Chapter 9, Solution 1.

First note: $$y = \frac{b_2 - b_1}{a}x + b_1$$

Have $$I_y = \int x^2 dA$$

$$= \int_0^a \int_0^{\frac{b_2 - b_1}{a}x + b_1} x^2 dy dx$$

$$= \int_0^a x^2 \left(\frac{b_2 - b_1}{a}x + b_1 \right) dx$$

$$= \left(\frac{1}{4}\frac{b_2 - b_1}{a}x^4 + \frac{1}{3}b_1 x^3 \right)\Bigg|_0^a$$

$$= \frac{1}{12}a^3 \left(b_1 + 3b_2 \right)$$

$$I_y = \frac{1}{12}a^3 \left(b_1 + 3b_2 \right) \blacktriangleleft$$

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Chapter 9, Solution 2.

At $x = a,\ y = b$: $\quad b = ka^{\frac{5}{2}} \quad$ or $\quad k = \dfrac{b}{a^{\frac{5}{2}}}$

$$\therefore\ y = \frac{b}{a^{\frac{5}{2}}}x^{\frac{5}{2}} \quad \text{or} \quad x = \frac{a}{b^{\frac{2}{5}}}y^{\frac{2}{5}}$$

$$dI_y = \frac{1}{3}x^3 dy$$

$$= \frac{1}{3}\frac{a^3}{b^{\frac{6}{5}}}y^{\frac{6}{5}}dy$$

Then

$$I_y = \frac{1}{3}\frac{a^3}{b^{\frac{6}{5}}}\int_0^b y^{\frac{6}{5}}dy$$

$$= \frac{1}{3}\frac{5}{11}\frac{a^3}{b^{\frac{6}{5}}}y^{\frac{11}{5}}\Big|_0^b$$

$$= \frac{5}{33}\frac{a^3}{b^{\frac{6}{5}}}b^{\frac{11}{5}}$$

or $I_y = \dfrac{5}{33}a^3 b$ ◀

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Chapter 9, Solution 3.

First note: At $x = 0$: $0 = k(b + c)^2$

or $c = -b$

At $x = a$: $a = k(2b - b)^2$

or $k = \dfrac{a}{b^2}$

$$\therefore\ x = \frac{a}{b^2}(y - b)^2$$

Have

$$I_y = \int x^2 dA$$

$$= \int_0^{2b} \int_0^{\frac{a}{b^2}(y-b)^2} x^2 dx dy$$

$$= \frac{1}{3}\int_0^{2b} \left[\frac{a}{b^2}(y - b)^2\right]^3 dy$$

$$= \frac{1}{3}\frac{a^3}{b^6} \times \frac{1}{7}(y - b)^7 \Big|_b^{2b}$$

$$= \frac{1}{21}a^3 b$$

$$I_y = \frac{1}{21}a^3 b \blacktriangleleft$$

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Chapter 9, Solution 4.

Have $$y = kx^2 + c$$

At $$x = 0,\quad y = b:\quad b = k(0) + c$$

or $$c = b$$

At $$x = 2a,\quad y = 0:\quad 0 = k(2a)^2 + b$$

or $$k = -\frac{b}{4a^2}$$

Then $$y = -\frac{b}{4a^2}x^2 + b$$

$$= \frac{b}{4a^2}\left(4a^2 - x^2\right)$$

Then $$I_y = \int x^2 dA,\qquad dA = ydx = \frac{b}{4a^2}\left(4a^2 - x^2\right)dx$$

$$I_y = \int_a^{2a} x^2 dA = \frac{b}{4a^2}\int_a^{2a} x^2\left(4a^2 - x^2\right)dx$$

$$= \frac{b}{4a^2}\left[4a^2\frac{x^3}{3} - \frac{x^5}{5}\right]_a^{2a}$$

$$= \frac{b}{3}\left(8a^3 - a^3\right) - \frac{b}{20a^2}\left(32a^5 - a^5\right)$$

$$= \frac{7a^3b}{3} - \frac{31a^3b}{20}$$

$$I_y = \frac{47}{60}a^3b \blacktriangleleft$$

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Chapter 9, Solution 5.

First note: $$y = \frac{b_2 - b_1}{a}x + b_1$$

Have $$I_x = \int y^2 dA$$

$$= \int_0^a \int_0^{\frac{b_2-b_1}{a}x+b_1} y^2 dy dx$$

$$= \frac{1}{3}\int_0^a \left(\frac{b_2 - b_1}{a}x + b_1\right)^3 dx$$

$$= \frac{1}{3} \times \left(\frac{1}{4}\frac{a}{b_2 - b_1}\right)\left(\frac{b_2 - b_1}{a}x + b_1\right)^4 \Bigg|_0^a$$

$$= \frac{1}{12}\frac{a}{b_2 - b_1}\left(b_2^4 - b_1^4\right)$$

$$= \frac{1}{12}\frac{a}{b_2 - b_1}(b_2 + b_1)(b_2 - b_1)\left(b_2^2 + b_1^2\right)$$

$$= \frac{1}{12}a(b_1 + b_2)\left(b_1^2 + b_2^2\right)$$

$$I_x = \frac{1}{12}a(b_1 + b_2)\left(b_1^2 + b_2^2\right) \blacktriangleleft$$

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Chapter 9, Solution 6.

SOLUTION

At $x = a,\ y = b$: $\quad b = ka^{\frac{5}{2}}$

or $\quad k = \dfrac{b}{a^{\frac{5}{2}}}$

$$\therefore\ y = \frac{b}{a^{\frac{5}{2}}}\, x^{\frac{5}{2}}$$

$$I_x = \int y^2 dA \qquad dA = x dy$$

$$= \int_0^b y^2 \left[\frac{a}{b^{\frac{2}{5}}} y^{\frac{2}{5}} dy \right]$$

$$= \frac{a}{b^{\frac{2}{5}}} \times \frac{5}{17} y^{\frac{17}{5}} \Bigg|_0^b = \frac{5a}{17} \frac{b^{\frac{17}{5}}}{b^{\frac{2}{5}}}$$

or $\ I_x = \dfrac{5}{17} ab^3$ ◀

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Chapter 9, Solution 7.

First note: At $x = 0$: $0 = k(b+c)^2$

or $c = -b$

At $x = a$: $a = k(2b-b)^2$

or $k = \dfrac{a}{b^2}$

$\therefore$ $x = \dfrac{a}{b^2}(y-b)^2$

Have

$$I_x = \int y^2 dA$$

$$= \int_b^{2b}\int_0^{\frac{a}{b^2}(y-b)^2} y^2 dx dy$$

$$= \frac{a}{b^2}\int_b^{2b} y^2 (y-b)^2 dy$$

$$= \frac{a}{b^2}\int_b^{2b}\left(y^4 - 2by^3 + b^2y^2\right)dy$$

$$= \frac{a}{b^2}\left(\frac{1}{5}y^5 - \frac{1}{2}by^4 + \frac{1}{3}b^2y^3\right)\Bigg|_b^{2b}$$

$$= \frac{a}{b^2}\left\{\left[\frac{1}{5}(2b)^5 - \frac{1}{2}(2b)^4 + \frac{1}{3}b^2(2b)^3\right] - \left[\frac{1}{5}b^5 - \frac{1}{2}b\left(b^4\right) + \frac{1}{3}b^2\left(b^3\right)\right]\right\}$$

$$= ab^3\left(\frac{32}{5} - 8 + \frac{8}{3} - \frac{1}{5} + \frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{31}{30}ab^3$$

$$I_x = \frac{31}{30}ab^3 \blacktriangleleft$$

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Chapter 9, Solution 8.

Have $y = kx^2 + c$

At $x = 0, \quad y = b: \quad b = k(0) + c$

or $c = b$

At $x = 2a, \quad y = 0: \quad 0 = k(2a)^2 + b$

or $k = -\dfrac{b}{4a^2}$

Then $y = \dfrac{b}{4a^2}\left(4a^2 - x^2\right)$

Now $dI_x = \dfrac{1}{3}y^3 dx$

$$= \frac{1}{3}\frac{b^3}{64a^6}\left(4a^2 - x^2\right)^3 dx$$

continued

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Then

$$I_x = \int dI_x$$

$$= \frac{1}{3}\frac{b^3}{64a^6}\int_a^{2a}\left(4a^2 - x^2\right)^3 dx$$

$$= \frac{b^3}{192a^6}\int_a^{2a}\left(64a^6 - 48a^4x^2 + 12a^2x^4 - x^6\right)dx$$

$$= \frac{b^3}{192a^6}\left[64a^6x - 16a^4x^3 + \frac{12}{5}a^2x^5 - \frac{x^7}{7}\right]_a^{2a}$$

$$= \frac{b^3}{192a^6}\left[64a^7(2-1) - 16a^7(8-1)\right.$$

$$\left. + \frac{12}{5}a^7(32-1) - \frac{1}{7}(128-1)\right]$$

$$= \frac{ab^3}{192}\left(64 - 112 + \frac{372}{5} - \frac{127}{7}\right) = 0.043006ab^3$$

$$I_x = 0.0430ab^3 \blacktriangleleft$$

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Chapter 9, Solution 9.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x = a\sqrt{1 - \frac{y^2}{b^2}}$$

$$dA = xdy$$

$$dI_x = y^2 dA = y^2 xdy$$

$$I_x = \int dI_x = \int_{-b}^{b} xy^2 dy = a\int_{-b}^{b} y^2 \sqrt{1 - \frac{y^2}{b^2}} dy$$

Set: $y = b\sin\theta \qquad dy = b\cos\theta d\theta$

$$I_x = a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} b^2 \sin^2\theta \sqrt{1 - \sin^2\theta}\; b\cos\theta d\theta$$

$$= ab^3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\theta \cos^2\theta d\theta = ab^3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4}\sin^2 2\theta d\theta$$

$$= \frac{1}{4}ab^3 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 4\theta) d\theta = \frac{1}{8}ab^3 \left[\theta - \frac{1}{4}\sin 4\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{8}ab^3 \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{8}ab^3 \qquad I_x = \frac{1}{8}\pi ab^3 \blacktriangleleft$$

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Chapter 9, Solution 10.

At $$x = 2a,\ y = b: \qquad 2a = kb^3$$

or $$k = \frac{2a}{b^3}$$

Then $$x = \frac{2a}{b^3}y^3$$

or $$y = \frac{b}{(2a)^{\frac{1}{3}}}x^{\frac{1}{3}}$$

Now $$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}\frac{b^3}{2a}xdx$$

Then $$I_x = \int dI_x = \frac{1}{3}\frac{b^3}{2a}\int_a^{2a} xdx = \frac{1}{6}\frac{b^3}{a}\frac{1}{2}x^2\Big|_a^{2a}$$

$$= \frac{b^3}{12a}\left(4a^2 - a^2\right)$$

$$I_x = \frac{1}{4}ab^3 \blacktriangleleft$$

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Chapter 9, Solution 11.

First note: At $x = a$: $b = k\left(1 - e^{\frac{-a}{a}}\right)$

or $$k = \frac{b}{1 - e^{-1}}$$

Have $$I_x = \int y^2 dA$$

$$= \int_0^a \int_0^{\frac{b}{1-e^{-1}}\left(1-e^{\frac{-x}{a}}\right)} y^2 dy dx$$

$$= \frac{1}{3}\left(\frac{b}{1 - e^{-1}}\right)^3 \int_0^a \left(1 - e^{\frac{-x}{a}}\right)^3 dx$$

$$= \frac{1}{3}\left(\frac{b}{1 - e^{-1}}\right)^3 \int_0^a \left(1 - 3e^{\frac{-x}{a}} + 3e^{\frac{-2x}{a}} - e^{\frac{-3x}{a}}\right) dx$$

$$= \frac{1}{3}\left(\frac{b}{1 - e^{-1}}\right)^3 \left[x - 3(-a)e^{\frac{-x}{a}} + 3\left(-\frac{a}{2}\right)e^{\frac{-2x}{a}} - \left(-\frac{a}{3}e^{\frac{-3x}{a}}\right)\right]\Bigg|_0^a$$

$$= \frac{1}{3}\left(\frac{b}{1 - e^{-1}}\right)^3 \left[\left(a + 3ae^{-1} - 1.5ae^{-2} + \frac{1}{3}ae^{-3}\right) - \left(3a - 1.5a + \frac{1}{3}a\right)\right]$$

$$= \frac{1}{3}\frac{ab^3}{\left(1 - e^{-1}\right)^3}\left(1.91723 - \frac{11}{6}\right)$$

$$= 0.1107ab^3$$

$I_x = 0.1107ab^3$ ◀

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Chapter 9, Solution 12.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$$dA = 2y\,dx$$

$$dI_y = x^2 dA = 2x^2 y\,dx$$

$$I_y = \int dI_y = \int_0^a 2x^2 y\,dx = 2b\int_0^a x^2\sqrt{1 - \frac{x^2}{a^2}}\,dx$$

Set: $x = a\sin\theta \qquad dx = a\cos\theta\,d\theta$

$$I_y = 2b\int_0^{\frac{\pi}{2}} a^2\sin^2\theta\sqrt{1 - \sin^2\theta}\,a\cos\theta\,d\theta$$

$$= 2a^3 b\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\,d\theta = 2a^3 b\int_0^{\frac{\pi}{2}} \frac{1}{4}\sin^2 2\theta\,d\theta$$

$$= \frac{1}{2}a^3 b\int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 4\theta)\,d\theta = \frac{1}{4}a^3 b\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}a^3 b\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{8}a^3 b$$

$$I_y = \frac{1}{8}\pi a^3 b \blacktriangleleft$$

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Chapter 9, Solution 13.

At $\quad x = 2a, \quad y = b: \quad 2a = kb^3$

Then $\quad x = \dfrac{2a}{b^3} y^3$

or $\quad y = \dfrac{b}{(2a)^{\frac{1}{3}}} x^{\frac{1}{3}}$

Now $\quad I_y = \int x^2 dA \qquad dA = y dx$

Then $\quad I_y = \int_a^{2a} x^2 \dfrac{b}{(2a)^{\frac{1}{3}}} x^{\frac{1}{3}} dx$

$$= \frac{b}{(2a)^{\frac{1}{3}}} \int_a^{2a} x^{\frac{7}{3}} dx$$

$$= \frac{b}{(2a)^{\frac{1}{3}}} \frac{3}{10} x^{\frac{10}{3}} \Bigg|_a^{2a}$$

$$= \frac{3b}{10(2a)^{\frac{1}{3}}} \left[(2a)^{\frac{10}{3}} - a^{\frac{10}{3}} \right]$$

$$= \frac{3ba^3}{10(2)^{\frac{1}{3}}} \left(2^{\frac{10}{3}} - 1^{\frac{10}{3}} \right)$$

$$= 2.1619 a^3 b$$

or $\quad I_y = 2.16 a^3 b$ ◀

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Chapter 9, Solution 14.

First note: At $x = a$: $b = k\left(1 - e^{\frac{-a}{a}}\right)$

or $$k = \frac{b}{1 - e^{-1}}$$

Have $$I_y = \int x^2 dA$$

$$= \int_0^a \int_0^{\frac{b}{1-e^{-1}}\left(1-e^{\frac{-x}{a}}\right)} x^2 dy dx$$

$$= \int_0^a x^2 \left[\frac{b}{1 - e^{-1}}\left(1 - e^{\frac{-x}{a}}\right)\right] dx$$

$$= \frac{b}{1 - e^{-1}} \left\{ \frac{1}{3}x^3 - \frac{e^{\frac{-x}{a}}}{\left(-\frac{1}{a}\right)^3}\left[\left(-\frac{1}{a}\right)^2 x^2 - 2\left(-\frac{1}{a}\right)x + 2\right]\right\}\Bigg|_0^a$$

$$= \frac{b}{1 - e^{-1}} \left\{\left[\frac{1}{3}a^3 + a^3 e^{-1}\left(\frac{a^2}{a^2} + 2\frac{a}{a} + 2\right)\right] - \left(a^3 \times 2\right)\right\}$$

$$= \frac{a^3 b}{1 - e^{-1}}\left(\frac{1}{3} + 5e^{-1} - 2\right)$$

$$= 0.273 a^3 b$$

$I_y = 0.273 a^3 b$ ◀

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Chapter 9, Solution 15.

At $\quad x = a,\ y = b: \quad b = k_1 a^4 \qquad b = k_2 a^{\frac{1}{4}}$

or $\quad k_1 = \dfrac{b}{a^4} \qquad k_2 = \dfrac{b}{a^{\frac{1}{4}}}$

Then $\quad y_1 = \dfrac{b}{a^4} x^4 \qquad y_2 = \dfrac{b}{a^{\frac{1}{4}}} x^{\frac{1}{4}}$

and $\quad x_1 = \dfrac{a}{b^{\frac{1}{4}}} y^{\frac{1}{4}} \qquad x_2 = \dfrac{a}{b^4} y^4$

Now
$$A = \int (y_2 - y_1)\,dx = b\int_0^a \left(\frac{x^{\frac{1}{4}}}{a^{\frac{1}{4}}} - \frac{x^4}{a^4} \right) dx$$
$$= b\left[\frac{4}{5}\frac{x^{\frac{5}{4}}}{a^{\frac{1}{4}}} - \frac{1}{5}\frac{x^5}{a^4} \right]_0^a = \frac{3}{5}ab$$

Then
$$I_x = \int y^2 dA \qquad dA = (x_1 - x_2)\,dy$$
$$I_x = \int_0^b y^2 \left(\frac{a}{b^{\frac{1}{4}}} y^{\frac{1}{4}} - \frac{a}{b^4} y^4 \right) dy$$
$$= a\left[\frac{4}{13}\frac{y^{\frac{13}{4}}}{b^{\frac{1}{4}}} - \frac{1}{7}\frac{y^7}{b^4} \right]_0^b$$
$$= ab^3\left(\frac{4}{13} - \frac{1}{7} \right)$$

or $\quad I_x = \dfrac{15}{91} ab^3$ ◀

Now
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{91}ab^3}{\frac{3}{5}ab}} = \sqrt{\frac{25}{91}b^2} = 0.52414b$$

or $\quad k_x = 0.524b$ ◀

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Chapter 9, Solution 16.

First note: At $x = a$: $2b = k\sqrt{a}$

or $k = \dfrac{2b}{\sqrt{a}}$ $\quad \therefore\ y_2 = \dfrac{2b}{\sqrt{a}}\sqrt{x}$

Straight line: $y_1 = \dfrac{b}{a}x$

Now:

$$A = 2\int_0^a \left(\frac{2b}{\sqrt{a}}x^{\frac{1}{2}} - \frac{b}{a}x\right)dx$$

$$= 2b\left(\frac{4}{3}\frac{x^{\frac{3}{2}}}{\sqrt{a}} - \frac{1}{2a}x^2\right)\Bigg|_0^a$$

$$= \frac{5}{3}ab$$

Have

$$I_x = \int y^2 dA$$

$$= 2\int_0^a \int_{\frac{b}{a}x}^{\frac{2b}{\sqrt{a}}x^{\frac{1}{2}}} y^2 dy dx$$

$$= \frac{2}{3}\int_0^a \left(\frac{8b^3}{a^{\frac{3}{2}}}x^{\frac{3}{2}} - \frac{b^3}{a^3}x^3\right)dx$$

$$= \frac{2b^3}{3}\left(\frac{2}{5}\times\frac{8}{a^{\frac{3}{2}}}x^{\frac{5}{2}} - \frac{1}{4a^3}x^4\right)\Bigg|_0^a$$

$$I_x = \frac{59}{30}ab^3 \blacktriangleleft$$

And

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$= \sqrt{\frac{\frac{59}{30}ab^3}{\frac{5}{3}ab}}$$

$$= b\sqrt{1.18}$$

$$k_x = 1.086\ b \blacktriangleleft$$

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Chapter 9, Solution 17.

At $\quad x = a,\ y = b: \quad b = k_1 a^4 \qquad b = k_2 a^{\frac{1}{4}}$

or $\quad k_1 = \dfrac{b}{a^4} \qquad k_2 = \dfrac{b}{a^{\frac{1}{4}}}$

Then $\quad y_1 = \dfrac{b}{a^4} x^4 \quad$ and $\quad y_2 = \dfrac{b}{a^{\frac{1}{4}}} x^{\frac{1}{4}}$

Now
$$A = \int (y_2 - y_1)\,dx = b\int_0^a \left(\frac{x^{\frac{1}{4}}}{a^{\frac{1}{4}}} - \frac{x^4}{a^4} \right) dx$$

$$= b\left[\frac{4}{5}\frac{x^{\frac{5}{4}}}{a^{\frac{1}{4}}} - \frac{1}{5}\frac{x^5}{a^4} \right]_0^a = \frac{3}{5}ab$$

Now $\quad I_y = \int x^2 dA \qquad dA = (y_2 - y_1)\,dx$

Then
$$I_y = \int_0^a x^2 \left(\frac{b}{a^{\frac{1}{4}}} x^{\frac{1}{4}} - \frac{b}{a^4} x^4 \right) dx$$

$$= b\int_0^a \left(\frac{x^{\frac{9}{4}}}{a^{\frac{1}{4}}} - \frac{x^6}{a^4} \right) dx$$

$$= b\left[\frac{4}{13}\frac{x^{\frac{13}{4}}}{a^{\frac{1}{4}}} - \frac{1}{7}\frac{x^7}{a^4} \right]_0^a$$

$$= b\left(\frac{4}{13}a^3 - \frac{1}{7}a^3 \right)$$

or $\ I_y = \dfrac{15}{91}a^3 b$ ◀

Now
$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{15}{91}a^3 b}{\frac{3}{5}ab}} = \sqrt{\frac{25}{91}}\,a = 0.52414a$$

or $\ k_y = 0.524a$ ◀

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Chapter 9, Solution 18.

First note: At $x = a$: $2b = k\sqrt{a}$

or $k = \dfrac{2b}{\sqrt{a}}$ $\therefore\ y_2 = \dfrac{2b}{\sqrt{a}}\sqrt{x}$

Straight line: $y_1 = \dfrac{b}{a}x$

Now:

$$A = 2\int_0^a \left(\frac{2b}{\sqrt{a}}x^{\frac{1}{2}} - \frac{b}{a}x\right)dx$$

$$= 2b\left(\frac{4}{3}\frac{x^{\frac{3}{2}}}{\sqrt{a}} - \frac{1}{2a}x^2\right)\Bigg|_0^a$$

$$= \frac{5}{3}ab$$

Have

$$I_y = \int x^2 dA$$

$$= 2\int_0^a \int_{\frac{b}{a}x}^{\frac{2b}{\sqrt{a}}x^{\frac{1}{2}}} x^2 dy dx$$

$$= 2\int_0^a x^2\left(\frac{2b}{\sqrt{a}}x^{\frac{1}{2}} - \frac{b}{a}x\right)dx$$

$$= 2b\left(2\times\frac{2}{7}\frac{x^{\frac{7}{2}}}{\sqrt{a}} - \frac{1}{4}\frac{x^4}{a}\right)\Bigg|_0^a$$

$$= \frac{9}{14}a^3 b$$

$I_y = \dfrac{9}{14}a^3 b$ ◀

continued

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And

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{\frac{9}{14}a^3 b}{\frac{5}{3}ab}}$$

$$= a\sqrt{\frac{27}{70}}$$

$$k_y = 0.621a \blacktriangleleft$$

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Chapter 9, Solution 19.

First note: At $x = 0$: $b = c\cos(0)$

or $c = b$

At $x = 2a$: $b = b\sin k(2a)$

or $2ka = \dfrac{\pi}{2}$

$$k = \frac{\pi}{4a}$$

Then
$$A = \int_a^{2a}\left(b\sin\frac{\pi}{4a}x - b\cos\frac{\pi}{4a}x\right)dx$$

$$= b\left(-\frac{4a}{\pi}\cos\frac{\pi}{4a}x - \frac{4a}{\pi}\sin\frac{\pi}{4a}x\right)\Bigg|_a^{2a}$$

$$= -\frac{4ab}{\pi}\left[(1) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4ab}{\pi}\left(\sqrt{2} - 1\right)$$

Have
$$I_x = \int y^2 dA$$

$$= \int_a^{2a}\int_{b\cos\frac{\pi}{4a}x}^{b\sin\frac{\pi}{4a}x} y^2 dy dx$$

$$= \frac{1}{3}\int_a^{2a}\left(b^3\sin^3\frac{\pi}{4a}x - b^3\cos^3\frac{\pi}{4a}x\right)dx$$

$$= \frac{b^3}{3}\left[\left(-\frac{4a}{\pi}\cos\frac{\pi}{4a}x + \frac{1}{3}\frac{4a}{\pi}\cos^3\frac{\pi}{4a}x\right) - \left(\frac{4a}{\pi}\sin\frac{\pi}{4a}x - \frac{1}{3}\frac{4a}{\pi}\sin^3\frac{\pi}{4a}x\right)\right]\Bigg|_a^{2a}$$

$$= \frac{4ab^3}{3\pi}\left\{\left[-1 + \frac{1}{3}\right] - \left[-\frac{1}{\sqrt{2}} + \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3 - \frac{1}{\sqrt{2}} + \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3\right]\right\}$$

$$= \frac{4ab^3}{3\pi}\left(\frac{5}{6}\sqrt{2} - \frac{2}{3}\right)$$

continued

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$$I_x = \frac{2}{9\pi} ab^3 \left(5\sqrt{2} - 4\right)$$

$$= 0.217\, ab^3 \qquad I_x = 0.217\, ab^3 \blacktriangleleft$$

And

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$= \sqrt{\frac{\frac{2}{9\pi} ab^3 \left(5\sqrt{2} - 4\right)}{\frac{4}{\pi} ab \left(\sqrt{2} - 1\right)}}$$

$$= 0.642\, b \qquad k_x = 0.642\, b \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 9, Solution 20.

First note: At $x = 0$: $b = c\cos(0)$

or $c = b$

At $x = 2a$: $b = b\sin k(2a)$

or $2ka = \dfrac{\pi}{2}$

$$k = \frac{\pi}{4a}$$

Then
$$A = \int_a^{2a}\left(b\sin\frac{\pi}{4a}x - b\cos\frac{\pi}{4a}x\right)dx$$

$$= b\left(-\frac{4a}{\pi}\cos\frac{\pi}{4a}x - \frac{4a}{\pi}\sin\frac{\pi}{4a}x\right)\Bigg|_a^{2a}$$

$$= -\frac{4ab}{\pi}\left[(1) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4ab}{\pi}\left(\sqrt{2} - 1\right)$$

Have
$$I_y = \int x^2 dA$$

$$= \int_a^{2a}\int_{b\cos\frac{\pi}{4a}x}^{b\sin\frac{\pi}{4a}x} x^2 dy dx$$

$$= \int_a^{2a} x^2\left(b\sin\frac{\pi}{4a}x - b\cos\frac{\pi}{4a}x\right)dx$$

continued

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$$= b\left\{\left[\frac{2x}{\left(\frac{\pi}{4a}\right)^2}\sin\frac{\pi}{4a}x + \frac{2}{\left(\frac{\pi}{4a}\right)^3}\cos\frac{\pi}{4a}x - \frac{x^2}{\left(\frac{\pi}{4a}\right)}\cos\frac{\pi}{4a}x\right]\right.$$

$$\left.-\left[\frac{2x}{\left(\frac{\pi}{4a}\right)^2}\cos\frac{\pi}{4a}x - \frac{2}{\left(\frac{\pi}{4a}\right)^3}\sin\frac{\pi}{4a}x + \frac{x^2}{\left(\frac{\pi}{4a}\right)}\sin\frac{\pi}{4a}x\right]\right\}\Bigg|_a^{2a}$$

$$I_y = \frac{64a^3b}{\pi^3}\left[\left(\sin\frac{\pi}{4a}x - \cos\frac{\pi}{4a}x\right)\left(\frac{\pi}{2a}x\right) + \left(\sin\frac{\pi}{4a}x + \cos\frac{\pi}{4a}x\right)\left(2 - \frac{\pi^2}{16a^2}x^2\right)\right]\Bigg|_a^{2a}$$

$$= \frac{64a^3b}{\pi^3}\left\{\left[(1)(\pi) + (1)\left(2 - \frac{\pi^2}{4}\right)\right] - \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\left(2 - \frac{\pi^2}{16}\right)\right]\right\}$$

$$= 1.48228a^3b \qquad I_y = 1.482a^3b \blacktriangleleft$$

And $$k_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{1.48228a^3b}{\frac{4ab}{\pi}\left(\sqrt{2} - 1\right)}}$$

$$= 1.676a \qquad k_y = 1.676a \blacktriangleleft$$

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Chapter 9, Solution 21.

(*a*)

$$dI_x = \frac{1}{3}a^3 dx$$

$$I_x = \frac{1}{3}a^3 \int_{-a}^{a} dx = \frac{a^3}{3}(2a) = \frac{2}{3}a^4$$

$$dI_y = x^2 dA = x^2 a dx$$

$$I_y = a\int_{-a}^{a} x^2 dx = a\left[\frac{x^3}{3}\right]_{-a}^{a} = \frac{2}{3}a^4$$

$$J_O = I_x + I_y = \frac{2}{3}a^4 + \frac{2}{3}a^4 \qquad J_O = \frac{4}{3}a^4 \blacktriangleleft$$

$$J_O = k_O^2 A \qquad k^2 = \frac{J_O}{A} = \frac{\frac{4}{3}a^4}{2a^2} = \frac{2}{3}a^2 \qquad k_O = a\sqrt{\frac{2}{3}} \blacktriangleleft$$

(*b*)

$$dI_x = \frac{1}{12}a^3 dx$$

$$I_x = \frac{a^3}{12}\int_0^{2a} dx = \frac{a^3}{12}[x]_0^{2a} = \frac{1}{6}a^4$$

$$dI_y = x^2 dA = x^2(a dx)$$

$$I_y = a\int_0^{2a} x^2 dx = a\left[\frac{x^3}{3}\right]_0^{2a} = \frac{8}{3}a^4$$

$$J_O = I_x + I_y = \frac{1}{6}a^4 + \frac{8}{3}a^4 = \frac{17}{6}a^4 \qquad J_O = \frac{17}{6}a^4 \blacktriangleleft$$

$$J_O = k_O^2 A \qquad k_O^2 = \frac{J_O}{A} = \frac{\frac{17}{6}a^4}{2a^2} = \frac{17}{12}a^2 \qquad k_O = a\sqrt{\frac{17}{12}} \blacktriangleleft$$

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Chapter 9, Solution 22.

Have: $A = (2a)(3b) - \frac{1}{2}(2a)(b)$

$= 5ab$

Have: $J_P = I_x + I_y$

Where: $I_x = \int y^2 dA$

$$= 2\int_0^a \int_{-\frac{b}{a}x}^{2b} y^2 dy dx$$

$$= \frac{2}{3}\int_0^a \left[(2b)^3 - \left(-\frac{b}{a}x\right)^3 \right] dx$$

$$= \frac{2}{3}b^3 \left(8x + \frac{x^4}{4a^3} \right)\Bigg|_0^a$$

$$= \frac{11}{2}ab^3$$

And $I_y = \int x^2 dA$

$$= 2\int_0^a \int_{-\frac{b}{a}x}^{2b} x^2 dy dx$$

$$= 2\int_0^a x^2 \left[2b - \left(-\frac{b}{a}x\right) \right] dx$$

$$= 2b\left(\frac{2}{3}x^3 + \frac{1}{4a}x^4 \right)\Bigg|_0^a$$

$$= \frac{11}{6}a^3 b$$

continued

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Then: $$J_P = \frac{11}{2}ab^3 + \frac{11}{6}a^3b$$

$$J_P = \frac{11}{6}ab\left(a^2 + 3b^2\right) \blacktriangleleft$$

Also: $$k_P = \sqrt{\frac{J_P}{A}}$$

$$= \sqrt{\frac{\frac{11}{6}ab\left(a^2 + 3b^2\right)}{5ab}}$$

$$k_P = \sqrt{\frac{11}{30}\left(a^2 + 3b^2\right)} \blacktriangleleft$$

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Chapter 9, Solution 23.

y_1: At $x = a,\quad y = 2b:\ \ 2b = ma$

or $$m = \frac{2b}{a}$$

Then $$y_1 = \frac{2b}{a}x$$

y_2: At $x = 0,\ y = b:\ \ b = k(0) + c$ or $c = b$

At $x = a,\ y = 2b:\ \ 2b = ka^2 + b$

or $$k = \frac{b}{a^2}$$

Then $$y_2 = \frac{b}{a^2}x^2 + b = \frac{b}{a^2}\left(x^2 + a^2\right)$$

Now $$A = \int(y_2 - y_1)\,dx = \int_0^a\left[\frac{b}{a^2}\left(x^2 + a^2\right) - \frac{2b}{a}x\right]dx$$

$$= \left[\frac{b}{a^2}\left(\frac{1}{3}x^3 + a^2x\right) - \frac{b}{a}x^2\right]_0^a$$

$$= \frac{b}{a^2}\left(\frac{1}{3}a^3 + a^3\right) - \frac{b}{a}a^2 = \frac{1}{3}ab$$

Now $$I_y = \int x^2 dA = \int_0^a x^2(y_2 - y_1)\,dx$$

$$= \int_0^a x^2\left[\frac{b}{a^2}\left(x^2 + a^2\right) - \frac{2b}{a}x\right]dx$$

$$= \frac{b}{a^2}\left[\left(\frac{1}{5}x^5 + \frac{1}{3}a^2x^3\right) - \frac{2b}{a}\frac{x^4}{4}\right]_0^a$$

$$= \frac{b}{a^2}\left(\frac{a^5}{5} + \frac{1}{3}a^5\right) - \frac{2b}{a}\frac{a^4}{4} = \frac{1}{30}a^3b$$

continued

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And $$I_x = \int dI_x = \int\left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3\right)dx$$

$$= \frac{1}{3}\int_0^a\left[\frac{b^3}{a^6}\left(x^2 + a^2\right)^3 - \frac{8b^3}{a^3}x^3\right]dx$$

$$= \frac{1}{3}\frac{b^3}{a^3}\int_0^a\left[\frac{1}{a^3}\left(x^6 + 3x^4a^2 + 3x^2a^4 + a^6\right) - 8x^3\right]dx$$

$$I_x = \frac{1}{3}\frac{b^3}{a^3}\left[\frac{1}{a^3}\left(\frac{x^7}{7} + \frac{3}{5}a^2x^5 + \frac{3}{3}a^4x^3 + a^6x\right) - \frac{8}{4}x^4\right]_0^a$$

$$= \frac{1}{3}\frac{b^3}{a^3}\left[\frac{1}{a^3}\left(\frac{a^7}{7} + \frac{3}{5}a^7 + a^7 + a^7\right) - 2a^4\right] = \frac{26}{105}ab^3$$

Finally $$J_P = I_x + I_y = \frac{26}{105}ab^3 + \frac{1}{30}a^3b$$

or $$J_P = \frac{ab}{210}\left(7a^2 + 52b^2\right) \blacktriangleleft$$

And $$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{ab}{210}\left(7a^2 + 52b^2\right)}{\frac{1}{3}ab}}$$

or $$k_P = \sqrt{\frac{7a^2 + 52b^2}{70}} \blacktriangleleft$$

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Chapter 9, Solution 24.

First note: $x = \sqrt{r^2 - y^2}$

$$J_P = I_x + I_y$$

$$I_x = \int y^2 dA = 2\int_{\frac{r}{2}}^{r}\int_0^{\sqrt{r^2-y^2}} y^2 dx dy$$

$$= 2\int_{\frac{r}{2}}^{r} y^2\sqrt{r^2 - y^2}\, dy$$

Let $y = r\sin\theta$

Then $dy = r\cos\theta d\theta$; $y = r$: $\theta = \frac{\pi}{2}$; $y = \frac{r}{2}$: $\theta = \frac{\pi}{6}$

Thus $I_x = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} r^2\sin^2\theta\left(r\cos\theta\right)\left(r\cos\theta d\theta\right)$

Now $\sin 2\theta = 2\sin\theta\cos\theta$ thus $\sin^2\theta\cos^2\theta = \frac{1}{4}\sin^2 2\theta$

$$I_x = \frac{1}{2}r^4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin^2 2\theta d\theta$$

$$= \frac{1}{2}r^4\left(\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}r^4\left(\frac{\pi}{4} - \frac{\pi}{12} + \frac{1}{8}\times\frac{\sqrt{3}}{2}\right)$$

$$= \frac{r^4}{4}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$$

$$I_y = \int x^2 dA = 2\int_{\frac{r}{2}}^{r}\int_0^{\sqrt{r^2-y^2}} x^2 dx dy$$

$$= \frac{2}{3}\int_{\frac{r}{2}}^{r}\left(r^2 - y^2\right)^{\frac{3}{2}} dy$$

continued

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Let $y = r\sin\theta$

Then $I_y = \frac{2}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(r^3\cos^3\theta\right)\left(r\cos\theta d\theta\right)$

Now $\cos^4\theta = \cos^2\theta\left(1-\sin^2\theta\right) = \cos^2\theta - \frac{1}{4}\sin^2 2\theta$

Thus $I_y = \frac{2}{3}r^4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\cos^2\theta - \frac{1}{4}\sin^2 2\theta\right)d\theta$

$$= \frac{2}{3}r^4\left[\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)-\frac{1}{4}\left(\frac{\theta}{2}-\frac{\sin 4\theta}{8}\right)\right]\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{1}{3}r^4\left[\left(\frac{\pi}{2}-\frac{1}{4}\times\frac{\pi}{2}\right)-\left(\frac{\pi}{6}+\frac{1}{2}\times\frac{\sqrt{3}}{2}-\frac{1}{4}\times\frac{\pi}{6}+\frac{1}{4}\times\frac{1}{4}\times\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{r^4}{4}\left(\frac{\pi}{3}-\frac{3\sqrt{3}}{8}\right)$$

Then $J_P = \frac{r^4}{4}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{8}\right)+\frac{r^4}{4}\left(\frac{\pi}{3}-\frac{3\sqrt{3}}{8}\right)$

$$= \frac{r^4}{48}\left(8\pi - 3\sqrt{3}\right)$$

$J_P = 0.415r^4$ ◀

Now $A = 2\int_{\frac{r}{2}}^{r} x dy$

$$= \int_{\frac{r}{2}}^{r}\sqrt{r^2 - y^2}\,dy$$

continued

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Let $$y = r\sin\theta$$

$$A = 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (r\cos\theta)(r\cos\theta d\theta)$$

$$= 2r^2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= r^2\left(\frac{\pi}{2} - \frac{\pi}{6} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right)$$

$$= \frac{r^2}{12}\left(4\pi - 3\sqrt{3}\right)$$

Have $$k_P = \sqrt{\frac{J_P}{A}}$$

$$= \sqrt{\frac{\frac{r^4}{48}\left(8\pi - 3\sqrt{3}\right)}{\frac{r^2}{12}\left(4\pi - 3\sqrt{3}\right)}}$$

$$k_P = 0.822r \blacktriangleleft$$

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Chapter 9, Solution 25.

(*a*) Have $J_O \equiv I_0 = \int r^2 dA$

Where $dA = \dfrac{3\pi}{2} r dr$

Then $J_O = \int_{R_1}^{R_2} r^2 \left(\dfrac{3\pi}{2} r \right) dr$

$$= \frac{3\pi}{8} r^4 \Big|_{R_1}^{R_2} = \frac{3\pi}{8}\left(R_2^4 - R_1^4\right)$$

$$J_O = \frac{3\pi}{8}\left(R_2^4 - R_1^4\right) \blacktriangleleft$$

(*b*) Now $I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

By inspection $(I_x)_1 = (I_x)_2 = (I_x)_3$

so that $I_x = 3(I_x)_1$

Similarly, $I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = 3(I_y)_1$

Symmetry implies $(I_x)_1 = (I_y)_1$

Then $I_x = I_y$

Now $J_O = I_x + I_y$

Then $I_x = I_y = \dfrac{J_O}{2} = \dfrac{3\pi}{16}\left(R_2^4 - R_1^4\right)$

or $I_x = I_y = \dfrac{3\pi}{16}\left(R_2^4 - R_1^4\right) \blacktriangleleft$

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Chapter 9, Solution 26.

(*a*) From Problem 9.25 $$J_O = \frac{3\pi}{8}\left(R_2^4 - R_1^4\right)$$

And $$A = \int dA = \int_{R_1}^{R_2} \frac{3\pi}{2} r dr = \frac{3\pi}{4}\left(R_2^2 - R_1^2\right)$$

Then $$k_O^2 = \frac{J_O}{A} = \frac{\frac{3\pi}{8}\left(R_2^4 - R_1^4\right)}{\frac{3\pi}{4}\left(R_2^2 - R_1^2\right)} = \frac{\left(R_2^2 + R_1^2\right)\left(R_2^2 - R_1^2\right)}{2\left(R_2^2 - R_1^2\right)} = \frac{1}{2}\left(R_2^2 + R_1^2\right)$$

Now $$R_m = \frac{1}{2}\left(R_1 + R_2\right) \qquad \text{and} \qquad t = R_2 - R_1$$

Then $$R_2 = R_m + \frac{t}{2} \qquad \text{and} \qquad R_1 = R_m - \frac{t}{2}$$

And $$k_O^2 = \frac{1}{2}\left[\left(R_m + \frac{t}{2}\right)^2 + \left(R_m - \frac{t}{2}\right)^2\right] = R_m^2 + \frac{1}{4}t^2$$

For t R_1, R_2 k_O^2 R_m^2

Or k_O R_m ◀

(*b*) Have $$\%\text{ error} = \frac{R_m - k_O}{k_O} \times 100\% = \frac{R_m - \sqrt{R_m^2 + \frac{1}{4}t^2}}{\sqrt{R_m^2 + \frac{1}{4}t^2}} \times 100\%$$

$$= \frac{1 - \sqrt{1 + \frac{1}{4}\left(\frac{t}{R_m}\right)^2}}{\sqrt{1 + \frac{1}{4}\left(\frac{t}{R_m}\right)^2}} \times 100\%$$

continued

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Then

$$\frac{t}{R_m} = 1: \quad \%\text{ error} = \frac{1-\sqrt{1+\frac{1}{4}}}{\sqrt{1+\frac{1}{4}}} \times 100\%$$

or $\%\text{ error} = -10.56\%$ ◀

$$\frac{t}{R_m} = \frac{1}{4}: \quad \%\text{ error} = \frac{1-\sqrt{1+\frac{1}{4}\left(\frac{1}{4}\right)^2}}{\sqrt{1+\frac{1}{4}\left(\frac{1}{4}\right)^2}} \times 100\%$$

or $\%\text{ error} = -0.772\%$ ◀

$$\frac{t}{R_m} = \frac{1}{16}: \quad \%\text{ error} = \frac{1-\sqrt{1+\frac{1}{4}\left(\frac{1}{16}\right)^2}}{\sqrt{1+\frac{1}{4}\left(\frac{1}{16}\right)^2}} \times 100\%$$

or $\%\text{ error} = -0.0488\%$ ◀

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Chapter 9, Solution 27.

Have:
$$A = 2\int_0^{\frac{\pi}{4}} \int_0^{a\cos 2\theta} r dr d\theta$$
$$= \int_0^{\frac{\pi}{4}} a^2 \cos^2 2\theta d\theta$$
$$= a^2 \left(\frac{\theta}{2} + \frac{\sin 4\theta}{8} \right) \Bigg|_0^{\frac{\pi}{4}}$$
$$= \frac{\pi}{8} a^2$$

Have:
$$J_O = \int r^2 dA$$
$$= 2\int_0^{\frac{\pi}{4}} \int_0^{a\cos 2\theta} r^2 \left(r dr d\theta \right)$$
$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} a^4 \cos^4 2\theta d\theta$$

Now:
$$\cos^4 2\theta = \cos^2 2\theta \left(1 - \sin^2 2\theta \right)$$
$$= \cos^2 2\theta - \frac{1}{4} \sin^2 4\theta$$

Then:
$$J_O = \frac{1}{2} a^4 \int_0^{\frac{\pi}{4}} \left(\cos^2 2\theta - \frac{1}{4} \sin^2 4\theta \right) d\theta$$
$$= \frac{1}{2} a^4 \left[\left(\frac{\theta}{2} + \frac{\sin 4\theta}{8} \right) - \frac{1}{4} \left(\frac{\theta}{2} - \frac{\sin 8\theta}{16} \right) \right] \Bigg|_0^{\frac{\pi}{4}}$$
$$= \frac{1}{4} a^4 \left(\frac{\pi}{4} - \frac{1}{4} \times \frac{\pi}{4} \right)$$

$$J_O = \frac{3\pi}{64} a^4 \blacktriangleleft$$

continued

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And:

$$k_O = \sqrt{\frac{J_O}{A}}$$

$$= \sqrt{\frac{\frac{3\pi}{64}a^4}{\frac{\pi}{8}a^2}}$$

$$k_O = \frac{a}{4}\sqrt{6} \blacktriangleleft$$

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Chapter 9, Solution 28.

By observation $$y = \frac{h}{\frac{b}{2}}x$$

or $$x = \frac{b}{2h}y$$

Now $$dA = xdy = \left(\frac{b}{2h}y\right)dy$$

And $$dI_x = y^2 dA = \frac{b}{2h}y^3 dy$$

Then $$I_x = \int dI_x = 2\int_0^h \frac{b}{2h}y^3 dy$$

$$= \frac{b}{h}\frac{y^4}{4}\bigg|_0^h = \frac{1}{4}bh^3$$

From above $$y = \frac{2h}{b}x$$

Now $$dA = (h - y)dx = \left(h - \frac{2h}{b}x\right)dx$$

$$= \frac{h}{b}(b - 2x)dx$$

And $$dI_y = x^2 dA = x^2\frac{h}{b}(b - 2x)dx$$

Then $$I_y = \int dI_y = 2\int_0^{\frac{b}{2}} \frac{h}{b}x^2(b - 2x)dx$$

$$= 2\frac{h}{b}\left[\frac{1}{3}bx^3 - \frac{1}{2}x^4\right]_0^{\frac{b}{2}}$$

$$= 2\frac{h}{b}\left[\frac{b}{3}\left(\frac{b}{2}\right)^3 - \frac{1}{2}\left(\frac{b}{2}\right)^4\right] = \frac{1}{48}b^3 h$$

continued

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Now $$J_O = I_x + I_y = \frac{1}{4}bh^3 + \frac{1}{48}b^3h$$

or $J_O = \frac{bh}{48}\left(12h^2 + b^2\right)$ ◀

And $$k_O^2 = \frac{J_O}{A} = \frac{\frac{bh}{48}\left(12h^2 + b^2\right)}{\frac{1}{2}bh} = \frac{1}{24}\left(12h^2 + b^2\right)$$

or $k_O = \sqrt{\frac{12h^2 + b^2}{24}}$ ◀

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Chapter 9, Solution 29.

First the circular area is divided into an increasing number of identical circular sectors. The sectors can be approximated by isosceles triangles. For a large number of sectors the approximate dimensions of one of the isosceles triangles are as shown.
For an isosceles triangle (see Problem 9.28)

$$J_O = \frac{bh}{48}\left(12h^2 + b^2\right)$$

Then with $\quad b = r\Delta\theta \quad$ and $\quad h = r$

$$\left(\Delta J_O\right)_{\text{sector}} \quad \frac{1}{48}(r\Delta\theta)(r)\left[12r^2 + (r\Delta\theta)^2\right]$$

$$= \frac{1}{48}r^4\Delta\theta\left[\left(12 + \Delta\theta^2\right)\right]$$

Now $\quad \dfrac{dJ_{O\,\text{sector}}}{d\theta} = \lim\limits_{\Delta\theta\to 0}\left(\dfrac{\Delta J_{O\,\text{sector}}}{\Delta\theta}\right) = \lim\limits_{\Delta\theta\to 0}\left\{\dfrac{1}{48}r^4\left[12 + (\Delta\theta)^2\right]\right\}$

$$= \frac{1}{4}r^4$$

Then $\quad \left(J_O\right)_{\text{circle}} = \int dJ_{O\,\text{sector}} = \int_0^{2\pi}\frac{1}{4}r^4 d\theta = \frac{1}{4}r^4\left[\theta\right]_0^{2\pi}$

or $\left(J_O\right)_{\text{circle}} = \dfrac{\pi}{2}r^4$ ◀

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Chapter 9, Solution 30.

