Chapter 7, Solution 1.

FBD FRAME:

$\circlearrowleft \Sigma M_A = 0$:

$$(0.3\text{ m})D_x - (0.4\text{ m})(900\text{ N}) = 0$$

$$\therefore \mathbf{D}_x = 1200\text{ N} \longrightarrow$$

FBD JEHDF:

$\longrightarrow \Sigma F_x = 0$: $\quad 1200\text{ N} + V = 0$

$$V = -1200\text{ N}$$

$\uparrow \Sigma F_y = 0$: $\quad F = 0$

$\circlearrowleft \Sigma M_J = 0$: $\quad (0.15\text{ m})(1200\text{ N}) - M = 0$

$$M = +180\text{ N}\cdot\text{m}$$

Thus, $\quad$ (on JE) $\quad \mathbf{F} = 0$ ◀

$\mathbf{V} = 1200\text{ N} \longleftarrow$ ◀

$\mathbf{M} = 180.0\text{ N}\cdot\text{m} \circlearrowright$ ◀

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Chapter 7, Solution 2.

FBD FRAME:

$\circlearrowleft \Sigma M_A = 0: \quad (0.95\text{ m})(480\text{ N}) - (0.25\text{ m})D_x = 0,$

$\mathbf{D}_x = 1824\text{ N} \leftarrow$

$\uparrow \Sigma F_y = 0: \quad A_y + D_y + 480\text{ N} = 0 \qquad (1)$

FBD ABC:

Note: BE is a two-force member

$\circlearrowleft \Sigma M_B = 0: \quad (0.75\text{ m})A_y = 0 \qquad \mathbf{A}_y = 0$

Then, from (1) above, $\quad D_y = -480\text{ N}$

$\mathbf{D}_y = 480\text{ N} \downarrow$

$\rightarrow \Sigma F_x = 0: \quad F - 1824\text{ N} = 0 \qquad \mathbf{F} = 1.824\text{ kN} \rightarrow \blacktriangleleft$

FBD sect. DJ:

$\uparrow \Sigma F_y = 0: \quad -480\text{ N} - V = 0 \qquad V = -480\text{ N}$

$\mathbf{V} = 480\text{ N} \uparrow \blacktriangleleft$

$\circlearrowleft \Sigma M_J = 0: \quad M + (0.25\text{ m})(480\text{ N}) = 0$

$M = -120\text{ N}\cdot\text{m} \qquad \mathbf{M} = 120.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$

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Chapter 7, Solution 3.

FBD CEF:

$\rightarrow \Sigma F_x = 0: \quad C_x - \frac{1}{\sqrt{2}}(180 \text{ N}) \qquad \mathbf{C}_x = 90\sqrt{2} \text{ N} \rightarrow$

$\circlearrowleft \Sigma M_E = 0: \quad (0.2 \text{ m})C_y - (0.05 \text{ m})(90\sqrt{2} \text{ N})$

$-(0.15 \text{ m} + 0.08 \text{ m})(180 \text{ N}) = 0$

$\mathbf{C}_y = 126\sqrt{2} \text{ N} \downarrow$

FBD sect. CJ:

$\rightarrow \Sigma F_x = 0: \quad 90\sqrt{2} \text{ N} - F = 0 \qquad \mathbf{F} = 127.3 \text{ N} \leftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad -126\sqrt{2} \text{ N} + V = 0 \qquad \mathbf{V} = 178.2 \text{ N} \uparrow$ ◀

$\circlearrowright \Sigma M_J = 0: \quad (0.05 \text{ m})(90\sqrt{2} \text{ N}) - (0.10 \text{ m})(126\sqrt{2} \text{ N}) - M = 0$

$\mathbf{M} = 11.46 \text{ N}\cdot\text{m} \circlearrowright$ ◀

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Chapter 7, Solution 4.

FBD Frame:

Note: AC is a two-force member, resolve F_{AC} at C:

$$\Sigma M_E = 0:\ (0.25\text{ m} + 0.25\text{ m})\frac{1}{\sqrt{2}}F_{AC} - (0.15\text{ m})(320\text{ N}) = 0$$

$$\mathbf{F}_{AC} = 96\sqrt{2}\text{ N}$$

FBD sect. AB:

$$\rightarrow \Sigma F_x = 0:\quad -96\text{ N} + \frac{3}{4}V + \frac{\sqrt{7}}{4}F = 0$$

$$\uparrow \Sigma F_y = 0:\ 96\text{ N} + \frac{\sqrt{7}}{4}V - \frac{3}{4}F = 0$$

Solving: $\mathbf{V} = 8.50\text{ N}\ \measuredangle\ 41.4°$ ◀

$\mathbf{F} = 135.5\text{ N}\ \measuredangle\ 46.8°$ ◀

$$\circlearrowleft \Sigma M_B = 0:\quad M - (0.3\text{ m})(96\text{ N}) + \left(\frac{4-\sqrt{7}}{10}\text{m}\right)(96\text{ N}) = 0$$

$M = 15.799\text{ N}\cdot\text{m}$, $\mathbf{M} = 15.80\text{ N}\cdot\text{m}\ \circlearrowright$ ◀

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Chapter 7, Solution 5.

FBD Frame:

Note: AB is a two-force member, so

$$\frac{A_x}{12} = \frac{A_y}{5} \qquad (1)$$

$\circlearrowleft \Sigma M_C = 0$: $\quad (15 \text{ in.})A_x - (24 \text{ in.})(78 \text{ lb}) = 0$

$\mathbf{A}_x = 124.8 \text{ lb} \leftarrow$

From (1) above, $\quad \mathbf{A}_y = 52.0 \text{ lb} \uparrow$

FBD AJ:

$\rightarrow \Sigma F_x = 0$: $\quad -124.8 \text{ lb} + F = 0 \qquad \mathbf{F} = 124.8 \text{ lb} \rightarrow$ ◀

$\uparrow \Sigma F_y = 0$: $\quad 52 \text{ lb} - V = 0 \qquad \mathbf{V} = 52.0 \text{ lb} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0$: $\quad M - (10 \text{ in.})(52 \text{ lb}) = 0 \qquad \mathbf{M} = 520 \text{ lb}\cdot\text{in.} \circlearrowright$ ◀

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Chapter 7, Solution 6.

Note: AB is a two-force member

FBD CD:

$$\circlearrowleft \Sigma M_C = 0: \quad (18\text{ in.})\left(\frac{5}{13}F_{AB}\right) + (7.5\text{ in.})\left(\frac{12}{13}F_{AB}\right) - (24\text{ in.})(78\text{ lb}) = 0$$

$$F_{AB} = 135.2\text{ lb}$$

$$\rightarrow \Sigma F_x = 0: \quad C_x - \frac{12}{13}(135.2\text{ lb}) = 0$$

$$\mathbf{C}_x = 124.8\text{ lb} \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y + \frac{5}{13}(135.2\text{ lb}) - (78\text{ lb}) = 0$$

$$\mathbf{C}_y = 26\text{ lb} \uparrow$$

FBD CK:

$$\nearrow \Sigma F_{x'} = 0: \quad -F + \frac{12}{13}(124.8\text{ lb}) + \frac{5}{13}(26\text{ lb}) = 0$$

$$\mathbf{F} = 125.2\text{ lb} \measuredangle 22.6° \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \quad V - \frac{5}{13}(124.8\text{ lb}) + \frac{12}{13}(26\text{ lb}) = 0$$

$$\mathbf{V} = 24.0\text{ lb} \measuredangle 67.4° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad (5\text{ in.})(124.8\text{ lb}) - (12\text{ in.})(26\text{ lb}) - M = 0$$

$$\mathbf{M} = 312\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 7.

FBD half-section:

By symmetry, $A_y = B_y = \dfrac{W}{2}$

Where $W = (9\text{ kg})(9.81\text{ N/kg}) = 88.29\text{ N}$

FBD AJ:

$\rightarrow \Sigma F_x = 0: \quad F = 0$ $\qquad$ $\mathbf{F} = 0$ ◀

$\uparrow \Sigma F_y = 0: \quad \dfrac{W}{2} - \dfrac{W}{2} - V = 0$ $\qquad$ $\mathbf{V} = 0$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad M - (r - \bar{x})\dfrac{W}{2} = 0$

but $\quad \bar{x} = \dfrac{2r}{\pi}$, so $M = \left(1 - \dfrac{2}{\pi}\right) r \dfrac{W}{2} = 2.406\text{ N}\cdot\text{m}$

$\mathbf{M} = 2.41\text{ N}\cdot\text{m}$ $\circlearrowleft$ ◀

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Chapter 7, Solution 8.

FBD AJ:

Note: Cut is just left of contact with ground.

Also $W = (9\text{ kg})(9.81\text{ N/kg}) = 88.29\text{ N}$

$$\bar{x} = \frac{2r}{\pi} \quad \text{and} \quad r = 0.15\text{ m}$$

$\rightarrow \Sigma F_x = 0: \quad F = 0$ $\qquad$ $\mathbf{F} = 0$ ◀

$\uparrow \Sigma F_y = 0: \quad -\frac{W}{2} + V = 0$ $\qquad$ $\mathbf{V} = 44.1\text{ N}\uparrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad \bar{x}\frac{W}{2} - M = 0$

$$M = \frac{2}{\pi}(0.15\text{ m})\left(\frac{88.29\text{ N}}{2}\right)$$ $\qquad$ $\mathbf{M} = 4.22\text{ N}\cdot\text{m}\circlearrowright$ ◀

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Chapter 7, Solution 9.

SOLUTION

FBD AB:

$\circlearrowleft \Sigma M_A = 0: \quad r\left(B_y - 12 \text{ lb}\right) = 0 \qquad \mathbf{B}_y = 12 \text{ lb} \uparrow$

FBD sect. BJ:

$\nwarrow \Sigma F_{y'} = 0: \quad (12 \text{ lb})\cos 30° - (12 \text{ lb})\sin 30° - F = 0$

$\mathbf{F} = 4.39 \text{ lb} \searrow 60°$ ◀

$\nearrow \Sigma F_{x'} = 0: \quad (12 \text{ lb})\cos 30° + (12 \text{ lb})\sin 30° - V = 0$

$\mathbf{V} = 16.39 \text{ lb} \nearrow 30°$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad (4 \text{ in.})\sin 30°(12 \text{ lb}) + (4 \text{ in.})(1 - \cos 30°)(12 \text{ lb}) - M = 0$

$\mathbf{M} = 30.4 \text{ lb}\cdot\text{in.} \circlearrowright$ ◀

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Chapter 7, Solution 10.

FBD AB:

$\circlearrowleft \Sigma M_A = 0: \quad r\left(B_y - 12 \text{ lb}\right) = 0 \qquad \mathbf{B}_y = 12 \text{ lb} \uparrow$

$\Sigma M_J = 0: \quad (4 \text{ in.})\sin\theta(12 \text{ lb}) + (4 \text{ in.})(1 - \cos\theta)(12 \text{ lb}) - M = 0$

$M = (48 \text{ lb}\cdot\text{in.})(1 - \cos\theta + \sin\theta) \qquad (1)$

FBD BJ:

(*a*) to maximize *M*, set $\dfrac{dM}{d\theta} = 0$

$\dfrac{dM}{d\theta} = (48 \text{ lb}\cdot\text{in.})(\sin\theta + \cos\theta) = 0$

so $\tan\theta = -1$

$\theta = -45°,\ 135°$

(only $\theta = 135°$ is on ring) $\qquad \theta = 135°$ ◀

(*b*) $\nwarrow \Sigma F_{y'} = 0: \quad F + (12 \text{ lb})\cos\theta - (12 \text{ lb})\sin\theta = 0 \qquad \theta = 135°$, so

$F = 16.97 \text{ lb}@135° \qquad \mathbf{F} = 16.97 \text{ lb} \swarrow 45°$ ◀

$\nearrow \Sigma F_{x'} = 0: \quad (12 \text{ lb})\cos\theta + (12 \text{ lb})\sin\theta - V = 0 \qquad \theta = 135°,\ \mathbf{V} = 0$ ◀

From (1) above, $M = (48 \text{ lb}\cdot\text{in.})(1 - \cos 135° + \sin 135°) = 115.88 \text{ lb}\cdot\text{in.}$

$\mathbf{M} = 115.9 \text{ lb}\cdot\text{in.} \circlearrowright$ ◀

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Chapter 7, Solution 11.

FBD Frame:

$$\circlearrowleft \Sigma M_A = 0: \quad (16.4 \text{ in.})\frac{24}{25}F_{EC} - (12.6 \text{ in.})(120 \text{ lb}) = 0$$

$$F_{EC} = 93.75 \text{ lb}$$

FBD CBD:

$$\circlearrowleft \Sigma M_C = 0: \quad (16.2 \text{ in.})(120 \text{ lb}) - (13.5 \text{ in.})B_y = 0$$

$$B_y = 144 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad 144 \text{ lb} + \frac{7}{25}(93.75 \text{ lb}) - 120 \text{ lb} - C_y = 0$$

$$C_y = 50.25 \text{ lb}$$

$$\rightarrow \Sigma F_x = 0: \quad C_x - \frac{24}{27}(93.75 \text{ lb}) = 0$$

$$C_x = 90.0 \text{ lb}$$

FBD CJ:

$$\nearrow \Sigma F_{y'} = 0: \quad F + (50.25 \text{ lb})\cos 30° - (90.0 \text{ lb})\sin 30° = 0$$

$$\mathbf{F} = 1.482 \text{ lb} \measuredangle 60° \blacktriangleleft$$

$$\searrow \Sigma F_{x'} = 0: \quad V - (90 \text{ lb})\cos 30° + (50.25 \text{ lb})\sin 30° = 0$$

$$\mathbf{V} = 103.1 \text{ lb} \measuredangle 30° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_O = 0: \quad (8.4 \text{ in.})(50.25 \text{ lb} + 1.482 \text{ lb}) - M = 0$$

$$\mathbf{M} = 435 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 12.

FBD Frame:

$$\circlearrowleft \Sigma M_A = 0: \quad (16.4 \text{ in.})\frac{24}{25}F_{EC} - (12.6 \text{ in.})(120 \text{ lb}) = 0$$

$$F_{EC} = 93.75 \text{ lb}$$

FBD CBD:

$$\circlearrowleft \Sigma M_C = 0: \quad (16.2 \text{ in.})(120 \text{ lb}) - (13.5 \text{ in.})B_y = 0$$

$$\mathbf{B}_y = 144 \text{ lb} \uparrow$$

FBD DK:

$$\nearrow \Sigma F_{x'} = 0: \quad (144 \text{ lb} - 120 \text{ lb})\sin 30° - F = 0$$

$$\mathbf{F} = 12.00 \text{ lb} \swarrow 30° \quad ◀$$

$$\nwarrow \Sigma F_{y'} = 0: \quad (144 \text{ lb} - 120 \text{ lb})\cos 30° - V = 0$$

$$\mathbf{V} = 20.8 \text{ lb} \searrow 60° \quad ◀$$

$$\circlearrowleft \Sigma M_O = 0: \quad (8.4 \text{ in.})(12 \text{ lb} - 144 \text{ lb}) + (11.1 \text{ in.})(120 \text{ lb}) - M = 0$$

$$\mathbf{M} = 223 \text{ lb}\cdot\text{in.} \circlearrowright \quad ◀$$

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Chapter 7, Solution 13.

