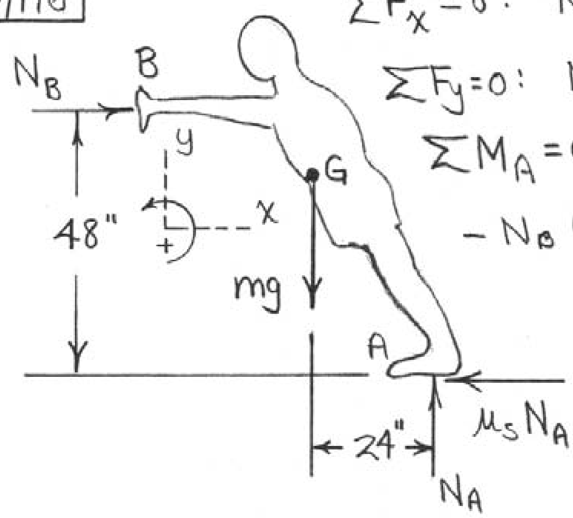
(c) $C = 200$ N

$\Sigma F_x = 0: \; 200 + F - 785 \sin 20° = 0$

$\underline{F = 68.4 \text{ N} \text{ up the incline}}$

($F < \mu_s N = 0.25\,(737) = 184.4$ N, so no motion occurs)

6/116



$\Sigma F_x = 0: \quad N_B - \mu_s N_A = 0 \quad (1)$

$\Sigma F_y = 0: \quad N_A - mg = 0 \quad (2)$

$\Sigma M_A = 0:$

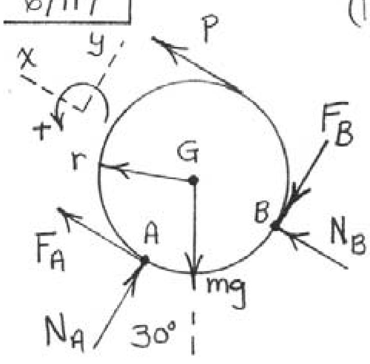
$-N_B(48) + mg(24) = 0 \quad (3)$

Solve Eqs. (1)-(3) to obtain $N_A = mg$

$N_B = \frac{1}{2} mg$

$\mu_s = \frac{1}{2}$

6/117



(1) Assume no slippage until contact at B is lost :

$F_B = N_B = 0$

$$\begin{cases} \Sigma F_x = 0 : P + F_A - mg \sin 30° = 0 \\ \Sigma F_y = 0 : N_A - mg \cos 30° = 0 \\ \Sigma M_G = 0 : Pr - F_A r = 0 \end{cases}$$

Solution : $P = F_A = \frac{mg}{4}$, $N_A = 0.866 mg$

$(F_A)_{max} = \mu_s N_A = 0.25(0.866 mg) = 0.217 mg < F_A$;

Assumption invalid

(2) Assume rotational slippage impends :

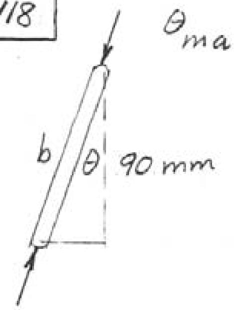
$F_A = \mu_s N_A = 0.25 N_A$, $F_B = \mu_s N_B = 0.25 N_B$

$$\left.\begin{array}{l} \Sigma F_x = 0 : P + 0.25 N_A - mg \sin 30° + N_B = 0 \\ \Sigma F_y = 0 : N_A - mg \cos 30° - 0.25 N_B = 0 \\ \Sigma M_G = 0 : Pr - 0.25 N_A r - 0.25 N_B r = 0 \end{array}\right\}$$

Solution : $P = 0.232 mg$, $N_A = 0.878 mg$, $N_B = 0.0487 mg$

<u>So rotational slippage occurs first, at $P = 0.232 mg$</u>

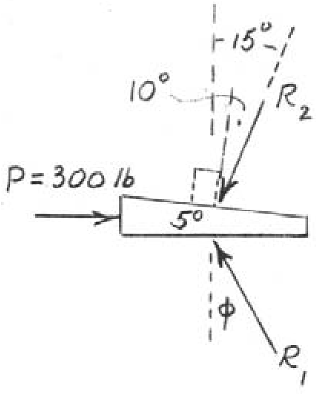
6/118

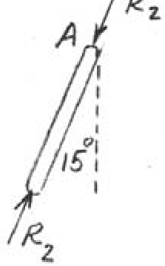


$\theta_{max} = \phi = \tan^{-1}\mu = \tan^{-1} 0.40 = 21.80°$

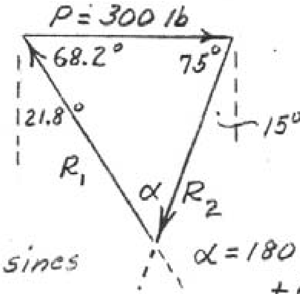
$b = 90/\cos 21.80° = \underline{96.9\ mm}$

6/119





Wedge $\phi = \tan^{-1} 0.40$
$= 21.80°$



$\alpha = 180 - (75 + 68.2) = 36.8°$

Law of sines

$$\frac{R_2}{\sin 68.2} = \frac{300}{\sin 36.8°}$$

$$R_2 = 300\ \frac{0.9285}{0.5990} = 465 \text{ lb}$$

Friction force at A is $F = R_2 \sin 15° = 465(0.2588)$

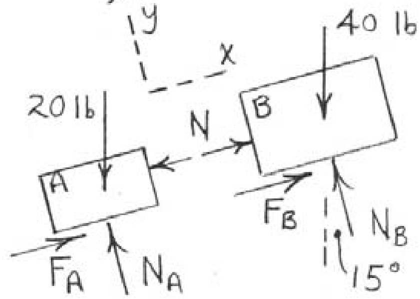
$= \underline{120.3 \text{ lb}}$

$10° < (\tan^{-1} 0.4 = 21.8°)$ so toggle does not slip & $F \neq \mu N$.

6/120 From $\theta_{max} = \tan^{-1}\mu_s$, we have

$$(\theta_{max})_A = \tan^{-1} 0.30 = 16.70^\circ$$
$$(\theta_{max})_B = \tan^{-1} 0.20 = 11.31^\circ$$
$$(\theta_{max})_C = \tan^{-1} 0.35 = 19.29^\circ$$

So C remains stationary. By themselves, B would slide, A would not.



From $\Sigma F_y = 0$:

$N_A = 19.32$ lb

$N_B = 38.6$ lb

Assume that slipping impends for B:

$\Sigma F_x = 0$: $N + 0.2(38.6) - 40 \sin 15^\circ = 0$, $N = 2.63$ lb

$\Sigma F_x = 0$ for A: $F_A - 20 \sin 15^\circ - 2.63 = 0$

$F_A = 7.80$ lb; $(F_A)_{max} = 0.30(19.32) = 5.80$ lb

Because $(F_A)_{max} < F$, both A and B slip.

6/121 Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

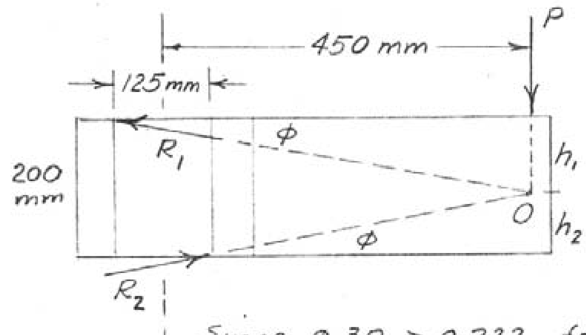
$\tan\alpha = \frac{L}{2\pi r}$; Critical when $\alpha = \phi$

$\therefore$ Lead $L = 2\pi r \tan\phi = 2\pi \frac{3/8}{2} \tan 8.53°$

$= 0.1767$ in. per revolution

$N = 1/L = 1/0.1767 = \underline{5.66 \text{ threads per inch}}$

6/122 Forces concurrent at O gives minimum ϕ



$h_1 = (450 + 62.5) \tan\phi$

$h_2 = (450 - 62.5) \tan\phi$

$h_1 + h_2 = 200$ mm

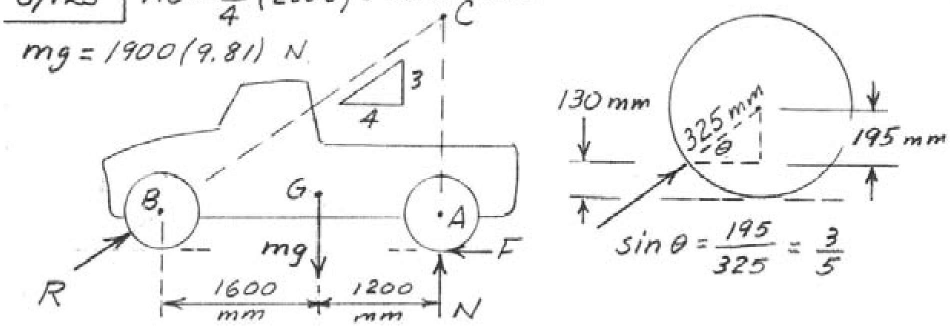
$(512.5 + 387.5) \tan\phi = 200$

$\tan\phi = \frac{200}{900} = 0.222$

Since $0.30 > 0.222$, friction will prevent slipping

6/123 $\overline{AC} = \frac{3}{4}(2800) = 2100$ mm

$mg = 1900(9.81)$ N



$\Sigma M_C = 0;\ F(2100 + 325) - 1200\,mg = 0,\ F = 0.495\,mg$

$\Sigma M_B = 0;\ 1600\,mg + 0.495\,mg\,(325) - 2800\,N = 0,\ N = 0.629\,mg$

Thus $\mu_{min} = F/N = 0.495/0.629 = \underline{0.787}$

$M = Fr = 0.495(1900)(9.81)(0.325) = 2998$ N·m

or $\underline{M = 3.00 \text{ kN·m}}$

6/124 $\phi = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.31°$

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{2.5}{2\pi(5)} = 4.55°$$

(a) Tighten: $M = Pr\tan(\phi + \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31° + 4.55°)$$

$$\underline{F = 8.52 \text{ N}}$$

(b) Loosen: $M = Pr\tan(\phi - \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31° - 4.55°)$$

$$\underline{F = 3.56 \text{ N}}$$

6/125 Helix angle $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{8}{2\pi\frac{25}{2}} = 5.82°$

$\phi = \tan^{-1} 0.25 = 14.04°$

Screw: (a) $M_s = Wr\tan(\alpha + \phi) = 4(\frac{25}{2})\tan 19.86°$

$= 18.05 \text{ N·m}$

(b) $M_s = Wr\tan(\phi - \alpha) = 4(\frac{25}{2})\tan 8.22°$

$= 7.22 \text{ N·m}$

Bearing (worn) $M_B = \frac{1}{2}\mu P(R_o + R_i) = \frac{1}{2}(0.25)(4)\frac{20+4}{2}$

$= 6.00 \text{ N·m}$

Total moment (a) $M = 18.05 + 6.00 = \underline{24.1 \text{ N·m}}$

(b) $M = 7.22 + 6.00 = \underline{13.22 \text{ N·m}}$

6/126 Helix angle $\alpha = \tan^{-1} \frac{24}{40\pi} = 10.81°$

Friction angle $\phi = \tan^{-1} \mu = \tan^{-1} 0.15 = 8.53°$

$\alpha > \phi$ so screw is not self-locking.

$\alpha + \phi = 19.34°$; $\alpha - \phi = 2.28°$

(a) $M = Pr \tan(\alpha - \phi)$: $60 = P(0.020) \tan 2.28°$

$P = 75\ 300$ N or $\underline{75.3 \text{ kN}}$

(b) $M = P r \tan(\alpha + \phi)$: $60 = P\ (0.020) \tan 19.34°$

$P = 8550$ N or $\underline{8.55 \text{ kN}}$

$\boxed{6/127}$ $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{13}{2\pi(78)} = 3.04°$

$\phi = \tan^{-1}\mu = \tan^{-1} 0.25 = 14.04°$

(a) To raise, $M = Wr \tan(\alpha + \phi)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(3.04° + 14.04°)$$

$$= \underline{129.3 \text{ N}\cdot\text{m}}$$

(b) To lower, $M = Wr \tan(\phi - \alpha)$

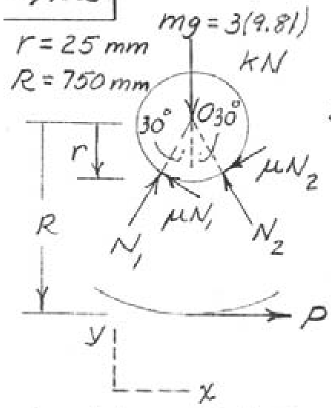
$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(14.04 - 3.04)$$

$$= \underline{81.8 \text{ N}\cdot\text{m}}$$

6/128

$r = 25\ mm$

$R = 750\ mm$



$$\Sigma M_O = 0;\ PR = \mu r(N_1 + N_2) \qquad (a)$$

$$\Sigma F_x = 0;\ P + (N_1 - N_2)\frac{1}{2} = \mu(N_1 + N_2)\frac{\sqrt{3}}{2} \qquad (b)$$

$$\Sigma F_y = 0;\ mg = (N_1 + N_2)\frac{\sqrt{3}}{2} + \mu(N_1 - N_2)\frac{1}{2} \qquad (c)$$

Combine (a) & (c) to give

$$\frac{1}{2}(N_1 - N_2) = \frac{mg}{\mu} - \frac{\sqrt{3}}{2}\frac{PR}{\mu^2 r} \qquad (d)$$

Combine (a) & (b) to give

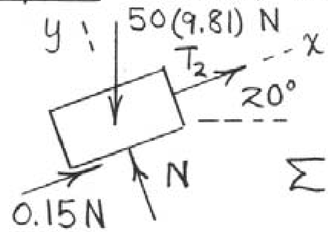
$$\frac{1}{2}(N_1 - N_2) = -P + \frac{PR}{r}\frac{\sqrt{3}}{2} \qquad (e)$$

Combine (d) & (e) & get

$$P = \frac{mg/\mu}{\frac{R}{r}\frac{\sqrt{3}}{2}\left(1 + \frac{1}{\mu^2}\right) - 1} = \frac{3(9.81)/0.30}{\frac{750}{25}\frac{\sqrt{3}}{2}\left(1 + \frac{1}{0.3^2}\right) - 1} = 0.313\ kN$$

or $\underline{313\ N}$

6/129 Motion impending down plane:



$\Sigma F_y = 0: \; N = 50(9.81)\cos 20°$

$N = 461$ N (throughout)

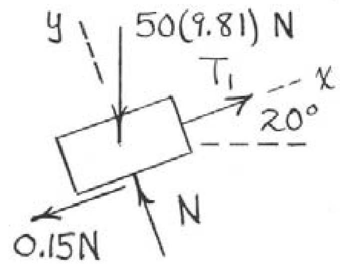
$\Sigma F_x = 0: \; T_2 - 50(9.81)\sin 20° + 0.15(461) = 0$

$T_2 = 98.6$ N

$T_2 = T_1 e^{\mu\beta}: \quad 98.6 = \frac{mg}{2} e^{0.25(110 \cdot \frac{\pi}{180})}$

$m = 12.44$ kg

Motion impending up plane:



$\Sigma F_x = 0: \; T_1 - 50(9.81)\sin 20°$
$- 0.15(461) = 0$

$T_1 = 237$ N

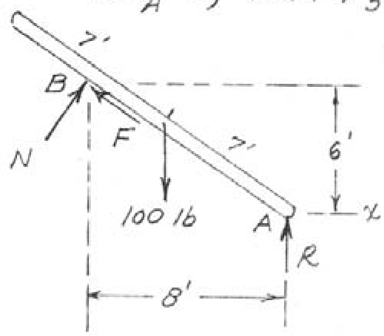
$T_2 = T_1 e^{\mu\beta}: \quad \frac{mg}{2} = 237 e^{0.25(110 \cdot \frac{\pi}{180})}$

$m = 78.0$ kg

So range is $\underline{12.44 \le m \le 78.0 \text{ kg}}$

6/130 Assume equilibrium

$\Sigma M_A = 0$; $100(7)\frac{4}{5} - N(10) = 0$, $N = 56$ lb



$\Sigma F_x = 0$; $\frac{4}{5}F - \frac{3}{5}(56) = 0$

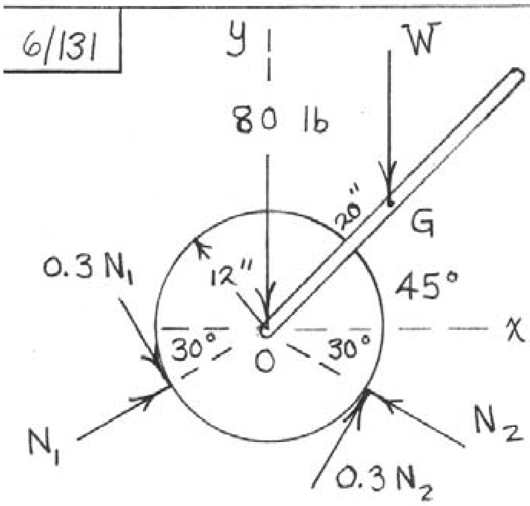
$F = 42$ lb

$\mu_{s_{min}} = \frac{F}{N} = \frac{42}{56} = 0.75 < 0.80$

so assumption is OK,

& $F = 42$ lb

6/131



$\begin{pmatrix} C = \cos \\ S = \sin \end{pmatrix}$

$\Sigma F_x = 0$: $N_1 c30° + 0.3N_1 s30° - N_2 c30° + 0.3N_2 s30° = 0$

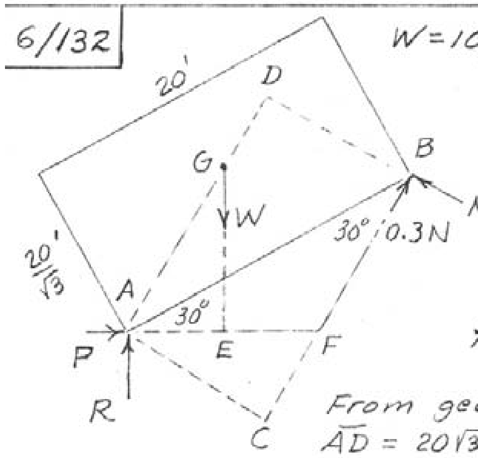
$\Sigma F_y = 0$: $N_1 s30° - 0.3N_1 c30° + N_2 s30° + 0.3N_2 c30° - 80 - W = 0$

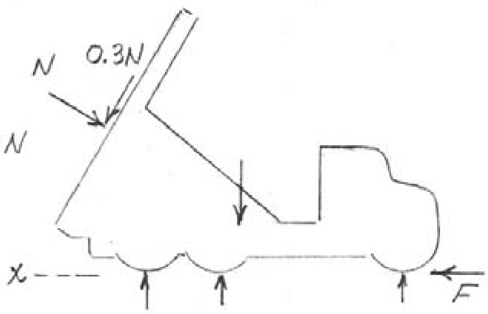
$\Sigma M_O = 0$: $(0.3N_1 + 0.3N_2)(12) - W(20c45°) = 0$

Solve to obtain $\begin{cases} N_1 = 113.9 \text{ lb} \\ N_2 = 161.6 \text{ lb} \\ \underline{W = 70.1 \text{ lb}} \end{cases}$

6/132

$W = 10{,}000$ lb





From geometry of 30-60 triangles

$\overline{AD} = 20\sqrt{3}/2 = 10\sqrt{3}$ ft, $\overline{AC} = 10$ ft

$\overline{AE} = \frac{1}{2}\overline{AF} = \frac{1}{2}\,10/\frac{\sqrt{3}}{2} = 10/\sqrt{3}$ ft.

Box:

$\Sigma M_A = 0$; $10{,}000(10/\sqrt{3}) - 10\sqrt{3}\,N - 10(0.3N) = 0$

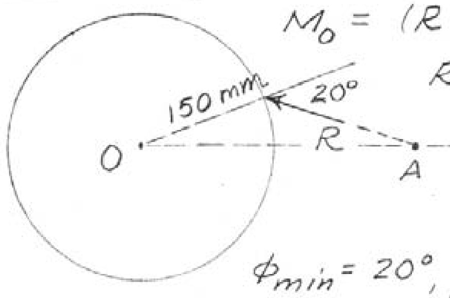
$N = 2840$ lb

Truck:

$\Sigma F_x = 0$; $F = N\left(\frac{\sqrt{3}}{2}\right) - 0.3N\left(\frac{1}{2}\right) = 0.7160(2840) = \underline{2030 \text{ lb}}$

6/133

$M_O = PL = 150(0.600) = 90 \text{ N}\cdot\text{m}$



$M_O = (R \sin 20^\circ)(0.150) = 90$

$R = \dfrac{90}{0.3420(0.150)} = 1754 \text{ N}$

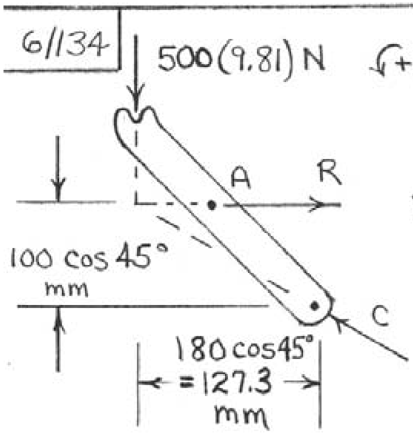
Force on pin at A is

$\underline{R = 1.754 \text{ kN}}$

$\phi_{min} = 20^\circ$, $\mu_{s(min)} = \tan\phi_{min} = \tan 20^\circ$

$= \underline{0.364}$

6/134



$\curvearrowleft + \sum M_C = 0: \; 500(9.81)(127.3) - R(100\cos 45°) = 0, \; R = 8830 \text{ N}$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\dfrac{0.002}{2\pi(0.005)} = 3.64°$

Raise load: $M = Rr\tan(\phi+\alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° + 3.64°)$

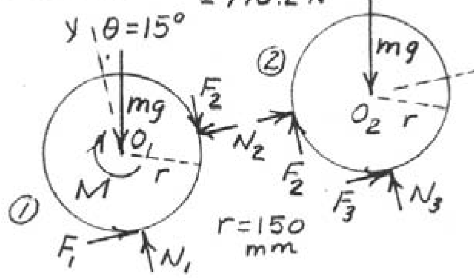
$\underline{P = 78.6 \text{ N}}$

Lower load: $M = Rr\tan(\phi-\alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° - 3.64°)$

$\underline{P = 39.6 \text{ N}}$

6/135 $mg = 20(9.81)$ $= 196.2\ N$ $\mu_s = 0.60,\ \mu_k = 0.50$



Assume slipping occurs only at contact between cylinders

so $F_2 = \mu_k N_2 = \mu N_2$

$\Sigma M_{O_1} = 0;\ M = (F_1 - F_2) r$

$\Sigma M_{O_2} = 0;\ F_2 = F_3$

① $\Sigma F_x = 0;\ F_1 - N_2 - mg \sin\theta = 0$

$\Sigma F_y = 0;\ N_1 - F_2 - mg\cos\theta = 0$

② $\Sigma F_x = 0;\ F_3 + N_2 - mg\sin\theta = 0$

$\Sigma F_y = 0;\ N_3 + F_2 - mg\cos\theta = 0$

7 eqs. in 7 unknowns

Solve, substitute values & get

$F_2 = \dfrac{\mu}{1+\mu} mg \sin\theta = F_3 = 16.93\ N$

$F_1 = \dfrac{2+\mu}{1+\mu} mg\sin\theta = 84.6\ N$

$N_1 = 206.4\ N$

$N_2 = 33.9\ N$

$N_3 = 172.6\ N$

$F_1 < \mu_s N_1$ &

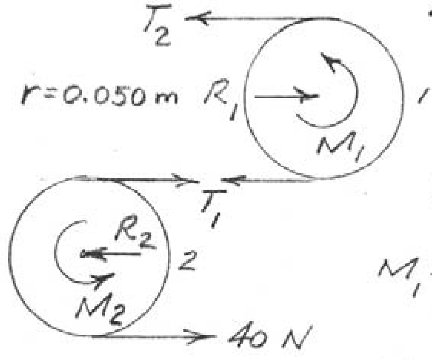
$F_3 < \mu_s N_3$ so

assumption valid &

$M = (84.6 - 16.93)(0.150)$

$= \underline{10.16\ N\cdot m}$

6/136



$$\frac{T_1}{40} = e^{0.2(\pi)} \; ; \quad \frac{T_1}{T_2} = e^{0.3(\pi)}$$

$$= 1.874 \qquad\qquad = 2.566$$

$$T_1 = 40(1.874) = 74.98 \text{ N}$$

$$T_2 = 74.98/2.566 = 29.22 \text{ N}$$

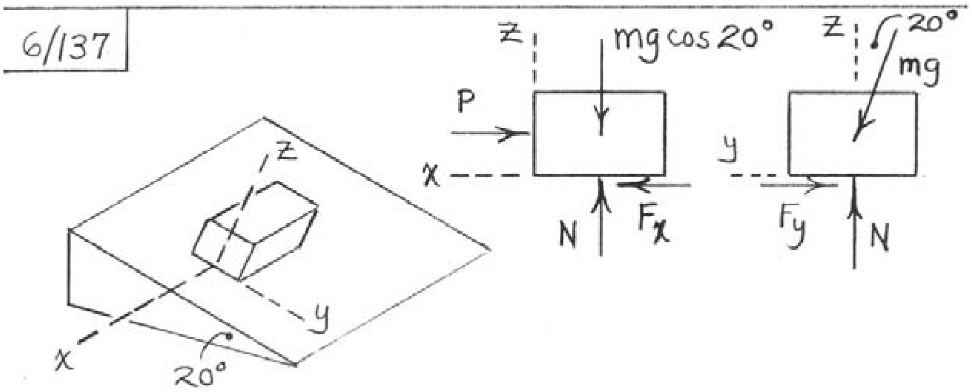
$$M_1 = (T_1 - T_2)r = (74.98 - 29.22)(0.050)$$

$$= \underline{2.29 \text{ N}\cdot\text{m}}$$

$$M_2 = (T_1 - 40)r = (74.98 - 40)(0.050)$$

$$= \underline{1.75 \text{ N}\cdot\text{m}}$$

6/137



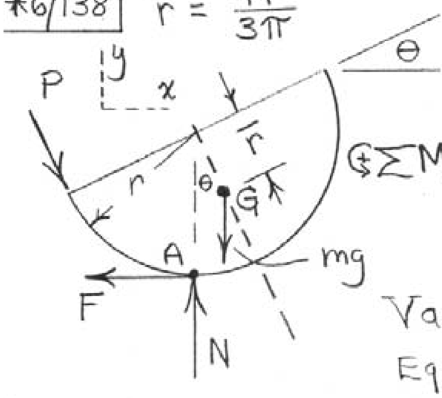
$(x-z)\begin{cases} \Sigma F_z = 0: \ N - 8(9.81)\cos 20° = 0, & N = 73.7 \text{ N} \\ \Sigma F_x = 0: \ F_x - P = 0, & F_x = P \end{cases}$

$(y-z)\ \{\ \Sigma F_y = 0: -F_y + 8(9.81)\sin 20° = 0,\ F_y = 26.8 \text{ N}$

$F = \sqrt{F_x^2 + F_y^2} = \mu_s N: \sqrt{P^2 + 26.8^2} = 0.5(73.7)$

$\underline{P = 25.3 \text{ N}}$

***6/138** $\bar{r} = \frac{4r}{3\pi}$

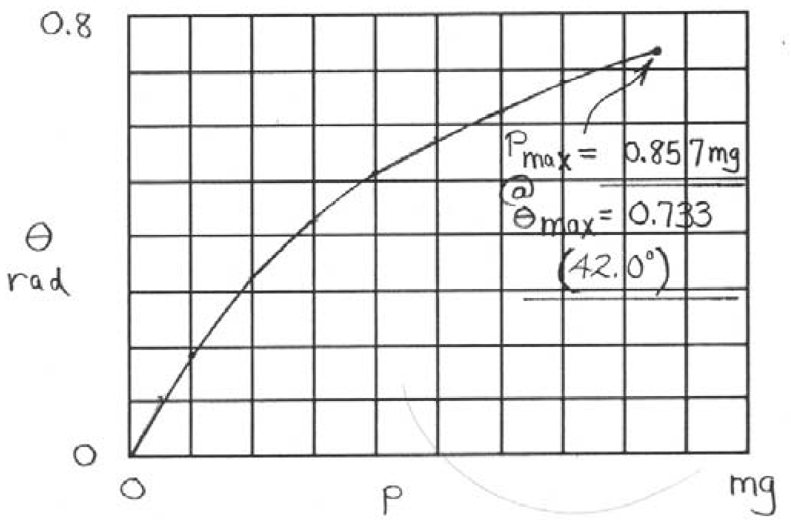


$\Sigma F_x = 0: \; P\sin\theta - F = 0 \quad (1)$

$\Sigma F_y = 0: \; N - mg - P\cos\theta = 0 \quad (2)$

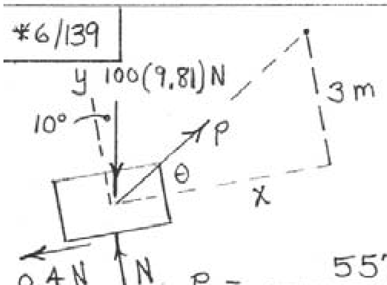
$\Sigma M_G = 0: \; Pr - N\bar{r}\sin\theta - F(r - \bar{r}\cos\theta) = 0 \quad (3)$

Vary P and numerically solve Eqs. (1), (2), and (3) for θ, F, and N. Plot for θ:



(Note: Could vary θ & solve for P; then plot!)

*6/139



$\Sigma F_x = 0$:

$P\cos\theta - 0.4N - 981\sin 10° = 0$

$\Sigma F_y = 0$:

$N + P\sin\theta - 981\cos 10° = 0$

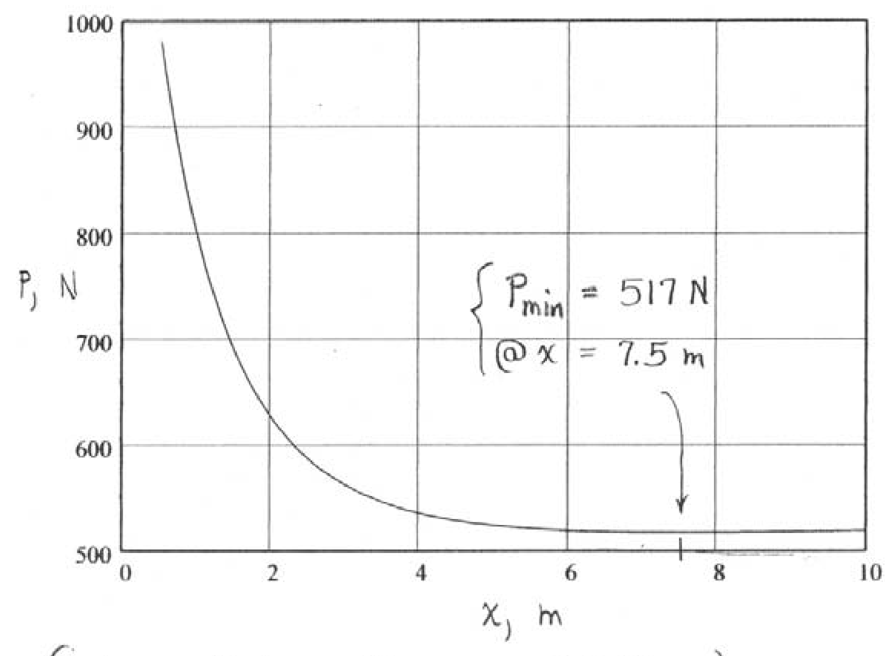
$P = \dfrac{557}{\cos\theta + 0.4\sin\theta}$, $N = \dfrac{-170.3\sin\theta + 966\cos\theta}{\cos\theta + 0.4\sin\theta}$

$$\frac{dP}{d\theta} = \frac{-557(-\sin\theta + 0.4\cos\theta)}{(\cos\theta + 0.4\sin\theta)^2} = 0$$

$\Rightarrow -\sin\theta + 0.4\cos\theta = 0$ or $\tan\theta = 0.4$, $\theta = 21.8°$

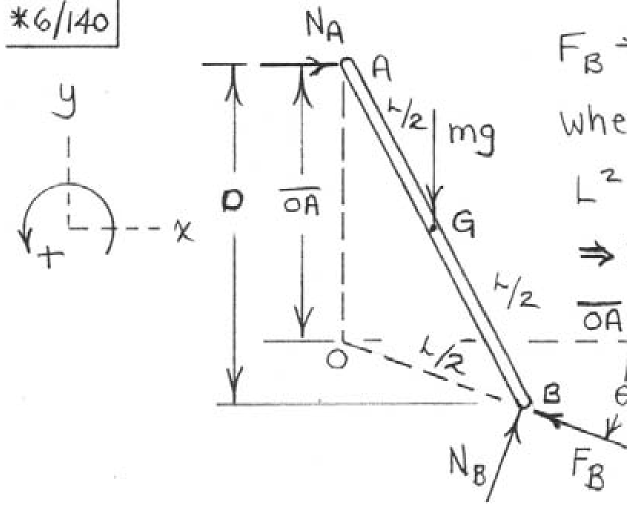
From $\tan\theta = \dfrac{3}{x}$, $x = \dfrac{3}{\tan\theta} = \dfrac{3}{0.4} = \underline{7.5\text{ m}}$

At $\theta = 21.8°$, $\underline{P = 517\text{ N}}$



(Note: $N \to 0$ @ $x = 0.529$ m)

*6/140



$F_B \rightarrow \mu_s N_B = 0.4 N_B$

when slipping impends.