From the solution to sample Problem 9.2, the centroidal polar moment of inertia of a circular area is

$$\left(J_C\right)_{\text{cir}} = \frac{\pi}{2} r^4$$

The area of the circle is

$$A_{\text{cir}} = \pi r^2$$

So that

$$\left[J_C\left(A\right)\right]_{\text{cir}} = \frac{A^2}{2\pi}$$

Two methods of solution will be presented. However, both methods depend upon the observation that as a given element of area dA is moved closer to some point C, The value of J_C will be decreased ($J_C = \int r^2 dA$; as r decreases, so must J_C).

Solution 1

Imagine taking the area A and drawing it into a thin strip of negligible width and of sufficient length so that its area is equal to A. To minimize the value of $\left(J_C\right)_A$, the area would have to be distributed as closely as possible about C. This is accomplished by winding the strip into a tightly wound roll with C as its center; any voids in the roll would place the corresponding area farther from C than is necessary, thus increasing the value of $\left(J_C\right)_A$. (The process is analogous to rewinding a length of tape back into a roll.) Since the shape of the roll is circular, with the centroid of its area at C, it follows that

$$\left(J_C\right)_A \geq \frac{A^2}{2\pi} \quad \text{Q.E.D.} \blacktriangleleft$$

where the equality applies when the original area is circular.

continued

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Solution 2

Consider an area A, with its centroid at point C, and a circular area of area A, with its center (and centroid) at point C. Without loss of generality, assume that

$$A_1 = A_2 \qquad A_3 = A_4$$

It then follows that

$$(J_C)_A = (J_C)_{\text{cir}} + \left[J_C(A_1) - J_C(A_2) + J_C(A_3) - J_C(A_4)\right]$$

Now observe that

$$J_C(A_1) - J_C(A_2) \geq 0$$

$$J_C(A_3) - J_C(A_4) \geq 0$$

since as a given area is moved farther away from C its polar moment of inertia with respect to C must increase.

$$\therefore (J_C)_A \geq (J_C)_{\text{cir}}$$

or $(J_C)_A \geq \dfrac{A^2}{2\pi}$ Q.E.D. ◀

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Chapter 9, Solution 31.

Area: $A = 2\left[(10\text{ mm})(40\text{ mm})\right] + (90\text{ mm})(10\text{ mm}) = 1700\text{ mm}^2$

Part ①: $I_x = \bar{I}_x + Ad^2 = \frac{1}{12}(10\text{ mm})(40\text{ mm})^3 + (10\text{ mm})(40\text{ mm})(25\text{ mm})^2$

$= 303.3 \times 10^3\text{ mm}^4$

Part ②: $I_x = \bar{I}_x = \frac{1}{12}(90\text{ mm})(10\text{ mm})^3 = 7.50 \times 10^3\text{ mm}^4$

Part ③: (Same as Part ①) $I_x = 303.3 \times 10^3\text{ mm}^4$

Thus for entire area:

$I_x = (303.3 + 7.50 + 303.3) \times 10^3 = 614.2 \times 10^3\text{ mm}^4$

$I_x = 614 \times 10^3\text{ mm}^4$ ◀

$k_x^2 = \frac{I_x}{A} = \frac{614.2 \times 10^3\text{ mm}^4}{1700\text{ mm}^2} = 361.27\text{ mm}^2$

$k_x = 19.01\text{ mm}$ ◀

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Chapter 9, Solution 32.

Have $A = A_1 - A_2 - A_3$

$$= \left[(12)(8) - (5)(4) - (2)(6)\right] \text{ in}^2$$

$$= 64 \text{ in}^2$$

Have $I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$

$$= \left\{\left[\frac{1}{12}(12)(8)^3\right] - \left[\frac{1}{12}(5)(4)^3\right] - \left[\frac{1}{12}(2)(6)^3\right]\right\} \text{ in}^4$$

$$= (512 - 26.667 - 36) \text{ in}^4$$

$$= 449.33 \text{ in}^4$$

$I_x = 449 \text{ in}^4$ ◀

And $k_x = \sqrt{\dfrac{I_x}{A}}$

$$= \sqrt{\frac{449.33 \text{ in}^4}{64 \text{ in}^2}}$$

$$= 2.65 \text{ in.}$$

$k_x = 2.65 \text{ in.}$ ◀

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Chapter 9, Solution 33.

Area: $A = 2\left[(10\text{ mm})(40\text{ mm})\right] + (90\text{ mm})(10\text{ mm}) = 1700\text{ mm}^2$

Part ①: $I_y = \bar{I}_y + Ad^2 = \frac{1}{12}(40\text{ mm})(10\text{ mm})^3 + (40\text{ mm})(10\text{ mm})(40\text{ mm})^2$

$= 643.3 \times 10^3\text{ mm}^4$

Part ②: $I_y = \bar{I}_y = \frac{1}{12}(10\text{ mm})(90\text{ mm})^3 = 607.5 \times 10^3\text{ mm}^4$

Part ③: (Same as Part ①) $I_y = 643.3 \times 10^3\text{ mm}^4$

Thus for entire area:

$I_y = (643.3 + 607.5 + 643.3) \times 10^3\text{ mm}^4$

$= 1.894 \times 10^6\text{ mm}^4$ $\qquad I_y = 1.894 \times 10^6\text{ mm}^4$ ◀

$k_y^2 = \frac{I_y}{A} = \frac{1.894 \times 10^6\text{ mm}^4}{1700\text{ mm}^2} = 1114.2\text{ mm}^2$

$k_y = 33.4\text{ mm}$ ◀

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Chapter 9, Solution 34.

Have $A = A_1 - A_2 - A_3$

$$= \left[(12)(8) - (5)(4) - (2)(6)\right] \text{ in}^2$$

$$= 64 \text{ in}^2$$

Have $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

$$= \left\{\left[\frac{1}{12}(8)(12)^3\right] - \left[\frac{1}{12}(4)(5)^3 + 20\left(\frac{1}{2}\right)^2\right] - \left[\frac{1}{12}(6)(2)^3 + 12(4)^2\right]\right\} \text{ in}^4$$

$$= \left[(1152) - (41.667 + 5) - (4 + 192)\right] \text{ in}^4$$

$$= 909.33 \text{ in}^4 \qquad I_y = 909 \text{ in}^4 \blacktriangleleft$$

And $k_y = \sqrt{\dfrac{I_y}{A}}$

$$= \sqrt{\frac{909.33 \text{ in}^4}{64 \text{ in}^2}}$$

$$= 3.77 \text{ in.} \qquad k_y = 3.77 \text{ in.} \blacktriangleleft$$

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Chapter 9, Solution 35.

Have $I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

$$= \left[\frac{1}{3}(2a)(4a)^3\right] + \left[\frac{1}{3}(a)(3a)^3\right]$$

$$+ \left\{\left[\frac{\pi}{16}a^4 - \frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{4}a^2\left(3a + \frac{4a}{3\pi}\right)^2\right\}$$

$$= \left(\frac{128}{3}a^4\right) + \left(\frac{27}{3}a^4\right) + \left(\frac{\pi}{16} - \frac{4}{9\pi} + \frac{9\pi}{4} + 2 + \frac{4}{9\pi}\right)a^4$$

$$= \left(\frac{161}{3} + \frac{37\pi}{16}\right)a^4 = 60.9316a^4$$

or $I_x = 60.9a^4$ ◀

Also $I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

$$= \left[\frac{1}{3}(4a)(2a)^3\right] + \left[\frac{1}{3}(3a)(a)^3\right] + \left[\frac{\pi}{16}a^4\right]$$

$$= \left(\frac{32}{3} + 1 + \frac{\pi}{16}\right)a^4 = 11.8630a^4$$

or $I_y = 11.86a^4$ ◀

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Chapter 9, Solution 36.

Have
$$I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$
$$= \left[\frac{1}{12}(3a)(2a)^3\right] - \left[\frac{\pi}{8}a^4\right] - \left[\frac{\pi}{8}a^4\right]$$
$$= \left(2 - \frac{\pi}{8} - \frac{\pi}{8}\right)a^4 = \left(2 - \frac{\pi}{4}\right)a^4$$

or $I_x = 1.215a^4$ ◀

Also
$$I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$$
$$= \left[\frac{1}{12}(2a)(3a)^3 + (3a)(2a)\left(\frac{a}{2}\right)^2\right]$$
$$-\left\{\left[\frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{2}a^2\left(2a - \frac{4a}{3\pi}\right)^2\right\}$$
$$-\left\{\left[\frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{2}a^2\left(a - \frac{4a}{3\pi}\right)^2\right\}$$
$$= \left(\frac{9}{2} + \frac{3}{2}\right)a^4 - \left(\frac{\pi}{8} - \frac{8}{9\pi} + 2\pi - \frac{8}{3} + \frac{8}{9\pi}\right)a^4$$
$$-\left(\frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} - \frac{4}{3} + \frac{8}{9\pi}\right)a^4 = \left(10 - \frac{11\pi}{4}\right)a^4$$
$$= 1.3606a^4$$

or $I_y = 1.361a^4$ ◀

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Chapter 9, Solution 37.

$$I_{AA'} = 2.2 \times 10^6 \text{ mm}^4 = \bar{I} + A(25 \text{ mm})^2$$

$$I_{BB'} = 4 \times 10^6 \text{ mm}^4 = \bar{I} + A(35 \text{ mm})^2$$

$$I_{BB'} - I_{AA'} = (4 - 2.2) \times 10^6 = A\left(35^2 - 25^2\right)$$

$$1.8 \times 10^6 = A(600) \qquad A = 3000 \text{ mm}^2 \blacktriangleleft$$

Then $\quad I_{AA'} = 2.2 \times 10^6 \text{ mm}^4 = \bar{I} + \left(3000 \text{ mm}^2\right)(25 \text{ mm})^2$

$$\bar{I} = 325 \times 10^3 \text{ mm}^4 \qquad \bar{I} = 325 \times 10^3 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 38.

Given: $A = 6000 \text{ mm}^2$

$$I_{AA'} = 18 \times 10^6 \text{ mm}^4 = \bar{I} + \left(6000 \text{ mm}^2\right)\left(50 \text{ mm}\right)^2$$

$$\bar{I} = 3 \times 10^6 \text{ mm}^4$$

$$I_{BB'} = \bar{I} + Ad^2 = 3 \times 10^6 \text{ mm}^4 + \left(6000 \text{ mm}^2\right)\left(60 \text{ mm}\right)^2$$

$$= 24.6 \times 10^6 \text{ mm}^4$$

$I_{BB'} = 24.6 \times 10^6 \text{ mm}^4$ ◀

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Chapter 9, Solution 39.

Have $J_A = \bar{J}_C + A d_1^2 \qquad d_1 = 2a$

$$J_B = \bar{J}_C + A\left(d_2^2 + a^2\right)$$

Then $J_A - J_B = A\left(3a^2 - d_2^2\right)$

Substituting $256 \text{ in}^4 - 190 \text{ in}^4 = \left(24 \text{ in}^2\right)\left[3a^2 - \left(2 \text{ in.}\right)^2\right]$

$$a = 1.500 \text{ in.} \blacktriangleleft$$

And $256 \text{ in}^4 = \bar{J}_C + \left(24 \text{ in}^2\right)\left[4\left(1.500 \text{ in.}\right)^2\right]$

$$\bar{J}_C = 40.0 \text{ in}^4 \blacktriangleleft$$

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Chapter 9, Solution 40.

Have $J_A = \bar{J}_C + A d_1^2$

$J_B = \bar{J}_C + A\left(d_2^2 + a^2\right)$

(a) $J_B = 3J_A;\ d_1 = d_2 = 2.5$ in.

Then $3\left(\bar{J}_C + A d_1^2\right) = \bar{J}_C + A\left(d_1^2 + a^2\right)$

or $a^2 = 2\dfrac{\bar{J}_C}{A} + 2d_1^2$

$$= 2\left[\frac{52.5\text{ in}^4}{30\text{ in}^2} + (2.5\text{ in.})^2\right]$$

$a = 4.00$ in. ◀

(b) Have $J_B = 52.5\text{ in}^4 + \left(30\text{ in}^2\right)\left[(2.5\text{ in})^2 + (4.00\text{ in.})^2\right]$

$J_B = 720\text{ in}^4$ ◀

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Chapter 9, Solution 41.

Determination of centroid:

$\bar{y} = 0$ by symmetry.

Part	Area	$\bar{x}$(in.)	$\bar{x}A$(in^3)
1	$3(0.75) = 2.25$	1.5	3.375
2	$6(0.75) = 4.50$	0.375	1.6875
3	$3(0.75) = 2.25$	1.5	3.375
Σ	9.00		8.4375

$$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{8.4375 \text{ in}^3}{9.00 \text{ in}^2} = 0.9375 \text{ in.}$$

Determination of $\bar{I}_x$:

Part ①: $\bar{I}_x = \frac{1}{12}(3 \text{ in.})(0.75 \text{ in.})^3 + (3 \text{ in.})(0.75 \text{ in.})(3.375 \text{ in.})^2 = 25.734 \text{ in}^4$

Part ②: $\bar{I}_x = \frac{1}{12}(0.75 \text{ in.})(6 \text{ in.})^3 = 13.50 \text{ in}^4$

Part ③: (Same as Part ①) $\bar{I}_x = 25.734 \text{ in}^4$

Entire Section: $\bar{I}_x = (25.734 + 13.50 + 25.734) \text{ in}^4$

$= 64.97 \text{ in}^4$ $\qquad \bar{I}_x = 65.0 \text{ in}^4$ ◀

Determination of $\bar{I}_y$:

Part ①: $\bar{I}_y = \frac{1}{12}(0.75 \text{ in.})(3 \text{ in.})^3 + (0.75 \text{ in.})(3 \text{ in.})\left[(1.5 - 0.9375) \text{ in.}\right]^2$

$= 2.3994 \text{ in}^4$

Part ②: $\bar{I}_y = \frac{1}{12}(6 \text{ in.})(0.75 \text{ in.})^3 + (6 \text{ in.})(0.75 \text{ in.})\left[(0.9375 - 0.375) \text{ in.}\right]^2$

$= 1.6348 \text{ in}^4$

Part ③: (Same as Part ①) $\bar{I}_y = 2.3994 \text{ in}^4$

Entire Section: $\bar{I}_y = (2.3994 + 1.6348 + 2.3994) \text{ in}^4$

$= 6.434 \text{ in}^4$ $\qquad \bar{I}_y = 6.43 \text{ in}^4$ ◀

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Chapter 9, Solution 42.

Locate centroid

$$\bar{y}_1 = 52.5 \text{ mm} \quad A_1 = (84 \text{ mm})(105 \text{ mm})$$

$$= 8820 \text{ mm}^2$$

$$\bar{y}_2 = 90 \text{ mm} \quad A_2 = -\frac{1}{2}(42 \text{ mm})(45 \text{ mm})$$

$$= -945 \text{ mm}^2$$

Then $$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$= \frac{(52.5 \text{ mm})(8820 \text{ mm}^2) + (90 \text{ mm})(-945 \text{ mm}^2)}{8820 \text{ mm}^2 - 945 \text{ mm}^2}$$

$$= 48.0 \text{ mm}$$

Have $$\bar{I}_x = (I_x)_1 - (I_x)_2$$

$$= \left[\frac{1}{12}(84 \text{ mm})(105 \text{ mm})^3 + (8820 \text{ mm}^2)(52.5 \text{ mm} - 48.0 \text{ mm})^2\right]$$

$$- \left[\frac{1}{36}(42 \text{ mm})(45 \text{ mm})^3 + (945 \text{ mm}^2)(90.0 \text{ mm} - 48.0 \text{ mm})^2\right]$$

$$= \left[(8103375 + 178605) - (106312.5 + 1666980)\right] \text{ mm}^4$$

$$\bar{I}_x = 6.51 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

And $$\bar{I}_y = (I_y)_1 - (I_y)_2$$

$$= \left[\frac{1}{12}(105 \text{ mm})(84 \text{ mm})^3\right] - 2\left[\frac{1}{12}(45 \text{ mm})(21 \text{ mm})^3\right]$$

$$= \left[5186160 - 2(34728.75)\right] \text{ mm}^4$$

$$\bar{I}_y = 5.12 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 43.

Locate centroid:

$\bar{x}_1 = 8 \text{ in.} \quad \bar{y}_1 = 3 \text{ in.} \quad A_1 = (16)(6) = 96 \text{ in}^2$

$\bar{x}_2 = -1.5 \text{ in.} \quad \bar{y}_2 = 2 \text{ in.} \quad A_2 = (3)(4) = 12 \text{ in}^2$

$\bar{x}_3 = 14 \text{ in.} \quad \bar{y}_3 = -5.25 \text{ in.} \quad A_3 = (4)(10.5) = 42 \text{ in}^2$

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A}$$

$$= \frac{\left[8(96) - 1.5(12) + 14(42)\right] \text{in}^3}{(96 + 12 + 42) \text{ in}^2} = 8.92 \text{ in.}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{\left[3(96) + 2(12) - 5.25(42)\right] \text{in}^3}{(96 + 12 + 42) \text{ in}^2} = 0.610 \text{ in.}$$

$$\bar{I}_x = \left(I_{x_1}\right) + \left(I_{x_2}\right) + \left(I_{x_3}\right)$$

$$= \left[\frac{1}{12}(16)(6)^3 + (96)(3 - 0.610)^2\right] \text{in}^4 + \left[\frac{1}{12}(3)(4)^3 + (12)(2 - 0.610)^2\right] \text{in}^4$$

$$+ \left[\frac{1}{12}(4)(10.5)^3 + (42)(-5.25 - 0.610)^2\right] \text{in}^4$$

$$= \left[(288 + 548.36) + (16 + 23.185) + (385.875 + 1442.26)\right] \text{in}^4$$

$$= 2703.7 \text{ in}^4$$

$\bar{I}_x = 2700 \text{ in}^4$ ◀

$$\bar{I}_y = \left(I_{y_1}\right) + \left(I_{y_2}\right) + \left(I_{y_3}\right)$$

$$= \left[\frac{1}{12}(6)(16)^3 + (96)(8 - 8.92)^2\right] \text{in}^4 + \left[\frac{1}{12}(4)(3)^3 + (12)(-1.5 - 8.92)^2\right] \text{in}^4$$

$$+ \left[\frac{1}{12}(10.5)(4)^3 + (42)(14 - 8.92)^2\right] \text{in}^4$$

$$= \left[(2048 + 81.254) + (9 + 1302.92) + (56 + 1083.87)\right] \text{in}^4$$

$$= 4581.0 \text{ in}^4$$

$\bar{I}_y = 4580 \text{ in}^4$ ◀

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Chapter 9, Solution 44.

Locate centroid:

$$\bar{x}_1 = 20 \text{ mm} \quad \bar{y}_1 = 45 \text{ mm} \quad A_1 = (40 \text{ mm})(90 \text{ mm}) = 3600 \text{ mm}^2$$

$$\bar{x}_2 = 50 \text{ mm} \quad \bar{y}_2 = 51 \text{ mm} \quad A_2 = \frac{1}{2}(48 \text{ mm})(30 \text{ mm}) = 720 \text{ mm}^2$$

Then $\bar{x} = \dfrac{\Sigma \bar{x}_i A_i}{\Sigma A_i}$

$$= \frac{(20 \text{ mm})(3600 \text{ mm}^2) + (50 \text{ mm})(720 \text{ mm}^2)}{3600 \text{ mm}^2 + 720 \text{ mm}^2}$$

$$= 25.0 \text{ mm}$$

And $\bar{y} = \dfrac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$

$$= \frac{(45 \text{ mm})(3600 \text{ mm}^2) + (51 \text{ mm})(720 \text{ mm}^2)}{3600 \text{ mm}^2 + 720 \text{ mm}^2}$$

$$= 46.0 \text{ mm}$$

Now $\bar{I}_x = (I_x)_1 + (I_x)_2$

$$= \left[\frac{1}{12}(40 \text{ mm})(90 \text{ mm})^3 + (3600 \text{ mm}^2)(1 \text{ mm})^2\right]$$

$$+\left[\frac{1}{36}(30 \text{ mm})(24 \text{ mm})^3 + \frac{1}{2}(720 \text{ mm}^2)(59 \text{ mm} - 46.0 \text{ mm})^2\right]$$

$$+\left[\frac{1}{36}(30 \text{ mm})(24 \text{ mm})^3 + \frac{1}{2}(720 \text{ mm}^2)(46.0 \text{ mm} - 43 \text{ mm})^2\right]$$

$$= \left[(2.430 \times 10^6 + 3600) + (11.520 \times 10^3 + 60.840 \times 10^3) + (11.520 \times 10^3 + 3240)\right] \text{mm}^4$$

$$\bar{I}_x = 2.52 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

continued

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And $$\bar{I}_y = \left(I_y\right)_1 + \left(I_y\right)_2$$

$$= \left[\frac{1}{12}(90\text{ mm})(40\text{ mm})^3 + \left(3600\text{ mm}^2\right)(5\text{ mm})^2\right]$$

$$+ \left[\frac{1}{36}(48\text{ mm})(30\text{ mm})^3 + \left(720\text{ mm}^2\right)(25\text{ mm})^2\right]$$

$$= \left[\left(480\times10^3 + 90\times10^3\right) + \left(36\times10^3 + 450\times10^3\right)\right]\text{mm}^4$$

$$\bar{I}_y = 1.056\times10^6\text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 45.

Dimensions in mm

Determination of centroid, C:

Part	Area mm^2	$\bar{y}$ mm	$\bar{y}A$ mm^3
1	$\frac{1}{2}(160)(80) = 6400$	$\frac{80}{3}$	170.67×10^3
2	$-\frac{1}{2}(80)(60) = -24$	20	-48.0×10^3
Σ	4000		122.67×10^3

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{122.67 \times 10^3 \text{ mm}^3}{4000 \text{ mm}^2} = 30.667 \text{ mm}$$

(a) Polar moment of inertia with respect to point O, J_O:

Part ①: $$I_x = \frac{1}{12}(160 \text{ mm})(80 \text{ mm})^3 = 6.8267 \times 10^6 \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(80 \text{ mm})(80 \text{ mm})^3\right] = 6.8267 \times 10^6 \text{ mm}^4$$

$$J_O = I_x + I_y = (6.8267 + 6.8267) \times 10^6 = 13.653 \times 10^6 \text{ mm}^4$$

Part ②: $$I_x = \frac{1}{12}(80 \text{ mm})(60 \text{ mm})^3 = 1.440 \times 10^6 \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(60 \text{ mm})(40 \text{ mm})^3\right] = 0.640 \times 10^6 \text{ mm}^4$$

$$J_O = I_x + I_y = (1.440 + 0.640) \times 10^6 = 2.080 \times 10^6 \text{ mm}^4$$

continued

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Entire Section:

$$J_O = \left(J_{O_1}\right) - \left(J_{O_2}\right) = \left[13.653 \times 10^6 - 2.080 \times 10^6\right] \text{mm}^4$$

$$= 11.573 \times 10^6 \text{ mm}^4 \qquad J_O = 11.57 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

(*b*) Polar moment of inertia with respect to centroid, J_C:

$$J_O = J_C + A\bar{y}^2$$

$$11.573 \times 10^6 \text{ mm}^4 = J_C + \left(4000 \text{ mm}^2\right)\left(30.667 \text{ mm}\right)^2$$

$$J_C = 7.8116 \times 10^6 \text{ mm}^4 \qquad J_C = 7.81 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 46.

All dimensions in mm

Locate centroid:

By symmetry, $\bar{y} = 0$

$\bar{x}_1 = 0 \qquad A_1 = \pi(100\text{ mm})(60\text{ mm}) = 6000\pi\text{ mm}^2$

$$\bar{x}_2 = \frac{4(45\text{ mm})}{3\pi} = \frac{60}{\pi}\text{ mm}$$

$$A_2 = -\frac{\pi}{2}(45\text{mm})^2 = -1012.5\pi\text{ mm}^2$$

$$\bar{x}_3 = \frac{-4(30\text{ mm})}{3\pi} = -\frac{40}{\pi}\text{ mm}$$

$$A_3 = -\frac{\pi}{2}(30\text{ mm})^2 = -450\pi\text{ mm}^2$$

$$\bar{x} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{\left[0(6000\pi) + \frac{60}{\pi}(-1012.5\pi) - \frac{40}{\pi}(-450\pi)\right]\text{mm}^3}{(6000\pi - 1012.5\pi - 450\pi)\text{ mm}^2} = -2.9990\text{ mm}$$

(*a*) Have $J_O = (J_O)_1 - (J_O)_2 - (J_O)_3$

$$= \left[\frac{1}{4}\pi(100)(60)(100^2 + 60^2)\right]\text{mm}^4 - \left[\frac{1}{4}\pi(45)^4\right]\text{mm}^4 - \left[\frac{1}{4}\pi(30)^4\right]\text{mm}^4$$

$$= \pi(20\ 400\ 000 - 1\ 025\ 156 - 202\ 500)\text{ mm}^4$$

$$= 60232000\text{ mm}^4$$

or $J_O = 60.2 \times 10^6\text{ mm}^4$ ◀

(*b*) Have $J_O = \bar{J}_C + A\bar{x}^2$

With $A = (6000\pi - 1012.5\pi - 450\pi)\text{ mm}^2$

$= 4537.5\pi\text{ mm}^2$

Then $\bar{J}_C = 60\ 232\ 000\text{ mm}^4 - (4537.5\pi\text{ mm}^2)(-2.9990\text{ mm})^2$

$= 60\ 232\ 000\text{ mm}^4 - 128209\text{ mm}^4$

$= 60\ 104\ 000\text{ mm}^4$

or $\bar{J}_C = 60.1 \times 10^6\text{ mm}^4$ ◀

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Chapter 9, Solution 47.

Locate centroid:

$\bar{x}_1 = 4$ in. $\quad \bar{y}_1 = -1.15$ in. $\quad A_1 = (16)(2.3) = 36.8 \text{ in}^2$

$\bar{x}_2 = 4$ in. $\quad \bar{y}_2 = 3.2$ in. $\quad A_2 = \frac{1}{2}(12)(9.6) = 57.6 \text{ in}^2$

$\bar{x}_3 = 2$ in. $\quad \bar{y}_3 = 1.6$ in. $\quad A_3 = \frac{1}{2}(6)(4.8) = -14.4 \text{ in}^2$

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left[4(36.8) + 4(57.6) + 2(-14.4)\right] \text{in}^3}{(36.8 + 57.6 - 14.4) \text{ in}^2} = -\frac{348.8 \text{ in}^3}{80 \text{ in}^2} = 4.360 \text{ in.}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{\left[-1.15(36.8) + 3.2(57.6) + 1.6(-14.4)\right] \text{in}^3}{80 \text{ in}^2} = -\frac{118.96 \text{ in}^3}{80 \text{ in}^2} = 1.487 \text{ in.}$$

(*a*) Have $I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$

$$= \frac{1}{3}(16)(2.3)^3 \text{ in}^4 + \frac{1}{12}(12)(9.6)^3 \text{ in}^4 - \frac{1}{12}(6)(4.8)^3 \text{ in}^4$$

$$= 64.891 \text{ in}^4 + 884.736 \text{ in}^4 - 55.296 \text{ in}^4$$

$$= 894.33 \text{ in}^4$$

$$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

$$= \left[\frac{1}{12}(2.3)(16)^3 + 2.3(16)(4)^2\right] \text{in}^4 + \frac{1}{12}(9.6)(12)^3 \text{ in}^4 - \frac{1}{12}(4.8)(6)^3 \text{ in}^4$$

$$= 1373.87 \text{ in}^4 + 1382.4 \text{ in}^4 - 86.4 \text{ in}^4$$

$$= 2669.9 \text{ in}^4$$

Now $J_O = I_x + I_y = 894.33 \text{ in}^4 + 2669.9 \text{ in}^4$

$$= 3564.2 \text{ in}^4$$

$J_O = 3560 \text{ in}^4$ ◀

(*b*) Have $J_O = \bar{J}_C + Ad^2$ where $d^2 = \bar{x}^2 + \bar{y}^2$

Then $\bar{J}_C = 3564.2 \text{ in}^4 - (80 \text{ in}^2)\left[(4.360 \text{ in.})^2 + (1.487 \text{ in.})^2\right]$

$$= 3564.2 \text{ in}^4 - 1697.66 \text{ in}^4$$

$$= 1866.54 \text{ in}^4$$

$\bar{J}_C = 1867 \text{ in}^4$ ◀

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Chapter 9, Solution 48.

Locate centroid:

$$\bar{y}_1 = \frac{4(12 \text{ in.})}{3\pi} = \frac{16}{\pi} \text{ in.} \qquad A_1 = \frac{\pi}{2}(12 \text{ in})^2 = 72\pi \text{ in}^2$$

$$\bar{y}_2 = 4 \text{ in.} \qquad A_2 = -(12 \text{ in.})(8 \text{ in.}) = -96 \text{ in}^2$$

Then $\bar{y} = \dfrac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$

$$= \frac{\left(\frac{16}{\pi} \text{ in.}\right)\left(72\pi \text{ in}^2\right) + (4 \text{ in.})\left(-96 \text{ in}^2\right)}{72\pi \text{ in}^2 - 96 \text{ in}^2}$$

$$= 5.8989 \text{ in.}$$

(*a*) Have $J_O = I_x + I_y$

Where $I_x = (I_x)_1 - (I_x)_2$

$$= \left[\frac{\pi}{8}(12 \text{ in.})^4 - \frac{1}{3}(12 \text{ in.})(8 \text{ in.})^3\right]$$

$$= 6095.0 \text{ in}^4$$

And $I_y = (I_y)_1 - (I_y)_2$

$$= \left[\frac{\pi}{8}(12 \text{ in.})^4 - \frac{1}{12}(8 \text{ in.})(12 \text{ in.})^3\right]$$

$$= 6991.0 \text{ in}^4$$

Then $J_O = (6095.0 + 6991.0) \text{ in}^4$

$$= 13086.0 \text{ in}^4$$

$J_O = 13.09 \times 10^3 \text{ in}^4$ ◀

(*b*) Have $J_O = \bar{J}_C + A\bar{y}^2$

$$13086.0 \text{ in}^4 = \bar{J}_C + \left[(72\pi - 96) \text{ in}^2\right](5.8989 \text{ in.})^2$$

$$\bar{J}_C = 8.555 \times 10^3 \text{ in}^4$$

$\bar{J}_C = 8.56 \times 10^3 \text{ in}^4$ ◀

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Chapter 9, Solution 49.

From Fig. 9.13A

Area $= A = 4.75 \text{ in}^2$

$I_{x'} = 17.4 \text{ in}^4$

$I_{y'} = 6.27 \text{ in}^4$

$$2\left[I_{x'} + Ad^2\right] = 2\left[17.4 \text{ in}^4 + \left(4.75 \text{ in}^2\right)\left(3.00 - 1.990\right)^2 \text{ in}^2\right]$$

$$= 2\left[17.4 \text{ in}^4 + 4.8455 \text{ in}^4\right] = 44.491 \text{ in}^4$$

$I_x = 44.5 \text{ in}^4$ ◀

$$k_x^2 = \frac{I_x}{A} = \frac{44.491 \text{ in}^4}{2\left(4.75 \text{ in}^2\right)} = 4.683 \text{ in}^2$$

$k_x = 2.16 \text{ in.}$ ◀

$$I_y = 2\left[I_{y'} + Ad^2\right] = 2\left[6.27 \text{ in}^4 + \left(4.75 \text{ in}^2\right)\left(2.25 - 0.987\right)^2 \text{ in}^2\right]$$

$$= 2\left[6.27 \text{ in}^4 + 7.5771 \text{ in}^4\right] = 27.694 \text{ in}^4$$

$I_y = 27.7 \text{ in}^4$ ◀

$$k_y^2 = \frac{I_y}{A} = \frac{27.694 \text{ in}^4}{2\left(4.75 \text{ in}^2\right)} = 2.9152 \text{ in}^2$$

$k_y = 1.707 \text{ in.}$ ◀

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Chapter 9, Solution 50.

Data for $C250 \times 30$:

$A = 3780 \text{ mm}^2$

$I_x = 32.6 \times 10^6 \text{ mm}^4$

$I_y = 1.14 \times 10^6 \text{ mm}^4$

Total Area, $A = 2\left[3780 \text{ mm}^2 + (10 \text{ mm})(375 \text{ mm})\right] = 15.06 \times 10^3 \text{ mm}^2$

$$\bar{I}_x = 2\left[32.6 \times 10^6 \text{ mm}^4\right] + 2\left[\frac{1}{12}(375 \text{ mm})(10 \text{ mm})^3 + (375 \text{ mm})(10 \text{ mm})(132 \text{ mm})^2\right]$$

$$= 65.2 \times 10^6 \text{ mm}^4 + 130.74 \times 10^6 \text{ mm}^4$$

$$= 195.94 \times 10^6 \text{ mm}^4 \qquad \text{or} \qquad \bar{I}_x = 195.9 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{k}_x^2 = \frac{\bar{I}_x}{A} = \frac{195.94 \times 10^6 \text{ mm}^4}{15.06 \times 10^3 \text{ mm}^2} = 13.01 \times 10^3 \text{ mm}^2 \qquad \text{or} \qquad \bar{k}_x = 114.0 \text{ mm} \blacktriangleleft$$

$$\bar{I}_y = 2\left[1.14 \times 10^6 \text{ mm}^4 + \left(3780 \text{ mm}^2\right)(115.3 \text{ mm})^2\right] + 2\left[\frac{1}{12}(10 \text{ mm})(375 \text{ mm})^3\right]$$

$$= 102.783 \times 10^6 \text{ mm}^4 + 87.891 \times 10^6 \text{ mm}^4$$

$$= 190.674 \times 10^6 \text{ mm}^4$$

$$\text{or} \qquad \bar{I}_y = 190.7 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{k}_y^2 = \frac{\bar{I}_y}{A} = \frac{190.674 \times 10^6 \text{ mm}^4}{15.06 \times 10^3 \text{ mm}^2} = 12.661 \times 10^3 \text{ mm}^2 \qquad \text{or} \qquad \bar{k}_y = 112.5 \text{ mm} \blacktriangleleft$$

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Chapter 9, Solution 51.

Shape

S10 × 35: $A = 10.3 \text{ in}^2$ C10 × 20: $A = 5.88 \text{ in}^2$

$I_x = 147 \text{ in}^4$ $I_x = 78.9 \text{ in}^4$

$I_y = 8.36 \text{ in}^4$ $I_y = 2.81 \text{ in}^4$

Combined section:

$$A = A_S + 2A_C = 10.3 \text{ in}^2 + 2\left(5.88 \text{ in}^2\right) = 22.06 \text{ in}^2$$

$$\bar{I}_x = \left(I_x\right)_S + 2\left(I_x\right)_C = 147 \text{ in}^4 + 2\left(78.9 \text{ in}^4\right) = 304.8 \text{ in}^4$$

or $\bar{I}_x = 305 \text{ in}^4$ ◀

$$\bar{I}_y = \left(I_y\right)_S + 2\left[\left(I_y\right)_C + A_C d^2\right]$$

$$= 8.36 \text{ in}^4 + 2\left[2.81 \text{ in}^4 + \left(5.88 \text{ in}^2\right)\left(\frac{4.944}{2} \text{ in.} + 2.739 \text{ in.} - 0.606 \text{ in.}\right)^2\right]$$

$$= \left(8.36 + 5.62 + 249.38\right) \text{ in}^4 = 263.36 \text{ in}^4$$

or $\bar{I}_y = 263 \text{ in}^4$ ◀

$$\bar{k}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{304.8 \text{ in}^4}{22.06 \text{ in}^2}}$$ or $\bar{k}_x = 3.72 \text{ in.}$ ◀

$$\bar{k}_y = \sqrt{\frac{\bar{I}_y}{A}} = \sqrt{\frac{263.36 \text{ in}^4}{22.06 \text{ in}^2}}$$ or $\bar{k}_y = 3.46 \text{ in.}$ ◀

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Chapter 9, Solution 52.

Channel:

$$A = 3780 \text{ mm}^2$$
$$\bar{I}_x = 32.6 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = 1.14 \times 10^6 \text{ mm}^4$$

Now

$$\bar{I}_x = 2\left(\bar{I}_x\right)_C + \left(\bar{I}_x\right)_{\text{plate}}$$
$$= 2\left(32.6 \times 10^6 \text{ mm}^4\right) + \frac{d}{12}(300 \text{ mm})^3$$
$$= \left(65.2 \times 10^6 + 2.25d \times 10^6\right) \text{mm}^4$$

And

$$\bar{I}_y = 2\left(I_y\right)_{\text{channel}} + \left(\bar{I}_y\right)_{\text{plate}}$$
$$= 2\left[1.14 \times 10^6 \text{ mm}^4 + \left(3780 \text{ mm}^2\right)\left(\frac{d}{2} + 15.3 \text{ mm}\right)^2\right] + \frac{(300 \text{ mm})d^3}{12}$$
$$= \left[\left(2.28 \times 10^6 + 1890d^2 + 115.668 \times 10^3 d + 1.7697 \times 10^6\right) + 25d^3\right] \text{mm}^4$$
$$= \left(25d^3 + 1890d^2 + 115.67 \times 10^3 d + 4.0497 \times 10^6\right) \text{mm}^4$$

Given $\bar{I}_x = 16\bar{I}_y$

Then $65.2 \times 10^6 + 2.25d \times 10^6$

$$= 16\left(25d^3 + 1890d^2 + 115.67 \times 10^3 d + 4.0497 \times 10^6\right)$$

or $25d^3 + 1890d^2 - 24955d - 25300 = 0$

Solving numerically $d = 12.2935$ mm

or $d = 12.29$ mm ◀

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Chapter 9, Solution 53.

Locate centroid:

$\bar{x}_1 = 15.3 \text{ mm}$ $\bar{y}_1 = 127 \text{ mm}$ $A_1 = 3780 \text{ mm}^2$

$\bar{x}_2 = -21.4 \text{ mm}$ $\bar{y}_2 = 76 \text{ mm} - 21.4 \text{ mm} = 54.6 \text{ mm}$ $A_2 = 932 \text{ mm}^2$

$\bar{x}_3 = -21.4 \text{ mm}$ $\bar{y}_3 = 76 \text{ mm} + 21.4 \text{ mm} = 97.4 \text{ mm}$ $A_3 = 932 \text{ mm}^2$

Then $$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i}$$

$$= \frac{(15.3 \text{ mm})(3780 \text{ mm}^2) + 2\left[(-21.4 \text{ mm})(932 \text{ mm}^2)\right]}{3780 \text{ mm}^2 + 2(932 \text{ mm}^2)}$$

$$= 3.1794 \text{ mm}$$

And $$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$= \frac{(127 \text{ mm})(3780 \text{ mm}^2) + (76 \text{ mm})(2 \times 932 \text{ mm}^2)}{3780 \text{ mm}^2 + 2(932 \text{ mm}^2)}$$

$$= 110.157 \text{ mm}$$

Now $$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= \left[32.6 \times 10^6 \text{ mm}^4 + (3780 \text{ mm}^2)(127 \text{ mm} - 110.157 \text{ mm})^2\right]$$
$$+ \left[0.517 \times 10^6 \text{ mm}^4 + (932 \text{ mm}^2)(110.157 \text{ mm} - 54.6 \text{ mm})^2\right]$$
$$+ \left[0.517 \times 10^6 \text{ mm}^4 + (932 \text{ mm}^2)(110.157 \text{ mm} - 97.4 \text{ mm})^2\right]$$

continued

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$$= \left[\left(32.6\times10^6 + 1.07234\times10^6\right) + \left(0.517\times10^6 + 2.8767\times10^6\right) + \left(0.517\times10^6 + 0.151675\times10^6\right)\right] \text{mm}^4$$

$$\bar{I}_x = 37.7\times10^6 \text{ mm}^4 \blacktriangleleft$$

And $$\bar{I}_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3$$

$$= \left[1.14\times10^6 \text{ mm}^4 + \left(3780 \text{ mm}^2\right)\left(15.3 \text{ mm} - 3.1794 \text{ mm}\right)^2\right] + 2\left[0.517\times10^6 \text{ mm}^4 + \left(932 \text{ mm}^4\right)\left(3.1794 \text{ mm} + 21.4 \text{ mm}\right)^2\right]$$

$$= \left[\left(1.14\times10^6 + 0.55532\times10^6\right) + 2\left(0.517\times10^6 + 0.56306\times10^6\right)\right] \text{mm}^4$$

$$\bar{I}_y = 3.86\times10^6 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 54.

Angle: $L3 \times 3 \times \frac{1}{4}$

$$A = 1.44 \text{ in}^2 \qquad \bar{I}_x = \bar{I}_y = 1.24 \text{ in}^4$$

$L6 \times 4 \times \frac{1}{2}$:

$$A = 4.75 \text{ in}^2 \qquad \bar{I}_x = 6.27 \text{ in}^4 \qquad \bar{I}_y = 17.4 \text{ in}^4$$

Plate:

$$A = (27 \text{ in.})(0.8 \text{ in.}) = 21.6 \text{ in}^2$$

$$\bar{I}_x = \frac{1}{12}(0.8 \text{ in.})(27 \text{ in.})^3 = 1312.2 \text{ in}^4$$

$$\bar{I}_y = \frac{1}{12}(27 \text{ in.})(0.8 \text{ in.})^3 = 1.152 \text{ in}^4$$

continued

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Centroid:

$$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or

$$\bar{Y} = \frac{2\left[\left(1.44\text{ in}^2\right)\left(27\text{ in.} - 0.84\text{ in.}\right)\right] + 2\left[\left(4.75\text{ in}^2\right)\left(0.987\text{ in.}\right)\right] + \left(21.6\text{ in}^2\right)\left(13.5\text{ in.}\right)}{2\left(1.44\text{ in}^2 + 4.75\text{ in}^2\right) + 21.6\text{ in}^2}$$

$$= \frac{376.31\text{ in}^3}{33.98\text{ in}^2} = 11.0745\text{ in.}$$

Now

$$\bar{I}_x = 2\left(I_x\right)_1 + 2\left(I_x\right)_3 + \left(I_x\right)_2$$

$$= 2\left[6.27 + 4.75\left(11.075 - 0.987\right)^2\right]\text{in}^4 + 2\left[1.24 + 1.44\left(27 - 0.842 - 11.075\right)^2\right]\text{in}^4$$

$$+\left[1312.2 + 21.6\left(13.5 - 11.075\right)^2\right]\text{in}^4$$

$$= 2\left(489.67\right)\text{in}^4 + 2\left(328.84\right)\text{in}^4 + 1439.22\text{ in}^4 = 3076.24\text{ in}^4$$

or $\bar{I}_x = 3076\text{ in}^4$ ◀

Also

$$\left(\bar{I}_y\right) = 2\left(I_y\right)_1 + 2\left(I_y\right)_3 + \left(I_y\right)_2$$

$$= 2\left[17.4 + 4.75\left(0.4 + 1.99\right)^2\right]\text{in}^4 + 2\left[1.24 + 1.44\left(0.4 + 0.842\right)^2\right]\text{in}^4 + 1.152\text{ in}^4$$

$$= 2\left(44.532\right)\text{in}^4 + 2\left(3.4613\right)\text{in}^4 + 1.152\text{ in}^4$$

$$= 97.139\text{ in}^4$$

or $\bar{I}_y = 97.1\text{ in}^4$ ◀

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Chapter 9, Solution 55.

Angle:

$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$$

Plate:

$$A = (200 \text{ mm})(10 \text{ mm}) = 2000 \text{ mm}^2$$

$$\bar{I}_x = \frac{1}{12}(200 \text{ mm})(10 \text{ mm})^3 = 0.01667 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{1}{12}(10 \text{ mm})(200 \text{ mm})^3 = 6.6667 \times 10^6 \text{ mm}^4$$

Centroid

$$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or

$$\bar{Y} = \frac{2\left(2420 \text{ mm}^2\right)(44.4 \text{ mm}) + 2000 \text{ mm}^2(-5 \text{ mm})}{\left[2(2420) + 2000\right]\text{mm}^2} = \frac{204896 \text{ mm}^3}{6840 \text{ mm}^2}$$

$$= 29.9556 \text{ mm}$$

Now

$$\bar{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$$

$$= 2\left[3.93 \times 10^6 + (2420)(44.4 - 29.9556)^2\right]\text{mm}^4$$

$$+ \left[0.01667 \times 10^6 + (2000)(29.9556 + 5)^2\right]\text{mm}^4$$

$$= 2\left(4.4349 \times 10^6\right)\text{mm}^4 + 2.4605 \times 10^6 \text{ mm}^4$$

$$= 11.3303 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 11.33 \times 10^6 \text{ mm}^4$ ◀

continued

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Also $$\bar{I}_y = 2\left(I_y\right)_{\text{angle}} + \left(\bar{I}_y\right)_{\text{plate}}$$

Where $$\left(\bar{I}_y\right)_{\text{angle}} = 1.06 \times 10^6 \text{ mm}^4 + \left(2420 \text{ mm}^2\right)\left(b - 19.0 \text{ mm}\right)^2$$

$$= \left[1.06 \times 10^6 + (2420)\left(b^2 - 38b + 361\right)\right] \text{mm}^4$$

$$= \left[2420b^2 - 91960b + 1.93362 \times 10^6\right] \text{mm}^4$$

and $$\left(\bar{I}_y\right)_{\text{plate}} = 6.6667 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_y = 3\left(\bar{I}_x\right)$$

Then $$2\left[2420b^2 - 91960b + 1.93362 \times 10^6\right] \text{mm}^4 + 6.6667 \times 10^6 \text{ mm}^4 = 3\left[\left(11.33 \times 10^6\right) \text{mm}^4\right]$$

or $$2420b^2 - 91960b + 1.93362 \times 10^6 - 13.662 \times 10^6 = 0$$

$$b^2 - 38.0b - 4846.5 = 0$$

$$b = 91.16 \text{ mm}$$

or $b = 91.2$ mm ◀

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Chapter 9, Solution 56.

(*a*)

$$x_A = 1.78 \text{ in.} \qquad A_A = 8.44 \text{ in}^2$$

$$x_C = -0.499 \text{ in.} \qquad A_C = 3.09 \text{ in}^2$$

$$x_P = \left(6 - \frac{a}{2}\right) \text{in.} \qquad A_P = (0.75a) \text{ in}^2$$

From the condition $\bar{x} = \dfrac{\Sigma \bar{x} A}{\Sigma A} = 0$ or $\Sigma \bar{x} A = 0$

$$(1.78 \text{ in.})(8.44 \text{ in}^2) - (0.499 \text{ in.})(3.09 \text{ in}^2) + \left[\left(6 - \frac{a}{2}\right) \text{in.}\right](0.75a \text{ in}^2) = 0$$

or $a^2 - 12a - 35.950 = 0$ $\qquad a = 14.4823$ in.

or $\qquad a = 14.48$ in. ◀

and $A_P = (0.75 \text{ in.})(14.4823 \text{ in.}) = 10.8617 \text{ in}^2$

(*b*) Locate centroid

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{(1.78 \text{ in.})(8.44 \text{ in}^2) + \left(\frac{6}{2} \text{ in.}\right)(3.09 \text{ in}^2) - (0.375 \text{ in.})(10.8617 \text{ in}^2)}{(8.44 + 3.09 + 10.8617) \text{ in}^2}$$

$$= 0.90302 \text{ in.}$$

continued

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Now $\bar{I}_x = (I_x)_A + (I_x)_C + (I_x)_P$

$$= \left[28.2 \text{ in}^4 + \left(8.44 \text{ in}^2\right)\left(1.78 \text{ in.} - 0.90302 \text{ in.}\right)^2\right]$$

$$+ \left[15.2 \text{ in}^4 + \left(3.09 \text{ in}^2\right)\left(3.0 \text{ in.} - 0.90302 \text{ in.}\right)^2\right]$$

$$+ \left[\frac{1}{12}(14.4823 \text{ in.})(0.75 \text{ in.})^3 + \left(10.8617 \text{ in}^2\right)\left(0.375 \text{ in.} + 0.90302 \text{ in.}\right)^2\right]$$

$$= \left[(28.2 + 6.4912) + (15.2 + 13.5877) + (0.5091 + 17.7408)\right] \text{ in}^4$$

$$= 81.729 \text{ in}^4$$

or $\bar{I}_x = 81.7 \text{ in}^4$ ◀

and $\bar{I}_y = (I_y)_A + (I_y)_C + (I_y)_P$

$$= \left[28.2 \text{ in}^4 + \left(8.44 \text{ in}^2\right)(1.78 \text{ in.})^2\right] + \left[0.866 \text{ in}^4 + \left(3.09 \text{ in}^2\right)(0.499 \text{ in.})^2\right]$$

$$+ \left[\frac{1}{12}(0.75 \text{ in.})(14.4823 \text{ in.})^3 + \left(10.8617 \text{ in}^2\right)\left(6 \text{ in.} - \frac{14.4823 \text{ in.}}{2}\right)^2\right]$$

$$= \left[(28.2 + 26.741) + (0.866 + 0.7694) + (189.842 + 16.7319)\right] \text{ in}^4$$

$$= 263.15 \text{ in}^4$$

or $\bar{I}_y = 263 \text{ in}^4$ ◀

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Chapter 9, Solution 57.