FBD AB:

$$\circlearrowleft \Sigma M_B = 0: \quad (0.8\text{ m})(1.8\text{ kN}) - (0.24\text{ m})A_x = 0$$

$$\mathbf{A}_x = 6.0\text{ kN} \rightarrow$$

Geometry:

$$y = kx^2;\ \text{at } B,\ 0.24\text{ m} = k(0.8\text{ m})^2$$

$$\text{so} \quad k = 0.375\,\frac{1}{\text{m}}$$

$$\text{at} \quad J,\ y_J = \left(0.375\frac{1}{\text{m}}\right)(0.48\text{ m})^2 = 0.0864\text{ m}$$

$$\text{slope of parabola} \quad \frac{dy}{dx} = 2kx$$

$$\text{at } J,\ \frac{dy}{dx} = 2\left(0.375\frac{1}{\text{m}}\right)(0.48\text{ m}) = 0.36 = \tan\theta_J$$

$$\theta_J = 19.799°$$

FBD AJ:

$$\searrow \Sigma F_{x'} = 0: \quad (6\text{ kN})\cos 19.799° - (1.8\text{ kN})\sin 19.799° - F = 0$$

$$\mathbf{F} = 6.26\text{ kN} \ \measuredangle\ 19.80° \ ◀$$

$$\nearrow \Sigma F_{y'} = 0: \quad (6\text{ kN})\sin 19.799° - (1.8\text{ kN})\cos 19.799° - V = 0$$

$$V = 0.3387\text{ kN} \qquad \mathbf{V} = 339\text{ N} \ \measuredangle\ 70.2° \ ◀$$

$$\circlearrowleft \Sigma M_J = 0: \quad (0.48\text{ m})(1.8\text{ kN}) - (0.0864\text{ m})(6\text{ kN}) - M = 0$$

$$M = 0.3456\text{ kN}\cdot\text{m} \qquad \mathbf{M} = 346\text{ N}\cdot\text{m} \ \circlearrowright \ ◀$$

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Chapter 7, Solution 14.

FBD AB:

$$\circlearrowleft \Sigma M_B = 0: \qquad LP - hA_x = 0, \quad A_x = \frac{L}{h}P$$

Geometry:

$$y = kx^2, \quad \text{at } B\text{:} \quad h = kL^2, \quad \text{so } k = \frac{h}{L^2}, \; y = \frac{hx^2}{L^2}$$

$$\text{at } J\text{:} \quad y_J = ka^2 = \frac{ha^2}{L^2}$$

$$\text{slope} = \frac{dy}{dx} = 2kx, \qquad \text{at } J\text{: slope} = \frac{2ha}{L^2} = \tan^{-1}\theta_J$$

FBD AJ:

$$\circlearrowleft \Sigma M_J = 0: \qquad aP - \frac{ha^2}{L^2}\frac{LP}{h} - M = 0$$

$$M = P\left(a - \frac{a^2}{L}\right)$$

To maximize set $\dfrac{dM}{da} = 0$

$$\text{or} \quad P\left(1 - 2\frac{a}{L}\right) = 0, \qquad a = \frac{L}{2}$$

$$\text{Then } M_{\max} = P\left[\frac{L}{2} - \frac{\left(\frac{L}{2}\right)^2}{L}\right] = \frac{PL}{4}$$

$$\mathbf{M}_{\max} = \frac{PL}{4} \circlearrowright \blacktriangleleft$$

$$\text{at } a = \frac{L}{2} \blacktriangleleft$$

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Chapter 7, Solution 15.

FBD Frame with pulley and cord:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.8\text{ m})B_x - (2.6\text{ m})(360\text{ N}) - (0.2\text{ m})(360\text{ N}) = 0$$

$$\mathbf{B}_x = 560\text{ N} \leftarrow$$

FBD BE:

Note: Cord forces have been moved to pulley hub as per Problem 6.91.

$$\circlearrowleft \Sigma M_E = 0: \quad (1.4\text{ m})(360\text{ N}) + (1.8\text{ m})(560\text{ N}) - (2.4\text{ m})B_y = 0$$

$$B_y = 630\text{ N} \uparrow$$

FBD BJ:

$$\searrow \Sigma F_{x'} = 0: \quad F + 360\text{ N} - \frac{3}{5}(630\text{ N} - 360\text{ N}) - \frac{4}{5}(560\text{ N}) = 0$$

$$\mathbf{F} = 250\text{ N} \ \measuredangle\ 36.9° \quad ◀$$

$$\nearrow \Sigma F_{y'} = 0: \quad V + \frac{4}{5}(630\text{ N} - 360\text{ N}) - \frac{3}{5}(560\text{ N}) = 0$$

$$\mathbf{V} = 120.0\text{ N} \ \measuredangle\ 53.1° \quad ◀$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (0.6\text{ m})(360\text{ N}) + (1.2\text{ m})(560\text{ N}) - (1.6\text{ m})(630\text{ N}) = 0$$

$$\mathbf{M} = 120.0\text{ N}\cdot\text{m} \ \circlearrowright \quad ◀$$

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Chapter 7, Solution 16.

FBD Frame with pulleys and cord:

$$\circlearrowleft \Sigma M_B = 0: \quad (1.8\text{ m})A_x - (2.0\text{ m})(360\text{ N}) - (2.6\text{ m})(360\text{ N}) = 0$$

$$\mathbf{A}_x = 920\text{ N} \rightarrow$$

FBD AE:

Note: Cord forces have been moved to pulley hub as per Problem 6.91.

$$\circlearrowleft \Sigma M_E = 0: \quad (2.4\text{ m})A_y - (1.8\text{ m})(360\text{ N}) = 0$$

$$\mathbf{A}_y = 270\text{ N} \downarrow$$

FBD AK:

$$\rightarrow \Sigma F_x = 0: \quad 920\text{ N} - 360\text{ N} - F = 0$$

$$\mathbf{F} = 560\text{ N} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad -270\text{ N} + 360\text{ N} - V = 0$$

$$\mathbf{V} = 90.0\text{ N} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad (1.6\text{ m})(270\text{ N}) - (1.0\text{ m})(360\text{ N}) - M = 0$$

$$\mathbf{M} = 72.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 17.

FBD Frame:

$\circlearrowleft \Sigma M_A = 0: \quad (1\text{ m})D_y - (2.675\text{ m})(360\text{ N}) = 0$

$\mathbf{D}_y = 963\text{ N} \downarrow$

FBD DEF:

$\circlearrowleft \Sigma M_F = 0: \quad (1\text{ m})(963\text{ N}) - (2.4\text{ m})D_x - (0.125\text{ m})(360\text{ N}) = 0$

$\mathbf{D}_x = 3825\text{ N} \leftarrow$

FBD DJ:

$\nearrow \Sigma F_{y'} = 0: \quad F - \frac{12}{13}(963\text{ N}) - \frac{5}{13}(382.5\text{ N}) = 0$

$F = 1036\text{ N} \qquad \mathbf{F} = 1.036\text{ kN} \measuredangle 67.4° \blacktriangleleft$

$\searrow \Sigma F_{x'} = 0: \quad \frac{5}{13}(963\text{ N}) - \frac{12}{13}(382.5\text{ N}) - V = 0$

$\mathbf{V} = 17.31\text{ N} \measuredangle 22.6° \blacktriangleleft$

$\circlearrowleft \Sigma M_J = 0: \quad (0.5\text{ m})(963\text{ N}) - (1.2\text{ m})(382.5\text{ N}) - M = 0$

$M = 22.5\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$

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Chapter 7, Solution 18.

FBD AC:

Note: Cord forces moved to pulley hub as per Problem 6.91.

To determine θ the coordinates of C are

$$x_C = 2.55 \text{ m},\ y_C = \frac{4}{3}(2.55 \text{ m}) = 3.40 \text{ m}, \text{ and}$$

$$x_G = 0,\ y_G = 3.75 \text{ m}$$

$$\therefore \theta = \tan^{-1}\frac{3.75 \text{ m} - 3.40 \text{ m}}{2.55 \text{ m}} = 7.8153°$$

$$\circlearrowleft \Sigma M_A = 0: \quad (3.75 \text{ m})\left[(360 \text{ N})\cos 7.8153°\right] - (2.55 \text{ m})(360 \text{ N}) - (3 \text{ m})F_{BG} = 0$$

$$F_{BG} = 139.820 \text{ N}$$

FBD KC:

$$\nearrow \Sigma F_{y'} = 0: \quad F - \frac{4}{5}(360 \text{ N}) - (360 \text{ N})\cos\left(7.8153° + \tan^{-1}\frac{4}{3}\right) = 0$$

$$F = 462.8 \text{ N} \qquad \mathbf{F} = 463 \text{ N} \ \measuredangle\ 53.1° \blacktriangleleft$$

$$\searrow \Sigma F_{x'} = 0: \quad -V + 139.820 \text{ N} + \frac{3}{5}(360 \text{ N}) - (360 \text{ N})\sin\left(7.8153° + \tan^{-1}\frac{4}{3}\right) = 0$$

$$\mathbf{V} = 41.1 \text{ N} \ \measuredangle\ 36.9° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad -M - (1.5 \text{ m})(139.820 \text{ N}) - \frac{4}{5}(2.75 \text{ m})(360 \text{ N}) + (2.75 \text{ m})(360 \text{ N})\sin\left(7.8153° + \tan^{-1}\frac{4}{3}\right) = 0$$

$$\mathbf{M} = 61.7 \text{ N}\cdot\text{m} \ \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 19.

FBD Frame and pipe:

$$W = (10\text{ ft})(18.5\text{ lb/ft}) = 185\text{ lb}$$

$$\Sigma M_A = (24\text{ in.})C_x - (12.6\text{ in.})185\text{ lb} = 0$$

$$\mathbf{C}_x = 97.125\text{ lb} \leftarrow$$

FBD pipe:

By symmetry $N_E = N_D = N$

$$\uparrow \Sigma F_y = 0: \quad 2\left(\frac{21}{29}N\right) - 185\ N = 0, \quad N = 127.738\text{ lb}$$

Also note $\quad a = r\tan\left(\tan^{-1}\frac{20}{21}\right) = (2.8\text{ in.})\frac{20}{21} = \frac{8}{3}\text{in.}$

FBD BC:

$$\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{8}{3}\text{in.}\right)(127.735\text{ lb}) + (12\text{ in.})(97.125\text{ lb}) - (12.6\text{ in.})C_y = 0$$

$$\mathbf{C}_y = 119.534\text{ lb}$$

$$\searrow \Sigma F_{x'} = 0: \quad F - \frac{21}{29}(97.125\text{ lb}) - \frac{20}{29}(119.534\text{ lb}) = 0$$

$$\mathbf{F} = 152.8\text{ lb} \ \searrow 46.4° \blacktriangleleft$$

FBD CJ:

$$\nearrow \Sigma F_{y'} = 0: \quad -V - \frac{20}{29}(97.125\text{ lb}) + \frac{21}{29}(119.534\text{ lb}) = 0$$

$$\mathbf{V} = 19.58\text{ lb} \ \swarrow 44.6° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_C = 0: \quad M - (8.7\text{ in.})(19.58\text{ lb}) = 0$$

$$\mathbf{M} = 170.3\text{ lb}\cdot\text{in.} \ \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 20.

FBD Frame:

$$W = (10\text{ ft})(18.5\text{ lb/ft}) = 185\text{ lb}$$

$$\circlearrowleft \Sigma M_C = 0: \quad (24\text{ in.})A_x - (12.6\text{ in.})(185\text{ lb}) = 0$$

$$\mathbf{A}_x = 97.125\text{ lb} \longrightarrow$$

FBD Pipe

By symmetry $N_E = N_D$

$$\uparrow \Sigma F_y = 0: \quad 2\left(\frac{21}{29}N_D\right) - 185\text{ lb} = 0, \qquad N_D = 127.738\text{ lb}$$

Also note $\quad a = r\tan\left(\tan^{-1}\frac{20}{21}\right) = (2.8\text{ in.})\left(\frac{20}{21}\right) = \frac{8}{3}\text{ in.}$

FBD AD:

$$\circlearrowleft \Sigma M_B = 0: \quad (12.6\text{ in.})(97.125\text{ lb}) - (12.6\text{ in.})A_y - \left(\frac{8}{3}\text{in.}\right)(127.738\text{ lb}) = 0$$

$$\mathbf{A}_y = 65.465\text{ lb} \uparrow$$

FBD AK:

$$\nearrow \Sigma F_{x'} = 0: \quad \frac{21}{29}(97.125\text{ lb}) + \frac{20}{29}(65.465\text{ lb}) - F = 0$$

$$\mathbf{F} = 115.5\text{ lb} \nearrow 44.6° \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \quad \frac{20}{29}(97.125\text{ lb}) - \frac{21}{29}(65.465\text{ lb}) + V = 0$$

$$\mathbf{V} = 19.58\text{ lb} \searrow 46.4° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_A = 0: \quad M - (8.7\text{ in.})(19.58\text{ lb}) = 0$$

$$\mathbf{M} = 170.3\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 21.

(*a*) **FBD Rod:**

$\rightarrow \Sigma F_x = 0: \quad A_x = 0$

$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA_y = 0 \qquad A_y = \dfrac{P}{2}$

FBD AJ:

$\rightarrow \Sigma F_x = 0: \mathbf{V} = 0$ ◀

$\uparrow \Sigma F_y = 0: \quad \dfrac{P}{2} - F = 0$

$\mathbf{F} = \dfrac{P}{2} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \mathbf{M} = 0$ ◀

(*b*) **FBD Rod:**

$\circlearrowleft \Sigma M_A = 0$

$2a\left(\dfrac{4}{5}D\right) + 2a\left(\dfrac{3}{5}D\right) - aP = 0 \qquad D = \dfrac{5P}{14}$

$\rightarrow \Sigma F_x = 0: \quad A_x - \dfrac{4}{5}\dfrac{5}{14}P = 0 \qquad A_x = \dfrac{2P}{7}$

$\uparrow \Sigma F_y = 0: \quad A_y - P + \dfrac{3}{5}\dfrac{5}{14}P = 0 \qquad A_y = \dfrac{11P}{14}$

continued

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FBD AJ:

$$\rightarrow \Sigma F_x = 0: \quad \frac{2}{7}P - V = 0$$

$$\mathbf{V} = \frac{2P}{7} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{11P}{14} - F = 0$$

$$\mathbf{F} = \frac{11P}{14} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad a\frac{2P}{7} - M = 0$$

$$\mathbf{M} = \frac{2}{7}aP \circlearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**

$$\circlearrowleft \Sigma M_A = 0: \quad \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$$

FBD AJ:

$$\rightarrow \Sigma F_x = 0: \quad 2P - V = 0$$

$$\mathbf{V} = 2P \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{5P}{2} - F = 0$$

$$\mathbf{F} = \frac{5P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad a(2P) - M = 0$$

$$\mathbf{M} = 2aP \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 22.