$L^2 = D^2 + \left(\frac{L}{2}\cos\theta\right)^2$

$\Rightarrow D = L\sqrt{1 - \frac{\cos^2\theta}{4}}$

$\overline{OA} = L\sqrt{1 - \frac{\cos^2\theta}{4}} - \frac{L}{2}\sin\theta$

$= \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$

$$\Sigma F_x = 0: \ N_A + N_B \sin\theta - 0.4 N_B \cos\theta = 0 \qquad (1)$$

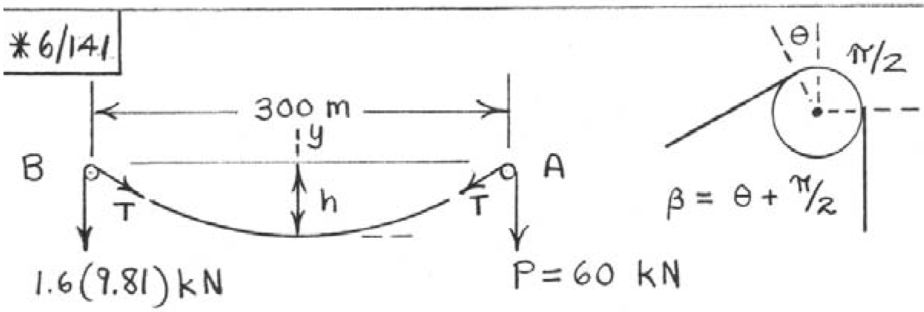
$$\Sigma F_y = 0: \ N_B \cos\theta + 0.4 N_B \sin\theta - mg = 0 \qquad (2)$$

$$\Sigma M_o = 0: \ N_B\left(\frac{L}{2}\right) - N_A \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$$

$$- mg \frac{L}{4}\cos\theta = 0 \qquad (3)$$

Numerical solution : $\begin{cases} N_A = 0.287 mg \\ N_B = 0.966 mg \\ \underline{\theta = 5.80°} \end{cases}$

*6/141



During slipping, Eq. 6/7 @ A: $60 = Te^{\mu_k \beta}$

Eq. 6/7 @ B: $T = 1.6(9.81)e^{\mu_k \beta}$

Eliminate $\mu_k \beta$ & get $\underline{T = 30.7 \text{ kN}}$

Then $e^{\mu_k \beta} = 60/30.7$, $\mu_k \beta = 0.670$

Eq. 5/21: $T = T_o \cosh \frac{\rho g x}{T_o}$: $30.7 = T_o \cosh \frac{12(9.81)10^{-3} \cdot 150}{T_o}$

or $30.7 = T_o \cosh \frac{17.66}{T_o}$. Solve numerically: $T_o = 23.8$ kN

$\frac{dy}{dx} = \sinh \frac{\rho g x}{T_o}$: $\tan\theta = \sinh \frac{17.66}{23.8} = 0.810$

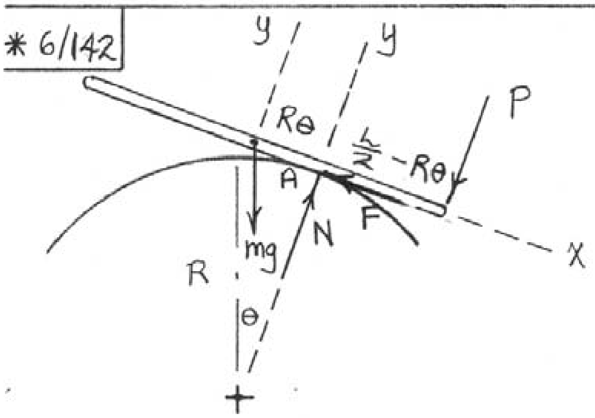
$\theta = 0.681$ rad, $\beta = \theta + \pi/2 = 2.25$ rad

$\mu_k = \frac{0.670}{2.25} = \underline{0.298}$

Eq. 5/22: $T = T_o + \rho g h$, $h = \frac{T - T_o}{\rho g}$

$= \frac{30.7 - 23.8}{12(9.81)(10^{-3})} = \underline{58.1 \text{ m}}$

*6/142



$$\Sigma F_x = 0: \quad mg\sin\theta - \mu_s N = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad N - mg\cos\theta - P = 0 \qquad (2)$$

$$\circlearrowleft + \ \Sigma M_A = 0: \quad P\left(\frac{L}{2} - R\theta\right) - mg\,R\theta\cos\theta = 0 \qquad (3)$$

$$(1): \quad N = \frac{mg\sin\theta}{\mu_s}$$

$$(2): \quad \frac{mg\sin\theta}{\mu_s} - mg\cos\theta = P$$

$$(3): \quad mg\left(\frac{\sin\theta}{\mu_s} - \cos\theta\right)\left(\frac{L}{2} - R\theta\right) - mg\,R\theta\cos\theta = 0$$

Simplify: $\tan\theta = \mu_s \dfrac{1}{1 - \frac{2R\theta}{L}} = 0.15 \dfrac{1}{1 - 2(0.6)\theta}$

or $\tan\theta - \dfrac{0.15}{1 - 1.2\theta} = 0$. Numerical solution: $\underline{\theta = 11.04°}$

*6/143 From Prob. 6/142, the equilibrium equations are

$$mg \sin\theta - \mu_s N = 0 \qquad (1)$$

$$N - mg\cos\theta - P = 0 \qquad (2)$$

$$P\left(\frac{L}{2} - R\theta\right) - mg R\theta \cos\theta = 0 \qquad (3)$$

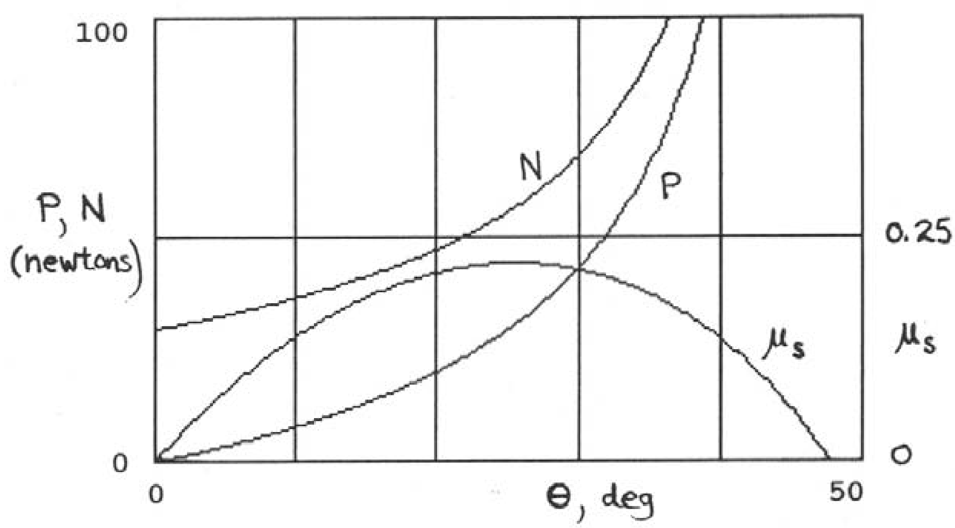
(2) & (3): $\left(N - mg\cos\theta\right)\left(\frac{L}{2} - R\theta\right) - mg R\theta\cos\theta = 0$

$$N = mg\cos\theta \frac{1}{1 - \frac{2R\theta}{L}} = 3(9.81)\frac{\cos\theta}{1 - \frac{2(1.2)\theta}{2}} = \underline{\frac{29.4\cos\theta}{1 - 1.2\theta}}$$

(3): $P = mg\dfrac{2R\theta}{L - 2R\theta}\cos\theta = \underline{\dfrac{35.3\theta\cos\theta}{1 - 1.2\theta}}$

(1) & (2): $\mu_s = \left(1 - \frac{2R\theta}{L}\right)\tan\theta = \underline{(1 - 1.2\theta)\tan\theta}$

$\underline{\mu_{s_{max}} = 0.222 \text{ @ } \theta = 25.5°}$



$*$6/144

$\mu = 0.4$

From geometry,

upper wheel :

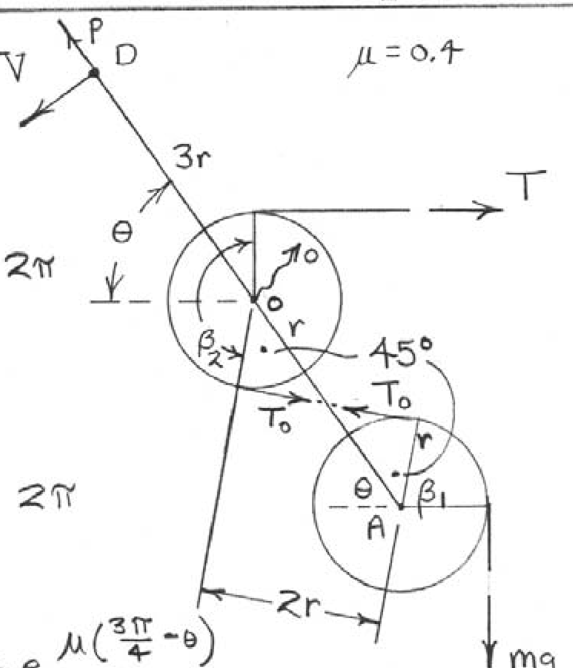
$$\theta + \frac{\pi}{4} + \frac{\pi}{2} + \beta_2 = 2\pi$$

$$\beta_2 = \frac{5\pi}{4} - \theta$$

lower wheel:

$$\theta + \frac{\pi}{4} + \beta_1 + \pi = 2\pi$$

$$\beta_1 = \frac{3\pi}{4} - \theta$$



Forces: $\begin{cases} mg = T_0 e^{\mu\left(\frac{3\pi}{4} - \theta\right)} \\ T_0 = T e^{\mu\left(\frac{5\pi}{4} - \theta\right)} \end{cases}$

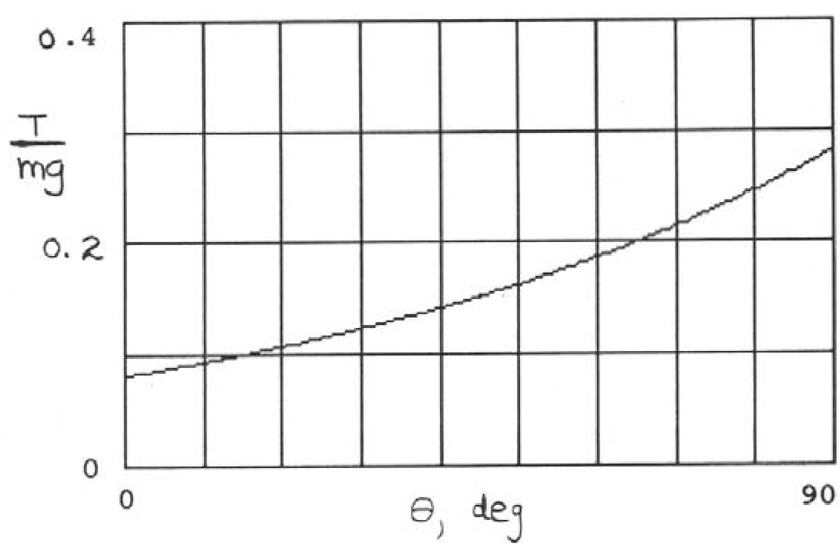
So $mg = Te^{2\mu(\pi - \theta)}$; $\dfrac{T}{mg} = e^{-0.8(\pi - \theta)}$

Device as a whole :

$$\curvearrowleft + \sum M_O = 0: \; 3rV - Tr - mg\left(r + 2r\sqrt{2}\cos\theta\right) = 0$$

$$V = \frac{1}{3}\left[T + mg\left(1 + 2\sqrt{2}\cos\theta\right)\right]$$

For $\theta = 60°$, $\dfrac{T}{mg} = 0.1872$, $\underline{V = 0.867mg}$



*6/145 Refer to Article 6/8. $dN = T d\theta$

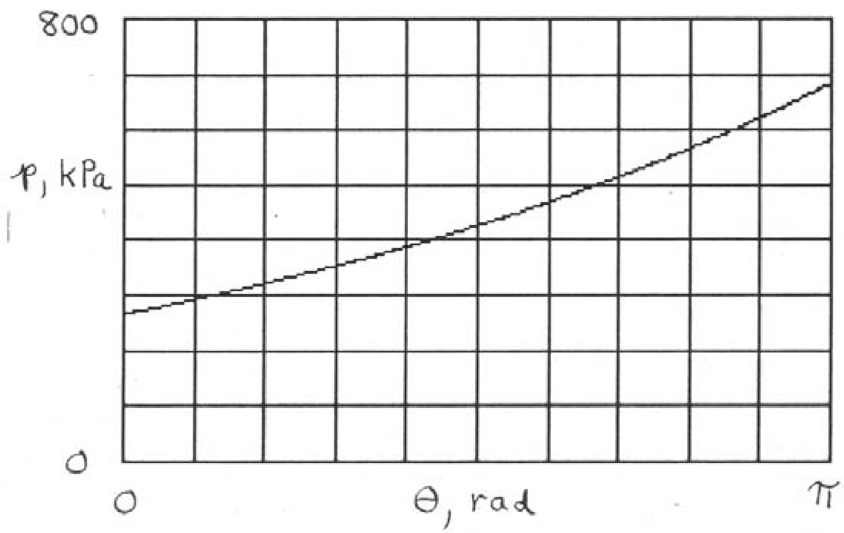
But $dN = pbr\,d\theta$ and $T = T_1 e^{\mu\theta}$ so

$pbr\,d\theta = T_1 e^{\mu\theta} d\theta$. Hence $p = \frac{T_1}{br} e^{\mu\theta}$

Numbers: $p = \frac{1000 e^{0.3\theta}}{(0.050)(0.075)}$ Pa or $p = 267 e^{0.3\theta}$ kPa

$T_2 = T_1 e^{\mu\pi} = 1000 e^{0.3\pi} = \underline{2570\text{ N}}$

$M = (T_2 - T_1) r = (2570 - 1000)(0.075) = \underline{117.5\text{ N}\cdot\text{m}}$



$\underline{p_1 = 267\text{ kPa}}$ $\underline{p_2 = 684\text{ kPa}}$

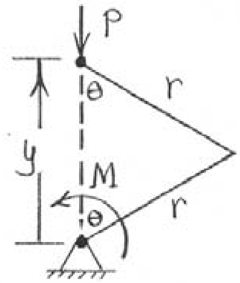
7/1

$$\delta U = 0: \ -M\delta\theta - P\delta y = 0$$

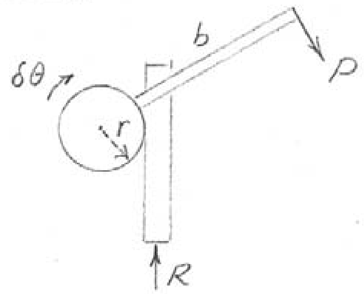
$$y = 2r\cos\theta, \quad \delta y = -2r\sin\theta\,\delta\theta$$

So $\quad M\delta\theta = P(2r\sin\theta\,\delta\theta)$

$$\underline{M = 2Pr\sin\theta}$$



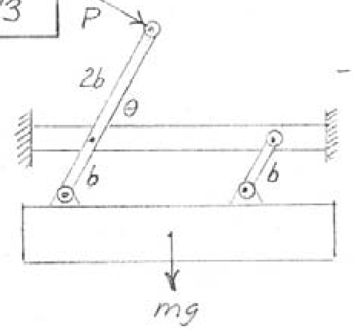
7/2



$\delta U = 0;\ Pb\delta\theta - Rr\delta\theta = 0$

$$\underline{R = P\frac{b}{r}}$$

7/3



$\delta U = 0;$

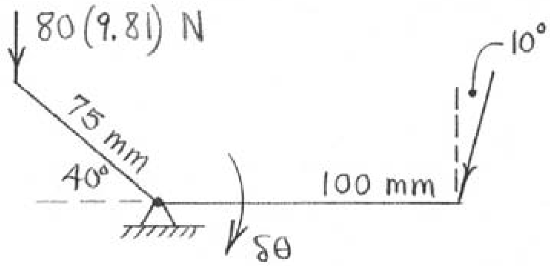
$-P(2b\,\delta\theta) + mg\,\delta(b\sin\theta) = 0$

$2Pb\,\delta\theta = mgb\cos\theta\,\delta\theta$

$\cos\theta = \frac{2P}{mg}$

$\underline{\theta = \cos^{-1}\frac{2P}{mg}}$

7/4

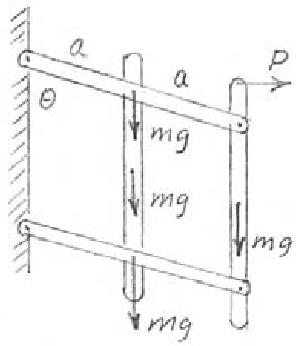


For a virtual displacement $\delta\theta$ of the lever,

$\delta U = 0$: $P\cos 10°(100\ \delta\theta) - 80(9.81)[75\ \delta\theta \cos 40°] = 0$

$\underline{P = 458\text{ N}}$

7/5



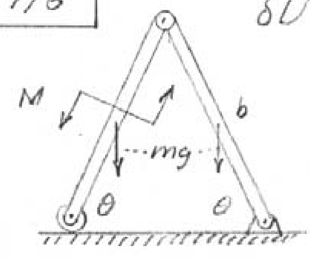
$\delta U = 0;\quad P\delta(2a\sin\theta)$

$+ mg\,\delta(2a\cos\theta) + 3mg\,\delta(a\cos\theta) = 0$

$2Pa\cos\theta\,\delta\theta = 5mga\sin\theta\,\delta\theta$

$P = \frac{5}{2}mg\tan\theta$

7/6

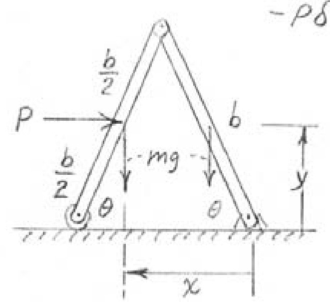


$\delta U = 0;\quad M\,\delta\theta - 2mg\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$

$M\,\delta\theta = mgb\cos\theta\,\delta\theta$

$\theta = \cos^{-1}\frac{M}{mgb}$

7/7 $\delta U = 0;\quad -P\,\delta x - 2mg\,\delta y = 0$



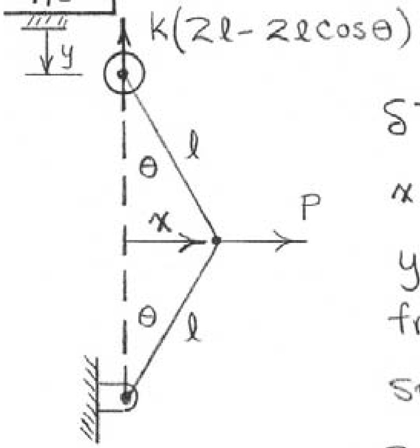
$$-P\delta\left(\frac{3b}{2}\cos\theta\right) - 2mg\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$$

$$\frac{3Pb}{2}\sin\theta\,\delta\theta - mgb\cos\theta\,\delta\theta = 0$$

$$\tan\theta = \frac{2mg}{3P}$$

$$\theta = \tan^{-1}\frac{2mg}{3P}$$

7/8



$\delta U = 0: \; P\delta x - k(2l - 2l\cos\theta)\delta y = 0$

$x = l\sin\theta, \quad \delta x = l\cos\theta\,\delta\theta$

$y = 2l - 2l\cos\theta$ (measured from wheel position when spring is unstretched)

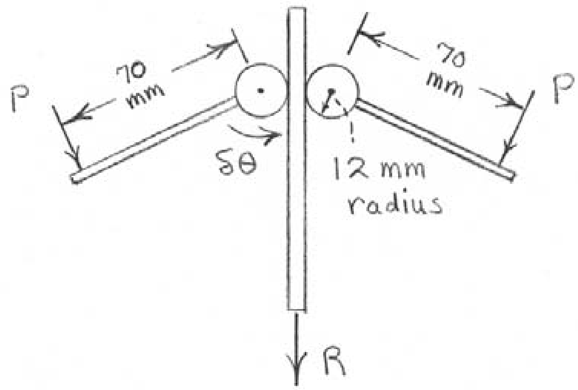
$\delta y = 2l\sin\theta\,\delta\theta$

So $P\,(l\cos\theta\,\delta\theta) - k(2l - 2l\cos\theta)(2l\sin\theta\,\delta\theta) = 0$

$\Rightarrow P = \dfrac{4kl\,(\sin\theta - \sin\theta\cos\theta)}{\cos\theta}$

or $\underline{P = 4kl(\tan\theta - \sin\theta)}$

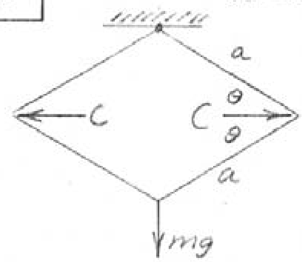
7/9



$$\delta U = 0: \; 2P(70)\delta\theta - R(12)\delta\theta = 0$$

$$\underline{R = 11.67\,P}$$

7/10

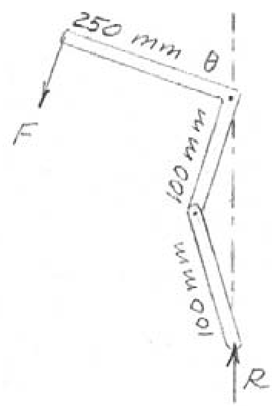


$\delta U = 0;\ 2C\,\delta(a\cos\theta) + mg\,\delta(2a\sin\theta) = 0$

$-2Ca\sin\theta\,\delta\theta + 2mga\cos\theta = 0$

$\underline{C = mg\,\mathrm{ctn}\,\theta}$

7/11



$\delta U = 0$

$F(250\,\delta\theta) - R\,\delta(2[100 \sin\theta]) = 0$

$250F\,\delta\theta = 200R\cos\theta\,\delta\theta$

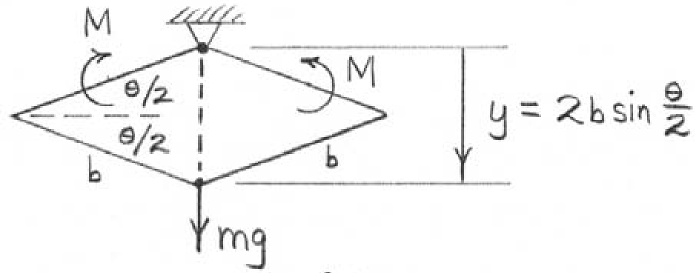
$\underline{F = 0.8R\cos\theta}$

$\boxed{7/12}$ $e = \dfrac{\text{output work}}{\text{input work}}$

To raise, $0.75 = \dfrac{250(1/4)}{P(1)}$, $\underline{P = 83.3 \text{ lb}}$

To lower, $0.75 = \dfrac{P'(1)}{250(1/4)}$, $\underline{P' = 46.9 \text{ lb}}$

7/13

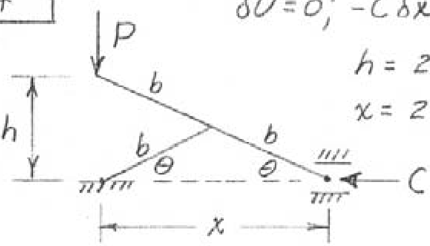


$\delta U = 0: \quad mg\,\delta y - 2M\,\delta\left(\frac{\theta}{2}\right) = 0$

$\delta y = b\cos\frac{\theta}{2}\,\delta\theta$, so $mgb\cos\frac{\theta}{2}\,\delta\theta = 2\left(\frac{1}{2}\right)M\delta\theta$

$\underline{M = mgb\cos\frac{\theta}{2}}$

7/14



$\delta U = 0; \; -C\delta x - P\delta h = 0$

$h = 2b\sin\theta, \; \delta h = 2b\cos\theta\,\delta\theta$

$x = 2b\cos\theta, \; \delta x = -2b\sin\theta\,\delta\theta$

Thus $\quad -C(-2b\sin\theta\,\delta\theta) - P(2b\cos\theta\,\delta\theta) = 0$

$C\sin\theta = P\cos\theta, \; C = P\,\text{ctn}\,\theta$

But $\quad \text{ctn}\,\theta = \sqrt{4b^2 - h^2}/h = \sqrt{(2b/h)^2 - 1}$, so $\underline{C = P\sqrt{(2b/h)^2 - 1}}$

7/15

$$\delta U = 0: \; 160 F \delta\theta - 0.4(160 F \delta\theta) - 100(9.81)\left(150 \delta \frac{\theta}{25}\right) = 0$$

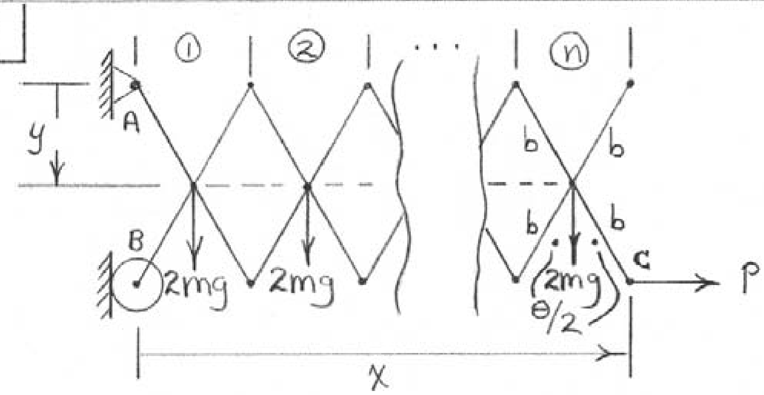
$$0.6(160)F = 981(6), \quad \underline{F = 61.3 \text{ N}}$$

$\boxed{7/16}$ Let $\delta\theta$ = virtual angle of input rotation

Then $\delta\theta/40$ = " " " output "

$$e = \frac{\text{output work}}{\text{input work}} = \frac{1180\,(\delta\theta/40)}{30\,\delta\theta} = \frac{1180}{30(40)} = \underline{0.983}$$

7/17



$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

$x = n\left(2b\sin\frac{\theta}{2}\right)$, $\delta x = nb\cos\frac{\theta}{2}\,\delta\theta$

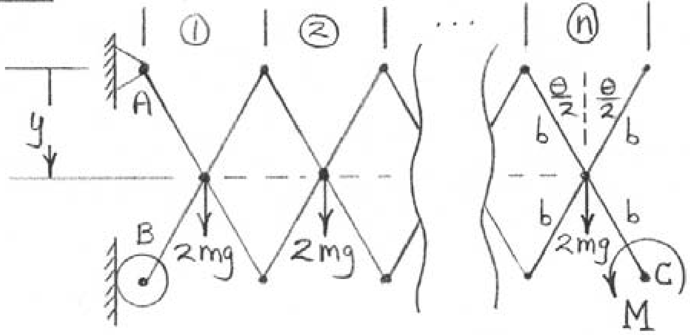
$\delta U = 0:\quad P\delta x + n(2mg)\delta y = 0$

$$P\left(nb\cos\frac{\theta}{2}\,\delta\theta\right) = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$$

$$\underline{P = mg\tan\frac{\theta}{2}}$$

P does <u>not depend</u> on the number n of sections present.

7/18



$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

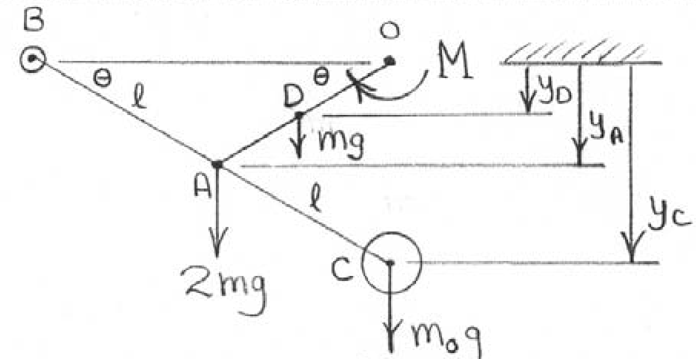
$\delta U = 0$: $M\,\delta\left(\frac{\theta}{2}\right) + n\,(2mg)\,\delta y = 0$

$$\frac{M}{2}\delta\theta = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$$

$$\underline{M = 2nmgb\sin\frac{\theta}{2}}$$

M <u>does depend</u> on the number n of sections present.

7/19



$y_D = \frac{\ell}{2}\sin\theta$, $\delta y_D = \frac{\ell}{2}\cos\theta\,\delta\theta$

$y_A = \ell\sin\theta$, $\delta y_A = \ell\cos\theta\,\delta\theta$

$y_C = 2\ell\sin\theta$, $\delta y_C = 2\ell\cos\theta\,\delta\theta$

$\delta U = 0: \; -M\delta\theta + mg\,\delta y_D + 2mg\,\delta y_A + m_0 g\,\delta y_C = 0$

$$-M\delta\theta + mg\left(\frac{\ell}{2}\cos\theta\,\delta\theta\right) + 2mg\left(\ell\cos\theta\,\delta\theta\right) + m_0 g\left(2\ell\cos\theta\,\delta\theta\right) = 0$$

$$\Rightarrow \; M = \left(\frac{5}{2}m + 2m_0\right)g\ell\cos\theta$$

For $\theta = 30°$: $\underline{M = \left(\frac{5}{4}m + m_0\right)g\ell\sqrt{3}}$

7/20 $e = \dfrac{\text{output work}}{\text{input work}}$

Let $\delta\theta$ = virtual crank angle, radians
δh = " movement of lifting pad, inches

where $\dfrac{\delta\theta}{\delta h} = \dfrac{12(2\pi)}{1}$, $\delta\theta = 24\pi\,\delta h$

To raise, $e = \dfrac{L\,\delta h}{Fr\,\delta\theta} = \dfrac{2700\,\delta h}{10(6)24\pi\,\delta h} = \underline{0.597}$

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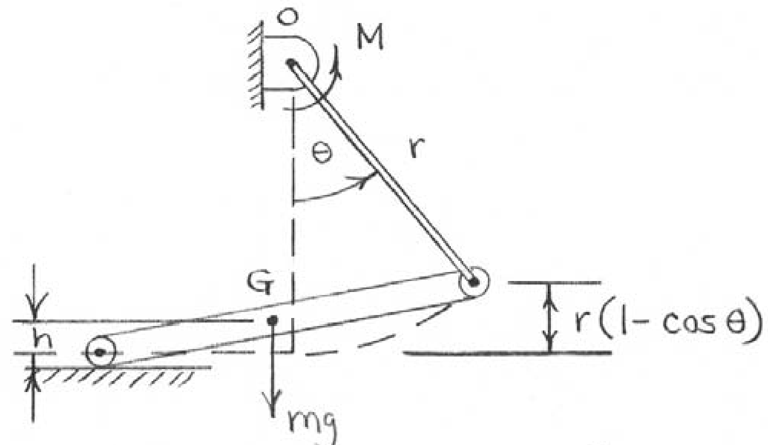


$$\delta U = 0: \quad M\,\delta\theta - mg\,\delta h = 0, \quad h = \frac{r}{2}(1 - \cos\theta)$$

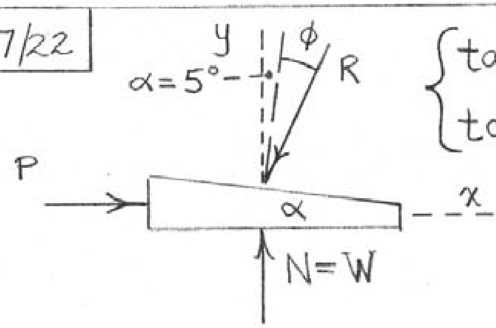
$$\delta h = \frac{r}{2}\sin\theta\,\delta\theta$$

$$\therefore\ M\,\delta\theta - mg\left(\frac{r}{2}\sin\theta\,\delta\theta\right) = 0$$

$$\left(M - \frac{mgr}{2}\sin\theta\right)\delta\theta = 0, \quad \underline{\theta = \sin^{-1}\left(\frac{2M}{mgr}\right)}$$

$M_{max} = \dfrac{mgr}{2}$ @ $\theta = 90°$. No equilibrium for $M > \dfrac{mgr}{2}$

7/22



$$\begin{cases} \tan\phi = 0.30, & \phi = 16.70° \\ \tan\alpha = \tan 5° = 0.0875 \end{cases}$$

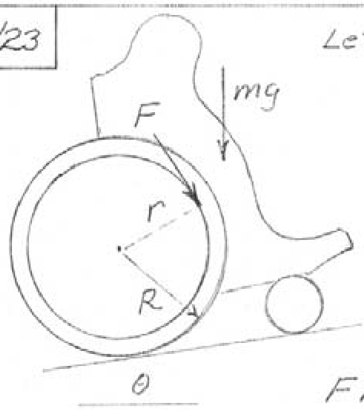
$$\left.\begin{aligned} \Sigma F_x = 0: \quad P &= R\sin(\phi+\alpha) \\ \Sigma F_y = 0: \quad W &= R\cos(\phi+\alpha) \end{aligned}\right\} \tan(\phi+\alpha) = \frac{P}{W}$$

$$\left.\begin{aligned} &\text{Work input: } P\delta x \\ &\text{Work output: } W\delta y \end{aligned}\right\} \text{ where } \delta y = \delta x \tan\alpha$$

$$e = \frac{W\delta y}{P\delta x} = \frac{\tan\alpha}{\tan(\phi+\alpha)} = \frac{\tan 5°}{\tan(16.70°+5°)}$$

$$\text{or} \quad \underline{e = 0.220}$$

7/23



Let β = angle through which wheel turns

s = corresponding displacement along incline.