From Sec. 9.2: $R = \gamma \int y dA$

$$M_{AA'} = \gamma \int y^2 dA$$

Let y_P = Distance of center of pressure from AA'.

We must have

$$Ry_P = M_{AA'}: \qquad y_P = \frac{M_{AA'}}{R} = \frac{\gamma \int y^2 dA}{\gamma \int y dA} = \frac{I_{AA'}}{\bar{y}A}$$

For a triangular panel:

$$I_{AA'} = \frac{1}{12}ah^3 \qquad \bar{y} = \frac{1}{3}h \qquad A = \frac{1}{2}ah$$

Thus $$y_P = \frac{\frac{1}{12}ah^3}{\left(\frac{1}{3}h\right)\left(\frac{1}{2}ah\right)} = \frac{1}{2}h$$

$$y_P = \frac{1}{2}h \blacktriangleleft$$

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Chapter 9, Solution 58.

From Sec. 9.2: $R = \gamma \int y dA$

$$M_{AA'} = \gamma \int y^2 dA$$

Let y_P = Distance of center of pressure from AA'.

We must have

$$Ry_P = M_{AA'}: \qquad y_P = \frac{M_{AA'}}{R} = \frac{\gamma \int y^2 dA}{\gamma \int y dA} = \frac{I_{AA'}}{\bar{y}A}$$

For a semiellipse $\quad I_{AA'} = \frac{\pi}{8} ab^3$

$$\bar{y} = \frac{4b}{3\pi}, \quad A = \frac{\pi}{2} ab$$

Then $\quad y_P = \dfrac{\frac{\pi}{8} ab^3}{\left(\frac{4b}{3\pi}\right)\left(\frac{\pi}{2} ab\right)}$

$$\text{or} \quad y_P = \frac{3\pi}{16} b \blacktriangleleft$$

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Chapter 9, Solution 59.

From Sec. 9.2: $R = \gamma \int y dA$

$$M_{AA'} = \gamma \int y^2 dA$$

Let y_P = Distance of center of pressure from AA':

We must have

$$Ry_P = M_{AA'}: \qquad y_P = \frac{M_{AA'}}{R} = \frac{\gamma \int y^2 dA}{\gamma \int y dA} = \frac{I_{AA'}}{\bar{y}A}$$

Divide Trapezoid as shown:

$$I_{AA'} = \frac{1}{12}(a-b)h^3 + \frac{1}{3}bh^3 = \frac{1}{12}ah^3 + \frac{1}{4}bh^3$$

$$\bar{y}A = y_1A_1 + y_2A_2 = \frac{1}{3}h\left[\frac{1}{2}(a-b)h\right] + \frac{1}{2}h(bh) = \frac{1}{6}ah^2 + \frac{1}{3}bh^2$$

$$y_P = \frac{I_{AA'}}{\bar{y}A} = \frac{\frac{1}{12}ah^3 + \frac{1}{4}bh^3}{\frac{1}{6}ah^2 + \frac{1}{3}bh^2} \qquad y_P = \frac{a+3b}{2a+4b}h \blacktriangleleft$$

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Chapter 9, Solution 60.

From Sec. 9.2: $R = \gamma \int y dA$

$$M_{AA'} = \gamma \int y^2 dA$$

Let y_P = Distance of center of pressure from AA'.

We must have

$$Ry_P = M_{AA'}: \qquad y_P = \frac{M_{AA'}}{R} = \frac{\gamma \int y^2 dA}{\gamma \int y dA} = \frac{I_{AA'}}{\bar{y}A}$$

where $I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$

$$= \left[\frac{1}{3}(2r)(r)^3\right] + \left\{\left[\frac{\pi}{8}r^4 - \frac{\pi}{2}r^2\left(\frac{4r}{3\pi}\right)^2\right] + \frac{\pi}{2}r^2\left(r + \frac{4r}{3\pi}\right)^2\right\}$$

$$= \frac{2}{3}r^4 + \left(\frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} + \frac{4}{3} + \frac{9}{8\pi}\right)r^4 = \left(2 + \frac{5\pi}{8}\right)r^4$$

And $\bar{Y}A = \Sigma \bar{y}A = \left[\frac{r}{2}(2r \times r)\right] + \left[\left(r + \frac{4r}{3\pi}\right)\left(\frac{\pi}{2}r^2\right)\right]$

$$= \left(1 + \frac{\pi}{2} + \frac{2}{3}\right)r^3 = \left(\frac{5}{3} + \frac{\pi}{2}\right)r^3$$

Then $$y_P = \frac{\left(2 + \frac{5\pi}{8}\right)r^4}{\left(\frac{5}{3} + \frac{\pi}{2}\right)r^3} = 1.2242r$$

or $y_P = 1.224r$ ◀

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Chapter 9, Solution 61.

Using equation developed on page 491 of text:

$$y_P = \frac{I_x}{\bar{y}A} = \frac{I_{AA'}}{\bar{y}A} \qquad R = \gamma\bar{y}A = \rho g\bar{y}A$$

Then $$R = 920\,\frac{\text{kg}}{\text{m}^3} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 3\text{ m} \times (0.55 \times 0.25)\text{m}^2$$

$$= 3722.9\text{ N}$$

and $$I_{AA'} = \frac{1}{12}(0.55\text{ m})(0.25\text{ m})^3 + (0.55\text{ m})(0.25\text{ m})(3\text{ m})^2$$

$$= 1.238216\text{ m}^4$$

and $$\bar{y}A = (3\text{ m})(0.55\text{ m})(0.25\text{ m}) = 0.4125\text{ m}^3$$

Then $$y_P = \frac{1.238216\text{ m}^4}{0.4125\text{ m}^3} = 3.001736\text{ m}$$

FBD Cover:

Symmetry implies $F_A = F_B$ and $F_C = F_D$.

Equilibrium:

$+\circlearrowleft\ \Sigma M_{CD} = 0$: $$-(0.125 - 0.001736)\text{ m} \times (3722.9\text{ N}) + (0.25\text{ m})(2F_A) = 0$$

$$F_A = 917.80\text{ N} \qquad \text{or} \quad F_A = F_B = 918\text{ N} \blacktriangleleft$$

$\xrightarrow{+}\ \Sigma F_x = 0$: $$-2(917.80\text{ N}) + 3722.9\text{ N} - 2F_C = 0$$

$$F_C = 943.65\text{ N} \qquad \text{or} \quad F_C = F_D = 944\text{ N} \blacktriangleleft$$

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Chapter 9, Solution 62.

$$d = (h + 79 \text{ in.})$$

Using equations developed on page 491 of text:

$$y_P = \frac{I_x}{\bar{y}A} = \frac{I_{ss'}}{\bar{y}A} \qquad R = \gamma\bar{y}A$$

Now $\bar{y}A = \Sigma\bar{y}A$

$$= (h + 17 \text{ in.})\left(\frac{1}{2} \times 120 \text{ in.} \times 51 \text{ in.}\right)$$

$$+ (h + 34 \text{ in.})\left(\frac{1}{2} \times 84 \text{ in.} \times 51 \text{ in.}\right)$$

$$= (5202h + 124848) \text{ in}^3$$

$$= (36.125h + 72.25) \text{ ft}^3$$

Then, $R = \gamma\bar{y}A = (62.4 \text{ lb/ft}^3)(36.125h + 72.75) \text{ ft}^3$

$$= 2254.2(h + 2) \text{ lb}$$

Also, $I_{ss'} = (I_{ss'})_1 + (I_{ss'})_2$

$$= \left\{\frac{1}{36}\left(\frac{120}{12} \text{ ft}\right)\left(\frac{51}{12} \text{ ft}\right)^3 + \left[\frac{1}{2}\left(\frac{120}{12} \text{ ft}\right)\left(\frac{51}{12} \text{ ft}\right)\right]\left(h + \frac{17}{12}\right)^2 \text{ ft}^2\right\}$$

$$+ \left\{\frac{1}{36}\left(\frac{84}{12} \text{ ft}\right)\left(\frac{51}{12} \text{ ft}\right)^3 + \left[\frac{1}{2}\left(\frac{84}{12} \text{ ft}\right)\left(\frac{51}{12} \text{ ft}\right)\right]\left(h + \frac{34}{12}\right)^2 \text{ ft}^2\right\}$$

$$= \left[21.324 + 21.25\left(h^2 + \frac{34}{12}h + \frac{289}{144}\right) + 14.9266 + 14.875\left(h^2 + \frac{68}{12}h + \frac{1156}{144}\right)\right] \text{ ft}^4$$

$$= (36.125h^2 + 144.5h + 198.311) \text{ ft}^4$$

continued

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Then
$$y_P = \frac{I_{ss'}}{\bar{y}A} = \frac{\left(36.125h^2 + 144.5h + 198.311\right)\text{ft}^4}{\left(36.125h + 72.25\right)\text{ft}^3}$$

$$= \frac{h^2 + 4h + 5.4896}{h + 2}\ \text{ft}$$

FBD of Gate: For gate to open:

$$+\circlearrowleft \Sigma M_{AB} = 0: \quad -M_{\text{open}} + \left(y_P - h\right)R = 0$$

$$2\left(8000\ \text{lb}\cdot\text{ft}\right) - \left[\left(\frac{h^2 + 4h + 5.4896}{h + 2} - h\right)\text{ft}\right]\left[2254.2\left(h + 2\right)\ \text{lb}\right] = 0$$

or $\quad 2h - 1.60826 = 0 \qquad h = 0.80413\ \text{ft}$

Thus $\quad d = h + \frac{79}{12}\ \text{ft} = \left(0.80413 + \frac{79}{12}\right)\text{ft}$

$= 7.3875\ \text{ft}$ $\qquad$ or $d = 7.39\ \text{ft}$ ◀

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Chapter 9, Solution 63.

Have $$\bar{x} = \frac{\int \bar{x}_{EL} dV}{\int dV}$$

where $dV = y dA$ and $\bar{x}_{EL} = x$

Now $$y = \frac{60}{300} x = \frac{1}{5} x$$

Then $$\bar{x} = \frac{\int x\left(\frac{1}{5}x\right) dA}{\int \left(\frac{1}{5}x\right) dA}$$

$$= \frac{\int x^2 dA}{\int x dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

where $(I_z)_A$ is the moment of inertia of the area with respect to the z axis, and $\bar{x}$ is analogous to y_p

Now $$(I_z)_A = \frac{1}{36}(240\text{ mm})(300\text{ mm})^3 + \frac{1}{2}\left[(240\text{ mm})(300\text{ mm})\right](200\text{ mm})^2$$

$$= 1.620 \times 10^9 \text{ mm}^4$$

and $$\bar{x}A = (200\text{ mm})\left[\frac{1}{2}(240\text{ mm})(300\text{ mm})\right] = 7.20 \times 10^6 \text{ mm}^3$$

Then $$\bar{x} = \frac{1.620 \times 10^9 \text{ mm}^4}{7.20 \times 10^6 \text{ mm}^3}$$

or $\bar{x} = 225$ mm ◀

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Chapter 9, Solution 64.

$\bar{x}_v$ of the volume is defined by

$$\bar{x}_v V = \int \bar{x}_{EL} dV$$

Selecting the element of volume shown

$$dV = y dA = kx dA$$

$$V = k\int x dA = k\bar{x}_A A$$

Where $\bar{x}_A =$ coordinate of the centroid of area A

$$\bar{x}_v V = \int \bar{x}_{EL} dV = \int x\left(kx\, dA\right) = k\int x^2 dA = kI_z$$

Where $I_z =$ moment of inertia of area with respect to z-axis.

Thus $\bar{x}_v = \dfrac{kI_z}{k\bar{x}_A A} = \dfrac{I_z}{\bar{x}_A A}$ which is the same as for center of pressure.

For circular area:

$$I_z = I_{z'} + Aa^2 = \frac{1}{4}\pi a^4 + \left(\pi a^2\right)a^2 = \frac{5}{4}\pi a^4$$

$$\bar{x}_A = a$$

$$A = \pi a^2$$

Thus $\bar{x}_v = \dfrac{I_z}{\bar{x}_A A} = \dfrac{\frac{5}{4}\pi a^4}{a\left(\pi a^2\right)}$ $\qquad \bar{x}_v = \dfrac{5}{4}a$ ◀

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Chapter 9, Solution 65.

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dF exerted on an infinitesimal area dA is

$$dF = (\gamma y\sin\theta)dA$$

Equivalence of the force **P** and the system of infinitesimal forces dF requires

$$\Sigma F: \quad P = \int dF = \int \gamma y\sin\theta dA = \gamma\sin\theta\int ydA$$

$$\text{or} \quad P = \gamma A\bar{y}\sin\theta \blacktriangleleft$$

Equivalence of the force and couple $(\mathbf{P}, \mathbf{M}_{x'} + \mathbf{M}_{y'})$ and the system of infinitesimal hydrostatic forces requires

$$\Sigma M_x: \quad -\bar{y}P - M_{x'} = \int(-ydF)$$

Now

$$-\int ydF = -\int y(\gamma y\sin\theta)dA = -\gamma\sin\theta\int y^2 dA$$

$$= -(\gamma\sin\theta)I_x$$

Then

$$-\bar{y}P - M_{x'} = -(\gamma\sin\theta)I_x$$

or

$$M_{x'} = (\gamma\sin\theta)I_x - \bar{y}(\gamma A\bar{y}\sin\theta)$$

$$= \gamma\sin\theta\left(I_x - A\bar{y}^2\right)$$

$$\text{or} \quad M_{x'} = \gamma\bar{I}_{x'}\sin\theta \blacktriangleleft$$

$$\Sigma M_y: \quad \bar{x}P + M_{y'} = \int xdF$$

Now

$$\int xdF = \int x(\gamma y\sin\theta)dA = \gamma\sin\theta\int xydA$$

$$= (\gamma\sin\theta)I_{xy} \ (\text{Equation 9.12})$$

continued

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Then $$\bar{x}P + M_{y'} = (\gamma \sin\theta) I_{xy}$$

or $$M_{y'} = (\gamma \sin\theta) I_{xy} - \bar{x}(\gamma A \bar{y} \sin\theta)$$

$$= \gamma \sin\theta \left(I_{xy} - A\bar{x}\,\bar{y} \right)$$

or, using Equation 9.13, or $M_{y'} = \gamma \bar{I}_{x'y'} \sin\theta$ ◀

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Chapter 9, Solution 66.

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dP exerted on an infinitesimal area dA is

$$dP = (\gamma y\sin\theta)dA$$

The magnitude **P** of the resultant force acting on the plane area is then

$$P = \int dP = \int \gamma y\sin\theta dA = \gamma\sin\theta\int ydA$$

$$= \gamma\sin\theta(\bar{y}A)$$

Now $$\bar{p} = \gamma\bar{y}\sin\theta \qquad \therefore\ P = \bar{p}A \blacktriangleleft$$

Next observe that the resultant **P** is equivalent to the system of infinitesimal forces d**P**. Equivalence then requires

$$\Sigma M_x:\quad -y_P P = -\int ydP$$

Now $$\int ydP = \int y(\gamma y\sin\theta)dA = \gamma\sin\theta\int y^2 dA$$

$$= (\gamma\sin\theta)I_x$$

Then $$y_P P = (\gamma\sin\theta)I_x$$

or $$y_P = \frac{(\gamma\sin\theta)I_x}{\gamma\sin\theta(\bar{y}A)}$$

$$\text{or}\quad y_P = \frac{I_x}{A\bar{y}} \blacktriangleleft$$

$$\Sigma M_y:\quad x_P P = \int xdP$$

Now $$\int xdP = \int x(\gamma y\sin\theta)dA = \gamma\sin\theta\int xydA$$

$$= (\gamma\sin\theta)I_{xy}\ (\text{Equation 9.12})$$

continued

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Then $$x_P P = (\gamma \sin\theta) I_{xy}$$

or $$x_P = \frac{(\gamma \sin\theta) I_{xy}}{\gamma \sin\theta (\bar{y} A)}$$

or $x_P = \dfrac{I_{xy}}{A\bar{y}}$ ◀

Now $$I_x = \bar{I}_{x'} + A\bar{y}^2$$

From above $$I_x = (A\bar{y}) y_P$$

By definition $$\bar{I}_{x'} = k_{x'}^2 A$$

Substituting $$(A\bar{y}) y_P = k_{x'}^2 A + A\bar{y}^2$$

Rearranging yields $y_P - \bar{y} = \dfrac{k_{x'}^2}{\bar{y}}$ ◀

Although $k_{x'}$ is not a function of the depth of the area (it depends only on the shape of A), $\bar{y}$ is dependent on the depth.

$$\therefore\ (y_P - \bar{y}) = f(\text{depth})$$

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Chapter 9, Solution 67.

First note
$$y = a\sqrt{1 - \frac{x^2}{4a^2}}$$
$$= \frac{1}{2}\sqrt{4a^2 - x^2}$$

Have
$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where
$$d\bar{I}_{x'y'} = 0 \qquad (\text{symmetry}) \qquad \bar{x}_{EL} = x$$
$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{4}\sqrt{4a^2 - x^2}$$
$$dA = ydx = \frac{1}{2}\sqrt{4a^2 - x^2}\,dx$$

Then
$$I_{xy} = \int dI_{xy} = \int_0^{2a} x\left(\frac{1}{4}\sqrt{4a^2 - x^2}\right)\left(\frac{1}{2}\sqrt{4a^2 - x^2}\right)dx$$
$$= \frac{1}{8}\int_0^{2a}\left(4a^2x - x^3\right)dx = \frac{1}{8}\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^{2a}$$
$$= \frac{a^4}{8}\left[2(2)^2 - \frac{1}{4}(2)^4\right]$$

or $I_{xy} = \frac{1}{2}a^4$ ◀

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Chapter 9, Solution 68.

First note: At $x = a$: $b = ka^3$

or $k = \dfrac{b}{a^3}$; $y = \dfrac{b}{a^3}x^3$

Now

$$I_{xy} = \int xydA$$

$$= \int_0^a \int_{\frac{b}{a^3}x^3}^b xydydx$$

$$= \frac{1}{2}\int_0^a x\left(b^2 - \frac{b^2}{a^6}x^6\right)dx$$

$$= \frac{b^2}{2}\left(\frac{1}{2}x^2 - \frac{1}{8a^6}x^8\right)\Bigg|_0^a$$

$$= \frac{3}{16}a^2b^2$$

$$I_{xy} = \frac{3}{16}a^2b^2 \blacktriangleleft$$

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Chapter 9, Solution 69.

Have

$$I_{xy} = \int xydA$$

$$= \int_{-b}^{0} \int_{\frac{h}{b}x}^{0} xydydx$$

$$= \frac{1}{2} \int_{-b}^{0} x\left(-\frac{h^2}{b^2}x^2\right)dx$$

$$= -\frac{1}{8}\frac{h^2}{b^2}x^4\Big|_{-b}^{0}$$

$$= \frac{1}{8}b^2h^2$$

$$I_{xy} = \frac{1}{8}b^2h^2 \blacktriangleleft$$

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Chapter 9, Solution 70.

First note: At $x = a$: $b = ke^{\frac{a}{a}}$

$$\text{or } k = \frac{b}{e}; \quad y = \frac{b}{e}e^{\frac{x}{a}}$$

Now $I_{xy} = \int xydA$

$$= \int_0^a \int_0^{\frac{b}{e}e^{\frac{x}{a}}} xydydx$$

$$= \frac{1}{2}\frac{b^2}{e^2}\int_0^a xe^{\frac{2x}{a}}dx$$

$$= \frac{1}{2}\frac{b^2}{e^2}\left[\frac{e^{\frac{2x}{a}}}{\left(\frac{2}{a}\right)^2}\left(\frac{2}{a}x - 1\right)\right]\Bigg|_0^a$$

$$= \frac{1}{2}\frac{b^2}{e^2}\left(\frac{a^2}{4}\right)\left[e^2(1) - (1)(-1)\right]$$

$$= \frac{a^2b^2}{8e^2}\left(e^2 + 1\right)$$

$$I_{xy} = 0.1419a^2b^2 \blacktriangleleft$$

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Chapter 9, Solution 71.

Have $\bar{I}_{xy} = (I_{xy})_1 - (I_{xy})_2 - (I_{xy})_3$

Where $(I_{xy})_1 = 0$

and $I_{xy} = I_{x'y'} + A\bar{x}\,\bar{y}$ for areas ② and ③ ($\bar{x}\,\bar{y} \to 0$)

Now $\bar{x}_2 = -15$ mm $\quad \bar{y}_2 = 10$ mm $\quad A_2 = (50\text{ mm})(20\text{ mm})$

$= 1000\text{ mm}^2$

$\bar{x}_3 = 15$ mm $\quad \bar{y}_3 = -10$ mm $\quad A_3 = (50\text{ mm})(20\text{ mm})$

$= 1000\text{ mm}^2$

Then $\bar{I}_{xy} = -(1000\text{ mm}^2)(-15\text{ mm})(10\text{ mm})$

$-(1000\text{ mm})(15\text{ mm})(-10\text{ mm})$

$= 300 \times 10^3\text{ mm}^4$

$\bar{I}_{xy} = 300 \times 10^3\text{ mm}^4$ ◀

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Chapter 9, Solution 72.

Note: Orientation of A_3 corresponding to a 180° rotation of the axes. Equation 9.20 then yields

$$I_{x'y'} = I_{xy}$$

- Symmetry implies $\left(I_{xy}\right)_1 = 0$

Using Sample Problem 9.6 $\left(\bar{I}_{x'y'}\right)_2 = -\frac{1}{72}(9\text{ in.})^2(4.5\text{ in.})^2 = -22.78125\text{ in}^4$

and $\bar{X}_2 = 9\text{ in.}$ $\bar{Y}_2 = 1.5\text{ in.}$ $A_2 = \frac{1}{2}(9\text{ in.})(4.5\text{ in.}) = 20.25\text{ in}^2$

Similarly, $\left(\bar{I}_{x'y'}\right)_3 = -\frac{1}{72}(9\text{ in.})^2(4.5\text{ in.})^2 = -22.78125\text{ in}^4$

and $\bar{X}_3 = -9\text{ in.}$ $\bar{Y}_2 = -1.5\text{ in.}$ $A_3 = \frac{1}{2}(9\text{ in.})(4.5\text{ in.}) = 20.25\text{ in}^2$

Then $\bar{I}_{xy} = \cancelto{0}{\left(I_{xy}\right)_1} + \left(I_{xy}\right)_2 + \left(I_{xy}\right)_3$ with $\left(I_{xy}\right)_2 = \left(I_{xy}\right)_3$

and $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}\,A$

Therefore, $\bar{I}_{xy} = 2\left[-22.78125 + (9)(1.5)(20.25)\right]\text{in}^4$

$= 501.1875\text{ in}^4$

or $\bar{I}_{xy} = 501\text{ in}^4$ ◀

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Chapter 9, Solution 73.

Have $\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2$

For each semicircle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}\,A$ and $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = \Sigma \bar{x}\,\bar{y}\,A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}\,\bar{y}$, mm^4
1	$\frac{\pi}{2}(120)^2 = 7200\pi$	-60	$-\frac{160}{\pi}$	69.12×10^6
2	$\frac{\pi}{2}(120)^2 = 7200\pi$	60	$\frac{160}{\pi}$	69.12×10^6
Σ				138.24×10^6

or $\bar{I}_{xy} = 138.2 \times 10^6$ mm^4 ◀

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Chapter 9, Solution 74.

Have $I_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2$

For each rectangle $I_{xy} = \bar{I}_{x'y'} + A\bar{x}\,\bar{y}$ and $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = \Sigma\,\bar{x}\,\bar{y}\,A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}\,\bar{y}$, mm^4
1	$76(6.4) = 486.4$	-12.9	9.4	$-58\,980.86$
2	$44.6(6.4) = 285.44$	21.9	-16.1	$-100\,643.29$
Σ				$-159\,624.15$

or $\bar{I}_{xy} = -0.1596 \times 10^6$ mm^4 ◀

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Chapter 9, Solution 75.

Have $\bar{I}_{xy} = \left(\bar{I}_{xy}\right)_1 + \left(I_{xy}\right)_2 + \left(I_{xy}\right)_3$

Now symmetry implies $\left(\bar{I}_{xy}\right)_1 = 0$

and for the other rectangles $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$ where $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = \left(\bar{x}\,\bar{y}A\right)_2 + \left(\bar{x}\,\bar{y}A\right)_3$

$$= (-69\text{ mm})(-25\text{ mm})\left[(12\text{ mm})(38\text{ mm})\right]$$

$$+ (69\text{ mm})(25\text{ mm})\left[(12\text{ mm})(38\text{ mm})\right]$$

$$= (786\,600 + 786\,600)\text{ mm}^4 = 1\,573\,200\text{ mm}^4$$

or $\bar{I}_{xy} = 1.573 \times 10^6\text{ mm}^4$ ◀

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Chapter 9, Solution 76.

Symmetry implies $\left(I_{xy}\right)_1 = 0$

Using Sample Problem 9.6 and Equation 9.20, note that the orientation of A_2 corresponds to a 90° rotation of the axes; thus $\left(\bar{I}_{x'y'}\right)_2 = \frac{1}{72}b^2h^2$

Also, the orientation of A_3 corresponds to a 270° rotation of the axes; thus $\left(\bar{I}_{x'y'}\right)_3 = \frac{1}{72}b^2h^2$

Then $$\left(\bar{I}_{x'y'}\right)_2 = \frac{1}{72}(9\text{ in.})^2(6\text{ in.})^2 = 40.5\text{ in}^4$$

and $$\bar{x}_2 = -6\text{ in.},\qquad \bar{y}_2 = 2\text{ in.},\qquad A_2 = \frac{1}{2}(9\text{ in.})(6\text{ in.}) = 27\text{ in}^2$$

Also $$\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{x'y'}\right)_2 = 40.5\text{ in}^4$$

and $$\bar{x}_3 = 6\text{ in.},\qquad \bar{y}_3 = -2\text{ in.},\qquad A_3 = A_2 = 27\text{ in}^2$$

Now $$\bar{I}_{xy} = \cancelto{0}{\left(I_{xy}\right)_1} - \left(I_{xy}\right)_2 - \left(I_{xy}\right)_3 \qquad \text{and} \qquad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \qquad \left(I_{xy}\right)_2 = \left(I_{xy}\right)_3$$

Then $$\bar{I}_{xy} = -2\left[40.5\text{ in}^4 + (6\text{ in.})(-2\text{ in.})\left(27\text{ in}^2\right)\right]$$

$$= -2(40.5 - 324)\text{ in}^4 \qquad \text{or } \bar{I}_{xy} = 567\text{ in}^4 \blacktriangleleft$$

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Chapter 9, Solution 77.

Ha

Where $\bar{I}_{x'y'} = 0$ for each rectangle

Then $\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2 + \left(I_{xy}\right)_3 = \Sigma \bar{x}\,\bar{y}A$

Section ①: $\bar{x}_1 = -(8.92 \text{ in.} - 8 \text{ in.}) = -0.92 \text{ in.}$

$\bar{y}_1 = 3 \text{ in.} - 0.61 \text{ in.} = 2.39 \text{ in.}$

$A_1 = (16 \text{ in.})(6 \text{ in.}) = 96 \text{ in}^2$

Section ②: $\bar{x}_2 = (-8.92 \text{ in.} - 1.5 \text{ in.}) = -10.42 \text{ in.}$

$\bar{y}_2 = 2 \text{ in.} - 0.61 \text{ in.} = 1.39 \text{ in.}$

$A_2 = (3 \text{ in.})(4 \text{ in.}) = 12 \text{ in}^2$

Section ③: $\bar{x}_3 = (16 \text{ in.} - 8.92 \text{ in.}) - 2 \text{ in.} = 5.08 \text{ in.}$

$\bar{y}_3 = -(0.61 \text{ in.} + 5.25 \text{ in.}) = -5.86 \text{ in.}$

$A_3 = (4 \text{ in.})(10.5 \text{ in.}) = 42 \text{ in}^2$

Then
$$\bar{I}_{xy} = \left[(-0.92 \text{ in.})(2.39 \text{ in.})\left(96 \text{ in}^2\right)\right] + \left[(-10.42 \text{ in.})(1.39 \text{ in.})\left(12 \text{ in}^2\right)\right] + \left[(5.08 \text{ in.})(-5.86 \text{ in.})\left(42 \text{ in}^2\right)\right]$$
$$= -(211.08 + 173.806 + 1250.29)\text{in}^4$$
$$= -1635.18 \text{ in}^4$$

$\bar{I}_{xy} = -1635 \text{ in}^4$ ◀

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Chapter 9, Solution 78.

Have $$\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2$$

For each rectangle

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \qquad \text{and} \qquad \bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

Then $$\bar{I}_{xy} = \Sigma\bar{x}\,\bar{y}A = (-0.75 \text{ in.})(-1.5 \text{ in.})\left[(3 \text{ in.})(0.5 \text{ in.})\right]$$

$$+ (0.5 \text{ in.})(1.00 \text{ in.})\left[(4.5 \text{ in.})(0.5 \text{ in.})\right]$$

$$= (1.6875 + 1.125)\text{in}^4 = 2.8125 \text{ in}^4$$

$$\text{or } \bar{I}_{xy} = 2.81 \text{ in}^4 \blacktriangleleft$$

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Chapter 9, Solution 79.

From Figure 9.12:

$$I_x = \frac{\pi}{16}(2a)(a)^3 = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{16}(2a)^3(a) = \frac{\pi}{2}a^4$$

From Problem 9.67:

$$I_{xy} = \frac{1}{2}a^4$$

First note

$$\frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4$$

$$\frac{1}{2}(I_x - I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right) = -\frac{3}{16}\pi a^4$$

Now use Equations (9.18), (9.19), and (9.20).

Equation (9.18):

$$I_{x'} = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4\cos 2\theta - \frac{1}{2}a^4\sin 2\theta$$

Equation (9.19):

$$I_{y'} = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta = \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4\cos 2\theta + \frac{1}{2}a^4\sin 2\theta$$

Equation (9.20):

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta = -\frac{3}{16}\pi a^4\sin 2\theta + \frac{1}{2}a^4\cos 2\theta$$

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Chapter 9, Solution 80.

From the solution to Problem 9.72

$$\bar{I}_{xy} = 501.1875 \text{ in}^4$$

$$A_2 = A_3 = 20.25 \text{ in}^2$$

First compute the moment of inertia

$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3 \qquad \text{with} \qquad (I_x)_2 = (I_x)_3$$

$$= \left[\frac{1}{12}(12 \text{ in.})(9 \text{ in.})^3\right] + 2\left[\frac{1}{12}(9 \text{ in.})(4.5 \text{ in.})^3\right]$$

$$= (729 + 136.6875) \text{ in}^4 = 865.6875 \text{ in}^4$$

and

$$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3 \qquad \text{with} \qquad (I_y)_2 = (I_y)_3$$

$$= \left[\frac{1}{12}(9 \text{ in.})(12 \text{ in.})^3\right] + 2\left[\frac{1}{36}(4.5 \text{ in.})(9 \text{ in.})^3 + (20.25 \text{ in}^2)(9 \text{ in.})^2\right]$$

$$= (1296 + 182.25 + 3280.5)\text{in}^4 = 4758.75 \text{ in}^4$$

From Equation 9.18

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \frac{865.6875 \text{ in}^4 + 4758.75 \text{ in}^4}{2} + \frac{865.6875 \text{ in}^4 - 4758.75 \text{ in}^4}{2}\cos\left[2(-45°)\right]$$

$$-501.1875 \text{ in}^4 \sin\left[2(-45°)\right]$$

$$= (2812.21875 + 501.1875)\text{in}^4 = 3313.4063 \text{ in}^4$$

$$\text{or } \bar{I}_{x'} = 3.31 \times 10^3 \text{ in}^4 \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Similarly

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= (2812.21875 - 501.1875)\text{in}^4 = 2311.0313 \text{ in}^4$$

$$\text{or } \bar{I}_{y'} = 2.31 \times 10^3 \text{ in}^4 \blacktriangleleft$$

and

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \frac{865.6875 \text{ in}^4 - 4758.75 \text{ in}^4}{2}\sin\left[2(-45°)\right]$$

$$+ 501.1875\cos\left[2(-45°)\right]$$

$$= (-1946.53125)(-1)\text{in}^4 = 1946.53125 \text{ in}^4$$

$$\text{or } \bar{I}_{x'y'} = 1.947 \times 10^3 \text{ in}^4 \blacktriangleleft$$

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Chapter 9, Solution 81.

From Problem 9.73, $\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$

$$\bar{I}_x = (I_x)_1 + (I_x)_2 \qquad (I_x)_1 = (I_x)_2$$

$$= 2\left[\frac{\pi}{8}(120 \text{ mm})^4\right]$$

$$= 51.84\pi \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = (I_y)_1 + (I_y)_2 \qquad (I_y)_1 = (I_y)_2$$

$$= 2\left[\frac{\pi}{8}(120 \text{ mm})^4 + \frac{\pi}{2}(120 \text{ mm})^2(60 \text{ mm})^2\right]$$

$$= 103.68\pi \times 10^6 \text{ mm}^4$$

Have $\bar{I}_x = 2\left(25.92\pi \times 10^6\right) = 51.84\pi \times 10^6 \text{ mm}^4$

and $\bar{I}_y = 2\left(51.84\pi \times 10^6\right) = 103.68\pi \times 10^6 \text{ mm}^4$

Then $\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 77.76\pi \times 10^6 \text{ mm}^4$

and $\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -25.92\pi \times 10^6 \text{ mm}^4$

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Now, from Equations 9.18, 9.19, and 9.20

Equation 9.18:
$$\bar{I}_{x'} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) + \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$
$$= \left[77.76\pi \times 10^6 - 25.92\pi \times 10^6 \cos\left(-60°\right) - 138.24 \times 10^6 \sin\left(-60°\right)\right]\text{mm}^4$$
$$= 323.29 \times 10^6 \text{ mm}^4$$
$$\text{or } \bar{I}_x = 323 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

Equation 9.19:
$$\bar{I}_{y'} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) - \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$
$$= \left[77.76\pi \times 10^6 + 25.92\pi \times 10^6 \cos\left(-60°\right) + 138.24 \times 10^6 \sin\left(-60°\right)\right]\text{mm}^4$$
$$= 165.29 \times 10^6 \text{ mm}^4$$
$$\text{or } \bar{I}_{y'} = 165.29 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

Equation 9.20:
$$\bar{I}_{x'y'} = \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$
$$= \left[-25.92\pi \times 10^6 \sin\left(-60°\right) + 138.24 \times 10^6 \cos\left(-60°\right)\right]\text{mm}^4$$
$$= 139.64 \times 10^6 \text{ mm}^4$$
$$\text{or } \bar{I}_{x'y'} = 139.6 \times 10^4 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 82.

From Problem 9.75 $\bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$

Now $\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

where $(I_x)_1 = \frac{1}{12}(150 \text{ mm})(12 \text{ mm})^3 = 21\,600 \text{ mm}^4$

and $(I_x)_2 = (I_x)_3 = \frac{1}{12}(12 \text{ mm})(38 \text{ mm})^3 + \left[(12 \text{ mm})(38 \text{ mm})\right](25 \text{ mm})^2$

$= 339\,872 \text{ mm}^4$

Then $\bar{I}_x = \left[21\,600 + 2(339\,872)\right]\text{mm}^4 = 701\,344 \text{ mm}^4 = 0.70134 \times 10^6 \text{ mm}^4$

Also $\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

where $(I_y)_1 = \frac{1}{12}(12 \text{ mm})(150 \text{ mm})^3 = 3.375 \times 10^6 \text{ mm}^4$

and $(I_y)_2 = (I_y)_3 = \frac{1}{12}(38 \text{ mm})(12 \text{ mm})^3 + \left[(12 \text{ mm})(38 \text{ mm})\right](69 \text{ mm})^2$

$= 2.1765 \times 10^6 \text{ mm}^4$

Then $\bar{I}_y = \left[(3.375 + 2(2.1765)\right] \times 10^6 \text{ mm}^4 = 7.728 \times 10^6 \text{ mm}^4$

Now $\frac{1}{2}(\bar{I}_x + \bar{I}_y) = 4.2146 \times 10^6 \text{ mm}^4$

and $\frac{1}{2}(\bar{I}_x - \bar{I}_y) = -3.5133 \times 10^6 \text{ mm}^4$

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Using Equations 9.18, 9.19, and 9.20

From Equation 9.18:

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[4.2147 \times 10^6 + \left(-3.5133 \times 10^6\right)\cos\left(120°\right) - 1.5732 \times 10^6 \sin\left(120°\right)\right]\text{mm}^4$$

$$= 4.6089 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 4.61 \times 10^6 \text{ mm}^4$ ◀

From Equation 9.19:

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[4.2147 \times 10^6 - \left(-3.5133 \times 10^6\right)\cos\left(120°\right) + 1.5732 \times 10^6 \sin\left(120°\right)\right]\text{mm}^4$$

$$= 3.8205 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 3.82 \times 10^6 \text{ mm}^4$ ◀

From Equation 9.20:

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[-3.5133 \times 10^6 \sin\left(120°\right) + 1.5732 \times 10^6 \cos\left(120°\right)\right]\text{mm}^4$$

$$= -3.8292 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = -3.83 \times 10^6 \text{ mm}^4$ ◀

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Chapter 9, Solution 83.

From Problem 9.74

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

From Figure 9.13

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

Now

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -0.1435 \times 10^6 \text{ mm}^4$$

Using Equations (9.18), (9.19), and (9.20)

Equation (9.18):

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[0.3095 \times 10^6 + \left(-0.1435 \times 10^6\right)\cos\left(-90°\right) - \left(-0.1596 \times 10^6\right)\sin\left(-90°\right)\right]\text{mm}^4$$

$$= 0.1499 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 0.1499 \times 10^6 \text{ mm}^4$ ◀

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Equation (9.19):

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[0.3095\times 10^6 - \left(-0.1435\times 10^6\right)\cos\left(-90°\right) + \left(-0.1596\times 10^6\right)\sin\left(-90°\right)\right]\text{mm}^4$$

$$= 0.4691\times 10^6\ \text{mm}^4$$

or $\bar{I}_{y'} = 0.469\times 10^6\ \text{mm}^4$ ◀

Equation (9.20):

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[-0.1435\times 10^6\sin\left(-90°\right) + 0.1596\times 10^6\cos\left(-90°\right)\right]\text{mm}^4$$

$$= 0.1435\times 10^6\ \text{mm}^4$$

or $\bar{I}_{x'y'} = 0.1435\times 10^6\ \text{mm}^4$ ◀

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Chapter 9, Solution 84.

From Problem 9.78

$$\bar{I}_{xy} = 2.8125 \text{ in}^4$$

From Figure 9.13

$$\bar{I}_x = 9.45 \text{ in}^4, \qquad \bar{I}_y = 2.58 \text{ in}^4$$

Now

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 6.015 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = 3.435 \text{ in}^4$$

Using Equations (9.18), (9.19), and (9.20)

Equation (9.18): $$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[6.015 + 3.435\cos\left(60°\right) - 2.8125\sin\left(60°\right)\right]\text{in}^4 = 5.2968 \text{ in}^4$$

or $\bar{I}_{x'} = 5.30 \text{ in}^4$ ◀

Equation (9.19): $$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[6.015 - 3.435\cos\left(60°\right) + 2.8125\sin\left(60°\right)\right]\text{in}^4 = 6.7332 \text{ in}^4$$

or $\bar{I}_{y'} = 6.73 \text{ in}^4$ ◀

Equation (9.20): $$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[3.435\sin\left(60°\right) + 2.8125\cos\left(60°\right)\right]\text{in}^4 = 4.3810 \text{ in}^4$$

or $\bar{I}_{x'y'} = 4.38 \text{ in}^4$ ◀

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Chapter 9, Solution 85.

From Problem 9.79: $I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $I_{xy} = \frac{1}{2}a^4$

Now, Equation (9.25):

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y} = -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= \frac{8}{3\pi} = 0.84883$$

Then $2\theta_m = 40.326°$ and $220.326°$

or $\theta_m = 20.2°$ and $110.2°$ ◀

Also, Equation (9.27):

$$I_{\max,\min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right)$$

$$\pm \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= (0.981748 \pm 0.772644)a^4$$

or $I_{\max} = 1.754a^4$ ◀

and $I_{\min} = 0.209a^4$ ◀

By inspection, the a axis corresponds to $I_{\min}$ and the b axis corresponds to $I_{\max}$.

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Chapter 9, Solution 86.

From the solutions to Problem 9.72 and 9.80

$$\bar{I}_{xy} = 501.1875 \text{ in}^4 \qquad \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

Then Equation (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{501.1875}{-1946.53125} = 0.257477$$

or

$$2\theta_m = 14.4387° \quad \text{and} \quad 194.4387°$$

or $\theta_m = 7.22°$ and $97.2°$ ◀

Equation (9.27):

$$\bar{I}_{\max,\min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 2812.21875 \pm \sqrt{(-1946.53125)^2 + (501.1875)^2}$$

$$= (2812.21875 \pm 2010.0181)\text{in}^4$$

or $\bar{I}_{\max} = 4.82 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 802 \text{ in}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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Chapter 9, Solution 87.

From Problems 9.73 and 9.81

$$\bar{I}_x = 51.84\pi \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 103.68\pi \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

Equation (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2\left(138.24 \times 10^6\right)}{51.84\pi \times 10^6 - 103.68\pi \times 10^6}$$

$$= 1.69765$$

$$2\theta_m = 59.500° \quad \text{and} \quad 239.500°$$

or $\theta_m = 29.7°$ and $119.7°$ ◀

Then

$$\bar{I}_{\max,\min} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= \frac{(51.84 + 103.68)\pi \times 10^6}{2} \pm \sqrt{\left[\frac{(51.84 - 103.68)\pi \times 10^6}{2}\right]^2 + \left(138.24 \times 10^6\right)^2}$$

$$= (244.29 \pm 160.44) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 405 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 83.9 \times 10^6 \text{ mm}^4$ ◀

Note: By inspection the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $I_{\max}$.

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Chapter 9, Solution 88.

From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 7.728 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Then $$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 4.2147 \times 10^6 \text{ mm}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -3.5133 \times 10^6 \text{ mm}^4$$

Equation (9.25): $$\tan 2\theta = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2\left(1.5732 \times 10^6\right)}{0.70134 \times 10^6 - 7.728 \times 10^6}$$

$$= 0.44778$$

Then $2\theta_m = 24.12°$ and $204.12°$

or $\theta_m = 12.06°$ and $102.1°$ ◀

Also, Equation (9.27): $$\bar{I}_{\max,\min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 \bar{I}_{xy}^2}$$

$$= 4.2147 \times 10^6 \pm \sqrt{\left(-3.5133 \times 10^6\right)^2 + \left(1.5732 \times 10^6\right)^2}$$

$$= \left(4.2147 \pm 3.8494\right) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 8.06 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.365 \times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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Chapter 9, Solution 89.

From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Then
$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -0.1435 \times 10^6 \text{ mm}^4$$

Equation (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{I_x - I_y} = -\frac{2\left(-0.1596 \times 10^6\right)}{\left(0.166 - 0.453\right) \times 10^6} = -1.1122$$

Then
$$2\theta_m = -48.041° \quad \text{and} \quad 131.96°$$

or
$$\theta_m = -24.0° \text{ and } 66.0° \blacktriangleleft$$

Also, Equation (9.27):
$$\bar{I}_{\max,\min} = \frac{\left(\bar{I}_x + \bar{I}_y\right)}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 0.3095 \times 10^6 \pm \sqrt{\left(-0.1435 \times 10^6\right)^2 + \left(-0.1596 \times 10^6\right)^2}$$

$$= \left(0.3095 \pm 0.21463\right) \times 10^6 \text{ mm}^4$$

or
$$\bar{I}_{\max} = 0.524 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{\min} = 0.0949 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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Chapter 9, Solution 90.

From Problems 9.78 and 9.84

$$\bar{I}_{xy} = 2.81 \text{ in}^4$$

$$\bar{I}_x = 9.45 \text{ in}^4$$

$$\bar{I}_y = 2.58 \text{ in}^4$$

Then $$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 6.015 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = 3.435 \text{ in}^4$$

Equation (9.25): $$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(2.81)}{9.45 - 2.58} = -0.8180$$

Then $$2\theta_m = -39.2849° \quad \text{and} \quad 140.7151°$$

or $\theta_m = -19.64°$ and $70.36°$ ◀

Also, Equation (9.27): $$\bar{I}_{\max,\min} = \frac{\left(\bar{I}_x + \bar{I}_y\right)}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2}$$

$$= 6.015 \pm \sqrt{3.435^2 - 2.81^2}$$

$$= (6.015 \pm 4.438)\text{in}^4$$

or $\bar{I}_{\max} = 10.45 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 1.577 \text{ in}^4$ ◀

Note: By inspection, the *a* axis corresponds to $I_{\max}$ and the *b* axis corresponds to $I_{\min}$.

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Chapter 9, Solution 91.

From Problem 9.79: $I_x = \frac{\pi}{8}a^4$

$I_y = \frac{\pi}{2}a^4$

Problem 9.67: $I_{xy} = \frac{1}{2}a^4$

The Mohr's circle is defined by the diameter XY, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \qquad \text{and} \qquad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(I_x + I_y\right) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4 = 0.98175a^4$$

and $$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $$2\theta_m = 40.326°$$

continued

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Then $\alpha = 90° - 40.326°$

$= 49.674°$

$\beta = 180° - (40.326° + 60°)$

$= 79.674°$

(*a*) $I_{x'} = I_{\text{ave}} - R\cos\alpha = 0.98175a^4 - 0.77264a^4 \cos 49.674°$

or $I_{x'} = 0.482a^4$ ◀

$I_{y'} = I_{\text{ave}} + R\cos\alpha = 0.98175a^4 + 0.77264a^4 \cos 49.674°$

or $I_{y'} = 1.482a^4$ ◀

$I_{x'y'} = -R\sin\alpha = -0.77264a^4 \sin 49.674°$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $I_{x'} = I_{\text{ave}} + R\cos\beta = 0.98175a^4 + 0.77264a^4 \cos 79.674°$

or $I_{x'} = 1.120a^4$ ◀

$I_{y'} = I_{\text{ave}} - R\cos\beta = 0.98175a^4 - 0.77264a^4 \cos 79.674°$

or $I_{y'} = 0.843a^4$ ◀

$I_{x'y'} = R\sin\beta = 0.77264a^4 \sin 79.674°$

or $I_{x'y'} = 0.760a^4$ ◀

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Chapter 9, Solution 92.

From the solution to

Problem 9.72: $\bar{I}_{xy} = 501.1875 \text{ in}^4$

Problem 9.80: $\bar{I}_x = 865.6875 \text{ in}^4$

$\bar{I}_y = 4758.75 \text{ in}^4$

Now $\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^4$

$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$

The Mohr's circle is defined by the points X and Y where

$$X: \left(\bar{I}_x, \bar{I}_{xy}\right) \qquad Y: \left(\bar{I}_y, -\bar{I}_{xy}\right)$$

Now $I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.2 \text{ in}^4$

and $R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left(-1946.53125\right)^2 + 501.1875^2} \text{ in}^4$

$= 2010.0 \text{ in}^4$

Also, $\tan 2\theta_m = \dfrac{\bar{I}_{xy}}{\left|\dfrac{\bar{I}_x - \bar{I}_y}{2}\right|} = \dfrac{501.1875}{1946.53125} = 0.2575$

or $2\theta_m = 14.4387°$

Then $\alpha = 180° - \left(14.4387° + 90°\right) = 75.561°$

Then $\bar{I}_{x'}, \bar{I}_{y'} = I_{\text{ave}} \pm R\cos\alpha = 2812.2 \pm 2010.0\cos 75.561°$

or $\bar{I}_{x'} = 3.31 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{y'} = 2.31 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{x'y'} = R\sin\alpha = 2010.0\sin 75.561°$

or $\bar{I}_{x'y'} = 1.947 \times 10^3 \text{ in}^4$ ◀

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Chapter 9, Solution 93.