(*a*) **FBD Rod:**

$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA = 0$

$$\mathbf{A} = \frac{P}{2} \leftarrow$$

$\rightarrow \Sigma F_x = 0: \quad V - \frac{P}{2} = 0$

$$\mathbf{V} = \frac{P}{2} \rightarrow \blacktriangleleft$$

$\uparrow \Sigma F_y = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$

FBD AJ:

$\circlearrowleft \Sigma M_J = 0: \quad M - a\frac{P}{2} = 0$

$$\mathbf{M} = \frac{aP}{2} \circlearrowright \blacktriangleleft$$

(*b*) **FBD Rod:**

$\circlearrowleft \Sigma M_D = 0: \quad aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

continued

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FBD AJ:

$$\rightarrow \Sigma F_x = 0: \quad \frac{3}{5}\frac{5P}{2} - V = 0$$

$$\mathbf{V} = \frac{3P}{2} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{2} - F = 0$$

$$\mathbf{F} = 2P \downarrow \blacktriangleleft$$

$$\mathbf{M} = \frac{3}{2}aP \circlearrowright \blacktriangleleft$$

$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$$

$$A = \frac{5P}{14}$$

(*c*) **FBD Rod:**

$$\rightarrow \Sigma F_x = 0: \quad V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

$$\mathbf{V} = \frac{3P}{14} \rightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{14} - F = 0$$

$$\mathbf{F} = \frac{2P}{7} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

$$\mathbf{M} = \frac{3}{14}aP \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 23.

FBD Rod:

$$\circlearrowleft \Sigma M_A = 0: \quad rB - \left(r - \frac{2r}{\pi}\right)W = 0 \qquad \mathbf{B} = \left(1 - \frac{2}{\pi}\right)W \longrightarrow$$

FBD JB:

$$\text{Note: } \bar{r} = \frac{r\sin\frac{\phi}{2}}{\frac{\phi}{2}} = \frac{r\sin\frac{60°}{2}}{30°\,\frac{\pi}{180°}} = \frac{3r}{\pi}$$

$$\bar{x} = \frac{\sqrt{3}r}{2} - \frac{1}{2}\frac{3r}{\pi} = \frac{r}{2}\left(\sqrt{3} - \frac{3}{\pi}\right)$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{r}{2}\left(1 - \frac{2}{\pi}\right)W - \frac{r}{2}\left(\sqrt{3} - \frac{3}{\pi}\right)\left(\frac{2}{3}W\right) = 0$$

$$\mathbf{M} = 0.774Wr \circlearrowright \blacktriangleleft$$

on BK

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Chapter 7, Solution 24.

FBD Rod:

$$\circlearrowleft \Sigma M_A = 0: \quad 2r\left(\frac{1}{\sqrt{2}}B\right) - \left(r - \frac{2r}{\pi}\right)W = 0$$

$$B = \sqrt{2}\left(\frac{1}{2} - \frac{1}{\pi}\right)W$$

FBD JB:

$$\bar{r} = \frac{r\sin\left(\frac{60°}{2}\right)}{30°\,\frac{\pi}{180°}} = \frac{3r}{\pi} \qquad \bar{x} = r\sin 60° - \bar{r}\sin 30°$$

$$\bar{x} = \frac{1}{2}\left(\sqrt{3} - \frac{3}{\pi}\right)r$$

$$\circlearrowleft \Sigma M_J = 0: \quad \left[r\sin 60° + r\left(1 - \cos 60°\right)\right]\frac{\sqrt{2}}{\sqrt{2}}\left(\frac{1}{2} - \frac{1}{\pi}\right)W$$

$$- \frac{1}{2}\left(\sqrt{3} - \frac{3}{\pi}\right)r\frac{2W}{3} - M = 0$$

$$M = -0.01085Wr \qquad \mathbf{M} = 0.01085Wr \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 25.

FBD Rod:

$$\uparrow \Sigma F_y = 0: \qquad A_y - W = 0$$

$$\mathbf{A}_y = W \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: \qquad r\left(A_x - W\right) + \frac{2r}{\pi}W = 0$$

$$\mathbf{A}_x = \left(1 - \frac{2}{\pi}\right)W \leftarrow$$

FBD AJ:

$$W' = \frac{\theta}{\frac{\pi}{2}}W = \frac{2\theta}{\pi}W$$

$$\bar{r} = r\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} = \frac{2r}{\theta}\sin\frac{\theta}{2}$$

$$\circlearrowleft \Sigma M_J = 0: \qquad r\sin\theta\left(1 - \frac{2}{\pi}\right)W - r\left(1 - \cos\theta\right)W$$

$$+\left[\left(\frac{2r}{\theta}\sin\frac{\theta}{2}\right)\cos\frac{\theta}{2} - r\cos\theta\right]\frac{2\theta}{\pi}W - M = 0$$

$$M = Wr\left(\sin\theta - 1 + \cos\theta - \frac{2\theta}{\pi}\cos\theta\right)$$

For $M_{\max}$, $\dfrac{dM}{d\theta} = Wr\left(\cos\theta - \sin\theta - \dfrac{2}{\pi}\cos\theta + \dfrac{2\theta}{\pi}\sin\theta\right) = 0$

$$\text{or } \tan\theta = \frac{\pi - 2}{\pi - 2\theta}$$

Solving numerically: $\theta = 0.48338 \text{ rod} = 27.7°$ ◀

$$\mathbf{M}_{\max} = 0.0777Wr \circlearrowright$$ ◀

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Chapter 7, Solution 26.

FBD Rod:

$$\circlearrowleft \Sigma M_A = 0: \quad \sqrt{2}rB - r\left(1 - \frac{2}{\pi}\right)W = 0 \qquad B = \sqrt{2}\left(\frac{1}{2} - \frac{1}{\pi}\right)W$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{1}{\sqrt{2}}\sqrt{2}\left(\frac{1}{2} - \frac{1}{\pi}\right)W = 0 \qquad \mathbf{A}_x = \left(\frac{1}{2} - \frac{1}{\pi}\right)W \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - W + \frac{1}{\sqrt{2}}\sqrt{2}\left(\frac{1}{2} - \frac{1}{\pi}\right)W = 0 \qquad \mathbf{A}_y = \left(\frac{1}{2} + \frac{1}{\pi}\right)W \uparrow$$

FBD AJ:

$$W' = \frac{\theta}{\frac{\pi}{2}}W = \frac{2\theta}{\pi}W$$

$$\bar{r} = \frac{2r}{\theta}\sin\frac{\theta}{2}$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + r\sin\theta\left(\frac{1}{2} - \frac{1}{\pi}\right)W - r\left(1 - \cos\theta\right)\left(\frac{1}{2} + \frac{1}{\pi}\right)W$$

$$+\left[\left(\frac{2r}{\theta}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) - r\cos\theta\right]\left(\frac{2\theta}{\pi}W\right) = 0$$

$$M = Wr\left[\left(\frac{1}{2} + \frac{1}{\pi}\right)\left(1 - \sin\theta - \cos\theta\right) + \frac{2\theta}{\pi}\cos\theta\right]$$

For $M_{\max}$,

$$\frac{dM}{d\theta} = Wr\left[\left(\frac{1}{2} + \frac{1}{\pi}\right)\left(-\cos\theta + \sin\theta\right) + \frac{2}{\pi}\cos\theta - \frac{2\theta}{\pi}\sin\theta\right] = 0$$

$$\text{or } \tan\theta = \frac{\pi - 2}{\pi + 2 - 4\theta}$$

Solving numerically $\theta = 0.27539 \text{ rad} = \theta_1$

or $\theta = 1.16164 \text{ rad} = \theta_2$

$$M\left(\theta_1\right) = -0.0230Wr, \; M\left(\theta_2\right) = 0.0362Wr$$

so $\mathbf{M}_{\max} = 0.0362Wr \circlearrowright$ ◀

at $\theta = \theta_2 = 66.6°$ ◀

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Chapter 7, Solution 27.

Note: $\alpha = \dfrac{180° - 60°}{2} = 60° = \dfrac{\pi}{3}$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{3r}{\pi}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi}r$$

$$\text{Weight of section} = W\frac{120}{270} = \frac{4}{9}W$$

$$\nwarrow \Sigma F_{y'} = 0: \quad F - \frac{4}{9}W\cos 30° = 0 \qquad F = \frac{2\sqrt{3}}{9}W$$

$$\circlearrowleft \Sigma M_O = 0: \quad rF - \left(\bar{r}\sin 60°\right)\frac{4W}{9} - M = 0$$

$$M = r\left[\frac{2\sqrt{3}}{9} - \frac{3\sqrt{3}}{2\pi}\frac{\sqrt{3}}{2}\frac{4}{9}\right]W = \left[\frac{2\sqrt{3}}{9} - \frac{1}{\pi}\right]Wr$$

$$\mathbf{M} = 0.0666Wr \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 28.

FBD Rod:

$$\rightarrow \Sigma F_x = 0: \quad A_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad rA_y + \frac{2r}{\pi}\frac{W}{3} - \frac{2r}{\pi}\frac{2W}{3} = 0$$

$$A_y = \frac{2W}{3\pi}$$

FBD AJ:

Note:

$$\alpha = \frac{60°}{2} = 30° = \frac{\pi}{6}$$

$$\text{Weight of segment} = W\frac{60}{270} = \frac{2W}{9}$$

$$F = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/6}\sin 30° = \frac{3r}{\pi}$$

$$\circlearrowleft \Sigma M_J = 0: \quad \left(\bar{r}\cos\alpha - r\sin 30°\right)\frac{2W}{9} + \left(r - r\sin 30°\right)\frac{2W}{3\pi} - M = 0$$

$$M = \frac{2W}{9}\left(\frac{3r}{\pi}\frac{\sqrt{3}}{2} - \frac{r}{2} + \frac{3r}{2\pi}\right) = Wr\left(\frac{\sqrt{3}}{3\pi} - \frac{1}{9} + \frac{1}{3\pi}\right)$$

$$\mathbf{M} = 0.1788Wr \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 29.

(*a*) **FBD Beam:**

$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0 \qquad \mathbf{C} = \frac{M_0}{L} \uparrow$

Along AB:

$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$

$\circlearrowleft \Sigma M_J = 0: \quad x\frac{M_0}{L} + M = 0 \qquad M = -\frac{M_0}{L}x$

straight with $M = -\frac{M_0}{2}$ at B

Along BC:

$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$

$\circlearrowleft \Sigma M_K = 0: \quad M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$

straight with $M = \frac{M_0}{2}$ at $B \qquad M = 0$ at C

(*b*) From diagrams:

$|V|_{\max} = \frac{M_0}{L}$ everywhere ◀

$|M|_{\max} = \frac{M_0}{2}$ at B ◀

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Chapter 7, Solution 30.

FBD Beam:

$\rightarrow \Sigma F_x = 0: \quad A_x = 0,$ by symmetry $A_y = B_y = P$

(*a*)

Along AB:

$\uparrow \Sigma F_y = 0: \quad P - V = 0$

$V = P$

$\circlearrowleft \Sigma M_J = 0: \quad M - xP = 0$

$M = Px$

Along BC:

$\uparrow \Sigma F_y = 0: \quad P - P - V = 0$

$V = 0$

$\circlearrowleft \Sigma M_K = 0: \quad M - xP + (x - a)P = 0$

$M = Pa$

Along CD:

$\uparrow \Sigma F_y = 0: \quad P - P - P - V = 0$

$V = -P$

$\circlearrowleft \Sigma M_L = 0: \quad M - xP + (x - a)P$

$+(x - L + a)P = 0$

$M = P(L - x)$

Note: Symmetry in *M* diag. follows symmetry of FBD

(*b*)

$|V|_{\max} = P$ along *AB* and *CD* ◀

$|M|_{\max} = Pa$ along *BC* ◀

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Chapter 7, Solution 31.

(*a*)

FBD Section:

$$\uparrow \Sigma F_y = 0: \quad -V - \frac{1}{2}x\left(w_0\frac{x}{L}\right) = 0$$

$$V = -\frac{1}{2}\frac{w_0}{L}x^2$$

$$V(L) = -\frac{1}{2}w_0L$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{1}{3}x\left[\frac{1}{2}x\left(w_0\frac{x}{L}\right)\right] = 0$$

$$M = -\frac{1}{6}\frac{w_0}{L}x^3$$

$$M(L) = -\frac{1}{6}w_0L^2$$

(*b*)

$$|V|_{\max} = \frac{1}{2}w_0L \text{ at } B \blacktriangleleft$$

$$|M|_{\max} = \frac{1}{6}w_0L^2 \text{ at } B \blacktriangleleft$$

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Chapter 7, Solution 32.

(*a*) **FBD Beam:**

$\rightarrow \Sigma F_x = 0: \quad B_x = 0$

$\circlearrowleft \Sigma M_B = 0: \quad aP + 2aC - 3.5aP = 0$

$C = 1.25P \uparrow$

$\uparrow \Sigma F_y = 0: \quad -P + B_y + 1.25P - P = 0$

$B_y = 0.75P$

Along AB:

$\uparrow \Sigma F_y = 0: \quad -P - V = 0 \qquad V = -P$

$\circlearrowleft \Sigma M_J = 0: \quad M + xP = 0 \qquad M = -Px$

Along BC:

$\uparrow \Sigma F_y = 0: \quad -P + 0.75P - V = 0 \qquad V = -0.25P$

$\circlearrowleft \Sigma M_K = 0: \quad M + xP - (x - a)(0.75P) = 0$

$M = -0.75Pa - 0.25Px \qquad M(3a) = -1.5Pa$

Along CD:

$\uparrow \Sigma F_y = 0: \quad V - P = 0 \qquad V = P$

$\circlearrowleft \Sigma M_L = 0: \quad -M - x_1 P = 0 \qquad M = -Px_1$

$M(1.5a) = -1.5Pa$

$|V|_{\max} = P$ along AB and CD ◀

(*b*)

$|M|_{\max} = 1.5Pa$ at C ◀

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Chapter 7, Solution 33.