$s = R\beta$ so $\delta s = R\,\delta\beta$

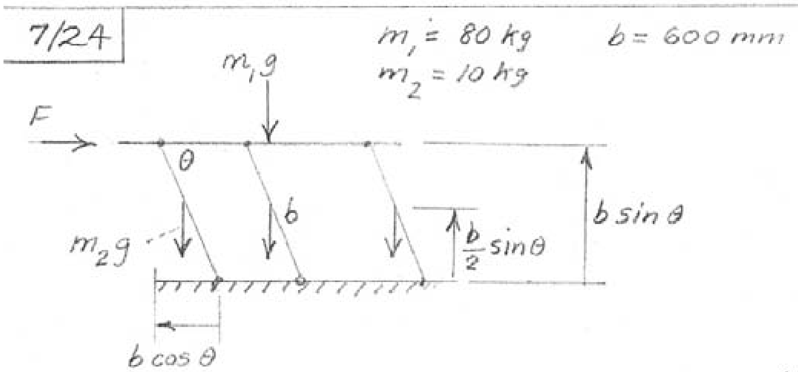
$\delta U = 0$

$F r \delta\beta - mg\,\delta s \sin\theta = 0$

$F r \delta\beta = mg R \sin\theta\,\delta\beta$

$F = mg \frac{R}{r} \sin\theta$

7/24



$$\delta U = 0; \quad -F\delta(b\cos\theta) - m_1 g\,\delta(b\sin\theta) - 3m_2 g\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$$

$$Fb\sin\theta\,\delta\theta = m_1 g b\cos\theta\,\delta\theta + \frac{3}{2}m_2 g b\cos\theta\,\delta\theta$$

$$F = g\,\text{ctn}\,\theta\left(m_1 + \frac{3}{2}m_2\right)$$

$$= 9.81\left(80 + \frac{3}{2}10\right)\text{ctn}\,\theta = \underline{932\,\text{ctn}\,\theta \text{ N}}$$

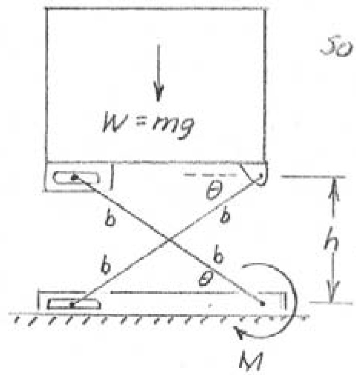
Solution by force and moment equilibrium would require dismemberment with four FBD's and eventual elimination of unwanted forces and dimensions

7/25 $\quad \delta U = 0; \quad M\delta\theta - mg\,\delta h = 0$

$h = 2b\sin\theta, \quad \delta h = 2b\cos\theta\,\delta\theta$

So $M\delta\theta = mg(2b\cos\theta)\delta\theta$

$M = 2mgb\cos\theta$



But since $\sin\theta = \dfrac{h}{2b}$,

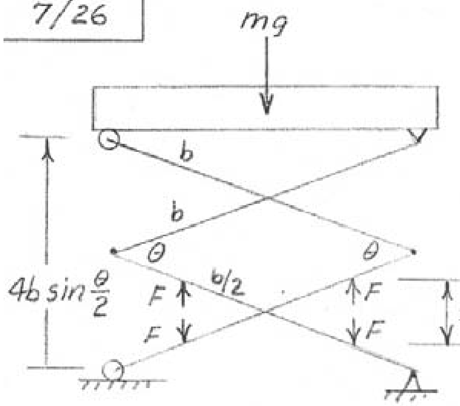
$\cos\theta = \sqrt{1 - \left(\dfrac{h}{2b}\right)^2}$

Thus

$M = 2mgb\sqrt{1 - \left(\dfrac{h}{2b}\right)^2}$

7/26

$F = pA$



$2\frac{b}{2}\sin\frac{\theta}{2} = b\sin\frac{\theta}{2}$

Work done by each cylinder is $F\,\delta\left(b\sin\frac{\theta}{2}\right)$

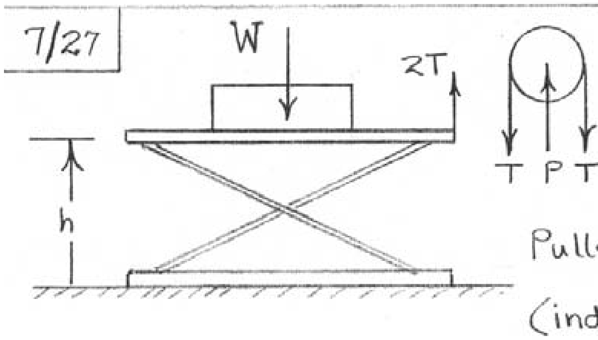
$$= \frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta$$

$$\delta U = 0;\quad -mg\,\delta\left(4b\sin\frac{\theta}{2}\right) + 2\,\frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta = 0$$

$$Fb\cos\frac{\theta}{2} = 2mgb\cos\frac{\theta}{2}, \quad F = pA = 2mg$$

so $\underline{p = 2mg/A}$ independent of b & θ

7/27

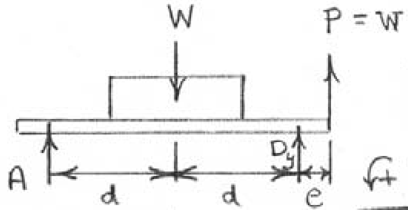


$\delta U = 0: \; 2T\delta h - W\delta h = 0$

$T = W/2$

Pulley: $P = 2T$, so $\underline{P = W}$

(independent of h)



D_y and A form a clockwise couple

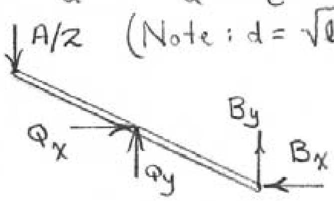
$D_y = -A$ (Note $D_x = 0$)

$\circlearrowleft + \Sigma M = 0: \; W(d+e) - A(2d) = 0$

(Note : $d = \sqrt{l^2 - h^2}/2$)

$A = \frac{W}{2}\left(1 + \frac{e}{d}\right)$

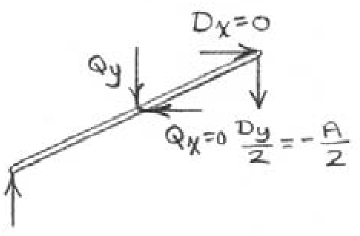
$= \frac{W}{2}\left(1 + \frac{2e}{\sqrt{l^2 - h^2}}\right)$



CD: $\Sigma M_C = 0$:

$Q_y d - \frac{A}{2}(2d) = 0$

$\underline{Q = Q_y = A = \frac{W}{2}\left(1 + \frac{2e}{\sqrt{l^2 - h^2}}\right)}$



(From drawing, $d > e$, so $Q \cong \frac{W}{2}$)

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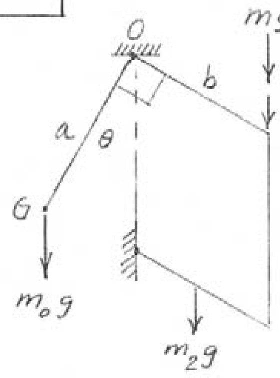


$\delta U = 0$; $m_o g\,\delta(a\cos\theta) + (m+m_1)g\,\delta(b\sin\theta) + m_2 g\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$

$-m_o a\sin\theta\,\delta\theta + (m+m_1)b\cos\theta\,\delta\theta + \frac{m_2 b}{2}\cos\theta\,\delta\theta = 0$

$m_o a\tan\theta = mb + \left(m_1 + \frac{m_2}{2}\right)b$ ---(1)

when $\theta = \theta_o$, $mg = 0$ so

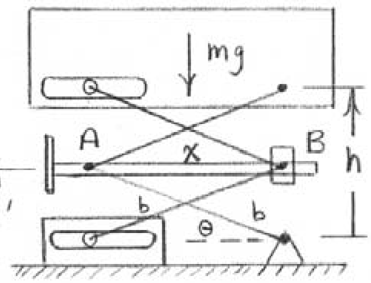
$m_o a\tan\theta_o = \left(m_1 + \frac{m_2}{2}\right)b$ ---(2)

Eliminate m_1 & m_2 from Eqs. (1) & (2) & get

$m_o a\tan\theta = mb + m_o a\tan\theta_o$

$m = \frac{a}{b}m_o(\tan\theta - \tan\theta_o)$

7/29 M' = necessary moment without friction



Let β = angle through which screw turns

$\delta U = 0: \quad M'\delta\beta - mg\,\delta h = 0$

$\frac{L}{2\pi} = \frac{-\delta(x)}{\delta\beta}, \quad \delta\beta = \frac{2\pi}{L}(-\delta x)$

$x = 2b\cos\theta, \quad \delta x = -2b\sin\theta\,\delta\theta$

$\delta\beta = \frac{4\pi b}{L}\sin\theta\,\delta\theta$

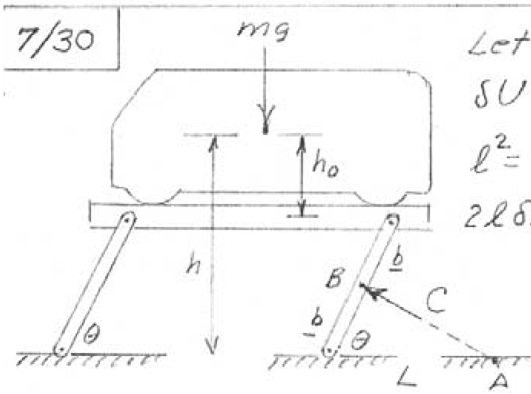
$h = 4b\sin\theta, \quad \delta h = 4b\cos\theta\,\delta\theta$

Thus $M'\frac{4\pi b}{L}\sin\theta\,\delta\theta - mg\,(4b\cos\theta\,\delta\theta) = 0$

$M' = \frac{mgL}{\pi}\cot\theta$

$\underline{M = M_f + \frac{mgL}{\pi}\cot\theta}$

7/30



Let $\overline{AB} = \ell$

$\delta U = 0$; $C\,\delta\ell - mg\,\delta h = 0$

$\ell^2 = (b\sin\theta)^2 + (L - b\cos\theta)^2$

$2\ell\,\delta\ell = 2b^2\sin\theta\cos\theta\,\delta\theta$

$\quad + 2(L - b\cos\theta)(b\sin\theta\,\delta\theta)$

$\quad = 2Lb\sin\theta\,\delta\theta$

$\delta\ell = \dfrac{Lb}{\ell}\sin\theta\,\delta\theta$

$h = 2b\sin\theta + h_o$, $\delta h = 2b\cos\theta\,\delta\theta + 0$

Thus $C\dfrac{Lb}{\ell}\sin\theta\,\delta\theta - mg\,(2b\cos\theta\,\delta\theta) = 0$

$$C = 2mg\,\frac{\ell}{L}\,\mathrm{ctn}\,\theta = \frac{2mg}{L}\sqrt{(b\sin\theta)^2 + (L - b\cos\theta)^2}\,\mathrm{ctn}\,\theta$$

$$C = 2mg\sqrt{1 + \left(\frac{b}{L}\right)^2 - 2\frac{b}{L}\cos\theta}\;\mathrm{ctn}\,\theta$$

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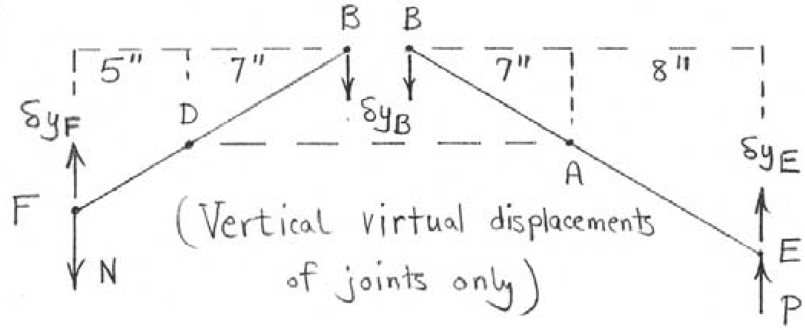


$\delta y_B = \frac{7}{8}\delta y_E$, $\delta y_F = \frac{5}{7}\delta y_B$

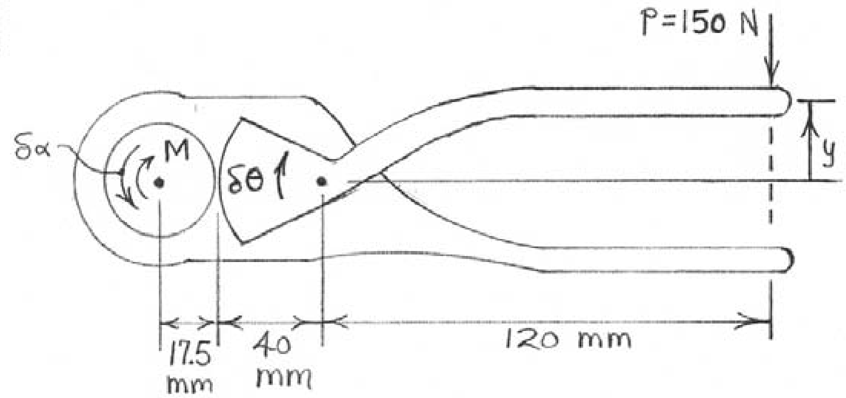
So $\delta y_F = \frac{5}{7}\cdot\frac{7}{8}\delta y_E = \frac{5}{8}\delta y_E$

$\delta U = 0$: $P\delta y_E - N\delta y_F = 0$

$$P\delta y_E = N\left(\frac{5}{8}\delta y_E\right)$$

$$N = \frac{8}{5}P = \underline{1.6P}$$

7/32



$\delta\alpha$ = rotation of socket on bolt head

$\delta\theta$ = rotation of upper handle (lower handle and frame taken as fixed)

$17.5\,\delta\alpha = 40\,\delta\theta\,,\quad \delta y = -120\,\delta\theta$

$\delta U = 0: \; -M\delta\alpha + P(-\delta y) = 0$

$$M\left(\frac{40}{17.5}\delta\theta\right) = 150\,(120\,\delta\theta)$$

$M = 7880$ N·mm or $\underline{M = 7.88 \text{ N}\cdot\text{m}}$

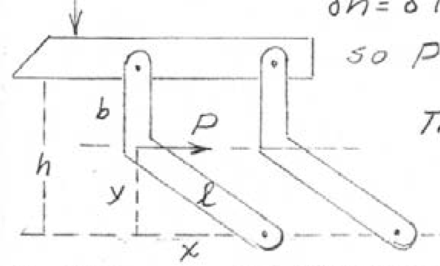
7/33

$\delta U = 0$; $\quad -P\,\delta x - W\,\delta h = 0$

$W = 250(9.81)$ N

$x^2 + y^2 = \ell^2, \quad x\,\delta x = -y\,\delta y$

$\delta h = \delta(b + y) = \delta y$



so $P\,\delta x = -W\,\delta y = -W\left(-\frac{x}{y}\right)\delta x$

Thus $P = W\frac{x}{y}$

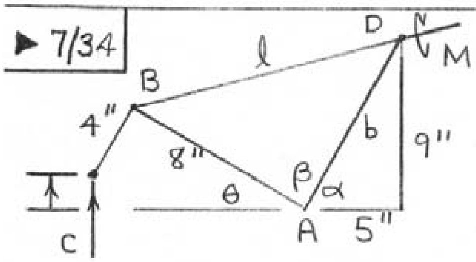
$P = 250(9.81)\,\frac{500}{350} = 3500$ N

or $\underline{P = 3.5 \text{ kN}}$

$h = 650$ mm, $b = 300$ mm

$y = 350$ mm, $x = 500$ mm

▶ 7/34



$\delta U = 0: \; C\delta h + M\,\delta(2\pi\gamma) = 0$

where $\delta\gamma$ = screw rotation in rev.

With L = lead = $\frac{1}{6}$ in.,

$\delta l = L\,\delta\gamma = \frac{1}{6}\delta\gamma$ in.

$b = \sqrt{5^2+9^2} = 10.30$ in.

$h = 8\sin\theta - 4\cos\theta, \quad \delta h = (8\cos\theta + 4\sin\theta)\delta\theta$

$l^2 = 8^2 + b^2 - 2(8)b\cos\beta, \quad 2l\delta l = 16b\sin\beta\,\delta\beta$

Because α = constant, $\delta\beta = -\delta\theta$: $l\delta l = -8b\sin\beta\,\delta\theta$

Thus $C(8\cos\theta + 4\sin\theta)\delta\theta + M\,2\pi\,6\,\frac{-8b}{l}\sin\beta\,\delta\theta = 0$

$$C = 2\pi M\,\frac{12b}{l}\,\frac{\sin\beta}{2\cos\theta + \sin\theta}$$

For $\theta = 30°$, $\beta = 180° - 30° - \tan^{-1}\frac{9}{5} = 89.1°$

$l = \sqrt{8^2 + 5^2 + 9^2 - 16(10.30)(0.0165)} = 12.93$ in.

$$C = 2\pi\,\frac{12\,(10.30)\sin 89.1°\;M}{12.93\,(2\cos 30° + \sin 30°)} = \underline{26.9\,M}$$

(M in lb-in., C in lb)

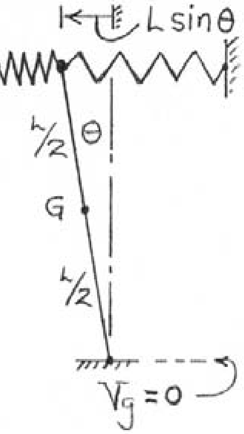
7/35 $V = 6x^3 - 9x^2 - 7$

$\frac{dV}{dx} = 18x^2 - 18x = 0$ for equil. $\underline{x = 0 \text{ or } x = 1}$

$\frac{d^2V}{dx^2} = 36x - 18 \quad = -18$ for $x = 0$ so unstable

$= +18$ " $x = 1$ so stable

7/36 | δ = initial spring compression



$$V = V_g + V_e = mg\frac{L}{2}\cos\theta$$
$$+\frac{1}{2}k(\delta + L\sin\theta)^2 + \frac{1}{2}k(\delta - L\sin\theta)^2$$
$$= \frac{1}{2}mgL\cos\theta + k(\delta^2 + L^2\sin^2\theta)$$
$$\frac{dV}{d\theta} = -\frac{1}{2}mgL\sin\theta + 2kL^2\sin\theta\cos\theta$$
$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL\cos\theta + 2kL^2\cos 2\theta$$

For equilibrium, $\frac{dV}{d\theta} = 0$, so

$$\left(-\frac{1}{2}mg + 2kL\cos\theta\right)\sin\theta = 0$$
$$\Rightarrow \theta = 0 \quad \text{or} \quad \cos\theta = \frac{mg}{4kL}$$

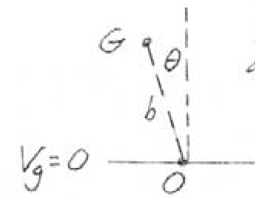
For $\theta = 0$, $\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL + 2kL^2$

> 0 (Stable) if $2kL^2 > \frac{1}{2}mgL$

So $\underline{k_{min} = \frac{mg}{4L}}$

7/37

$$V = V_g = mgb\cos\theta$$



$$\frac{dV}{d\theta} = -mgb\sin\theta = 0 \text{ for equil.}$$

$$\theta = 0° \text{ or } \theta = 180°$$

$$\frac{d^2V}{d\theta^2} = -mgb\cos\theta$$

$\theta = 0$, $\dfrac{d^2V}{d\theta^2} = -mgb$ so <u>unstable</u>

$\theta = 180°$, $\dfrac{d^2V}{d\theta^2} = -mgb(-1) = +mgb$ <u>stable</u>

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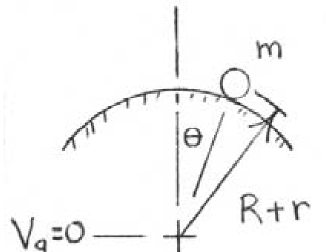
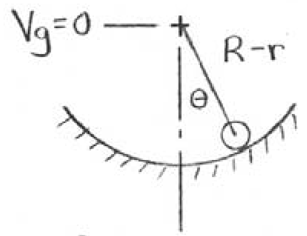


$$V = V_g = (R+r)\cos\theta$$

$$\frac{dV}{d\theta} = -(R+r)\sin\theta = 0 \quad \left(\begin{matrix}\text{for} \\ \text{equil.}\end{matrix}\right.$$

$$\theta = 0, \pi \ (\text{reject})$$

$$\frac{d^2V}{d\theta^2} = -(R+r)\cos\theta < 0 \quad @ \ \theta = 0 : \ \underline{\text{unstable}}$$



$$V = V_g = -(R-r)\cos\theta$$

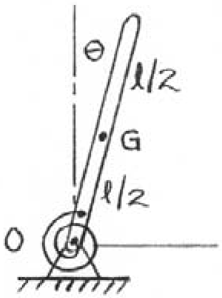
$$\frac{dV}{d\theta} = (R-r)\sin\theta = 0 \quad \left(\begin{matrix}\text{for} \\ \text{equil.}\end{matrix}\right.$$

$$\theta = 0, \pi \ (\text{reject})$$

$$\frac{d^2V}{d\theta^2} = (R-r)\cos\theta > 0 \ @ \ \theta = 0 : \ \underline{\text{stable}}$$

7/39

$$V = V_e + V_g = \frac{1}{2} K\theta^2 + mg\frac{l}{2}\cos\theta$$



$$\frac{dV}{d\theta} = K\theta - mg\frac{l}{2}\sin\theta$$

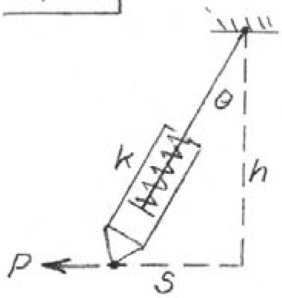
$$\frac{d^2V}{d\theta^2} = K - mg\frac{l}{2}\cos\theta$$

$V_g = 0$ $\quad \frac{d^2V}{d\theta^2} > 0$ @ $\theta = 0$:

$$K - mg\frac{l}{2} > 0$$

or $\quad \underline{l < \frac{2K}{mg}}$

7/40



x = spring compression

$= \frac{h}{\cos\theta} - h = h(\sec\theta - 1)$

$\delta U = \delta V_e$; $P\delta s = \delta(\frac{1}{2}kx^2)$

$= kx\,\delta x$

$s = h\tan\theta$ so that

$P\delta(h\tan\theta) = k\,h^2(\sec\theta - 1)\sec\theta\tan\theta\,\delta\theta$

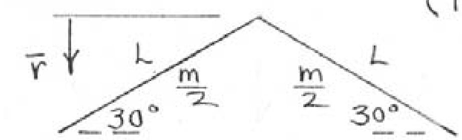
$Ph\sec^2\theta\,\delta\theta = kh^2(\sec\theta - 1)\sec^2\theta\sin\theta\,\delta\theta$

$P = kh(\sec\theta - 1)\sin\theta$

or $\underline{P = kh\tan\theta(1 - \cos\theta)}$

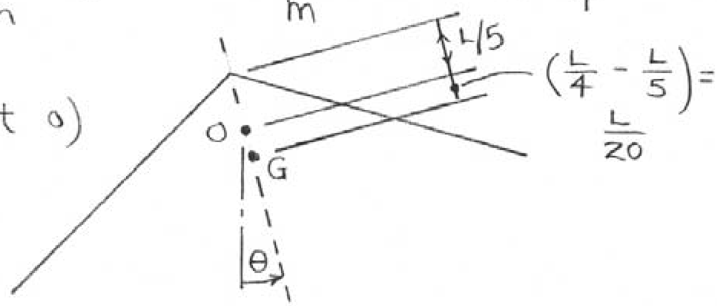
7/41

(Total mass m)



$$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2\left[\frac{m}{2}\frac{L}{2}\sin 30°\right]}{m} = \frac{L}{4}$$

(Datum at o)



$$V = V_g = -mg\frac{L}{20}\cos\theta$$

$$\frac{dV}{d\theta} = \frac{mgL}{20}\sin\theta \quad (\text{Zero } @ \ \theta = 0)$$

$$\frac{d^2V}{d\theta^2} = \frac{mgL}{20}\cos\theta > 0 \ @ \ \theta = 0 \quad \underline{\text{so stable}}.$$

7/42 Take $V_g = 0$ through AO & $V_e = 0$ when $\theta = 0$

So $V_g = -mgh = -60(9.81)(0.7\sin\theta) = -412.0\sin\theta$

$V_e = \frac{1}{2}kx^2 = \frac{1}{2}(160)\left[2(1.4)\sin\frac{\theta}{2}\right]^2 = 627.2\sin^2\frac{\theta}{2}$

$V = V_e + V_g = 627.2\sin^2\frac{\theta}{2} - 412.0\sin\theta$

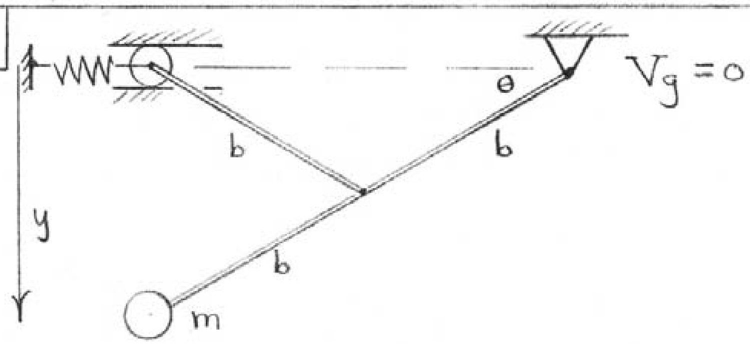
$\frac{dV}{d\theta} = \frac{2}{2}(627.2)\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 412.0\cos\theta$

$= 313.6\sin\theta - 412.0\cos\theta = 0$ for equil.

or $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{412.0}{313.6} = 1.314$

$\underline{\theta = 52.7°}$

7/43



Spring stretch $= 2b - 2b\cos\theta = 2b(1-\cos\theta)$

$V_e = \frac{1}{2}k[2b(1-\cos\theta)]^2 = 2kb^2(1-\cos\theta)^2$

$V_g = -mg(2b\sin\theta) = -2mgb\sin\theta$

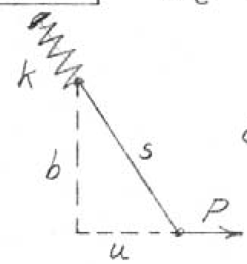
$V = 2kb^2(1-\cos\theta)^2 - 2mgb\sin\theta$

$\frac{dV}{d\theta} = 4kb^2(1-\cos\theta)\sin\theta - 2mgb\cos\theta = 0$

(for equilibrium)

$2kb(1-\cos\theta)\sin\theta = mg\cos\theta$, $\underline{k = \frac{mg}{2b}\frac{\cot\theta}{1-\cos\theta}}$

7/44



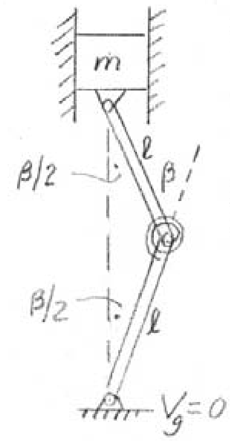
$\delta V_e = kx\,\delta x$ where $x = s - b = \sqrt{b^2+u^2} - b$

$\&\ \delta x = \dfrac{u\,\delta u}{\sqrt{b^2+u^2}}$

$\delta U' = \delta V_e;\ P\delta u = k\left(\sqrt{b^2+u^2} - b\right)\dfrac{u\,\delta u}{\sqrt{b^2+u^2}}$

$P = \left(1 - \dfrac{b}{\sqrt{b^2+u^2}}\right)ku$

7/45



$V_g = 2mg\ell \cos\frac{\beta}{2}$, $V_e = \frac{1}{2}K\beta^2$

$V = V_g + V_e = 2mg\ell\cos\frac{\beta}{2} + \frac{1}{2}K\beta^2$

$\frac{dV}{d\beta} = -mg\ell\sin\frac{\beta}{2} + K\beta$, $\frac{dV}{d\beta} = 0$ for $\beta = 0$

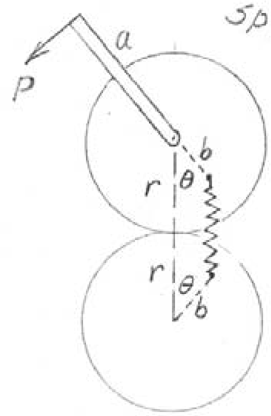
$\frac{d^2V}{d\beta^2} = -\frac{1}{2}mg\ell\cos\frac{\beta}{2} + K$

$= -\frac{1}{2}mg\ell + K$ for $\beta = 0$

$= (+)$ stable if $K > \frac{1}{2}mg\ell$

Thus $\underline{K_{min} = \frac{1}{2}mg\ell}$

7/46



$\delta U' = \delta V_e$; $\delta U' = Pa\,\delta\theta$; $\delta V_e = kx\,\delta x$

Spring stretch $x = (2r - 2b\cos\theta) - (2r - 2b)$

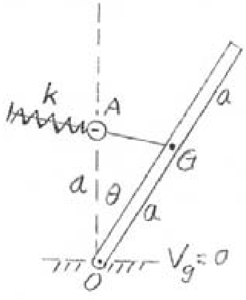
$= 2b(1-\cos\theta)$

$\delta x = 2b\sin\theta\,\delta\theta$

Thus $Pa\,\delta\theta = 4b^2k(1-\cos\theta)\sin\theta\,\delta\theta$

$$\underline{P = \frac{4kb^2}{a}\sin\theta(1-\cos\theta)}$$

7/47 $V_e = \frac{1}{2} k x^2$ where $x = 2a \sin\frac{\theta}{2}$



$V_g = mg\, a \cos\theta$

$V = V_e + V_g = 2ka^2 \sin^2\frac{\theta}{2} + mga\cos\theta$

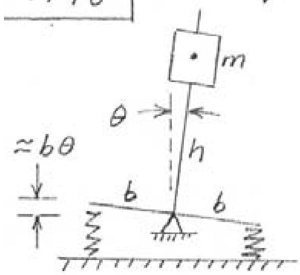
$\frac{dV}{d\theta} = 2ka^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2} - mga\sin\theta$

$= ka^2\sin\theta - mga\sin\theta = (ka - mg)a\sin\theta$

$\frac{dV}{d\theta} = 0$ for equil. gives $\underline{k = mg/a \text{ for all } \theta}$

7/48

$$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta)^2 + \frac{1}{2}k(\Delta - b\theta)^2 + mgh\cos\theta$$



for θ small

$$V = k(\Delta^2 + b^2\theta^2) + mgh\cos\theta$$

$$\frac{dV}{d\theta} = 2kb^2\theta - mgh\sin\theta$$

$$\frac{d^2V}{d\theta^2} = 2kb^2 - mgh\cos\theta$$

Let preset of springs be Δ when $\theta = 0$

For $\theta \to 0$, $\frac{d^2V}{d\theta^2}$ is (+) if $2kb^2 > mgh$

Thus $\theta = 0$ is stable if $\underline{h < \frac{2kb^2}{mg}}$

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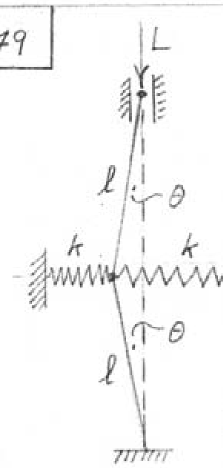


Let Δ = initial compression in each spring

Then for small angles

$$V = V_e + V_g = 2\left[\frac{1}{2}k(\Delta - \ell\sin\theta)^2 + \frac{1}{2}k(\Delta + \ell\sin\theta)^2\right] + 2L\ell\cos\theta$$

$$V = 2k(\Delta^2 + \ell^2\sin^2\theta) + 2L\ell\cos\theta$$

$$\frac{dV}{d\theta} = 4k\ell^2\sin\theta\cos\theta - 2L\ell\sin\theta$$

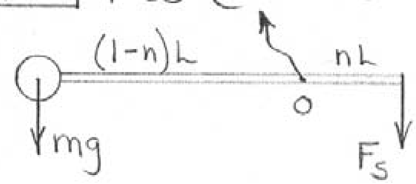
$$\frac{d^2V}{d\theta^2} = 4k\ell^2\cos 2\theta - 2L\ell\cos\theta$$

$\frac{dV}{d\theta} = 0$ for equil. $\theta = 0$ & $\theta = \cos^{-1}\frac{L}{2k\ell}$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 4k\ell^2(1) - 2L\ell(1) = (+)$ stable if

$$\underline{k > \frac{L}{2\ell}}$$

7/50 FBD @ $\theta = 0$:



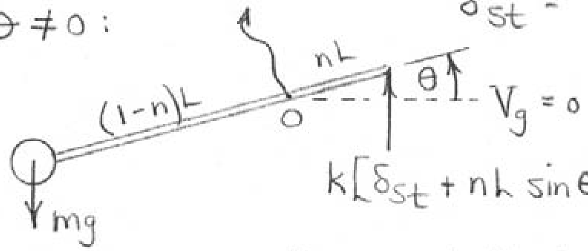
$\circlearrowleft + \; \Sigma M_o = 0$:

$$mg(1-n)L - F_s(nL) = 0$$

$$F_s = \frac{mg(1-n)}{n} = k\delta_{st}$$

$$\delta_{st} = \frac{mg(1-n)}{nk}$$

$\theta \neq 0$:



$$V = V_g + V_e = -mg\left[(1-n)L\sin\theta\right] + \tfrac{1}{2}k\left[\frac{mg(1-n)}{kn} + nL\sin\theta\right]^2$$

$$= \frac{m^2g^2(1-n)^2}{2kn^2} + \frac{n^2L^2k\sin^2\theta}{2}$$

$$\frac{dV}{d\theta} = n^2L^2k\sin\theta\cos\theta = 0 \quad \text{at} \quad \theta = 0$$

$$\frac{d^2V}{d\theta^2} = n^2L^2k\left[\cos^2\theta - \sin^2\theta\right] > 0 \quad \text{at} \quad \theta = 0$$

for all $0 < n < 1$

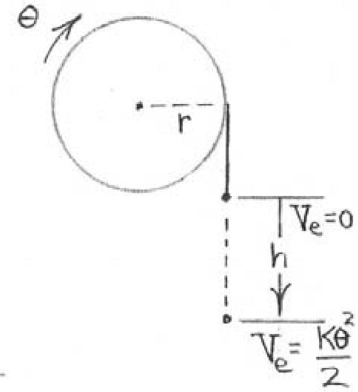
7/51

$$\delta U' = \delta V:\ 0 = -mgr\,\delta\theta + \delta\left(\tfrac{1}{2}K\theta^2\right)$$

$$mgr = K\theta$$

With $h = r\theta$: $\quad mgr = K\left(\frac{h}{r}\right)$

$$h = \frac{mgr^2}{K}$$



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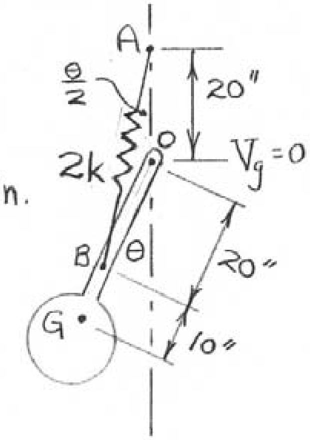


Length $AB = 2(20)\cos\frac{\theta}{2}$ (in.)

Unstretched length = 40 − 4 = 36 in.