From Problems 9.73 and 9.81

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

$$\bar{I}_x = 51.84\pi \times 10^6 \text{ mm}^4$$

$$= 162.86 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 103.68\pi \times 10^6 \text{ mm}^4$$

$$= 325.72 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right)$$

$$= 244.29 \times 10^6 \text{ mm}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 160.4405 \times 10^6 \text{ mm}^4$$

From Problem 9.87 $\quad 2\theta_m = 59.5°$

Then $\quad \alpha = 180 - 60° - 2\theta_m = 60.5°$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\cos\alpha = \left(244.29 + 160.4405\cos 60.5°\right) \times 10^6$$

$$= 323.29 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 323 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\cos\alpha = \left(244.24 - 160.4405\cos 60.5°\right) \times 10^6$$

$$= 165.29 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 165.3 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = 160.44 \times 10^6 \sin 60.5° = 139.6 \times 10^6 \text{ mm}^4$$ ◀

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Chapter 9, Solution 94.

From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 7.728 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 4.2147 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = 3.8494 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(1.5732)}{0.70134 - 7.728}\right] = 24.12°$$

and $$\alpha = 120° - 24.12° - 90° = 5.88°$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\sin\alpha = \left(4.2147 + 3.8494\sin 5.88°\right) \times 10^6 \text{ mm}^4$$

$$= 4.6091 \times 10^6 \text{ mm}^4$$

$$\text{or } \bar{I}_{x'} = 4.61 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\sin\alpha = \left(4.2147 - 3.8494\sin 5.88°\right) \times 10^6 \text{ mm}^4$$

$$= 3.8203 \times 10^6 \text{ mm}^4$$

$$\text{or } \bar{I}_{y'} = 3.82 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = -R\cos\alpha = -3.8494\cos 5.88° = -3.8291 \times 10^6 \text{ mm}^4$$

$$\text{or } \bar{I}_{x'y'} = -3.83 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 95.

From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 0.21463 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(-0.1596)}{0.166 - 0.453}\right] = -48.04°$$

and $$\alpha + 90° - 2\theta = 90°; \ \alpha = 2\theta_m$$

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\sin\alpha = \left(0.3095 - 0.21463\sin 48.04°\right) \times 10^6 \text{ mm}^4$$

$$= 0.14989 \times 10^6 \text{ mm}^4$$

$$\text{or } \bar{I}_{x'} = 0.1499 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

and

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\sin\alpha = \left(0.3095 + 0.21463\sin 48.04°\right) \times 10^6 \text{ mm}^4$$

$$= 0.46910 \times 10^6 \text{ mm}^4$$

$$\text{or } \bar{I}_{y'} = 0.4690 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

and

$$\bar{I}_{x'y'} = R\cos\alpha = 0.21463\cos 48.04° = 0.1435 \times 10^6 \text{ mm}^4$$

$$\text{or } \bar{I}_{x'y'} = 0.1435 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

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Chapter 9, Solution 96.

Have $\bar{I}_x = 9.45 \text{ in}^4$

$\bar{I}_y = 2.58 \text{ in}^4$

From Problem 9.78 $\bar{I}_{xy} = 2.8125 \text{ in}^4$

Now $\bar{I}_{\text{ave}} = \dfrac{\bar{I}_x + \bar{I}_y}{2} = 6.015 \text{ in}^4$

and
$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2}$$

$$= 4.43952 \text{ in}^4$$

Then
$$2\theta_m = \tan^{-1}\left[\frac{-2(2.8125)}{9.45 - 2.58}\right] = -39.31°$$

$$2\theta_m + 60° + \alpha = 180°, \qquad \alpha = 80.69°$$

Then
$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 6.015 \text{ in}^4 - \left(4.43952 \text{ in}^4\right)\cos 80.69°$$

$$= 5.29679 \text{ in}^4$$

or $\bar{I}_{x'} = 5.30 \text{ in}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = 6.015 \text{ in}^4 + \left(4.43952 \text{ in}^4\right)\cos 80.69°$$

$$= 6.73321 \text{ in}^4$$

or $\bar{I}_{y'} = 6.73 \text{ in}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = \left(4.43952 \text{ in}^4\right)\sin 80.69° = 4.38104 \text{ in}^4$$

or $\bar{I}_{x'y'} = 4.38 \text{ in}^4$ ◀

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Chapter 9, Solution 97.

From Problem 9.79: $I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $I_{xy} = \frac{1}{2}a^4$

The Mohr's circle is defined by the diameter XY, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(I_x + I_y\right) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = 0.98175a^4$$

and

$$R = \sqrt{\left[\frac{1}{2}\left(I_x - I_y\right)\right]^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $$2\theta_m = 40.326°$$

and $$\theta_m = 20.2°$$

$\therefore$ The principal axes are obtained by rotating the xy axes through 20.2° counterclockwise ◀

Now $$I_{\max,\min} = I_{\text{ave}} \pm R = 0.98175a^4 \pm 0.77264a^4$$

or $I_{\max} = 1.754a^4$ ◀

and $I_{\min} = 0.209a^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to $I_{\min}$ and the b axis corresponds to $I_{\max}$.

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Chapter 9, Solution 98.

From the solution to Problem 9.72:

$$\bar{I}_{xy} = 501.1875 \text{ in}^4$$

From the solution to Problem 9.80:

$$\bar{I}_x = 865.6875 \text{ in}^4$$

$$\bar{I}_y = 4758.75 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

The Mohr's circle is defined by the point $X: \left(\bar{I}_x, \bar{I}_{xy}\right), \qquad Y: \left(\bar{I}_y, -\bar{I}_{xy}\right)$

Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.2 \text{ in}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left(-1946.53125\right)^2 + 501.1875^2} = 2010.0 \text{ in}^4$$

continued

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$$\tan 2\theta_m = -\frac{\bar{I}_{xy}}{\left(\dfrac{\bar{I}_x - \bar{I}_y}{2}\right)} = -\frac{501.1875}{-1946.53125} = 0.2575, \qquad 2\theta_m = 14.4387°$$

or $\theta_m = 7.22°$ counterclockwise ◀

Then $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (2812.2 \pm 2010.0)\text{in}^4$$

or $\bar{I}_{\max} = 4.82 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 802 \text{ in}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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Chapter 9, Solution 99.

From the solution to Problem 9.76 $\qquad \bar{I}_{xy} = 567 \text{ in}^4$

Now
$$\bar{I}_x = (I_x)_1 - (I_x)_2 - (I_x)_3, \quad \text{where} \quad (I_x)_2 = (I_x)_3$$
$$= \frac{\pi}{4}(15 \text{ in.})^4 - 2\left[\frac{1}{12}(9 \text{ in.})(6 \text{ in.})^3\right] = (39761 - 324)\text{in}^4$$
$$= 39{,}437 \text{ in}^4$$

and
$$\bar{I}_y = (I_y)_1 - (I_y)_2 - (I_y)_3, \quad \text{where} \quad (I_y)_2 = (I_y)_3$$
$$= \frac{\pi}{4}(15 \text{ in.})^4 - 2\left[\frac{1}{36}(6 \text{ in.})(9 \text{ in.})^3 + \frac{1}{2}(9 \text{ in.})(6 \text{ in.})(6 \text{ in.})^2\right]$$
$$= (39{,}761 - 243 - 1944)\text{in}^4 = 37{,}574 \text{ in}^4$$

The Mohr's circle is defined by the point (*X, Y*) where
$$X: \ (\bar{I}_x, \bar{I}_{xy}) \qquad Y: \ (\bar{I}_y, -\bar{I}_{xy})$$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(39{,}437 + 37{,}574)\text{in}^4 = 38{,}506 \text{ in}^4$$

and
$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(39{,}437 - 37{,}574)\right]^2 + 567^2} = 1090.5 \text{ in}^4$$

continued

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$$\tan 2\theta_m = \frac{-\bar{I}_{xy}}{\dfrac{\bar{I}_x - \bar{I}_y}{2}} = \frac{-567}{\frac{1}{2}(39{,}437 - 37{,}574)} = -0.6087$$

or $\theta_m = -15.66°$ clockwise ◀

Then $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (38{,}506 \pm 1090.50)\,\text{in}^4$$

or $\bar{I}_{\max} = 39.6 \times 10^3\ \text{in}^4$ ◀

and $\bar{I}_{\min} = 37.4 \times 10^3\ \text{in}^4$ ◀

Note: From the Mohr's circle it is seen that the a axis corresponds to the $\bar{I}_{\max}$ and the b axis corresponds to $\bar{I}_{\min}$.

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Chapter 9, Solution 100.

From Problems 9.73 and 9.81

$$\bar{I}_x = 162.86 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 325.72 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

Define points $X(162.86, 138.24) \times 10^6 \text{ mm}^4 \qquad Y(325.72, -138.24) \times 10^6 \text{ mm}^4$

Now

$$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(162.86 + 325.72) \times 10^6 \text{ mm}^4$$

$$= 244.29 \times 10^6 \text{ mm}^4$$

and

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{(162.86 - 325.72)}{2} \times 10^6\right]^2 + \left(138.24 \times 10^6\right)^2}$$

$$= 160.44 \times 10^6 \text{ mm}^4$$

and

$$2\theta_m = \tan^{-1}\left[\frac{-2(138.24) \times 10^6}{(162.86 - 325.72) \times 10^6}\right] = 59.4999°$$

or $\theta_m = 29.7°$ counterclockwise ◀

Then

$$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = \left(244.29 \times 10^6 \pm 160.44 \times 10^6\right) \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 405 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 83.9 \times 10^6 \text{ mm}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

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Chapter 9, Solution 101.

From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 0.453 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Define points $\quad X(0.166, -0.1596) \times 10^6 \text{ mm}^4 \quad$ and $\quad Y(0.453, 0.1596) \times 10^6 \text{ mm}^4$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(0.166 + 0.453) \times 10^6 \text{ mm}^4$$
$$= 0.3095 \times 10^6 \text{ mm}^4$$

and
$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{(0.166 - 0.453) \times 10^6}{2}\right]^2 + \left(-0.1596 \times 10^6\right)^2}$$
$$= 0.21463 \times 10^6 \text{ mm}^4$$

Also
$$2\theta_m = \tan^{-1}\left(\frac{-2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}\right) = \tan^{-1}\left[\frac{-2(-0.1596)}{0.166 - 0.453}\right] = -48.04°$$
$$\theta_m = -24.02°$$

or $\theta = -24.0°$ clockwise ◀

Then
$$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (0.3095 \pm 0.21463) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 0.524 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.0949 \times 10^6 \text{ mm}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

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Chapter 9, Solution 102.

From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 7.728 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(0.70134 + 7.728) \times 10^6 \text{ mm}^4 = 4.2147 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{(0.70134 - 7.728) \times 10^6}{2}\right]^2 + \left(1.5732 \times 10^6\right)^2}$$

$$= 3.8495 \times 10^6 \text{ mm}^4$$

Define points $$X(0.70134,\ 15732) \times 10^6 \text{ mm}$$

$$Y(7.728,\ -1.5732) \times 10^6 \text{ mm}$$

Also $$2\theta_m = \tan^{-1}\left[\frac{-2(1.5732)}{0.70134 - 7.728}\right] = 24.122°,\ \theta_m = 12.06°$$

or $\theta_m = 12.06°$ counterclockwise ◀

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (4.2147 \pm 3.8495) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 8.06 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.365 \times 10^6 \text{ mm}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

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Chapter 9, Solution 103.

From the solution to Problem 9.71

$$\bar{I}_{xy} = 300 \times 10^3 \text{ mm}^4$$

Now $\bar{I}_x = (I_x)_1 - (I_x)_2 - (I_x)_3$

$$= \left[\frac{1}{12}(120 \text{ mm})(80 \text{ mm})^3\right] - 2\left[\frac{1}{12}(50 \text{ mm})(\quad)(10 \text{ mm})^2\right]$$

$$= \left[5.120 \times 10^6 - 2\left(33.333 \times 10^3 + 100 \times 10^3\right)\right] \text{ mm}^4$$

$$= 4.8533 \times 10^6 \text{ mm}^4$$

and $\bar{I}_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

$$= \left[\frac{1}{12}(80 \text{ mm})(120 \text{ mm})^3\right] - 2\left[\frac{1}{12}(20 \text{ mm})(50 \text{ mm})^3 + \left(1000 \text{ mm}^2\right)(15 \text{ mm})^2\right]$$

$$= \left[11.520 \times 10^6 - 2\left(208.33 \times 10^3 + 225 \times 10^3\right)\right] \text{mm}^4$$

$$= 10.6533 \times 10^6 \text{ mm}^4$$

Center: $I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right)$

$$= \frac{1}{2}(4.8533 + 10.6533) \times 10^6 \text{ mm}^4$$

$$= 7.7533 \times 10^6 \text{ mm}^4$$

Radius: $R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$

$$= 10^6 \sqrt{\left[\frac{1}{2}(4.8533 - 10.6533)\right]^2 + (0.300)^2} \text{ mm}$$

$$= 2.9155 \times 10^6 \text{ mm}$$

continued

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Chapter 9, Solution 104.

From Prob. 9.43 and 9.77

$\bar{I}_x = 2703.7 \text{ in}^4$ $\qquad \bar{I}_y = 4581.0 \text{ in}^4$ $\qquad I_{xy} = -1635.18 \text{ in}^4$

Define Points: $\quad x(2703.7, -1635.18)\text{in}^4 \quad y(4581.0, 1635.18)\text{in}^4$

Then $\quad \bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(2703.7 + 4581.0)\text{in}^4 = 3642.4 \text{ in}^4$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left(\frac{2703.7 - 4581.0}{2}\right)^2 + (-1635.18)^2} = 1885.44 \text{ in}^4$$

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-1635.18)}{2703.7 - 4581.0} = -1.74206$$

$\theta_m = -30.1° \blacktriangleleft$

$$I_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (3642.4 \pm 1885.44)\text{in}^4$$

$\bar{I}_{\max} = 5530 \text{ in}^4 \blacktriangleleft$

$\bar{I}_{\min} = 1757 \text{ in}^4 \blacktriangleleft$

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Chapter 9, Solution 105.

Given: $\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4, \quad \bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$ and $\bar{I}_{xy} < 0$

Note: A review of a table of rolled-steel shapes reveals that the given values of $\bar{I}_x$ and $\bar{I}_y$ are obtained when the 102 mm leg of the angle is parallel to the x axis. For $\bar{I}_{xy} < 0$ the angle must be oriented as shown.

(a) Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(0.166 + 0.453\right) \times 10^6 \text{ mm}^4$$
$$= 0.3095 \times 10^6 \text{ mm}^4$$

Now
$$\bar{I}_{\min} = \bar{I}_{\text{ave}} - R \qquad \text{or} \qquad R = \bar{I}_{\text{ave}} - \bar{I}_{\min}$$

Then
$$R = \left(0.3095 - 0.051\right) \times 10^6 \text{ mm}^4$$
$$= 0.2585 \times 10^6 \text{ mm}^4$$

From
$$R^2 = \left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2$$
$$\bar{I}_{xy} = \sqrt{\left[\left(0.2585\right)^2 - \left(\frac{0.166 - 0.453}{2}\right)^2\right]} \times 10^6 \text{ mm}^4$$
$$\bar{I}_{xy} = \pm 0.21501 \times 10^6 \text{ mm}^4$$

Since $\bar{I}_{xy} < 0, \quad \bar{I}_{xy} = -0.21501 \times 10^6 \text{ mm}^4$ or $\bar{I}_{xy} = -0.215 \times 10^6 \text{ mm}^4$ ◀

(b)
$$2\theta_m = \tan^{-1}\left[\frac{-2\left(-0.21501\right)}{0.166 - 0.453}\right] = -56.28°$$

or $\theta_m = -28.1°$ clockwise ◀

(c)
$$\bar{I}_{\max} = \bar{I}_{\text{ave}} + R = \left(0.3095 + 0.2585\right) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 0.568 \times 10^6 \text{ mm}^4$ ◀

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Chapter 9, Solution 106.

From Figure 9.13 $\bar{I}_x = 9.45 \text{ in}^4$

$\bar{I}_y = 2.58 \text{ in}^4$

From Problem 9.78 $\bar{I}_{xy} = -2.81 \text{ in}^4$

The Mohr's circle is defined by the diameter XY where

$$X\left(9.45,\ -2.81\right) \text{ in}^4$$

$$Y\left(2.58,\ 2.81\right) \text{ in}^4$$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(9.45 \text{ in}^4 + 2.58 \text{ in}^4\right)$$
$$= 6.015 \text{ in}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\right]^2 + \bar{I}_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}\left(9.45 \text{ in}^4 - 2.58 \text{ in}^4\right)\right]^2 + \left(2.81 \text{ in}^4\right)^2}$$
$$= 4.438 \text{ in}^4$$

$$\tan 2\theta_m = \frac{-2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = \frac{-2\left(-2.81 \text{ in}^4\right)}{9.45 \text{ in}^4 - 2.58 \text{ in}^4} = 0.81805$$

or
$$2\theta_m = 32.285°$$
or $\theta_m = 19.64°$ counterclockwise ◀

Now
$$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = \left(6.015 \pm 4.438\right)\text{in}^4$$
or $\bar{I}_{\max} = 10.45 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 1.577 \text{ in}^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\max}$ and the b axis corresponds to $\bar{I}_{\min}$.

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Chapter 9, Solution 107.

From Figure 9.13B: $\bar{I}_x = 7.20 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 2.64 \times 10^6 \text{ mm}^4$

Have $\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2, \qquad$ where $\qquad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \qquad$ and $\qquad \bar{I}_{x'y'} = 0$

Now $\bar{x}_1 = \dfrac{102}{2} - 25.3 = 25.7 \text{ mm}, \qquad \bar{y}_1 = 50.3 - \dfrac{12.7}{2} = 43.95 \text{ mm}$

$$A_1 = 102 \times 12.7 = 1295.4 \text{ mm}^2$$

$$\bar{x}_2 = -25.3 + \frac{12.7}{2} = -18.95 \text{ mm} \qquad \bar{y}_2 = -\left[\frac{1}{2}(152 - 12.7) - (50.3 - 12.7)\right] = -32.05 \text{ mm}$$

$$A_2 = (12.7)(152 - 12.7) = 1769.11 \text{ mm}^2$$

Then $\bar{I}_{xy} = \left\{\left[(25.7 \text{ mm})(43.95 \text{ mm})\left(1295.4 \text{ mm}^2\right)\right] + \left[(-18.95 \text{ mm})(-32.05 \text{ mm})\left(1769.11 \text{ mm}^2\right)\right]\right\} \times 10^6$

$$= (1.46317 + 1.07446) \times 10^6 \text{ mm}^4 = 2.5376 \times 10^6 \text{ mm}^4$$

The Mohr's circle is defined by points X and Y, where

$$X\left(\bar{I}_x, \bar{I}_{xy}\right), \; Y\left(\bar{I}_y, -\bar{I}_{xy}\right)$$

Now $\bar{I}_{\text{ave}} = \dfrac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \dfrac{1}{2}(7.20 + 2.64) \times 10^6 \text{ mm}^4 = 4.92 \times 10^6 \text{ mm}^4$

continued

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and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \left[\sqrt{\left[\frac{1}{2}(7.20 - 2.64)\right]^2 + 2.5376^2}\right] \times 10^6 \text{ mm}^4$$

$$= 3.4114 \times 10^6 \text{ mm}^4$$

$$\tan\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(2.5376)}{(7.20 - 2.64)} = -1.11298, \qquad 2\theta = -48.0607°$$

or $\theta = -24.0°$ clockwise ◀

Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (4.92 \pm 3.4114) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 8.33 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 1.509 \times 10^6 \text{ mm}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\max}$ and the *b* axis corresponds to $\bar{I}_{\min}$.

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Chapter 9, Solution 108.

Have $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(640 \text{ in}^4 + 280 \text{ in}^4\right) = 460 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = \frac{1}{2}\left(640 \text{ in}^4 - 280 \text{ in}^4\right) = 180 \text{ in}^4$$

Also have $$\bar{I}_{x'y'} = -180 \text{ in}^4, \qquad 2\theta = -120°, \qquad I_x > I_y$$

Letting the points $\left(\bar{I}_x, \bar{I}_{xy}\right)$ and $\left(\bar{I}_{x'}, \bar{I}_{x'y'}\right)$ be denoted by X an X', respectively, three possible Mohr's circles can be constructed

Assume the first case applies

Then $$\frac{\bar{I}_x - \bar{I}_y}{2} = R\cos 2\theta_m \qquad \text{or} \qquad R\cos 2\theta_m = 180 \text{ in}^4$$

Also $$\left|\bar{I}_{x'y'}\right| = R\cos\alpha \qquad \text{or} \qquad R\cos\alpha = 180 \text{ in}^4$$

$$\therefore \quad \alpha = \pm 2\theta_m$$

Also have $$120° = 2\theta_m + \left(90° - \alpha\right) \qquad \text{or} \qquad 2\theta_m - \alpha = 30°$$

$$\therefore \quad \alpha = -2\theta_m \qquad \text{and} \qquad 2\left(2\theta_m\right) = 30° \qquad \text{or} \qquad 2\theta_m = \left|\alpha\right| = 15°$$

Note $$\left.\begin{array}{r} 2\theta_m > 0 \\ \alpha < 0 \end{array}\right\} \text{ implies case 2 applies}$$

continued

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(*a*) Therefore, $\theta_m = 7.5°$ clockwise ◀

(*b*) Have $R\cos 15° = 180$ or $R = 186.35 \text{ in}^4$

Then $\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 460 \pm 186.350$

or $\bar{I}_{\max} = 646 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 274 \text{ in}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\max}$ and the *b* axis corresponds to $\bar{I}_{\min}$.

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Chapter 9, Solution 109.

First assume $\bar{I}_x > \bar{I}_y$

(Note: Assuming $\bar{I}_x < \bar{I}_y$ is not consistent with the requirement that the axis corresponding to the $(\bar{I}_{xy})_{max}$ is obtained by rotating the x axis through $67.5°$ counterclockwise)

From Mohr's circle have $2\theta_m = 2(67.5°) - 90° = 45°$

(a) From $\tan 2\theta_m = \dfrac{2|\bar{I}_{xy}|}{\bar{I}_x - \bar{I}_y}$

$$\text{Have } \bar{I}_x = \bar{I}_y + 2\frac{|\bar{I}_{xy}|}{\tan 2\theta_m} = 300 \text{ in}^4 + 2\frac{125 \text{ in}^4}{\tan 45°} = 550 \text{ in}^4$$

or $\bar{I}_x = 550 \text{ in}^4$ ◀

(b) Now $I_{ave} = \dfrac{1}{2}(\bar{I}_x + \bar{I}_y) = \dfrac{550 + 300}{2} \text{ in}^4 = 425 \text{ in}^4$

and $R = \dfrac{|\bar{I}_{xy}|}{\sin 2\theta_m} = \dfrac{125 \text{ in}^4}{\sin 45°} = 176.78 \text{ in}^4$

Then $\bar{I}_{max, min} = \bar{I}_{ave} \pm R = (425 \pm 176.76)\text{in}^4$

$= (601.78,\ 248.22)\text{in}^4$

or $\bar{I}_{max} = 602 \text{ in}^4$ ◀

and $\bar{I}_{min} = 248 \text{ in}^4$ ◀

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Chapter 9, Solution 110.

Consider the regular pentagon shown, with centroidal axes x and y.

Because the y axis is an axis of symmetry, it follows that $\bar{I}_{xy} = 0$. Since $\bar{I}_{xy} = 0$, the x and y axes must be principal axes. Assuming $\bar{I}_x = \bar{I}_{\max}$ and $\bar{I}_y = \bar{I}_{\min}$, the Mohr's circle is then drawn as shown.

Now rotate the coordinate axes through an angle α as shown; the resulting moments of inertia, $\bar{I}_{x'}$ and $\bar{I}_{y'}$, and product of inertia, $\bar{I}_{x'y'}$, are indicated on the Mohr's circle. However, the x' axis is an axis of symmetry, which implies $\bar{I}_{x'y'} = 0$. For this to be possible on the Mohr's circle, the radius R must be equal to zero (thus, the circle degenerates into a point). With $R = 0$, it immediately follows that

(*a*) $\bar{I}_x = \bar{I}_y = \bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_{\text{ave}}$ (for all moments of inertia with respect to an axis through C) ◀

(*b*) $\bar{I}_{xy} = \bar{I}_{x'y'} = 0$ (for all products of inertia with respect to all pairs of rectangular axes with origin at C)◀

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Chapter 9, Solution 111.

First observe that for a given area A and origin O of a rectangular coordinate system, the values of I_{ave} and R are the same for all orientations of the coordinate axes. Shown below is a Mohr's circle, with the moments of inertia, $I_{x'}$ and $I_{y'}$, and the product of inertia. $I_{x'y'}$, having been computed for an arbitrary orientation of the $x'y'$ axes.

From the Mohr's circle

$$I_{x'} = I_{\text{ave}} + R\cos 2\theta$$

$$I_{y'} = I_{\text{ave}} - R\cos 2\theta$$

$$I_{x'y'} = R\sin 2\theta$$

Then, forming the expression $\qquad I_{x'}I_{y'} - I_{x'y'}^2$

$$I_{x'}I_{y'} - I_{x'y'}^2 = \left(I_{\text{ave}} + R\cos 2\theta\right)\left(I_{\text{ave}} - R\cos 2\theta\right) - \left(R\sin 2\theta\right)^2$$

$$= \left(I_{\text{ave}}^2 - R^2\cos^2 2\theta\right) - \left(R^2\sin^2 2\theta\right)$$

$$= I_{\text{ave}}^2 - R^2 \quad \text{which is a constant}$$

$\therefore \quad I_{x'}I_{y'} - I_{x'y'}^2$ is independent of the orientation of the coordinate axes

Q.E.D.◀

Shown is a Mohr's circle, with line $\overline{OA}$, of length L, the required tangent. Noting that $\measuredangle OAC$ is a right angle, it follows that

$$L^2 = I_{\text{ave}}^2 - R^2$$

or $\quad L^2 = I_{x'}I_{y'} - I_{x'y'}^2 \qquad$ Q.E.D.◀

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Chapter 9, Solution 112.

From Problem 9.111 have, $I_{x'}I_{y'} - I_{x'y'} = \text{constant}$

Now consider the following two cases

Case 1: $I_{x'} = I_x, \quad I_{y'} = I_y, \quad I_{x'y'} = I_{xy}$

Case 2: $I_{x'} = I_{\max}, \quad I_{y'} = I_{\min}, \quad I_{x'y'} = 0$

Then $$I_x I_y - I_{xy}^2 = I_{\max} I_{\min}$$

or $$I_{xy} = \pm\sqrt{I_x I_y - I_{\max} I_{\min}} \blacktriangleleft$$

From Figure 9.13B: $\bar{I}_x = 453 \times 10^3 \text{ mm}^4 \qquad \bar{I}_y = 166 \times 10^3 \text{ mm}^4$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(\bar{I}_{\max} + \bar{I}_{\min}\right)$$

With $$\bar{I}_{\max} = 524 \times 10^3 \text{ mm}^4$$

then $$\bar{I}_{\min} = (453 + 166 - 524) \times 10^3 \text{ mm}^4 = 95.0 \times 10^3 \text{ mm}^4$$

Finally $$\bar{I}_{xy} = \pm\left[\sqrt{(453)(166) - (524)(95.0)}\right] \times 10^3 \text{ mm}^4$$

$$= \pm 159.43 \times 10^3 \text{ mm}^4$$

The two roots corresponding to the following orientations of the cross section:

(*a*)

(*b*)

(*a*) $\bar{I}_{xy} = -159.4 \times 10^3 \text{ mm}^4 \blacktriangleleft$

(*b*) $\bar{I}_{xy} = 159.4 \times 10^3 \text{ mm}^4 \blacktriangleleft$

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Chapter 9, Solution 113.

$$\text{Mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

$$\text{Area} = A = \frac{1}{2}\pi a^2$$

$$I_{AA',\text{area}} = I_{DD',\text{area}} = \frac{1}{2}\left(\frac{\pi}{4}a^4\right) = \frac{1}{8}\pi a^4$$

$$I_{AA',\text{mass}} = I_{DD',\text{mass}} = \frac{m}{A} I_{AA',\text{area}} = \frac{m}{\frac{1}{2}\pi a^2}\left(\frac{1}{8}\pi a^4\right) = \frac{1}{4}ma^2$$

(*a*)
$$I_{BB'} = I_{DD'} - m(AC)^2 = \frac{1}{4}ma^2 - m\left(\frac{4a}{3\pi}\right)^2$$

$$= (0.25 - 0.1801)ma^2 \qquad I_{BB'} = 0.0699\,ma^2 \blacktriangleleft$$

(*b*) Eq. (9.38):

$$I_{CC'} = I_{AA'} + I_{BB'}$$

$$= \frac{1}{4}ma^2 + 0.0699ma^2 \qquad I_{CC'} = 0.320\,ma^2 \blacktriangleleft$$

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Chapter 9, Solution 114.

$$\text{Mass} = m = \rho V = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

$$\text{Area} = A = \pi r_2^2 - \pi r_1^2 = \pi\left(r_2^2 - r_1^2\right)$$

$$I_{AA',\text{area}} = \frac{\pi}{4} r_2^4 - \frac{\pi}{4} r_1^4 = \frac{\pi}{4}\left(r_2^4 - r_1^4\right)$$

$$(a)\ I_{AA',\text{mass}} = \frac{m}{A} I_{AA',\text{area}} = \frac{m}{\pi\left(r_2^2 - r_1^2\right)} \frac{\pi}{4}\left(r_2^4 - r_1^4\right) = \frac{1}{4} m\left(r_2^2 + r_1^2\right)$$

$$I_{AA'} = \frac{1}{4} m\left(r_2^2 + r_1^2\right) \blacktriangleleft$$

(b) By Symmetry: $\quad I_{BB'} = I_{AA'}$

Eq. (9.38): $\quad I_{CC'} = I_{AA'} + I_{BB'} = 2 I_{AA'}$

$$I_{CC'} = \frac{1}{2} m\left(r_2^2 + r_1^2\right) \blacktriangleleft$$

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Chapter 9, Solution 115.

First note $\quad \text{mass} = m = \rho V = \rho t A$

$$= \rho t \frac{\pi}{4}\left(r_2^2 - r_1^2\right)$$

Also $\quad I_{\text{mass}} = \rho t I_{\text{area}}$

$$= \frac{m}{\frac{\pi}{4}\left(r_2^2 - r_1^2\right)} I_{\text{area}}$$

(*a*) Using Figure 9.12, $\quad I_{AA',\,\text{area}} = \frac{\pi}{16}\left(r_2^4 - r_1^4\right)$

Then $\quad I_{AA',\,\text{mass}} = \frac{m}{\frac{\pi}{4}\left(r_2^2 - r_1^2\right)} \times \frac{\pi}{16}\left(r_2^4 - r_1^4\right)$

$$= \frac{m}{4}\left(r_2^2 + r_1^2\right)$$

$$= \frac{m}{4}\left[r_2^2 + \left(\frac{1}{2}r_2\right)^2\right] = \frac{m}{4}\left(\frac{5}{4}r_2^2\right)$$

or $I_{AA'} = \frac{5}{16} m r_2^2$ ◀

(*b*) Symmetry implies $\quad I_{BB',\,\text{mass}} = I_{AA',\,\text{mass}}$

Then, $\quad I_{OO'} = I_{AA'} + I_{BB'}$

$$= 2\left(\frac{5}{16} m r_2^2\right)$$

$$= \frac{5}{8} m r_2^2$$

continued

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Now locate centroid C.

$$\bar{X}\Sigma A = \Sigma \bar{x} A$$

or $$\bar{X}\left(\frac{\pi}{4}r_2^2 - \frac{\pi}{4}r_1^2\right) = \frac{4r_2}{3\pi}\left(\frac{\pi}{4}r_2^2\right) - \frac{4r_1}{3\pi}\left(\frac{\pi}{4}r_1^2\right)$$

or $$\bar{X} = \frac{4}{3\pi}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$$

Now $$\bar{r} = \bar{X}\sqrt{2}$$

$$= \frac{4\sqrt{2}}{3\pi}\frac{r_2^3 - \left(\frac{1}{2}r_2\right)^3}{r_2^2 - \left(\frac{1}{2}r_2\right)^2} = \frac{14}{9}\frac{\sqrt{2}}{\pi}r_2$$

Finally $$I_{OO'} = \bar{I}_{CC'} + m\bar{r}^2$$

or $$\frac{5}{8}mr_2^2 = \bar{I}_{CC'} + m\left(\frac{14}{9}\frac{\sqrt{2}}{\pi}r_2\right)^2$$

or $\bar{I}_{CC'} = 0.1347mr_2^2$ ◀

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Chapter 9, Solution 116.

Locate centroid:

$$\bar{y}_1 = \frac{b}{2} \qquad A_1 = 3ab$$

$$\bar{y}_2 = \frac{3}{2}b \qquad A_2 = ab$$

Then $$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$= \frac{\frac{b}{2}(3ab) + \frac{3}{2}b(ab)}{3ab + ab}$$

$$= \frac{3}{4}b$$

Uniform thickness: m is proportional to A $\quad \therefore m_1 = \frac{3}{4}m \quad m_2 = \frac{1}{4}m$

(a) $$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$$

$$= \left[\frac{1}{12}\left(\frac{3}{4}m\right)b^2 + \left(\frac{3}{4}m\right)\left(\frac{b}{2}\right)^2\right] + \left[\frac{1}{12}\left(\frac{1}{4}m\right)b^2 + \left(\frac{1}{4}m\right)\left(\frac{3}{2}b\right)^2\right]$$

$$= mb^2\left[\frac{3}{48} + \frac{3}{16} + \frac{1}{48} + \frac{9}{16}\right]$$

$$I_{AA'} = \frac{5}{6}mb^2 \blacktriangleleft$$

(b) Have $$I_{AA'} = \bar{I}_x + m\bar{y}^2$$

or $$\bar{I}_x = \frac{5}{6}mb^2 - m\left(\frac{3}{4}b\right)^2$$

$$= \frac{13}{48}mb^2$$

continued

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Chapter 9, Solution 117.

$$\text{Mass} = m = \rho V = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

$$A = \frac{1}{2} bh$$

(*a*) Axis AA':

$$I_{AA',\,\text{area}} = 2\left[\frac{1}{12} h \left(\frac{b}{2}\right)^3\right] = \frac{1}{48} hb^3$$

$$I_{AA',\,\text{mass}} = \frac{m}{A} I_{AA',\,\text{area}} = \frac{m}{\frac{1}{2}bh} \cdot \frac{1}{48} hb^3 = \frac{1}{24} mb^2$$

$$I_{AA'} = \frac{1}{24} mb^2 \blacktriangleleft$$

Axis BB':

$$I_{BB',\,\text{area}} = \frac{1}{36} bh^3$$

$$I_{BB',\,\text{mass}} = \frac{m}{A} I_{BB',\,\text{area}} = \frac{m}{\frac{1}{2}bh} \cdot \frac{1}{36} bh^3 = \frac{1}{18} mh^2$$

$$I_{BB'} = \frac{1}{18} mh^2 \blacktriangleleft$$

(*b*) Axis CC':

Eq. (9.38):

$$I_{CC'} = I_{AA'} + I_{BB'}$$

$$= \frac{1}{24} mb^2 + \frac{1}{18} mh^2 \qquad I_{CC'} = \frac{m}{72}\left(3b^2 + 4h^2\right) \blacktriangleleft$$

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Chapter 9, Solution 118.

From Prob. 9.117:

$$I_{AA'} = \frac{1}{24}mb^2$$

$$I_{BB'} = \frac{1}{18}mh^2$$

Note that AA' and BB' are centroidal axes.

Hence $I_{DD'} = I_{AA'} + md^2 = \frac{1}{24}mb^2 + md^2$

$$I_{DD'} = \frac{m}{24}\left(b^2 + 24d^2\right) \blacktriangleleft$$

$$I_{EE'} = I_{BB'} + md^2 = \frac{1}{18}mh^2 + md^2$$

$$I_{EE'} = \frac{m}{18}\left(h^2 + 18d^2\right) \blacktriangleleft$$

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Chapter 9, Solution 119.

First note $\quad \text{mass} = m = \rho V = \rho t A$

$$= \rho t\left[(2a)(a) + \frac{1}{2}(2a)(a)\right]$$

$$= 3\rho t a^2$$

Also $\quad I_{\text{mass}} = \rho t I_{\text{area}}$

$$= \frac{m}{3a^2} I_{\text{area}}$$

(*a*) Now $\quad I_{x,\text{ area}} = (I_x)_{1,\text{ area}} + (I_x)_{2,\text{ area}}$

$$= \frac{1}{3}(2a)(a)^3 + \frac{1}{12}(2a)(a)^3$$

$$= \frac{5}{6}a^4$$

Then $\quad I_{x,\text{ mass}} = \frac{m}{3a^2} \times \frac{5}{6}a^4$

or $I_{x,\text{ mass}} = \frac{5}{18}ma^2$ ◀

(*b*) Have

$$I_{z,\text{ area}} = (I_z)_{1,\text{ area}} + (I_z)_{2,\text{ area}}$$

$$= \left[\frac{1}{3}(a)(2a)^3\right] + \left[\frac{1}{36}(a)(2a)^3 + \frac{1}{2}(2a)(a)\left(2a + \frac{1}{3}\times 2a\right)^2\right]$$

$$= 10a^4$$

Then $\quad I_{z,\text{ mass}} = \frac{m}{3a^2} \times 10a^4$

$$= \frac{10}{3}ma^2$$

Finally, $\quad I_{y,\text{ mass}} = I_{x,\text{ mass}} + I_{z,\text{ mass}}$

$$= \frac{5}{18}ma^2 + \frac{10}{3}ma^2$$

$$= \frac{65}{18}ma^2$$

or $I_{y,\text{ mass}} = 3.61ma^2$ ◀

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Chapter 9, Solution 120.

First locate the centroid C

$$\bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}\left(2a^2 + a^2\right) = a\left(2a^2\right) + \left(2a + \frac{1}{3} \times 2a\right)\left(a^2\right)$$

or

$$\bar{X} = \frac{14}{9}a$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A: \quad \bar{Z}\left(2a^2 + a^2\right) = \left(\frac{1}{2}a\right)\left(2a^2\right) + \left(\frac{1}{3}a\right)\left(a^2\right)$$

or

$$\bar{Z} = \frac{4}{9}a$$

(*a*) Have

$$I_{y,\text{ mass}} = \bar{I}_{CC',\text{ mass}} + m\left(\bar{X}^2 + \bar{Z}^2\right)$$

From the solution to Problem 9.119

$$I_{y,\text{ mass}} = \frac{65}{18}ma^2$$

Then

$$\bar{I}_{cc',\text{ mass}} = \frac{65}{18}ma^2 - m\left[\left(\frac{14}{9}a\right)^2 + \left(\frac{4}{9}a\right)^2\right]$$

or $\bar{I}_{cc'} = 0.994ma^2$ ◀

(*b*) Have

$$I_{x,\text{ mass}} = \bar{I}_{BB',\text{ mass}} + m\left(\bar{Z}\right)^2$$

and

$$I_{AA',\text{ mass}} = \bar{I}_{BB',\text{ mass}} + m\left(1.5a\right)^2$$

Then

$$I_{AA',\text{ mass}} = I_{x,\text{ mass}} + m\left[\left(1.5a\right)^2 - \left(\frac{4}{9}a\right)^2\right]$$

From the solution to Problem 9.119

$$I_{x,\text{ mass}} = \frac{5}{18}ma^2$$

Then

$$I_{AA',\text{ mass}} = \frac{5}{18}ma^2 + m\left[\left(1.5a\right)^2 - \left(\frac{4}{9}a\right)^2\right]$$

or $I_{AA'} = 2.33ma^2$ ◀

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Chapter 9, Solution 121.

At $x = a,\ y = b$: $\quad b = ka^2 \quad$ or $\quad k = \dfrac{b}{a^2}$

Then $\quad y = \dfrac{b}{a^2}x^2$

Now $\quad dm = \rho\left(\pi r^2\right)dx$

$$= \pi\rho\left(\frac{b}{a^2}x^2\right)^2 dx$$

Then $\quad m = \pi\rho\dfrac{b^2}{a^4}\int_0^a x^4 dx$

$$= \frac{1}{5}\pi\rho\frac{b^2}{a^4}x^5\Big|_0^a$$

$$= \frac{1}{5}\pi\rho ab^2 \quad \text{or} \quad \pi\rho = \frac{5m}{ab^2}$$

Now $\quad d\bar{I}_x = \left(\dfrac{1}{2}r^2\right)dm = \dfrac{1}{2}\left(\dfrac{b}{a^2}x^2\right)^2\left[\pi\rho\left(\dfrac{b}{a^2}x^2\right)^2 dx\right]$

$$= \frac{5m}{ab^2}\times\frac{1}{2}\frac{b^2}{a^4}x^4\times\frac{b^2}{a^4}x^4 dx = \frac{5}{2}m\frac{b^2}{a^9}x^8 dx$$

Then.. $\quad \bar{I}_x = \dfrac{5}{2}m\dfrac{b^2}{a^9}\int_0^a x^8 dx = \dfrac{5}{2}m\dfrac{b^2}{a^9}\times\dfrac{1}{9}x^9\Big|_0^a$

or $\bar{I}_x = \dfrac{5}{18}mb^2$ ◀

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Chapter 9, Solution 122.

$r_1 = 2r_2 \qquad \therefore a = L$

Now $\quad m = \rho V$

$$= \rho\left[\frac{\pi}{3}(2L)(2r_2)^2 - \frac{\pi}{3}(L)(r_2)^2\right]$$

$$= \frac{7\pi}{3}\rho L r_2^2$$

or $\quad \pi\rho = \frac{3}{7}\frac{m}{Lr_2^2}$

Now $\quad r = \frac{r_2 - r_1}{L}x + r_1$

$$= r_2\left(2 - \frac{x}{L}\right)$$

Have $\quad dI_z = dI_{z'} + x^2 dm$

$$= \frac{1}{4}(dm)r^2 + x^2 dm$$

Where $\quad dm = \rho dV$

$$= \rho\left(\pi r^2 dx\right)$$

$\therefore \quad dI_z = \pi\rho\left(\frac{1}{4}r^4 + r^2x^2\right)dx$

$$= \pi\rho\left\{\frac{1}{4}\left[r_2\left(2 - \frac{x}{L}\right)\right]^4 + x^2\left[r_2\left(2 - \frac{x}{L}\right)\right]^2\right\}$$

Then $\quad I_z = \frac{3}{7}\frac{m}{Lr_2^2}\left(r_2^2\right)\int_0^L\left[\frac{1}{4}r_2^2\left(2 - \frac{x}{L}\right)^4 + x^2\left(4 - 4\frac{x}{L} + \frac{x^2}{L^2}\right)\right]dx$

$$= \frac{3}{7}\frac{m}{L}\left[\frac{1}{4}r_2^2\left(\frac{1}{5}\right)(-L)\left(2 - \frac{x}{L}\right)^5 + \frac{4}{3}x^3 - \frac{x^4}{L} + \frac{1}{5}\frac{x^5}{L^2}\right]\Bigg|_0^L$$

$$I_z = \frac{m}{140}\left(93r_2^2 + 32L^2\right) \blacktriangleleft$$

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Chapter 9, Solution 123.

At $\quad x = 2a \qquad 2a = k(2a)^3 \qquad$ or $\qquad k = \dfrac{1}{4a^2}$

Then $\quad y = \dfrac{1}{4a^2}x^3$

Now $\quad dm = \rho\left(\pi r^2 dx\right)$

$$= \pi\rho\left(\frac{1}{4a^2}x^3\right)^2 dx = \frac{\pi\rho}{16a^4}x^6 dx$$

Then $\quad m = \dfrac{\pi\rho}{16a^4}\int_a^{2a} x^6 dx$

$$= \frac{\pi\rho}{16a^4}\frac{1}{7}x^7\Big|_a^{2a} = \frac{\pi\rho}{112a^4}\left[(2a)^7 - (a)^7\right] = \frac{127}{112}\pi\rho a^3$$

or $\pi\rho = \dfrac{112m}{127a^3}$

(a) Now $\quad d\bar{I}_x = \left(\dfrac{1}{2}r^2\right)dm = \dfrac{1}{2}\left(\dfrac{1}{4a^2}x^3\right)^2\left(\dfrac{\pi\rho}{16a^4}x^6 dx\right)$

$$= \frac{1}{32a^4}x^6 \times \frac{112m}{127a^3} \times \frac{x^6}{16a^4}dx = \frac{7m}{4064a^{11}}x^{12}dx$$

Then

$$\bar{I}_x = \frac{7m}{4064a^{11}}\int_a^{2a} x^{12}dx = \frac{7m}{4064a^{11}}\frac{1}{13}x^{13}\Big|_a^{2a}$$

$$= \frac{7m}{52832a^{11}}\left[(2a)^{13} - (a)^{13}\right] = \frac{57337}{52832}ma^2 = 1.0853ma^2$$

or $\bar{I}_x = 1.085ma^2$ ◀

continued

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(*b*) Have $$d\bar{I}_y = \left(\frac{1}{4}r^2 + x^2\right)dm = \left[\frac{1}{4}\left(\frac{1}{4a^2}x^3\right)^2 + x^2\right]\frac{\pi\rho}{16a^4}x^6dx$$

$$= \frac{1}{16a^4} \times \frac{112m}{127a^3}\left(\frac{1}{64a^4}x^{12} + x^8\right)dx$$

Then $$\bar{I}_y = \frac{7m}{127a^7}\int_a^{2a}\left(\frac{1}{64a^4}x^{12} + x^8\right)dx = \frac{7m}{127a^7}\left(\frac{1}{832a^4}x^{13} + \frac{1}{9}x^9\right)\Bigg|_a^{2a}$$

$$= \frac{7m}{127a^7}\left[\frac{1}{832a^4}(2a)^{13} + \frac{1}{9}(2a)^9 - \frac{1}{832a^4}(a)^{13} - \frac{1}{9}(a)^9\right]$$

$$= \frac{7m}{127a^7}\left(\frac{8191}{832}a^9 + \frac{511}{9}a^9\right) = 3.67211ma^2$$

or $\bar{I}_y = 3.67\,ma^2$ ◀

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Chapter 9, Solution 124.

For the element shown:

$$dm = \rho dV = \rho\left(a\frac{y}{h}\right)\left(b\frac{y}{h}\right)dy = \rho\frac{ab}{h^2}y^2dy$$

$$d\bar{I}x' = \frac{1}{12}\left(b\frac{y}{h}\right)^2 dm = \frac{1}{12}\frac{b^2}{h^2}y^2\left(\rho\frac{ab}{h^2}\right)y^2dy = \frac{1}{12}\rho\frac{ab^3}{h^4}y^4dy$$

Parallel-axis theorem

$$dI_x = d\bar{I}_{x'} + d^2dm \qquad \text{where } d^2 = y^2 + \left(\frac{1}{2}b\frac{y}{h}\right)^2$$

$$= \frac{1}{12}\rho\frac{ab^3}{h^4}y^4dy + \left[y^2 + \frac{1}{4}b^2\frac{y^2}{h^2}\right]\rho\frac{ab}{h^2}y^2dy$$

$$= \left(\frac{\rho}{3}\frac{ab^3}{h^4} + \rho\frac{ab}{h^2}\right)y^4dy$$

$$I_x = \int dI_x = \left(\frac{\rho}{3}\frac{ab^3}{h^4} + \rho\frac{ab}{h^2}\right)\int_0^h y^4dy = \frac{\rho ab^3h}{15} + \frac{\rho abh^3}{5}$$

For pyramid, $\quad m = \rho v = \frac{1}{3}\rho abh$

Thus $\quad I_x = \left(\frac{1}{3}\rho abh\right)\left(\frac{b^2}{5} + \frac{3h^2}{5}\right) \qquad I_x = \frac{1}{5}m\left(b^2 + 3h^2\right)$ ◀

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Chapter 9, Solution 125.