(*a*) **FBD Beam:**

$\rightarrow \Sigma F_x = 0: \quad C_x = 0$

$\circlearrowleft \Sigma M_C = 0: \quad (3.6\text{ ft})(1\text{ kip}) - (3\text{ ft})(4\text{ kips}) + (6\text{ ft})(2\text{ kips}) - (9.6\text{ ft})B = 0$

$\mathbf{B} = 0.375\text{ kip}\downarrow$

$\uparrow \Sigma F_y = 0: \quad -1\text{ kip} + C_y - 4\text{ kips} + 2\text{ kips} - 0.375\text{ kip} = 0$

$\mathbf{C}_y = 3.375\text{ kips}\uparrow$

Along AC:

$\uparrow \Sigma F_y = 0: \quad -1\text{ kip} - V = 0$

$V = -1\text{ kip}$

$\circlearrowleft \Sigma M_J = 0: \quad M + x(1\text{ kip}) = 0$

$M = -(1\text{ kip})x$

$M(3.6\text{ ft}) = 3.6\text{ kip}\cdot\text{ft}$

Along CD:

$\uparrow \Sigma F_y = 0: \quad -1\text{ kip} + 3.375\text{ kips} - V = 0$

$V = 2.375\text{ kips}$

$\circlearrowleft \Sigma M_K = 0: \quad M + x(1\text{ kip}) - (x - 3.6\text{ ft})(3.375\text{ kips}) = 0$

$M = -12.15\text{ kip}\cdot\text{ft} + (2.375\text{ kips})x$

$M(6.6\text{ ft}) = 3.525\text{ kip}\cdot\text{ft}$

continued

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Along EB:

$\uparrow \Sigma F_y = 0: \quad V - 0.375 \text{ kips} = 0$

$$V = 0.375 \text{ kips}$$

$\circlearrowleft \Sigma M_L = 0: \quad -M - x_1(0.375 \text{ kips}) = 0$

$$M = -(0.375 \text{ kips})x_1$$

$$M(3.6 \text{ ft}) = -1.35 \text{ kip}\cdot\text{ft}$$

Along DE:

$\uparrow \Sigma F_y = 0: \quad V + 2 \text{ kips} - 0.375 \text{ kips} = 0$

$$V = -1.625 \text{ kips}$$

Also M is linear

(*b*) $|V|_{\max} = 2.38$ kips along CD ◀

$|M|_{\max} = 3.60$ kips at C ◀

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Chapter 7, Solution 34.

(*a*)

FBD Beam:

$\circlearrowleft \Sigma M_B = 0$:

$$(.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips} \uparrow$$

$\uparrow \Sigma F_y = 0$: $\quad 12.625\text{ kips} - 10\text{ kips} - 8\text{ kips} - 4\text{ kips} + B = 0$

$$\mathbf{B} = 9.375\text{ kips} \uparrow$$

Along AC:

$\uparrow \Sigma F_y = 0$: $\quad 12.625\text{ kips} - V = 0$

$$V = 12.625\text{ kips}$$

$\circlearrowleft \Sigma M_J = 0$: $\quad M - x(12.625\text{ kips}) = 0$

$$M = (12.625\text{ kips})x$$

$$M = 22.725\text{ kip}\cdot\text{ft at } C$$

Along CD:

$\uparrow \Sigma F_y = 0$: $\quad 12.625\text{ kips} - 10\text{ kips} - V = 0$

$$V = 2.625\text{ kips}$$

$\circlearrowleft \Sigma M_K = 0$: $\quad M + (x - 1.8\text{ ft})(10\text{ kips}) - x(12.625\text{ kips}) = 0$

$$M = 18\text{ kip}\cdot\text{ft} + (2.625\text{ kips})\,x$$

$$M = 29.8125\text{ kip}\cdot\text{ft at } D\ (x = 4.5\text{ ft})$$

continued

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Along DE:

$$\uparrow \Sigma F_y = 0: \quad (12.625 - 10 - 8)\text{ kips} - V = 0 \qquad V = -5.375\text{ kips}$$

$$\circlearrowleft \Sigma M_L = 0: \; M + x_1(8\text{ kips}) + (2.7\text{ ft} + x_1)(10\text{ kips})$$

$$- (4.5\text{ ft} + x_1)(12.625\text{ kips}) = 0$$

$$M = 29.8125\text{ kip}\cdot\text{ft} - (5.375\text{ kips})\, x_1$$

$$M = 5.625\text{ kip}\cdot\text{ft at } E \quad (x_1 = 4.5\text{ ft})$$

Along EB:

$$\uparrow \Sigma F_y = 0: \quad V + 9.375\text{ kips} = 0 \qquad V = 9.375\text{ kips}$$

$$\circlearrowleft \Sigma M_N = 0: \quad x_2(9.375\text{ kip}) - M = 0$$

$$M = (9.375\text{ kips})\, x_2$$

$$M = 5.625\text{ kip}\cdot\text{ft at } E$$

From diagrams: $|V|_{\max} = 12.63$ kips on AC ◀

$|M|_{\max} = 29.8$ kip·ft at D ◀

(*b*)

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Chapter 7, Solution 35.

(*a*)

FBD Beam: $\circlearrowleft \Sigma M_E = 0$:

$$(1.1\text{ m})(540\text{ N}) - (0.9\text{ m})C_y + (0.4\text{ m})(1350\text{ N}) - (0.3\text{ m})(540\text{ N}) = 0$$

$$\mathbf{C}_y = 1080\text{ N} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad -540\text{ N} + 1080\text{ N} - 1350\text{ N}$$

$$-540\text{ N} + E_y = 0 \qquad \mathbf{E}_y = 1350\text{ N} \uparrow$$

Along AC:

$$\uparrow \Sigma F_y = 0: \quad -540\text{ N} - V = 0$$

$$V = -540\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x(540\text{ N}) + M = 0 \qquad M = -(540\text{ N})x$$

Along CD:

$$\uparrow \Sigma F_y = 0: \quad -540\text{ N} + 1080\text{ N} - V = 0 \qquad V = 540\text{ N}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + (0.2\text{ m} + x_1)(540\text{ N}) - x_1(1080\text{ N}) = 0$$

$$M = -108\text{ N}\cdot\text{m} + (540\text{ N})x_1$$

$$M = 162\text{ N}\cdot\text{m at } D \ (x_1 = 0.5\text{ m})$$

continued

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Along DE:

$\uparrow \Sigma F_y = 0: \quad V + 1350\text{ N} - 540\text{ N} = 0 \qquad V = -810\text{ N}$

$\circlearrowleft \Sigma M_N = 0: \quad M + (x_3 + 0.3\text{ m})(540\text{ N}) - x_3(1350\text{ N}) = 0$

$$M = -162\text{ N}\cdot\text{m} + (810\text{ N})x_3$$

$$M = 162\text{ N}\cdot\text{m at } D \ (x_3 = 0.4)$$

Along EB:

(*b*) $\uparrow \Sigma F_y = 0: \quad V - 540\text{ N} = 0 \qquad V = 540\text{ N}$

$\circlearrowleft \Sigma M_L = 0: \quad M + x_2(540\text{ N}) = 0 \qquad M = -540\text{ N}\,x_2$

$$M = -162\text{ N}\cdot\text{m at } E \quad (x_2 = 0.3\text{ m})$$

From diagrams $\qquad |V|_{\max} = 810\text{ N on } DE$ ◀

$|M|_{\max} = 162.0\text{ N}\cdot\text{m at } D \text{ and } E$ ◀

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Chapter 7, Solution 36.

(*a*) **FBD Beam:**

$a = 1\text{ m}$ $\quad \rightarrow \Sigma F_x = 0: \quad B_x = 0$

$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kN} + 2\text{ kN} - 4\text{ kN} + 5\text{ kN} - B_y = 0$

$B_y = 1.5\text{ kN}\downarrow$

$\circlearrowleft \Sigma M_B = 0: \quad a\left[4(1.5\text{ kN}) - 3(2\text{ kN}) + 2(4\text{ kN}) - 1(5\text{ kN})\right] - M_B = 0$

Along AC: $\quad M_B = (3\text{ kN})a = 3\text{ kN}\cdot\text{m}\circlearrowright$

$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kN} - V = 0 \quad V = -1.5\text{ kN}$

$\circlearrowleft \Sigma M_J = 0: \quad M - x(1.5\text{ kN}) = 0 \quad M = -(1.5\text{ kN})x$

Along CD: $\quad M(1\text{ m}) = -1.5\text{ kN}\cdot\text{m}$

$\uparrow \Sigma F_y = 0: \quad -1.5\text{ kN} + 2\text{ kN} - V = 0 \quad V = 0.5\text{ kN}$

$\circlearrowleft \Sigma M_K = 0: \quad M + x(1.5\text{ kN}) - (x - 1\text{ m})(2\text{ kN}) = 0$

$M = -2\text{ kN}\cdot\text{m} + (0.5\text{ kN})x \quad M(2\text{ m}) = -1\text{ kN}\cdot\text{m}$

continued

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Along EB:

$\uparrow \Sigma F_y = 0: \quad V - 1.5 \text{ kN} = 0 \quad V = 1.5 \text{ kN}$

$\circlearrowleft \Sigma M_L = 0: \quad -M - x_1(1.5 \text{ kN}) - 3 \text{ kN}\cdot\text{m} = 0$

$M = -3 \text{ kN}\cdot\text{m} - (1.5 \text{ kN})x_1, \quad M(1 \text{ m}) = -4.5 \text{ kN}\cdot\text{m}$

Along DE:

$\uparrow \Sigma F_y = 0: \quad V + 5 \text{ kN} - 1.5 \text{ kN} = 0 \quad V = -3.5 \text{ kN}$

Also M is linear here

$|V|_{\max} = 3.50 \text{ kN}$ along DE ◀

(*b*)

$|M|_{\max} = 4.50 \text{ kN}\cdot\text{m}$ at E ◀

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Chapter 7, Solution 37.

(*a*) **FBD Beam:**

$$\circlearrowleft \Sigma M_A = 0: \quad -(1.3\text{ m})\left[(1.8\text{ kN/m})(2.6\text{ m})\right] - (1.6\text{ m})(4\text{ kN}) + (4\text{ m})B = 0$$

$$\mathbf{B} = 3.121\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - (1.8\text{ kN/m})(2.6\text{ m}) - 4\text{ kN} + 3.121\text{ kN} = 0$$

$$\mathbf{A}_y = 5.559\text{ kN} \uparrow$$

Along AC:

$$\uparrow \Sigma F_y = 0: \quad 5.559\text{ kN} - (1.8\text{ kN/m})x - V = 0$$

$$V = 5.559\text{ kN} - (1.8\text{ kN/m})x$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}\left[(1.8\text{ km})x\right] - x(5.559\text{ kN}) = 0$$

$$M = (5.559\text{ kN})x - (0.9\text{ kN/m})x^2$$

Along CD:

$$\uparrow \Sigma F_y = 0: \quad 5.559\text{ kN} - x(1.8\text{ kN/m}) - 4\text{ kN} - V = 0$$

$$V = (1.559\text{ kN}) - (1.8\text{ kN/m})x$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 1.6\text{ m})(4\text{ kN}) + \frac{x}{2}\left[(1.8\text{ kN/m})x\right] - x(5.559\text{ kN}) = 0$$

$$M = 6.4\text{ kN}\cdot\text{m} + (1.559\text{ kN})x - (0.9\text{ kN/m})x^2$$

continued

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Along DB:

(*b*)

$\uparrow \Sigma F_y = 0: \quad V + 3.121 \text{ kN} = 0$

$V = -3.121 \text{ kN}$

$\circlearrowleft \Sigma M_L = 0: \quad -M + x_1(3.121 \text{ kN}) = 0 \qquad M = (3.121 \text{ kN})x_1$

$|V|_{\max} = 5.56 \text{ kN}$ at A ◀

$|M|_{\max} = 6.59 \text{ kN}\cdot\text{m}$ at C ◀

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Chapter 7, Solution 38.

(*a*) **FBD Beam:**

$\leftarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

$\uparrow \Sigma F_y = 0: \quad A_y + (2\text{ m})(24\text{ kN/m}) - 48\text{ kN} - 8\text{ kN} = 0$

$\mathbf{A}_y = 8\text{ kN}\uparrow$

$\circlearrowleft \Sigma M_A = 0: \quad M_A + (1\text{ m})(2\text{ m})(24\text{ kN/m}) - (3.5\text{ m})(48\text{ kN})$

$-(2\text{ m})(8\text{ kN}) = 0, \qquad \mathbf{M}_A = 152\text{ kN}\cdot\text{m}\circlearrowright$

Along AC:

$\uparrow \Sigma F_y = 0: \quad 8\text{ kN} + x(24\text{ kN}\cdot\text{m}) - V = 0$

$V = 8\text{ kN} + (24\text{ kN/m})x$

$\circlearrowleft \Sigma M_J = 0: \quad M + 152\text{ kN}\cdot\text{m} - x(8\text{ kN}) - \frac{x}{2}(24\text{ kN/m})x = 0$

$M = (12\text{ kN/m})x^2 + (8\text{ kN})x - 152\text{ kN}\cdot\text{m}$

Along DB:

$\uparrow \Sigma F_y = 0: \quad V - 8\text{ kN} = 0$

$V = 8\text{ kN}$

$\circlearrowleft \Sigma M_K = 0: \quad M + x_1(8\text{ kN}) = 0, \qquad M = -(8\text{ kN})x_1$

continued

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Along CD:

$\uparrow \Sigma F_y = 0: \quad V - 48\text{ kN} - 8\text{ kN} = 0, \quad V = 56\text{ kN}$

$\circlearrowleft \Sigma M_L = 0: \quad M + (x_1 - 0.5\text{ m})(48\text{ kN}) + x_1(8\text{ kN}) = 0$

$$M = 24\text{ kN}\cdot\text{m} - (56\text{ kN})x_1$$

(*b*) $|V|_{\max} = 56.0\text{ kN}$ along CD ◀

$|M|_{\max} = 152.0\text{ kN}\cdot\text{m}$ at A ◀

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Chapter 7, Solution 39.