Spring stretch for arbitrary θ

is $x = 40\cos\frac{\theta}{2} - 36$ in.

$$V_e = 2\tfrac{1}{2}k\left(40\cos\tfrac{\theta}{2} - 36\right)^2$$

$$= k\left[1600\cos^2\tfrac{\theta}{2} - 2880\cos\tfrac{\theta}{2} + 1296\right] \text{ in.-lb}$$

$$V_g = -3(20\cos\theta) = -60\cos\theta \text{ in.-lb}$$

$$V = V_e + V_g = k\left[1600\cos^2\tfrac{\theta}{2} - 2880\cos\tfrac{\theta}{2} + 1296\right] - 60\cos\theta \text{ in.-lb}$$

$$\frac{dV}{d\theta} = k\left[-1600\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2} + 1440\sin\tfrac{\theta}{2}\right] + 60\sin\theta$$

$$= k\left[-800\sin\theta + 1440\sin\tfrac{\theta}{2}\right] + 60\sin\theta$$

$$\frac{d^2V}{d\theta^2} = k\left[-800\cos\theta + 720\cos\tfrac{\theta}{2}\right] + 60\cos\theta$$

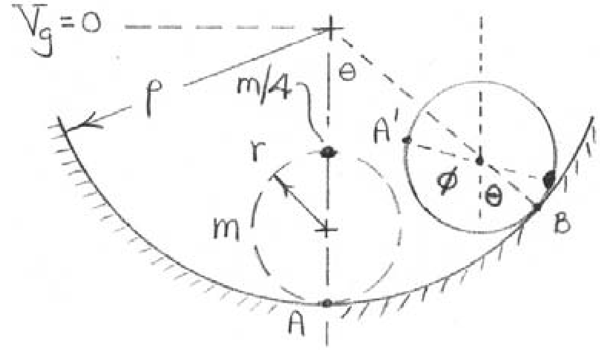
$$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0} = k[-800 + 720] + 60 > 0 \text{ (Stable) if } k \text{ does}$$

not exceed $\frac{60}{80} = \underline{0.75 \text{ lb/in.} = k_{max}}$

7/53 Arc $\widehat{AB}$ = Arc $\widehat{A'B}$: $\rho\theta = r(\theta + \phi)$

$$(\rho - r)\theta = r\phi$$

$$\phi = \frac{\rho - r}{r}\theta$$



$$V = V_g = -mg(\rho - r)\cos\theta - \frac{m}{4}g\left[(\rho - r)\cos\theta - r\cos\phi\right]$$

$$= \frac{mg}{4}\left[-5(\rho - r)\cos\theta + r\cos\left(\frac{\rho - r}{r}\theta\right)\right]$$

$$\frac{dV}{d\theta} = \frac{mg}{4}\left[5(\rho - r)\sin\theta - (\rho - r)\sin\left(\frac{\rho - r}{r}\theta\right)\right]$$

$= 0$ for equilibrium, $\theta = 0$ is desired solution

$$\frac{d^2V}{d\theta} = \frac{mg}{4}\left[5(\rho - r)\cos\theta - \frac{(\rho - r)^2}{r}\cos\left(\frac{\rho - r}{r}\theta\right)\right]$$

At $\theta = 0$: $5(\rho - r) - \frac{(\rho - r)^2}{r} > 0$ for stability

$$\rho^2 - 7\rho r + 6r^2 < 0$$

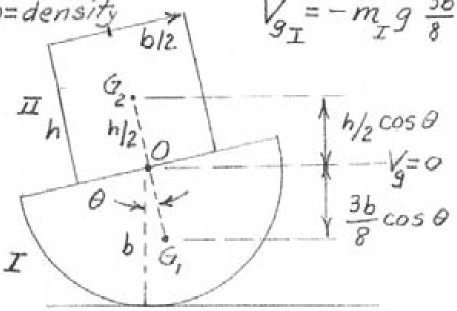
$(\rho - 6r)(\rho - r) < 0$ ($\rho = r$ not possible)

$\Rightarrow$ $(\rho - 6r) < 0$ & $(\rho - r) > 0$ (or the reverse which is impossible)

So $\underline{\rho < 6r}$

7/54 For solid hemisphere $\overline{OG_1} = 3b/8$

ρ = density



$$V_{g_I} = -m_I g \frac{3b}{8}\cos\theta = -\frac{2}{3}\pi b^3 \rho g \frac{3b}{8}\cos\theta$$

$$= -\pi\rho g \frac{b^4}{4}\cos\theta$$

$$V_{g_{II}} = +m_{II} g \frac{h}{2}\cos\theta$$

$$= \pi\left(\frac{b}{2}\right)^2 h\rho g \frac{h}{2}\cos\theta$$

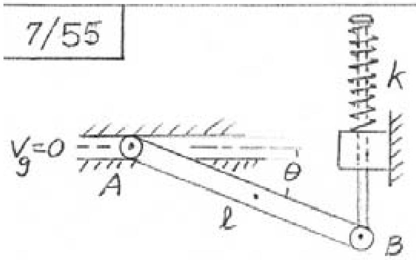
$$= \pi\rho g \frac{b^2h^2}{8}\cos\theta$$

$$V = V_{II} + V_I = \pi\rho g \frac{b^2}{4}\left(\frac{h^2}{2} - b^2\right)\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -\pi\rho g \frac{b^2}{4}\left(\frac{h^2}{2} - b^2\right)\cos\theta = +\pi\rho g \frac{b^2}{4}\left(b^2 - \frac{h^2}{2}\right)\cos\theta$$

$= (+)$ stable if $\frac{h^2}{2} < b^2$ or $\underline{h < b\sqrt{2}}$

7/55



Spring compressed $\ell \sin\theta$

so $V_e = \frac{1}{2} k (\ell \sin\theta)^2$

$V_g = -mg\frac{\ell}{2}\sin\theta$

$V = V_e + V_g = \frac{k}{2}\ell^2 \sin^2\theta - mg\frac{\ell}{2}\sin\theta$

$$\frac{dV}{d\theta} = k\ell^2 \sin\theta\cos\theta - mg\frac{\ell}{2}\cos\theta = \frac{k\ell^2}{2}\sin 2\theta - mg\frac{\ell}{2}\cos\theta$$

$$= \ell\cos\theta\left(k\ell\sin\theta - mg/2\right) = 0 \text{ for equil.}$$

(1) $\cos\theta = 0$, $\theta = \pi/2$

(2) $\underline{\sin\theta = \frac{mg}{2k\ell}}$

$$\frac{d^2V}{d\theta^2} = k\ell^2 \cos 2\theta + mg\frac{\ell}{2}\sin\theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(1)} = k\ell^2\cos\pi + mg\frac{\ell}{2}(1) = k\ell^2\left(-1 + \left[\frac{mg}{2k\ell}\right]\right)$$

$$= (+) \text{ stable if } k < \frac{mg}{2\ell}$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(2)} = k\ell^2\left(1 - 2\left[\frac{mg}{2k\ell}\right]^2\right) + \frac{mg\ell}{2}\frac{mg}{2k\ell} = k\ell^2\left[1 - \left(\frac{mg}{2k\ell}\right)^2\right]$$

$$= (+) \text{ stable if } k > \frac{mg}{2\ell}$$

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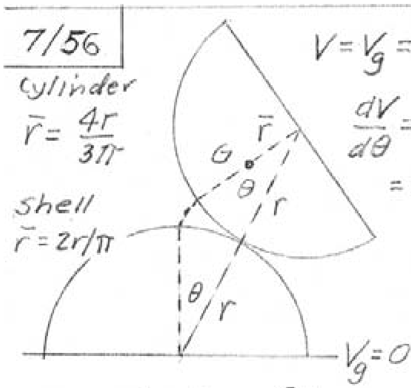
cylinder
$\bar{r} = \frac{4r}{3\pi}$

shell
$\bar{r} = 2r/\pi$



$V = V_g = mg(2r\cos\theta - \bar{r}\cos 2\theta)$

$\frac{dV}{d\theta} = mg(-2r\sin\theta + 2\bar{r}\sin 2\theta)$

$= 2mg(-r\sin\theta + \bar{r}\sin 2\theta)$

$\frac{d^2V}{d\theta^2} = 2mg(-r\cos\theta + 2\bar{r}\cos 2\theta)$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{2\bar{r}}{r}\right)$

For cylinder $\bar{r}/r = \frac{4}{3\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{8}{3\pi}\right) = (-)$

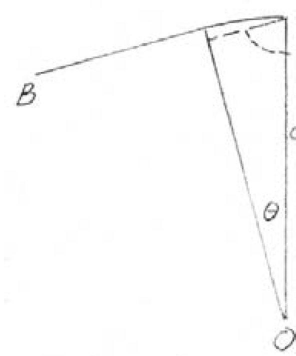
unstable

For shell $\bar{r}/r = \frac{2}{\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{4}{\pi}\right) = (+)$

stable

7/57

$$\delta V_g = \delta(0.3\, mg \cos\theta) = -0.3(80)(9.81)\sin\theta\,\delta\theta$$
$$= -235 \sin\theta\,\delta\theta$$



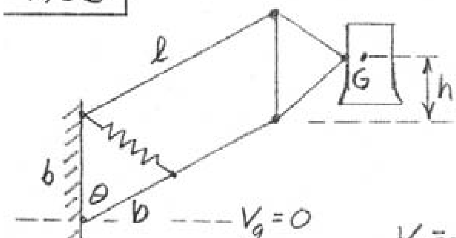
-- $0.05 \sin\theta \approx$ spring compression x

$$\delta V_e = k x\,\delta x$$
$$= (96 \times 10^3)(0.05 \sin\theta)\,\delta(0.05 \sin\theta)$$
$$= 240 \sin\theta \cos\theta\,\delta\theta$$

$$\delta V_e + \delta V_g = 0;$$
$$240 \sin\theta\cos\theta\,\delta\theta - 235 \sin\theta\,\delta\theta = 0$$

$\sin\theta = 0$ or $\cos\theta = \dfrac{235}{240} = 0.9810$, $\underline{\theta = 11.19°}$

7/58



$V_g = mg(l\cos\theta + h)$

Spring length $= 2b\sin\frac{\theta}{2}$

" stretch $x = 2b\sin\frac{\theta}{2} - \frac{b}{2}$

$= \frac{b}{2}(4\sin\frac{\theta}{2} - 1)$

$V_e = \frac{1}{2}kx^2 = \frac{kb^2}{8}(4\sin\frac{\theta}{2} - 1)^2$

$V = V_e + V_g$; $\frac{dV}{d\theta} = -mgl\sin\theta + \frac{kb^2}{4}(4\sin\frac{\theta}{2} - 1)\,2\cos\frac{\theta}{2}$

$= (kb^2 - mgl)\sin\theta - \frac{kb^2}{2}\cos\frac{\theta}{2} = 0$ for equil.

Thus $\left[2(kb^2 - mgl)\sin\frac{\theta}{2} - \frac{kb^2}{2}\right]\cos\frac{\theta}{2} = 0$

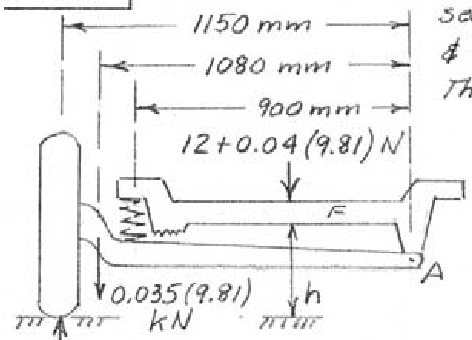
& $[\ \ ] = 0$ gives $k = \frac{mgl}{b^2}\frac{1}{1 - \frac{1}{4}\csc\frac{\theta}{2}}$

$\frac{d^2V}{d\theta^2} = (kb^2 - mgl)\cos\theta + \frac{kb^2}{4}\sin\frac{\theta}{2}$; substitute $\sin\frac{\theta}{2} = \frac{kb^2}{4(kb^2 - mgl)}$

& get $\frac{d^2V}{d\theta^2} = +$ (stable) within range of

$\theta = 29^\circ,\ k = \infty$ to $\theta = 180^\circ,\ k = \frac{4mgl}{3b^2}$

▶7/59 Let x = compression of spring (most easily seen by considering A fixed & wheels moving up)



Thus $x = \frac{900}{1150}(0.35 - h)$ meters

With F & hence A fixed,

$$\delta U' = -2(6.54)\delta h + 2(0.035)9.81\,\frac{1080}{1150}\,\delta h$$

$$= (-13.08 + 0.645)\delta h$$

$$= -12.43\,\delta h$$

$P = 6 + 0.035(9.81) + 0.02(9.81)$

$= 6.54$ kN

$$\delta V_e = 2(kx\,\delta x) = 2(120)\frac{900}{1150}(0.35-h)\frac{900}{1150}(-\delta h)$$

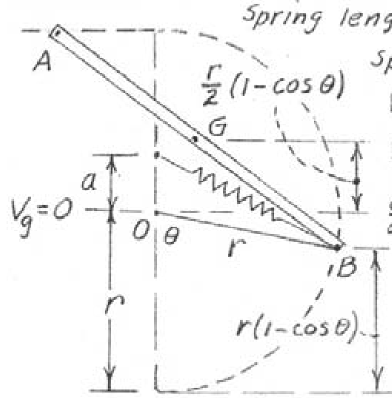
$$= -147.0(0.35-h)\delta h$$

$\delta U' = \delta V_e$; $-12.43\,\delta h = -147.0(0.35-h)\delta h$

$$h = 0.35 - \frac{12.43}{147.0} = 0.265 \text{ m} \quad \text{or} \quad \underline{h = 265 \text{ mm}}$$

▶ 7/60 For $\frac{1}{2}$ of door $V_g = \frac{mg}{2}\frac{r}{2}(1-\cos\theta)$

Spring length $\ell = \sqrt{r^2+a^2+2ra\cos\theta}$



Spring stretch is $\ell - (r-a)$

$V_e = \frac{1}{2}k(\ell - [r-a])^2$

$V = V_g + V_e = \frac{mgr}{4}(1-\cos\theta) + \frac{k}{2}(\ell - [r-a])^2$

$\frac{dV}{d\theta} = \frac{mgr}{4}\sin\theta + k(\ell - [r-a])\frac{-2ra\sin\theta}{2\ell}$

$= \left(\frac{mgr}{4} - kar\right)\sin\theta + \frac{kar(r-a)\sin\theta}{\sqrt{r^2+a^2+2ra\cos\theta}}$

$$\frac{d^2V}{d\theta^2} = \left(\frac{mgr}{4} - kar\right)\cos\theta + kar(r-a)\frac{\ell\cos\theta - \sin\theta(-ra\sin\theta)/\ell}{\ell^2}$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = \frac{mgr}{4} - kar + kar(r-a)\left(\frac{1}{r+a} + 0\right)$

$= 0$ for neutral equil. (insensitive response)

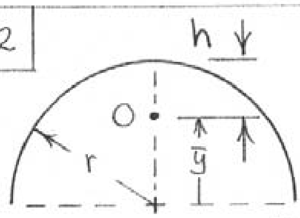
Thus $\frac{mgr}{4} = kar\left(1 - \frac{r-a}{r+a}\right)$, $k = \frac{mg(r+a)}{8a^2}$

7/61 For system $\delta U = 0$

$P_1 \delta x - P_2 \delta x/4 = 0, \quad P_2 = 4P_1 = 4(100)$

$= \underline{400 \text{ N}}$

7/62



The pivot O must be at or above the mass center G of the shell, which is located at $\bar{y} = \frac{2r}{\pi}$. So

$$h_{max} = r - \bar{y} = r\left(1 - \frac{2}{\pi}\right) = \underline{0.363r}$$

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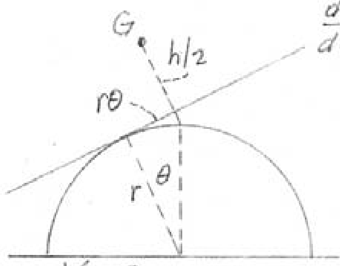
$$V = V_g = mg\left(r\cos\theta + r\theta\sin\theta + \frac{h}{2}\cos\theta\right)$$



$$\frac{dV}{d\theta} = mg\left(-r\sin\theta + r\sin\theta + r\theta\cos\theta - \frac{h}{2}\sin\theta\right)$$

$$= mg\left(r\theta\cos\theta - \frac{h}{2}\sin\theta\right)$$

$$\frac{d^2V}{d\theta^2} = mg\left(r\cos\theta - r\theta\sin\theta - \frac{h}{2}\cos\theta\right)$$

For $\theta = 0$ position, $\frac{d^2V}{d\theta^2} = mg\left(r - 0 - \frac{h}{2}\right)$

$= (+)$ stable if $\underline{h < 2r}$

7/64 Force & moment equilibrium (A)

(a), (b), (d)

Virtual work (B)

(c), (e), (f)

(c) $\delta U' = \delta V_g$

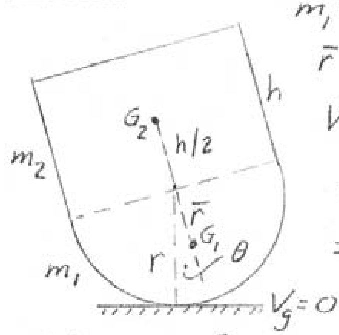
(e) $\delta U' = \delta V_g + \delta V_e$

(f) $d^2V/d\theta^2$ must be (+)

7/65 Let ρ = mass per unit area of shell

$m_1 = 2\pi r^2 \rho$, $m_2 = 2\pi r h \rho$

$\bar{r} = r/2$ for hemispherical shell



$V = V_{g_1} + V_{g_2}$

$= 2\pi r^2 \rho g (r - \bar{r}\cos\theta) + 2\pi r h \rho g (r + \frac{h}{2}\cos\theta)$

$= 2\pi r \rho g [(r^2 + hr) - \frac{1}{2}(r^2 - h^2)\cos\theta]$

$\frac{dV}{d\theta} = 2\pi r \rho g [0 + \frac{1}{2}(r^2 - h^2)\sin\theta]$

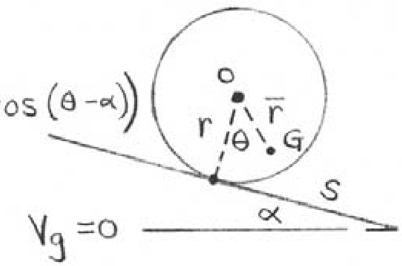
$\frac{d^2V}{d\theta^2} = \pi r \rho g (r^2 - h^2)\cos\theta$

For equil. $\frac{dV}{d\theta} = 0$ gives $\theta = 0$ & $h = r$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = (+)$ if $\underline{h < r}$

For $h = r$, neutral equilibrium

7/66



$V_g = W(s\sin\alpha + r\cos\alpha - \bar{r}\cos(\theta-\alpha))$

But $s = s_0 + r\theta$, $\frac{ds}{d\theta} = r$

So $\frac{dV_g}{d\theta} = W(r\sin\alpha + \bar{r}\sin(\theta-\alpha))$

For $\alpha = 10°$, $r = 100$ mm, $\bar{r} = 60$ mm:

$$\frac{dV_g}{d\theta} = W[0.1\sin 10° + 0.060\sin(\theta - 10°)]$$

$= 0$ for equilibrium

$$-0.1\sin 10° = 0.060\sin(\theta-10°)$$

$$\theta - 10° = \sin^{-1}\left(-\frac{0.1}{0.06}\sin 10°\right) = -16.82° \text{ or } 196.8°$$

$\Rightarrow$ $\underline{\theta = -6.82°}$ or $\underline{\theta = 207°}$

$$\frac{d^2V_g}{d\theta^2} = W[0 + \bar{r}\cos(\theta-\alpha)]$$

$\theta = -6.82°$: $d^2V_g/d\theta^2 = W\bar{r}\cos(-16.82°) > 0$ Stable

$\theta = 207°$: $d^2V_g/d\theta^2 = W\bar{r}\cos(196.8°) < 0$ Unstable

7/67 $V = V_e = \frac{1}{2}k\left(2a\sin\frac{\theta}{2}\right)^2 = 2ka^2\sin^2\frac{\theta}{2}$

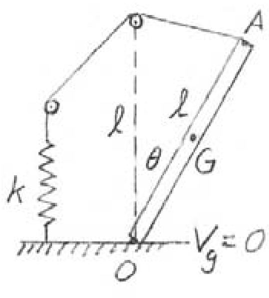
$\delta U = \delta V_e$; $P\delta(2a\sin\theta) = \delta\left(2ka^2\sin^2\frac{\theta}{2}\right)$

$$2Pa\cos\theta\,\delta\theta = 2ka^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\delta\theta$$

$$2Pa\cos\theta = ka^2\sin\theta$$

$\tan\theta = \frac{2P}{ka}$, $\quad \underline{\theta = \tan^{-1}\frac{2P}{ka}}$

7/68 $V = V_e + V_g = \frac{1}{2}k\left(2l\sin\frac{\theta}{2}\right)^2 + mg\frac{l}{2}\cos\theta$



$$\frac{dV}{d\theta} = 2kl^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - mg\frac{l}{2}\sin\theta$$

$$= kl^2\sin\theta - mg\frac{l}{2}\sin\theta$$

$$= l\left(kl - \frac{1}{2}mg\right)\sin\theta = 0 \text{ for equil.}$$

$\theta = 0$ and $\underline{k = \frac{mg}{2l}}$ independent of θ

$$\frac{d^2V}{d\theta^2} = l\left(kl - \frac{1}{2}mg\right)\cos\theta$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = l\left(kl - \frac{1}{2}mg\right)$ = (+) stable if $k > \frac{mg}{2l}$

(−) unstable if $k < \frac{mg}{2l}$

For $k = \frac{mg}{2l}$, $V = \frac{mgl}{2}$ const., <u>neutral equil</u>

7/69 Total length of door is $2.5 + 0.6\pi/2 = 3.44$ m

Unit mass is $135/3.44 = 39.22$ kg/m

Take $V_g = 0$ through A

Let potential energy of cylindrical portion be $-V_0$ which remains constant

So $V_g = 0 - V_0 - 39.22(9.81)x\left(0.6 + \frac{x}{2}\right)$

$= -V_0 - 384.7\,x\left(0.6 + \frac{x}{2}\right)$ J

$V_e = 2\left(\frac{1}{2}k\theta^2\right) = \frac{10}{2\pi}\theta^2 = \frac{5}{\pi}\left(\frac{x}{0.080}\right)^2 = 248.7x^2$ J

$V = V_e + V_g = 248.7x^2 - V_0 - 384.7\,x\left(0.6 + \frac{x}{2}\right)$

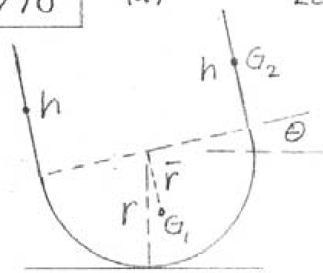
$\frac{dV}{dx} = 497.4x - 230.8 - 384.7x = 112.7x - 230.8$

$= 0$ for equilibrium, so $x = \frac{230.8}{112.7} = \underline{2.05 \text{ m}}$

$\frac{d^2V}{dx^2} = 112.7$ (+) so $\underline{\text{stable}}$

7/70

(a)



(a)

Let ρ = mass per unit periphery of shell

$\bar{r} = 2r/\pi$

$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + r\sin\theta + \frac{h}{2}\cos\theta\right) + h\left(r - r\sin\theta + \frac{h}{2}\cos\theta\right)\right]$

$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr + h^2\cos\theta\right]$

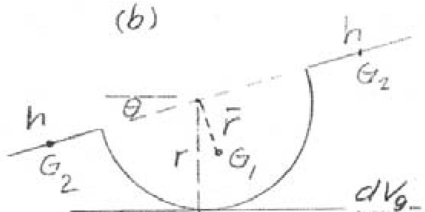
$\frac{dV_g}{d\theta} = \rho\left[2r^2 - h^2\right]\sin\theta$, $\frac{d^2V_g}{d\theta^2} = \rho\left[2r^2 - h^2\right]\cos\theta$

Equil. at $\theta = 0$ stable if $\underline{h < r\sqrt{2}}$

unstable if $h > r\sqrt{2}$

Neutral equil. if $h = r\sqrt{2}$ for any θ

(b)



$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + \left[r + \frac{h}{2}\sin\theta\right]\right) + h\left(r - \left[r + h/2\sin\theta\right]\right)\right]$

$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr\right]$

$\frac{dV_g}{d\theta} = 2\rho r^2\sin\theta$, $\theta = 0$ for stable equil. independent of θ

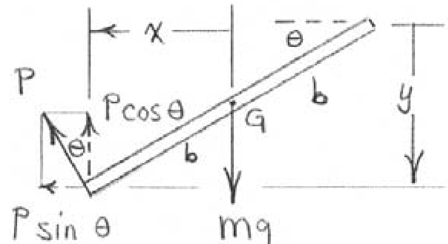
7/71

$y = 2b\sin\theta$

$\delta y = 2b\cos\theta\ \delta\theta$

$x = b\cos\theta$

$\delta x = -b\sin\theta\ \delta\theta$



$\delta U = 0:\ mg\,\delta\left(\frac{y}{2}\right) - P\cos\theta\,(\delta y) + P\sin\theta\,(\delta x) = 0$

$mgb\cos\theta\ \delta\theta - P\cos\theta\,(2b\cos\theta\ \delta\theta) + P\sin\theta\,(-b\sin\theta\ \delta\theta) = 0$

$mg\cos\theta = P(\sin^2\theta + 2\cos^2\theta) = P(1+\cos^2\theta)$

$$P = \frac{mg\cos\theta}{1+\cos^2\theta}$$

7/72 In displaced position

$V_g = mg(3R\cos\theta)$
$\quad + mg(3R\cos\theta - b\cos[\theta+\beta])$

But $s = 2R\theta = R\beta$

so $\beta = 2\theta$

& $V_g = 6mgR\cos\theta - mgb\cos 3\theta$

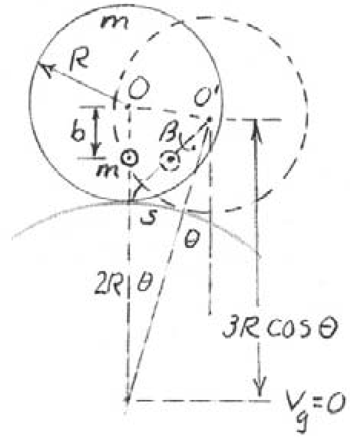
$\frac{dV_g}{d\theta} = -6mgR\sin\theta + 3mgb\sin 3\theta$

$(= 0 \text{ for } \theta = 0)$

$\frac{d^2V_g}{d\theta^2} = -6mgR\cos\theta + 9mgb\cos 3\theta$

$\left(\frac{d^2V_g}{d\theta^2}\right)_{\theta=0} = 3mg(-2R + 3b) = +$ stable if $3b > 2R$

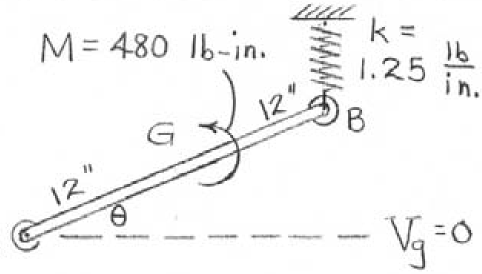
so $\underline{b_{min} = \frac{2}{3}R}$



*7/73

$\delta U' = \delta V_g$

$\delta U' = 480\ \delta\theta$



$V = V_g + V_e = 20\,(12\sin\theta) + \frac{1}{2}(1.25)(24\sin\theta)^2$

$\delta V = 240\cos\theta\ \delta\theta + 360(2\sin\theta\cos\theta)\,\delta\theta$

So $480\,\delta\theta = 240\cos\theta\ \delta\theta + 360\sin 2\theta\ \delta\theta$

or $4 = 2\cos\theta + 3\sin 2\theta$

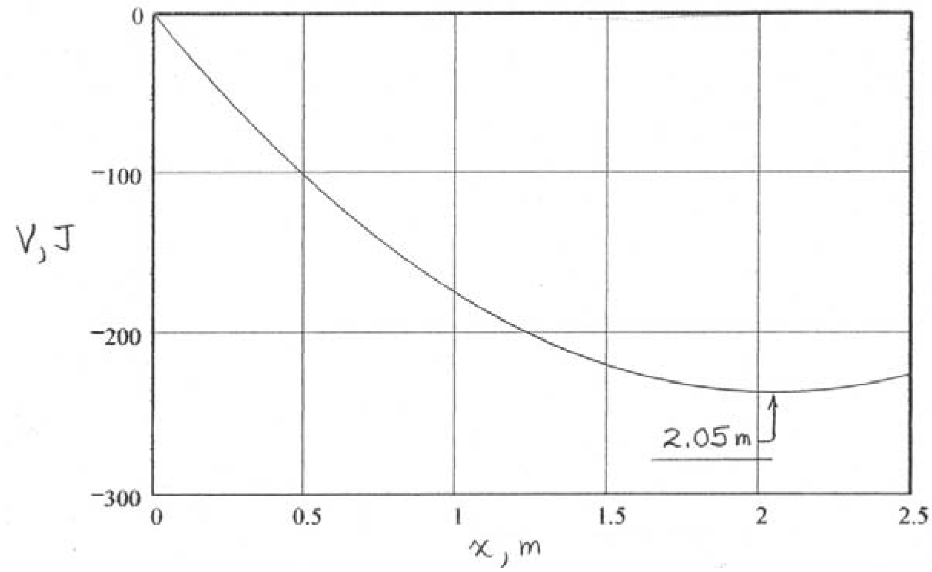
Numerically solve to obtain $\underline{\theta = 23.0°}$

*7/74 From Prob. 7/69,

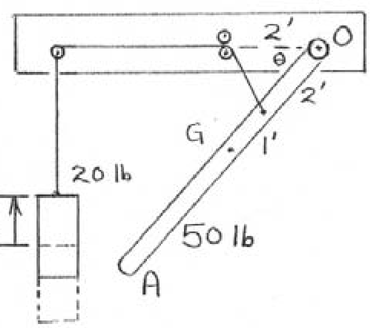
$V = 248.7x^2 - V_0 - 384.7x(0.6 + \frac{x}{2})$ in Joules

$= 56.33x^2 - 230.8x$ with $V_0 = 0$

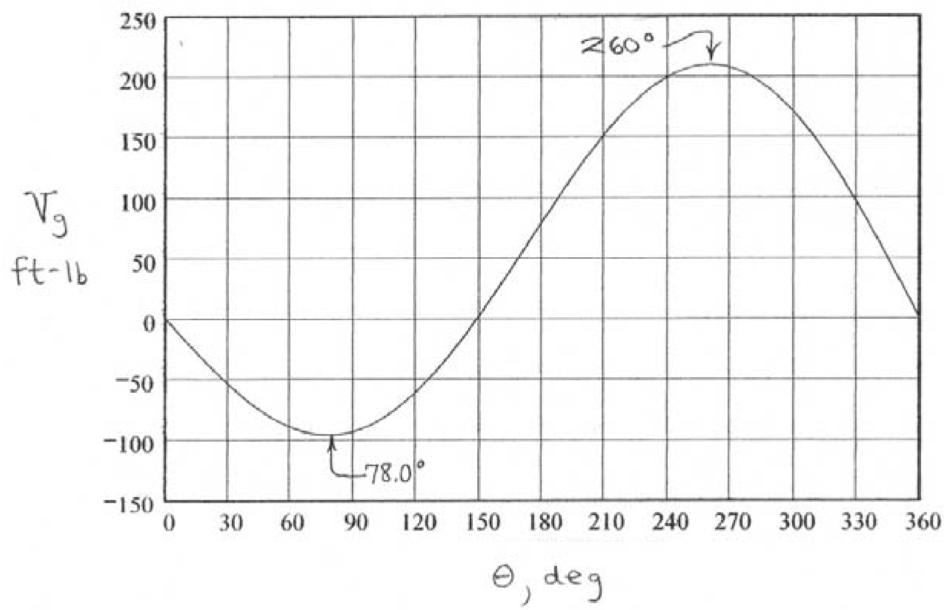
where V_0 is the potential energy of the cylindrical section. Compute V and get



*7/75 $\quad h = 2\left(2\sin\frac{\theta}{2}\right) = 4\sin\frac{\theta}{2}$



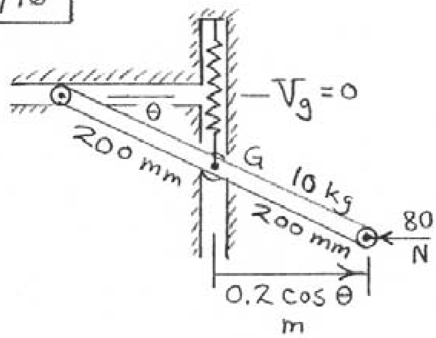
$$V_g = 20\left(4\sin\frac{\theta}{2}\right) - 50\,(3\sin\theta)$$
$$= 80\sin\frac{\theta}{2} - 150\sin\theta \text{ ft-lb}$$



$\underline{\theta = 78.0° \text{ stable}}$

$\underline{\theta = 260° \text{ unstable}}$

*7/76



$\delta U = \delta V_g + \delta V_e$

$\delta U = -80\,\delta(0.2\cos\theta) = 16\sin\theta\,\delta\theta$

$\delta V_g = \delta(-mgh) = -\delta\left[10(9.81)(0.2\sin\theta)\right]$

$= -19.62\cos\theta\,\delta\theta$

$\delta V_e = \delta\left(\frac{1}{2}kx^2\right) = kx\,\delta x$

$= 1500\,(0.2\sin\theta)\,\delta(0.2\sin\theta)$

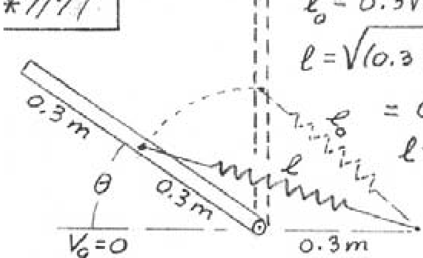
$= 60\sin\theta\cos\theta\,\delta\theta = 30\sin 2\theta\,\delta\theta$

Thus $16\sin\theta\,\delta\theta = -19.62\cos\theta\,\delta\theta + 30\sin 2\theta\,\delta\theta$

or $(16\sin\theta + 19.62\cos\theta - 30\sin 2\theta)\,\delta\theta = 0$

Numerical solution : $\underline{\theta = 27.9°}$

*7/77



$V_g = 0$

$k = 1250$ N/m

$\ell_0 = 0.3\sqrt{2} = 0.4243$ m

$\ell = \sqrt{(0.3 + 0.3\cos\theta)^2 + (0.3\sin\theta)^2}$

$= 0.3\sqrt{2}\sqrt{1+\cos\theta} = 0.4243\sqrt{1+\cos\theta}$

$\ell - \ell_0 = 0.4243\left(\sqrt{1+\cos\theta} - 1\right)$

$V_e = \frac{1}{2}k(\ell - \ell_0)^2$

$= \frac{1250}{2} \times 0.18\left[2 + \cos\theta - 2\sqrt{1+\cos\theta}\right]$

$= 112.5\left[2 + \cos\theta - 2\sqrt{1+\cos\theta}\right]$

$V_g = mgh = 10(9.81)(0.3\sin\theta)$

$= 29.43\sin\theta$

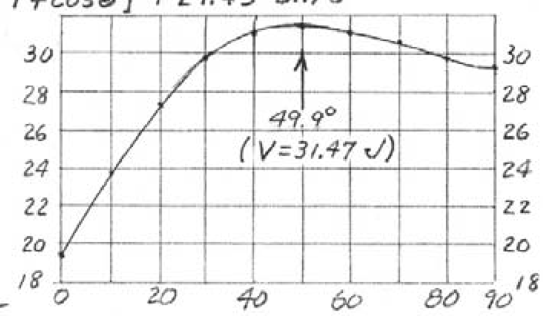
$V = V_e + V_g$

$V = 112.5\left[2 + \cos\theta - 2\sqrt{1+\cos\theta}\right] + 29.43\sin\theta$

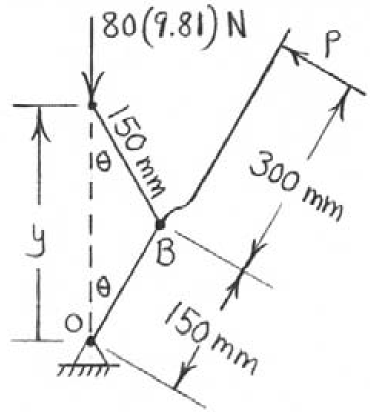
Compute V as a function of θ & get →

unstable equilibrium occurs at $\theta = 49.9°$

Stable equilibrium at $\theta = \pm 90°$



*7/78



$y = 2(150)\cos\theta$

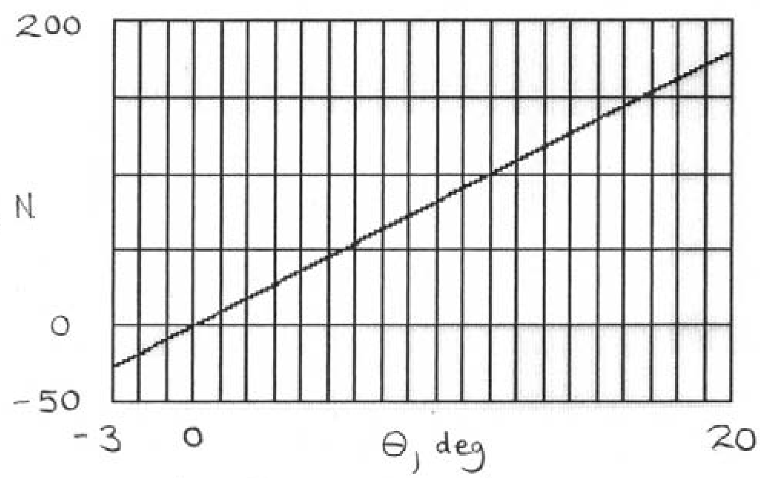
$\delta y = -300 \sin\theta \,\delta\theta$

$\delta V = 0:$

$-P(450\,\delta\theta) - 80(9.81)\delta y = 0$

$$P = \frac{80(9.81)(300 \sin\theta)}{450}$$

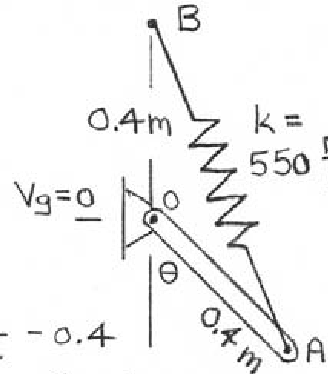
$= \underline{523 \sin\theta \text{ (in newtons)}}$



(At $\theta = -3°$, $P = -27.4$ N)

(At $\theta = 20°$, $P = 178.9$ N)

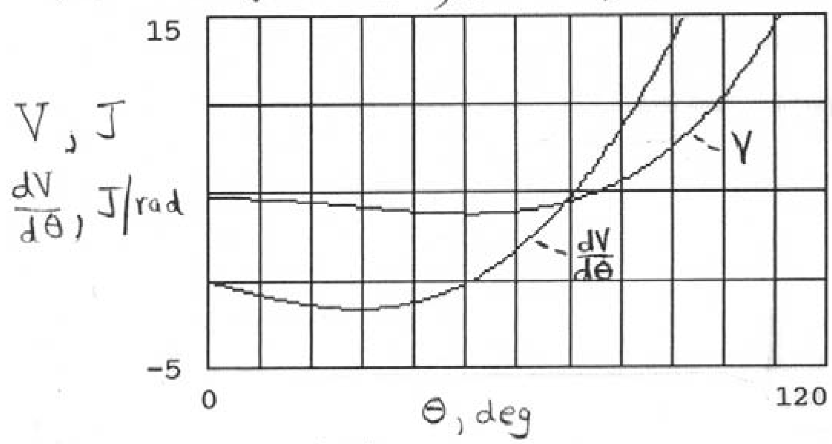
***7/79**



$$V_g = 20(9.81)(-0.2\cos\theta)$$
$$= -39.24\cos\theta \quad \text{J}$$
$$\overline{AB} = 2(0.4)\sin\left(\frac{180-\theta}{2}\right)$$
$$= 0.8\cos\frac{\theta}{2}$$

Stretch of spring $= 0.8\cos\frac{\theta}{2} - 0.4$
$$= 0.4\left(2\cos\frac{\theta}{2} - 1\right)$$

$$V_e = \frac{1}{2}550\left[0.4\left(2\cos\frac{\theta}{2}-1\right)\right]^2 = 44\left(2\cos\frac{\theta}{2}-1\right)^2$$
$$V = V_e + V_g = 44\left(2\cos\frac{\theta}{2}-1\right)^2 - 39.24\cos\theta$$
$$dV/d\theta = 88\left(2\cos\frac{\theta}{2}-1\right)\left(-\sin\frac{\theta}{2}\right) + 39.24\sin\theta$$

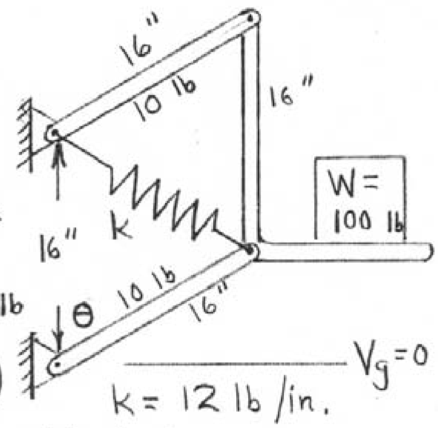


$\theta = 0$, unstable
$\theta = 51.1°$, stable

*7/80

Spring stretch $= 2\left(16\sin\frac{\theta}{2}\right) - 8$

$= 8\left(4\sin\frac{\theta}{2} - 1\right)$ in.