For the element shown:

$$dm = \rho\left(b\frac{y}{h}\right)\left(a\frac{y}{h}\right)dy = \rho\frac{ab}{h^2}y^2dy$$

For thin plate:

$$dI_y = dI_{x''} + dI_{z''}$$

$$= \frac{1}{3}\left(b\frac{y}{h}\right)^2 dm + \frac{1}{3}\left(a\frac{y}{h}\right)^2 dm = \frac{1}{3h^2}\left(b^2 + a^2\right)y^2dm$$

$$= \frac{1}{3h^2}\left(a^2 + b^2\right)y^2\left(\rho\frac{ab}{h^2}y^2dy\right) = \frac{\rho ab}{3h^4}\left(a^2 + b^2\right)y^4dy$$

$$I_y = \int dI_y = \frac{\rho ab}{3h^4}\left(a^2 + b^2\right)\int_0^h y^4dy = \frac{\rho ab}{15}\left(a^2 + b^2\right)h$$

For pyramid, $m = \rho V = \frac{1}{3}\rho abh$

$$I_y = \left(\frac{1}{3}\rho abh\right)\frac{1}{5}\left(a^2 + b^2\right) \qquad I_y = \frac{1}{5}m\left(a^2 + b^2\right) \blacktriangleleft$$

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Chapter 9, Solution 126.

$r^2 = y^2 + z^2 = kx$:

At $x = h$, $r = a$; $a^2 = kh$; $k = \dfrac{a^2}{h}$

Thus, $r^2 = \dfrac{a^2}{h}x$

Have $dm = \rho\pi r^2 dx = \rho\pi \dfrac{a^2}{h} x dx$

Then $m = \int_0^h dm = \rho\pi \dfrac{a^2}{h}\int_0^h x dx$

$$= \frac{1}{2}\rho\pi a^2 h$$

Now $$dI_y = dI_{y'} + x^2 dm = \frac{1}{4}r^2 dm + x^2 dm = \left(\frac{1}{4}\frac{a^2}{h}x + x^2\right)dm$$

So $$I_y = \int_0^h dI_y = \int_0^h \left(\frac{1}{4}\frac{a^2}{h}x + x^2\right)\rho\pi\frac{a^2}{h}x dx$$

$$= \rho\pi\frac{a^2}{h}\int_0^h\left(\frac{1}{4}\frac{a^2}{h}x^2 + x^3\right)dx$$

$$= \rho\pi\frac{a^2}{h}\left(\frac{1}{4}\frac{a^2}{h}\frac{h^3}{3} + \frac{h^4}{4}\right)$$

$$= \frac{1}{12}\rho\pi a^2 h\left(a^2 + 3h^2\right)$$

Recall: $$m = \frac{1}{2}\rho\pi a^2 h$$

$$I_y = \left(\frac{1}{2}\rho\pi a^2 h\right)\frac{1}{6}\left(a^2 + 3h^2\right) \quad \text{or} \quad I_y = m\left(a^2 + 3h^2\right)/6 \blacktriangleleft$$

And $$k_y^2 = \frac{I_y}{m} = \left(a^2 + 3h^2\right)/6 \quad \text{or} \quad k_y = \sqrt{\left(a^2 + 3h^2\right)/6} \blacktriangleleft$$

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Chapter 9, Solution 127.

First note $$\frac{dy}{dx} = -x^{-\frac{1}{3}}\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^{\frac{1}{2}}$$

Then $$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-\frac{2}{3}}\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)$$

$$= \left(\frac{a}{x}\right)^{\frac{2}{3}}$$

For the element shown $$dm = m'dL = m'\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$= m'\left(\frac{a}{x}\right)^{\frac{1}{3}} dx$$

Then $$m = \int dm = \int_0^a m'\frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx = \frac{3}{2}m'a^{\frac{1}{3}}\left[x^{\frac{2}{3}}\right]_0^a = \frac{3}{2}m'a$$

Now $$I_x = \int y^2 dm = \int_0^a \left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^3 \left(m'\frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx\right)$$

$$= m'a^{\frac{1}{3}}\int_0^a \left(\frac{a^2}{x^{\frac{1}{3}}} - 3a^{\frac{4}{3}}x^{\frac{1}{3}} + 3a^{\frac{2}{3}}x - x^{\frac{5}{3}}\right)dx$$

$$= m'a^{\frac{1}{3}}\left[\frac{3}{2}a^2x^{\frac{2}{3}} - \frac{9}{4}a^{\frac{4}{3}}x^{\frac{4}{3}} + \frac{3}{2}a^{\frac{2}{3}}x^2 - \frac{3}{8}x^{\frac{8}{3}}\right]_0^a$$

$$= m'a^3\left(\frac{3}{2} - \frac{9}{4} + \frac{3}{2} - \frac{3}{8}\right) = \frac{3}{8}m'a^3$$

or $I_x = \frac{1}{4}ma^2$ ◀

Symmetry implies $I_y = \frac{1}{4}ma^2$ ◀

continued

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Alternative Solution

$$I_y = \int x^2 dm = \int_0^a x^2 \left(m' \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}} dx \right) = m' a^{\frac{1}{3}} \int_0^a x^{\frac{5}{3}} dx$$

$$= m' a^{\frac{1}{3}} \times \frac{3}{8} \left[x^{\frac{8}{3}} \right]_0^a = \frac{3}{8} m' a^3$$

$$= \frac{1}{4} m a^2$$

Also $$I_z = \int \left(x^2 + y^2 \right) dm = I_y + I_x$$

or $I_z = \frac{1}{2} m a^2$ ◀

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Chapter 9, Solution 128.

For line BC

$$\zeta = -\frac{h}{\frac{a}{2}}x + h$$

$$= \frac{h}{a}(a - 2x)$$

Also

$$m = \rho V = \rho t\left(\frac{1}{2}ah\right)$$

$$= \frac{1}{2}\rho tah$$

(a) Have

$$dI_x = \frac{1}{12}\zeta^2 dm' + \left(\frac{\zeta}{2}\right)^2 dm'$$

$$= \frac{1}{3}\zeta^2 dm'$$

where

$$dm' = \rho t\zeta dx$$

Then

$$I_x = \int dI_x = 2\int_0^{\frac{a}{2}} \frac{1}{3}\zeta^2(\rho t\zeta dx)$$

$$= \frac{2}{3}\rho t\int_0^{\frac{a}{2}}\left[\frac{h}{a}(a - 2x)\right]^3 dx$$

$$= \frac{2}{3}\rho t\frac{h^3}{a^3} \times \frac{1}{4}\left(-\frac{1}{2}\right)\left[(a - 2x)^4\right]_0^{\frac{a}{2}}$$

$$= -\frac{1}{12}\rho t\frac{h^3}{a^3}\left[(a - a)^4 - (a)^4\right]$$

$$= \frac{1}{12}\rho tah^3$$

or $I_x = \frac{1}{6}mh^2$ ◀

continued

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Now $$I_\zeta = \int x^2 dm$$

and $$I_\zeta = \int x^2 dm' = 2\int_0^{\frac{a}{2}} x^2 \left(\rho t \zeta dx\right)$$

$$= 2\rho t \int_0^{\frac{a}{2}} x^2 \left[\frac{h}{a}\left(a - 2x\right)\right] dx$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}x^3 - \frac{1}{4}x^4\right]_0^{\frac{a}{2}}$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}\left(\frac{a}{2}\right)^3 - \frac{1}{4}\left(\frac{a}{2}\right)^4\right]$$

$$= \frac{1}{48}\rho t a^3 h = \frac{1}{24} m a^2$$

(*b*) Have $$I_y = \int r_y^2 dm = \int\left[x^2 + \left(\zeta \sin\theta\right)^2\right] dm$$

$$= \int x^2 dm + \sin^2\theta \int \zeta^2 dm$$

Now $$I_x = \int \zeta^2 dm \Rightarrow I_y = I_\zeta + I_x \sin^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \sin^2\theta$$

or $$I_y = \frac{m}{24}\left(a^2 + 4h^2 \sin^2\theta\right) \blacktriangleleft$$

(*c*) Have $$I_z = \int r_z^2 dm = \int\left(x^2 + y^2\right) dm$$

$$= \int\left[x^2 + \left(\zeta \cos\theta\right)^2\right] dm$$

$$= \int x^2 dm + \cos^2\theta \int \zeta^2 dm$$

$$= I_\zeta + I_x \cos^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \cos^2\theta$$

or $$I_z = \frac{m}{24}\left(a^2 + 4h^2 \cos^2\theta\right) \blacktriangleleft$$

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Chapter 9, Solution 129.

Mass of cylindrical ring:

$$m = \rho V$$

$$= \frac{\gamma}{g}\left(\frac{\pi}{4}d_2^2 - \frac{\pi}{4}d_1^2\right)t$$

$$= \frac{\pi}{4}\frac{\gamma}{g}\left(d_2^2 - d_1^2\right)t$$

Now treat the wheel as a series of 4 concentric rings. (Note - the steel is treated as a large ring minus two smaller rings.)

$$m_{\text{wheel}} = \sum m_{\text{ring}}$$

$$= \frac{\pi}{4}\frac{0.310\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times 1.5\ \text{in.} \times \left(0.7^2 - 0.5^2\right)\text{in}^2$$

$$+ \frac{\pi}{4}\frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times 1.5\ \text{in.} \times \left(4.4^2 - 0.7^2\right)\text{in}^2$$

$$-2 \times \frac{\pi}{4}\frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times \frac{1.1}{2}\ \text{in.} \times \left(4^2 - 1.2^2\right)\text{in}^2$$

$$+ \frac{\pi}{4}\frac{0.043\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times 1.5\ \text{in.} \times \left(5^2 - 4.4^2\right)\text{in}^2$$

$$m_{\text{wheel}} = \left(2.7221 \times 10^{-3} + 196.072 \times 10^{-3} - 110.945 \times 10^{-3} + 8.8730 \times 10^{-3}\right)\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$= 96.722 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

continued

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For a cylindrical ring:

$$\begin{aligned}
(I_{AA'})_{\text{ring}} &= \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2 \\
&= \frac{1}{8}\left(\frac{\pi}{4}\frac{\gamma t}{g}d_2^2\right)d_2^2 - \frac{1}{8}\left(\frac{\pi}{4}\frac{\gamma t}{g}d_1^2\right)d_1^2 \\
&= \frac{\pi}{32}\frac{\gamma t}{g}\left(d_2^4 - d_1^4\right) \\
&= \frac{\pi}{32}\frac{\gamma t}{g}\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right) \\
&= \frac{1}{8}m_{\text{ring}}\left(d_2^2 + d_1^2\right)
\end{aligned}$$

Then:

$$\begin{aligned}
\bar{I}_{AA'} &= \sum (I_{AA'})_{\text{ring}} \\
&= \frac{1}{8}\left(2.7221\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(0.7^2 + 0.5^2\right)\text{ in}^2 \times \left(\frac{1\text{ft}}{12\text{ in.}}\right)^2 \\
&\quad + \frac{1}{8}\left(196.072\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(4.4^2 + 0.7^2\right)\text{ in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 \\
&\quad - \frac{1}{8}\left(110.945\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(4^2 + 1.2^2\right)\text{ in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 \\
&\quad + \frac{1}{8}\left(8.8730\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(5^2 + 4.4^2\right)\text{ in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 \\
&= \left(1.74857\times10^{-6} + 3.3785\times10^{-3} - 1.67958\times10^{-3} + 341.67\times10^{-6}\right)\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \\
&= 2.0423\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2
\end{aligned}$$

$$\bar{I}_{AA'} = 2.04\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

And

$$\begin{aligned}
\bar{k}_{AA'} &= \sqrt{\frac{\bar{I}_{AA'}}{m}} \\
&= \sqrt{\frac{2.04\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2}{96.722\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}}} \\
&= 0.145311\text{ ft}
\end{aligned}$$

$$\bar{k}_{AA'} = 1.744\text{ in.} \blacktriangleleft$$

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Chapter 9, Solution 130.

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\rho t\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that

$$\begin{aligned} I_{AA'} &= \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2 \\ &= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right] \\ &= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right) \\ &= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right) \\ &= \frac{1}{8}m\left(d_1^2 + d_2^2\right) \end{aligned}$$

Now treat the roller as three concentric rings and, working from the bronze outward, have

Have
$$\begin{aligned} m &= \frac{\pi}{4}\Big\{\left(8580\text{ kg/m}^3\right)\left(0.0195\text{ m}\right)\left[\left(0.009\text{ m}\right)^2 - \left(0.006\text{ m}\right)^2\right] \\ &\quad + \left(2770\text{ kg/m}^3\right)\left(0.0165\text{ m}\right)\left[\left(0.012\text{ m}\right)^2 - \left(0.009\text{ m}\right)^2\right] \\ &\quad + \left(1250\text{ kg/m}^3\right)\left(0.0165\text{ m}\right)\left[\left(0.027\text{ m}\right)^2 - \left(0.012\text{ m}\right)^2\right]\Big\} \\ &= \frac{\pi}{4}\left[7.52895 + 2.87942 + 12.06563\right]\times 10^{-3}\text{ kg} \\ &= 5.9132\times 10^{-3}\text{ kg} + 2.26149\times 10^{-3}\text{ kg} \\ &\quad + 9.47632\times 10^{-3}\text{ kg} \\ &= 17.6510\times 10^{-3}\text{ kg} \end{aligned}$$

continued

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And $$I_{AA'} = \frac{1}{8}\left\{\left(5.9132\times10^{-3}\text{ kg}\right)\left[\left(0.006\right)^2+\left(0.009\right)^2\right]\text{m}^2\right.$$

$$+\left(2.26149\times10^{-3}\text{ kg}\right)\left[\left(0.009\right)^2+\left(0.012\right)^2\right]\text{m}^2$$

$$\left.+\left(9.47632\times10^{-3}\text{ kg}\right)\left[\left(0.012\right)^2+\left(0.027\right)^2\right]\text{m}^2\right\}$$

$$=\frac{1}{8}\left(691.844+508.835+8272.827\right)10^{-9}\text{ kg}\cdot\text{m}^2$$

$$=1.18419\times10^{-6}\text{ kg}\cdot\text{m}^2$$

or $I_{AA'} = 1.184\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

Now $$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.18419\times10^{-6}\text{ kg m}^2}{17.6510\times10^{-3}\text{ kg}}$$

$$=67.08902\times10^{-6}\text{ m}^2$$

$$k_{AA'} = 8.19079\times10^{-3}\text{ m}$$

or $k_{AA'} = 8.19\text{ mm}$ ◀

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Chapter 9, Solution 131.

fro

Consider shell to be formed by removing hemisphere of radius r hemisphere of radius $r + t$.

For hemisphere:

$$I = \frac{2}{5}mr^2 \qquad \text{Area} = \frac{1}{2}\left(4\pi r^2\right) = 2\pi r^2$$

$$m = \rho V = \frac{1}{2}\rho\frac{4}{3}\pi r^3 = \frac{2}{3}\rho\pi r^3$$

Thus $$I = \frac{2}{5}\left(\frac{2}{3}\rho\pi r^3\right)r^2 = \frac{4}{15}\rho\pi r^5$$

For hemispherical shell:

$$I = \frac{4}{15}\rho\pi\left[(r+t)^5 - r^5\right] = \frac{4}{15}\rho\pi\left[r^5 + 5r^4t + 10r^3t^2 + \cdots - r^5\right]$$

Neglect terms with powers of $t > 1$,

$$I = \frac{4}{3}\rho\pi r^4 t$$

Mass of shell: $m = \rho V = \rho t A = \rho t\left(2\pi r^2\right) = 2\rho\pi r^2 t$

$$I = \left(2\rho\pi r^2 t\right)\frac{2}{3}r^2 = \frac{2}{3}mr^2 \qquad I = \frac{2}{3}mr^2 \blacktriangleleft$$

Radius of gyration:

$$k^2 = \frac{I}{m} = \frac{2}{3}r^2 \qquad k = 0.816r \blacktriangleleft$$

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Chapter 9, Solution 132.

For solid cylinder:

$$\bar{I} = \frac{1}{2}ma^2 \qquad m = \rho V = \rho\pi a^2 h$$

$$\therefore \bar{I} = \frac{1}{2}\rho\pi a^4 h$$

For ring:

$$\bar{I}_{AA'} = \frac{1}{2}\rho\pi a_2^4 h - \frac{1}{2}\rho\pi a_1^4 h = \frac{1}{2}\rho\pi h\left(a_2^4 - a_1^4\right)$$

$$m = \rho\pi h\left(a_2^2 - a_1^2\right)$$

(a) By parallel-axis theorem

$$I_{BB'} = I_{AA'} + ma_1^2 = \frac{1}{2}\rho\pi h\left(a_2^4 - a_1^4\right) + \rho\pi h\left(a_2^2 - a_1^2\right)a_1^2$$

$$I_{BB'} = \frac{1}{2}\rho\pi h\left(a_2^4 + 2a_2^2a_1^2 - 3a_1^4\right) \blacktriangleleft$$

(b) For Maximum $I_{BB'}$:

$$\frac{dI_{BB'}}{da_1} = \frac{1}{2}\rho\pi h\left(4a_2^2a_1 - 12a_1^3\right) = 0$$

$$a_1 = 0; \qquad 4\left(a_2^2 - 3a_1^2\right) = 0 \qquad a_1^2 = \frac{1}{3}a_2^2 \qquad a_1 = \frac{1}{\sqrt{3}}a_2 \blacktriangleleft$$

(c) Maximum $I_{BB'}$:

$$I_{BB'} = \frac{1}{2}\rho\pi h\left[a_2^4 + 2a_2^2\left(\frac{a_2}{\sqrt{3}}\right)^2 - 3\left(\frac{a_2}{\sqrt{3}}\right)^4\right]$$

$$= \frac{1}{2}\rho\pi h\left(a_2^4 + \frac{2}{3}a_2^4 - \frac{1}{3}a_2^4\right)$$

$$= \frac{1}{2}\rho\pi h\left(\frac{4}{3}a_2^4\right)$$

$$I_{BB'} = \frac{2}{3}\rho\pi h a_2^4 \blacktriangleleft$$

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Chapter 9, Solution 133.

First note $\dfrac{L}{120} = \dfrac{L - 240}{80}$ or $L = 720$ mm

and $m = \rho_{st} V$

Now $m_1 = \rho_{st}\left(\dfrac{1}{3}\pi a_1^2 h_1\right) = \dfrac{\pi}{3} \times 7850 \text{ kg/m}^3 \times (0.120 \text{ m})^2 (0.720 \text{ m})$

$= 85.230$ kg

$m_2 = \rho_{st}\left(\dfrac{2}{3}\pi a_2^3\right) = \dfrac{2}{3}\pi \times 7850 \text{ kg/m}^3 \times (0.090 \text{ m})^3 = 11.9855$ kg

$m_3 = \rho_{st}\left(\dfrac{1}{3}\pi a_3^2 h_3\right) = \dfrac{\pi}{3} \times 7850 \text{ kg/m}^3 \times (0.080 \text{ m})^2 (0.720 - 0.240)\text{m}$

$= 25.253$ kg

Now $\bar{I}_y = (\bar{I}_y)_1 - (\bar{I}_y)_2 - (\bar{I}_y)_3$

continued

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(*b*) Have $$\frac{(I_{GG'})_{\text{cup}}}{(I_{AA'})_{\text{cup}}} = 0.01$$

or $$\frac{5}{12}m_{\text{cup}}a^2 = 0.01m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) \quad \text{(From Part a)}$$

Now let $\zeta = \dfrac{a}{l}$

Then $$5\zeta^2 = 0.12\left(\frac{5}{3}\zeta^2 + 2\zeta + 1\right)$$

or $$40\zeta^2 - 2\zeta - 1 = 0$$

Then $$\zeta = \frac{2 \pm \sqrt{(-2)^2 - 4(40)(-1)}}{2(40)}$$

or $\zeta = 0.1851$ and $\zeta = -0.1351$

$$\therefore \quad \frac{a}{l} = 0.1851 \blacktriangleleft$$

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Chapter 9, Solution 134.

Have

(*a*) $$d_B = \frac{4}{12} - d_A = (0.33333 - d_A)\text{ft}$$

and $$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

$$I_{BB'} = \bar{I}_{GG'} + md_B^2$$

Then $$I_{BB'} - I_{AA'} = m\left(d_B^2 - d_A^2\right)$$

$$= m\left[(0.33333 - d_A)^2 - d_A^2\right]$$

$$= m(0.11111 - 0.66666d_A)$$

Then $$(1.26 - 0.6)\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= \frac{0.40\ \text{lb}}{32.2\ \text{ft/s}^2}(0.11111 - 0.66666d_A)\ \text{ft}^2$$

or $$d_A = 0.08697\ \text{ft}$$

or $$d_A = 1.044\ \text{in.}$$ ◀

(*b*) $$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

or $$\bar{I}_{GG'} = 0.6\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 - \frac{0.4\ \text{lb}}{32.2\ \text{ft/s}^2}(0.08697\,\text{ft})^2$$

$$= 0.50604\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$\bar{k}_{GG'}^2 = \frac{\bar{I}_{GG'}}{m} = \frac{0.50604\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2}{\dfrac{0.4\ \text{lb}}{32.2\ \text{ft/s}^2}}$$

$$= 0.04074\ \text{ft}^2$$

$$\bar{k}_{GG'} = 0.20183\ \text{ft} = 2.4219\ \text{in.}$$

or $$\bar{k}_{GG'} = 2.42\ \text{in.}$$ ◀

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Chapter 9, Solution 135.

(*a*) First note $m_{\text{arm}} = \rho V_{\text{arm}} = \rho \times \frac{\pi}{4} d^2 l$

and $dm_{\text{cup}} = \rho dV_{\text{cup}}$

$$= \rho\left[(2\pi a\cos\theta)(t)(ad\theta)\right]$$

Then $m_{\text{cup}} = \int dm_{\text{cup}} = \int_0^{\frac{\pi}{2}} 2\pi\rho a^2 t\cos\theta d\theta$

$$= 2\pi\rho a^2 t\left[\sin\theta\right]_0^{\frac{\pi}{2}}$$

$$= 2\pi\rho a^2 t$$

Now $(I_{AA'})_{\text{anem.}} = (I_{AA'})_{\text{cups}} + (I_{AA'})_{\text{arms}}$

Using the parallel-axis theorem and assuming the arms are slender rods, have

$$(I_{AA'})_{\text{anem.}} = 3\left[(I_{GG'})_{\text{cup}} + m_{\text{cup}} d_{AG}^2\right]$$

$$+ 3\left[\bar{I}_{\text{arm}} + m_{\text{arm}} d_{AG_{\text{arm}}}^2\right]$$

$$= 3\left\{\frac{5}{12} m_{\text{cup}} a^2 + m_{\text{cup}}\left[(l+a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$+3\left[\frac{1}{2} m_{\text{arm}} l^2 + m_{\text{arm}}\left(\frac{l}{2}\right)^2\right]$$

$$= 3m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) + m_{\text{arm}} l^2$$

$$= 3\left(2\pi\rho a^2 t\right)\left(\frac{5}{3}a^2 + 2la + l^2\right) + \left(\frac{\pi}{4}\rho d^2 l\right)\left(l^2\right)$$

or $(I_{AA'})_{\text{anem}} = \pi\rho l^2\left[6a^2 t\left(\frac{5}{3}\frac{a^2}{l^2} + 2\frac{a}{l} + 1\right) + \frac{d^2 l}{4}\right]$ ◀

continued

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Chapter 9, Solution 136.

First note $m_1 = \rho V_1 = \rho b^2 L$

And $m_2 = \rho V_2 = \rho a^2 L$

(a) Using Figure 9.28 and the parallel-axis theorem, have

$$I_{AA'} = (I_{AA'})_1 - (I_{AA'})_2$$

$$= \left[\frac{1}{12} m_1 \left(b^2 + b^2\right) + m_1 \left(\frac{a}{2}\right)^2\right]$$

$$- \left[\frac{1}{12} m_2 \left(a^2 + a^2\right) + m_2 \left(\frac{a}{2}\right)^2\right]$$

$$= \left(\rho b^2 L\right)\left(\frac{1}{6} b^2 + \frac{1}{4} a^2\right) - \left(\rho a^2 L\right)\left(\frac{5}{12} a^2\right)$$

$$= \frac{\rho L}{12}\left(2b^4 + 3b^2 a^2 - 5a^4\right)$$

Then $$\frac{dI_{AA'}}{da} = \frac{\rho L}{12}\left(6b^2 a - 20a^3\right) = 0$$

or $a = 0$ and $a = b\sqrt{\frac{3}{10}}$

Also $$\frac{d^2 I_{AA'}}{da^2} = \frac{\rho L}{12}\left(6b^2 - 60a^2\right) = \frac{1}{2}\rho L\left(b^2 - 10a^2\right)$$

Now, for

$a = 0$, $\frac{d^2 I_{AA'}}{da^2} > 0$ and for $a = b\sqrt{\frac{3}{10}}$, $\frac{d^2 I_{AA'}}{da^2} < 0$

$\therefore$ $(I_{AA'})_{\max}$ occurs when $a = b\sqrt{\frac{3}{10}}$

$$a = 84\sqrt{\frac{3}{10}} = 46.009 \text{ mm}$$

or $a = 46.0$ mm ◀

continued

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(*b*) From part (*a*)

$$\left(I_{AA'}\right)_{\text{mass}} = \frac{\rho L}{12}\left[2b^4 + 3b^2\left(b\sqrt{\frac{3}{10}}\right)^2 - 5\left(b\sqrt{\frac{3}{10}}\right)^4\right]$$

$$= \frac{49}{240}\rho L b^4 = \frac{49}{240}\left(2800\ \text{kg/m}^3\right)\left(0.3\ \text{m}\right)\left(0.084\ \text{m}\right)^4$$

$$= 8.5385 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $\left(I_{AA'}\right)_{\text{mass}} = 8.54 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

and $$k_{AA'}^2 = \frac{\left(I_{AA'}\right)_{\text{mass}}}{m}$$

where

$$m = m_1 - m_2 = \rho L\left(b^2 - a^2\right) = \rho L\left[b^2 - \left(b\sqrt{\frac{3}{10}}\right)^2\right] = \frac{7}{10}\rho L b^2$$

Then

$$k_{AA'}^2 = \frac{\frac{49}{240}\rho L b^4}{\frac{7}{10}\rho L b^2} = \frac{7}{24}b^2 = \frac{7}{24}\left(84\ \text{mm}\right)^2 = 2058\ \text{mm}^2$$

$$k_{AA'} = 45.3652\ \text{mm}$$

or $k_{AA'} = 45.4\ \text{mm}$ ◀

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Chapter 9, Solution 137.

$$m = \rho V$$

$$= \frac{\gamma}{g} tA$$

Then:

$$m_1 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.08 \text{ in.} \times (3.6 \text{ in.})(1.2 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 3.0435 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.08 \text{ in.} \times \frac{1}{2}(1.8 \text{ in.})(1.2 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 760.87 \times 10^{-6} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_3 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.08 \text{ in.} \times \frac{\pi}{2}(1.8 \text{ in.})^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 3.5855 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

Now $\quad I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

$$I_x = \left[\frac{1}{3}\left(3.0435 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.2 \text{ in.})^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left[\frac{1}{18}\left(760.87 \times 10^{-6} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.8^2 + 1.2^2)\text{in}^2 + \left(760.87 \times 10^{-6} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(0.6^2 + 0.4^2)\text{in}^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left[\left(3.5855 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{1}{4}\right)(1.8 \text{ in.})^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= \left[\left(10.1450 \times 10^{-6}\right) + \left(1.37379 \times 10^{-6} + 2.7476 \times 10^{-6}\right) + \left(20.168 \times 10^{-6}\right)\right] \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 34.435 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_x = 34.4 \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

continued

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$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left[\frac{1}{3}\left(3.0435\times10^{-3}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(3.6\text{ in.})^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left[\frac{1}{18}\left(760.87\times10^{-6}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.8\text{ in.})^2 + \left(760.87\times10^{-6}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(1.8^2+0.6^2\right)\text{in}^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left\{\left(3.5855\times10^{-3}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(1.8\text{ in.})^2 + \left(3.5855\times10^{-3}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\left(\frac{4\times1.8}{3\pi}\right)^2+1.8^2\right]\text{in}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= \left[\left(91.305\times10^{-6}\right)+\left(0.95109\times10^{-6}+19.0218\times10^{-6}\right)+\left(25.805\times10^{-6}+92.205\times10^{-6}\right)\right]\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 232\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 232\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left[\frac{1}{3}\left(3.0435\times10^{-3}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(3.6^2+1.2^2\right)\text{in}^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left[\frac{1}{18}\left(760.87\times10^{-6}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.2\text{ in.})^2 + \left(760.87\times10^{-6}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(1.8^2+0.4^2\right)\text{in}^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left[\frac{1}{4}\left(3.5855\times10^{-3}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.8\text{ in.})^2 + \left(3.5855\times10^{-3}\,\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.8\text{ in.})^2\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= \left[\left(101.45\times10^{-6}\right)+\left(0.42271\times10^{-6}+17.9650\times10^{-6}\right)+\left(100.842\times10^{-6}\right)\right]\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 221\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 221\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

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To the Instructor:

The following formulas for the mass moment of inertia of thin plates and a half cylindrical shell are derived at this time for use in the solutions of Problems 9.137–9.142.

Thin rectangular plate

$$\left(I_x\right)_m = \left(\bar{I}_{x'}\right)_m + md^2$$

$$= \frac{1}{12}m\left(b^2 + h^2\right) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right]$$

$$= \frac{1}{3}m\left(b^2 + h^2\right)$$

$$\left(I_y\right)_m = \left(\bar{I}_{y'}\right)_m + md^2$$

$$= \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2$$

$$= \frac{1}{3}mb^2$$

$$I_z = \left(\bar{I}_{z'}\right)_m + md^2$$

$$= \frac{1}{12}mh^2 + m\left(\frac{h}{2}\right)^2$$

$$= \frac{1}{3}mh^2$$

Thin triangular plate

Have $\quad m = \rho V = \rho\left(\frac{1}{2}bht\right)$

and $\quad \bar{I}_{z,\text{ area}} = \frac{1}{36}bh^3$

Then $\quad \bar{I}_{z,\text{ mass}} = \rho t I_{z,\text{ area}}$

$$= \rho t \times \frac{1}{36}bh^3$$

$$= \frac{1}{18}mh^2$$

continued

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Similarly, $\bar{I}_{y,\text{ mass}} = \frac{1}{18}mb^2$

Now $\bar{I}_{x,\text{ mass}} = \bar{I}_{y,\text{ mass}} + \bar{I}_{z,\text{ mass}} = \frac{1}{18}m\left(b^2 + h^2\right)$

Thin semicircular plate

Have $m = \rho V = \rho\left(\frac{\pi}{2}a^2 t\right)$

And $\bar{I}_{y,\text{ area}} = I_{z,\text{ area}} = \frac{\pi}{8}a^4$

Then $\bar{I}_{y,\text{ mass}} = I_{z,\text{ mass}} = \rho t \bar{I}_{y,\text{ area}}$

$$= \rho t \times \frac{\pi}{8}a^4$$

$$= \frac{1}{4}ma^2$$

Now $I_{x,\text{ mass}} = \bar{I}_{y,\text{ mass}} + I_{z,\text{ mass}} = \frac{1}{2}ma^2$

Also $I_{x,\text{ mass}} = \bar{I}_{x',\text{ mass}} + m\bar{y}^2$ or $\bar{I}_{x',\text{ mass}} = m\left(\right.$

And $I_{z,\text{ mass}} = \bar{I}_{z',\text{ mass}} + m\bar{y}^2$ or $\bar{I}_{z',\text{ mass}} = m\left(\right.$

Thin quarter-circular plate

$$\bar{y} = \bar{z} = \frac{4a}{3\pi}$$

Have $m = \rho V = \rho\left(\frac{\pi}{4}a^2 t\right)$

and $I_{y,\text{ area}} = I_{z,\text{ area}} = \frac{\pi}{16}a^4$

Then $I_{y,\text{ mass}} = I_{z,\text{ mass}} = \rho t I_{y,\text{ area}}$

$$= \rho t \times \frac{\pi}{16}a^4$$

$$= \frac{1}{4}ma^2$$

continued

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Now	$I_{x,\text{ mass}} = I_{y,\text{ mass}} + I_{z,\text{ mass}} = \frac{1}{2}ma^2$
Also	$I_{x,\text{ mass}} = I_{x',\text{ mass}} + m\left(\bar{y}^2 + \bar{z}^2\right)$
or	$\bar{I}_{x',\text{ mass}} = m\left(\frac{1}{2} - \frac{32}{9\pi^2}\right)a^2$
and	$I_{y,\text{ mass}} = I_{y',\text{ mass}} + m\bar{z}^2$
or	$\bar{I}_{y',\text{ mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$

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Chapter 9, Solution 138.

First compute the mass of each component.

Have $m = \rho_{st}V = \rho_{st}tA$

$$m_1 = \left(7850\ \text{kg/m}^3\right)\left(0.003\ \text{m}\right)\left(0.70\ \text{m}\right)\left(0.780\ \text{m}\right) = 12.858\ \text{kg}$$

$$m_2 = \left(7850\ \text{kg/m}^3\right)\left(0.003\ \text{m}\right)\left[\left(\frac{\pi}{2}\right)\left(0.39\ \text{m}\right)^2\right] = 5.6265\ \text{kg}$$

$$m_3 = \left(7850\ \text{kg/m}^3\right)\left(0.003\ \text{m}\right)\left[\left(\frac{1}{2}\right)\left(0.780\ \text{m}\right)\left(0.3\ \text{m}\right)\right] = 2.7554\ \text{kg}$$

Using Fig. 9.28 for component ① and the equations derived above for components ② and ③ have

$$\begin{aligned}
I_x &= \left(I_x\right)_1 + \left(I_x\right)_2 + \left(I_x\right)_3 \\
&= \left(12.858\ \text{kg}\right)\left[\frac{1}{12}\left(0.78\right)^2 + \left(\frac{0.78}{2}\right)^2\right]\text{m}^2 \\
&\quad + \left(5.6265\ \text{kg}\right)\left\{\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\left(0.39\right)^2 + \left[\left(\frac{4 \times 0.39}{3\pi}\right)^2 + \left(0.39\right)^2\right]\right\}\text{m}^2 \\
&\quad + \left(2.7554\ \text{kg}\right)\left\{\frac{1}{18}\left[\left(0.78\right)^2 + \left(0.30\right)^2\right] + \left[\left(\frac{0.78}{3}\right)^2 + \left(\frac{0.30}{3}\right)^2\right]\right\}\text{m}^2 \\
&= \left(2.6076 + 1.2836 + 0.3207\right)\text{kg}\cdot\text{m}^2 \\
&= 4.2119\ \text{kg}\cdot\text{m}^2
\end{aligned}$$

or $I_x = 4.21\ \text{kg}\cdot\text{m}^2$ ◀

continued

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And

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= (12.858\text{ kg})\left\{\frac{1}{12}\left[(0.7)^2 + (0.78)^2\right] + \frac{1}{4}\left[(0.7)^2 + (0.78)^2\right]\right\}\text{m}^2$$

$$+(5.6265\text{ kg})\left[\frac{1}{4}(0.39)^2 + (0.39)^2\right]\text{m}^2$$

$$+(2.7554\text{ kg})\left\{\frac{1}{18}(0.78)^2 + \left[(0.7)^2 + \left(\frac{0.78}{3}\right)^2\right]\right\}\text{m}^4$$

$$= (4.7077 + 1.0697 + 1.6295)\text{ kg}\cdot\text{m}^2$$

$$= 7.4069\text{ kg}\cdot\text{m}^2$$

or $I_y = 7.41\text{ kg}\cdot\text{m}^2$ ◀

And

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= (12.858\text{ kg})\left[(0.7)^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]\text{m}^2$$

$$+(5.6265\text{ kg})\left[\frac{1}{4}(0.39)^2\right]\text{m}^2$$

$$+(2.7554\text{ kg})\left\{\frac{1}{18}(0.3)^2 + \left[(0.70)^2 + \left(\frac{0.30}{3}\right)^2\right]\right\}\text{m}^2$$

$$= (2.1001 + 0.21395 + 1.39145)\text{ kg}\cdot\text{m}^2 = 3.7055\text{ kg}\cdot\text{m}^2$$

or $I_z = 3.71\text{ kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 139.

$t = 2$ mm

$\rho = 7850 \text{ kg/m}^3$

Part ①: $m_1 = \rho V = \left(7850 \text{ kg/m}^3\right)\left(0.08 \text{ m}\right)\left(0.1 \text{ m}\right)\left(0.002 \text{ m}\right) = 0.1256 \text{ kg}$

$$I_x = \bar{I}_x + md^2 = \frac{1}{12}\left(0.1256 \text{ kg}\right)\left(0.08 \text{ m}\right)^2 + \left(0.1256 \text{ kg}\right)\left[\left(0.1 \text{ m}\right)^2 + \left(0.04 \text{ m}\right)^2\right]$$

$$= \left(66.99 \times 10^{-6} + 1.457 \times 10^{-3}\right)\text{kg}\cdot\text{m}^2 = 1.524 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_y = \bar{I}_y + md^2 = \frac{1}{12}\left(0.1256 \text{ kg}\right)\left(0.1 \text{ m}\right)^2 + \left(0.1256 \text{ kg}\right)\left[\left(0.05 \text{ m}\right)^2 + \left(0.1 \text{ m}\right)^2\right]$$

$$= \left(0.1047 \times 10^{-3} + 1.57 \times 10^{-3}\right)\text{kg}\cdot\text{m}^2 = 1.675 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_z = \bar{I}_z + md^2 = \frac{1}{12}\left(0.1256 \text{ kg}\right)\left[\left(0.1 \text{ m}\right)^2 + \left(0.08 \text{ m}\right)^2\right] + \left(0.1256 \text{ kg}\right)\left[\left(0.05 \text{ m}\right)^2 + \left(0.04 \text{ m}\right)^2\right]$$

$$= \left(0.1717 \times 10^{-3} + 0.515 \times 10^{-3}\right)\text{kg}\cdot\text{m}^2 = 0.687 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

Part ②: $m_2 = \rho V = \left(7850 \text{ kg/m}^3\right)\left(0.2 \text{ m}\right)\left(0.2 \text{ m}\right)\left(0.002 \text{ m}\right) = 0.628 \text{ kg}$

$$I_x = \frac{1}{12}ma^2 = \frac{1}{12}\left(0.628 \text{ kg}\right)\left(0.2 \text{ m}\right)^2 = 2.093 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

continued

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$$I_y = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(0.628\text{ kg})\left[(0.2\text{ m})^2 + (0.2\text{ m})^2\right] = 4.187\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{12}mb^2 = \frac{1}{12}(0.628\text{ kg})(0.2\text{ m})^2 = 2.093\times10^{-3}\text{ kg}\cdot\text{m}^2$$

Part ③: Same values as Part ①

Total mass moment of inertia:

$$I_x = 2\left(1.524\times10^{-3}\text{ kg}\cdot\text{m}^2\right) + 2.093\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad I_x = 5.14\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_y = 2\left(1.675\times10^{-3}\text{ kg}\cdot\text{m}^2\right) + 4.187\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad I_y = 7.54\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_z = 2\left(0.687\times10^{-3}\text{ kg}\cdot\text{m}^2\right) + 2.093\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad I_z = 3.47\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

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Chapter 9, Solution 140.

First compute the mass of each component

Have $$m = \rho V = \rho A t$$

Now $$m_1 = \left(7530\ \text{kg/m}^3\right)\left(0.002\ \text{m}\right)\left(0.045\ \text{m}\right)\left(0.070\ \text{m}\right) = 0.047439\ \text{kg}$$

$$m_2 = \left(7530\ \text{kg/m}^3\right)\left(0.002\ \text{m}\right)\left(0.045\ \text{m}\right)\left(0.020\ \text{m}\right) = 0.013554\ \text{kg}$$

$$m_3 = \left(7530\ \text{kg/m}^3\right)\left(0.002\ \text{m}\right)\times\frac{1}{2}\left(0.04\ \text{m}\right)\left(0.095\ \text{m}\right) = 0.028614\ \text{kg}$$

Using Fig. 9.28 for components ① and ② and the equations derived above for components ③, have

$$\begin{aligned} I_x &= \left(I_x\right)_1 + \left(I_x\right)_2 + \left(I_x\right)_3 \\ &= \left(0.047439\ \text{kg}\right)\left[\frac{1}{3}\left(0.07\right)^2\right]\text{m}^2 + \left(0.013554\ \text{kg}\right)\left\{\frac{1}{12}\left(0.020\right)^2 + \left[\left(0.07\right)^2 + \left(0.01\right)^2\right]\right\}\text{m}^2 \\ &\quad + \left(0.028614\ \text{kg}\right)\left\{\frac{1}{18}\left[\left(0.095\right)^2 + \left(0.04\right)^2\right] + \frac{1}{9}\left[\left(2\times 0.095\right)^2 + \left(0.040\right)^2\right]\right\}\text{m}^2 \\ &= \left[\left(0.077484 + 0.068222 + 0.136751\right)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.282457\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

or $I_x = 0.2825\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

continued

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continued

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= (0.047439\text{ kg})\left[\frac{1}{3}(0.045)^2\right]\text{m}^2 + (0.013554\text{ kg})\left\{\left[(0.045)^2 + (0.02)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2$$

$$+(0.028614\text{ kg})\left[\frac{1}{18}(0.04)^2 + (0.045)^2 + \frac{1}{9}(0.04)^2\right]\text{m}^2$$

$$= \left[(0.03202 + 0.010956 + 0.065574)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.10855\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_y = 0.1086\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= (0.047439\text{ kg})\left\{\left[(0.045)^2 + (0.070)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2 + (0.013554\text{ kg})\left[(0.045)^2\left(\frac{1}{3}\right) + (0.070)^2\right]\text{m}^2$$

$$+(0.028614\text{ kg})\left[\frac{1}{18}(0.095)^2 + 0.045^2 + \left(\frac{2}{3}0.095\right)^2\right]\text{m}^2$$

$$= \left[(0.0109505 + 0.075564 + 0.187064)\times 10^{-3}\right]\text{kg}\cdot\text{m}^3$$

$$= 0.37213\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_z = 0.372\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 141.

$$t = 0.1 \text{ in.}$$

$$\gamma_{st} = 490 \text{ lb/ft}^3$$

Have $\quad m = \rho_{st} V = \dfrac{\gamma_{st}}{g} tA$

$$m = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left(\frac{0.1}{12}\right)\text{ft} \times A$$

$$= \left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\cdot\text{ft}^2\right)A$$

Then $\quad m_1 = \left(0.126812 \dfrac{\text{lb}\cdot\text{s}^2}{\text{ft}}\cdot\text{ft}^2\right)\left(\dfrac{16}{12}\times\dfrac{12}{12}\right)\text{ft}^2 = 0.169083 \dfrac{\text{lb}\cdot\text{s}^2}{\text{ft}}$

$$m_2 = \left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\cdot\text{ft}^2\right)\left(\frac{16}{12}\times\frac{9}{12}\right)\text{ft}^2 = 0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_3 = \left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\cdot\text{ft}^2\right)\left(\frac{1}{2}\times\frac{12}{12}\times\frac{9}{12}\right)\text{ft}^2 = 0.047555 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_4 = \left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\cdot\text{ft}^2\right)\left(\frac{\pi}{2}\left(\frac{5}{12}\right)^2\right)\text{ft}^2 = 0.034583 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

Using Fig. 9.28 for components ① and ② and the equations derived previously for components ③ and ④, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4$$

Where $\quad (I_x)_1 = \dfrac{1}{3}\left(0.169083 \dfrac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\dfrac{12}{12}\text{ ft}\right)^2 = 0.056361 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$$(I_x)_2 = \frac{1}{3}\left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{9}{12}\text{ ft}\right)^2 = 0.023777 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_x)_3 = \left(0.047555 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left\{\frac{1}{18}\left[\left(\frac{12}{12}\right)^2 + \left(\frac{9}{12}\right)^2\right] + \left[\left(\frac{4}{12}\right)^2 + \left(\frac{3}{12}\right)^2\right]\right\}\text{ft}^2$$

continued

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$$= 0.0123841 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_x)_4 = \frac{1}{4}\left(0.034583 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{5}{12}\text{ ft}\right)^2 = 0.00150100 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $I_x = (0.056361 + 0.023777 + 0.0123841 - 0.00150100) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_x = 91.0 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 - (I_y)_4$$

Where $(I_y)_1 = \frac{1}{3}\left(0.169083 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\left(\frac{16}{12}\right)^2 + \left(\frac{12}{12}\right)^2\right]\text{ft}^2 = 0.156558 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$$(I_y)_2 = \frac{1}{3}\left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{16}{12}\text{ ft}\right)^2 = 0.075148 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_y)_3 = \left(0.047555 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\frac{1}{18}\left(\frac{12}{12}\right)^2 + \left(\frac{4}{12}\right)^2\right]\text{ft}^2 = 0.0079258 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_y)_4 = \left(0.034583 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left\{\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\left(\frac{5}{12}\text{ft}\right)^2 + \left[\left(\frac{8}{12}\right)^2 + \left(\frac{4}{3\pi} \times \frac{5}{12}\right)^2\right]\text{ft}^2\right\}$$

$$= 0.0183722 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $I_y = (0.156558 + 0.075148 + 0.0079258 - 0.0183722) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_y = 221 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

Where $(I_z)_1 = \frac{1}{3}\left(0.169083 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{16}{12}\text{ft}\right)^2 = 0.100197 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$$(I_z)_2 = \frac{1}{3}\left(0.126812 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\left(\frac{16}{12}\right)^2 + \left(\frac{9}{12}\right)^2\right]\text{ft}^2 = 0.098925 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_z)_3 = \left(0.047555 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\frac{1}{18}\left(\frac{9}{12}\right)^2 + \left(\frac{3}{12}\right)^2\right]\text{ft}^2 = 0.0044583 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_z)_4 = \left(0.034583 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\frac{1}{4}\left(\frac{5}{12}\text{ft}\right)^2 + \left(\frac{8}{12}\text{ft}\right)^2\right] = 0.0168712 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $I_z = (0.100197 + 0.098925 + 0.0044583 - 0.0168712) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_z = 186.7 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

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Chapter 9, Solution 142.