(*a*) **FBD Beam:**

by symmetry, $\mathbf{C}_x = 0$, and

$$C_y = G_y = \frac{1}{2}\left[2(12\text{ lb/in})(10\text{ in}) + 2(100\text{ lb}) + (150\text{ lb})\right]$$

$$\mathbf{C}_y = \mathbf{G}_y = 295\text{ lb}\uparrow$$

Along AC:

$$\uparrow \Sigma F_y = 0: \quad -(12\text{ lb/in.})x - V = 0$$

$$V = -\left(12\,\frac{\text{lb}}{\text{in.}}\right)x$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}(12\text{ lb/in.})x = 0, \quad M = -\left(6\frac{\text{lb}}{\text{in.}}\right)x^2$$

Along CD:

$$\uparrow \Sigma F_y = 0: \quad -(12\text{ lb/in.})(10\text{ in.}) + 295\text{ lb} - V = 0$$

$$V = 175\text{ lb}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 5\text{ in.})(12\text{ lb/in.})(10\text{ in.}) - (x - 10\text{ in.})(295\text{ lb}) = 0$$

$$M = -2350\text{ lb}\cdot\text{in.} + (175\text{ lb})x$$

continued

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Along DE:

$\uparrow \Sigma F_y = 0: \quad -(12 \text{ lb/in.})(10 \text{ in.}) + 295 \text{ lb} - 100 \text{ lb} - V = 0, \quad V = 75 \text{ lb}$

$\circlearrowleft \Sigma M_N = 0: \quad M + (x - 16 \text{ in.})(100 \text{ lb}) - (x - 10 \text{ in.})(295 \text{ lb})$

$+ (x - 5 \text{ in.})(12 \text{ lb/in.})(10 \text{ in.}) = 0$

$M = -750 \text{ lb}\cdot\text{in} + (75 \text{ lb})x$

Complete diagrams using symmetry.

(*b*)

$|V|_{\max} = 175.0 \text{ lb}$ along *CD* and *FG* ◀

$|M|_{\max} = 900 \text{ lb}\cdot\text{in.}$ at *E* ◀

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Chapter 7, Solution 40.

(*a*) **FBD Beam:**

$$\circlearrowleft \Sigma M_D = 0: \quad (6\text{ ft})(1\text{ kip/ft})(6\text{ ft}) - (7.5\text{ ft})(2\text{ kips/ft})(9\text{ ft}) - (12\text{ ft})F_y + (15\text{ ft})(33\text{ kips}) = 0$$

$$\mathbf{F}_y = 33\text{ kips}\downarrow$$

$$\uparrow \Sigma F_y = 0: \quad -(1\text{ kip/ft})(6\text{ ft}) + D_y - (2\text{ kips/ft})(9\text{ ft}) - 33\text{ kips} + 33\,kips = 0$$

$$\mathbf{D}_y = 24\text{ kips}\uparrow$$

Along AC:

$$\uparrow \Sigma F_y = 0: \quad -(1\text{ kip/ft})x - V = 0, \quad V = -(1\text{ kip/ft})x$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}(1\text{ kip}\cdot\text{ft})x = 0, \quad M = -\left(\frac{1}{2}\text{kip/ft}\right)x^2$$

Along CD:

$$\uparrow \Sigma F_y = 0: \quad -(1\text{ kip/ft})(6\text{ ft}) - V = 0, \quad V = -6\text{ kips}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 3\text{ ft})(1\text{ kip/ft})(6\text{ ft}) = 0$$

$$M = 18\text{ kip}\cdot\text{ft} - (6\text{ kips})x$$

continued

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Along DE:

$\uparrow \Sigma F_y = 0: \quad -6 \text{ kips} + 24 \text{ kips} - V = 0, \qquad V = 18 \text{ kips}$

$\circlearrowleft \Sigma M_L = 0: \quad (x - 3 \text{ ft})(6 \text{ kips}) - (x - 9 \text{ ft})(24 \text{ kips}) + M = 0$

$$M = -198 \text{ kip}\cdot\text{ft} + (18 \text{ kips})x$$

Along FB:

$\uparrow \Sigma F_y = 0: \quad V + 33 \text{ kips} = 0, \quad V = -33 \text{ kips}$

$\circlearrowleft \Sigma M_N = 0: \quad x_1(33 \text{ kips}) - M = 0, \quad M = (33 \text{ kips})x_1$

Along EF:

$\uparrow \Sigma F_y = 0: \quad V - (2 \text{ kips/ft})x_2 - 33 \text{ kips} + 33 \text{ kips} = 0$

$$V = (2 \text{ kips/ft})x_2$$

$\circlearrowleft \Sigma M_O = 0: \quad M + \frac{x_2}{2}(2 \text{ kips/ft})x_2 - (3 \text{ ft})(33 \text{ kips}) = 0$

$$M = 99 \text{ kip}\cdot\text{ft} - (1 \text{ kip/ft})x_2^2$$

$|V|_{\max} = 33.0 \text{ kips along } FB$ ◀

$|M|_{\max} = 99.0 \text{ kip}\cdot\text{ft at } F$ ◀

(*b*)

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Chapter 7, Solution 41.

(*a*) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: \qquad (4\text{ m})(w) - (2\text{ m})(12\text{ kN/m}) = 0$$

$$w = 6\text{ kN/m}$$

Along AC:

$$\uparrow \Sigma F_y = 0: \qquad -x(6\text{ kN/m}) - V = 0, \qquad V = -(6\text{ kN/m})x$$

$$V = -6\text{ kN at } C(x = 1\text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: \qquad M + \frac{x}{2}(6\text{ kN/m})(x) = 0$$

$$M = -(3\text{ kN/m})x^2 \qquad M = -3\text{ kN}\cdot\text{m at } C$$

Along CD:

$$\uparrow \Sigma F_y = 0: \qquad -(1\text{ m})(6\text{ kN/m}) + x_1(6\text{ kN/m}) - V = 0$$

$$V = (6\text{ kN/m})(1\text{ m} - x_1), \qquad V = 0 \text{ at } x_1 = 1\text{ m}$$

$$\circlearrowleft \Sigma M_K = 0: \qquad M + (0.5\text{ m} + x_1)(6\text{ kN/m})(1\text{ m}) - \frac{x_1}{2}(6\text{ kN/m})x_1 = 0$$

$$M = -3\text{ kN}\cdot\text{m} - (6\text{ kN})x_1 + (3\text{ kN/m})x_1^2$$

$$M = -6\text{ kN}\cdot\text{m at center } (x_1 = 1\text{ m})$$

Finish by symmetry.

(*b*) From diagrams: $|V|_{\max} = 6.00\text{ kN at } C \text{ and } D$ ◀

$|M|_{\max} = 6.00\text{ kN at center}$ ◀

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Chapter 7, Solution 42.

(*a*) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: \qquad (12\text{ m})w - (6\text{ m})(3\text{ kN/m}) = 0$$

$$w = 1.5\text{ kN/m}$$

Along AC:

$$\uparrow \Sigma F_y = 0: \qquad x(1.5\text{ kN/m}) - V = 0, \qquad V = (1.5\text{ kN/m})x$$

$$V = 4.5\text{ kN at } C$$

$$\circlearrowleft \Sigma M_J = 0: \qquad M - \frac{x}{2}(1.5\text{ kN/m})(x) = 0$$

$$M = (0.75\text{ kN/m})x^2, \qquad M = 6.75\text{ N}\cdot\text{m at } C$$

Along CD:

$$\uparrow \Sigma F_y = 0: \qquad x(1.5\text{ kN/m}) - (x - 3\text{ m})(3\text{ kN/m}) - V = 0$$

$$V = 9\text{ kN} - (1.5\text{ kN/m})x, \qquad V = 0 \text{ at } x = 6\text{ m}$$

$$\circlearrowleft \Sigma M_K = 0: \qquad M + \left(\frac{x - 3\text{ m}}{2}\right)(3\text{ kN/m})(x - 3\text{ m}) - \frac{x}{2}(1.5\text{ kN/m})x = 0$$

$$M = -13.5\text{ kN}\cdot\text{m} + (9\text{ kN})x - (0.75\text{ kN/m})x^2$$

$$M = 13.5\text{ kN}\cdot\text{m at center } (x = 6\text{ m})$$

Finish by symmetry.

(*b*) From diagrams: $|V|_{\max} = 4.50\text{ kN at } C \text{ and } D$ ◀

$|M|_{\max} = 13.50\text{ kN}\cdot\text{m at center}$ ◀

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Chapter 7, Solution 43.

(*a*) **FBD Beam:**

$\uparrow \Sigma F_y = 0: \quad (8\text{ m})w - 2(6\text{ kN}) - (4\text{ m})(5\text{ kN/m}) = 0$

$$w = 4\text{ kN/m} \uparrow$$

Along AC:

$\uparrow \Sigma F_y = 0: \quad (4\text{ kN/m})x - V = 0$

$$V = (4\text{ kN/m})x$$

$\circlearrowleft \Sigma M_J = 0: \quad M - \frac{x}{2}(4\text{ kN/m})x = 0, \qquad M = (2\text{ kN/m})x^2$

Along CD:

$\uparrow \Sigma F_y = 0: \quad (4\text{ kN/m})x - 6\text{ kN} - V = 0$

$$V = (4\text{ kN/m})x - 6\text{ kN}$$

$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 1\text{ m})(6\text{ kN}) - \frac{x}{2}(4\text{ kN/m})x = 0$

$$M = (2\text{ kN/m})x^2 - (6\text{ kN})x + 6\text{ kN}\cdot\text{m}$$

Note: $V = 0$ at $x = 1.5$ m where $M = 1.5$ kN/m

Along DE:

$\uparrow \Sigma F_y = 0: \quad (4\text{ kN/m})(2\text{ m}) - 6\text{ kN} - (1\text{ kN/m})(x - 2\text{ m}) - V = 0$

$$V = 4\text{ kN} - (1\text{ kN/m})x$$

$\circlearrowleft \Sigma M_L = 0: \quad M + \left(\frac{x - 2\text{ m}}{2}\right)(1\text{ kN/m})(x - 2\text{ m}) + (x - 1\text{ m})6\text{ kN}$

$$-(x - 1\text{ m})(4\text{ kN/m})(2\text{ m}) = 0$$

continued

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$$M = -\left(\frac{1}{2}\text{ kN/m}\right)x^2 + \left(4\text{ kN}\right)x - 4\text{ kN}\cdot\text{m}$$

Note: $V = 0$ at $x = 4$ m, where $M = 4\text{ kN}\cdot\text{m}$

Complete diagrams using symmetry.

(*b*) $|V|_{\max} = 4$ kN at C and F ◀

$|M|_{\max} = 4\text{ kN}\cdot\text{m}$ at center ◀

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Chapter 7, Solution 44.

(*a*) **FBD Beam:**

$\uparrow \Sigma F_y = 0: \quad w(1.5\text{ m}) - 2(3.6\text{ kN}) = 0$

$w = 4.8\text{ kN/m}$

Along AC:

$\uparrow \Sigma F_y = 0: \quad (4.8\text{ kN/m})x - V = 0, \quad V = (4.8\text{ kN/m})x$

$\circlearrowleft \Sigma M_J = 0: \quad M - \frac{x}{2}(4.8\text{ kN/m})x = 0, \quad M = (2.4\text{ kN/m})x^2$

Along CD:

$\uparrow \Sigma F_y = 0: \quad (4.8\text{ kN/m})x - 3.6\text{ kN} - V = 0$

$V = (4.8\text{ kN/m})x - 3.6\text{ kN}$

$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 0.3\text{ m})(3.6\text{ kN}) - \frac{x}{2}(4.8\text{ kN/m})x = 0$

$M = 1.08\text{ kN}\cdot\text{m} - (3.6\text{ kN})x + (2.4\text{ kN/m})x^2$

Note: $V = 0$ at $x = 0.75$ m, where $M = -0.27\text{ kN}\cdot\text{m}$

Complete diagrams using symmetry.

$|V|_{\max} = 2.16\text{ kN at } C \text{ and } D$ ◀

$|M|_{\max} = 270\text{ N}\cdot\text{m at center}$ ◀

(*b*)

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Chapter 7, Solution 45.

FBD CE:

$\rightarrow \Sigma F_x = 0: \quad \mathbf{C}_y = 0$

$\uparrow \Sigma F_y = 0: \quad C_y - 4\text{ kN} = 0 \qquad \mathbf{C}_y = 4\text{ kN}\uparrow$

$\circlearrowleft \Sigma M_C = 0: \quad M_C - (0.5\text{ m})(4\text{ kN}) = 0$

$M_C = 2\text{ kN}\cdot\text{m}$

Beam AB:

$\rightarrow \Sigma F_x = 0: \quad A_x = 0$

$\uparrow \Sigma F_y = 0: \quad A_y - 4\text{ kN} - 2\text{ kN} - 1\text{ kN} = 0 \qquad \boldsymbol{A}_y = 7\text{ kN}\uparrow$

$\circlearrowright \Sigma M_A = 0: \quad M_A - 2\text{ kN}\cdot\text{m} - (0.5\text{ m})(4\text{ kN}) - (1\text{ m})(2\text{ kN})$

$-(1.5\text{ m})(1\text{ kN}) = 0, \qquad \mathbf{M}_A = 7.5\text{ kN}\cdot\text{m}\circlearrowright$

Along AC:

$\uparrow \Sigma F_y = 0: \quad 7\text{ kN} - V = 0$

$V = 7\text{ kN}$

$\circlearrowright \Sigma M_J = 0: \quad M + 7.5\text{ kN}\cdot\text{m} - x(7\text{ kN}) = 0$

$M = (7\text{ kN})x - 7.5\text{ kN}\cdot\text{m}$

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Along DB:

$\uparrow \Sigma F_y = 0: \quad V - 1\text{ kN} = 0$

$$V = 1\text{ kN}$$

$\circlearrowleft \Sigma M_K = 0: \quad -M + x_1(1\text{ kN}) = 0$

$$M = (1\text{ kN})x_1$$

Along CD:

$\uparrow \Sigma F_y = 0: \quad V - 2\text{ kN} - 1\text{ kN} = 0 \qquad V = 3\text{ kN}$

$\circlearrowright \Sigma M_M = 0: \quad M + (x_1 - 0.5\text{ m})(2\text{ kN}) + x_1(1\text{ kN}) = 0$

$$M = 1\text{ kN}\cdot\text{m} - (3\text{ kN})x_1$$

Note: M exhibits a discontinuity at C, equal to $2\text{ kN}\cdot\text{m}$, the value of M_C.

From the diagrams, $|V|_{\max} = 7.00\text{ kN along } AC$ ◀

$|M|_{\max} = 7.50\text{ kN at } A$ ◀

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Chapter 7, Solution 46.