$V_e = \frac{1}{2}(12)(8)^2\left(4\sin\frac{\theta}{2} - 1\right)^2$

$= 384\left(4\sin\frac{\theta}{2} - 1\right)^2$ in.-lb

$V_g = 10(8\cos\theta) + 10(16 + 8\cos\theta)$

$+100(16\cos\theta) = 1760\cos\theta + 160$ in.-lb

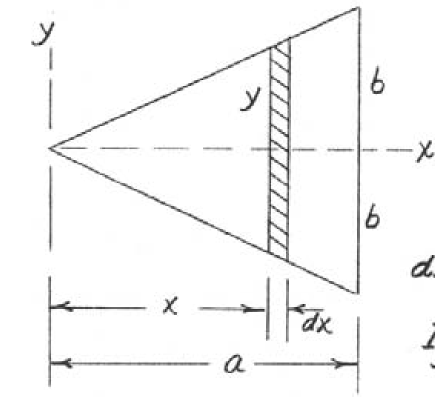
$V = V_e + V_g = 384\left(4\sin\frac{\theta}{2} - 1\right)^2 + 1760\cos\theta + 160$ in.-lb

$\frac{dV}{d\theta} = 768\left(4\sin\frac{\theta}{2} - 1\right)2\cos\frac{\theta}{2} - 1760\sin\theta$

$= 0$ for equilibrium

Numerical solution : $\underline{\theta = 71.7°}$

A/1 Using the results of Sample Prob. A/1



$$dI_x = \frac{1}{12}(2y)^3 dx = \frac{2}{3}y^3 dx$$

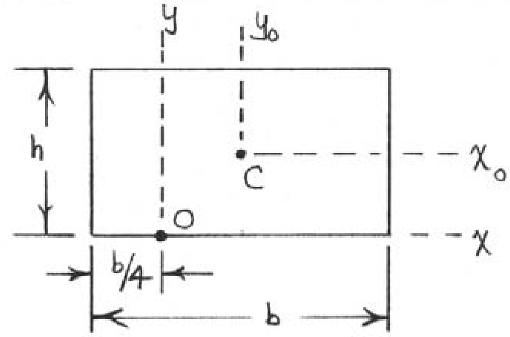
But $y = \frac{b}{a}x$ so

$$I_x = \frac{2b^3}{3a^3}\int_0^a x^3 dx = \underline{\frac{1}{6}ab^3}$$

$$dI_y = x^2(2y\,dx) = 2\frac{b}{a}x^3 dx$$

$$I_y = \frac{2b}{a}\int_0^a x^3 dx = \frac{2b}{a}\frac{a^4}{4} = \underline{\frac{1}{2}ba^3}$$

A/2



From Sample Prob. A/1,

$$I_{x_0} = \frac{1}{12} bh^3$$

$$I_{y_0} = \frac{1}{12} hb^3$$

So

$$I_x = \frac{1}{12} bh^3 + bh\left(\frac{h}{2}\right)^2 = \underline{\frac{1}{3} bh^3}$$

$$I_y = \frac{1}{12} hb^3 + bh\left(\frac{b}{4}\right)^2 = \underline{\frac{7}{48} hb^3}$$

$$I_o = I_x + I_y = \underline{\frac{bh}{3}\left(h^2 + \frac{7}{16} b^2\right)}$$

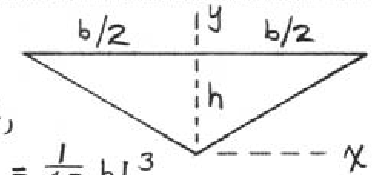
A/3

$$I_x \approx Ad^2 = 300(15)^2 = 67.5(10^3) \text{ mm}^4$$

$$J_O = I_x + I_y = 67.5(10^3) + 35(10^3) = 102.5(10^3) \text{ mm}^4$$

$$k_O = \sqrt{J_O/A} = \sqrt{\frac{102.5(10^3)}{300}} = \underline{18.48 \text{ mm}}$$

A/4

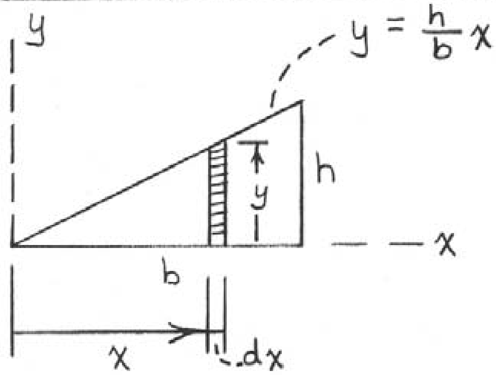


From Sample Problem A/2,

$I_x = \frac{1}{4} bh^3$, $I_y = 2\left\{\frac{1}{12} h \left(\frac{b}{2}\right)^3\right\} = \frac{1}{48} hb^3$

$I_x = I_y$ if $\frac{1}{4} bh^3 = \frac{1}{48} hb^3$, $\underline{\frac{b}{h} = 2\sqrt{3}}$

A/5



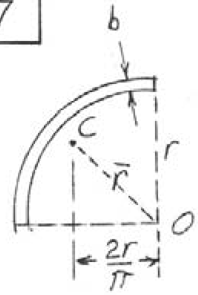
$$I_y = \int x^2\,dA = \int x^2 y\,dx = \int x^2\left(\frac{h}{b}x\right)dx$$

$$= \frac{h}{b}\int_0^b x^3\,dx = \frac{h}{4b}\,x^4\Big|_0^b = \underline{\frac{hb^3}{4}}$$

A/6 $I_p = I_c + A(3)^2, \quad I_{p'} = I_c + A(2)^2$

$I_p - I_{p'} = 50 = A(3^2 - 2^2), \quad \underline{A = 10 \text{ in.}^2}$

A/7



$$I_C = I_O - A\bar{r}^2$$

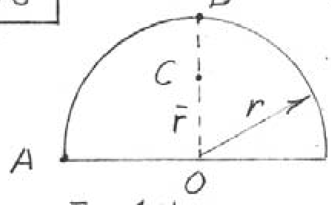
$$= Ar^2 - A\left(\frac{2r}{\pi}\sqrt{2}\right)^2$$

$$= Ar^2\left(1 - \frac{8}{\pi^2}\right) = \frac{\pi b r^3}{2}\left(1 - \frac{8}{\pi^2}\right)$$

$$= br^3\left(\frac{\pi}{2} - \frac{4}{\pi}\right)$$

$$= \underline{0.298\, br^3}$$

A/8



$\bar{r} = 4r/3\pi$

For complete circle

$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2$

$= \frac{3}{2}Ar^2$

For half circle

$I_A = \frac{1}{2}\left(\frac{3}{2}\pi r^4\right) = \underline{\frac{3}{4}\pi r^4}$

For half circle, $I_O = \frac{1}{4}\pi r^4$

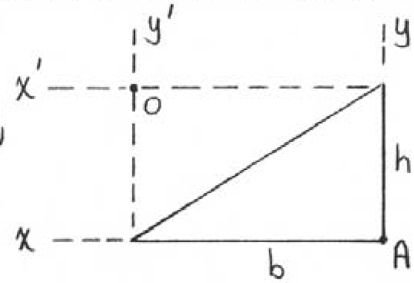
$I_B = I_C + A(r-\bar{r})^2 = I_O - A\bar{r}^2 + A(r-\bar{r})^2$

$= I_O + A(r^2 - 2r\bar{r})$

$= \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2}\left(1 - \frac{8}{3\pi}\right) = \underline{r^4\left(\frac{3\pi}{4} - \frac{4}{3}\right)}$

A/9

From Sample Problem A/2,



$I_x = \frac{1}{12} bh^3$, $I_y = \frac{1}{12} b^3 h$

$I_A = I_x + I_y$

$= \frac{1}{12} bh(h^2 + b^2)$

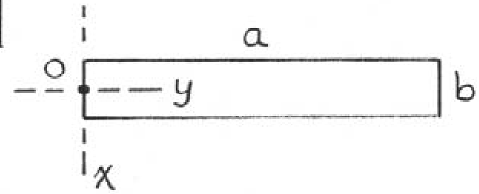
$k_A = \sqrt{I_A/A} = \sqrt{\frac{1}{12} bh(h^2+b^2) \Big/ \frac{bh}{2}} = \underline{\sqrt{(h^2+b^2)/6}}$

$I_{x'} = \frac{1}{4} bh^3$, $I_{y'} = \frac{1}{4} b^3 h$

$I_o = I_{x'} + I_{y'} = \frac{1}{4} bh(h^2+b^2)$

$k_o = \sqrt{I_o/A} = \sqrt{\frac{1}{4} bh(h^2+b^2) \Big/ \frac{bh}{2}} = \underline{\sqrt{(h^2+b^2)/2}}$

A/10



$I_x = \frac{1}{3} A a^2 = \frac{1}{3} a^3 b$, $I_y = \frac{1}{12} a b^3$

$I_O = I_x + I_y = \frac{1}{3} ab\left(a^2 + \frac{b^2}{4}\right)$

$n = \%\text{ error} = \frac{I_x - I_O}{I_O}(100) = \frac{-\frac{1}{12} ab^3}{\frac{1}{3} ab\left(a^2 + \frac{b^2}{4}\right)} 100$

$= -\frac{1}{4} \frac{b^2}{a^2 + \frac{b^2}{4}} 100$

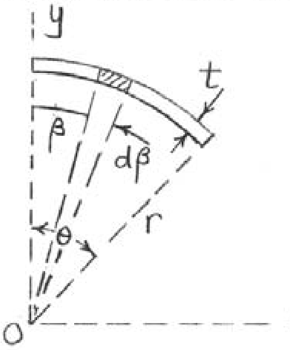
For $\frac{b}{a} = \frac{1}{10}$, $n = -25 \frac{1}{10^2 + \frac{1}{4}} = \underline{-0.249\ \%}$

A/11

$$A \cong \frac{90-30-15}{360} 2\pi (300)(10) = 2360 \text{ mm}^2$$

$$I_o \cong Ar^2 = 2360(300)^2 = \underline{212(10^6) \text{ mm}^4}$$

A/12



$I_x = \int y^2 dA$, $I_y = \int x^2 dA$

where $y = r\cos\beta$, $x = r\sin\beta$,

and $dA = rt\,d\beta$

$$I_x = \int_0^\theta r^2\cos^2\beta\; rt\,d\beta$$

$$= r^3 t\left[\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right]_0^\theta$$

But $A = r\theta t$, so $\underline{I_x = \frac{Ar^2}{4}\left(2 + \frac{\sin 2\theta}{\theta}\right)}$

$$I_y = \int_0^\theta r^2\sin^2\beta\; rt\,d\beta = r^3 t\left[\frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]_0^\theta$$

$$= \underline{\frac{Ar^2}{4}\left(2 - \frac{\sin 2\theta}{\theta}\right)}$$

$I_o = I_x + I_y = Ar^2$

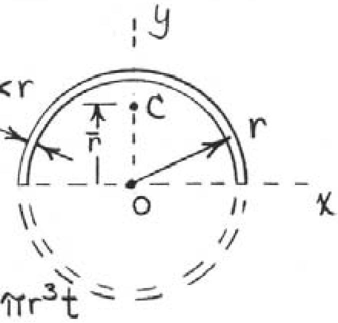
For $\theta = 45°$, $r = 300\ mm^2$, and $t = 10$ mm

from Prob. A/11, we have

$$A = 45\left(\frac{\pi}{180}\right)(300)(10) = 2360\ mm^2$$

So $I_o = 2360(300)^2 = \underline{212(10^6)\ mm^4}$

A/13



For complete ring,

$I_o = Ar^2 = 2\pi r t r^2 = 2\pi r^3 t$

and $I_o = I_x + I_y$, $I_x = I_y$

So for complete ring, $I_x = \frac{I_o}{2} = \pi r^3 t$

For half-ring, $\underline{I_x = \frac{1}{2}\pi r^3 t}$ and $I_y = I_x$

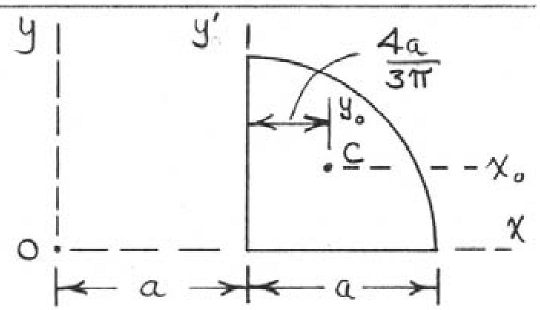
by symmetry so $\underline{I_y = \frac{1}{2}\pi r^3 t}$

For half-ring, $I_o = \frac{1}{2}(2\pi r^3 t) = \pi r^3 t$

$I_C = I_o - A\bar{r}^2 = \pi r^3 t - \pi r t\left(\frac{2r}{\pi}\right)^2$

$= \underline{\pi r^3 t\left(1 - \frac{4}{\pi^2}\right)}$

A/14



From Table D/3

$$\underline{I_x} = I_{y'} = \underline{\frac{\pi a^4}{16}}$$

So $I_{x_o} = I_{y_o} =$

$$\frac{\pi a^4}{16} - \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right)^2 = a^4\left[\frac{\pi}{16} - \frac{4}{9\pi}\right]$$

Then $I_y = I_{y_o} + Ad^2 = a^4\left[\frac{\pi}{16} - \frac{4}{9\pi}\right] + \frac{\pi a^2}{4}\left[a + \frac{4a}{3\pi}\right]^2$

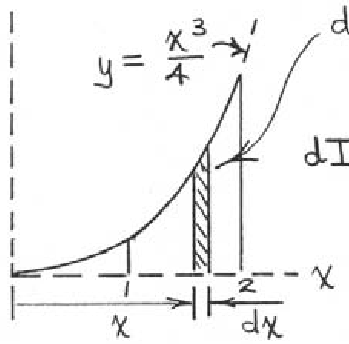
$$= \underline{a^4\left[\frac{5\pi}{16} + \frac{2}{3}\right]}$$

$$I_o = I_x + I_y = a^4\left[\frac{3\pi}{8} + \frac{2}{3}\right]$$

$$k_o = \sqrt{I_o/A} = \sqrt{a^4\left[\frac{3\pi}{8} + \frac{2}{3}\right]\Big/\frac{\pi a^2}{4}} = \underline{a\sqrt{\frac{8}{3\pi} + \frac{3}{2}}}$$

Decimal forms: $\begin{cases} \underline{I_x = 0.1963\,a^4}, \ \underline{I_y = 1.648a^4} \\ \underline{k_o = 1.533a} \end{cases}$

A/15



$$dA = y\,dx = \frac{x^3}{4}\,dx$$

$$dI_y = x^2\,dA = \frac{x^5}{4}\,dx$$

$$dI_x = \frac{1}{3}y^3\,dx = \frac{x^9}{192}\,dx$$

$$A = \int dA = \int_1^2 \frac{x^3}{4}\,dx = \frac{1}{4}\frac{x^4}{4}\Big|_1^2 = \frac{15}{16}$$

$$I_y = \int dI_y = \int_1^2 \frac{x^5}{4}\,dx = \frac{1}{4}\frac{x^6}{6}\Big|_1^2 = \frac{63}{24}$$

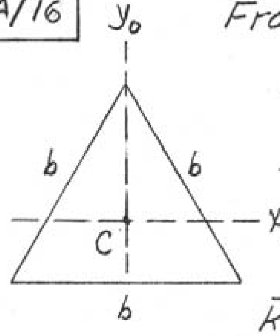
$$I_x = \int dI_x = \int_1^2 \frac{x^9}{192}\,dx = \frac{1}{192}\frac{x^{10}}{10}\Big|_1^2 = \frac{1023}{1920}$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = \underline{1.673}$$

$$k_x = \sqrt{I_x/A} = \sqrt{\frac{1023/1920}{15/16}} = \underline{0.754}$$

$$k_z = \sqrt{k_x^2 + k_y^2} = \sqrt{1.673^2 + 0.754^2} = \underline{1.835}$$

A/16



From Sample Problem A/2

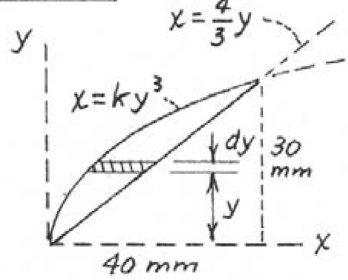
$$\bar{I}_x = \frac{1}{36} b \left(b \frac{\sqrt{3}}{2} \right)^3 = \frac{b^4}{96}\sqrt{3}$$

$$\bar{I}_y = 2\left(\frac{1}{12} b \frac{\sqrt{3}}{2} \left[\frac{b}{2} \right]^3 \right) = \frac{b^4}{96}\sqrt{3}$$

$$\bar{J} = \bar{I}_x + \bar{I}_y = \frac{b^4}{48}\sqrt{3}$$

$$\bar{k} = \sqrt{\bar{J}/A} = \sqrt{\frac{b^4\sqrt{3}}{48} \Big/ \left(\frac{b^2}{2}\frac{\sqrt{3}}{2} \right)} = \underline{\frac{b}{2\sqrt{3}}}$$

A/17



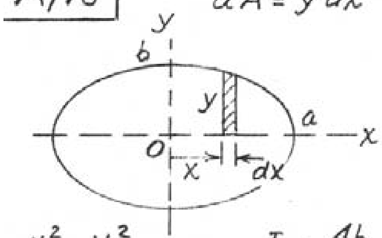
For $x = 40$ mm & $y = 30$ mm, $k = \frac{40}{27(10^3)}$

$$dI_x = y^2 dA = y^2\left(\frac{4}{3}y - \frac{4}{2700}y^3\right)dy$$

$$I_x = \int_0^{30}\left(\frac{4}{3}y^3 - \frac{4}{2700}y^5\right)dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050}\right]_0^{30} = \underline{9(10^4)\ mm^4}$$

A/18



$dA = y\,dx = \frac{b}{a}\sqrt{a^2-x^2}\,dx$

$A = \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx = \pi a b$

$dI_y = x^2 y\,dx = \frac{b}{a}x^2\sqrt{a^2-x^2}\,dx$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$I_y = \frac{4b}{a}\int_0^a x^2\sqrt{a^2-x^2}\,dx$

$= \frac{4b}{a}\left[-\frac{x}{4}\sqrt{(a^2-x^2)^3}+\frac{a^2}{8}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)\right]_0^a$

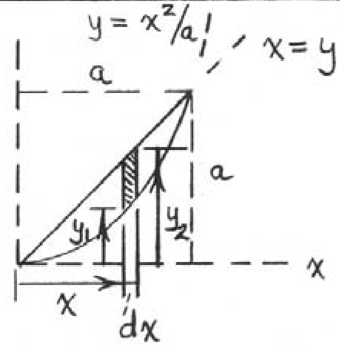
$= \frac{b}{a}\left[0+\frac{a^2}{2}\left(0+a^2\frac{\pi}{2}\right)\right] = \underline{\frac{\pi a^3 b}{4}}$

Similarly $\quad I_x = \frac{\pi a b^3}{4}$

So $I_o = I_x + I_y = \frac{\pi a b}{4}(a^2+b^2)$

& $k_o = \sqrt{I_o/A} = \underline{\frac{1}{2}\sqrt{a^2+b^2}}$

A/19



$$dA = (y_2 - y_1)\,dx = \left(x - \frac{x^2}{a}\right)dx$$

$$I_x = \int \frac{1}{3} y_2^{\,3}\,dx - \int \frac{1}{3} y_1^{\,3}\,dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right)dx$$

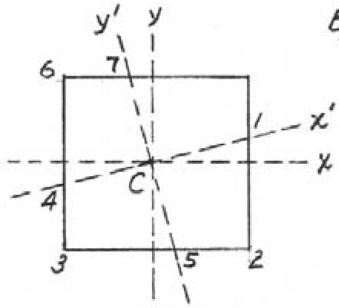
$$= \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \underline{a^4/28}$$

$$I_y = \int x^2\,dA = \int x^2\left(x - \frac{x^2}{a}\right)dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{a^4/20}$$

A/20

$$I_C = I_x + I_y = I_{x'} + I_{y'}$$

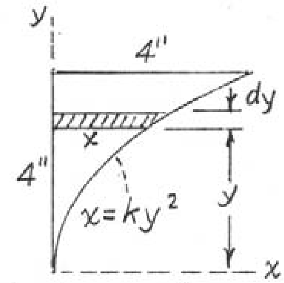


By symmetry of the figures 1-2-3-4 and 5-3-6-7

$I_{x'} = I_{y'}$. Thus since $I_x = I_y$,

$2I_x = I_C = 2I_{x'}$, so $\underline{I_{x'} = I_x}$

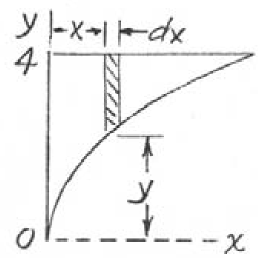
A/21 $dI_x = y^2 dA = y^2 x\,dy = y^2(y^2/4)\,dy = (y^4/4)\,dy$



$$I_x = \frac{1}{4}\int_0^4 y^4\,dy = \frac{1}{4}\frac{y^5}{5}\Big]_0^4 = \frac{256}{5}$$

$$= \underline{51.2 \text{ in.}^4}$$

$k = 1/4$, $x = y^2/4$



$$dI_x = \frac{1}{3}(dx)\left(4^3 - y^3\right)$$

$$= \frac{1}{3}\left(4^3 - [2\sqrt{x}]^3\right)dx$$

$$I_x = \frac{1}{3}\int_0^4 \left(4^3 - 8x^{3/2}\right)dx$$

$$= \frac{1}{3}\left[4^3x - 8\,\frac{2}{5}x^{5/2}\right]_0^4$$

$$= \frac{256}{3}\left(1 - \frac{2}{5}\right) = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$

A/22 (a) $k_O^2 = k_C^2 + \overline{OC}^2$ where $k_C^2 = I_C/A = \frac{40(10^4)}{1600}\ mm^2$

$= 250\ mm^2$

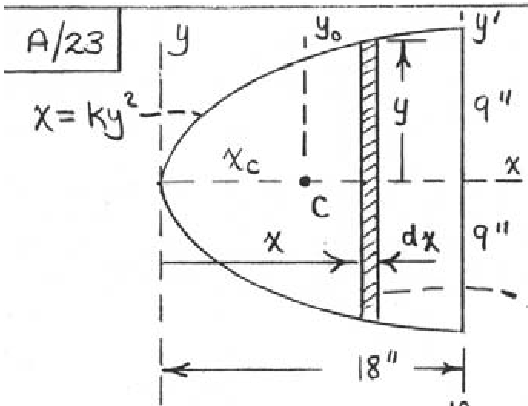
$k_O^2 = 250 + (30\sqrt{2})^2 = 2050\ mm^2$

$k_O = \sqrt{2050} = \underline{45.3\ mm}$

(b) $I_{x_O} = k_{x_O}^2 A$ & $I_{x_O} + I_{y_O} = I_C$ & $I_{y_O} = I_{x_O}$ so $I_{x_O} = \frac{1}{2} I_C$

so $k_{x_O}^2 = I_C/2A$, $k_{x_O} = \sqrt{\frac{40(10^4)}{2(1600)}} = \underline{11.18\ mm}$

A/23



$x = ky^2$: $18 = k(9^2)$

$k = \frac{2}{9}$ in.$^{-1}$

$x = \frac{2}{9}y^2$, $y = 3\sqrt{x/2}$

$$\begin{cases} dA = 2y\,dx \\ dI_y = dA\,x^2 = 2yx^2\,dx \end{cases}$$

$$A = \int dA = \int 2y\,dx = \int_0^{18} 2(3\sqrt{x/2})\,dx = \frac{6}{\sqrt{2}}\int_0^{18} x^{1/2}\,dx$$

$$= \frac{6}{\sqrt{2}}\frac{x^{3/2}}{3/2}\Big|_0^{18} = \frac{4}{\sqrt{2}}\,18^{3/2} = 216 \text{ in.}^2$$

$$I_y = \int dI_y = \int 2(3\sqrt{x/2})x^2\,dx = \frac{6}{\sqrt{2}}\int_0^{18} x^{5/2}\,dx$$

$$= \frac{6}{\sqrt{2}}\frac{x^{7/2}}{7/2}\Big|_0^{18} = \underline{30{,}000 \text{ in.}^4}$$

$$\int x\,dA = \int 2(3\sqrt{x/2})x\,dx = \frac{6}{\sqrt{2}}\int_0^{18} x^{3/2}\,dx = 2330 \text{ in.}^3$$

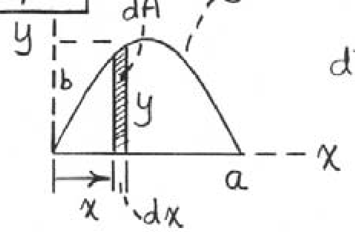
$$x_c = \int x\,dA/A = 2330/216 = 10.80 \text{ in.}$$

$$I_{y_0} = I_y - Ax_c^2 = 30{,}000 - 216(10.80)^2 = 4800 \text{ in.}^4$$

$$I_{y'} = I_{y_0} + A(18 - x_c)^2 = 4800 + 216(18 - 10.8)^2 = \underline{16{,}000 \text{ in.}^4}$$

A/24

$y = b \sin \frac{\pi}{a} x$



$$dI_x = \frac{1}{3}\, dA\, y^2 = \frac{1}{3}(y\, dx) y^2 = \frac{1}{3} y^3 dx$$

$$dI_y = dA\, x^2 = (y\, dx)\, x^2 = x^2 y\, dx$$

$$I_x = \int dI_x = \int \frac{1}{3} y^3 dx = \frac{1}{3}\int_0^a \left(b \sin \frac{\pi}{a} x\right)^3 dx$$

$$= \frac{b^3}{3}\left[-\frac{a}{\pi}\cos\frac{\pi}{a}x + \frac{a}{3\pi}\cos^3\frac{\pi}{a}x\right]_0^a$$

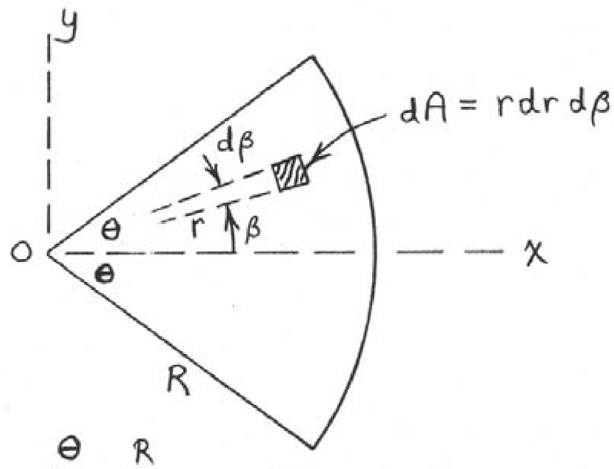
$$= \underline{\frac{4}{9}\frac{ab^3}{\pi}}$$

$$I_y = \int dI_y = \int_0^a x^2 b \sin\frac{\pi}{a} x\, dx$$

$$= b\left[\frac{2x}{(\pi/a)^2}\sin\frac{\pi}{a}x + \frac{2}{(\pi/a)^3}\cos\frac{\pi}{a}x - \frac{x^2}{\pi/a}\cos\frac{\pi}{a}x\right]_0^a$$

$$= \underline{\frac{ba^3}{\pi}\left(1 - \frac{4}{\pi^2}\right)}$$

A/25



$$I_x = \int y^2 dA = \int_{-\theta}^{\theta}\int_0^R (r\sin\beta)^2 \, r\,dr\,d\beta$$

$$= \frac{R^4}{4}\left(\frac{\beta}{2} - \frac{\sin 2\beta}{4}\right)_{-\theta}^{\theta} = \underline{\frac{R^4}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right)}$$

$$I_y = \int x^2 dA = \int_{-\theta}^{\theta}\int_0^R (r\cos\beta)^2 \, r\,dr\,d\beta$$

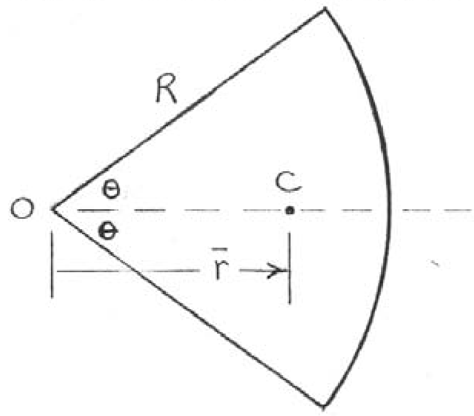
$$= \frac{R^4}{4}\left(\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right)_{-\theta}^{\theta} = \underline{\frac{R^4}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right)}$$

A/26

$$I_C = I_O - A\bar{r}^2$$

$$\bar{r} = \frac{2}{3}\,\frac{R\sin\theta}{\theta}$$

$$A = \frac{2\theta}{2\pi}\pi R^2 = \theta R^2$$



From Prob. A/25,

$$I_O = I_x + I_y = \frac{R^4}{4}\left(\theta - \frac{\sin\theta}{2} + \theta + \frac{\sin\theta}{2}\right) = \frac{1}{2}R^4\theta$$

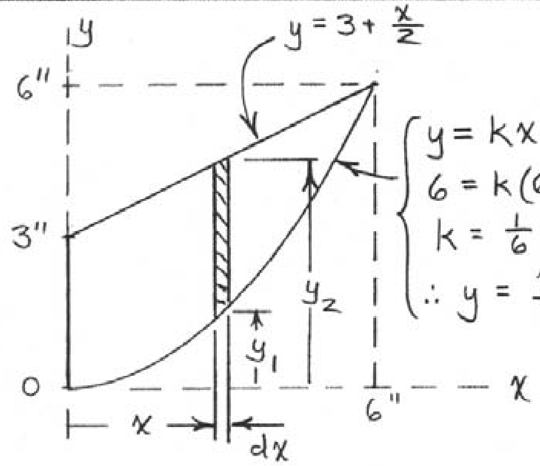
So $$I_C = \frac{1}{2}R^4\theta - \theta R^2\left(\frac{2}{3}\,\frac{R\sin\theta}{\theta}\right)^2$$

$$= \underline{R^4\theta\left(\frac{1}{2} - \frac{4}{9}\,\frac{\sin^2\theta}{\theta^2}\right)}$$

A/27

$y_2 = 3 + \frac{x}{2}$

$y_1 = x^2/6$



$y = kx^2$: $6 = k(6)^2$, $k = \frac{1}{6}$ in.$^{-1}$, $\therefore y = \frac{x^2}{6}$

From Sample Problem A/1:

$$dI_x = \frac{1}{3}(y_2\,dx)y_2^2 - \frac{1}{3}(y_1\,dx)y_1^2$$

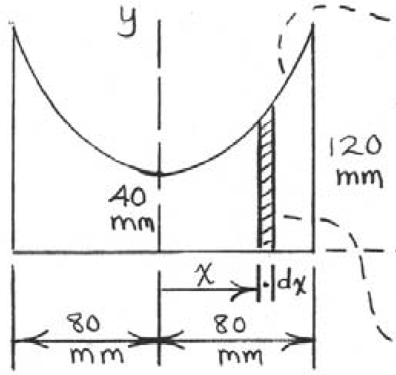
$$= \frac{1}{3}(y_2^3 - y_1^3)\,dx$$

$$I_x = \frac{1}{3}\int_0^6 \left[\left(3 + \frac{x}{2}\right)^3 - \left(\frac{x^2}{6}\right)^3\right] dx$$

$$= \frac{1}{3}\left[27x + \frac{27}{4}x^2 + \frac{3}{4}x^3 + \frac{1}{32}x^4 - \frac{1}{1512}x^7\right]_0^6$$

$$= \underline{140.8 \text{ in.}^4}$$

A/28



$y = a + bx^2$

$$\begin{cases} 40 = a + b(0)^2 \\ 120 = a + b(80)^2 \end{cases}$$

$$\begin{cases} a = 40 \text{ mm} \\ b = \dfrac{1}{80 \text{ mm}} \end{cases}$$

$$\begin{cases} dA = y\,dx \\ dI_y = dA\,x^2 \end{cases}$$

$$A = \int dA = \int y\,dx = 2\int_0^{80}\left(40 + \frac{1}{80}x^2\right)dx$$

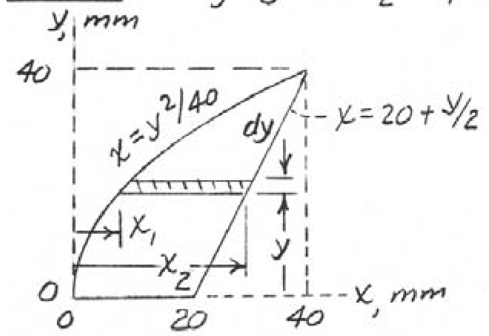
$$= 2\left[40x + \frac{x^3}{240}\right]_0^{80} = 10\,670 \text{ mm}^2$$

$$I_y = \int dI_y = \int x^2 y\,dx = 2\int_0^{80}\left[40x^2 + \frac{x^4}{80}\right]dx$$

$$= 2\left[\frac{40}{3}x^3 + \frac{x^5}{400}\right]_0^{80} = 30.0(10^6) \text{ mm}^4$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{30.0(10^6)}{10\,670}} = \underline{53.1 \text{ mm}}$$