Have $\quad m = \rho_{\text{cu}} V = \rho_{\text{cu}}\, tA$

Then $\quad m_1 = m_2 = \left(8940\ \text{kg/m}^3\right)\left(0.0008\ \text{m}\right)\left(1.2\ \text{m}\right)\left(0.15\ \text{m}\right)$

$\quad = 1.28736\ \text{kg}$

Using Fig. 9.28 for components ① and ②, have

$$I_x = \left(I_x\right)_1 + \left(I_x\right)_2 \text{ and } \left(I_x\right)_1 = \left(I_x\right)_2$$

Then $\quad I_x = 2\left[\frac{1}{3}\left(1.28736\ \text{kg}\right)\left(0.15\ \text{m}\right)^2\right] = 0.0193104\ \text{kg}\cdot\text{m}^2$

$$I_x = 19.31\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

Also $\quad I_y = \left(I_y\right)_1 + \left(I_y\right)_2$

Where $\quad \left(I_y\right)_1 = \frac{1}{3}\left(1.28736\ \text{kg}\right)\left[\left(1.2\ \text{m}\right)^2 + \left(0.15\ \text{m}\right)^2\right] = 0.62759\ \text{kg}\cdot\text{m}^2$

and $\quad \left(I_y\right)_2 = \int r_y^2\, dm$

$$r_y^2 = x^2 + z^2 = x^2 + \left(\zeta\cos 30°\right)^2$$

$$dm = \rho dV = \rho_{\text{cu}}\, t d\zeta\, dx$$

Then $\quad \left(I_y\right)_2 = \rho_{\text{cu}}\, t\int_0^L \int_0^a \left(x^2 + \zeta^2\cos^2 30°\right) d\zeta\, dx$

continued

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$$= \rho_{\text{cu}}\, t \int_0^L \left(ax^2 + \frac{1}{3} a^3 \cos^2 30° \right) dx$$

$$= \frac{1}{3} \rho_{\text{cu}}\, t \left(aL^3 + a^3 L \cos^2 30° \right) \qquad \text{where } V = aLt$$

$$= \frac{1}{3} m_2 \left(L^2 + a^2 \cos^2 30° \right)$$

$$= \frac{1}{3} (1.28736 \text{ kg}) \left[(1.2 \text{ m})^2 + (0.15 \text{ m})^2 \cos^2 30° \right] = 0.62517 \text{ kg}\cdot\text{m}^2$$

Thus $\quad I_y = (0.62759 + 0.62517) \text{ kg}\cdot\text{m}^2$

$I_y = 1.253 \text{ kg}\cdot\text{m}^2$ ◀

Also $\quad I_z = (I_z)_1 + (I_z)_2$

Where $\quad (I_z)_1 = \frac{1}{3}(1.28736 \text{ kg})(1.2 \text{ m})^2 = 0.61793 \text{ kg}\cdot\text{m}^2$

and $\quad (I_z)_2 = \int r_z^2 dm$

$$r_z^2 = x^2 + y^2 = x^2 + (\zeta \sin 30°)^2$$

$$dm = \rho dV = \rho_{\text{cu}}\, t d\zeta\, dx$$

Then $\quad (I_z)_2 = \rho_{\text{cu}}\, t \int_0^L \int_0^a \left(x^2 + \zeta^2 \sin^2 30° \right) d\zeta\, dx$

$$= \rho_{\text{cu}}\, t \int_0^L \left(ax^2 + \frac{1}{3} a^3 \sin^2 30° \right) dx$$

$$= \frac{1}{3} \rho_{\text{cu}}\, t \left(aL^3 + a^3 L \sin^2 30° \right) \qquad \text{where } V = aLt$$

$$= \frac{1}{3} m_2 \left(L^2 + a^2 \sin^2 30° \right)$$

$$= \frac{1}{3} (1.28736 \text{ kg}) \left[(1.2 \text{ m})^2 + (0.15 \text{ m})^2 \sin^2 30° \right]$$

$$= 0.62035 \text{ kg}\cdot\text{m}^2$$

Thus $\quad I_z = (0.61793 + 0.62035) \text{ kg}\cdot\text{m}^2$

$I_z = 1.238 \text{ kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 143.

Have $$m = \rho V = \frac{\gamma}{g}V = \frac{0.284 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times V$$

$$= \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)V$$

Then $$m_1 = \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(4)^2(2)\right]\text{in}^3 = 0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(1)^2(3)\right]\text{in}^3 = 0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(1)^2(2)\right]\text{in}^3 = 0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Using Fig. 9-28 and the parallel theorem, have

(*a*)

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(4)^2 + (2)^2\right] + (1)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (3)^2\right] + (1.5)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$-\left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (2)^2\right] + (1)^2\right\}\text{in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.034106 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 0.0341 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

continued

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(*b*)

$$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(4)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(1)^2 + (2)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$-\left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(1)^2 + (2)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 5.0125 \times 10^{-2} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 0.0501 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*c*)

$$I_z = (I_z)_1 + (I_z)_2 - (I_z)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(4)^2 + (2)^2\right] + (1)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (3)^2\right] + \left[(2)^2 + (1.5)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$-\left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (2)^2\right] + \left[(2)^2 + (1)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.034876 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 0.0349 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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Chapter 9, Solution 144.

Have $$m = \rho_{\text{steel}} V$$

Then $$m_1 = 7850 \text{ kg/m}^3 (0.200 \times 0.120 \times 0.600)\text{m}^3$$

$$= 113.040 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \times (0.200 \times 0.080 \times 0.360)\text{m}^3$$

$$= 45.216 \text{ kg}$$

$$m_3 = 7850 \text{ kg/m}^3 \times \left[\frac{\pi}{2}(0.100)^2(0.120)\right]\text{m}^3$$

$$= 14.7969 \text{ kg}$$

$$m_4 = 7850 \text{ kg/m}^3 \times \left[\pi(0.050)^2(0.120)\right]\text{m}^3$$

$$= 7.3985 \text{ kg}$$

Using Figure 9.28 for components ① and ② and the equations derived above for components ③ and ④, have

Now $$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

where $$(I_y)_1 = (113.040 \text{ kg})\left\{\frac{1}{12}\left[(0.600)^2 + (0.200)^2\right] + \left[\left(\frac{0.600}{2}\right)^2 + \left(\frac{0.200}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 15.0720 \text{ kg} \cdot \text{m}^2$$

continued

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$$\left(I_y\right)_2 = \left(45.216\text{ kg}\right)\left\{\frac{1}{12}\left[\left(0.360\right)^2 + \left(0.200\right)^2\right] + \left[\left(\frac{0.360}{2}\right)^2 + \left(\frac{0.200}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 2.5562\text{ kg}\cdot\text{m}^2$$

$$\left(I_y\right)_3 = \left(14.7969\text{ kg}\right)\left\{\left[\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\left(0.100\right)^2\right] + \left[\left(0.100\right)^2 + \left(0.600 + \frac{4\times 0.100}{3\pi}\right)^2\right]\right\}\text{m}^2$$

$$= 6.3024\text{ kg}\cdot\text{m}^2$$

$$\left(I_y\right)_4 = \left(7.3985\text{ kg}\right)\left\{\left[\frac{1}{2}\left(0.050\right)^2\right] + \left[\left(0.100\right)^2 + \left(0.600\right)^2\right]\right\}\text{m}^2$$

$$= 2.7467\text{ kg}\cdot\text{m}^2$$

Then $$I_y = \left(15.0720 + 2.5562 + 6.3024 - 2.7467\right)\text{ kg}\cdot\text{m}^2$$

$$= 21.1839\text{ kg}\cdot\text{m}^2$$

or $I_y = 21.2\text{ kg}\cdot\text{m}^2$ ◀

continued

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To the Instructor:

The following formulas for the mass moment of inertia of a semicylinder are derived at this time for use in the solutions of Problems 9.144–9.147.

From Figure 9.28

Cylinder: $$\left(I_x\right)_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2$$

$$\left(I_y\right)_{\text{cyl}} = \left(I_z\right)_{\text{cyl}} = \frac{1}{12}m_{\text{cyl}}\left(3a^2 + L^2\right)$$

Symmetry and the definition of the mass moment of inertia $\left(I = \int r^2 dm\right)$ imply

$$\left(I\right)_{\text{semicylinder}} = \frac{1}{2}\left(I\right)_{\text{cylinder}}$$

$$\therefore \quad \left(I_x\right)_{\text{sc}} = \frac{1}{2}\left(\frac{1}{2}m_{\text{cyl}}a^2\right)$$

and $$\left(I_y\right)_{\text{sc}} = \left(I_z\right)_{\text{sc}} = \frac{1}{2}\left[\frac{1}{12}m_{\text{cyl}}\left(3a^2 + L^2\right)\right]$$

However, $$m_{\text{sc}} = \frac{1}{2}m_{\text{cyl}}$$

Thus, $$\left(I_x\right)_{\text{sc}} = \frac{1}{2}m_{\text{sc}}a^2$$

and $$\left(I_y\right)_{\text{sc}} = \left(I_z\right)_{\text{sc}} = \frac{1}{12}m_{\text{sc}}\left(3a^2 + L^2\right)$$

Also, using the parallel axis theorem find

$$\bar{I}_{x'} = m_{\text{sc}}\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

$$\bar{I}_{z'} = m_{\text{sc}}\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2 + \frac{1}{12}L^2\right]$$

where x' and z' are centroidal axes.

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Chapter 9, Solution 145.

See machine elements shown in Problem 9.144

Also note

$$m_1 = 113.040 \text{ kg}$$

$$m_2 = 45.216 \text{ kg}$$

$$m_3 = 14.7969 \text{ kg}$$

$$m_4 = 7.3985 \text{ kg}$$

Using Fig. 9.28 for components ① and ② and the equations derived above for components ③ and ④, have

Now

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

where

$$(I_z)_1 = (113.040 \text{ kg})\left\{\frac{1}{12}\left[(0.200)^2 + (0.120)^2\right] + \left[\left(\frac{0.200}{2}\right)^2 + \left(\frac{0.120}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 2.0498 \text{ kg}\cdot\text{m}^2$$

$$(I_z)_2 = (45.216 \text{ kg})\left\{\frac{1}{12}\left[(0.200)^2 + (0.080)^2\right] + \left[(0.100)^2 + (0.160)^2\right]\right\}\text{m}^2$$

$$= 1.78453 \text{ kg}\cdot\text{m}^2$$

$$(I_z)_3 = (14.7969 \text{ kg})\left\{\frac{1}{12}\left[3(0.100)^2 + (0.120)^2\right] + \left[(0.100)^2 + (0.060)^2\right]\right\}\text{m}^2$$

$$= 0.25599 \text{ kg}\cdot\text{m}^2$$

$$(I_z)_4 = (7.3985 \text{ kg})\left\{\frac{1}{12}\left[3(0.050)^2 + (0.120)^2\right] + \left[(0.100)^2 + (0.060)^2\right]\right\}\text{m}^2$$

$$= 0.114122 \text{ kg}\cdot\text{m}^2$$

Then

$$I_z = (2.0498 + 1.78453 + 0.25599 - 0.114122) \text{ kg}\cdot\text{m}^2$$

$$= 3.97629 \text{ kg}\cdot\text{m}^2$$

or $I_z = 3.98 \text{ kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 146.

$$m = \rho V$$

$$= \frac{\gamma}{g} V$$

$$m_1 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2}(4 \text{ in.})(1 \text{ in.})(3 \text{ in.})$$

$$= 37.267 \times 10^{-3} \ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_2 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2}(2 \text{ in.})(1.2 \text{ in.})(3 \text{ in.})$$

$$= 22.360 \times 10^{-3} \ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_3 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times \frac{\pi}{2}(0.9 \text{ in.})^2(2 \text{ in.})$$

$$= 7.9028 \times 10^{-3} \ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_4 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times \frac{\pi}{2}(0.5 \text{ in.})^2(3 \text{ in.})$$

$$= 3.6587 \times 10^{-3} \ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

continued

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Have

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$= \left[\frac{1}{12}\left(37.267\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(4^2+1^2\right)\text{in}^2 + \left(37.267\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(2^2+0.5^2\right)\text{in}^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left[\frac{1}{12}\left(22.360\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(2^2+1.2^2\right)\text{in}^2 + \left(22.360\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(1^2+1.6^2\right)\text{in}^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\left(7.9028\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[\left(\frac{1}{4}-\frac{16}{9\pi^2}\right)(0.9\ \text{in.})^2 + \frac{1}{12}(2\ \text{in.})^2\right]\right.$$

$$\left.+\left(7.9028\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[(1\ \text{in.})^2 + \left(2.2+\frac{4\times0.9}{3\pi}\right)^2\text{in}^2\right]\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\left(3.6587\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(0.5\ \text{in.})^2\right.$$

$$\left.+\left(3.6587\times10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left[(3.5\ \text{in.})^2 + \left(1+\frac{4\times0.5}{3\pi}\right)^2\text{in}^2\right]\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= \left[\left(1.46653\times10^{-3}\right) + \left(70.393\times10^{-6} + 552.79\times10^{-6}\right)\right.$$

$$\left.+\left(21.400\times10^{-6} + 420.75\times10^{-6}\right) + \left(2.0318\times10^{-6} + 348.58\times10^{-6}\right)\right]\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 2.88\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 2.88\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 9, Solution 147.

Have $$m = \rho_{\text{st}} V = \frac{\delta_{\text{st}}}{g} V$$

Then $$m_1 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (3 \times 1 \times 4) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 105.676 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (1.5 \times 1 \times 2) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 26.419 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2}(0.5)^2 \times 1.5\right] \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 5.1874 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2}(1.4)^2 \times 0.4\right] \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 10.8491 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

(*a*) Using Fig. 9.28 for components ① and ② and the equations derived above for components ③ and ④, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4$$

where $$(I_x)_1 = \left(105.676 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1)^2 + (4)^2\right] + \left[\left(\frac{1}{2}\right)^2 + \left(\frac{4}{2}\right)^2\right] \text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2\right\}$$

$$= 4.1585 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$(I_x)_2 = \left(26.419 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1)^2 + (2)^2\right] + \left[(0.5)^2 + (5)^2\right]\right\} \text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 4.7089 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

continued

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$$(I_x)_3 = \left(5.1874\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(0.5)^2\right]+\left[(0.5)^2+\left(6+\frac{4\times0.5}{3\pi}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.40209\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_x)_4 = \left(10.8451\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1.4)^2+(0.4)^2\right]+\left(2^2+0.8^2\right)\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.38736\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_x = \left[(4.1585+4.7089+1.40209-0.38736)\times10^{-3}\right]\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 9.8821\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 9.88\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*b*) Have $$I_y = (I_y)_1+(I_y)_2+(I_y)_3-(I_y)_4$$

where

$$(I_y)_1 = \left(105.676\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3)^2+(4)^2\right]+\left[\left(\frac{3}{2}\right)^2+\left(\frac{4}{2}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 6.1155\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_y)_2 = \left(26.419\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1.5)^2+(2)^2\right]+\left[(0.75)^2+(5)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 4.7854\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_y)_3 = \left(5.1874\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{4}-\frac{16}{9\pi^2}\right)(0.5)^2+\frac{1}{12}(1.5)^2\right]+\left[(0.75)^2+\left(6+\frac{4\times0.5}{3\pi}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.41785\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_y)_4 = \left(10.8451\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(1.4)^2\right]+\left[\left(3-\frac{4\times1.4}{3\pi}\right)^2+(2)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.78438\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

continued

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Then $$I_y = \left[\left(6.1155 + 4.7854 + 1.41785 - 0.78438\right)\times 10^{-3}\right]\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 11.5344\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 11.53\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*c*) Have $$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

where

$$(I_z)_1 = \left(105.676\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3)^2 + (1)^2\right] + \left[\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 2.4462\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_z)_2 = \left(26.419\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1.5)^2 + (1)^2\right] + \left[\left(\frac{1.5}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 0.198754\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_z)_3 = \left(5.1874\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(0.5)^2 + (1.5)^2\right] + \left[(0.75)^2 + (0.5)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 0.038275\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_z)_4 = \left(10.8451\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(1.4)^2 + \frac{1}{12}(0.4)^2\right] + \left[\left(3 - \frac{4\times 1.4}{3\pi}\right)^2 + (0.8)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 0.49543\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_z = \left[\left(2.4462 + 0.198754 + 0.038275 - 0.49543\right)\times 10^{-3}\right]\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 2.1878\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 2.19\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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Chapter 9, Solution 148.

First compute the mass of each component.

Have $m = \rho L$

Then $m_1 = (0.049 \text{ kg/m})\left[2\pi(0.32 \text{ m})\right]$

$= 0.09852 \text{ kg}$

$m_2 = m_3 = m_4 = m_5$

$= (0.049 \text{ kg/m})(0.160 \text{ m}) = 0.00784 \text{ kg}$

Using the equation given above and the parallel axis theorem, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5$$

$$= (0.09852 \text{ kg})\left[\left(\frac{1}{2}\right)(0.32 \text{ m})^2\right] + (0.00784 \text{ kg})\left[\left(\frac{1}{3}\right)(0.160 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[0 + (0.160 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[\left(\frac{1}{12}\right)(0.16 \text{ m})^2 + (0.08 \text{ m})^2 + (0.32 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[\frac{1}{12}(0.16 \text{ m})^2 + (0.16 \text{ m})^2 + (0.32 \text{ m} - 0.08 \text{ m})^2\right]$$

$$= \left[(5.0442 + 0.06690 + 0.2007 + 0.86972 + 0.66901) \times 10^{-3}\right] \text{ kg} \cdot \text{m}^2$$

$$= 6.8505 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or} \quad I_x = 6.85 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

continued

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Have $I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5$

where $(I_y)_2 = (I_y)_4$ and $(I_y)_3 = (I_y)_5$

Then
$$I_y = (0.09852\,\text{kg})\left[(0.32\text{ m})^2\right] + 2(0.00784\,\text{kg})\left[0 + (0.32\,\text{m})^2\right]$$
$$+ 2(0.00784\,\text{kg})\left[\frac{1}{12}(0.16\,\text{m})^2 + (0.24\,\text{m})^2\right]$$
$$= \left[10.088 + 2(0.80282) + 2(0.46831)\right] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$= 12.6303 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_y = 12.63 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

By symmetry $I_z = I_x$ or $I_z = 6.85 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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To the Instructor:

The following formulas for the mass moment of inertia of wires are derived or summarized at this time for use in the solutions of Problems 9.148–9.150.

Slender Rod

$$I_x = 0 \qquad \bar{I}_{y'} = \bar{I}_{z'} = \frac{1}{12}mL^2 \text{ (Fig. 9.28)}$$

$$I_y = I_z = \frac{1}{3}mL^2 \text{ (Sample Problem 9.9)}$$

Circle

Have $\bar{I}_y = \int r^2 dm = ma^2$

Now $\bar{I}_y = \bar{I}_x + \bar{I}_z$

And symmetry implies $\bar{I}_x = \bar{I}_z$

$$\therefore \quad \bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2$$

Semicircle

Following the above arguments for a circle, have

$$\bar{I}_x = I_z = \frac{1}{2}ma^2 \qquad I_y = ma^2$$

Using the parallel-axis theorem

$$I_z = \bar{I}_{z'} + m\bar{x}^2 \qquad x = \frac{2a}{\pi}$$

or $$I_{z'} = m\left(\frac{1}{2} - \frac{4}{\pi^2}\right)a^2$$

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Chapter 9, Solution 149.

Have $\qquad m = \rho V = \rho AL$

Then

$$m_1 = m_2 = \left(7850\ \text{kg/m}^3\right)\left[\pi\left(0.0015\ \text{m}\right)^2\right]\times\left(\pi\times 0.36\ \text{m}\right)$$

$$m_2 = m_1 = 0.062756\ \text{kg}$$

$$m_3 = m_4 = \left(7850\ \text{kg/m}^3\right)\left[\pi\left(0.0015\ \text{m}\right)^2\right]\times\left(0.36\ \text{m}\right)$$

$$= 0.019976\ \text{kg}$$

Using the equations given above and the parallel axis theorem, have

$$I_x = \left(I_x\right)_1 + \left(I_x\right)_2 + \left(I_x\right)_3 + \left(I_x\right)_4$$

where $\qquad \left(I_x\right)_3 = \left(I_x\right)_4$

Then

$$I_x = \left(0.062756\ \text{kg}\right)\left[\frac{1}{2}\left(0.36\ \text{m}\right)^2\right] + \left(0.062756\right)\left[\frac{1}{2}\left(0.36\ \text{m}\right)^2 + \left(0.36\ \text{m}\right)^2\right]$$

$$+2\left(0.019976\ \text{kg}\right)\left[\frac{1}{12}\left(0.36\ \text{m}\right)^2 + \left(0.18\ \text{m}\right)^2 + \left(0.36\ \text{m}\right)^2\right]$$

$$= \left[4.06659 + 12.19977 + 2\left(3.45185\right)\right]\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= 23.1701\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $\quad I_x = 23.2\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

Have $\qquad I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3 + \left(I_y\right)_4$

where $\qquad \left(I_y\right)_1 = \left(I_y\right)_2 \qquad$ and $\qquad \left(I_y\right)_3 = \left(I_y\right)_4$

continued

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Then

$$I_y = 2(0.062756\text{ kg})\left[(0.36\text{ m})^2\right] + 2(0.019976\text{ kg})\left[0 + (0.36\text{ m})^2\right]$$

$$= 2\left(8.13318\times10^{-3}\text{ kg}\cdot\text{m}^2\right) + 2\left(2.58889\times10^{-3}\text{ kg}\cdot\text{m}^2\right)$$

$$= 21.44414\times10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_y = 21.4\times10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

Have

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

where $(I_z)_3 = (I_z)_4$

Then

$$I_z = (0.062756\text{ kg})\left[\frac{1}{2}(0.36\text{ m})^2\right]$$

$$+(0.062756\text{ kg})\left[\left(\frac{1}{2}-\frac{4}{\pi^2}\right)(0.36\text{ m})^2 + \left(\frac{2\times0.36\text{ m}}{\pi}\right)^2 + (0.36\text{ m})^2\right]$$

$$+2(0.019976\text{ kg})\left[\frac{1}{3}(0.36\text{ m})^2\right]$$

$$= \left[4.06659 + 12.1998 + 2(0.86296)\right]10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 17.9923\times10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_z = 17.99\times10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 150.

First compute the mass of each component. Mass of each component is indentical

$$m = \frac{(m/L)}{g} L$$

Have $$= \frac{(0.041 \text{ lb/ft})(1.5 \text{ ft})}{32.2 \text{ ft/s}^2}$$

$$= 0.00190994 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Using the equations given above and the parallel axis theorem, have

$$(I_x)_1 = (I_x)_3 = (I_x)_4 = (I_x)_6 \qquad \text{and} \qquad (I_x)_2 = (I_x)_5$$

Then $$I_x = 4(I_x)_1 + 2(I_x)_2$$

$$I_x = 4\left(0.00190994 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{3}(1.5 \text{ ft})^2\right]$$

$$+2\left(0.00190994 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[0 + (1.5 \text{ ft})^2\right]$$

$$= 0.0143246 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 14.32 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now $$(I_y)_1 = 0 \qquad (I_y)_2 = (I_y)_6 \qquad (I_y)_4 = (I_y)_5$$

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Then

$$I_y = 2\left(I_y\right)_2 + \left(I_y\right)_3 + 2\left(I_y\right)_4$$

$$= \left(0.0019094\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[2\left(\frac{1}{3}\right)\left(1.5\ \text{ft}\right)^2\right] + \left[0 + \left(1.5\ \text{ft}\right)^2\right]\right.$$

$$\left. + 2\left[\frac{1}{12}\left(1.5\ \text{ft}\right)^2 + \left(1.5\ \text{ft}\right)^2 + \left(0.75\ \text{ft}\right)^2\right]\right\}$$

$$= 0.0019094\left(1.5 + 2.25 + 6\right)\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 = 0.0186219\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

By symmetry $\quad I_z = I_y$

$$I_z = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

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Chapter 9, Solution 151.

From the solution to Problem 9.147

$$m_1 = 105.676 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft} \qquad m_3 = 5.1874 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = 26.419 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft} \qquad m_4 = 10.8451 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

First note that symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each component

Now $$I_{uv} = \cancelto{0}{I_{u'v'}} + m\bar{u}\,\bar{v} = m\bar{u}\,\bar{v}$$

so that $$\left(I_{uv}\right)_{\text{body}} = \Sigma m\bar{u}\,\bar{v}$$

Then $$I_{xy} = \Sigma m\bar{x}\,\bar{y} = \left(105.676 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{1.5}{12}\ \text{ft}\right)\left(\frac{0.5}{12}\ \text{ft}\right) + \left(26.419 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.75}{12}\ \text{ft}\right)\left(\frac{0.5}{12}\ \text{ft}\right)$$

$$+\left(5.1874 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.75}{12}\ \text{ft}\right)\left(\frac{0.5}{12}\ \text{ft}\right)$$

$$-\left(10.8451 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(3\ \text{in.} - \frac{4 \times 1.4\ \text{in.}}{3\pi}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)\right]\left(\frac{0.8}{12}\ \text{ft}\right)$$

$$= \left(550.40 + 68.799 + 13.5089 - 144.952\right) \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 487.76 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{xy} = 0.488 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

continued

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Chapter 9, Solution 152.

From the solution to Problem 9.146

$$m_1 = 37.267 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_2 = 22.360 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_3 = 7.9028 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$m_4 = 3.6587 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

Have $\qquad I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$

Symmetry implies $\qquad \bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$

for each element.

$$\therefore \quad I_{uv} = \sum m_i \bar{u}_i \bar{v}_i$$

Then:

$$I_{xy} = \left(37.267 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(2\text{ in.})(0.5\text{ in.})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left(22.360 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1\text{ in.})(1.6\text{ in.})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left(7.9028 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1\text{ in.})\left(2.2 + \frac{4 \times 0.9}{3\pi}\right)\text{in.} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+\left(3.6587 \times 10^{-3}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(3.5\text{ in.})\left(1 + \frac{4 \times 0.5}{3\pi}\right)\text{in.} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

continued

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$$= \left(258.80 \times 10^{-6} + 248.44 \times 10^{-6} + 141.700 \times 10^{-6} + 107.80 \times 10^{-6}\right) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 757 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{xy} = 757 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$I_{yz} = \left(37.267 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(0.5 \text{ in.})(1.5 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(22.360 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.6 \text{ in.})(1.5 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(7.9028 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(2.2 + \frac{4 \times 0.9}{3\pi}\right) \text{in.} \times (1.5 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(3.6587 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(1 + \frac{4 \times 0.5}{3\pi}\right) \text{in.} \times (1.5 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= \left(194.099 \times 10^{-6} + 372.67 \times 10^{-6} + 212.55 \times 10^{-6} + 46.199 \times 10^{-6}\right) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 826 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = 826 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$I_{zx} = \left(37.267 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.5 \text{ in.})(2 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(22.360 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.5 \text{ in.})(1 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(7.9028 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.5 \text{ in.})(1 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(3.6587 \times 10^{-3} \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)(1.5 \text{ in.})(3.5 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= \left(776.40 \times 10^{-6} + 232.92 \times 10^{-6} + 82.321 \times 10^{-6} + 133.390 \times 10^{-6}\right) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 1.225 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{zx} = 1.225 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

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Chapter 9, Solution 153.

Have $m = \rho_{\text{al}} V$

Then
$$m_1 = \left(2700\ \frac{\text{kg}}{\text{m}^3}\right)(0.350 \times 0.100 \times 0.030)\,\text{m}^3$$
$$= 2.8350\,\text{kg}$$
$$m_2 = \left(2700\ \frac{\text{kg}}{\text{m}^3}\right)(0.200 \times 0.100 \times 0.050)\,\text{m}^3$$
$$= 2.7000\,\text{kg}$$
$$m_3 = \left(2700\ \frac{\text{kg}}{\text{m}^3}\right)\left[\pi(0.025)^2 \times 0.100\right]\text{m}^3$$
$$= 0.53014\,\text{kg}$$

First note that symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = I_{z'x'} = 0$ for each component

Now $$I_{uv} = I_{u'v'} + m\bar{u}\,\bar{v}$$

where $$\bar{I}_{u'v'} = 0$$

$$I_{xy} = \Sigma m\bar{x}\,\bar{y} = (2.8350\,\text{kg})(0.175\,\text{m})(0.050\,\text{m})$$
$$+(2.7000\,\text{kg})(0.100\,\text{m})(0.050\,\text{m}) - (0.53014\,\text{kg})(0.080\,\text{m})(0.050\,\text{m})$$
$$= (24.806 + 13.500 - 2.1206) \times 10^{-3}\ \text{kg}\cdot\text{m}^2 = 36.1854 \times 10^{-3}\ \text{kg}\,\text{m}^2$$

or $I_{xy} = 36.2 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{yz} = \Sigma m\bar{y}\,\bar{z} = (2.8350\ \text{kg})(0.050\,\text{m})(0.015\,\text{m})$$
$$+(\ 2.7000\ \text{kg})(0.050\,\text{m})(0.055\,\text{m}) - (0.53014\,\text{kg})(0.050\,\text{m})(0.040\,\text{m})$$
$$= (2.1263 + 7.4250 - 1.06028) \times 10^{-3}\ \text{kg}\cdot\text{m}^2 = 8.49102 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{yz} = 8.49 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

continued

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$$I_{zx} = \Sigma m\bar{z}\,\bar{x} = (2.8350\,\text{kg})(0.015\,\text{m})(0.175\,\text{m})$$

$$+(2.7000\,\text{kg})(0.055\,\text{m})(0.100\,\text{m}) - (0.53014\,\text{kg})(0.040\,\text{m})(0.080\,\text{m})$$

$$= (7.4419 + 14.850 - 1.69645)10^{-3}\ \text{kg}\cdot\text{m}^2 = 20.59545 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{zx} = 20.6 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 154.

Have $m = \rho V$

Then
$$m_1 = \left(2700\ \text{kg/m}^3\right)\left(0.118 \times 0.036 \times 0.044\right)\text{m}^3 = 0.50466\ \text{kg}$$

$$m_2 = \left(2700\ \text{kg/m}^3\right)\left[\frac{\pi}{2}\left(0.022\right)^2 \times 0.036\right]\text{m}^3 = 0.07389\ \text{kg}$$

$$m_3 = \left(2700\ \text{kg/m}^3\right)\left(0.028 \times 0.022 \times 0.024\right)\text{m}^3 = 0.03992\ \text{kg}$$

Now observe that $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$ and $\bar{I}_{z'x'}$ are zero because of symmetry

Now
$$\bar{x}_2 = -\left(0.118 + \frac{4 \times 0.022}{3\pi}\right)\text{m} = -0.12734\ \text{m}$$

$$\bar{y}_3 = \left(0.036 - \frac{0.022}{2}\right)\text{m} = 0.025\ \text{m}$$

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$ kg·m^2	$m\bar{y}\,\bar{z}$ kg·m^2	$m\bar{z}\,\bar{x}$ kg·m^2
1	0.50466	–0.059	0.018	0.022	-0.53595×10^{-3}	0.19985×10^{-3}	-0.65505×10^{-3}
2	0.07389	–0.12734	0.018	0.022	-0.16932×10^{-3}	0.02926×10^{-3}	-0.20695×10^{-3}
3	0.03992	–0.014	0.025	0.026	-0.01397×10^{-3}	0.02594×10^{-3}	-0.01453×10^{-3}

continued

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And

$$I_{xy} = \Sigma\left(\cancelto{0}{I_{x'y'}} + m\bar{x}\,\bar{y}\right)$$

$$I_{yz} = \Sigma\left(\cancelto{0}{I_{y'z'}} + m\bar{y}\,\bar{z}\right)$$

$$I_{zx} = \Sigma\left(\cancelto{0}{I_{z'x'}} + m\bar{x}\,\bar{z}\right)$$

Finally

$$I_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2 - \left(I_{xy}\right)_3 = -0.6913 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\text{or}\quad I_{xy} = -0.691 \times 10^{-3}\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{yz} = \left(I_{yz}\right)_1 + \left(I_{yz}\right)_2 - \left(I_{yz}\right)_3 = 0.20317 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\text{or}\quad I_{yz} = 0.203 \times 10^{-3}\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{zx} = \left(I_{zx}\right)_1 + \left(I_{zx}\right)_2 - \left(I_{zx}\right)_3 = -0.84747 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\text{or}\quad I_{zx} = -0.848 \times 10^{-3}\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

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Chapter 9, Solution 155.

Have $m = \rho V$

$= \rho t A$

Then $m_1 = \left(7860\ \dfrac{\text{kg}}{\text{m}^3}\right)(0.003\ \text{m})(0.2 \times 0.090)\ \text{m}^2$

$= 424.44 \times 10^{-3}\ \text{kg}$

$m_2 = \left(7860\ \dfrac{\text{kg}}{\text{m}^3}\right)(0.003\ \text{m}) \times \dfrac{\pi}{2}(0.045\ \text{m})^2$

$= 75.005 \times 10^{-3}\ \text{kg}$

Now $I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$

Symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for both elements.

$\therefore\ I_{uv} = \sum m_i \bar{u}_i \bar{v}_i$

Then $I_{xy} = \left(424.44 \times 10^{-3}\ \text{kg}\right)(0.050\ \text{m})(0.045\ \text{m}) + \left(75.005 \times 10^{-3}\ \text{kg}\right)(-0.050\ \text{m})(0.045\ \text{m})$

$= \left(954.99 \times 10^{-6} - 168.761 \times 10^{-6}\right)\ \text{kg}\cdot\text{m}^2$

$= 786 \times 10^{-6}\ \text{kg}\cdot\text{m}^2$

$I_{xy} = 786 \times 10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

continued

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$$I_{yz} = \left(424.44 \times 10^{-3}\ \text{kg}\right)\left(0.045\ \text{m}\right)\left(0\right) + \left(75.005 \times 10^{-3}\ \text{kg}\right)\left(0.045\ \text{m}\right)\left(\frac{4 \times 0.045}{3\pi}\text{m}\right)$$

$$= 64.5 \times 10^{-6}\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = 64.5 \times 10^{-6}\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{zx} = \left(424.44 \times 10^{-3}\ \text{kg}\right)\left(0\right)\left(0.050\ \text{m}\right) + \left(75.005 \times 10^{-3}\ \text{kg}\right)\left(\frac{4 \times 0.045}{3\pi}\ \text{m}\right)\left(-0.050\ \text{m}\right)$$

$$= -71.6 \times 10^{-6}\ \text{kg} \cdot \text{m}^2$$

$$I_{zx} = -71.6 \times 10^{-6}\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 9, Solution 156.

First compute the mass of each component

Have $$m = \rho_{\text{st}} V = \rho_{\text{st}} t A$$

Then $$m_1 = \left(7860 \text{ kg/m}^3\right)\left[(0.003)(0.08)(0.09)\right] \text{m}^3$$

$$= 0.169776 \text{ kg}$$

$$m_2 = 7860 \text{ kg/m}^3 \left[(0.003)\left(\frac{1}{2} \times 0.09 \times 0.036\right)\right] \text{m}^3$$

$$= 0.03820 \text{ kg}$$

Now observe that

$$\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{y'z'}\right)_1 = \left(\bar{I}_{z'x'}\right)_1 = 0$$

$$\left(\bar{I}_{y'z'}\right)_2 = \left(\bar{I}_{z'x'}\right)_2 = 0$$

From Sample Problem 9.6 $$\left(\bar{I}_{x'y'}\right)_{2,\text{area}} = -\frac{1}{72} b_2^2 h_2^2$$

Then $$\left(\bar{I}_{x'y'}\right)_2 = \rho_{sT} t \left(I_{x'y'}\right)_{2,\text{area}} = \rho_{\text{st}} t \left(-\frac{1}{72} b_2^2 h_2^2\right) = -\frac{1}{36} m_2 b_2 h_2$$

Also $$\bar{x}_1 = \bar{y}_1 = \bar{z}_2 = 0 \qquad \bar{x}_2 = \left(-0.045 + \frac{0.09}{3}\right) \text{m} = -0.015 \text{ m}$$

continued

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Finally

$$I_{xy} = \Sigma\left(\bar{I}_{xy} + m\bar{x}\,\bar{y}\right) = (0+0) + \left[-\frac{1}{36}(0.03820\,\text{kg})(0.09\,\text{m})(0.036\,\text{m})\right.$$

$$\left. + (0.03820\,\text{kg})(-0.015\,\text{m})\left(\frac{0.036\,\text{m}}{3}\right)\right]$$

$$= \left(-3.4379\times10^{-6} - 6.876\times10^{-6}\right)\,\text{kg}\cdot\text{m}^2 = -10.3139\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

or $I_{xy} = -10.31\times10^{-6}\,\text{kg}\cdot\text{m}^2$ ◀

And $I_{yz} = \Sigma\left(\bar{I}_{y'z'} + m\,\bar{y}\,\bar{z}\right) = (0+0) + (0+0) = 0$ or $I_{yz} = 0$ ◀

$I_{zx} = \Sigma\left(\bar{I}_{z'x'} + m\,\bar{z}\,\bar{x}\right) = (0+0) + (0+0) = 0$ or $I_{zx} = 0$ ◀

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Chapter 9, Solution 157.

First compute the mass of each component

Have $$m = \rho_{st} V = \rho_{st} t A$$

Then $$m_1 = \left(7860\ \text{kg/m}^3\right)\left[(0.003)(0.7)(0.78)\right]\text{m}^3 = 12.875\ \text{kg}$$

$$m_2 = \left(7860\ \text{kg/m}^3\right)\left[(0.003)\left(\frac{\pi}{2} \times 0.39^2\right)\right]\text{m}^3 = 5.6337\ \text{kg}$$

$$m_3 = \left(7860\ \text{kg/m}^3\right)\left[(0.003)\left(\frac{1}{2} \times 0.78 \times 0.3\right)\right]\text{m}^3 = 2.7589\ \text{kg}$$

Now observe that because of symmetry the centroidal products of inertia of components ① and ② are zero and $\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{z'x'}\right)_3 = 0$

Also $$\left(\bar{I}_{y'z'}\right)_{3,\text{mass}} = \rho_{st} t \left(\bar{I}_{y'z'}\right)_{3,\text{area}}$$

Using the results of Sample Problem 9.6 and noting that the orientation of the axes corresponds to a $90°$ rotation, have

$$\left(\bar{I}_{y'z'}\right)_{3,\text{area}} = \frac{1}{72} b_3^2 h_3^2$$

Then

$$\left(\bar{I}_{y'z'}\right)_3 = \rho_{st} t \left(\frac{1}{72} b_3^2 h_3^2\right) = \frac{1}{36} m_3 b_3 h_3$$

Also $$\bar{y}_1 = \bar{x}_2 = 0 \qquad \bar{y}_2 = \frac{4 \times 0.39\ \text{m}}{3\pi} = 0.16552\ \text{m}$$

Finally $$I_{xy} = \Sigma\left(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}\right) = (0 + 0) + (0 + 0)$$

$$+ \left[0 + (2.7589\ \text{kg})(0.7\ \text{m})\left(\frac{-0.3\,\text{m}}{3}\right)\right] = -0.19312\ \text{kg}\cdot\text{m}^2$$

or $I_{xy} = -0.1931\ \text{kg}\cdot\text{m}^2$ ◀

continued

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$$I_{yz} = \Sigma\left(\bar{I}_{y'z'} + m\bar{y}\,\bar{z}\right)$$

$$= (0 + 0) + \left[0 + (5.6337\text{ kg})(0.16552\text{ m})(0.39\text{ m})\right]$$

$$+ (2.7589\text{ kg})\left[\frac{1}{36}(0.78\text{ m})(0.3\text{ m}) + \left(\frac{-0.3\text{ m}}{3}\right)\left(\frac{0.78\text{ m}}{3}\right)\right]$$

$$= (0.36367 + 0.017933 - 0.07173)\text{ kg}\cdot\text{m}^2 = 0.30987\text{ kg}\cdot\text{m}^2$$

$$\text{or } I_{yz} = 0.310\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{zx} = \Sigma\left(\bar{I}_{z'x'} + m\bar{z}\,\bar{x}\right) = \left[0 + (12.875\text{ kg})(0.35\text{ m})(0.39\text{ m})\right]$$

$$+ (0 + 0) + \left[0 + (2.7589\text{ kg})\left(\frac{0.78\text{ m}}{3}\right)(0.7\text{ m})\right]$$

$$= (1.75744 + 0.50212)\text{ kg}\cdot\text{m}^2 = 2.25956\text{ kg}\cdot\text{m}^2$$

$$\text{or } I_{zx} = 2.26\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

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Chapter 9, Solution 158.

First note $m = \rho_{st} V = \dfrac{\gamma_{st}}{g} tA$

Then $m_1 = \left(\dfrac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.08 \text{ in.})\left[(6\times 3.6)\text{in}^2\right]\left(\dfrac{1\text{ft}}{12\text{in.}}\right)^3$

$= 15.2174\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$

$m_2 = \left(\dfrac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.08 \text{ in.})\left[\dfrac{\pi}{2}(1.8 \text{ in.})^2\right]\left(\dfrac{1 \text{ ft}}{12 \text{ in.}}\right)^3$

$= 3.5855\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$

$m_3 = \left(\dfrac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.08 \text{ in.})\left[\dfrac{\pi}{4}(3.6 \text{ in.})^2\right]\left(\dfrac{1 \text{ ft}}{12 \text{ in.}}\right)^3$

$= 7.1710\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$

Note that symmetry implies $\left(\bar{I}_{x'y'}\right)_{1,2} = \left(\bar{I}_{y'z'}\right)_{1,2} = \left(\bar{I}_{z'x'}\right)_{1,2} = 0$

$\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{y'z'}\right)_3 = 0$

Now $I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$

continued

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Thus $\quad I_{xy} = \Sigma m\bar{x}\,\bar{y}$

$$= m_1\bar{x}_1\bar{y}_1 - m_2\bar{x}_2\bar{y}_2 + m_3x_3\cancelto{0}{\bar{y}_3}$$

$$= \left(15.2174\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(-\frac{0.6}{12}\ \text{ft}\right)\left(\frac{1.8}{12}\ \text{ft}\right)$$

$$-\left(3.5855\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(2.4-\frac{4\times1.8}{3\pi}\right)\text{in.}\times\frac{1\ \text{ft}}{12\ \text{in.}}\right]\left(\frac{1.8}{12}\ \text{ft}\right)$$

$$= \left(-114.131-73.326\right)\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or}\quad I_{xy} = -187.5\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

Now $\quad I_{yz} = \Sigma m\bar{y}\,\bar{z} = m_1\cancelto{0}{\bar{y}_1\bar{z}_1} - m_2\cancelto{0}{\bar{y}_2\bar{z}_2} + m_3\cancelto{0}{\bar{y}_3\bar{z}_3} \qquad \text{or}\quad I_{yz} = 0 \blacktriangleleft$

Also $\quad I_{zx} = \left(I_{zx}\right)_1 - \left(I_{zx}\right)_2 + \left(I_{zx}\right)_3$

$$= m_1\bar{z}_1\bar{x}_1 - \cancelto{0}{m_2z_2x_2} + \cancelto{0}{\left(I_{zx}\right)_3}$$

Now determine $\left(I_{zx}\right)_3$

Have $\quad \left(dI_{zx}\right)_3 = \cancelto{0}{\left(d\bar{I}_{z'x'}\right)_3} + \bar{z}\,\bar{x}\,dm$

$$= (z)\left(-\frac{x}{2}\right)\left(\frac{\gamma_{\text{st}}}{g}t|x|dz\right)$$

$$= -\frac{1}{2}\frac{\gamma_{\text{st}}}{g}tz\left(a_3^2-z^2\right)dz$$

Now $\quad m_3 = \frac{\gamma_{\text{st}}}{g}t\left(\frac{\pi}{4}a_3^2\right)$

or $\quad \frac{\gamma_{\text{st}}}{g}t = \frac{4m_3}{\pi a_3^2}$

Therefore, $\quad \left(I_{zx}\right)_3 = -\frac{2m_3}{\pi a_3^2}\int_0^a\left(a_3^2z-z^3\right)dz = -\frac{2m_3}{\pi a_3^2}\left(\frac{1}{2}a_3^2z^2-\frac{1}{4}z^4\right)\Big|_0^a$

$$= -\frac{1}{2\pi}m_3a_3^2$$

Finally $\quad I_{zx} = -\frac{1}{2\pi}\left(7.1710\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{3.6}{12}\text{ft}\right)^2 \qquad \text{or}\quad I_{zx} = -102.7\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$

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Chapter 9, Solution 159.

Have $m = \dfrac{wL}{g}$

Then
$$m_1 = \frac{w}{g}\left(\pi \times \frac{3}{2}a\right) = \frac{3\pi}{2}\frac{wa}{g}$$
$$m_2 = \frac{w}{g}(2a) = 2\frac{wa}{g}$$
$$m_3 = \frac{w}{g}(\pi \times a) = \pi\frac{wa}{g}$$

Now $I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$

Symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each element.

$$\therefore \quad I_{uv} = \sum m_i \bar{u}_i \bar{v}_i$$

Then
$$I_{xy} = \left(\frac{3\pi}{2}\frac{wa}{g}\right)\left(\frac{3}{2}a\right)\left(\frac{2}{\pi} \times \frac{3}{2}a\right) + \left(\pi\frac{wa}{g}\right)(3a)\left(\frac{2}{\pi} \times a\right)$$
$$= \frac{wa^3}{g}\left(\frac{27}{4} + 6\right) \qquad I_{xy} = 12.75\frac{w}{g}a^3 \blacktriangleleft$$
$$I_{yz} = \left(\frac{3\pi}{2}\frac{wa}{g}\right)\left(\frac{2}{\pi} \times \frac{3}{2}a\right)(2a) + \left(\pi\frac{wa}{g}\right)\left(\frac{2 \times a}{\pi}\right)(-a)$$
$$= \frac{wa^3}{g}(9 - 2) \qquad I_{yz} = 7\frac{wa^3}{g} \blacktriangleleft$$
$$I_{zx} = \left(\frac{3\pi}{2}\frac{wa}{g}\right)(2a)\left(\frac{3}{2}a\right) + \left(2\frac{wa}{g}\right)(a)(3a) + \left(\pi\frac{wa}{g}\right)(-a)(3a)$$
$$= \frac{wa^3}{g}\left(\frac{9\pi}{2} + 6 - 3\pi\right) \qquad I_{zx} = 1.5\frac{w}{g}a^3(\pi + 4) \blacktriangleleft$$

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Chapter 9, Solution 160.

First compute the mass of each component. Have

$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then

$$m_1 = \frac{w}{g}(2\pi \times a) = 2\pi \frac{w}{g} a$$

$$m_2 = \frac{w}{g}(a) = \frac{w}{g} a$$

$$m_3 = \frac{w}{g}(2a) = 2\frac{w}{g} a$$

$$m_4 = \frac{w}{g}\left(2\pi \times \frac{3}{2} a\right) = 3\pi \frac{w}{g} a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'},$ of each component are zero because of symmetry.

continued

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	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$2\pi\frac{w}{g}a$	$2a$	a	$-a$	$4\pi\frac{w}{g}a^3$	$-2\pi\frac{w}{g}a^3$	$-4\pi\frac{w}{g}a^3$
2	$\frac{w}{g}a$	$2a$	$\frac{1}{2}a$	0	$\frac{w}{g}a^3$	0	0
3	$2\frac{w}{g}a$	$2a$	0	a	0	0	$4\frac{w}{g}a^3$
4	$3\pi\frac{w}{g}a$	$2a$	$-\frac{3}{2}a$	$2a$	$-9\pi\frac{w}{g}a^3$	$-9\pi\frac{w}{g}a^3$	$12\pi\frac{w}{g}a^3$
Σ					$\frac{w}{g}(1-5\pi)a^3$	$-11\pi\frac{w}{g}a^3$	$4\frac{w}{g}(1+2\pi)a^3$

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) \qquad \text{or} \quad I_{xy} = \frac{w}{g}a^3(1-5\pi) \blacktriangleleft$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}\right) \qquad \text{or} \quad I_{yz} = -11\pi\frac{w}{g}a^3 \blacktriangleleft$$

$$I_{zx} = \Sigma\left(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}\right) \qquad \text{or} \quad I_{zx} = 4\frac{w}{g}a^3(1+2\pi) \blacktriangleleft$$

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Chapter 9, Solution 161.

First note $$m = \rho V = \frac{\gamma}{g} V = \frac{\gamma}{g} AL$$

Specific weight of aluminium $= 0.10 \text{ lb/in}^3 = 172.8 \text{ lb/ft}^3$

Then $$m = \frac{172.8 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\left[\frac{\pi}{4}\left(\frac{0.075}{12} \text{ ft}\right)^2\right] L$$

$$= \left(0.16464 \times 10^{-3}\right) L \text{ lb}\cdot\text{s}^2/\text{ft}^2$$

Now $$L_1 = L_4 = 12.5 \text{ in.} = 1.04167 \text{ ft}$$

$$m_1 = m_4 = 0.17150 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$L_2 = L_5 = 9 \text{ in.} = 0.75 \text{ ft}$$

$$m_2 = m_5 = 0.12348 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$L_3 = L_6 = 15 \text{ in.} = 1.25 \text{ ft}$$

$$m_3 = m_6 = 0.20580 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

and $$\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$$

continued

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	m, lb·s²/ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$, lb·ft·s²	$m\bar{y}\,\bar{z}$, lb·ft·s²	$m\bar{z}\,\bar{x}$, lb·ft·s²
1	0.17150×10^{-3}	0.75	0.5208	0	0.06699×10^{-3}	0	0
2	0.12348×10^{-3}	0.375	1.04167	0	0.04823×10^{-3}	0	0
3	0.20580×10^{-3}	0	1.04167	0.625	0	0.13398×10^{-3}	0
4	0.17150×10^{-3}	0	0.5208	1.25	0	0.111646×10^{-3}	0
5	0.12348×10^{-3}	0.375	0	1.25	0	0	0.05788×10^{-3}
6	0.20580×10^{-3}	0.75	0	0.625	0	0	0.09647×10^{-3}
Σ					0.11522×10^{-3}	0.24563×10^{-3}	0.15435×10^{-3}

$$I_{xy} = \Sigma\left(\cancelto{0}{I_{xy'}} + m\bar{x}\,\bar{y}\right) = 0.115222 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{xy} = 0.1152 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{yz} = \Sigma\left(\cancelto{0}{I_{y'z'}} + m\bar{y}\,\bar{z}\right) = 0.24563 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{yz} = 0.246 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{zx} = \Sigma\left(\cancelto{0}{I_{z'x'}} + m\bar{z}\,\bar{x}\right) = 0.15435 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{zx} = 0.1543 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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Chapter 9, Solution 162.

$$m_{\text{ring}} = 2\pi(0.5\text{ m})(1.8\text{ kg/m}) = 5.655\text{ kg}$$

$$m_{\text{rod}} = (0.8\text{ m})(1.8\text{ kg/m}) = 1.44\text{ kg}$$

For each ring $\bar{x} = \bar{z} = 0$ and $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0,$ thus the mass product of inertia of the rings is zero with respect to each pair of coordinate axes.