FBD CE or DF:

$\rightarrow \Sigma F_x = 0: \quad C_x,\ D_x = 0$

$\uparrow \Sigma F_y = 0: \quad C_y - 750\text{ N} = 0, \qquad C_y = 750\text{ N}$

$D_y = 750\text{ N}$

$\circlearrowleft \Sigma M_C = 0: \quad M_C - (0.3\text{ m})(750\text{ N}) = 0$

$M_C = 225\text{ N}\cdot\text{m} = M_D$

Beam AB:

$\circlearrowleft \Sigma M_A = 0: \quad (0.9\text{ m})D_y - 2(225\text{ N}\cdot\text{m}) - (0.3\text{ m})(750\text{N})$

$-(0.9\text{ m})(750\text{ N}) - (1.2\text{ m})(540\text{ N}) = 0$

$D_y = 2220\text{ N}$

$\uparrow \Sigma F_y = 0: \quad A_y - 2(750\text{ N}) - 540\text{ N} + 2220\text{ N} = 0$

$A_y = -180\text{ N} \qquad \mathbf{A}_y = 180\text{ N} \downarrow$

Along AC:

$\uparrow \Sigma F_y = 0: -180\text{ N} - V = 0$

$V = -180\text{ N}$

$\circlearrowleft \Sigma M_J = 0: \quad M + x(180\text{ N}) = 0 \qquad M = (180\text{ N})x$

Along CD:

$\uparrow \Sigma F_y = 0: \quad -180\text{ N} - 750\text{ N} - V = 0, \qquad V = -930\text{ N}$

$\circlearrowleft \Sigma M_K = 0: \quad M - 225\text{ N}\cdot\text{m} + (x - 0.3\text{ m})(750\text{ N}) + x(180\text{ N}) = 0$

$M = 450\text{ N}\cdot\text{m} - (930\text{ N})x$

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Along DB:

$\uparrow \Sigma F_y = 0: \quad V - 540 \text{ N} = 0$

$V = 540 \text{ N}$

$\circlearrowleft \Sigma M_N = 0: \quad M + x_1(540 \text{ N}) = 0 \qquad M = -(540 \text{ N})x_1$

Note: The discontinuities in M, at C and D, equal 225 $\text{N}\cdot\text{m}$, M_C and M_D

From the diagrams $|V|_{\max} = 930 \text{ N}$ along CD ◀

$|M|_{\max} = 387 \text{ N}\cdot\text{m}$ at D ◀

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Chapter 7, Solution 47.

FBD Angle:

(*a*)

Beam AB:

(*b*)

$\uparrow \Sigma F_y = 0: \quad T - C_y = 0 \quad C_y = T$

$\circlearrowleft \Sigma M_C = 0: \quad (0.3 \text{ ft})T - M_C = 0, \quad M_C = (0.3 \text{ ft})T$

By symmetry, $D_y = T$, $M_D = (0.3 \text{ ft})T$

$\uparrow \Sigma F_y = 0: \quad 2T - 810 \text{ lb} - (100 \text{ lb/ft})(9 \text{ ft}) = 0$

$$T = 855 \text{ lb.}$$

From above $M_C = M_D = (0.3 \text{ ft})(855 \text{ lb}) = 256.5 \text{ lb}\cdot\text{ft}$

Along AC:

$\uparrow \Sigma F_y = 0: \quad -x(100 \text{ lb/ft}) - V = 0$

$$V = -(100 \text{ lb/ft})x$$

$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}(100 \text{ lb/ft})x = 0, \quad M = -(50 \text{ lb/ft})x^2$

Along CI:

$\mathbf{M}_C = 256.5 \text{ lb}\cdot\text{ft} \circlearrowright$

$\uparrow \Sigma F_y = 0: \quad -(100 \text{ lb/ft})x + 855 \text{ lb} - V = 0$

$$V = 855 \text{ lb} - (100 \text{ lb/ft})x$$

$\circlearrowleft \Sigma M_K = 0: \quad M - 256.5 \text{ lb}\cdot\text{ft} + \frac{x}{2}(100 \text{ lb/ft})x$

$$-(x - 3.6 \text{ ft})(855 \text{ lb}) = 0$$

$$M = -(50 \text{ lb/ft})x^2 + (855 \text{ lb})x - 2821.5 \text{ lb}\cdot\text{ft}$$

Complete diagrams using symmetry

Note: Discontinuities in *M*, at *C* and *D*, equal M_C and M_D

$|V|_{\max} = 495$ lb at *C* and *D* ◀

$|M|_{\max} = 648 \text{ lb}\cdot\text{ft}$ at *C* and *D* ◀

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Chapter 7, Solution 48.

FBD Angle:

(*a*)

$\uparrow \Sigma F_y = 0: \quad T - C_y = 0 \qquad C_y = T$

$\circlearrowleft \Sigma M_C = 0: \quad M_C - (0.3 \text{ ft})T = 0, \qquad M_C = (0.3 \text{ ft})T$

By symmetry, $D_y = T$ and $M_D = (0.3 \text{ ft})T$

$\uparrow \Sigma F_y = 0: \quad 2T - (100 \text{ lb/ft})(9 \text{ ft}) - 810 \text{ lb} = 0 \qquad T = 855 \text{ lb}$

From above, $M_C = M_D = (0.3 \text{ ft})(855 \text{ lb}) = 256.5 \text{ lb}\cdot\text{ft}$

Beam AB:

Along AC:

$\uparrow \Sigma F_y = 0: \quad -(100 \text{ lb/ft})x - V = 0$

$V = -(100 \text{ lb/ft})x$

$\circlearrowleft \Sigma M_J = 0: \quad M + \frac{x}{2}(100 \text{ lb/ft})x = 0 \qquad M = (50 \text{ lb/ft})x^2$

Along CI:

$\mathbf{M}_C = 256.5 \text{ lb}\cdot\text{ft} \circlearrowright$

$\uparrow \Sigma F_y = 0: \quad 855 \text{ lb} - (100 \text{ lb/ft})x - V = 0$

$V = 855 \text{ lb} - (100 \text{ lb/ft})x$

$\circlearrowleft \Sigma M_K = 0: \quad M - 256.5 \text{ lb}\cdot\text{ft} + \frac{x}{2}(100 \text{ lb/ft})x$

$-(x - 2.7 \text{ ft})(855 \text{ lb}) = 0$

$M = -(50 \text{ lb/ft})x^2 + (855 \text{ lb})x - 2052 \text{ lb}\cdot\text{ft}$

Complete diagrams using symmetry

Note: The discontinuities in *M*, at *C* and *D*, equal M_C and M_D

(*b*)

$|V|_{\max} = 585 \text{ lb at } C \text{ and } D$ ◀

$|M|_{\max} = 783 \text{ lb}\cdot\text{ft at } I$ ◀

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Chapter 7, Solution 49.

FBD Whole:

Note: **D** passes through C, so $\dfrac{D_y}{D_x} = \dfrac{0.35\text{ m}}{0.5\text{ m}} = 0.7$

$$\circlearrowleft \Sigma M_H = 0:\ (0.65\text{ m})(200\text{ N}) - (0.5\text{ m})(0.7\,D_x) + (0.1\text{ m})(D_x)$$
$$+ (0.25\text{ m})(400\text{ N}) - (0.15\text{ m})(200\text{ N}) = 0$$

$$\mathbf{D}_x = 800\text{ N} \rightarrow$$

$$\mathbf{D}_y = 560\text{ N} \uparrow$$

$$\rightarrow \Sigma F_x = 0:\ 800\text{ N} - H_x = 0 \qquad H_x = 800\text{ N} \leftarrow$$

$$\uparrow \Sigma F_y = 0:\ 560\text{ N} - 400\text{ N} - 2(200\text{ N}) + H_y = 0$$

$$\mathbf{H}_y = 240\text{ N} \uparrow$$

Beam AB with forces at D & H replaced by forces and couples at E and G. Horizontal forces not shown to avoid clutter.

Along AE:

$$\uparrow \Sigma F_y = 0:\ -200\text{ N} - V = 0$$

$$V = -200\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0:\ x(200\text{ N}) + M = 0$$

$$M = -(200\text{ N})x$$

Along EF:

$$\uparrow \Sigma F_y = 0:\ -200\text{ N} + 560\text{ N} - V = 0$$

$$V = 360\text{ N}$$

$$\circlearrowleft \Sigma M_K = 0:\ M - 80\text{ N}\cdot\text{m} + x(200\text{ N}) - (x - 0.15\text{ m})(560\text{ N}) = 0$$

$$M = (360\text{ N})x - 4\text{ N}\cdot\text{m}$$

Along GB:

$$\uparrow \Sigma F_y = 0:\ V - 400\text{ N} = 0$$

$$V = 200\text{ N}$$

$$\circlearrowleft \Sigma M_L = 0:\ M + x_1(200\text{ N}) = 0 \qquad M = (200\text{ N})x_1$$

continued

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Along FG: $\uparrow \Sigma F_y = 0: \; V + 240 \text{ N} - 200 \text{ N} = 0$

$V = -40 \text{ N}$

$\circlearrowleft \Sigma M_N = 0: \quad M - 160 \text{ N}\cdot\text{m} + \left(x_1 - 0.15 \text{ m}\right)\left(240 \text{ N}\right) - x_1\left(200\right)\text{N} = 0$

$M = 124 \text{ N}\cdot\text{m} + \left(40 \text{ N}\right)x_1$

From diagrams, $\left|V\right|_{\max} = 360 \text{ N along } EF$ ◀

$\left|M\right|_{\max} = 140.0 \text{ N}\cdot\text{m at } F$ ◀

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Chapter 7, Solution 50.

FBD AB + Pulley & Cord:

(*a*)

$\circlearrowleft \Sigma M_A = 0$: $(48 \text{ in.})\left(\frac{5}{13}D\right) + (20 \text{ in.})\left(\frac{12}{13}D\right) - (100 \text{ in.})(120 \text{ lb}) = 0$

$\mathbf{D} = 325 \text{ lb} \nearrow$ so $\mathbf{D}_x = 300 \text{ lb} \rightarrow$, $\mathbf{D}_y = 125 \text{ lb} \uparrow$

$\uparrow \Sigma F_y = 0$: $-A_y + 125 \text{ lb} - 120 \text{ lb} = 0$ $\quad \mathbf{A}_y = 5 \text{ lb} \downarrow$

Neglecting the diameter of pulley *G*, the cord *EG* has slope 3/4, and tension 120 lb, $\mathbf{E}_x = 96 \text{ lb} \rightarrow$, $\quad \mathbf{E}_y = 72 \text{ lb} \uparrow$

Beam *AB* with forces at *D* and *G* replaced by forces and couples at *E* and *F*. Horizontal forces are omitted to avoid clutter.

Along AE:

$\uparrow \Sigma F_y = 0$: $-5 \text{ lb} - V = 0$, $\quad V = -5 \text{ lb}$

$\circlearrowleft \Sigma M_J = 0$: $x(5 \text{ lb}) + M = 0$, $\quad M = (-5 \text{ lb})x$

Along EF:

$\uparrow \Sigma F_y = 0$: $-5 \text{ lb} + 197 \text{ lb} - V = 0$, $\quad V = 192 \text{ lb}$

$\circlearrowleft \Sigma M_K = 0$: $M + 6000 \text{ lb}\cdot\text{in.} + x(5 \text{ lb}) - (x - 48 \text{ in.})(197 \text{ lb}) = 0$

$M = (192 \text{ lb})x - 15456 \text{ lb}\cdot\text{in.}$

Along FB:

$\uparrow \Sigma F_y = 0$: $V - 120 \text{ lb} = 0$, $V = 120 \text{ lb}$

$\circlearrowleft \Sigma M_L = 0$: $-M - x_1(120 \text{ lb}) = 0$

$M = -(120 \text{ lb})x_1$

(*b*)

From diagrams, $\quad |V|_{\max} = 192.0 \text{ lb along } EF$ ◀

$|M|_{\max} = 6240 \text{ lb}\cdot\text{in.} = 520 \text{ lb}\cdot\text{ft at } E$ ◀

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Chapter 7, Solution 51.

$\uparrow \Sigma F_y = 0: \quad Lw - 2P = 0 \qquad w = \dfrac{2P}{L}$

Along AC:

$\circlearrowleft \Sigma M_J = 0: \quad M - \dfrac{x}{2}\left(\dfrac{2P}{L}\right)x = 0 \qquad M = \dfrac{P}{L}x^2$

Along CD:

$\circlearrowleft \Sigma M_K = 0: \quad M + (x - a)P - \dfrac{x}{2}\left(\dfrac{2P}{L}\right)x = 0$

$$M = \frac{P}{L}x^2 - Px + Pa$$

Complete diagram using symmetry

Note: $M_{\min} = Pa - \dfrac{1}{4}PL$ at $x = \dfrac{L}{2}$ (center)

Setting $M_{\max} = -M_{\min}$: $\quad \dfrac{P}{L}a^2 = -Pa + \dfrac{PL}{4}$ or $a^2 + La - \dfrac{L^2}{4} = 0$

Solving $a = -\dfrac{L}{2} \pm \dfrac{\sqrt{2}}{2}L$, positive root $a = \dfrac{\sqrt{2} - 1}{2}L$

(*a*) Then, with $L = 1.5$ m, $\quad a = 0.31066$ m $\qquad a = 0.311$ m ◀

(*b*) and, with $P = 3.6$ kN, $\quad M_{\max} = \dfrac{Pa^2}{L} \qquad |M|_{\max} = 232$ N·m ◀

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Chapter 7, Solution 52.

FBD Angle CE:

$\uparrow \Sigma F_y = 0: \quad T - C_y = 0 \qquad \mathbf{C}_y = T \uparrow$

$\circlearrowleft \Sigma M_C = 0: \quad M_C - (0.3 \text{ ft})T = 0, \qquad \mathbf{M}_C = (0.3 \text{ ft})T \circlearrowright$

by symmetry $\mathbf{D}_y = T \uparrow$ and $\mathbf{M}_D = (0.3 \text{ ft})T \circlearrowright$

$\uparrow \Sigma F_y = 0: \; 2T - 810 \text{ lb} - (9 \text{ ft})(100 \text{ lb/ft}) = 0 \qquad T = 855 \text{ lb}$

From above $M_C = M_D = 256.5 \text{ lb}\cdot\text{ft}$

Beam AB:

Along AC:

$\circlearrowleft \Sigma M_J = 0: \; M - \frac{x}{2}(100 \text{ lb/ft})x = 0$

$M = -(50 \text{ lb/ft})x^2$

Along CI:

$\mathbf{M}_C = 256.5 \text{ lb}\cdot\text{ft} \circlearrowright$

$\circlearrowleft \Sigma M_K = 0: \quad M - 256.5 \text{ lb}\cdot\text{ft} + \frac{x}{2}(100 \text{ lb/ft})x$

$- (x - 4.5 \text{ ft} + a)(855 \text{ lb}) = 0$

$M = \left[-50\,x^2 + 855(x + a) - 3591\right] \text{lb}\cdot\text{ft}$ with a in ft

Complete M diagram using symmetry

At $x = (4.5 - a)\text{ft}, \; M_{\min} = -50(4.5 - a)^2 \text{ lb}\cdot\text{ft}$

At $x = 4.5 \text{ ft}, \; M_{\max} = (855a - 756)\text{lb}\cdot\text{ft}$

Setting $M_{\max} = -M_{\min}: \; a^2 - 26.1a + 35.37 = 0$

Solving: $a = 13.05 \pm \sqrt{11.6160}, \quad a < 4.5$ so $\qquad a = 1.434 \text{ ft} \blacktriangleleft$

(*a*)

(*b*) giving $|M|_{\max} = 470 \text{ lb}\cdot\text{ft} \blacktriangleleft$

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Chapter 7, Solution 53.