A/29 $dI_y = \frac{1}{3}dy(x_2^3 - x_1^3) = \frac{1}{3}\left[(20 + \frac{y}{2})^3 - (\frac{y^2}{40})^3\right]dy$



$$\int_0^{40}(20 + \frac{y}{2})^3 dy$$

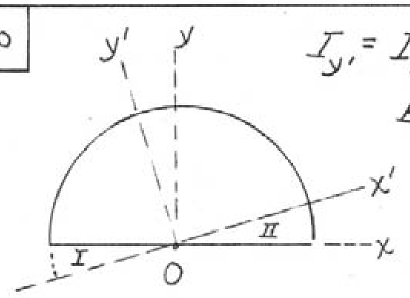
$$= \frac{1}{2}(20 + \frac{y}{2})^4\Big]_0^{40} = 120(10^4) \text{ mm}^4$$

$$\int_0^{40}\frac{y^6}{40^3}dy = \frac{1}{40^3}\frac{y^7}{7}\Big]_0^{40}$$

$$= 36.57(10^4) \text{ mm}^4$$

$I_y = \frac{1}{3}(120 - 36.6)10^4 = \underline{27.8\,(10^4)\text{ mm}^4}$

A/30



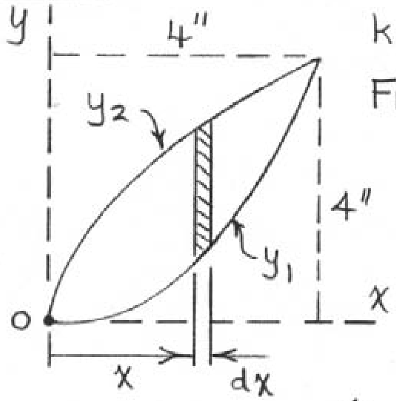
$I_{y'} = I_{half\ circle} - I_{I} + I_{II}$

But $I_{I} = I_{II}$ by symmetry

Similarly for $I_{x'}$

So $\underline{I_{x'} = I_{y'} = I_x = I_y}$

A/31



From $y_1 = k_1 x^3$: $4 = k_1 4^3$

$k_1 = \frac{1}{16}$ in.$^{-2}$ & $y_1 = \frac{x^3}{16}$

From $y_2 = k_2\sqrt{x}$: $4 = k_2\sqrt{4}$

$k_2 = 2$ in.$^{1/2}$ & $y_2 = 2\sqrt{x}$

$$dA = (y_2 - y_1)\,dx = \left(2\sqrt{x} - \frac{x^3}{16}\right)dx$$

$$I_y = \int x^2 dA = \int_0^4 x^2\left(2x^{\frac{1}{2}} - \frac{x^3}{16}\right)dx$$

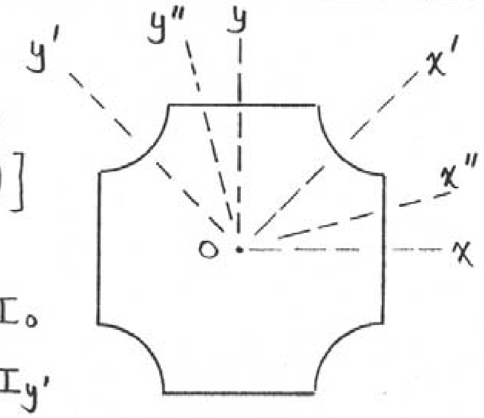
$$= \left[\frac{4}{7}x^{7/2} - \frac{x^6}{96}\right]_0^4 = \underline{30.5 \text{ in.}^4}$$

$$I_x = \int\left[\frac{1}{3}y_2^3\,dx - \frac{1}{3}y_1^3\,dx\right] = \frac{1}{3}\int_0^4\left[(2\sqrt{x})^3 - \left(\frac{x^3}{16}\right)^3\right]dx$$

$$= \frac{1}{3}\left[8\cdot\frac{2}{5}x^{5/2} - \frac{x^{10}}{10\cdot 16^3}\right]_0^4 = \underline{25.6 \text{ in.}^4}$$

$$I_o = I_x + I_y = \underline{56.1 \text{ in.}^4}$$

A/32



$I_x + I_y = I_o$, $I_x = I_y$

So $I_x = \frac{1}{2} I_o = \frac{1}{2}\left[16(10^6)\right]$

$= \underline{8(10^6)\ mm^4}$

Similarly, $I_{x'} + I_{y'} = I_o$

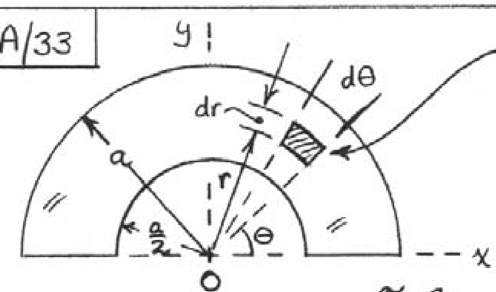
and $I_{x'} = I_{y'}$

So $I_{x'} = \frac{1}{2} I_o = I_x = \underline{8(10^6)\ mm^4}$

By symmetry it is seen that $I_{x''} = I_{y''}$

and because $I_{x''} + I_{y''} = I_o$, it follows that the moment of inertia about every axis through O has the same value, $8(10^6)\ mm^4$.

A/33



$$dA = r\,dr\,d\theta$$

$$A = \frac{1}{2}\left[\pi a^2 - \pi\left(\frac{a}{2}\right)^2\right] = \frac{3}{8}\pi a^2$$

$$I_x = \int y^2 dA = \int_0^{\pi}\int_{a/2}^{a} (r\sin\theta)^2\, r\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{15}{64} a^4 \sin^2\theta\, d\theta = \frac{15}{128}\pi a^4$$

$$I_y = \int x^2 dA = 2\int_0^{\pi/2}\int_{a/2}^{a} (r\cos\theta)^2\, r\,dr\,d\theta$$

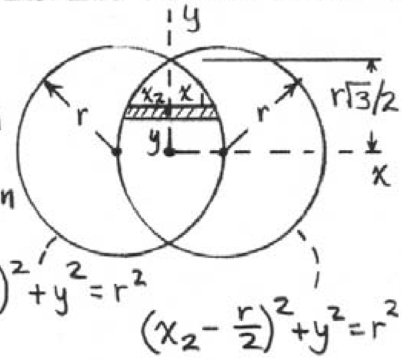
$$= 2\int_{a/2}^{a} \frac{15}{64} a^4 \cos^2\theta\, d\theta = \frac{15}{128}\pi a^4$$

$$\underline{k_x} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{128}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{5}}{4} a = k_y}$$

$$k_z^2 = k_x^2 + k_y^2 = 2\left(\frac{5}{16} a^2\right)$$

$$\underline{k_z = \frac{\sqrt{10}}{4} a}$$

▶ A/34



$x_2 = \frac{r}{2} \pm \sqrt{r^2-y^2}$ use (−) sign

$x_1 = -\frac{r}{2} \pm \sqrt{r^2-y^2}$ use (+) sign

$$(x_1 - x_2) = -\frac{r}{2} + \sqrt{r^2-y^2} - \frac{r}{2} + \sqrt{r^2-y^2} = 2\sqrt{r^2-y^2} - r$$

$$dA = \left(2\sqrt{r^2-y^2} - r\right) dy$$

$$I_x = \int y^2 dA = 2\int_0^{r\sqrt{3}/2} y^2 \left(2\sqrt{r^2-y^2} - r\right) dy$$

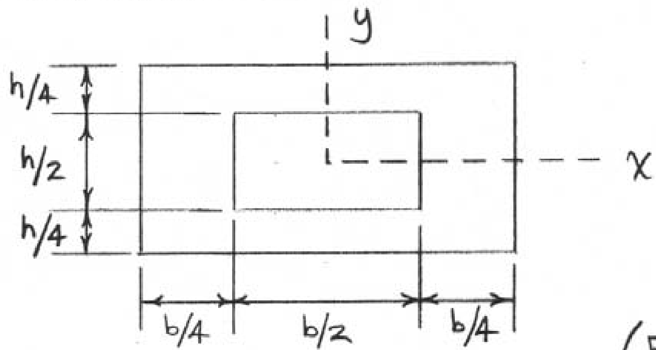
$$= 4\left\{-\frac{y}{4}\sqrt{(r^2-y^2)^3} + \frac{r^2}{8}\left(y\sqrt{r^2-y^2} + r^2\sin^{-1}\frac{y}{r}\right)\right\} - \frac{2r}{3}y^3\Bigg|_0^{\frac{r\sqrt{3}}{2}}$$

$$= 4\left\{-\frac{r\sqrt{3}}{8}\frac{r^3}{8} + \frac{r^2}{8}\left(\frac{r\sqrt{3}}{2}\frac{r}{2} + r^2\frac{\pi}{3}\right)\right\} - \frac{2\sqrt{3}}{8}r^4 - 0$$

$$= \frac{r^4}{2}\left\{-\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right\} = \frac{r^4}{2}\left\{\frac{\pi}{3} - \frac{3\sqrt{3}}{8}\right\}$$

$$= 0.1988\,r^4$$

A/35



For solid plate : $I_z = \frac{1}{12} bh(b^2+h^2)$ (From Sample Problem A/1

For plate with hole :

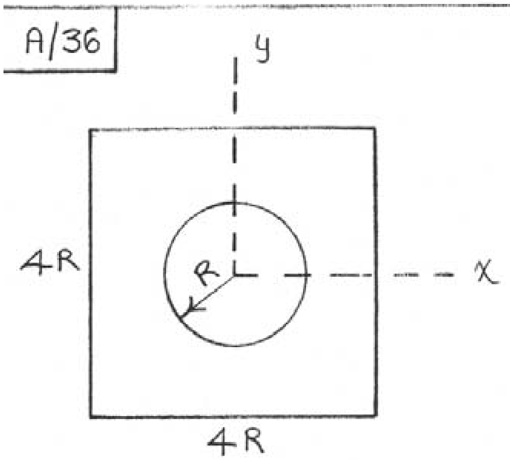
$$(I_z)_h = \frac{1}{12}bh(b^2+h^2) - \frac{1}{12}\frac{b}{2}\frac{h}{2}\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right]$$

$$= \frac{15}{16}\left[\frac{1}{12}bh(b^2+h^2)\right]$$

The percent reduction $n = \frac{I_z - (I_z)_h}{I_z}(100)$

$$n = \left(\frac{1-\frac{15}{16}}{1}\right)100 = \underline{6.25\%}$$

A/36



Without hole, $I_x = \frac{1}{12}(4R)(4R)^3 = \frac{64}{3}R^4$

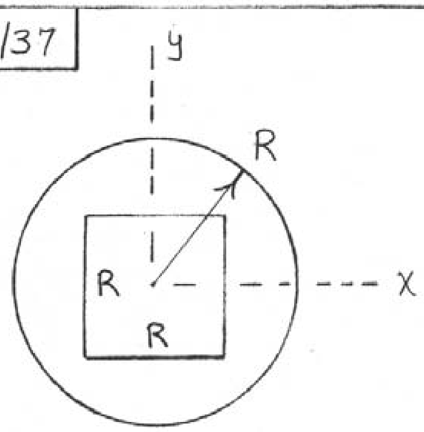
$(21.3R^4)$

With hole, $I_x = \frac{64}{3}R^4 - \frac{1}{4}(\pi R^2)R^2$

$= \underline{20.6R^4}$

(a 3.68% reduction)

A/37



Without square hole:

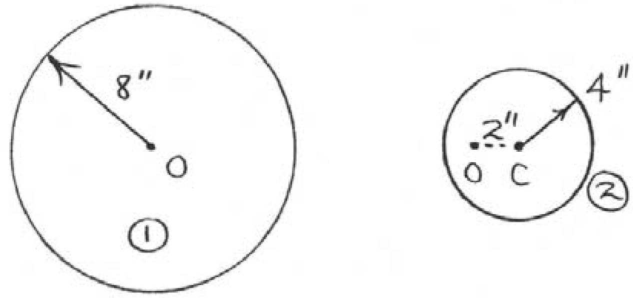
$I_z = 2I_x = 2\left(\frac{1}{4}\pi R^2 \cdot R^2\right) = \underline{1.571 R^4}$

With hole:

$I_z = 1.571R^4 - 2\left(\frac{1}{12} R \cdot R^3\right) = \underline{1.404 R^4}$

(a reduction of 10.61%)

A/38



Area $A = A_1 - A_2 = \pi\left(8^2 - 4^2\right) = 48\pi \text{ in.}^2$

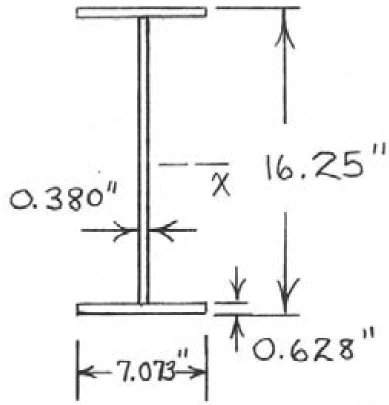
① $I_{o_1} = \frac{1}{2}\left(\pi 8^2\right) 8^2 = 2048\pi \text{ in.}^4$

② $I_{o_2} = +\frac{1}{2}\left(\pi 4^2\right) 4^2 + \pi (4)^2 \left(2^2\right) = +192\pi \text{ in.}^4$

So $I_o = I_{o_1} - I_{o_2} = 1856\pi \text{ in.}^4$

$k_o = \sqrt{I_o/A} = \sqrt{\frac{1856\pi}{48\pi}} = \underline{6.22 \text{ in.}}$

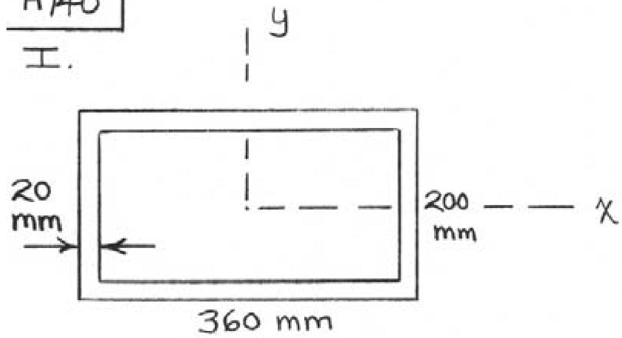
A/39



$$\bar{I}_x = \frac{1}{12}(0.380)\left[16.25 - 2(0.628)\right]^3$$
$$+2\left\{\frac{1}{12}(7.073)(0.628)^3 + 7.073(0.628)\left[\frac{16.25}{2} - \frac{0.628}{2}\right]^2\right\}$$
$$= \underline{649 \text{ in.}^4}$$

A/40

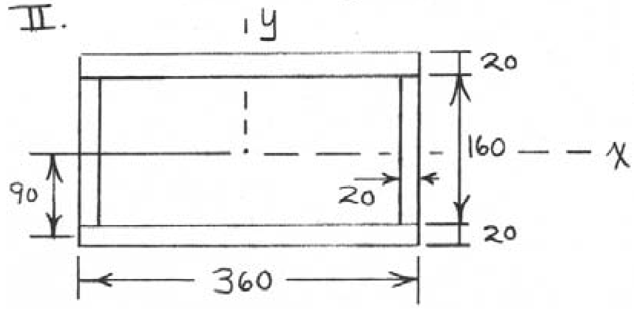
I.



$$I_x = \frac{1}{12}(360)(200)^3 - \frac{1}{12}(320)(160)^3$$

$$= \underline{130.8\,(10^6)\ \text{mm}^4}$$

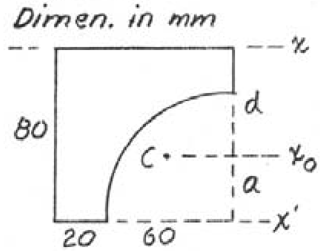
II.



(Dim. in mm)

$$I_x = 2\left[\frac{1}{12}(360)(20^3) + 360(20)(90)^2\right] + 2\left[\frac{1}{12}(20)(160)^3\right] = \underline{130.8\,(10^6)\ \text{mm}^4}$$

A/41



Square: $I_x = \frac{1}{3}b^4 = \frac{1}{3}(80)^4 = 13.65(10^6)\ mm^4$

Quarter-circle: $a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi}$

$= 25.46\ mm$

$d = 80 - 25.46 = 54.54\ mm$

$I_x = I_{x_0} + Ad^2 = I_{x'} - Aa^2 + Ad^2$

$= \frac{-1}{4}\frac{\pi r^4}{4} - \frac{\pi r^2}{4}(d^2 - a^2) = -\frac{\pi r^2}{4}\left(\frac{r^2}{4} + d^2 - a^2\right)$

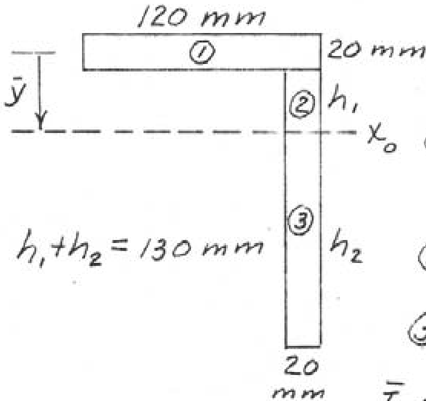
$= -\frac{\pi(60)^2}{4}\left[\frac{60^2}{4} + (54.54)^2 - (25.46)^2\right]$

$= -9.120\,(10^6)\ mm^4$

Total $I_x = (13.65 - 9.120)(10^6) = \underline{4.53\,(10^6)\ mm^4}$

A/42

$$\bar{y} = \Sigma A y / \Sigma A = \frac{0 + (130)(20)(65+10)}{250(20)} = 39.0 \text{ mm}$$



so $h_1 = 29.0$ mm

$h_2 = 101.0$ mm

① $\bar{I}_x = \frac{1}{12}(120)(20^3) + (120)(20)(39.0)^2$
$= 3.73(10^6)$ mm^4

② $\bar{I}_x = \frac{1}{3}(20)(29.0)^3 = 0.163(10^6)$ mm^4

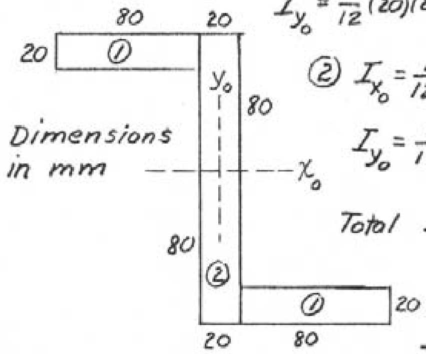
③ $\bar{I}_x = \frac{1}{3}(20)(101.0)^3 = 6.87(10^6)$ mm^4

$\bar{I}_x = (3.73 + 0.163 + 6.87)10^6$
$= \underline{10.76(10^6) \text{ mm}^4}$

A/43

① $I_{x_o} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6)\ mm^4$

$I_{y_o} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6)\ mm^4$



② $I_{x_o} = \frac{1}{12}(20)(160)^3 = 6.83(10^6)\ mm^4$

$I_{y_o} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6)\ mm^4$

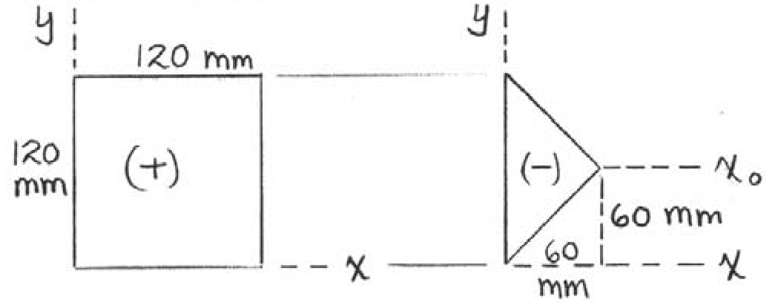
Total $\bar{I}_x = [2(7.89) + 6.83](10^6)$

$= \underline{22.6(10^6)\ mm^4}$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$

$= \underline{9.81(10^6)\ mm^4}$

A/44



Positive area:

$$I_x = I_y = \frac{1}{3}Ah^2 = \frac{1}{3}(120)^2(120)^2 = 69.1\,(10^6)\text{ mm}^4$$

$$I_z = I_x + I_y = 2(69.1)(10^6) = 138.2(10^6)\text{ mm}^4$$

Negative area:

$$I_x = I_{x_0} + A\bar{y}^2 = -2\left(\frac{bh^3}{12}\right) - A(60)^2$$

$$= -2\left(\frac{60 \times 60^3}{12}\right) - \frac{1}{2}(60)(120)(60)^2 = -15.12(10^6)\text{ mm}^4$$

$$I_y = -\frac{bh^3}{12} = -\frac{120(60)^3}{12} = -2.16(10^6)\text{ mm}^4$$

$$I_z = I_x + I_y = -15.12(10^6) - 2.16(10^6) = -17.28(10^6)\text{ mm}^4$$

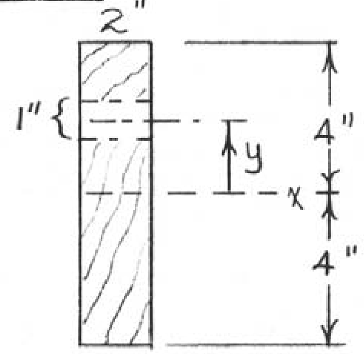
For composite area:

$$I_x = [69.1 - 15.12]\,10^6 = \underline{54\,(10^6)\text{ mm}^4}$$

$$I_y = [69.1 - 2.16]\,10^6 = \underline{67.0\,(10^6)\text{ mm}^4}$$

$$I_z = I_x + I_y = [54 + 67.0]\,10^6 = \underline{121.0\,(10^6)\text{ mm}^4}$$

A/45



Without hole,

$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3$$
$$= 85.3 \text{ in.}^4$$

With hole, $I_x' = I_x - (\bar{I}_{hole} + Ay^2)$

$$I_x' = 85.3 - \left[\frac{1}{12}(2)(1)^3 + (2)(1)y^2\right]$$
$$= 85.2 - 2y^2 \quad (y \text{ in in., } I_x' \text{ in in.}^4)$$

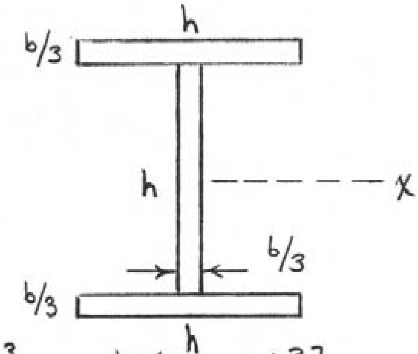
Percent reduction $n = \dfrac{I_x - I_x'}{I_x}(100\%)$

$$n = \frac{85.3 - (85.2 - 2y^2)}{85.3}(100\%) = \underline{0.1953 + 2.34y^2}$$
(in percent)

For $y = 2$ in.,

$$n = 0.1953 + 2.34(2)^2 = \underline{9.57\%}$$

A/46



For area (a),

$I_x = \frac{1}{12} bh^3$

For area (b),

$$I_x = \frac{1}{12}\frac{b}{3}h^3 + 2\left[\frac{1}{12}h\left(\frac{b}{3}\right)^3 + h\frac{b}{3}\left(\frac{h}{2}+\frac{b}{6}\right)^2\right]$$

$$= \frac{hb}{9}\left(\frac{7}{4}h^2 + \frac{2}{9}b^2 + hb\right)$$

If $h = 200$ mm and $b = 60$ mm, we have

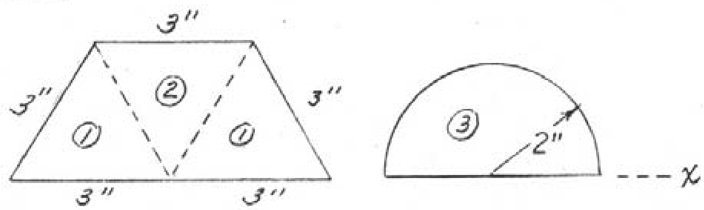
(a) $I_x = \frac{1}{12}(60)(200)^3 = 40(10^6)\ \text{mm}^4$

(b) $I_x = \frac{200(60)}{9}\left(\frac{7}{4}(200)^2 + \frac{2}{9}(60)^2 + 200(60)\right)$

$= 110.4(10^6)\ \text{mm}^4$

Percent increase $n = \frac{110.4 - 40}{40}(100\%) = \underline{176.0\%}$

A/47



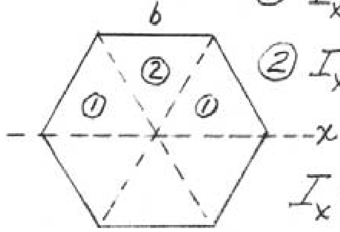
① $I_x = 2\frac{1}{12}(3)(3\sqrt{3}/2)^3 = \frac{81}{16}\sqrt{3}$ in.^{4}

② $I_x = \frac{1}{4}(3)(3\sqrt{3}/2)^3 = \frac{243}{32}\sqrt{3}$ in.^{4}

③ $I_x = \frac{-1}{2}\left(\frac{1}{4}\pi 2^4\right) = -2\pi = -6.28$ in.^{4}

Total $I_x = \frac{81}{16}\sqrt{3} + \frac{243}{32}\sqrt{3} - 6.28 = \underline{15.64 \text{ in.}^4}$

A/48 From Sample Problem A2



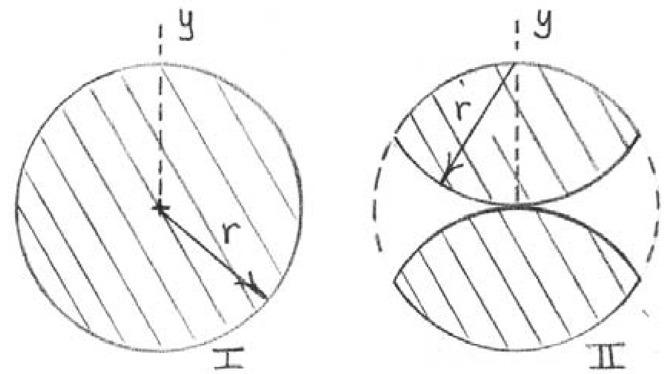
① $I_x = \frac{1}{12} b\left(b\sqrt{3}/2\right)^3 = \frac{\sqrt{3}}{32} b^4$

② $I_x = \frac{1}{4} b h^3 = \frac{1}{4} b\left(b\sqrt{3}/2\right)^3 = \frac{3\sqrt{3}}{32} b^4$

$$I_x = 4 I_{①} + 2 I_{②} = \frac{\sqrt{3}}{8} b^4 + \frac{3\sqrt{3}}{16} b^4$$

$$= \underline{\frac{5\sqrt{3}}{16} b^4}$$

A/49

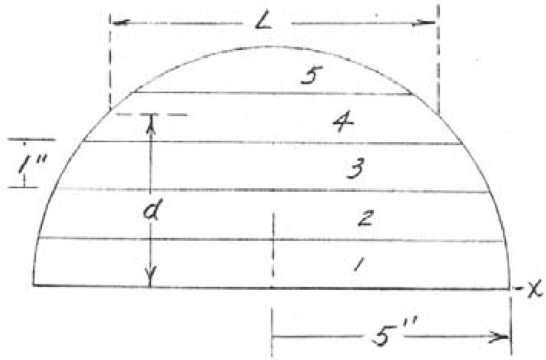


$$I_{I} = \frac{1}{4}\pi r^4; \quad I_{II} = 2(0.1988\, r^4) = 0.398\, r^4$$

$$I_y = I_{I} - I_{II} = 0.785 r^4 - 0.398 r^4 = \underline{0.388 r^4}$$

(Refer to Prob. A/34 for I_{II}.)

A/50



	L in.	A in.^{2}	d in.	Ad^2 in.^{4}
1	9.9	9.9	0.5	2.5
2	9.5	9.5	1.5	21.5
3	8.7	8.7	2.5	54.1
4	7.1	7.1	3.5	87.5
5	4.4	4.4	4.5	88.3

$I_x \approx \Sigma Ad^2 = \underline{253.8 \text{ in.}^4}$

$I_{x\,exact} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right) = \frac{\pi}{8}5^4 = \underline{245.4 \text{ in.}^4}$; Error $= \frac{253.8-245.4}{245.4}100$

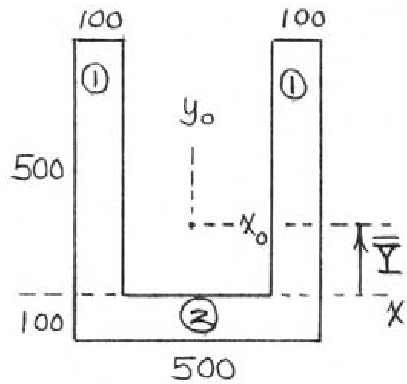
$= \underline{+3\ \%}$

A/51

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{2[(100)(500)(250)] + 500(100)(-50)}{2(100)(500) + 100(500)}$$

$$= 150 \text{ mm}$$



(Dim. in mm)

$$A = 2(100)(500) + 100(500)$$

$$= 15(10^4) \text{ mm}^2$$

① + ① $I_{x_o} = 2\left[\frac{1}{12}(100)(500)^3 + 100(500)(250-150)^2\right]$

$$= 30.8(10^8) \text{ mm}^4$$

$I_{y_o} = 2\left[\frac{1}{12}(500)(100)^3 + 100(500)(150+50)^2\right] = 40.8(10^8) \text{ mm}^4$

② $I_{x_o} = \frac{1}{12}(500)(100)^3 + 100(500)(50+150)^2 = 20.4(10^8) \text{ mm}^4$

$I_{y_o} = \frac{1}{12}(100)(500)^3 = 10.42(10^8) \text{ mm}^4$

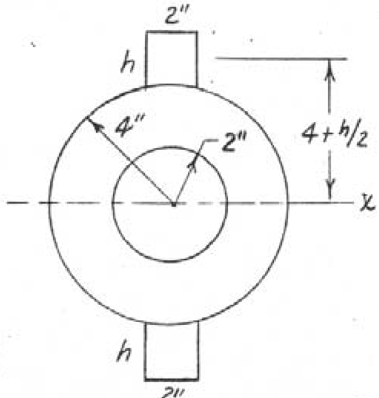
Totals ① + ① + ② : $I_{x_o} = 51.2(10^8) \text{ mm}^4$

$I_{y_o} = 51.2(10^8) \text{ mm}^4$

$$I_C = I_{x_o} + I_{y_o} = 102.5(10^8) \text{ mm}^4$$

$$k_C = \sqrt{I_C/A} = \sqrt{\frac{102.5(10^8)}{15(10^4)}} = \underline{261 \text{ mm}}$$

A/52



Circular section

$I_x = \pi r_2^4/4 - \pi r_1^4/4$

$= \frac{\pi}{4}(4^4 - 2^4) = 188.50 \text{ in.}^4$

Rectangular strips

$I_x = \bar{I}_x + Ad^2$

$I_x = 2\left[\frac{1}{12}(2)(h)^3 + 2h\left(4 + \frac{h}{2}\right)^2\right]$

$= \frac{h^3}{3} + 64h + 16h^2 + h^3$

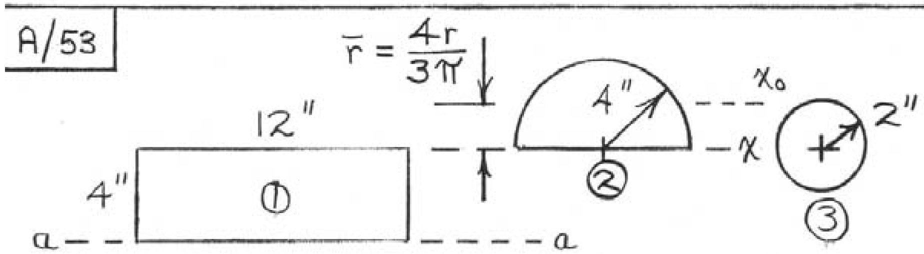
$= \frac{4h^3}{3} + 16h^2 + 64h$

To double stiffness $(I_x)_\square = [I_x]_o$

so $\frac{4h^3}{3} + 16h^2 + 64h = 188.50$

solve for h & get $\underline{h = 1.900 \text{ in.}}$

A/53



Part 1 : $I_{a\text{-}a} = \frac{1}{3}(12)4^3 = 256 \text{ in.}^4$

Part 2 : $I_{a\text{-}a} = I_{x_0} + A\left(4 + \frac{4 \cdot 4}{3\pi}\right)^2$

where $I_{x_0} = I_x - A\bar{r}^2 = \frac{1}{8}\pi 4^4 - \pi \frac{4^2}{2}\left(\frac{4 \cdot 4}{3\pi}\right)^2$

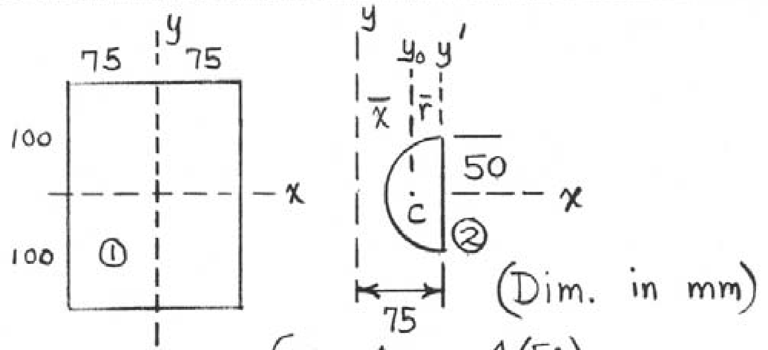
$= 28.1 \text{ in.}^4$

So $I_{a\text{-}a} = 28.1 + \frac{\pi 4^2}{2}(32.5) = 844 \text{ in.}^4$

Part 3 : $I_{a\text{-}a} = I_x + A(4)^2 = \frac{1}{4}\pi 2^4 + \pi 2^2 (4)^2$
$= 214 \text{ in.}^4$

Combined : $I_{a\text{-}a} = 256 + 844 - 214 = 886 \text{ in.}^4$

A/54



(Dim. in mm)

$$\begin{cases} \bar{r} = \dfrac{4r}{3\pi} = \dfrac{4(50)}{3\pi} = 21.2 \text{ mm} \\ \bar{x} = 75 - 21.2 = 53.8 \text{ mm} \end{cases}$$

Part I: $I_x = \frac{1}{12}(150)(200)^3 = 100(10^6) \text{ mm}^4$

$I_y = \frac{1}{12}(200)(150)^3 = 56.2(10^6) \text{ mm}^4$

Parts II: $I_x = \frac{1}{4}\pi(50)^4 = 4.91(10^6) \text{ mm}^4$ (for both together

$I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2 = I_{y'} + A(\bar{x}^2 - \bar{r}^2)$

$= \frac{1}{2}\left(\frac{1}{4}\pi\, 50^4\right) + \frac{\pi(50)^2}{2}(53.8^2 - 21.2^2)$

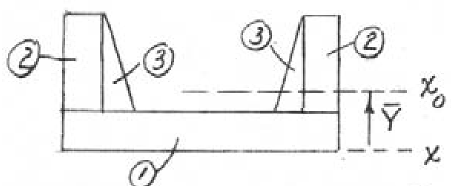
$= 12.04(10^6) \text{ mm}^4$ for each, $24.1(10^6) \text{ mm}^4$ for both