For each rod:

Since each rod lies in the *x*-*y* plane, $I_{yz} = I_{zx} = 0$

Thus for entire wire figure $\qquad I_{yz} = I_{zx} = 0$ ◀

$$I_{xy} = \sum\left(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}\right) \quad \text{where } \bar{I}_{x'y'} = 0$$

Hence $\qquad I_{xy} = \sum m\bar{x}\,\bar{y}$

$$= (1.44\text{ kg})(-0.5\text{ m})(0.4\text{ m}) + (1.44\text{ kg})(0.5\text{ m})(-0.4\text{ m})$$
$$= -0.576\text{ kg}\cdot\text{m}^2$$

$I_{xy} = -0.576\text{ kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 163.

Have $I_{xy} = \int xydm \qquad I_{yz} = \int yzdm \qquad I_{zx} = \int zxdm$ (9.45)

and $x = x' + \bar{x} \qquad y = y' + \bar{y} \qquad z = z' + \bar{z}$ (9.31)

Consider $I_{xy} = \int xydm$

Substituting for x and for y

$$I_{xy} = \int (x' + \bar{x})(y' + \bar{y})\,dm$$

$$= \int x'y'dm + \bar{y}\int x'dm + \bar{x}\int y'dm + \bar{x}\,\bar{y}\int dm$$

By definition $\bar{I}_{x'y'} = \int x'y'dm$

and $\int x'dm = m\bar{x}'$

$$\int y'dm = m\bar{y}'$$

However, the origin of the primed coordinate system coincides with the mass center G, so that

$$\bar{x}' = \bar{y}' = 0$$

$\therefore\ I_{xy} = \bar{I}_{x'y'} + m\bar{x}\,\bar{y}$ Q.E.D.◀

The expressions for I_{yz} and I_{zx} are obtained in a similar manner.

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Chapter 9, Solution 164.

(*a*) First divide the tetrahedron into a series of thin vertical slices of thickness *dz* as shown.

Now $$x = -\frac{a}{c}z + a = a\left(1 - \frac{z}{c}\right)$$

and $$y = -\frac{b}{c}z + b = b\left(1 - \frac{z}{c}\right)$$

The mass *dm* of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xydz\right) = \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz$$

Then $$m = \int dm = \int_0^c \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz = \frac{1}{2}\rho ab\left[\left(-\frac{c}{3}\right)\left(1 - \frac{z}{c}\right)^3\right]_0^c$$

$$= \frac{1}{6}\rho abc$$

Now $$dI_{zx} = d\bar{I}_{z'x'} + \bar{z}_{EL}\bar{x}_{EL}dm$$

where $$d\bar{I}_{z'x'} = 0 \qquad \text{(symmetry)}$$

and $$\bar{z}_{EL} = z \qquad \bar{x}_{EL} = \frac{1}{3}x = \frac{1}{3}a\left(1 - \frac{z}{c}\right)$$

Then $$I_{zx} = \int dI_{zx} = \int_0^c z\left[\frac{1}{3}a\left(1 - \frac{z}{c}\right)\right]\left[\frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz\right]$$

$$= \frac{1}{6}\rho a^2 b\int_0^c\left(z - 3\frac{z^2}{c} + 3\frac{z^3}{c^2} - \frac{z^4}{c^3}\right)dz$$

$$= \frac{m}{c}a\left[\frac{1}{2}z^2 - \frac{z^3}{c} + \frac{3}{4}\frac{z^4}{c^2} - \frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

or $$I_{zx} = \frac{1}{20}mac \blacktriangleleft$$

continued

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(*b*) Because of the symmetry of the body, I_{xy} and I_{yz} can be deduced by considering the circular permutation of (x, y, z) and (a, b, c). Thus

$$I_{xy} = \frac{1}{20}mab \blacktriangleleft$$

$$I_{yz} = \frac{1}{20}mbc \blacktriangleleft$$

Alternative solution for part *a*

First divide the tetrahedron into a series of thin horizontal slices of thickness dy as shown.

Now $$x = -\frac{a}{b}y + a = a\left(1 - \frac{y}{b}\right)$$

and $$z = -\frac{c}{b}y + c = c\left(1 - \frac{y}{b}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ac\left(1 - \frac{y}{b}\right)^2 dy$$

Now $$dI_{zx} = \rho t dI_{zx,\,\text{Area}}$$

where $$t = dy$$

and $dI_{zx,\,\text{Area}} = \frac{1}{24}x^2z^2$ from the results of Sample Problem 9.6

Then $$dIzx = \rho(dy)\left\{\frac{1}{24}\left[a\left(1 - \frac{y}{b}\right)^2\right]\left[c\left(1 - \frac{y}{b}\right)\right]^2\right\}$$

$$= \frac{1}{24}\rho a^2c^2\left(1 - \frac{y}{b}\right)^4 dy = \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

Finally $$I_{zx} = \int dI_{zx} = \int_0^b \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

$$= \frac{1}{4}\frac{m}{b}ac\left[\left(-\frac{b}{5}\right)\left(1 - \frac{y}{b}\right)^5\right]_0^b$$

or $$I_{zx} = \frac{1}{20}mac \blacktriangleleft$$

continued

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Alternative solution for part a

The equation of the included face of the tetrahedron is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

so that $$y=b\left(1-\frac{x}{a}-\frac{z}{c}\right)$$

For an infinitesimal element of sides dx, dy, and dz

$$dm=\rho dV=\rho dydxdz$$

From part a $$x=a\left(1-\frac{z}{c}\right)$$

Now

$$I_{zx}=\int zxdm=\int_0^c\int_0^{a\left(1-\frac{z}{c}\right)}\int_0^{b\left(1-\frac{x}{a}-\frac{z}{c}\right)}zx\left(\rho dydxdz\right)$$

$$=\rho\int_0^c\int_0^{a\left(1-\frac{z}{c}\right)}zx\left[b\left(1-\tfrac{x}{a}-\tfrac{z}{c}\right)\right]dxdz$$

$$=\rho b\int_0^c z\left[\frac{1}{2}x^2-\frac{1}{3}\frac{x^3}{a}-\frac{1}{2}\frac{z}{c}x^2\right]_0^{a\left(1-\frac{z}{c}\right)}dz$$

$$=\rho b\int_0^c z\left[\frac{1}{2}a^2\left(1-\frac{z}{c}\right)^2-\frac{1}{3a}a^3\left(1-\frac{z}{c}\right)^3-\frac{1}{2}\frac{z}{c}a^2\left(1-\frac{z}{c}\right)^2\right]dz$$

$$=\rho b\int_0^c\frac{1}{6}a^2z\left(1-\frac{z}{c}\right)^3dz$$

$$=\frac{1}{6}\rho a^2b\int_0^c\left(z-3\frac{z^2}{c}+3\frac{z^3}{c^2}-\frac{z^4}{c^3}\right)dz$$

$$=\frac{m}{c}a\left[\frac{1}{2}z^2-\frac{z^3}{c}+\frac{3}{4}\frac{z^4}{c^2}-\frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

or $I_{zx}=\dfrac{1}{20}mac$ ◀

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Chapter 9, Solution 165.

From Figure 9.28 $\qquad I_y = \frac{1}{2}ma^2$

and using the parallel-axis theorem

$$I_x = I_z = \frac{1}{12}m\left(3a^2 + h^2\right) + m\left(\frac{h}{2}\right)^2 = \frac{1}{12}m\left(3a^2 + 4h^2\right)$$

Symmetry implies $\qquad I_{xy} = I_{yz} = I_{zx} = 0$

For convenience, let point A lie in the yz plane. Then

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}\left(h\mathbf{j} + a\mathbf{k}\right)$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = \cancelto{0}{I_x\lambda_x^2} + I_y\lambda_y^2 + I_z\lambda_z^2$$

$$= \frac{1}{2}ma^2\left(\frac{h}{\sqrt{h^2 + a^2}}\right)^2 + \frac{1}{12}m\left(3a^2 + 4h^2\right)\left(\frac{a}{\sqrt{h^2 + a^2}}\right)^2$$

$$\text{or} \quad I_{OA} = \frac{1}{12}ma^2\frac{10h^2 + 3a^2}{h^2 + a^2} \blacktriangleleft$$

Note: For point A located at an arbitrary point on the perimeter of the top surface, λ_{OA} is given by

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}\left(a\cos\phi\mathbf{i} + h\mathbf{j} + a\sin\phi\mathbf{k}\right)$$

which results in the same expression for I_{OA}

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Chapter 9, Solution 166.

First note that $$d_{OA} = \sqrt{\left(\frac{3}{2}a\right)^2 + (-3a)^2 + (3a)^2} = \frac{9}{2}a$$

Then $$\boldsymbol{\lambda}_{OA} = \frac{1}{\frac{9}{2}a}\left(\frac{3}{2}a\mathbf{i} - 3a\mathbf{j} + 3a\mathbf{k}\right) = \frac{1}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

For a rectangular coordinate system with origin at point A and axes aligned with the given x, y, z axes, have (using Figure 9.28)

$$I_x = I_z = \frac{3}{5}m\left[\frac{1}{4}a^2 + (3a)^2\right] \qquad I_y = \frac{3}{10}ma^2$$

$$= \frac{111}{20}ma^2$$

Also, symmetry implies $$I_{xy} = I_{yz} = I_{zx} = 0$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2$$

$$= \frac{111}{20}ma^2\left(\frac{1}{3}\right)^2 + \frac{3}{10}ma^2\left(-\frac{2}{3}\right)^2 + \frac{111}{20}ma^2\left(\frac{2}{3}\right)^2$$

$$= \frac{193}{60}ma^2$$

or $I_{OA} = 3.22ma^2$ ◀

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Chapter 9, Solution 167.

First compute the mass of each component

Have $$m = \rho_{st} V = \frac{0.284 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} V = \left(0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right) V$$

Then

$$m_1 = 0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3 \left[\pi (4 \text{ in.})^2 (2 \text{ in.})\right] = 0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = 0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3 \left[\pi (1 \text{ in.})^2 (3 \text{ in.})\right] = 0.083125 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = 0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3 \left[\pi (1 \text{ in.})^2 (2 \text{ in.})\right] = 0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Symmetry implies $$I_{yz} = I_{zx} = 0 \quad (I_{xy})_1 = 0$$

and $$(\bar{I}_{x'y'})_2 = (\bar{I}_{x'y'})_3 = 0$$

Now

$$I_{xy} = \Sigma\left(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 - m_3\bar{x}_3\bar{y}_3$$

$$= \left(0.083125 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[(2 \text{ in.})(1.5 \text{ in.})\right] \times \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$-\left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[(-2 \text{ in.})(-1 \text{ in.})\right] \times \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$= 0.96209 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

From the solution to Problem 9.143:

$$I_x = 34.106 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 50.125 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 34.876 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

continued

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By observation $\boldsymbol{\lambda}_{OA} = \frac{1}{\sqrt{13}}(2\mathbf{i} + 3\mathbf{j})$

Then

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + \cancelto{0}{I_z\lambda_z^2} - 2I_{xy}\lambda_x\lambda_y - \cancelto{0}{2I_{yz}\lambda_y\lambda_z} - \cancelto{0}{2I_{zx}\lambda_z\lambda_x}$$

$$= \left(34.106 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2\right)\left(\frac{2}{\sqrt{13}}\right)^2$$

$$+ \left(50.125 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2\right)\left(\frac{3}{\sqrt{13}}\right)^2$$

$$- 2\left(0.96209 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2\right)\left(\frac{2}{\sqrt{13}}\right)\left(\frac{3}{\sqrt{13}}\right)$$

$$= \left(10.4942 + 34.7019 - 0.8881\right) \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 44.308 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{OA} = 44.3 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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Chapter 9, Solution 168.

From Problem 9.147:

$$I_x = 9.8821 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 11.5344 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 2.1878 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Problem 9.151:

$$I_{xy} = 0.48776 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = 1.18391 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{zx} = 2.6951 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Now $\lambda_x = \lambda_y = \lambda_z$

and $\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$

Therefore, $3\lambda_x^2 = 1$

or $\lambda_x = \lambda_y = \lambda_z = \dfrac{1}{\sqrt{3}}$

Equation 9.46

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[9.8821\left(\frac{1}{\sqrt{3}}\right)^2 + 11.5344\left(\frac{1}{\sqrt{3}}\right)^2 + 2.1878\left(\frac{1}{\sqrt{3}}\right)^2 - 2(0.48776)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right.$$

$$\left. - 2(1.18391)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) - 2(2.6951)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right] \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.95692 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{OL} = 4.96 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 9, Solution 169.

First note that $m_1 = m_2 = \frac{1}{2}\frac{W}{g}$

And that $\boldsymbol{\lambda}_{OA} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$

Using Figure 9.28 and the parallel-axis theorem have

$$I_x = (I_x)_1 + (I_x)_2$$

$$= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right]$$

$$+ \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)\left(a^2 + a^2\right) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12} + \frac{1}{4}\right)a^2 + \left(\frac{1}{6} + \frac{1}{2}\right)a^2\right] = \frac{1}{2}\frac{W}{g}a^2$$

$$I_y = (I_y)_1 + (I_y)_2$$

$$= \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)\left(a^2 + a^2\right) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$+ \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{6} + \frac{1}{2}\right)a^2 + \left(\frac{1}{12} + \frac{5}{4}\right)a^2\right] = \frac{W}{g}a^2$$

continued

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$$I_z = (I_z)_1 + (I_z)_2$$

$$= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right]$$

$$+\left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12}+\frac{1}{4}\right)a^2 + \left(\frac{1}{12}+\frac{5}{4}\right)a^2\right] = \frac{5}{6}\frac{W}{g}a^2$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of both components are zero because of symmetry. Also, $\bar{y}_1 = 0$

Then $$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 = \frac{1}{2}\frac{W}{g}(a)\left(\frac{a}{2}\right) = \frac{1}{4}\frac{W}{g}a^2$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}\right) = m_2\bar{y}_2\bar{z}_2 = \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{8}\frac{W}{g}a^2$$

$$I_{zx} = \Sigma\left(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}\right) = m_1\bar{z}_1\bar{x}_1 + m_2\bar{z}_2\bar{x}_2$$

$$= \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)(a) = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Equation (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

Noting that

$$\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \frac{1}{3}$$

Have

$$I_{OA} = \frac{1}{3}\left[\frac{1}{2}\frac{W}{g}a^2 + \frac{W}{g}a^2 + \frac{5}{6}\frac{W}{g}a^2\right.$$

$$\left. - 2\left(\frac{1}{4}\frac{W}{g}a^2 + \frac{1}{8}\frac{W}{g}a^2 + \frac{3}{8}\frac{W}{g}a^2\right)\right]$$

$$= \frac{1}{3}\left[\frac{14}{6} - 2\left(\frac{3}{4}\right)\right]\frac{W}{g}a^2$$

or $I_{OA} = \dfrac{5}{18}\dfrac{W}{g}a^2$ ◀

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Chapter 9, Solution 170.

Have $\qquad m = \rho V = \rho t A$

Then $\qquad m_1 = \rho t a^2 \qquad m_2 = \frac{1}{2}\rho t a^2$

Compute moments and moments of inertia with respect to point A

Now

$$I_{x''} = (I_{x''})_1 + (I_{x''})_2$$

$$= \rho t a^2 \left\{ \left(\frac{1}{12} a^2 \right) + \left[\left(\frac{a}{2} \right)^2 + (a)^2 \right] \right\} + \frac{1}{2} \rho t a^2 \left\{ \left[\frac{1}{18} a^2 \right] + \left[\left(\frac{2}{3} a \right)^2 \right] \right\}$$

$$= \frac{19}{12} \rho t a^4$$

$$I_{y''} = (I_{y''})_1 + (I_{y''})_2$$

$$= \rho t a^2 \left\{ \left[\frac{1}{12} a^2 \right] + \left[\left(\frac{a}{2} \right)^2 + (a)^2 \right] \right\}$$

$$+ \frac{1}{2} \rho t a^2 \left\{ \frac{1}{18} \left[a^2 + a^2 \right] + \left[\left(\frac{a}{3} \right)^2 + \left(\frac{2a}{3} \right)^2 \right] \right\}$$

$$= \frac{5}{3} \rho t a^4$$

$$I_{z''} = (I_{z''})_1 + (I_{z''})_2$$

$$= \rho t a^2 \left[\frac{1}{3} \left(a^2 + a^2 \right) \right] + \frac{1}{2} \rho t a^2 \left(\frac{1}{6} a^2 \right)$$

$$= \frac{3}{4} \rho t a^4$$

Now note symmetry implies $\qquad (\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = (\bar{I}_{z'x'})_1 = 0$

$$(\bar{I}_{x'y'})_2 = (\bar{I}_{y'z'})_2 = 0$$

continued

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Now $$I_{uv} = I_{u'v'} + m\bar{u}\,\bar{v}$$

Therefore $$I_{x''y''} = m_1\bar{x}''_1\bar{y}''_1 + m_2\cancelto{0}{\bar{x}''_2\bar{y}''_2} = \rho ta^2\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{4}\rho ta^4$$

$$I_{y''z''} = m_1\bar{y}''_1\,\bar{z}''_1 + m_2\cancelto{0}{\bar{y}''_2\bar{z}''_2} = \rho ta^2\left(\frac{a}{2}\right)(-a) = -\frac{1}{2}\rho ta^4$$

$$I_{z''x''} = m_1\bar{z}''_1\,\bar{x}''_1 + \left[\left(\bar{I}_{z'x'}\right)_2 + m_2\bar{z}''_2\,\bar{x}''_2\right]$$

From Sample Problem 9.6 $$\left[\left(I_{z'x'}\right)_2\right]_{\text{area}} = -\frac{1}{72}a^4$$

Then $$\left(\bar{I}_{z''x''}\right)_2 = \rho t\left[\left(\bar{I}_{z'x'}\right)_2\right]_{\text{area}} = -\frac{1}{72}\rho ta^4$$

Then $$I_{z''x''} = \rho ta^2(-a)\left(\frac{a}{2}\right)$$

$$+\left[-\frac{1}{72}\rho ta^4 + \frac{1}{2}\rho ta^2\left(-\frac{2}{3}a\right)\left(\frac{1}{3}a\right)\right]$$

$$= -\frac{5}{8}\rho ta^4$$

By observation $$\boldsymbol{\lambda}_{AB} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$$

Now, Equation 9.46

$$I_{AB} = I_{x''}\lambda_{x''}^2 + I_{y''}\lambda_{y''}^2 + I_{z''}\lambda_{z''}^2 - 2I_{x''y''}\lambda_{x''}\lambda_{y''} - 2I_{y''z''}\lambda_{y''}\lambda_{z''} - 2I_{z''x''}\lambda_{z''}\lambda_{x''}$$

$$= \rho ta^4\left[\frac{19}{12}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{5}{3}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{3}{4}\left(-\frac{1}{\sqrt{3}}\right)^2\right.$$

$$-2\left(\frac{1}{4}\right)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) - 2\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right)$$

$$\left.-2\left(-\frac{5}{8}\right)\left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right]$$

or $$I_{AB} = \frac{5}{12}\rho ta^4 \blacktriangleleft$$

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Chapter 9, Solution 171.

From Problem 9.138:

$$I_x = 4.212\ \text{kg}\cdot\text{m}^2$$

$$I_y = 7.407\ \text{kg}\cdot\text{m}^2$$

$$I_z = 3.7055\ \text{kg}\cdot\text{m}^2$$

From Problem 9.157:

$$I_{xy} = -0.19312\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = 0.30987\ \text{kg}\cdot\text{m}^2$$

$$I_{zx} = 2.25956\ \text{kg}\cdot\text{m}^2$$

Now

$$\boldsymbol{\lambda}_{OL} = \frac{1}{9}\left(-4\mathbf{i} + 8\mathbf{j} + \mathbf{k}\right)$$

Eq. (9.46):

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[4.212\left(-\frac{4}{9}\right)^2 + 7.407\left(\frac{8}{9}\right)^2 + 3.7055\left(\frac{1}{9}\right)^2\right.$$

$$- 2\left(-0.19312\right)\left(-\frac{4}{9}\right)\left(\frac{8}{9}\right) - 2\left(0.3098\right)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)$$

$$\left. - 2\left(2.25956\right)\left(\frac{1}{9}\right)\left(-\frac{4}{9}\right)\right]\text{kg}\cdot\text{m}^2$$

$$= \left(0.832 + 5.85244 + 0.04575 - 0.15259 - 0.061195 + 0.22317\right)\text{kg}\cdot\text{m}^2$$

$$= 6.73957\ \text{kg}\cdot\text{m}^2$$

$$I_{OL} = 6.74\ \text{kg}\cdot\text{m}^2 \blacktriangleleft$$

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Chapter 9, Solution 172.

Mass of each leg is identical:

$$m = \left(\frac{W/L}{g}\right)L$$

$$= \frac{0.041\ \text{lb/ft}\left(1.5\ \text{ft}\right)}{32.2\ \text{ft/s}^2} = 0.00190994\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Also, $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each leg,

and $\bar{x}_1 = \bar{x}_6 = 0 \qquad \bar{y}_4 = \bar{y}_5 = \bar{y}_6 = 0 \qquad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$

Now $I_{xy} = \Sigma\left(\cancelto{0}{I_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$

$$= \left(0.00190994\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(0.75\right)\left(1.5\right) + \left(1.5\right)\left(0.75\right)\right]\text{ft}^2$$

$$= 0.0042974\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.2974\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = 0$$

$$I_{zx} = \Sigma\left(I_{z'x'} + m\bar{z}\,\bar{x}\right) = m_4\bar{z}_4\,\bar{x}_4 + m_5\bar{z}_5\bar{x}_5$$

$$= \left(0.00190994\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(0.75\right)\left(1.5\right) + \left(1.5\right)\left(0.75\right)\right]\text{ft}^2$$

$$= 0.0042974\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.2974\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

continued

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From Problem 9.150 $\qquad I_x = 14.32 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_y = I_z = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Now $\qquad \boldsymbol{\lambda}_{OL} = \frac{1}{7}\left(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\right)$ and then

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\cancelto{0}{I_{yz}}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x \qquad \left[\text{Eq. } (9.46)\right]$$

$$= \left\{\left[\left(14.32 \times 10^{-3}\right)\left(-\frac{3}{7}\right)^2 + \left(18.62 \times 10^{-3}\right)\left[\left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2\right] - 2\left(4.2974 \times 10^{-3}\right)\left[\left(-\frac{3}{7}\right)\left(\frac{-6}{7}\right)\right]\right.\right.$$

$$\left.\left. - 2\left(4.2974 \times 10^{-3}\right)\left[\left(\frac{2}{7}\right)\left(\frac{-3}{7}\right)\right]\right]\right\}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= \left(2.6302 \times 10^{-3} + 15.20 \times 10^{-3} - 3.1573 \times 10^{-3} + 1.05242 \times 10^{-3}\right)\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 15.725 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $\quad I_{OL} = 15.73 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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Chapter 9, Solution 173.

First compute the mass of each component

Have $\quad m = \rho_{st} V = mAL$

$$= \left(7850 \text{ kg/m}^3\right)\left[\pi\left(0.0015 \text{ m}\right)^2\right]L$$

$$= \left(0.055488L\right) \text{ kg/m}$$

Then $\quad m_1 = m_2 = 0.055488 \text{ kg/m}\left(\pi \times 0.36 \text{ m}\right)$

$$= 0.062756 \text{ kg}$$

$$m_3 = m_4 = 0.055488 \text{ kg/m}\left(0.36 \text{ m}\right)$$

$$= 0.019976 \text{ kg}$$

Now observe that the centroidal products of inertia $\bar{I}_{x'y'} = \bar{I}_{y'z'}$ $= \bar{I}_{z'x'} = 0$ for each component.

Also $\quad \bar{x}_3 = \bar{x}_4 = 0, \quad \bar{y}_1 = 0, \quad \bar{z}_1 = \bar{z}_2 = 0$

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{I_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2$$

$$= \left(0.062756 \text{ kg}\right)\left(-\frac{2 \times 0.36 \text{ m}}{\pi}\right)\left(0.36 \text{ m}\right) = -5.1777 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = \Sigma\left(\cancelto{0}{I_{y'z'}} + m\bar{y}\,\bar{z}\right) = m_3\bar{y}_3\bar{z}_3 + m_4\bar{y}_4\bar{z}_4$$

where $\quad m_3 = m_4, \quad \bar{y}_3 = \bar{y}_4, \quad \bar{z}_4 = -\bar{z}_3, \qquad$ so that $\qquad I_{yz} = 0$

$$I_{zx} = \Sigma\left(I_{z'x'} + m\bar{z}\,\bar{x}\right) = m_1\bar{z}_1\bar{x}_1 + m_2\bar{z}_2\bar{x}_2 = 0$$

continued

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From the solution to Problem 9.149

$$I_x = 23.170 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_y = 21.444 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_z = 17.992 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Now $$\boldsymbol{\lambda}_{OL} = \frac{1}{7}\left(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\right)$$

Have

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - \overset{0}{2I_{yz}}\lambda_y\lambda_z - \overset{0}{2I_{zx}}\lambda_z\lambda_x \quad \left[\text{Eq. } (9.46)\right]$$

$$= \left[23.170\left(-\frac{3}{7}\right)^2 + 21.444\left(-\frac{6}{7}\right)^2 + 17.992\left(\frac{2}{7}\right)^2 - 2(-5.1777)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right)\right] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= \left(4.2557 + 15.755 + 1.4687 + 3.8040\right) \times 10^{-3}\ \text{kg}\cdot\text{m}$$

$$= 25.283 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{OL} = 25.3 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 174.

First compute the mass of each component.

Have $$m = (m/L)L$$

$$= (0.049 \text{ kg/m})L$$

Then $$m_1 = (0.049 \text{ kg/m})(2\pi \times 0.32 \text{ m})$$

$$= 0.09852 \text{ kg}$$

$$m_2 = m_3 = m_4 = m_5 = (0.049 \text{ kg})(0.160 \text{ m})$$

$$= 0.00784 \text{ kg}$$

Now observe that $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each component.

Also, $\bar{x}_1 = \bar{x}_4 = \bar{x}_5 = 0, \quad \bar{y}_1 = 0, \quad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$

$$= (0.00784 \text{ kg})\left[(0.32 \text{ m})(0.08 \text{ m}) + (0.24 \text{ m})(0.16 \text{ m})\right]$$

$$= 0.50176 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

By symmetry $$I_{yz} = I_{xy} = 0.50176 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

Now $$I_{zx} = \Sigma\left(I_{z'x'} + m\bar{z}\,\bar{x}\right) = 0$$

From the solution to Problem 9.148

$$I_x = I_z = 6.8505 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_y = 12.630 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

continued

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Now $$\boldsymbol{\lambda}_{OL} = \frac{1}{7}\left(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\right)$$

Have

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2\cancelto{0}{I_{zx}}\lambda_z\lambda_x$$

$$\left[\text{Eq.}(9.46)\right]$$

$$= \left\{(6.8505)\left[\left(-\frac{3}{7}\right)^2 + \left(\frac{2}{7}\right)^2\right]\right\} \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$+ \left\{(12.63)\left(-\frac{6}{7}\right)^2\right] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$-2(0.50176)\left[\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right) + \left(-\frac{6}{7}\right)\left(\frac{2}{7}\right)\right]\right\} \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= (1.8175 + 9.2792 - 0.12288) \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= 10.9738 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{OL} = 10.97 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

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Chapter 9, Solution 175.

(*a*) Using Figure 9.28 and the parallel-axis theorem have at point *A*.

$$I_{x'} = \frac{1}{12}m\left(b^2 + c^2\right)$$

$$I_{y'} = \frac{1}{12}m\left(a^2 + c^2\right) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m\left(4a^2 + c^2\right)$$

$$I_{z'} = \frac{1}{12}m\left(a^2 + b^2\right) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m\left(4a^2 + b^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is then

$$I_{x'}x^2 + I_{y'}y^2 + I_{z'}z^2 = 1$$

or $\frac{1}{12}m\left(b^2 + c^2\right)x^2 + \frac{1}{12}m\left(4a^2 + c^2\right)y^2 + \frac{1}{12}m\left(4a^2 + b^2\right)z^2 = 1$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{1}{12}m\left(b^2 + c^2\right) = \frac{1}{12}m\left(4a^2 + c^2\right) = \frac{1}{12}m\left(4a^2 + b^2\right)$$

Then $\quad b^2 + c^2 = 4a^2 + c^2 \quad$ or $\frac{b}{a} = 2$ ◀

and $\quad b^2 + c^2 = 4a^2 + b^2 \quad$ or $\frac{c}{a} = 2$ ◀

(*b*) Using Figure 9.28 and the parallel-axis theorem, we have at point *B*

$$I_{x''} = \frac{1}{12}m\left(b^2 + c^2\right) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m\left(b^2 + 4c^2\right)$$

$$I_{y''} = \frac{1}{12}m\left(a^2 + c^2\right) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m\left(a^2 + 4c^2\right)$$

$$I_{z''} = \frac{1}{12}m\left(a^2 + b^2\right)$$

continued

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Now observe that symmetry implies

$$I_{x''y''} = I_{y''z''} = I_{z''x''} = 0$$

From part *a* it then immediately follows that

$$\frac{1}{12}m\left(b^2 + 4c^2\right) = \frac{1}{12}m\left(a^2 + 4c^2\right) = \frac{1}{12}m\left(a^2 + b^2\right)$$

Then $\quad b^2 + 4c^2 = a^2 + 4c^2 \quad$ or $\dfrac{b}{a} = 1$ ◀

and $\quad b^2 + 4c^2 = a^2 + b^2 \quad$ or $\dfrac{c}{a} = \dfrac{1}{2}$ ◀

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Chapter 9, Solution 176.

(*a*) From sample Problem 9.11, we have at the apex *A*

$$I_x = \frac{3}{10}ma^2$$

$$I_y = I_z = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Equation (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1$$

or $$\frac{3}{10}ma^2x^2 + \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)y^2 + \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{3}{10}ma^2 = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right) \qquad \text{or } \frac{a}{h} = 2 \blacktriangleleft$$

(*b*) From Sample Problem 9.11, we have

$$I_{x'} = \frac{3}{10}ma^2$$

and at the centroid *C* $$\bar{I}_{y'} = \frac{3}{20}m\left(a^2 + \frac{1}{4}h^2\right)$$

Then $$I_{y'} = I_{z'} = \frac{3}{20}m\left(a^2 + \frac{1}{4}h^2\right) + m\left(\frac{h}{4}\right)^2$$

$$= \frac{1}{20}m\left(3a^2 + 2h^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From part *a* it then immediately follows that

$$\frac{3}{10}ma^2 = \frac{1}{20}m\left(3a^2 + 2h^2\right) \qquad \text{or } \frac{a}{h} = \sqrt{\frac{2}{3}} \blacktriangleleft$$

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Chapter 9, Solution 177.

(*a*) From Figure 9.28

$$\bar{I}_x = \frac{1}{2}ma^2 \qquad \bar{I}_y = \bar{I}_z = \frac{1}{12}m\left(3a^2 + L^2\right)$$

Now observe that symmetry implies

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1\text{:}\ \frac{1}{2}ma^2x^2 + \frac{1}{12}m\left(3a^2 + L^2\right)y^2 + \frac{1}{12}m\left(3a^2 + L^2\right) = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{1}{2}ma^2 = \frac{1}{12}m\left(3a^2 + L^2\right) \qquad \text{or} \quad \frac{a}{L} = \frac{1}{\sqrt{3}} \blacktriangleleft$$

(*b*) Using Fig. 9.28 and the parallel-axis theorem

Have $I_{x'} = \frac{1}{2}ma^2$

$$I_{y'} = I_{z'} = \frac{1}{12}m\left(3a^2 + L^2\right) + m\left(\frac{L}{4}\right)^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part a it then immediately follows that

$$\frac{1}{2}ma^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right) \qquad \text{or} \quad \frac{a}{L} = \sqrt{\frac{7}{12}} \blacktriangleleft$$

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Chapter 9, Solution 178.

(i) To prove $I_y + I_z \geq I_x$

By definition

$$I_y = \int\left(z^2 + x^2\right)dm \qquad I_z = \int\left(x^2 + y^2\right)dm$$

Then $$I_y + I_z = \int\left(z^2 + x^2\right)dm + \int\left(x^2 + y^2\right)dm$$

$$= \int\left(y^2 + z^2\right)dm + 2\int x^2 dm$$

Now.. $$\int\left(y^2 + z^2\right)dm = I_x \qquad \text{and} \qquad \int x^2 dm \geq 0$$

$$\therefore \quad I_y + I_z \geq I_x \qquad \text{Q.E.D.}$$

The proofs of the other two inequalities follow similar steps.

(ii) If the x axis is the axis of revolution, then

$$I_y = I_z$$

and from part (i) $I_y + I_z \geq I_x$

or $2I_y \geq I_x$

or $I_y \geq \frac{1}{2} I_x$ Q.E.D.

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Chapter 9, Solution 179.

(*a*) At the center of the cube have (using Figure 9.28)

$$I_x = I_y = I_z = \frac{1}{12}m\left(a^2 + a^2\right) = \frac{1}{6}ma^2$$

Now observe that symmetry implies $\quad I_{xy} = I_{yz} = I_{zx} = 0$

Using Equation (9.48), the equation of the ellipsoid of inertia is

$$\left(\frac{1}{6}ma^2\right)x^2 + \left(\frac{1}{6}ma^2\right)y^2 + \left(\frac{1}{6}ma^2\right)z^2 = 1$$

or $$x^2 + y^2 + z^2 = \frac{6}{ma^2}\left(= R^2\right) \blacktriangleleft$$

which is the equation of a sphere.

Since the ellipsoid of inertia is a sphere, the moment of inertia with respect to any axis *OL* through the center *O* of the cube must always be the same $\left(R = \frac{1}{\sqrt{I_{OL}}}\right)$.

$$\therefore \quad I_{OL} = \frac{1}{6}ma^2 \blacktriangleleft$$

(*b*) The above sketch of the cube is the view seen if the line of sight is along the diagonal that passes through corner *A*. For a rectangular coordinate system at *A* and with one of the coordinate axes aligned with the diagonal, an ellipsoid of inertia at *A* could be constructed. If the cube is then rotated 120° about the diagonal, the mass distribution will remain unchanged. Thus, the ellipsoid will also remain unchanged after it is rotated. As noted at the end of section 9.17, this is possible only if the ellipsoid is an ellipsoid of revolution, where the diagonal is both the axis of revolution and a principal axis.

It then follows that $$I_{x'} = I_{OL} = \frac{1}{6}ma^2 \blacktriangleleft$$

continued

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In addition, for an ellipsoid of revolution, the two transverse principal moments of inertia are equal and any axis perpendicular to the axis of revolution is a principal axis. Then, applying the parallel-axis theorem between the center of the cube and corner A for any perpendicular axis

$$I_{y'} = I_{z'} = \frac{1}{6}ma^2 + m\left(\frac{\sqrt{3}}{2}a\right)^2$$

$$\text{or } I_{y'} = I_{z'} = \frac{11}{12}ma^2 \blacktriangleleft$$

Note: Part b can also be solved using the method of Section 9.18. First note that at corner A

$$I_x = I_y = I_z = \frac{2}{3}ma^2 \qquad I_{xy} = I_{yz} = I_{zx} = \frac{1}{4}ma^2$$

Substituting into Equation (9.56) yields

$$k^3 - 2ma^2k^2 + \frac{55}{48}m^2a^6k - \frac{121}{864}m^3a^9 = 0$$

For which the roots are

$$k_1 = \frac{1}{6}ma^2 \qquad k_2 = k_3 = \frac{11}{12}ma^2$$

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Chapter 9, Solution 180.

(i) Using Equation (9.30), we have

$$I_x + I_y + I_z = \int\left(y^2 + z^2\right)dm + \int\left(z^2 + x^2\right)dm + \int\left(x^2 + y^2\right)dm$$

$$= 2\int\left(x^2 + y^2 + z^2\right)dm$$

$$= 2\int r^2 dm$$

where r is the distance from the origin O to the element of mass dm. Now assume that the given body can be formed by adding and subtracting appropriate volumes V_1 and V_2 from a sphere of mass m and radius a which is centered at O; it then follows that $m_1 = m_2 \left(m_{\text{body}} = m_{\text{sphere}} = m\right)$.

Then

$$\left(I_x + I_y + I_z\right)_{\text{body}} = \left(I_x + I_y + I_z\right)_{\text{sphere}} + \left(I_x + I_y + I_z\right)_{V_1} - \left(I_x + I_y + I_z\right)_{V_2}$$

or

$$\left(I_x + I_y + I_z\right)_{\text{body}} = \left(I_x + I_y + I_z\right)_{\text{sphere}} + 2\int_{m_1} r^2 dm - 2\int_{m_2} r^2 dm$$

Now, $m_1 = m_2$ and $r_1 \geq r_2$ for all elements of mass dm in volumes 1 and 2.

$$\therefore \quad \int_{m_1} r^2 dm - \int_{m_2} r^2 dm \geq 0$$

so that $\left(I_x + I_y + I_z\right)_{\text{body}} \geq \left(I_x + I_y + I_z\right)_{\text{sphere}}$ Q.E.D.

continued

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(ii) First note from Figure 9.28 that for a sphere

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$

Thus, $$\left(I_x + I_y + I_z\right)_{\text{sphere}} = \frac{6}{5}ma^2$$

For a solid of revolution, where the x axis is the axis of revolution, have

$$I_y = I_z$$

Then, using the results of part i

$$\left(I_x + 2I_y\right)_{\text{body}} \geq \frac{6}{5}ma^2$$

From Problem 9.178 have $$I_y \geq \frac{1}{2}I_x$$

or $$\left(2I_y - I_x\right)_{\text{body}} \geq 0$$

Adding the last two inequalities yields

$$\left(4I_y\right)_{\text{body}} \geq \frac{6}{5}ma^2$$

or $$\left(I_y\right)_{\text{body}} \geq \frac{3}{10}ma^2 \quad \text{Q.E.D.}$$

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Chapter 9, Solution 181.

(*a*) First compute the moments of inertia using Figure 9.28 and the parallel-axis theorem.

$$I_x = I_z = \frac{1}{12}m\left(3a^2 + a^2\right) + m\left[\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2\right] = \frac{13}{12}ma^2$$

$$I_y = \frac{1}{2}ma^2 + m(a)^2 = \frac{3}{2}ma^2$$

Next observe that the centroidal products of inertia are zero because of symmetry. Then

$$I_{xy} = \cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y} = m\left(\frac{a}{\sqrt{2}}\right)\left(-\frac{a}{2}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{yz} = \cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z} = m\left(-\frac{a}{2}\right)\left(\frac{a}{\sqrt{2}}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{zx} = \cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x} = m\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{1}{2}ma^2$$

Substituting into Equation (9.56)

$$K^3 - \left(\frac{13}{12} + \frac{3}{2} + \frac{13}{12}\right)ma^2K^2$$

$$+\left[\left(\frac{13}{12}\times\frac{3}{2}\right) + \left(\frac{3}{2}\times\frac{13}{12}\right) + \left(\frac{13}{12}\times\frac{13}{12}\right) - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2\right]\left(ma^2\right)^2 K$$

$$-\left[\left(\frac{13}{12}\times\frac{3}{2}\times\frac{13}{12}\right) - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)^2 - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - 2\left(-\frac{1}{2\sqrt{2}}\right)\left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{2}\right)\right]\left(ma^2\right)^3 = 0$$

Simplifying and letting $K = ma^2\zeta$ yields

$$\zeta^3 - \frac{11}{3}\zeta^2 + \frac{565}{144}\zeta - \frac{95}{96} = 0$$

continued

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Solving yields

$$\zeta_1 = 0.363383 \qquad \zeta_2 = \frac{19}{12} \qquad \zeta_3 = 1.71995$$

The principal moments of inertia are then

$$K_1 = 0.363ma^2 \blacktriangleleft$$

$$K_2 = 1.583ma^2 \blacktriangleleft$$

$$K_3 = 1.720ma^2 \blacktriangleleft$$

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, we use two of the equations of Equations (9.54) and Equation (9.57). Thus

$$\left(I_x - K\right)\lambda_x - I_{xy}\lambda_y - I_{zx}\lambda_z = 0 \qquad (9.54a)$$

$$-I_{zx}\lambda_x - I_{yz}\lambda_y + \left(I_z - K\right)\lambda_z = 0 \qquad (9.54c)$$

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \qquad (9.57)$$

Note: Since $I_{xy} = I_{yz}$, Equations (9.54a) and (9.54c) were chosen to simplify the "elimination" of λ_y during the solution process.

Substituting for the moments and products of inertia in Equations (9.54a) and (9.54c)

$$\left(\frac{13}{12}ma^2 - K\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y - \left(\frac{1}{2}ma^2\right)\lambda_z = 0$$

$$-\left(\frac{1}{2}ma^2\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y + \left(\frac{13}{12}ma^2 - K\right)\lambda_z = 0$$

or

$$\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_z = 0 \qquad \text{(i)}$$

and

$$-\frac{1}{2}\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y + \left(\frac{13}{12} - \zeta\right)\lambda_z = 0 \qquad \text{(ii)}$$

Observe that these equations will be identical, so that one will need to be replaced, if

$$\frac{13}{12} - \zeta = -\frac{1}{2} \qquad \text{or} \qquad \zeta = \frac{19}{12}$$

Thus, a third independent equation will be needed when the direction cosines associated with K_2 are determined. Then for K_1 and K_3

continued

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Eq.(i) – Eq.(ii) $\left[\frac{13}{12} - \zeta - \left(-\frac{1}{2}\right)\right]\lambda_x + \left[-\frac{1}{2} - \left(\frac{13}{12} - \zeta\right)\right]\lambda_z = 0$

or $\lambda_z = \lambda_x$

Substituting into Eq.(i) $\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_x = 0$

or $\lambda_y = 2\sqrt{2}\left(\zeta - \frac{7}{12}\right)\lambda_x$

Substituting into Equation (9.57)

$$\lambda_x^2 + \left[2\sqrt{2}\left(\zeta - \frac{7}{12}\right)\lambda_x\right]^2 + (\lambda_x)^2 = 1$$

or $$\left[2 + 8\left(\zeta - \frac{7}{12}\right)^2\right]\lambda_x^2 = 1 \qquad \text{(iii)}$$

$\mathbf{K}_1$: Substituting the value of ζ_1 into Eq.(iii)

$$\left[2 + 8\left(0.363383 - \frac{7}{12}\right)^2\right](\lambda_x)_1^2 = 1$$

or $(\lambda_x)_1 = (\lambda_z)_1 = 0.647249$

and then $(\lambda_y)_1 = 2\sqrt{2}\left(0.363383 - \frac{7}{12}\right)(0.647249)$

$= -0.402662$

$\therefore \quad (\theta_x)_1 = (\theta_z)_1 = 49.7° \quad (\theta_y)_1 = 113.7°$ ◀

$\mathbf{K}_3$: Substituting the value of ζ_3 into Eq.(iii)

$$\left[2 + 8\left(1.71995 - \frac{7}{12}\right)^2\right](\lambda_x)_3^2 = 1$$

or $(\lambda_x)_3 = (\lambda_z)_3 = 0.284726$

and then $(\lambda_y)_3 = 2\sqrt{2}\left(1.71995 - \frac{7}{12}\right)(0.284726)$

$= 0.915348$

$\therefore \quad (\theta_x)_3 = (\theta_z)_3 = 73.5° \quad (\theta_y)_3 = 23.7°$ ◀

continued

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$\mathbf{K}_2$: For this case, the set of equations to be solved consists of Equations (9.54a), (9.54b), and (9.57). Now

$$-I_{xy}\lambda_x + \left(I_y - K\right)\lambda_y - I_{yz}\lambda_z = 0 \qquad (9.54b)$$

Substituting for the moments and products of inertia.

$$-\left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_x + \left(\frac{3}{2}ma^2 - K\right)\lambda_y - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_z = 0$$

or $$\frac{1}{2\sqrt{2}}\lambda_x + \left(\frac{3}{2} - \zeta\right)\lambda_y + \frac{1}{2\sqrt{2}}\lambda_z = 0 \qquad \text{(iv)}$$

Substituting the value of ζ_2 into Eqs.(i) and (iv)

$$\left(\frac{13}{12} - \frac{19}{12}\right)(\lambda_x)_2 + \frac{1}{2\sqrt{2}}(\lambda_y)_2 - \frac{1}{2}(\lambda_z)_2 = 0$$

$$\frac{1}{2\sqrt{2}}(\lambda_x)_2 + \left(\frac{3}{2} - \frac{19}{12}\right)(\lambda_y)_2 + \frac{1}{2\sqrt{2}}(\lambda_z)_2 = 0$$

or $$-(\lambda_x)_2 + \frac{1}{\sqrt{2}}(\lambda_y)_2 - (\lambda_z)_2 = 0$$

and $$(\lambda_x)_2 - \frac{\sqrt{2}}{6}(\lambda_y)_2 + (\lambda_z)_2 = 0$$

Adding yields $$(\lambda_y)_2 = 0$$

and then $$(\lambda_z)_2 = -(\lambda_x)_2$$

Substituting into Equation (9.57)

$$(\lambda_x)_2^2 + \cancelto{0}{(\lambda_y)_2^2} + (-\lambda_x)_2^2 = 1$$

or $$(\lambda_x)_2 = \frac{1}{\sqrt{2}} \quad \text{and} \quad (\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$

$$\therefore \quad (\theta_x)_2 = 45.0° \quad (\theta_y)_2 = 90.0° \quad (\theta_z)_2 = 135.0° \blacktriangleleft$$

(*c*) Principal axes 1 and 3 lie in the vertical plane of symmetry passing through points *O* and *B*. Principal axis 2 lies in the *xz* plane.

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Chapter 9, Solution 182.