Replacing the 1500 N force with equivalent force and couple at D,

$$\circlearrowleft M_C = 0: \quad (1.25\text{ m})P - (1.2\text{ m})(1500\text{ N}) - 1800\text{ N}\cdot\text{m} + (2.4\text{ m})E_y - (3.65\text{ m})2P = 0$$

assume P in N: $\mathbf{E}_y = (1500 + 2.5208P)\text{N} \uparrow$

$$\uparrow \Sigma F_y = 0: \quad C_y - P - 2P - 1500 + (1500 + 2.5208\,P) = 0$$

$$\mathbf{C}_y = 0.47917\,P \uparrow$$

Along AC:

$$\circlearrowleft \Sigma M_J = 0: \quad M + xP = 0$$

$$M = -Px$$

Along CD:

$$\circlearrowleft \Sigma M_K = 0:$$

$$M + xP - (x - 1.25\text{ m})(0.47917\,P) = 0$$

$$M = -0.5208\,Px - 0.5990\,P$$

at $x = 2.45^-$ (left of D), $M = -1.875\,P$

at $x = 2.45^+$ (right of D), $M = 1800 - 1.875\,P$

At E:

$$\circlearrowleft \Sigma M_E = 0: \quad -M - (1.25\text{ m})(2P) = 0$$

$$M = -2.5\,P$$

(*a*) Setting $M_{\max} = -M_{\min}$: $1800 - 1.875\,P = 2.5\,P \qquad P = 411.43$

$$P = 411\text{ N} \blacktriangleleft$$

(*b*) $M_{\max} = 2.5\,P \qquad M_{\max} = 1029\text{ N}\cdot\text{m}$

$$|M|_{\max} = 1.029\text{ kN}\cdot\text{m} \blacktriangleleft$$

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Chapter 7, Solution 54.

Since there are no distributed loads, M is piecewise linear, and only pts A, C, and D need be considered.

At A: $\circlearrowleft \Sigma M_A = 0: \quad M + (0.75\text{ m})(4\text{ kN}) + (1.75\text{ m})(16\text{ kN})$

$$-(1.75\text{ m} + a)(8\text{ kN}) = 0$$

(With a in m) $M = (8a - 17)\text{kN}\cdot\text{m}$

At C:

$\circlearrowleft \Sigma M_C = M + (1\text{ m})(16\text{ kN}) - (1\text{ m} + a\text{ m})(8\text{ kN}) = 0$

$$M = (8a - 8)\text{kN}\cdot\text{m}$$

At D:

$\circlearrowleft \Sigma M_D = 0: \quad M - (a\text{ m})(8\text{ kN}) = 0$

$$M = 8a\text{ kN}\cdot\text{m}$$

(*a*)

Apparently $M_{\max} = 8a\text{ kN}\cdot\text{m}$ at D,

and $M_{\min} = (8a - 17)\text{kN}\cdot\text{m}$ at A

Setting $M_{\max} = -M_{\min}$: $\quad 8a = 17 - 8a \qquad a = \dfrac{17}{16}\text{m}$

$a = 1.063\text{ m}$ ◀

(*b*)

and $M_{\max} = 8a = \dfrac{17}{2}\text{ kN}\cdot\text{m} \qquad |M|_{\max} = 8.50\text{ kN}\cdot\text{m}$ ◀

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Chapter 7, Solution 55.

$$\circlearrowleft \Sigma M_A = 0: \quad a D_y - (5\text{ ft})(500\text{ lb}) - (10\text{ ft})(500\text{ lb}) = 0$$

$$D_y = \frac{7500\text{ lb}\cdot\text{ft}}{a}$$

Since there are no distributed loads, M is piecewise linear, so only points C and D need be considered. Assume a in ft.

At D:

$$\circlearrowleft \Sigma M_D = 0:$$

$$M + \left[(10-a)\text{ ft}\right](500\text{ lb}) = 0$$

$$M_D = -500(10-a)\text{ lb}\cdot\text{ft}$$

At C:

$$\circlearrowleft \Sigma M_C = 0: \quad M + (5\text{ ft})(500\text{ lb}) - \left[5 - (10-a)\right]\text{ft}\,\frac{7500}{a}\text{lb} = 0$$

$$M_C = \left(5000 - \frac{37500}{a}\right)\text{lb}\cdot\text{ft}$$

(*a*) Apparently $M_{\max} = M_C$ and $M_{\min} = M_D$ (recall $5 < a < 10$)

Setting $M_C = -M_D$: $\quad 5000 - \dfrac{37500}{a} = 5000 - 500a$

$$37500 = 500a^2$$

$$a = \sqrt{75} \qquad a = 8.66\text{ ft} \blacktriangleleft$$

(*b*) $$|M|_{\max} = 500\left(10 - \sqrt{75}\right)\text{lb}\cdot\text{ft} \qquad |M|_{\max} = 670\text{ lb}\cdot\text{ft} \blacktriangleleft$$

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Chapter 7, Solution 56.

$$\circlearrowleft \Sigma M_A = 0: \quad aD_y - (5\text{ ft})(250\text{ lb}) - (10\text{ ft})(500\text{ lb}) = 0$$

$$D_y = \frac{6250\text{ lb}\cdot\text{ft}}{a}$$

With a in ft, $D_y = \dfrac{6250}{a}\text{lb}$

Since there are no distributed loads, M is piecewise linear, so only points C and D need be considered.

At D:

$$\Sigma M_D = 0: \quad \left[(10-a)\text{ft}\right](500\text{ lb}) + M_D = 0$$

$$M_D = -500(10-a)\text{lb}\cdot\text{ft}$$

At C:

$$\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ ft})(500\text{ lb}) - \left[5 - (10-a)\right]\text{ft}\left(\frac{6250}{a}\text{lb}\right) = 0$$

$$M_C = \left(3750 - \frac{31250}{a}\right)\text{lb}\cdot\text{ft}$$

(a) Apparently $M_{\max}$ is M_C and $M_{\min}$ is M_D $(5 < a < 10)$

Setting $M_C = -M_D$: $3750 - \dfrac{31250}{a} = -500a + 5000$

$$500a^2 - 1250 - 31250 = 0 \quad \text{or} \quad a^2 - 2.5a - 62.5 = 0$$

$$a = 1.25 \pm 8.004, \text{ positive root } a = 9.254\text{ ft}$$

$a = 9.25$ ft ◀

(b) $|M|_{\max} = 500(10-a)\text{lb}\cdot\text{ft}$ $\qquad |M|_{\max} = 373\text{ lb}\cdot\text{ft}$ ◀

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Chapter 7, Solution 57.

M due to distributed load:

$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

M due to counter weight:

$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(a) **Both applied:**

$$M = W_x - \frac{w}{2}x^2 \qquad \frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \dfrac{W^2}{2w} > 0$ so $M_{\max}$; $M_{\min}$ must be at $x = L$

So $M_{\min} = WL - \dfrac{1}{2}wL^2$. For minimum $|M|_{\max}$ set $M_{\max} = -M_{\min}$, so

$$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \text{ or } W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need +)}$$

$$W = \left(\sqrt{2} - 1\right)wL = 0.414wL \blacktriangleleft$$

(b) ***w* may be removed**

$$M_{\max} = \frac{W^2}{2w} = \frac{\left(\sqrt{2} - 1\right)^2}{2}wL^2 \qquad M_{\max} = 0.858wL^2 \blacktriangleleft$$

Without w, $\quad M = Wx,\ M_{\max} = WL$ at A

With w (see part a) $\quad M = Wx - \dfrac{w}{2}x^2, \quad M_{\max} = \dfrac{W^2}{2w}$ at $x = \dfrac{W}{w}$

$$M_{\min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

For minimum $M_{\max}$, set $M_{\max}(\text{no } w) = -M_{\min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{\max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $\qquad W = \dfrac{1}{4}wL \blacktriangleleft$

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Chapter 7, Solution 58.

(*a*) **FBD Beam:**

$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0 \qquad \mathbf{C} = \frac{M_0}{L} \uparrow$

Shear Diag: $V = -\frac{M_0}{L}$ at A, and remains constant $\left(\frac{dV}{dx} = w = 0\right)$.

Moment Diag: M starts at zero at A and decreases linearly $\left(\frac{dM}{dx} = V = -\frac{M_0}{L}\right)$ to $-\frac{M_0}{2}$ at B, where M jumps by M_0 to $+\frac{M_0}{2}$.

M continues to decrease with slope $-\frac{M_0}{L}$ to zero at C.

(*b*) From diagrams:

$$|V|_{\max} = \frac{M_0}{L} \text{ everywhere} \blacktriangleleft$$

$$|M|_{\max} = \frac{M_0}{2} \text{ at } B \blacktriangleleft$$

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Chapter 7, Solution 59.

(*a*) and (*b*)

By symmetry $A_y = B_y$

$\uparrow \Sigma F_y = 0: \quad 2A_y - 2P = 0 \qquad A_y = P \uparrow \qquad \mathbf{B}_y = P \uparrow$

Shear Diag:

V is piecewise constant with discontinuities equal to P at A, B, C and D in the direction of the loads.

Moment Diag:

M is piecewise linear with slope equal to + P on AB, 0 on BC, $-P$ on CD.

$$M_B = Pa$$

$|V|_{\max} = P$ along AB and CD ◀

$|M|_{\max} = Pa$ along BC ◀

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Chapter 7, Solution 60.

(*a*) and (*b*)

Shear Diag:

Since w is linear, V is quadratic $\left(\dfrac{dV}{dx} = -w\right)$ starting at 0 at A, and decreasing to $-\dfrac{1}{2}w_0 L$ at B.

Moment Diag:

M is zero at A and decreases cubically $\left(\dfrac{dm}{dx} = V\right)$ to

$$\frac{1}{3}\left(-\frac{1}{2}w_0 L\right)L = -\frac{1}{6}w_0L^2 \text{ at } B.$$

$$|V|_{\max} = \frac{1}{2}w_0L \text{ at } B \quad ◀$$

$$|M|_{\max} = \frac{1}{6}w_0L^2 \text{ at } B \quad ◀$$

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Chapter 7, Solution 61.

(*a*) and (*b*)

$\circlearrowleft \Sigma M_B = 0: \quad aP + 2aC_y - 3.5aP = 0$

$$C_y = 1.25P \uparrow$$

$\uparrow \Sigma F_y = 0: \quad B_y - 2P + 1.25P = 0 \qquad B_y = 0.75P \uparrow$

Shear Diag:

V is piecewise constant, equal to –*P* from *A* to *B*, jumping up 0.75*P*, at *B,* to $-0.25P$, and jumping up 1.25 *P*, at *C*, to $+P$.

Moment Diag:

M is zero at *A*, decreasing linearly $\left(\dfrac{dM}{dx} = V = -P\right)$ to $-Pa$ at *B*, and further, $\left(\dfrac{dM}{dx} = V = -0.25P\right)$ to $-Pa - (0.25P)(2a) = -1.5Pa$ at *C*. *M* then increases linearly $\left(\dfrac{dM}{dx} = V = P\right)$ to $-1.5Pa + P(1.5a) = 0$ at *D*, as it must.

$|V|_{\max} = P$ along *AB* and *CD* ◀

$|M|_{\max} = 1.5Pa$ at *C* ◀

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Chapter 7, Solution 62.

(*a*) and (*b*)

$$\circlearrowleft \Sigma M_B = 0: \quad (0.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0\right)$ with discontinuities at each concentrated force. (equal to force)

$$|V|_{\max} = 12.63\text{ kips} \blacktriangleleft$$

Moment Diag:

M is zero at A, and piecewise linear $\left(\dfrac{dM}{dx} = V\right)$ throughout.

$$M_C = (1.8\text{ ft})(12.625\text{ kips}) = 22.725\text{ kip}\cdot\text{ft}$$

$$M_D = 22.725\text{ kip}\cdot\text{ft} + (2.7\text{ ft})(2.625\text{ kips}) = 29.8125\text{ kip}\cdot\text{ft}$$

$$M_E = 29.8125\text{ kip}\cdot\text{ft} - (4.5\text{ ft})(5.375\text{ kips}) = 5.625\text{ kip}\cdot\text{ft}$$

$$M_B = 5.625\text{ kip}\cdot\text{ft} - (0.6\text{ ft})(9.375\text{ kips}) = 0$$

$$|M|_{\max} = 29.8\text{ kip}\cdot\text{ft} \blacktriangleleft$$

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Chapter 7, Solution 63.

(*a*) and (*b*)

FBD Beam:

$$\circlearrowleft \Sigma M_E = 0: \quad (1.1\text{ m})(0.54\text{ kN}) - (0.9\text{ m})C_y + (0.4\text{ m})(1.35\text{ kN}) - (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$\mathbf{C}_y = 1.08\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad -0.54\text{ kN} + 1.08\text{ kN} - 1.35\text{ kN} + E - 0.54\text{ kN} = 0$$

$$\mathbf{E} = 1.35\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0 \text{ everywhere}\right)$ with discontinuities at each concentrated force. (equal to the force)

$$|V|_{\max} = 810\text{ N} \blacktriangleleft$$

Moment Diag:

M is piecewise linear starting with $M_A = 0$

$$M_C = 0 - 0.2\text{ m}(0.54\text{ kN}) = 0.108\text{ kN}\cdot\text{m}$$
$$M_D = 0.108\text{ kN}\cdot\text{m} + (0.5\text{ m})(0.54\text{ kN}) = 0.162\text{ kN}\cdot\text{m}$$
$$M_E = 0.162\text{ kN}\cdot\text{m} - (0.4\text{ m})(0.81\text{ kN}) = -0.162\text{ kN}\cdot\text{m}$$
$$M_B = 0.162\text{ kN}\cdot\text{m} + (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$|M|_{\max} = 0.162\text{ kN}\cdot\text{m} = 162.0\text{ N}\cdot\text{m} \blacktriangleleft$$

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Chapter 7, Solution 64.

$\circlearrowleft \Sigma M_A = 0: \quad (1.5\text{ m})E_y - \left(\frac{1}{2}\text{m}\right)(360\text{ N/m})(1\text{ m}) - (1.3\text{ m})(600\text{ N})$

$-(2\text{ m})(420\text{ N}) = 0 \qquad \mathbf{E}_y = 1200\text{ N} \uparrow$

$\uparrow \Sigma F_y = 0: \quad A_y + 1200\text{ N} - (360\text{ N/m})(1\text{ m}) - 600\text{ N} - 420\text{ N} = 0$

$\mathbf{A}_y = 180.0\text{ N} \uparrow$

(*a*)

Shear Diag:

V jumps to $A_y = 180\text{ N}$ at *A*, then decreases linearly $\left(\frac{dV}{dx} = -w = -360\text{ N/m}\right)$ to $180\text{ N} - (360\text{ N/m})(1\text{ m}) = -180\text{ N}$ at *C*. From *C*, *V* is piecewise constant $(w = 0)$ with jumps of -600 N at *D*, $+1200\text{ N}$ at *E*, -420 N at *B*.