Combined: $I_x = 100(10^6) - 4.91(10^6) = \underline{95.1(10^6) \text{ mm}^4}$

$I_y = 56.2(10^6) - 24.1(10^6) = \underline{32.2(10^6) \text{ mm}^4}$

A/55

Part	A $in.^2$	$\bar{y}$ $in.$	$\bar{y}A$ $in.^3$	$\bar{I}_x$ $in.^4$	d $in.$	Ad^2 $in.^4$
1	8.40	0.35	2.94	0.343	0.696	4.07
2	4.29	2.35	10.08	3.894	1.304	7.30
3	0.33	1.80	0.59	0.200	0.754	0.19
Totals	13.02		13.61	4.437		11.56



$$\bar{Y} = \frac{13.61}{13.02} = 1.046 \text{ in.}$$

$$\bar{I}_x = I_{x_0} = \Sigma\bar{I}_x + \Sigma Ad^2$$

$$= 4.437 + 11.56$$

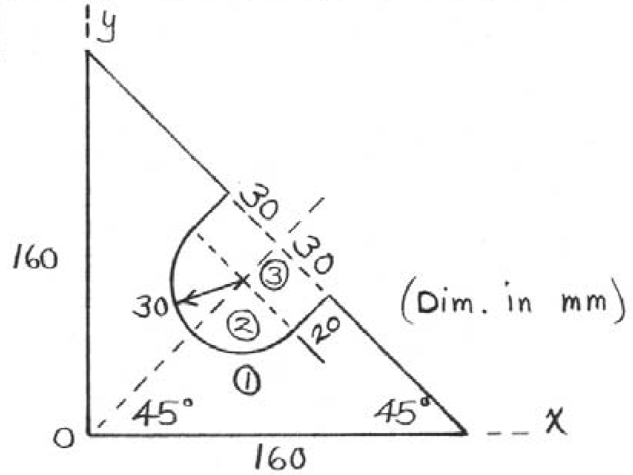
$$= \underline{16.00 \text{ in.}^4}$$

① $A_1 = 0.70 \times 12 = 8.40 \text{ in.}^2$

② $A_2 = 0.65 \times 3.30 \times 2 = 4.29 \text{ in.}^2$

③ $A_3 = 2\left(\frac{1}{2}\right)(0.10)(3.30) = 0.33 \text{ in.}^2$

A/56



Triangle ①: $I_o = I_x + I_y = 2I_x = 2\left[\dfrac{160(160)^3}{12}\right]$
$= 109.2(10^6)\ \text{mm}^4$

Semicircular area ②:

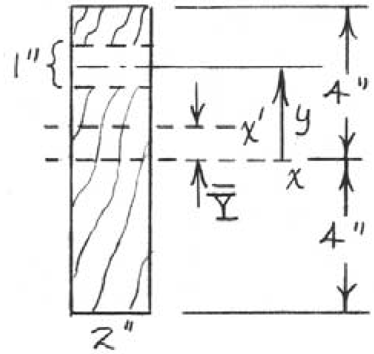
$$I_o = -\frac{\pi(30)^4}{4} + \frac{\pi(30)^2}{2}\left(\frac{4\cdot 30}{3\pi}\right)^2 - \frac{\pi(30)^2}{2}\left(\frac{160}{\sqrt{2}} - 20 - \frac{4\cdot 30}{3\pi}\right)^2$$
$$= -9.55(10^6)\ \text{mm}^4$$

Rectangular area ③:

$$I_o = -\frac{60(20)}{12}(60^2+20^2) - 60(20)\left(\frac{160}{\sqrt{2}} - 10\right)^2$$
$$= -13.16(10^6)\ \text{mm}^4$$

Total $I_o = \underline{86.5(10^4)\ \text{mm}^4}$

▶A/57 Without hole, $x' = x$ and $I_x = I_{x'} = \frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3 = 85.3 \text{ in.}^4$



Centroid location:

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{8(2)(0) - 2(1)y}{8(2) - 2(1)}$$

$$= -0.1429y \quad \left(\text{or } -\frac{1}{7}y\right)$$

$$I'_{x'} = \frac{1}{3}(2)(4+\bar{Y})(4+\bar{Y})^2 + \frac{1}{3}(2)(4-\bar{Y})(4-\bar{Y})^2 - \left[\frac{1}{12}(2)(1)^3 + 2(1)(y-\bar{Y})^2\right]$$

$$= 85.2 + 16\bar{Y}^2 - 2(y-\bar{Y})^2$$

With $\bar{Y} = -\frac{1}{7}y$: $I'_{x'} = 85.2 + 16\left(-\frac{1}{7}y\right)^2 - 2\left(y + \frac{1}{7}y\right)^2$

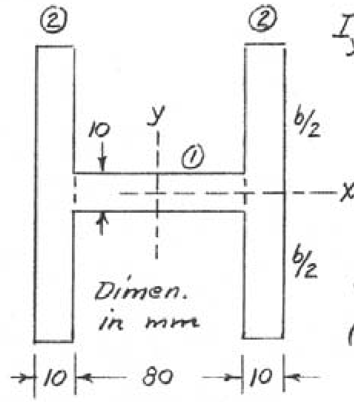
or $I'_{x'} = 85.2 - \frac{112}{49}y^2 = 85.2 - 2.29y^2$

Percent reduction $n = \frac{I_{x'} - I'_{x'}}{I_{x'}}(100\%)$

$$= (100\%)\frac{85.3 - (85.2 - 2.29y^2)}{85.3} = \underline{0.1953 + 2.68y^2 \text{ (in percent)}}$$

For $y = 2$ in., $\underline{n = 10.91\%}$

▸A/58



① $I_x = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ mm^4$

$I_y = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ mm^4$

② $I_x = 2\left[\frac{1}{12}(10)\,b^3\right] = 1.667b^3$

$I_y = 2\left[\frac{1}{12}b(10)^3\right] + (10b)(45)^2\Big]$

$= 0.0407(10^6)b$

Total I_x = Total I_y

$(0.00667)(10^6) + 1.667b^3$

$= (0.427 + 0.0407)(10^6)$

or $b^3 - 0.0244(10^6)b - 0.252(10^6) = 0$

Solve by cubic formula; $\left[\frac{0.252(10^6)}{2}\right]^2 < \left[\frac{0.0244(10^6)}{3}\right]^3$ so 3 real roots

$\cos u = \frac{q}{p\sqrt{p}}$ where $q = \frac{252(10^3)}{2} = 126(10^3)$, $p = \frac{24.4(10^3)}{3} = 8.13(10^3)$

$\cos u = 126(10^3)/[8.13(10^3)90.2] = 0.1718$, $u = 80.11°$

$b_1 = 2\sqrt{p}\cos u/3 = 2(90.2)(0.8933) = 161.1\ mm$ or $\underline{b = 161.1\ mm}$

$b_2 = 2\sqrt{p}\cos\left(\frac{u}{3} + 120°\right) = (-)$, $b_3 = 2\sqrt{p}\cos\left(\frac{u}{3} + 240°\right) = -(\ )$

$\boxed{A/59}$ $A = (30)(60) = 1800 \text{ mm}^2$ for each area.

$\bar{I}_{xy} = 0$ for each area, so $I_{xy} = 0 + A d_x d_y$.

(a) $I_{xy} = 50(40)(1800) = 360(10^4) \text{ mm}^4$

(b) $I_{xy} = 50(-40)(1800) = -360(10^4) \text{ mm}^4$

(c) $I_{xy} = (-50)(10)(1800) = \underline{-90(10^4) \text{ mm}^4}$

A/60

$$I_{xy} = -3(3)\left[(-3.5)(3.5) + (3.5)(-3.5)\right]$$
$$= \underline{220 \text{ in.}^4}$$

A/61

$$I_x = \frac{1}{12}(400)(200)^3 - 3\left[\frac{1}{4}\pi(30^4) + \pi(30)^2(50)^2\right]$$

$$= \underline{2.44(10^8)\ mm^4}$$

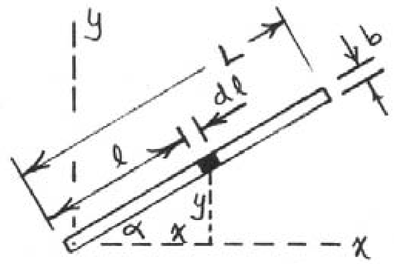
$$I_y = \frac{1}{12}(200)(400)^3 - 3\left[\frac{1}{4}\pi(30^4) + \pi(30)^2(100)^2\right]$$

$$= \underline{9.80(10^8)\ mm^4}$$

$$I_{xy} = -\pi(30)^2\left[(100)(50) + (-100)(50) + (-100)(-50)\right]$$

$$= \underline{-14.14(10^6)\ mm^4}$$

A/62



$$I_{xy} = \int xy\,dA = \int_0^L (l\cos\alpha)(l\sin\alpha)\,b\,dl$$

$$= b\sin\alpha\cos\alpha\int_0^L l^2\,dl$$

$$= \frac{bL^3}{3}\sin\alpha\cos\alpha \quad \text{or} \quad \underline{\frac{1}{6}bL^3\sin 2\alpha}$$

A/63

(a) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50)$
$= \underline{9.60(10^6)\ mm^4}$

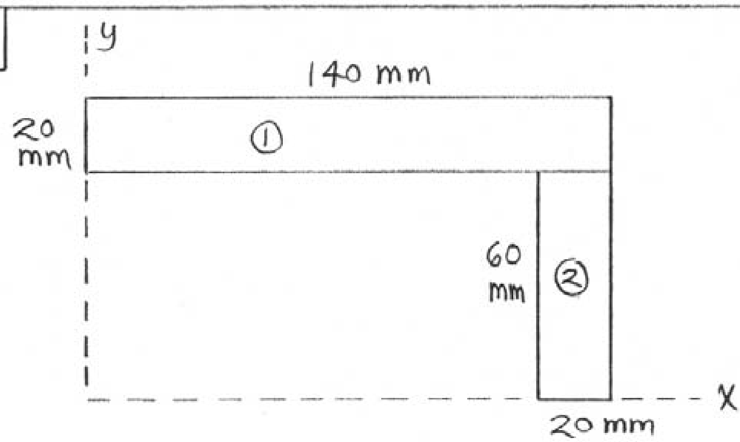
(b) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2)$
$= \underline{-4.71\ (10^6)\ mm^4}$

(c) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50)$
$= \underline{9.60\ (10^6)\ mm^4}$

(d) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left(-40 - \frac{4(25)}{3\pi}\right)$
$\times\ (\pi \cdot 25^2)/2$

$= \underline{-2.98(10^6)\ mm^4}$

A/64

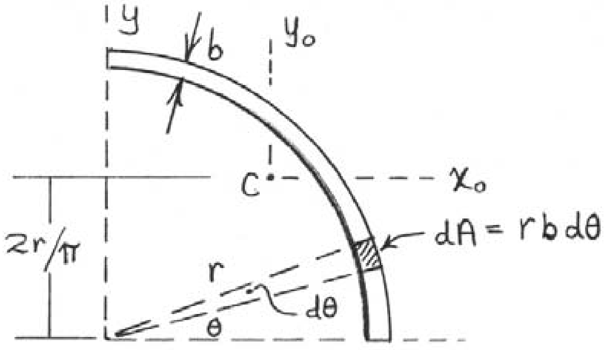


$$I_{xy_1} = 20(140)(70)(70) = 13.72(10^6)\ \text{mm}^4$$

$$I_{xy_2} = 60(20)(130)(30) = 4.68(10^6)\ \text{mm}^4$$

$$\text{Total}: \underline{I_{xy} = 18.40(10^6)\ \text{mm}^4}$$

A/65



$$I_{xy} = \int xy\,dA = \int_0^{\pi/2} (r\cos\theta)(r\sin\theta)\,rb\,d\theta$$

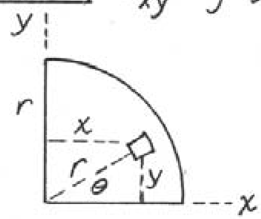
$$= r^3 b \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2} = \underline{\tfrac{1}{2} r^3 b}$$

$$I_{x_o y_o} = I_{xy} - d_x d_y A$$

$$= \tfrac{1}{2} r^3 b - \left(\frac{2r}{\pi}\right)\left(\frac{2r}{\pi}\right) rb\,\frac{\pi}{2} = \underline{\left(\frac{1}{2} - \frac{2}{\pi}\right) r^3 b}$$

A/66

$$I_{xy} = \int xy\,dA = \int_0^{\pi/2}\int_0^r (r\cos\theta)(r\sin\theta)\, r\,dr\,d\theta$$



$$= \int_0^{\pi/2} \frac{\sin 2\theta}{2}\frac{r^4}{4}\,d\theta = \frac{r^4}{16}\left(-\cos 2\theta\right)_0^{\pi/2}$$

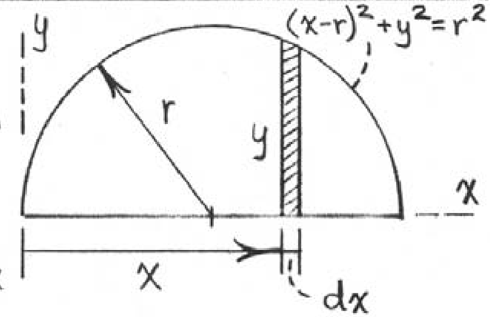
$$= \frac{r^4}{16}\left(1-[-1]\right) = \underline{r^4/8}$$

$$\bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{r^4}{8} - \frac{4r}{3\pi}\left(\frac{4r}{3\pi}\right)\frac{\pi r^2}{4} = \frac{r^4}{8}\left(1-\frac{32}{9\pi}\right)$$

$$= \underline{-0.01647\, r^4}$$

A/67

(1) By direct integration



For elemental strip,

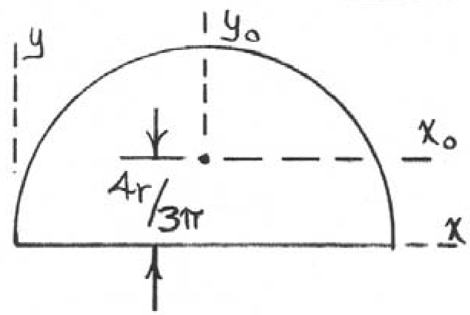
$$dI_{xy} = x\,\frac{y}{2}\,dA = \frac{xy}{2}\,y\,dx$$

$$= \frac{x}{2}\left[r^2 - (x-r)^2\right]dx$$

$$I_{xy} = \frac{1}{2}\int_0^{2r}\left(xr^2 - x^3 + 2rx^2 - r^2x\right)dx$$

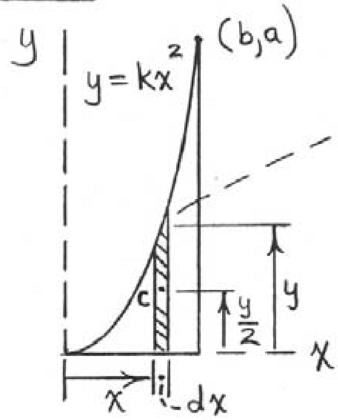
$$= \frac{1}{2}\left[\frac{x^2r^2}{2} - \frac{x^4}{4} + \frac{2rx^3}{3} - \frac{r^2x^2}{2}\right]_0^{2r} = \underline{\frac{2}{3}r^4}$$

(2) By axis transfer



$$I_{xy} = I_{x_0y_0} + A\,d_x\,d_y = 0 + \frac{\pi r^2}{2}(r)\left(\frac{4r}{3\pi}\right) = \underline{\frac{2}{3}r^4}$$

A/68



$y = kx^2$: $a = kb^2$, $k = a/b^2$

$\Rightarrow y = \frac{a}{b^2}x^2$

$dA = y\,dx = \frac{a}{b^2}x^2\,dx$

$d\bar{I}_{xy} = 0$

$dI_{xy} = d\bar{I}_{xy} + dA\,(x)\left(\frac{y}{2}\right)$

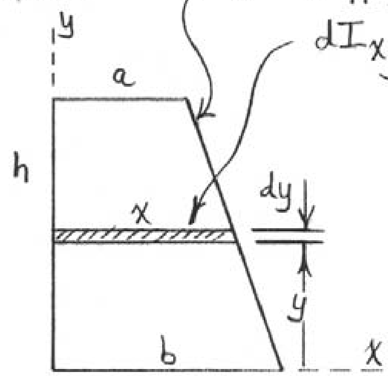
$= 0 + y\,dx\,(x)\left(\frac{y}{2}\right)$

$= \frac{1}{2}\frac{a^2}{b^4}x^5\,dx$

$I_{xy} = \int dI_{xy} = \int_0^b \frac{1}{2}\frac{a^2}{b^4}x^5\,dx$

$= \frac{1}{2}\frac{a^2}{b^4}\frac{x^6}{6}\Big|_0^b = \underline{\frac{1}{12}a^2b^2}$

A/69



$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dI_{xy} = x\,dy\left(\frac{x}{2}\right)(y) = x^2\frac{y}{2}\,dy$$

$$= \left[\left(\frac{a-b}{h}\right)y+b\right]^2\frac{y}{2}\,dy$$

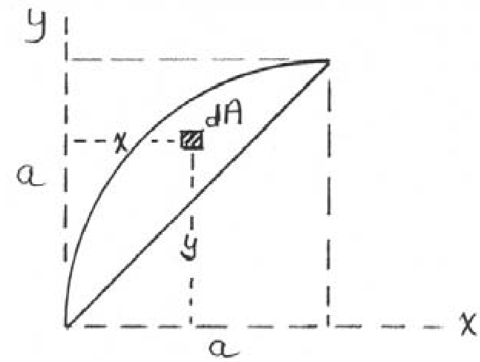
$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2y\right]dy$$

$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_0^h\left[\left(\frac{a-b}{h}\right)^2y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2y\right]dy$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2\frac{y^4}{4} + 2\left(\frac{a-b}{h}\right)b\,\frac{y^3}{3} + b^2\frac{y^2}{2}\right]_0^h$$

$$= \underline{\frac{h^2}{4}\left(\frac{a^2}{2} + \frac{ab}{3} + \frac{b^2}{6}\right)} \text{ or } \underline{\frac{h^2}{24}\left(3a^2 + 2ab + b^2\right)}$$

A/70



$$I_{xy} = \int xy\, dA$$
$$= \iint xy\, dx\, dy$$
$$= \int_0^a y \left[\int_{x_1}^{x_2} x\, dx \right] dy$$

where $x_2 = y$ and $(x_1 - a)^2 + y^2 = a^2$

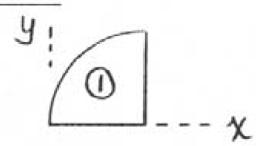
$$x_1 = a - \sqrt{a^2 - y^2}$$

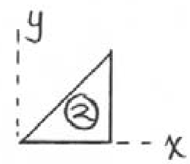
(note minus sign)

$$\int_{x_1}^{x_2} x\, dx = \frac{x^2}{2}\Big|_{a-\sqrt{a^2-y^2}}^{y} = y^2 - a^2 + a\sqrt{a^2-y^2}$$

So $$I_{xy} = \int_0^a y\left(y^2 - a^2 + a\sqrt{a^2-y^2}\right) dy$$
$$= \left[\frac{y^4}{4} - \frac{a^2y^2}{2} - \frac{a}{3}\sqrt{(a^2-y^2)^3}\right]_0^a$$
$$= \frac{a^4}{12}$$

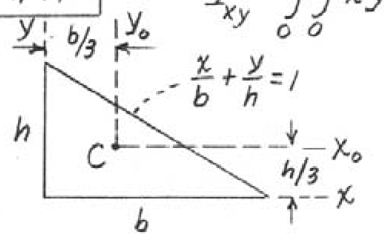
Alternatively





$$I_{xy} = (I_{xy})_1 - (I_{xy})_2$$

A/71



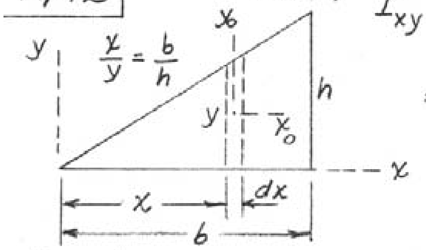
$$I_{xy} = \int_0^h \int_0^x xy\,dx\,dy = \int_0^h y\left(\frac{x^2}{2}\right)\Big]_0^{b(1-y/h)} dy$$

$$= \int_0^h \frac{b^2}{2}\left(y - \frac{2y^2}{h} + \frac{y^3}{h^2}\right) dy$$

$$= \frac{b^2}{2}\left[\frac{h^2}{2} - \frac{2h^2}{3} + \frac{h^2}{4}\right] = \underline{\frac{b^2h^2}{24}}$$

$$I_{x_0 y_0} = \bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{b^2h^2}{24} - \frac{bh}{2}\left(\frac{h}{3}\right)\left(\frac{b}{3}\right) = \underline{-\frac{b^2h^2}{72}}$$

A/72



First: $I_{xy} = \int_0^b \int_0^y xy\,dy\,dx$

$$= \int_0^b \frac{x}{2} y^2 \Big]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$$

$$= \frac{h^2}{2b^2}\,\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$$

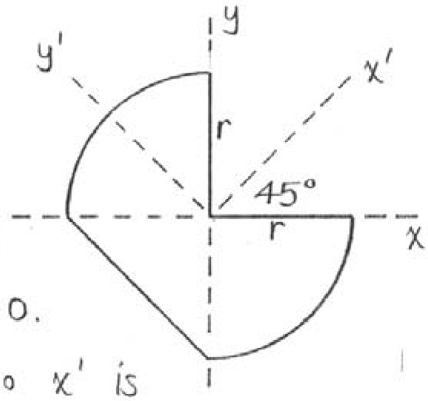
Second: $dI_{xy} = dI_{x_0 y_0} + d_x d_y\,(dA)$

$$= 0 + \frac{y}{2}\,x\,(y\,dx) = \frac{h^2}{2b^2} x^3 dx$$

$$I_{xy} = \frac{h^2}{2b^2}\int_0^b x^3 dx = \frac{h^2}{2b^2}\,\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$$

A/73

By inspection, x' is an axis of symmetry, so x' and y' are principal axes of inertia where $I_{x'y'} = 0$.



By inspection $I_{x'} > I_{y'}$ so x' is an axis of maximum inertia.

For each quarter-circular area,

$I_x = I_y = \frac{1}{16}\pi r^4$, $I_{xy} = -\frac{1}{8} r^4$ (from Prob. A/66)

For triangular area

$I_x = I_y = \frac{1}{12} r^4$, $I_{xy} = \frac{1}{24} r^4$ (from Prob. A/71 with $h = b = r$)

For Composite area

$$I_x = I_y = \left(\frac{1}{12} + 2\frac{\pi}{16}\right) r^4 = 0.476\, r^4$$

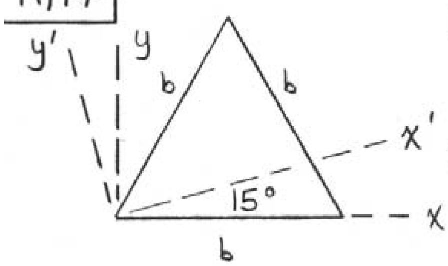
$$I_{xy} = \left(-\frac{1}{8}(2) + \frac{1}{24}\right) r^4 = -0.208\, r^4$$

Eq. A/11:

$$I_{max} = 0.476\, r^4 + \frac{1}{2}\sqrt{0 + 4(-0.208)^2 r^8}$$

$$= \underline{0.684\, r^4}$$

A/74



$$I_x = \frac{1}{12} b \left(\frac{b\sqrt{3}}{2}\right)^3 = \frac{\sqrt{3}}{32} b^4$$

$$\bar{I}_y = \bar{I}_x = \frac{\sqrt{3}}{32} b^4 - \frac{1}{2} b^2 \frac{\sqrt{3}}{2} \times \left(\frac{1}{3} b \frac{\sqrt{3}}{2}\right)^2 = \frac{\sqrt{3}}{96} b^4$$

$$I_y = \bar{I}_y + A d_y^2 = \frac{\sqrt{3}}{96} b^4 + b\left(\frac{b\sqrt{3}}{4}\right)\left(\frac{b}{2}\right)^2 = \frac{7\sqrt{3}}{96} b^4$$

$$I_{xy} = \bar{I}_{xy} + A d_x d_y = 0 + b\left(\frac{b\sqrt{3}}{4}\right)\left(\frac{b}{2}\right)\left(\frac{1}{3}\frac{\sqrt{3}}{2} b\right) = \frac{3}{48} b^4$$

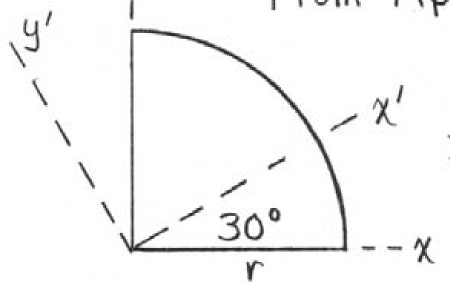
Eqs. A/9 & A/9a, with $\theta = 15°$:

$$I_{x'} = \frac{5\sqrt{3}}{96} b^4 - \frac{\sqrt{3}}{48} b^4 \cos 30° - \frac{3}{48} b^4 \sin 30° = \underline{0.0277\, b^4}$$

$$I_{y'} = \frac{5\sqrt{3}}{96} b^4 + \frac{\sqrt{3}}{48} b^4 \cos 30° + \frac{3}{48} b^4 \sin 30° = \underline{0.1527\, b^4}$$

$$I_{x'y'} = -\frac{\sqrt{3}}{48} b^4 \sin 30° + \frac{3}{48} b^4 \cos 30° = \underline{0.0361\, b^4}$$

A/75



From Appendix D: $I_x = I_y = \dfrac{\pi r^4}{16}$

From Prob. A/67:

$$I_{xy} = \frac{r^4}{8}$$

Eq. A/9: $I_{x'} = \dfrac{\pi r^4}{16} + 0 - \dfrac{r^4}{8}\sin 60° = \underline{\dfrac{r^4}{16}\left[\pi - \sqrt{3}\right]}$

$I_{y'} = \dfrac{\pi r^4}{16} - 0 + \dfrac{r^4}{8}\sin 60° = \underline{\dfrac{r^4}{16}\left[\pi + \sqrt{3}\right]}$

Eq. A/9a :

$I_{x'y'} = 0 + \dfrac{r^4}{8}\cos 60° = \underline{\dfrac{r^4}{16}}$

A/76 $I_x = \frac{1}{3}b(b^3) = \frac{1}{3}b^4$; $I_y = \frac{1}{3}b^4$

$$I_{xy} = 0 + \frac{b}{2}\frac{b}{2}b^2 = \frac{1}{4}b^4$$

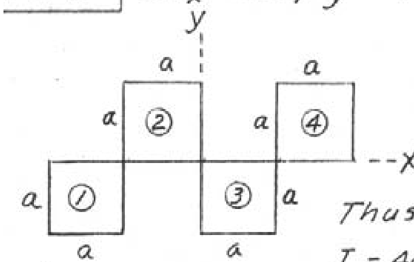
with $\theta = 30°$, Eqs. A/9 & A/9a give

$$I_{x'} = \frac{b^4}{3} + 0 - \frac{1}{4}b^4 \sin 60° = \left(\frac{1}{3} - \frac{\sqrt{3}}{8}\right)b^4 = \underline{0.1168\,b^4}$$

$$I_{y'} = \frac{b^4}{3} + 0 + \frac{1}{4}b^4 \sin 60° = \left(\frac{1}{3} + \frac{\sqrt{3}}{8}\right)b^4 = \underline{0.5498\,b^4}$$

$$I_{x'y'} = 0 + \frac{b^4}{4}\frac{1}{2} = \frac{b^4}{8} = \underline{0.1250\,b^4}$$

A/77 | ① $I_x = a^4/3$, $I_y = a^4/12 + a^2(3a/2)^2 = 7a^4/3$; $I_{xy} = +3a^4/4$



② $I_x = a^4/3$, $I_y = a^4/3$, $I_{xy} = -a^4/4$

③ same as ②

④ same as ①

Thus for composite area

$I_x = 4a^4/3$, $I_y = 16a^4/3$, $I_{xy} = +a^4$

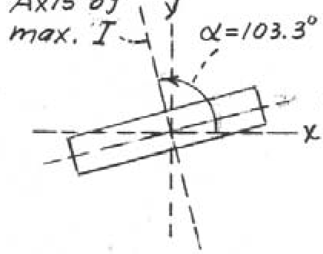
From Eq. A/11,

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} + \sqrt{5}\right)a^4 = \underline{5.57a^4}$$

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} - \sqrt{5}\right)a^4 = \underline{1.097a^4}$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2a^4}{(16/3 - 4/3)a^4} = +\frac{1}{2}$$

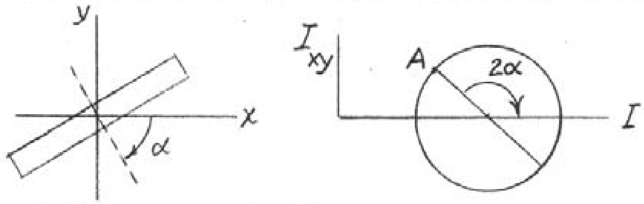


$2\alpha = 26.6°$ or $206.6°$

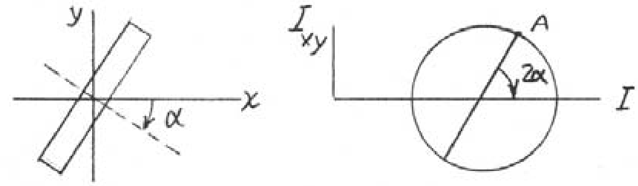
$\alpha = 13.3°$ or $\underline{103.3°}$

A/78

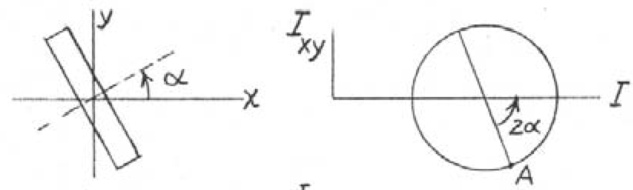
(a) $I_x < I_y$, I_{xy} (+)



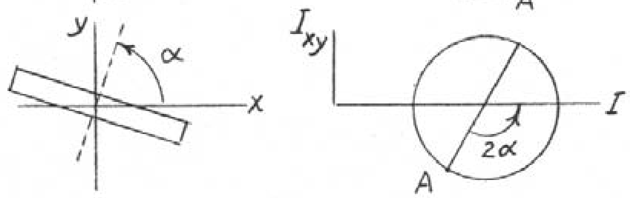
(b) $I_x > I_y$, I_{xy} (+)



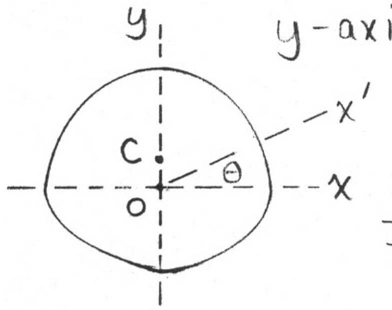
(c) $I_x > I_y$, I_{xy} (−)



(d) $I_x < I_y$, I_{xy} (−)

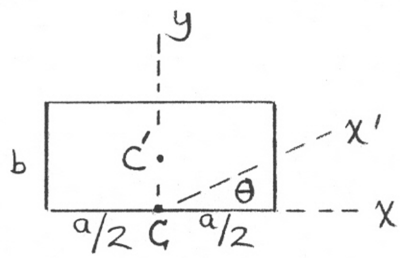


A/79 For an area symmetrical about the y-axis, $I_{xy} = 0$ and if $I_x = I_y$, Eq. A/9 becomes



$$I_{x'} = \frac{I_x + I_y}{2} + 0 - 0 = I_x$$

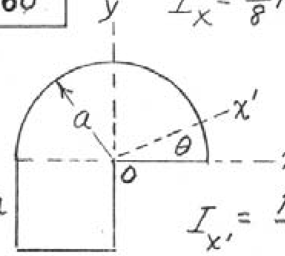
So $I_{x'}$ is independent of θ.



The rectangular area meets the condition of y-axis symmetry, so $I_{xy} = 0$. $I_x = I_y$

if $\frac{1}{3}ab^3 = \frac{1}{12}ba^3 \Rightarrow \underline{a = 2b}$.