From the solution to Problem 9.143 and 9.167

$$I_x = 34.106 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 50.125 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 34.876 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{xy} = 0.96211 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = I_{zx} = 0$$

(*a*) From Equation 9.55

$$\begin{vmatrix} I_x - K & I_{xy} & 0 \\ I_{xy} & I_y - K & 0 \\ 0 & 0 & I_z - K \end{vmatrix} = 0$$

or $$(I_x - K)(I_y - K)(I_z - K) - (I_z - K)I_{xy}^2 = 0$$

or $$(I_z - K)\left[(I_x - K)(I_y - K) - I_{xy}^2\right] = 0$$

Then $I_z - K = 0$ and $I_x I_y - (I_x + I_y)K + K^2 - I_{xy}^2 = 0$

Now $K_1 = I_z = 34.876 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

or $K_1 = 34.9 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

and

$$\left(34.106 \times 10^{-3}\right)\left(50.125 \times 10^{-3}\right) - \left(34.106 \times 10^{-3} + 50.125 \times 10^{-3}\right)K$$
$$+ K^2 - \left(0.96211 \times 10^{-3}\right)^2 = 0$$

or $$1.70864 \times 10^{-3} - 84.231 \times 10^{-3} K + K^2 = 0$$

Solving yields $K_2 = 34.0486 \times 10^{-3}$ $K_3 = 50.1824 \times 10^3$

or $K_2 = 34.0 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

and $K_3 = 50.2 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

continued

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(*b*) To determine the directions cosines λ_x, λ_y and λ_z of each principal axis use two of the Equations 9.54 and Equation 9.57

$\mathbf{K}_1$: Using Equation 9.54(a) and Equation 9.54(b) with $I_{yz} = I_{zx} = 0$, we have

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 = 0$$

$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 = 0$$

Substituting

$$\left(34.106\times10^{-3} - 34.876\times10^{-3}\right)(\lambda_x)_1 - 0.96211\times10^{-3}(\lambda_y)_1 = 0$$

$$-0.96211\times10^{-3}(\lambda_x)_1 + \left(50.125\times10^{-3} - 34.876\times10^{-3}\right)(\lambda_y)_1 = 0$$

or

$$-0.770\times10^{-3}(\lambda_x)_1 - 0.96211\times10^{-3}(\lambda_y)_1 = 0$$

$$-0.96211\times10^{-3}(\lambda_x)_1 + 15.249\times10^{-3}(\lambda_y)_1 = 0$$

Solving yields $\quad (\lambda_x)_1 = (\lambda_y)_1 = 0$

From Equation 9.57 $\quad \cancelto{0}{(\lambda_x)_1^2} + \cancelto{0}{(\lambda_y)_1^2} + (\lambda_z)_1^2 = 1 \quad$ or $\quad (\lambda_z)_1 = 1$

and $\quad (\theta_x)_1 = 90.0°, \quad (\theta_y)_1 = 90.0°, \quad (\theta_z)_1 = 0°$ ◀

$\mathbf{K}_2$: Using Equation 9.54(b) and Equation 9.54(c) with $I_{yz} = I_{zx} = 0$

$$-I_{xz}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 = 0$$

$$(I_z - K_2)(\lambda_z)_2 = 0$$

Now $\quad I_z \neq K_2 \Rightarrow (\lambda_z)_2 = 0$

Substituting

$$-0.96211\times10^{-3}(\lambda_x)_2 + \left(50.125\times10^{-3} - 34.0486\times10^{-3}\right)(\lambda_y)_2 = 0$$

or $\quad (\lambda_y)_2 = 0.05985(\lambda_x)_2$

continued

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Then $$(\lambda_x)_2^2 + \left[0.05985(\lambda_x)_2\right]^2 + (\lambda_z)_2 = 1$$

$$(\lambda_x)_2 = 0.99821$$

$$(\lambda_y)_2 = 0.05974$$

and $$(\theta_x)_2 = 3.43°, \quad (\theta_y)_2 = 86.6°, \quad (\theta_z)_2 = 90.0° \blacktriangleleft$$

$\mathbf{K}_3$: $$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 = 0$$

$$(I_z - K_3)(\lambda_z)_3 = 0$$

Now $$I_z \neq K_3 \Rightarrow (\lambda_z)_3 = 0$$

Substituting

$$-0.96211 \times 10^{-3}(\lambda_x)_3 + \left(50.125 \times 10^{-3} - 50.1824 \times 10^{-3}\right)(\lambda_y)_3 = 0$$

$$-0.96211 \times 10^{-3}(\lambda_x)_3 - 0.0574 \times 10^{-3}(\lambda_y)_3 = 0$$

or $$(\lambda_y)_3 = -16.7615(\lambda_x)_3$$

Have $$(\lambda_x)_3^2 + \left[-16.7615(\lambda_x)_3\right]^2 + \cancelto{0}{(\lambda_z)_3^2} = 1$$

yields $(\lambda_x)_3 = -0.059555$ and $(\lambda_y)_3 = 0.998231$

and $$(\theta_x)_3 = 93.4°, \quad (\theta_y)_3 = 3.41°, \quad \theta_z = 90.0° \blacktriangleleft$$

(*c*) Principal axis 1 coincides with the *z* axis, while the principal axes 2 and 3 lie in the *xy* plane

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Chapter 9, Solution 183.

From Problem 9.147:

$$I_x = 9.8821 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 11.5344 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 2.1878 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

From Problem 9.151:

$$I_{xy} = 0.48776 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = 1.18391 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{zx} = 2.6951 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

(*a*) From Equation 9.56

$$K^3 - \left(I_x + I_y + I_z\right)K^2 + \left(I_xI_y + I_yI_z + I_zI_x - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\right)K$$
$$-\left(I_xI_yI_z - I_xI_{yz}^2 - I_yI_{zx}^2 - I_zI_{xy}^2 - 2I_{xy}I_{yz}I_{zx}\right) = 0$$

Substituting

$$K^3 - \left[\left(9.8821 + 11.5344 + 2.1878\right)\times 10^{-3}\right]K^2 + \left\{\left[\left(9.8821\right)\left(11.5344\right) + \left(11.5344\right)\left(2.1878\right)\right.\right.$$
$$\left.\left. + \left(2.1878\right)\left(9.8821\right) - \left(0.48776\right)^2 - \left(1.18391\right)^2 - \left(2.6951\right)^2\right]\times 10^{-6}\right\}K$$
$$-\left[\left(9.8821\right)\left(11.5344\right)\left(2.1878\right) - \left(9.8821\right)\left(1.18391\right)^2 - \left(11.5344\right)\left(2.6951\right)^2\right.$$
$$\left. - \left(2.1878\right)\left(0.48776\right)^2 - 2\left(0.48776\right)\left(1.18391\right)\left(2.6951\right)\right]\times 10^{-9} = 0$$

or

$$K^3 - \left(23.6043 \times 10^{-3}\right)K^2 + \left(151.9360 \times 10^{-6}\right)K - 148.1092 \times 10^{-9} = 0$$

Solving numerically

$$K_1 = 1.180481 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad \text{or} \quad K_1 = 1.180 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ ◀$$

$$K_2 = 10.72017 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad \text{or} \quad K_2 = 10.72 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ ◀$$

$$K_3 = 11.70365 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad \text{or} \quad K_3 = 11.70 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ ◀$$

continued

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(*b*) From Equations 9.54(a) and 9.54(b)

$$(I_x - K)(\lambda_x) - I_{xy}(\lambda_y) - I_{zx}(\lambda_z) = 0$$

$$-I_{xy}(\lambda_x) + (I_y - K_1)(\lambda_y) - I_{yz}(\lambda_z) = 0$$

$\mathbf{K}_1$: Substitute K_1 and solve for λ to get $(\lambda_x)_1$, $(\lambda_y)_1$ and $(\lambda_z)_1$.

$$\left[(9.8821 - 1.180481)(\lambda_x)_1 - 0.48776(\lambda_y)_1 - 2.6951(\lambda_z)_1\right] \times 10^{-3} = 0$$

$$\left[-0.48776(\lambda_x)_1 + (11.5344 - 1.180481)(\lambda_y)_1 - 1.18391(\lambda_z)_1\right] \times 10^{-3} = 0$$

or

$$17.83996(\lambda_x)_1 - (\lambda_y)_1 - 5.52546(\lambda_z)_1 = 0$$

$$-0.0471(\lambda_x)_1 + (\lambda_y)_1 - 0.11434(\lambda_z)_1 = 0$$

Then

$$(\lambda_z)_1 = 3.1549(\lambda_x)_1$$

and

$$(\lambda_y)_1 = 0.40769(\lambda_x)_1$$

Equation 9.57:

$$(\lambda_x)_1^2 + (\lambda_y)_1^2 + (\lambda_z)_1^2 = 1$$

Substituting

$$(\lambda_x)_1^2 + \left[0.40769(\lambda_x)_1\right]^2 + \left[3.1549(\lambda_x)_1\right]^2 = 1$$

or $(\lambda_x)_1 = 0.29989$ then $(\theta_x)_1 = 72.5°$ ◀

and $(\lambda_y)_1 = 0.122262$ then $(\theta_y)_1 = 83.0°$ ◀

$(\lambda_z)_1 = 0.94612$ then $(\theta_z)_1 = 18.89°$ ◀

$\mathbf{K}_2$: Substitute K_2 and solve for λ.

$$\left[(9.8821 - 10.72017)(\lambda_x)_2 - 0.48776(\lambda_y)_2 - 2.6951(\lambda_z)_2\right] \times 10^{-3} = 0$$

$$\left[-0.48776(\lambda_x)_2 + (11.5344 - 10.72017)(\lambda_y)_2 - 1.18391(\lambda_z)_2\right] \times 10^{-3} = 0$$

continued

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or $$-1.718202(\lambda_x)_2 - (\lambda_y)_2 - 5.52546(\lambda_z)_2 = 0$$

$$-0.599045(\lambda_x)_2 + (\lambda_y)_2 - 1.45402(\lambda_z)_2 = 0$$

Then $$(\lambda_z)_2 = -0.33201(\lambda_x)_2$$

and $$(\lambda_y)_2 = 0.116306(\lambda_x)_2$$

Then $$(\lambda_x)_2^2 + \left[0.116306(\lambda_x)_2\right]^2 + \left[-0.33201(\lambda_x)_2\right]^2 = 1$$

or $(\lambda_x)_2 = 0.94333$ then $(\theta_x)_2 = 19.38°$ ◀

And $(\lambda_y)_2 = 0.109715$ then $(\theta_y)_2 = 83.7°$ ◀

$(\lambda_z)_2 = -0.31320$ then $(\theta_z)_2 = 108.3°$ ◀

$\mathbf{K}_3$: Substitute K_3 and solve for λ.

$$\left[(9.8821 - 11.70365)(\lambda_x)_3 - 0.48776(\lambda_y)_3 - 2.6951(\lambda_z)_3\right]\times 10^{-3} = 0$$

$$\left[-0.48776(\lambda_x)_3 + (11.5344 - 11.70365)(\lambda_y)_3 - 1.18391(\lambda_z)_3\right]\times 10^{-3} = 0$$

or $$-3.73452(\lambda_x)_3 - (\lambda_y)_3 - 5.52546(\lambda_z)_3 = 0$$

$$2.88189(\lambda_x)_3 + (\lambda_y)_3 + 6.99504(\lambda_z)_3 = 0$$

Then $$(\lambda_z)_3 = 0.58019(\lambda_x)_3$$

and $$(\lambda_y)_3 = -6.9403(\lambda_x)_3$$

Then $(\lambda_x)_3^2 + \left[-6.9403(\lambda_x)_3\right]^2 + \left[0.58019(\lambda_x)_3\right]^2 =$

or $(\lambda_x)_3 = -0.142128^*$ then $(\theta_x)_3 = 98.2°$ ◀

$(\lambda_y)_3 = 0.98641$ then $(\theta_y)_3 = 9.46°$ ◀

$(\lambda_z)_3 = -0.082461$ then $(\theta_z)_3 = 94.7°$ ◀

*Note: the negative root of $(\lambda_x)_3$ is taken so that axes 1, 2, 3 form a right-handed set.

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Chapter 9, Solution 184.

(*a*) From the solution of Problem 9.169 have

$$I_x = \frac{1}{2}\frac{W}{g}a^2 \qquad I_{xy} = \frac{1}{4}\frac{W}{g}a^2$$

$$I_y = \frac{W}{g}a^2 \qquad I_{yz} = \frac{1}{8}\frac{W}{g}a^2$$

$$I_z = \frac{5}{6}\frac{W}{g}a^2 \qquad I_{zx} = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Equation (9.56)

$$K^3 - \left[\left(\frac{1}{2}+1+\frac{5}{6}\right)\left(\frac{W}{g}a^2\right)\right]K^2 + \left[\left(\frac{1}{2}\right)(1)+(1)\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{4}\right)^2-\left(\frac{1}{8}\right)^2-\left(\frac{3}{8}\right)^2\right]\left(\frac{W}{g}a^2\right)^2 K$$

$$-\left[\left(\frac{1}{2}\right)(1)\left(\frac{5}{6}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{8}\right)^2-(1)\left(\frac{3}{8}\right)^2-\left(\frac{5}{6}\right)\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)\left(\frac{1}{8}\right)\left(\frac{3}{8}\right)\right]\left(\frac{W}{g}a^2\right)^3 = 0$$

Simplifying and letting $K = \frac{W}{g}a^2\zeta$ yields

$$\zeta^3 - 2.33333\zeta^2 + 1.53125\zeta - 0.192708 = 0$$

Solving yields

$$\zeta_1 = 0.163917 \qquad \zeta_2 = 1.05402 \qquad \zeta_3 = 1.11539$$

The principal moments of inertia are then

$$K_1 = 0.1639\frac{W}{g}a^2 \blacktriangleleft$$

$$K_2 = 1.054\frac{W}{g}a^2 \blacktriangleleft$$

$$K_3 = 1.115\frac{W}{g}a^2 \blacktriangleleft$$

continued

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(*b*) To determine the direction cosines $\lambda_x,\ \lambda_y,\ \lambda_z$ of each principal axis, use two of the equations of Equations (9.54) and (9.57). Then

$\mathbf{K}_1$: Begin with Equations (9.54a) and (9.54b).

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$

$$-I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting

$$\left[\left(\frac{1}{2} - 0.163917\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_1 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_1 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_1 + \left[(1 - 0.163917)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_1 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

Simplifying yields

$$1.34433(\lambda_x)_1 - (\lambda_y)_1 - 1.5(\lambda_z)_1 = 0$$

$$-0.299013(\lambda_x)_1 + (\lambda_y)_1 - 0.149507(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$

$(\lambda_z)_1 = 0.633715(\lambda_x)_1$

and then $\quad (\lambda_y)_1 = [1.34433 - 1.5(0.633715)](\lambda_x)_1$

$= 0.393758(\lambda_x)_1$

Now substitute into Equation (9.57)

$$(\lambda_x)_1^2 + [0.393758(\lambda_x)_1]^2 + [0.633715(\lambda_x)_1]^2 = 1$$

or $\quad (\lambda_x)_1 = 0.801504$

and $\quad (\lambda_y)_1 = 0.315599 \qquad (\lambda_z)_1 = 0.507925$

$\therefore\ (\theta_x)_1 = 36.7° \qquad (\theta_y)_1 = 71.6° \qquad (\theta_z)_1 = 59.5°$ ◀

$\mathbf{K}_2$: Begin with Equations (9.54a) and (9.54b).

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

continued

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Substituting

$$\left[\left(\frac{1}{2}-1.05402\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_2-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_2-\left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_2=0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_2+\left[(1-1.05402)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_2-\left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_2=0$$

Simplifying yields

$$-2.21608(\lambda_x)_2-(\lambda_y)_2-1.5(\lambda_z)_2=0$$

$$4.62792(\lambda_x)_2+(\lambda_y)_2+2.31396(\lambda_z)_2=0$$

Adding and solving for $(\lambda_z)_2$

$(\lambda_z)_2=-2.96309(\lambda_x)_2$

and then $(\lambda_y)_2=\left[-2.21608-1.5(-2.96309)\right](\lambda_x)_2$

$=2.22856(\lambda_x)_2$

Now substitute into Equation (9.57)

$$(\lambda_x)_2^2+\left[2.22856(\lambda_x)_2\right]^2+\left[-2.96309(\lambda_x)_2\right]^2=1$$

or $(\lambda_x)_2=0.260410$

and $(\lambda_y)_2=0.580339 \qquad (\lambda_z)_2=-0.771618$

$$\therefore\ (\theta_x)_2=74.9^\circ \qquad (\theta_y)_2=54.5^\circ \qquad (\theta_z)_2=140.5^\circ \blacktriangleleft$$

$\mathbf{K}_3$: Begin with Equations (9.54a) and (9.54b).

$$(I_x-K_3)(\lambda_x)_3-I_{xy}(\lambda_y)_3-I_{zx}(\lambda_z)_3=0$$

$$-I_{xy}(\lambda_x)_3+(I_y-K_3)(\lambda_y)_3-I_{yz}(\lambda_z)_3=0$$

Substituting

$$\left[\left(\frac{1}{2}-1.11539\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_3-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_3-\left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_3=0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_3+\left[(1-1.11539)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_3-\left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_3=0$$

continued

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Simplifying yields

$$-2.46156(\lambda_x)_3 - (\lambda_y)_3 - 1.5(\lambda_z)_3 = 0$$

$$2.16657(\lambda_x)_3 + (\lambda_y)_3 + 1.08328(\lambda_z)_3 = 0$$

Adding and solving for $(\lambda_z)_3$

$(\lambda_z)_3 = -0.707885(\lambda_x)_3$

and then $(\lambda_y)_3 = \left[-2.46156 - 1.5(-0.707885)\right](\lambda_x)_3$

$= -1.39973(\lambda_x)_3$

Now substitute into Equation (9.57)

$(\lambda_x)_3^2 + \left[-1.39973(\lambda_x)_3\right]^2 + \left[-0.707885(\lambda_x)_3\right]^2 = 1$ (i)

or $(\lambda_x)_3 = 0.537577$

and $(\lambda_y)_3 = -0.752463 \qquad (\lambda_z)_3 = -0.380543$

$\therefore \quad (\theta_x)_3 = 57.5° \qquad (\theta_y)_3 = 138.8° \qquad (\theta_z)_3 = 112.4°$ ◀

(*c*) Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Equation (i) must be chosen; that is, $(\lambda_x)_3 = -0.537577$

Then $(\theta_x)_3 = 122.5° \qquad (\theta_y)_3 = 41.2° \qquad (\theta_z)_3 = 67.6°$

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Chapter 9, Solution 185.

From Problem 9.170 $\qquad m_1 = \rho t a^2 \qquad m_2 = \frac{1}{2}\rho t a^2$

Now
$$I_x = (I_x)_1 + (I_x)_2 = \frac{1}{3}\left(\rho t a^2\right)a^2 + \frac{1}{6}\left(\frac{1}{2}\rho t a^2\right)a^2 = \frac{5}{12}\rho t a^4$$

$$I_y = (I_y)_1 + (I_y)_2 = \frac{1}{3}\left(\rho t a^2\right)a^2 + \frac{1}{6}\left(\frac{1}{2}\rho t a^2\right)\left(a^2 + a^2\right) = \frac{1}{2}\rho t a^4$$

$$I_z = (I_z)_1 + (I_z)_2 = \frac{1}{3}\left(\rho t a^2\right)\left(a^2 + a^2\right) + \frac{1}{6}\left(\frac{1}{2}\rho t a^2\right)a^2 = \frac{3}{4}\rho t a^4$$

Now note that symmetry implies
$$\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{y'z'}\right)_1 = \left(\bar{I}_{z'x'}\right)_1 = 0$$

$$\left(\bar{I}_{x'y'}\right)_2 = \left(\bar{I}_{y'z'}\right)_2 = 0$$

Have
$$I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$$

continued

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Then
$$I_{xy} = m_1\bar{x}_1\bar{y}_1 + m_2\bar{x}_2\bar{y}_2 = \rho ta^2\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{4}\rho ta^4$$

$$I_{yx} = m_1\bar{y}_1\cancelto{0}{\bar{z}_1} + m_2\bar{y}_2\cancelto{0}{\bar{z}_2}$$

$$I_{zx} = m_1\cancelto{0}{\bar{z}_1}\bar{x}_1 + \left[\left(\bar{I}_{z'x'}\right)_2 + m_2\bar{z}_2\bar{x}_2\right]$$

From Problem 9.170
$$\left(\bar{I}_{z'x'}\right)_2 = -\frac{1}{72}\rho ta^4$$

Then
$$I_{zx} = -\frac{1}{72}\rho ta^4 + \left(\frac{1}{2}\rho ta^2\right)\left(\frac{1}{3}a\right)\left(\frac{1}{3}a\right) = \frac{1}{24}\rho ta^4$$

(*a*) Equation 9.56

$$K^3 - \left(I_x + I_y + I_z\right)K^2 + \left(I_xI_y + I_yI_z + I_zI_x\right) - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\Big)K$$
$$-\left(I_xI_yI_z - I_xI_{yz}^2 - I_yI_{zx}^2 - I_zI_{xy}^2 - 2I_{xy}I_{yz}I_{zx}\right. = 0$$

Substituting

$$K^3 - \left[\left(\frac{5}{12} + \frac{1}{2} + \frac{3}{4}\right)\rho ta^4\right]K^2 + \left\{\left[\left(\frac{5}{12}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{5}{12}\right) - \left(\frac{1}{4}\right)^2 - 0 - \left(\frac{1}{24}\right)^2\right]\left(\rho ta^4\right)^2\right\}K$$
$$-\left\{\left[\left(\frac{5}{12}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right) - 0 - \left(\frac{1}{2}\right)\left(\frac{1}{24}\right)^2 - \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 - 0\right]\left(\rho ta^4\right)^3\right\} = 0$$

Simplifying and letting
$$K = \rho ta^4\zeta$$

yields
$$\zeta^3 - \frac{5}{3}\zeta^2 + \frac{479}{576}\zeta - \frac{125}{1152} = 0$$

Solving numerically... $\zeta_1 = 0.203032$ or $K_1 = 0.203\rho ta^4$ ◀

$\zeta_2 = 0.698281$ or $K_2 = 0.698\rho ta^4$ ◀

$\zeta_3 = 0.765354$ or $K_3 = 0.765\rho ta^4$ ◀

(*b*) Equations 9.54a and 9.54b

$$\left(I_x - K\right)\left(\lambda_x\right) - I_{xy}\left(\lambda_y\right) - I_{zx}\left(\lambda_z\right) = 0$$

$$-I_{xy}\left(\lambda_x\right) + \left(I_y - K\right)\left(\lambda_y\right) - I_{yz}\left(\lambda_z\right) = 0$$

continued

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Substituting K_1

$$\left[\left(\frac{5}{12} - 0.203032\right)(\lambda_x)_1 - \frac{1}{4}(\lambda_y)_1 - \frac{1}{24}(\lambda_z)_1\right]\rho ta^4 = 0$$

$$\left[-\frac{1}{4}(\lambda_x)_1 + \left(\frac{1}{2} - 0.203032\right)(\lambda_y)_1 - 0\right]\rho ta^4 = 0$$

or $$(\lambda_y)_1 = 0.841842(\lambda_x)_1$$

and $$(\lambda_z)_1 = 0.0761800(\lambda_x)_1$$

Equation 9.57 $$(\lambda_x)_1^2 + (\lambda_y)_1^2 + (\lambda_z)_1^2 = 1$$

Substituting $$(\lambda_x)_1^2 + \left[0.841842(\lambda_x)_1\right]^2 + \left[0.0761800(\lambda_x)_1\right]^2 = 1$$

or $(\lambda_x)_1 = 0.763715$ then $(\theta_x)_1 = 40.2°$ ◀

$(\lambda_y)_1 = 0.642927$ then $(\theta_y)_1 = 50.0°$ ◀

$(\lambda_z)_1 = 0.0581798$ then $(\theta_z)_1 = 86.7°$ ◀

Substituting K_2

$$\left[\left(\frac{5}{12} - 0.698281\right)(\lambda_x)_2 - \frac{1}{4}(\lambda_y)_2 - \frac{1}{24}(\lambda_z)_2\right]\rho ta^4 = 0$$

$$\left[-\frac{1}{4}(\lambda_x)_2 + \left(\frac{1}{2} - 0.698281\right)(\lambda_y)_2 - 0\right]\rho ta^4 = 0$$

or $$(\lambda_y)_2 = -1.260837(\lambda_x)_2$$

and $$(\lambda_z)_2 = 0.806278(\lambda_x)_2$$

Then $$(\lambda_x)_2^2 + \left[-1.260837(\lambda_x)_2\right]^2 + \left[0.806278(\lambda_x)_2\right]^2 = 1$$

or $(\lambda_x)_2 = 0.555573$ then $(\theta_x)_2 = 56.2°$ ◀

$(\lambda_y)_2 = -0.700487$ then $(\theta_y)_2 = 134.5°$ ◀

$(\lambda_z)_2 = 0.447946$ then $(\theta_z)_2 = 63.4°$ ◀

continued

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Substituting K_3

$$\left[\left(\frac{5}{12} - 0.765354\right)(\lambda_x)_3 - \frac{1}{4}(\lambda_y)_3 - \frac{1}{24}(\lambda_z)_3\right]\rho t a^4 = 0$$

$$\left[-\frac{1}{4}(\lambda_x)_3 + \left(\frac{1}{2} - 0.765354\right)(\lambda_y)_3 - 0\right]\rho t a^4 = 0$$

or $(\lambda_y)_3 = -0.942138(\lambda_x)_3$

And $$(\lambda_z)_3 = -2.71567(\lambda_x)_3$$

Then $$(\lambda_x)_3^2 + \left[-0.942138(\lambda_x)_3\right]^2 + \left[-2.71567(\lambda_x)_3\right]^2 = 1$$

or $(\lambda_x)_3 = 0.328576$ then $(\theta_x)_3 = 70.8°$ ◀

$(\lambda_y)_3 = -0.309564$ then $(\theta_y)_3 = 108.0°$ ◀

$(\lambda_z)_3 = -0.892304$ then $(\theta_z)_3 = 153.2°$ ◀

(c)

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Chapter 9, Solution 186.

(*a*) From the solutions to Problem 9.150

$I_x = 14.32 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$I_y = I_z = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

From Problem 9.172 $\quad I_{xy} = I_{zx} = 4.297 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2,\ I_{yz} = 0$

Substituting into Eq. (9.56) and using $\quad I_y = I_z, \quad I_{xy} = I_{zx}, \quad I_{yz} = 0$

$$K^3 - \left(I_x + 2I_y\right)K^2 + \left[I_x\left(2I_y\right) + I_y^2 - 2\left(I_{xy}\right)^2\right]K - \left(I_x I_y^2 - 2I_y I_{xy}^2\right) = 0$$

$$K^3 - \left[14.32 \times 10^{-3} + 2\left(18.62 \times 10^{-2}\right)\right]K^2 + \left[\left(14.32 \times 10^{-3}\right)(2)\left(18.62 \times 10^{-3}\right) + \left(18.62 \times 10^{-3}\right)^2\right.$$

$$\left. -2\left(4.297 \times 10^{-3}\right)^2\right]K - \left[\left(14.32 \times 10^{-3}\right)\left(18.62 \times 10^{-3}\right)^2 - 2\left(18.62 \times 10^{-3}\right)\left(4.297 \times 10^{-3}\right)^2\right] = 0$$

or $$K^3 - 51.56 \times 10^{-3}K^2 + 0.84305 \times 10^{-3}K - 0.004277 \times 10^{-3} = 0$$

Solving: $K_1 = 0.010022\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ or $K_1 = 10.02 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$K_2 = 0.018624\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ or $K_2 = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$K_3 = 0.022914\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ or $K_3 = 22.9 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*b*) To determine the direction cosines $\lambda_x,\ \lambda_y,\ \lambda_z$ of each principal axis, use two of the equations of Equations (9.54) and Equation (9.57). Then

$\mathbf{K}_1$: Begin with Equations (9.54b) and (9.54c):

$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - \cancelto{0}{I_{yz}}(\lambda_z)_1 = 0$$

$$-I_{zx}(\lambda_x)_1 - I_{yz}(\lambda_y)_1 + (I_z - K_1)(\lambda_z)_1 = 0$$

or $$-4.297 \times 10^{-3}(\lambda_x)_1 + \left(18.62 \times 10^{-3} - 10.02 \times 10^{-3}\right)(\lambda_y)_1 = 0$$

$$-4.297 \times 10^{-3}(\lambda_x)_1 + \left(18.62 \times 10^{-3} - 10.02 \times 10^{-3}\right)(\lambda_z)_1 = 0$$

continued

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$$\left(\lambda_y\right)_1 = \left(\lambda_z\right)_1 = 0.49965\left(\lambda_x\right)_1$$

$$\left(\lambda_x\right)_1^2 + 2\left[0.49965\left(\lambda_x\right)_1\right]^2 = 1$$

$$\left(\lambda_x\right)_1 = 0.81669$$

$$\left(\lambda_y\right)_1 = \left(\lambda_z\right)_1 = 0.40806$$

$$\left(\theta_x\right)_1 = 35.2°; \quad \left(\theta_y\right)_1 = \left(\theta_z\right)_1 = 65.9° \blacktriangleleft$$

K$_2$: Begin with Equations (9.54a) and (9.54b):

$$\left(I_x - K_2\right)\left(\lambda_x\right)_2 - I_{xy}\left(\lambda_y\right)_2 - I_{zx}\left(\lambda_z\right)_2 = 0$$

$$-I_{xy}\left(\lambda_x\right)_2 + \left(I_y - K_2\right)\left(\lambda_y\right)_2 - \cancelto{0}{I_{yz}}\left(\lambda_z\right)_2 = 0$$

Substituting:

$$\left(14.32\times10^{-3} - 18.62\times10^{-3}\right)\left(\lambda_x\right)_2 - 4.297\times10^{-3}\left[\left(\lambda_y\right)_2 - \left(\lambda_z\right)_2\right] = 0 \qquad \text{(i)}$$

$$-4.297\times10^{-3}\left(\lambda_x\right)_2 + \left(18.62\times10^{-3} - 18.62\times10^{-3}\right)\left(\lambda_y\right)_2 = 0 \qquad \text{(ii)}$$

From (ii) $\quad \left(\lambda_x\right)_2 = 0$

From (i) $\quad \left(\lambda_y\right)_2 = -\left(\lambda_z\right)_2$

Substituting: $\quad \left(\lambda_x\right)_2^2 + \left(\lambda_y\right)_2^2 + \left[-\left(\lambda_z\right)_2\right]^2 = 1$

$$\left(\lambda_y\right)_2 = \frac{1}{\sqrt{2}}$$

$$\left(\theta_x\right)_2 = 90.0°, \left(\theta_y\right)_2 = 45.0°, \left(\theta_z\right)_2 = 135.0° \blacktriangleleft$$

K$_3$: Begin with Equations (9.54b) and (9.54c)

$$-I_{xy}\left(\lambda_x\right)_3 + \left(I_y - K_3\right)\left(\lambda_y\right)_3 + I_{yz}\left(\lambda_z\right)_3 = 0$$

$$-I_{zx}\left(\lambda_x\right)_3 - \cancelto{0}{I_{yz}}\left(\lambda_y\right)_3 + \left(I_z - K_3\right)\left(\lambda_z\right)_3 = 0$$

Substituting:

$$-4.297\times10^{-3}\left(\lambda_x\right)_3 + \left(18.62\times10^{-3} - 22.9\times10^{-3}\right)\left(\lambda_z\right)_3 = 0$$

$$-4.297\times10^{-3}\left(\lambda_x\right)_3 + \left(18.62\times10^{-3} - 22.9\times10^{-3}\right)\left(\lambda_z\right)_3 = 0$$

Simplifying: $\quad \left(\lambda_y\right)_3 = \left(\lambda_z\right)_3 = -\left(\lambda_x\right)_3$

$$\left(\lambda_x\right)_3^2 + 2\left[-\left(\lambda_x\right)_3\right]^2 = 1 \Rightarrow \left(\lambda_x\right)_3 = \frac{1}{\sqrt{3}} \qquad \text{and} \qquad \left(\lambda_y\right)_3 = \left(\lambda_z\right)_3 = -\frac{1}{\sqrt{3}}$$

$$\left(\theta_x\right)_3 = 54.7°, \left(\theta_y\right)_3 = \left(\theta_z\right)_3 = 125.3° \blacktriangleleft$$

continued

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Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set to obtain the direction cosines corresponding to the labeled axis, the negative root of Equation (i) must be chosen; that is:

$$(\lambda_x)_3 = -\frac{1}{\sqrt{3}}$$

Then: $(\theta_x)_3 = 125.3°$ $(\theta_y)_3 = (\theta_z)_3 = 54.7°$ ◀

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Chapter 9, Solution 187.

At $\quad x = 0,\ y = 0: \quad 0 = ka^2 + c$

$$\text{or} \quad k = -\frac{c}{a^2}$$

$$x = a, \quad y = b: \quad b = c$$

$$\therefore k = -\frac{b}{a^2}$$

$$y = -\frac{b}{a^2}(x-a)^2 + b$$

$$= -\frac{b}{a^2}\left(x^2 - 2ax + a^2\right) + b$$

Now $\quad dI_y = x^2 dA = x^2\left(y_2 - y_1\right)dx$

$$= x^2\left(b + \frac{b}{a^2}x^2 - \frac{2b}{a}x + b - b\right)dx$$

$$= \left(\frac{b}{a^2}x^4 - \frac{2b}{a}x^3 + bx^2\right)dx$$

$$I_y = \int_0^a \left(\frac{b}{a^2}x^4 - \frac{2b}{a}x^3 + bx^2\right)dx$$

$$= \left(\frac{1}{5}\frac{b}{a^2}x^5 - \frac{b}{2a}x^4 + \frac{b}{3}x^3\right)\Bigg|_0^a$$

$$= a^3 b\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right)$$

$$= \frac{1}{30}a^3 b$$

$$I_y = \frac{1}{30}a^3 b \ ◀$$

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Chapter 9, Solution 188.

At $\quad x = 0,\ y = 0:\ \ 0 = ka^2 + c$

$$k = -\frac{c}{a^2}$$

$$x = a,\ \ y = b:\quad b = c$$

$$k = -\frac{b}{a^2}$$

Then $\quad y = b - \frac{b}{a^2}(x - a)^2$

Now $\quad dI_x = y^2 dA = y^2(xdy)$

From above $\quad (x - a)^2 = \frac{a^2}{b}(b - y)$

Then $\quad x - a = a\sqrt{1 - \frac{y}{b}}$

and $\quad x = a\sqrt{1 - \frac{y}{b}} + a$

Then $\quad dI_x = ay^2\left(1 + \sqrt{1 - \frac{y}{b}}\right)dy$

and $\quad I_x = \int dI_x = a\int_0^b y^2\left(1 + \sqrt{1 - \frac{y}{b}}\right)dy$

$$= a\frac{y^3}{3}\bigg|_0^b + a\int_0^b y^2\left(\sqrt{1 - \frac{y}{b}}\right)dy$$

continued

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Chapter 9, Solution 189.

First note that $A = A_1 - A_2 - A_3$

$$= (100\text{ mm})(120\text{ mm}) - (80\text{ mm})(40\text{ mm}) - (80\text{ mm})(20\text{ mm})$$

$$= (12\,000 - 3200 - 1600)\text{ mm}^2 = 7200\text{ mm}^2$$

Now $$I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$

where $$(I_x)_1 = \frac{1}{12}(100\text{ mm})(120\text{ mm})^3 = 14.4 \times 10^6\text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(80\text{ mm})(40\text{ mm})^3 + (3200\text{ mm}^2)(40\text{ mm})^2 = 5.5467 \times 10^6\text{ mm}^4$$

$$(I_x)_3 = \frac{1}{12}(80\text{ mm})(20\text{ mm})^2 + (1600\text{ mm}^2)(30\text{ mm})^2 = 1.4933 \times 10^6\text{ mm}^4$$

Then $$I_x = (14.4 - 5.5467 - 1.4933) \times 10^6\text{ mm}^4 = 7.36 \times 10^6\text{ mm}^4$$

or $I_x = 7.36 \times 10^6\text{ mm}^4$ ◀

and $$k_x^2 = \frac{I_x}{A} = \frac{7.36 \times 10^6}{7200} = 1022.2\text{ mm}^2$$

or $k_x = 32.0\text{ mm}$ ◀

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Chapter 9, Solution 190.

First note that $A = A_1 - A_2 - A_3$

$$= (100\text{ mm})(120\text{ mm}) - (80\text{ mm})(40\text{ mm}) - (80\text{ mm})(20\text{ mm})$$

$$= (12\,000 - 3200 - 1600)\text{mm}^2 = 7200\text{ mm}^2$$

Now $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

where $(I_y)_1 = \frac{1}{12}(120\text{ mm})(100\text{ mm})^3 = 10 \times 10^6\text{ mm}^4$

$$(I_y)_2 = \frac{1}{12}(40\text{ mm})(80\text{ mm})^3 = 1.7067 \times 10^6\text{ mm}^4$$

$$(I_y)_3 = \frac{1}{12}(20\text{ mm})(80\text{ mm})^3 = 0.8533 \times 10^6\text{ mm}^4$$

Then $I_y = (10 - 1.7067 - 0.8533) \times 10^6\text{ mm}^4 = 7.44 \times 10^6\text{ mm}^4$

or $I_y = 7.44 \times 10^6\text{ mm}^4$ ◀

And $k_y^2 = \frac{I_y}{A} = \frac{7.44 \times 10^6\text{mm}^4}{7200\text{ mm}^2} = 1033.33\text{ mm}^2$

$$k = 32.14550\text{ mm}$$

or $k_y = 32.1\text{ mm}$ ◀

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Chapter 9, Solution 191.

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{2}(120)^2 = 22\ 619.5$	50.9296	1.1520×10^6
2	$-\frac{1}{2}(240)(90) = -10\ 800$	30	-0.324×10^6
Σ	11 819.5		0.828×10^6

Now $$\bar{Y} = \frac{\Sigma A\bar{Y}}{\Sigma A} = \frac{0.828 \times 10^6 \text{ mm}^3}{11819.5 \text{ mm}^2} = 70.054 \text{ mm}$$

(a) $$J_O = (J_O)_1 - (J_O)_2$$

where $$(J_O)_1 = \frac{\pi}{4}(120 \text{ mm}^4) = 162.86 \times 10^6 \text{ mm}^4$$

and $$(J_O)_2 = (I_{x'})_2 + (I_{y'})_2 = \frac{1}{12}(240 \text{ mm})(90 \text{ mm})^3 + 2\left[\frac{1}{12}(90 \text{ mm})(120 \text{ mm})^3\right]$$

$$= 40.5 \times 10^6 \text{ mm}^4$$

Then $$J_O = (162.86 - 40.5) \times 10^6 \text{ mm}^4 = 122.36 \times 10^6 \text{ mm}^4$$

or $J_O = 122.4 \times 10^6 \text{ mm}^4$ ◀

(b) $$J_O = \bar{J}_C + A\bar{y}^2$$

or $$\bar{J}_C = 122.36 \times 10^6 \text{ mm}^4 - (11\ 819.5 \text{ mm}^2)(70.054 \text{ mm})^2$$

$$= (122.36 - 58.005)10^6 \text{ mm}^4$$

or $\bar{J}_C = 64.4 \times 10^6 \text{ mm}^4$ ◀

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Chapter 9, Solution 192.

W section (fig. 9.13A):

$$A = 7.08 \text{ in}^2$$

$$\bar{I}_x = 18.3 \text{ in}^4$$

$$\bar{I}_y = 82.8 \text{ in}^4$$

$$A = 2A_{\text{W}} + 2A_{\text{plate}}$$

$$= 2\left[7.08 \text{ in}^2 + (7.93 \text{ in.})(0.3 \text{ in.})\right]$$

$$= 18.918 \text{ in}^2$$

Now

$$\bar{I}_x = 2\left(\bar{I}_x\right)_{\text{W}} + 2\left(\bar{I}_x\right)_{\text{plate}}$$

$$= 2\left[18.3 \text{ in}^4 + \left(7.08 \text{ in}^2\right)\left(\frac{6.495 \text{ in.}}{2}\right)^2\right]$$

$$+2\left\{\frac{(7.93 \text{ in.})(0.3 \text{ in.})^3}{12} + \left[(7.93 \text{ in.})(0.3 \text{ in.})\right](6.495 \text{ in.} + 0.15 \text{ in.})^2\right\}$$

$$= 2\left[92.967 \text{ in}^4\right] + 2\left[105.07 \text{ in}^4\right] = 396.07 \text{ in}^4$$

or $\bar{I}_x = 396 \text{ in}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A} = \dfrac{396.07 \text{ in}^4}{18.918 \text{ in}^2} = 20.936 \text{ in}^2$

or $\bar{k}_x = 4.58 \text{ in.}$ ◀

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Chapter 9, Solution 193.

Angle: $A = 1.44 \text{ in}^2$

$$\bar{I}_x = \bar{I}_y = 1.24 \text{ in}^4$$

Channel: $A = 5.88 \text{ in}^2$

$$\bar{I}_x = 2.81 \text{ in}^4 \qquad \bar{I}_y = 78.9 \text{ in}^4$$

Locate the centroid

$$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2\left[\left(1.44 \text{ in}^2\right)\left(0.842 \text{ in.}\right)\right] + \left(5.88 \text{ in}^2\right)\left(-0.606 \text{ in.}\right)}{2\left(1.44 \text{ in}^2\right) + 5.88 \text{ in}^2}$$

$$= \frac{\left(2.42496 - 3.5638\right) \text{in}^3}{8.765 \text{ in}^4} = -0.12995 \text{ in.}$$

Now
$$\left(\bar{I}_x\right) = 2\left(I_x\right)_{\text{L}} + \left(I_x\right)_{\text{C}} = 2\left[1.24 \text{ in}^4 + \left(1.44 \text{ in}^2\right)\left(0.842 \text{ in.} + 0.12995 \text{ in.}\right)^2\right]$$

$$+ \left[2.81 \text{ in}^4 + \left(5.88 \text{ in}^2\right)\left(0.606 \text{ in.} - 0.12995 \text{ in.}\right)^2\right]$$

$$= 2\left(2.6003\right) \text{in}^4 + 4.1425 \text{ in}^4 = 9.3431 \text{ in}^4$$

or $\bar{I}_x = 9.34 \text{ in}^4$ ◀

Also
$$\left(\bar{I}_y\right) = 2\left(I_y\right)_{\text{L}} + \left(\bar{I}_y\right)_{\text{C}} = 2\left[1.24 \text{ in}^4 + 1.44 \text{ in}^2\left(5 \text{ in.} - 0.842 \text{ in.}\right)^2\right] + 7.89 \text{ in}^4$$

$$= 2\left(26.136\right) \text{in}^4 + 78.9 \text{ in}^4 = 131.17 \text{ in}^4$$

or $\bar{I}_y = 131.2 \text{ in}^4$ ◀

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Chapter 9, Solution 194.

(*a*) Have $I_{AA'} = \bar{I}_{GG'} + mr_a^2 \qquad r_a + r_b = 290 \text{ mm}$

$= 0.29 \text{ m}$

and $I_{BB'} = \bar{I}_{GG'} + mr_b^2$

Subtracting $I_{BB'} - I_{AA'} = m\left(r_b^2 - r_a^2\right)$

$$\left(41 - 78\right)\text{g} \cdot \text{m}^2 = (2000 \text{ g})\left(r_b + r_a\right)\left(r_b - r_a\right)$$

or $-37 = (2000)(0.29)\left(r_b - r_a\right)$

or $r_a - r_b = 63.793 \times 10^{-3} \text{ m}$

now $r_a + r_b = 0.29 \text{ m}$

so that $2r_a = 0.35379 \text{ m}$

$r_a = 0.17689 \text{ m}$

or $r_a = 176.9 \text{ mm}$ ◀

(*b*) Have $I_{AA'} = \bar{I}_{GG'} + mr_a^2$

Then $\bar{I}_{GG'} = 78 \text{ g} \cdot \text{m}^2 - (2000 \text{ g})(0.17689 \text{ m})^2$

$= 15.420 \text{ g} \cdot \text{m}^2$

Finally, $\bar{k}_{GG'}^2 = \dfrac{\bar{I}_{GG'}}{m} = \dfrac{15.420 \text{ g} \cdot \text{m}^2}{2000 \text{ g}} = 0.007710 \text{ m}^2$

$\bar{k}_{GG'} = 0.08781 \text{ m}$

$\bar{k}_{GG'} = 87.8 \text{ mm}$ ◀

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Chapter 9, Solution 195.

Have $\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(\bar{I}_{xy}\right)_2 + \left(I_{xy}\right)_3$

Symmetry implies $\left(I_{xy}\right)_2 = 0$

For the other rectangles $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

Where symmetry implies $I_{x'y'} = 0$

	A in^2	$\bar{x}$, in.	$\bar{y}$, in.	$A\bar{x}\,\bar{y}$ in^4
1	$4(0.5) = 2$	-2.75	1.0	-5.5
3	$4(0.5) = 2$	2.75	-1.0	-5.5
Σ				-11.00

or $\bar{I}_{xy} = -11.00 \text{ in}^4$ ◀

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Chapter 9, Solution 196.

From Problem 9.195 $\qquad \bar{I}_{xy} = -11.0 \text{ in}^4$

Compute $\bar{I}_x$ and $\bar{I}_y$ for area of Problem 9.195

$$\bar{I}_x = \frac{5 \text{ in.} \times (0.5 \text{ in.})^3}{12} + 2\left[\frac{(0.5 \text{ in.})(4 \text{ in.})^3}{12} + (4 \text{ in.} \times 0.5 \text{ in.})(1.0 \text{ in.})^2\right]$$

$$= 9.38542 \text{ in}^4$$

$$\bar{I}_y = 2\left[\frac{(0.5 \text{ in.})^3 (4 \text{ in.})}{12} + (4 \text{ in.} \times 0.5 \text{ in.})(2.75 \text{ in.})^2\right] + \frac{0.5 \text{ in.} \times (5 \text{ in.})^3}{12}$$

$$= 35.54167 \text{ in}^4$$

Define points $\qquad X(9.38542,\ -11), \qquad$ and $\qquad Y(35.54167,\ 11)$

Now $\qquad I_{\text{ave}} = \dfrac{\bar{I}_x + \bar{I}_y}{2} = \dfrac{9.38542 \text{ in}^4 + 35.54167 \text{ in}^4}{2} = 22.46354 \text{ in}^4$

and $\qquad R = \sqrt{\left(\dfrac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2} = \sqrt{\left(\dfrac{9.38542 - 35.54167}{2}\right)^2 + (11.0)^2}$

$$= 17.08910 \text{ in}^4$$

Also $\qquad 2\theta_m = \tan^{-1}\left[\dfrac{-2(-11.0)}{9.38542 - 35.54167}\right] = -40.067 \qquad$ or $\theta_m = -20.0°$ clockwise ◀

Then $\qquad \bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 22.46354 \pm 17.08910$

$$= 39.55264,\ 5.37444 \qquad \text{or } \bar{I}_{\max} = 39.6 \text{ in}^4 \ ◀$$

$$\bar{I}_{\min} = 5.37 \text{ in}^4 \ ◀$$

Note: The *a* axis corresponds to $\bar{I}_{\min}$ and *b* axis corresponds to $\bar{I}_{\max}$.

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Chapter 9, Solution 197.

For the cylinder $m = \rho V = \rho \pi a^2 L$

For the element shown $dm = \rho \pi a^2 dx$

$$= \frac{m}{L} dx$$

and $dI_z = d\bar{I}_z + x^2 dm$

$$= \frac{1}{4} a^2 dm + x^2 dm$$

Then $I_z = \int dI_z = \int_0^L \left(\frac{1}{4} a^2 + x^2 \right) \left(\frac{m}{L} dx \right) = \frac{m}{L} \left[\frac{1}{4} a^2 x + \frac{1}{3} x^3 \right]_0^L$

$$= \frac{m}{L} \left(\frac{1}{4} a^2 L + \frac{1}{3} L^3 \right)$$

or $I_z = \frac{1}{12} m \left(3a^2 + 4L^2 \right)$ ◀

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Chapter 9, Solution 198.

First compute the mass of each component $m = \dfrac{\gamma}{g}V$

Then
$$m_1 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}(5\ \text{in.} \times 4.5\ \text{in.} \times 0.9\ \text{in.}) = 0.1786\ \text{lb}\cdot\text{s}^2/\text{ft}$$
$$m_2 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}(3\ \text{in.} \times 2.5\ \text{in.} \times 0.8\ \text{in.}) = 0.05292\ \text{lb}\cdot\text{s}^2/\text{ft}$$
$$m_3 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}\left[\pi(0.6\ \text{in.})^2 \times 0.5\ \text{in.}\right] = 0.0049875\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because

of symmetry. Now $I_{uv} = \cancelto{0}{I_{u'v'}} + m\bar{u}\bar{v}$ so that $(I_{uv})_{\text{body}} = \Sigma m\bar{u}\,\bar{v}$.

	m, lb·s²/ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$ lb·ft·s²	$m\bar{y}\,\bar{z}$ lb·ft·s²	$m\bar{z}\,\bar{x}$ lb·ft·s²
1	0.1786	0.20833	0.0375	0.1875	1.39531×10^{-3}	1.25578×10^{-3}	6.97656×10^{-3}
2	0.05292	0.38333	0.20	0.1875	4.0572×10^{-3}	1.98451×10^{-3}	3.80362×10^{-3}
3	0.0049875	0.4375	0.225	0.1875	0.49095×10^{-3}	0.21041×10^{-3}	0.40913×10^{-3}
Σ					5.94347×10^{-3}	3.45069×10^{-3}	11.18909×10^{-3}

Then

or $I_{xy} = 5.94 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

or $I_{yz} = 3.45 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

or $I_{zx} = 11.19 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

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