A/80



$I_x = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4, \quad I_y = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4 = I_x$

$I_{xy} = 0 + a^2(-\frac{a}{2})(-\frac{a}{2}) = \frac{a^4}{4}$

From Eq. A/9

$I_{x'} = \frac{\pi a^4}{8} + \frac{a^4}{3} + 0 - \frac{a^4}{4}\sin 2\theta$

$= a^4(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4}\sin 2\theta)$

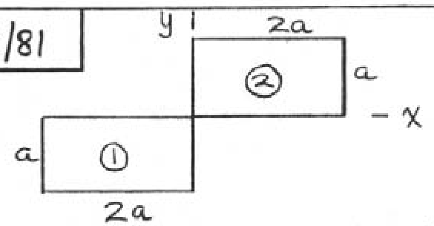
$\frac{dI_{x'}}{d\theta} = a^4(-\frac{1}{2}\cos 2\theta) = 0$ for max. or min.

so $2\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ so $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\underline{\theta = \pi/4}$, $I_{x'} = a^4(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4}[1]) = \underline{0.476a^4}$ (min)

$I_{y'} = a^4(\frac{\pi}{8} + \frac{1}{3} + \frac{1}{4}[1]) = \underline{0.976a^4}$ (max)

A/81



① $I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4, \quad I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$

$I_{xy} = (2a^2)(a)(a/2) = a^4$

② $I_x = \frac{2}{3}a^4, \quad I_y = \frac{8}{3}a^4, \quad I_{xy} = a^4$

Eq. A/11 : $I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{10}{3}a^4 - \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{0.505\,a^4}$

$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x^2 - I_y^2) + 4I_{xy}^2}$

$= \frac{10}{3}a^4 + \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{6.16\,a^4}$

Eq. A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4}$

$2\alpha = 45°$ or $225°$

$\alpha = 22.5°$ for I_{min}

or $\alpha = \underline{112.5°}$ for I_{max}

A/82 | From figure, I_{xy} is (−)

Add Eqs. A/11 & get $I_{max} + I_{min} = I_x + I_y$

so $I_x + I_y = (12+2)10^6 = 14(10^6)\ mm^4$

From the 1st of Eqs. A/11,

$$(I_x - I_y)^2 = [2I_{max} - (I_x + I_y)]^2 - 4I_{xy}^2$$

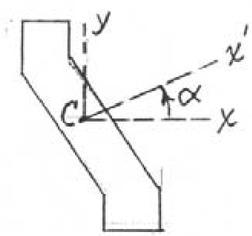
$$= [2(12) - 14]^2 10^{12} - 4(-4)^2 10^{12} = 36(10^{12})\ mm^8$$

$$\left.\begin{aligned} I_x - I_y &= 6(10^6)\ mm^4 \\ I_x + I_y &= 14(10^6)\ mm^4 \end{aligned}\right\} \text{add \& get } \underline{I_x = 10(10^6)\ mm^4}$$

$$\& \ \ I_y = 4(10^6)\ mm^4$$

From Eq. A/10,

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(-4)(10^6)}{-6(10^6)} = 4/3$$

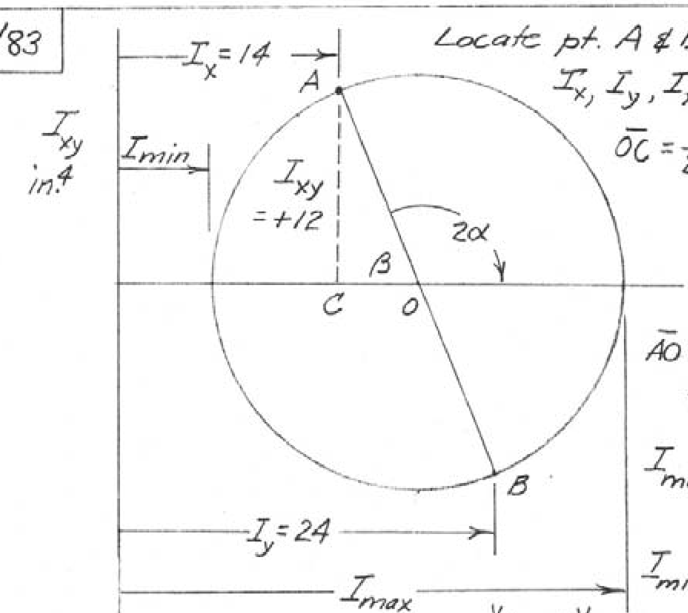


$2\alpha = 53.13°$, $\underline{\alpha = 26.6°}$

A/83

Locate pt. A & B from I_x, I_y, I_{xy} values

$\overline{OC} = \frac{1}{2}(24 - 14) = 5$ in.^{4}



$\overline{AO} = \sqrt{12^2 + 5^2}$

$= 13$ in.^{4}

$I_{max} = 14 + 5 + 13$

$\underline{= 32 \text{ in.}^4}$

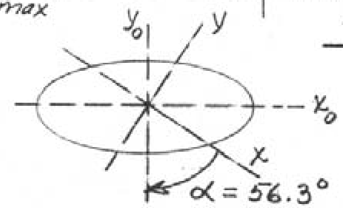
$I_{min.} = 14 + 5 - 13$

$\underline{= 6 \text{ in.}^4}$

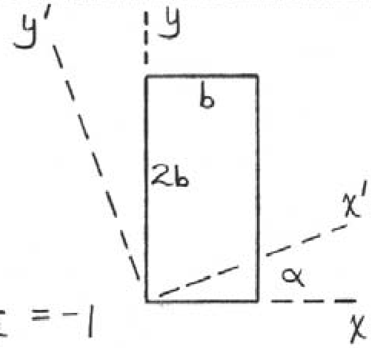
$\beta = \sin^{-1}\frac{12}{13} = 67.38°$,

$2\alpha = 180 - 67.38 = 112.6°$

$\underline{\alpha = 56.3°}$ Clockwise



A/84



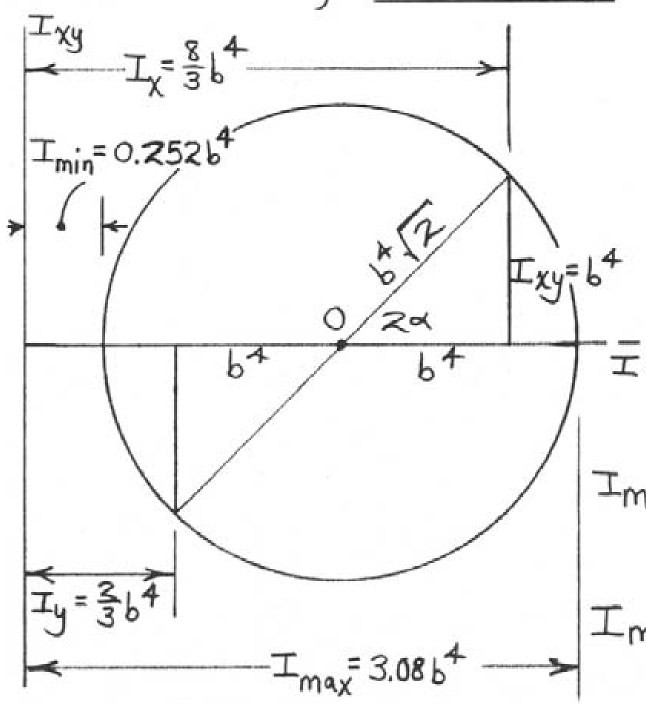
$I_x = \frac{1}{3} 2b^2 (2b)^2 = \frac{8}{3} b^4$

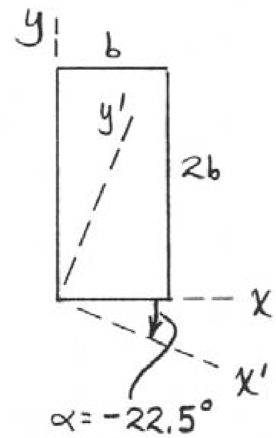
$I_y = \frac{1}{3} 2b^2 (b)^2 = \frac{2}{3} b^4$

$I_{xy} = 2b^2 \left(\frac{b}{2}\right)(b) = b^4$

$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2b^4}{\frac{2}{3}b^4 - \frac{8}{3}b^4} = -1$

$\Rightarrow 2\alpha = -45°$, $\underline{\alpha = -22.5°}$





$I_{max} = \frac{5}{3} b^4 + b^4\sqrt{2}$

$= \underline{3.08 b^4}$

$I_{min} = \frac{5}{3} b^4 - b^4\sqrt{2}$

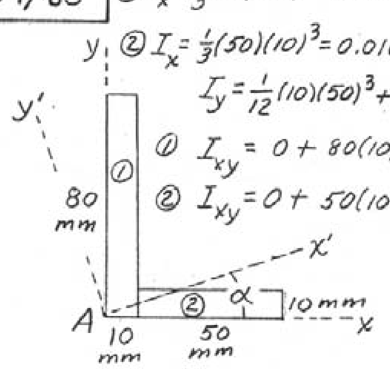
$= \underline{0.252 b^4}$

A/85

① $I_x = \frac{1}{3}(10)(80)^3 = 1.707(10^6)\ mm^4$, $I_y = \frac{1}{3}(80)(10)^3 = 0.0267(10^6)\ mm^4$

② $I_x = \frac{1}{3}(50)(10)^3 = 0.0167(10^6)\ mm^4$,

$I_y = \frac{1}{12}(10)(50)^3 + 10(50)(35)^2 = 0.7167(10^6)\ mm^4$



① $I_{xy} = 0 + 80(10)(40)(5) = 0.1600(10^6)\ mm^4$

② $I_{xy} = 0 + 50(10)(35)(5) = 0.0875(10^6)\ mm^4$

Totals: $I_x = 1.723(10^6)\ mm^4$

$I_y = 0.743(10^6)\ mm^4$

$I_{xy} = 0.248(10^6)\ mm^4$

From Eqs. A/11

$$I_{max} = \frac{I_x+I_y}{2} + \frac{1}{2}\sqrt{(I_x-I_y)^2+4I_{xy}^2},\quad I_{min} = \frac{I_x+I_y}{2} - \frac{1}{2}\sqrt{(I_x-I_y)^2+4I_{xy}^2}$$

$$I_{max} = \left[\frac{1.723+0.743}{2} + \frac{1}{2}\sqrt{(1.723-0.743)^2+4(0.248)^2}\right]10^6 = \underline{1.782(10^6)\ mm^4}$$

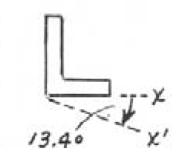
$$I_{min} = \left[\frac{1.723+0.743}{2} - \frac{1}{2}\sqrt{(1.723-0.743)^2+4(0.248)^2}\right]10^6 = \underline{0.684(10^6)\ mm^4}$$

From Eq. A/10

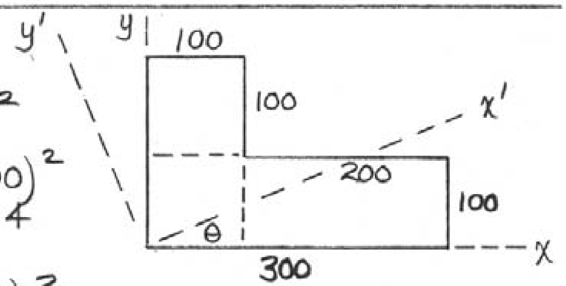
$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(0.248)}{0.743-1.723} = -0.5051$$

$2\alpha = -26.8°$

$\underline{\alpha = -13.40°}$



*A/86



$$I_x = \frac{1}{3}(100\cdot 200)(100)^2 + \frac{1}{3}(100\cdot 200)(200)^2 = 3.33(10^8)\ \text{mm}^4$$

(Dim. in mm)

$$I_y = \frac{1}{3}(100\cdot 300)(300)^2 + \frac{1}{3}(100\cdot 100)(100)^2 = 9.33(10^8)\text{mm}^4$$

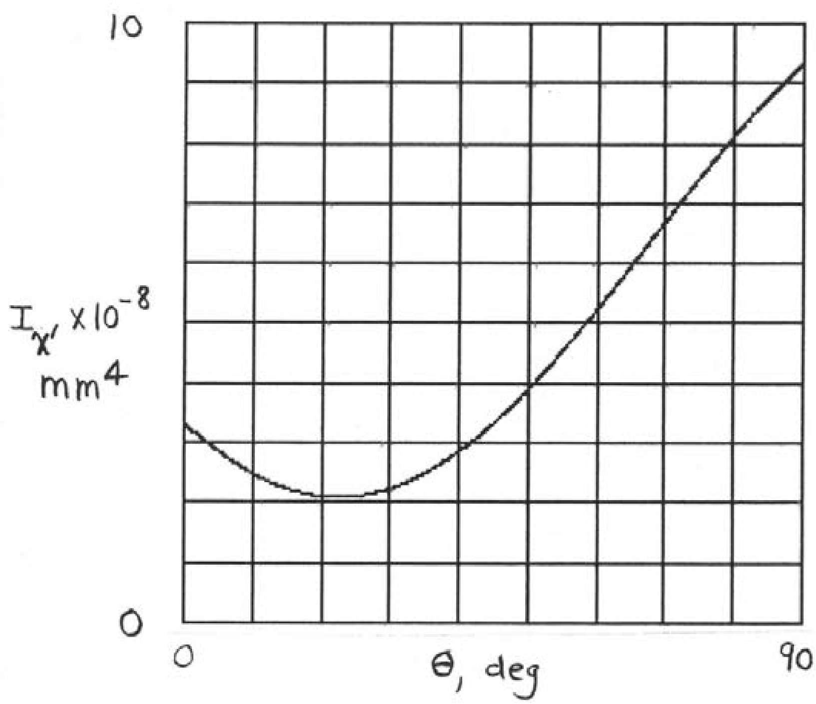
$$I_{xy} = (100\cdot 200)(50)(100) + (100\cdot 200)(200)(50) = 3(10^8)\text{mm}^4$$

Eq. A/9: $I_{x'} = \left\{\frac{3.33+9.33}{2} + \frac{3.33-9.33}{2}\cos 2\theta - 3\sin 2\theta\right\}10^8$

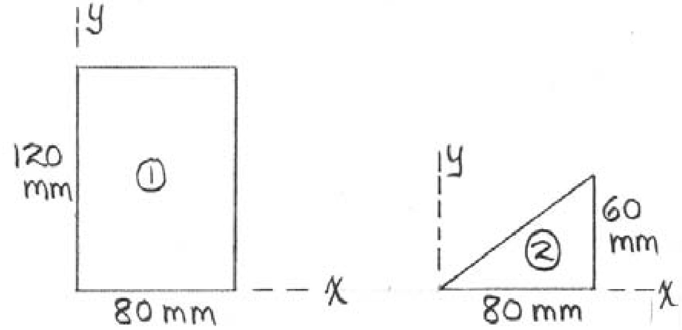
$$= \left\{6.33 - 3\cos 2\theta - 3\sin 2\theta\right\}10^8\ \text{mm}^4$$

$$\frac{dI_{x'}}{d\theta} = 6\left\{\sin 2\theta - \cos 2\theta\right\}10^8\ \text{mm}^4/\text{rad}$$

Set $\frac{dI_{x'}}{d\theta} = 0$ to find $(I_{x'})_{min} = 2.09(10^8)\text{mm}^4$
@ $\theta = 22.5°$



*A/87



① $I_x = \frac{1}{3}(80)(120)^3 = 46.1(10^6)\ mm^4$

$I_y = \frac{1}{3}(120)(80)^3 = 20.5\ (10^6)\ mm^4$

$I_{xy} = 80(120)(40)(60) = 23.0(10^6)\ mm^4$

② $I_x = -\frac{1}{12}(80)(60)^3 = -1.440\ (10^6)\ mm^4$

$I_y = -\frac{1}{4}(60)(80)^3 = -7.68\ (10^6)\ mm^4$

$I_{xy} = -\frac{b^2h^2}{8} = -\frac{80^2 60^2}{8} = -2.88(10^6)\ mm^4$

(See Prob. A/72)

So for the composite body:

$$\begin{cases} I_x = (46.1 - 1.440)10^6 = 44.6(10^6)\ mm^4 \\ I_y = (20.5 - 7.68)10^6 = 12.80\ (10^6)\ mm^4 \\ I_{xy} = (23.0 - 2.88)10^6 = 20.2\ (10^6)\ mm^4 \end{cases}$$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

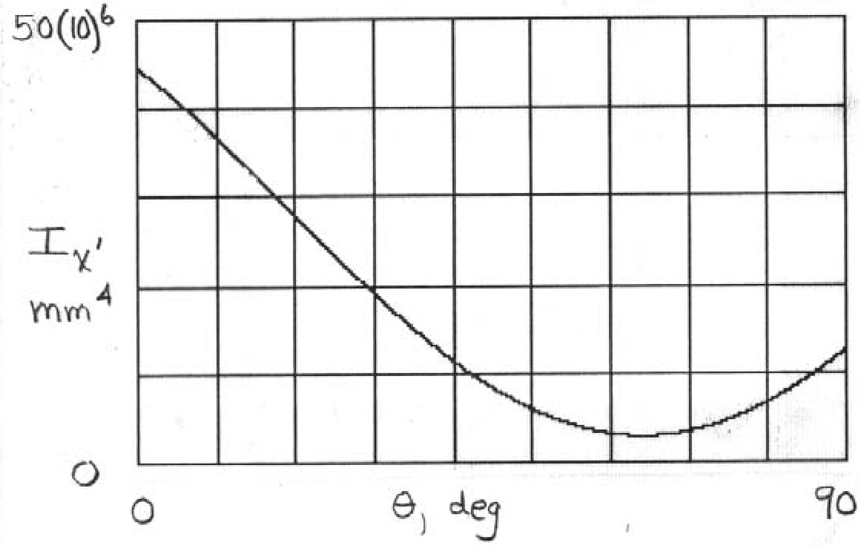
(See plot below)

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

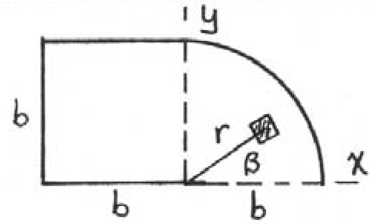
$$= \left\{\frac{44.6 + 12.80}{2} - \frac{1}{2}\sqrt{(44.6 - 12.80)^2 + 4(20.2)^2}\right\}10^6$$

$$= \underline{3.03\,(10^6)\ \text{mm}^4}$$

$$\tan 2\theta_{cr} = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.2)}{12.80 - 44.6}, \quad \underline{\theta_{cr} = 64.1^\circ}$$



* A/88



$$I_x = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$$

$$I_y = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$$

Quarter circle: $I_{xy} = \int_0^{\pi/2}\int_0^b (r\cos\beta)(r\sin\beta)\,r\,dr\,d\beta$

$$= \frac{r^4}{4}\Big|_0^b \times \left(-\frac{1}{4}\cos 2\beta\right)\Big|_0^{\pi/2} = \frac{b^4}{4}\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{b^4}{8}$$

Square: $I_{xy} = b^2\left(-\frac{b}{2}\right)\left(\frac{b}{2}\right) = -\frac{b^4}{4} = -0.25\,b^4$

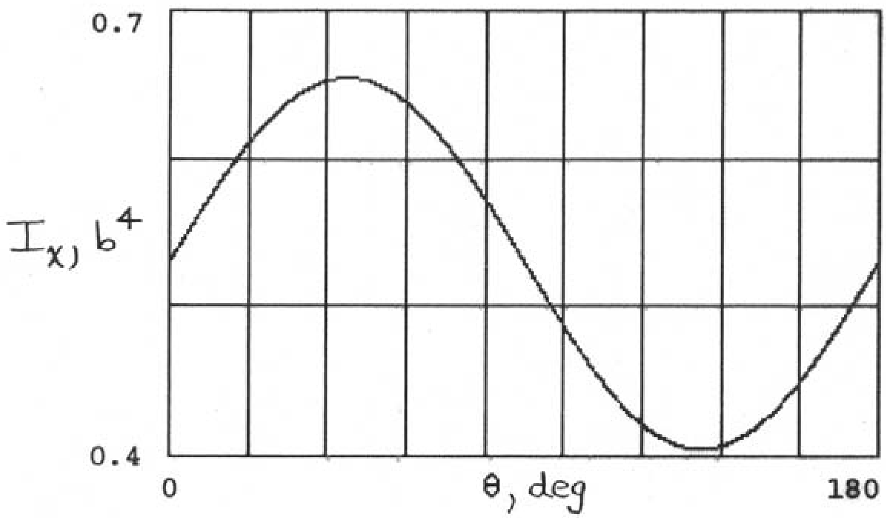
Combined: $I_{xy} = \frac{b^4}{8} - \frac{b^4}{4} = -\frac{b^4}{8} = -0.125\,b^4$

Eq. A/9: $I_{x'} = \frac{2(0.530\,b^4)}{2} + 0 - (-0.125b^4)\sin 2\theta$

$$= (0.530 + 0.125\sin 2\theta)\,b^4$$

For critical angle $\theta = \alpha$, Eq. A/10 gives

$$\tan 2\alpha = \frac{2(0.530\,b^4)}{0}, \quad 2\alpha = \frac{\pi}{2}, \quad \alpha = \frac{\pi}{4}$$



$$I_{max} = 0.655\,b^4 \quad @ \quad \theta = 45^\circ$$

$$I_{min} = 0.405\,b^4 \quad @ \quad \theta = 135^\circ$$

Eqs. A/11:

$$I_{max} = 0.530\,b^4 + \frac{1}{2}\sqrt{0^2 + 4\left(-0.125 b^4\right)^2}$$

$$= 0.655\,b^4 \quad \checkmark$$

$$I_{min} = 0.530\,b^4 - \frac{1}{2}\sqrt{0^2 + 4\left(-0.125 b^4\right)^2}$$

$$= 0.405\,b^4 \quad \checkmark$$

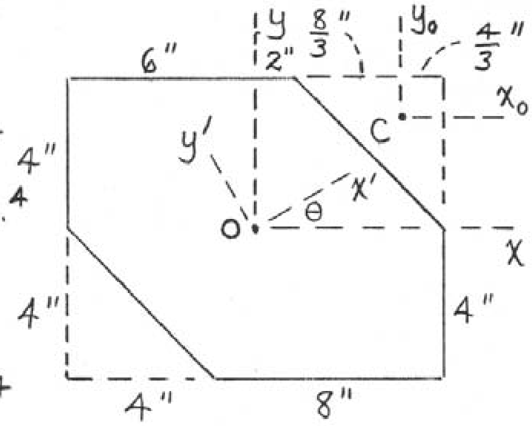
*** A/89**

Complete rectangle:

$I_x = \frac{1}{12}(8 \cdot 12)8^2 = 512 \text{ in.}^4$

$I_y = \frac{1}{12}(8 \cdot 12)12^2 = 1152 \text{ in.}^4$

$I_{xy} = 0$



Each dashed triangle:

$I_x = \frac{1}{4}(4)(4^3) = 64 \text{ in.}^4$
(see Sample Problem A/2)

$I_y = \frac{1}{36}4(4)^3 + \frac{1}{2}(4)(4)\left(2+\frac{8}{3}\right)^2 = 181.3 \text{ in.}^4$
(see Sample Problem A/2 and Eq. A/6)

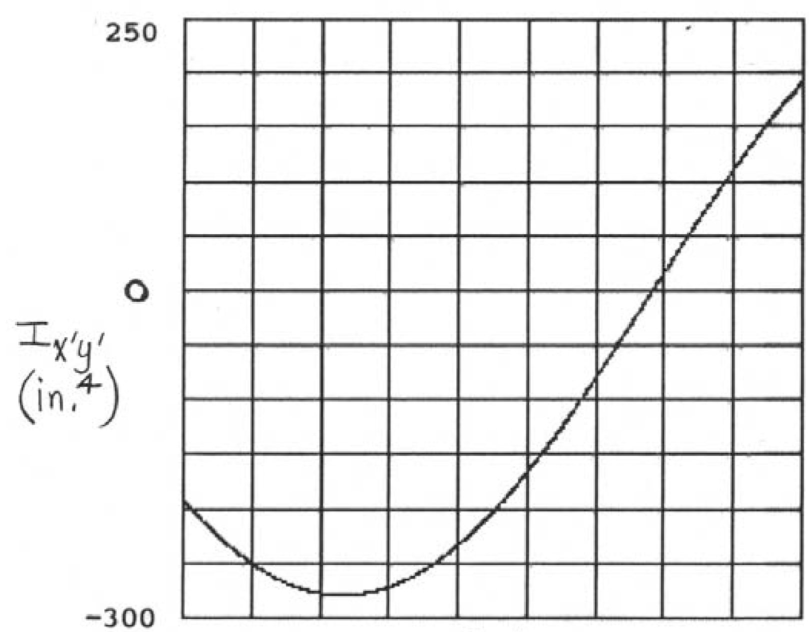
$I_{xy} = -\frac{4^4}{72} + \frac{1}{2}4^2\left(2+\frac{8}{3}\right)\left(\frac{8}{3}\right) = 96 \text{ in.}^4$
(see Prob. A/69 and Eq. A/8)

Totals: $I_x = 384 \text{ in.}^4$ $\quad I_{xy} = -2(96)$

$I_y = 789 \text{ in.}^4$ $\quad = -192 \text{ in.}^4$

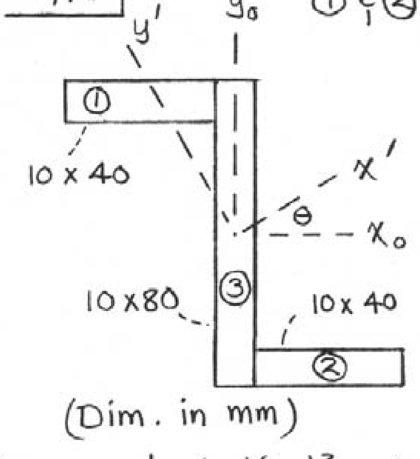
Eq. A/9a: $I_{x'y'} = \frac{384-789}{2}\sin 2\theta - 192\cos 2\theta$

$= \underline{-203 \sin 2\theta - 192 \cos 2\theta}$



$I_{x'y'} = 0$ @ $\underline{\theta = 68.3°}$

*A/90



① & ②: $I_{x_o y_o} = \bar{I}_{xy} + d_x d_y A$

$= 0 + (35)(-25)(10 \times 40)$

$= -0.350(10^6)$ mm^4

③: $I_{x_o y_o} = 0$

① & ②: $I_{x_o} = \frac{1}{12}(40)(10^3)$

$+ (10 \times 40)\, 35^2 = 0.493(10^6)$ mm^4

$I_{y_o} = \frac{1}{12}(10)(40)^3 + (10 \times 40)\, 25^2 = 0.303(10^6)$ mm^4

③: $I_{x_o} = \frac{1}{12}(10)(80)^3 = 0.427(10^6)$ mm^4

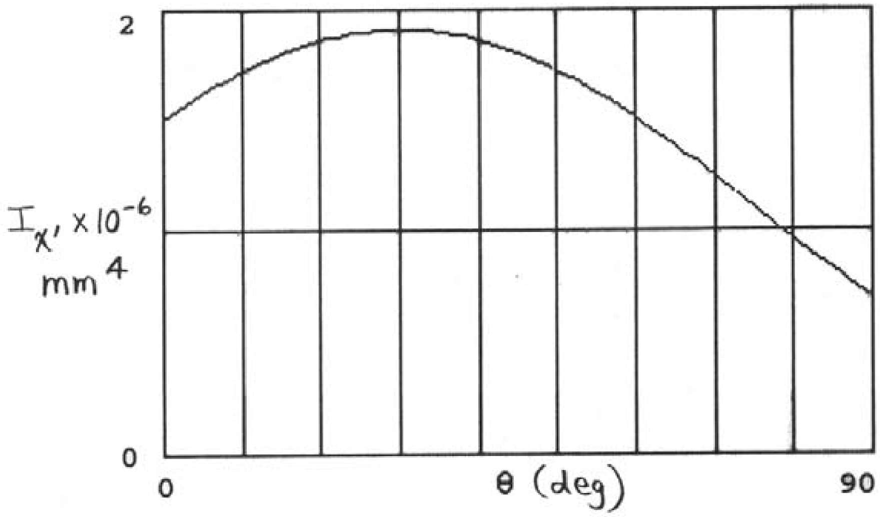
$I_{y_o} = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)$ mm^4

Totals: $\begin{cases} I_{x_o} = 1.413\,(10^6) \text{ mm}^4 \\ I_{y_o} = 0.613\,(10^6) \text{ mm}^4 \\ I_{x_o y_o} = -0.700\,(10^6) \text{ mm}^4 \end{cases}$

From Eq. A/9:

$$I_{x'} = \left[\frac{1.413 + 0.613}{2} + \frac{1.413 - 0.613}{2}\cos 2\theta + 0.700 \sin 2\theta\right] 10^6$$

$$= \left[1.013 + 0.4 \cos 2\theta + 0.7 \sin 2\theta\right] 10^6$$



Eq. (A/11): $I_{max} = \dfrac{I_x + I_y}{2} + \dfrac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \dfrac{1.413 + 0.613}{2} 10^6 + \dfrac{1}{2}\sqrt{(1.413 - 0.613)^2(10^6)^2 + 4(-0.7 \times 10^6)^2}$

$= \underline{1.820(10^6)\ mm^4}$

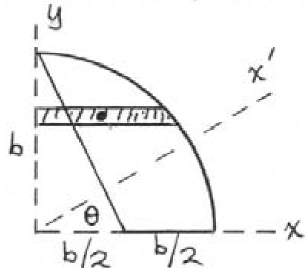
Eq. (A/10): $\tan 2\alpha = \dfrac{2I_{xy}}{I_y - I_x}$

$= \dfrac{2(-0.7)}{0.613 - 1.413}$

$\Rightarrow \alpha = \underline{30.1^\circ}, 120.1^\circ$

(Values from Eqs. A/10 & A/11 agree with plot.)

*A/91 Quarter circle $I_x = I_y = \frac{1}{16}\pi r^4$



For horizontal strip:

$$dI_{xy} = 0 + (x\,dy)\,y\,\frac{x}{2} = \frac{b^2y - y^3}{2}dy$$

$$I_{xy} = \frac{1}{2}\int_0^b (b^2y - y^3)\,dy = \frac{b^4}{8}$$

Triangle $I_x = \frac{1}{12}\frac{b}{2}b^3 = \frac{b^4}{24}$

$$I_y = \frac{1}{12}b\left(\frac{b}{2}\right)^3 = \frac{b^4}{96}, \quad I_{xy} = \frac{1}{24}\left(\frac{b}{2}\right)^2 b^2 = \frac{b^4}{96}$$

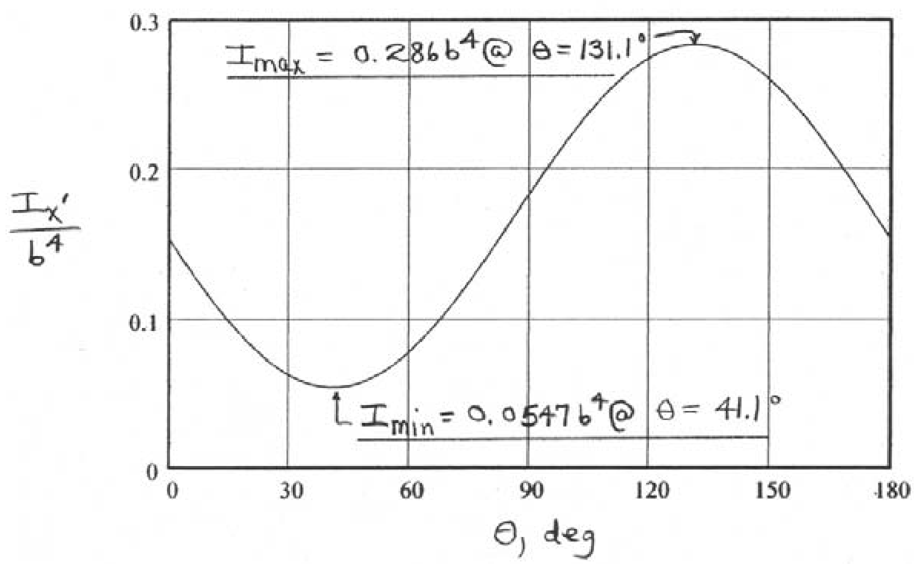
(Prob. A/71)

Composite
$$\begin{cases} I_x = \left(\frac{\pi}{16} - \frac{1}{24}\right)b^4 = 0.1547\,b^4 \\ I_y = \left(\frac{\pi}{16} - \frac{1}{96}\right)b^4 = 0.1859\,b^4 \\ I_{xy} = \left(\frac{1}{8} - \frac{1}{96}\right)b^4 = 0.1146\,b^4 \end{cases}$$

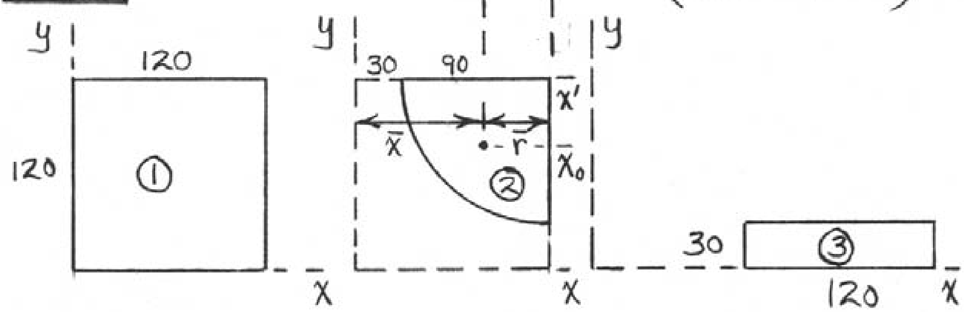
From Eq. A/9:

$$I_{x'} = \left[\frac{0.1547 + 0.1859}{2} + \frac{0.1547 - 0.1859}{2}\cos 2\theta - 0.1146\sin 2\theta\right]b^4$$

$$= \left[0.1703 - 0.01562\cos 2\theta - 0.1146\sin 2\theta\right]b^4$$



*A/92 (Dim. in mm)



① $I_x = I_y = \frac{1}{3}\,120^4 = 69.1(10^6)\text{ mm}^4, \quad A = 120^2 = 1.44(10^4)\text{ mm}^2$

$I_{xy} = 120^2(60)(60) = 51.8\text{ mm}^4$

② $\bar{r} = \frac{4r}{3\pi} = \frac{4(90)}{3\pi} = 38.2\text{ mm}, \quad \bar{x} = 120 - \bar{r} = 81.8\text{ mm}$

$A = \frac{1}{4}\pi(90^2) = 0.636(10^4)\text{ mm}^2$

$I_{x'} = I_{y'} = \frac{1}{4}\left(\frac{1}{4}\pi \times 90^4\right) = 12.88(10^6)\text{ mm}^4$

$I_x = I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2$

$= \left[12.88 + 0.00636(81.8^2 - 38.2^2)\right]10^6 = 46.2(10^6)\text{ mm}^4$

$I_{xy} = I_{x_0y_0} + A\bar{x}\bar{y} = I_{x'y'} - A\bar{r}\bar{r} + A\bar{x}\bar{y}$

where $I_{x'y'} = \frac{r^4}{8} = \frac{90^4}{8} = 8.20(10^6)\text{ mm}^4$ $\left\{\begin{array}{l}\text{See Sol. to}\\ \text{Prob. A/65}\end{array}\right.$

$\therefore I_{xy} = \left[8.20 + 0.00636(81.8^2 - 38.2^2)\right]10^6 = 41.5(10^6)\text{ mm}^4$

③ $I_x = \frac{1}{3}(120)(30)^3 = 1.08(10^6)\ mm^4$

$I_y = \frac{1}{12}(30)(120)^3 + 30(120)(180^2) = 121.0\ (10^6)\ mm^4$

$I_{xy} = 30(120)(180)(15) = 9.72\ (10^6)\ mm^4$

Combined: $\begin{cases} I_x = (69.1 - 46.2 + 1.08)\ 10^6 = 24.0(10^6)\ mm^4 \\ I_y = (69.1 - 46.2 + 121.0)10^6 = 143.9(10^6)\ mm^4 \\ I_{xy} = (51.8 - 41.5 + 9.72)10^6 = 20.1(10^6)\ mm^4 \end{cases}$

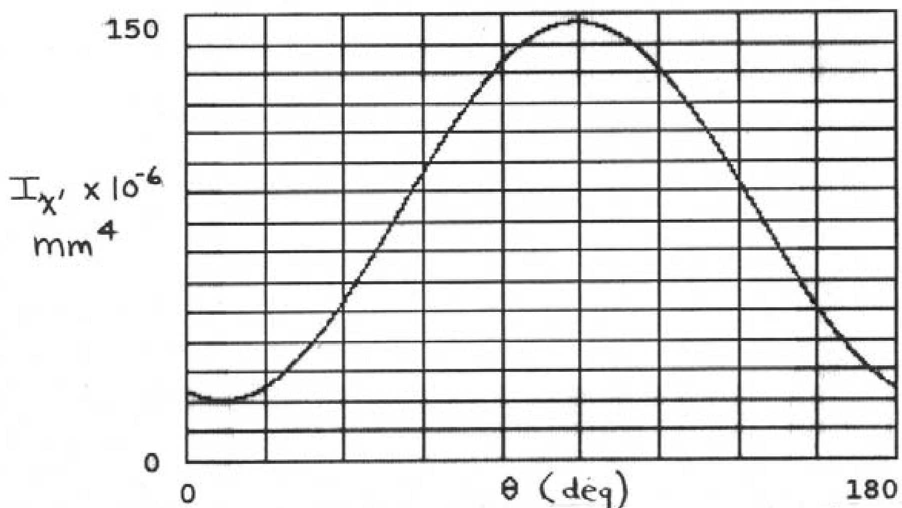
A/9: $I_{x'} = \left\{\frac{24.0 + 143.9}{2} + \frac{24.0 - 143.9}{2}\cos 2\theta - 20.1 \sin 2\theta\right\} \times 10^6$

$I_{x'}(10^{-6}) = 84.0 - 59.9 \cos 2\theta - 20.1 \sin 2\theta \quad mm^4$

A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.1)}{143.9 - 24.0} = 0.335$

$2\alpha = 18.51°, \quad \alpha = 9.26°$ (minimum I)

$\alpha = 9.26 + 90 = 99.3°$ (maximum I)



$I_{min} = 20.8(10^6)\ mm^4$ @ $\theta = 9.26°$ $\qquad$ $I_{max} = 147.2\ (10^6)\ mm^4$ @ $\theta = 99.3°$