Moment Diag:

M starts at zero at *A* with slope $\frac{dM}{dx} = V = 180\text{ N/m}$, decreasing to zero at $x = 0.5\text{ m}$. There $M = \frac{1}{2}(180\text{ N})(0.5\text{ m}) = 45\text{ N}\cdot\text{m}$. *M* is zero again at *C*, decreasing to $-(180\text{ N})(0.3\text{ m}) = -54\text{ N}\cdot\text{m}$ at *D*. *M* then decreases by $(780\text{ N})(0.2\text{ m}) = 156\text{ N}\cdot\text{m}$ to $-210\text{ N}\cdot\text{m}$ at *E*, and increases by $(420\text{ N})(0.5\text{ m}) = 210\text{ N}\cdot\text{m}$ to zero at *B*.

(*b*) From the diagrams, $|V|_{\max} = 780\text{ N}$ along *EB* ◀

$|M|_{\max} = 210\text{ N}\cdot\text{m}$ at *E* ◀

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Chapter 7, Solution 65.

(*a*)

(*b*)

Shear Diag:

V is zero at A with constant slope $\left(\dfrac{dV}{dx} = -w = -1 \text{ kip/ft}\right)$ decreasing to -3.6 kips at C. V then jumps 9 kips to 5.4 kips and is constant to D. Then V increases with constant slope 1.5 kips/ft for 3 ft, to 9.9 kips at B. This is also equal to B_y.

Moment Diag:

M is zero at A, with zero slope $\left(\dfrac{dM}{dx} = V\right)$ decreasing to -3.6 kips at C, where $M = \dfrac{1}{2}(-3.6 \text{ kips})(3.6 \text{ ft})$, $M_C = -6.48$ kip·ft. M then increases linearly with slope 5.4 kips to $-6.48 \text{ kip}\cdot\text{ft} + (5.4 \text{ kips})(1.8 \text{ ft})$ $= 3.24$ kip·ft at D. Finally, M increases, with increasing slope, to

$$M_B = 3.24 \text{ kip}\cdot\text{ft} + \left(\frac{5.4 \text{ kips} + 9.9 \text{ kips}}{2}\right)(3 \text{ ft}),$$

$$M_B = 26.19 \text{ kip}\cdot\text{ft}.$$

From the diagrams, $|V|_{\max} = 9.90$ kips at B ◀

$|M|_{\max} = 26.2$ kip·ft at B ◀

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Chapter 7, Solution 66.

(*a*)

$$\circlearrowleft \Sigma M_B = 0: (4\text{ m})A_y - (2.7\text{ m})(1.8\text{ kN/m})(2.6\text{ m}) - (2.4\text{ m})(4\text{ kN}) = 0$$

$$\mathbf{A}_y = 5.559\text{ kN} \uparrow$$

Shear Diag:

At A, V jumps up 5.559 kN, then decreases with uniform slope of -1.8 kN/m to 2.679 kN at C. V then jumps down 4 kN to -1.321 kN, and continues with uniform slope -1.8 kN/m to -3.121 kN at D. V is then constant to B. Note: $B_y = 3.121\text{ kN}$

Moment Diag:

M is zero at A, with slope $\dfrac{dM}{dx} = V = 5.559\text{ kN}$. The slope decreases to 2.679 kN at C, where $M = \left(\dfrac{5.559 + 2.679}{2}\text{ kN}\right)(1.6\text{ m})$,

$M_C = 6.59\text{ kN}\cdot\text{m}$. At C the slope drops to -1.321 kN and continues to decrease, $M_D = 6.59\text{ kN}\cdot\text{m} - \left(\dfrac{1.321 + 3.121}{2}\right)\text{kN}(1\text{ m}) = 4.37\text{ kN}\cdot\text{m}$.

M then decreases with uniform slope -3.121 kN, to zero at B.

From the diagrams, $|V|_{\max} = 5.56\text{ kN}$ at A ◀

(*b*) $|M|_{\max} = 6.59\text{ kN}\cdot\text{m}$ at C ◀

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Chapter 7, Solution 67.

(*a*)

$\uparrow \Sigma F_y = 0$: $\quad A_y + (24\text{ kN/m})(2\text{ m}) - 48\text{ kN} - 8\text{ kN} = 0$

$$\mathbf{A}_y = 8\text{ kN} \uparrow$$

$\circlearrowleft \Sigma M_A = 0$: $\quad M_A - (1\text{ m})(24\text{ kN/m})(2\text{ m}) + (3.5\text{ m})(48\text{ kN})$

$$+(4\text{ m})(8\text{ kN}) = 0$$

$$M_A = -152\text{ kN}\cdot\text{m}$$

Shear Diag:

V jumps to 8 kN at A, and increases with uniform slope $\dfrac{dV}{dx} = -w = 24\text{ kN/m}$ to 56 kN at C. V is constant at 56 kN to D, then drops by 8 kN to 8 kN at D, is then constant at 8 kN to B.

Moment Diag:

M starts at $M_A = -152\text{ kN}\cdot\text{m}$, with slope 8 kN, which increases to 56 kN at C, where $M = -152\text{ kN}\cdot\text{m} + \left(\dfrac{8+56}{2}\text{ kN}\right)(2\text{ m}) = -88\text{ kN}\cdot\text{m}$. Then M increases with uniform slope 56 kN to $-88\text{ kN}\cdot\text{m} + (56\text{ kN})(1.5\text{ m}) = -4\text{ kN}\cdot\text{m}$ at D, and finally, with slope 8 kN, to zero at B.

From the diagrams, $\quad |V|_{\max} = 56\text{ kN}$ along CD ◀

(*b*) $\quad |M|_{\max} = 152.0\text{ kN}\cdot\text{m}$ at A ◀

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Chapter 7, Solution 68.

Note: C_x is omitted to avoid clutter.

(*a*)

By symmetry $C_y = G_y$

$\uparrow \Sigma F_y = 0: \quad 2C_y - 2(12 \text{ lb/in.})(10 \text{ in.}) - 2(100 \text{ lb}) - 150 \text{ lb} = 0$

$\mathbf{C}_y = 295 \text{ lb} \uparrow \qquad \mathbf{G}_y = 295 \text{ lb} \uparrow$

Shear Diag:

At A, $V = 0$ and has slope $\dfrac{dV}{dx} = -w = -12$ lb/in. which is uniform to C, where $V = -(12 \text{ lb/in.})(10 \text{ in.}) = -120 \text{ lb}\cdot\text{in.}$ V jumps 295 lb to $+175$ lb at C, is constant to D where it drops 100 lb to 75 lb, is constant to E where it drops 150 lb to -75 lb. The diagram can be completed using symmetry.

Moment Diag:

M is zero at A, with zero slope, which decreases linearly to -120 lb at C, where $M = -\frac{1}{2}(120 \text{ lb})(10 \text{ in.}) = -600 \text{ lb}\cdot\text{in.}$ M then increases, with uniform slope 175 lb, to $-600 \text{ lb}\cdot\text{in.} + (175 \text{ lb})(6 \text{ in.}) = 450 \text{ lb}\cdot\text{in.}$ at D. M then increases, at uniform slope 75 lb, to 450 lb + (75 lb)(6 in.) $= 900 \text{ lb}\cdot\text{in.}$ at E. The diagram can be completed using symmetry.

From the diagrams, $|V|_{\max} = 175.0$ lb along CD and FG ◀

(*b*) $|M|_{\max} = 900 \text{ lb}\cdot\text{in.}$ at center E ◀

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Chapter 7, Solution 69.

(*a*)

$\circlearrowleft \Sigma M_F = 0$: $(18\text{ ft})(1\text{ kip/ft})(6\text{ ft}) - (12\text{ ft})D_y$

$+ (4.5\text{ ft})(2\text{ kips/ft})(9\text{ ft}) + (3\text{ ft})(33\text{ kips}) = 0$

$\mathbf{D}_y = 24\text{ kips} \uparrow$

$\uparrow \Sigma F_y = 0$: $24\text{ kips} + F_y + 33\text{ kips} - (1\text{ kip/ft})(6\text{ ft})$

$- (2\text{ kips/ft})(9\text{ ft}) = 0 \qquad F_y = -33\text{ kips}$

$\mathbf{F}_y = 33\text{ kips} \downarrow$

Shear Diag:

$V = 0$ at A and $\dfrac{dV}{dx} = -1\text{ kip/ft}$ is uniform to C, where $V = -6\text{ kips}$. Then V is constant to D where it jumps up 24 kips to $+18\text{ kips}$, and remains constant to E. From E to F, $\dfrac{dV}{dx} = -2\text{ kips/ft}$ and V decreases by 18 kips to zero at F, where it drops 33 kips, is constant to B, and jumps 33 kips to zero.

Moment Diag:

At A, $M = 0$ and $\dfrac{dM}{dx}$ starts at zero, decreasing to -6 kips at C, where $M = \frac{1}{2}(6\text{ kips})(6\text{ ft}) = -18\text{ kip}\cdot\text{ft}$. M then decreases linearly by $(6\text{ kips})(3\text{ ft})$ to $-36\text{ kip}\cdot\text{ft}$ at D, and increases linearly by $(18\text{ kip})(3\text{ ft})$ to $+18\text{ kip}\cdot\text{ft}$ at E. From E to F, $\dfrac{dM}{dx}$ decreases from 18 kips to zero as M increases by $\frac{1}{2}(18\text{ kips})(9\text{ ft})$ to $99\text{ kip}\cdot\text{ft}$, at F. M then decreases linearly to zero at B.

From the diagrams, $|V|_{\max} = 33.0\text{ kips}$ along FB ◀

$|M|_{\max} = 99.0\text{ kips}$ at F ◀

(*b*)

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Chapter 7, Solution 70.

(*a*)

$\uparrow \Sigma F_y = 0$: $\quad A_y + (200\text{ N/m})(0.8\text{ m}) - (120\text{ N/m})(0.3\text{ m}) = 0$

$$A_y = -124\text{ N} \qquad \mathbf{A}_y = 124\text{ N} \downarrow$$

$\circlearrowleft \Sigma M_A = 0$: $\quad M_A - 60\text{ N}\cdot\text{m} - (0.4\text{ m})(200\text{ N/m})(0.8\text{ m})$

$$+ (1.35\text{ m})(120\text{ N/m})(0.3\text{ m}) = 0 \qquad M_A = 75.4\text{ N}\cdot\text{m}$$

Shear Diag:

V drops to 124 N at A. $\dfrac{dV}{dx} = 200\text{ N/m}$ from A to C and V increases by $(200\text{ N/m})(0.8\text{ m}) = 160\text{ N}$ to $+36\text{ N}$ at C. It remains at 36 N to D, then decreases linearly to zero at B. Note $V = 0$ where $\dfrac{x}{0.8\text{ m}} = \dfrac{124\text{ N}}{160\text{ N}}$, or $x = 0.62$ m.

Moment Diag:

M jumps to 75.4 at A where $\dfrac{dM}{dx} = -124$ N. The slope increases to zero at $x = 0.62$ m, where $M = 75.4\text{ N}\cdot\text{m} - \dfrac{1}{2}(124\text{ N})(0.62\text{ m})$ $= 36.96\text{ N}\cdot\text{m}$. The slope then increases as M increases by $\dfrac{1}{2}(36\text{ N})(0.18\text{ m}) = 3.24\text{ N}\cdot\text{m}$ to $40.2\text{ N}\cdot\text{m}$ at C, where it drops $60\text{ N}\cdot\text{m}$ to $-19.8\text{ N}\cdot\text{m}$. M increases linearly by $(36\text{ N})(0.4\text{ m}) = 14.4\text{ N}\cdot\text{m}$ to $-5.4\text{ N}\cdot\text{m}$, and finally M increases quadratically by $\dfrac{1}{2}(36\text{ N})(0.3\text{ m}) = 5.4\text{ N}\cdot\text{m}$ to zero at B where $\dfrac{dM}{dx}$ is also zero.

From the diagrams, $\quad |V|_{\max} = 124.0\text{ N}$ at A ◀

$|M|_{\max} = 75.4\text{ N}$ at A ◀

(*b*)

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Chapter 7, Solution 71.

(*a*)

$$\circlearrowleft \Sigma M_A = 0: \quad -(3\text{ m})(9\text{ kN}) - 27\text{ kN}\cdot\text{m} - (9\text{ m})(12\text{ kN}) + (12\text{ m})F$$

$$+ (16.5\text{ m})(3\text{ kN}) + 22.5\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{F} = 7.5\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 9\text{ kN} - 12\text{ kN} + 7.5\text{ kN} + 3\text{ kN} = 0$$

$$\mathbf{A}_y = 10.5\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, with jumps at A, C, E, F, and B, equal to the forces there.

Moment Diag:

M is piecewise linear with jumps at D and B equal to the couples there.

$$M_C = (10.5\text{ kN})(3\text{ m}) = 31.5\text{ kN}\cdot\text{m}$$

$$M_{D^-} = 31.5\text{ kN}\cdot\text{m} + (1.5\text{ kN})(3\text{ m}) = 36.0\text{ kN}\cdot\text{m}$$

$$M_{D^+} = 36\text{ kN}\cdot\text{m} + 27\text{ kN}\cdot\text{m} = 63\text{ kN}\cdot\text{m}$$

$$M_E = 63\text{ kN}\cdot\text{m} + (1.5\text{ kN})(3\text{ m}) = 67.5\text{ kN}\cdot\text{m}$$

$$M_F = 67.5\text{ kN}\cdot\text{m} - (10.5\text{ kN})(3\text{ m}) = 36\text{ kN}\cdot\text{m}$$

$$M_{B^-} = 36\text{ kN}\cdot\text{m} - (3\text{ kN})(4.5\text{ m}) = 22.5\text{ kN}\cdot\text{m}$$

Finally M drops $22.5\text{ kN}\cdot\text{m}$ to zero at B

(*b*) From the diagrams, $|V|_{\max} = 10.50\text{ kN}$ along AC and EF ◀

$|M|_{\max} = 67.5\text{ kN}\cdot\text{m}$ at E ◀

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Chapter 7, Solution 72.

(*a*)

$\circlearrowleft \Sigma M_A = 0: \quad (3\text{ m})B_y - (2.1\text{ m})(2.5\text{ kN/m})(4.2\text{ m}) = 0$

$\mathbf{B}_y = 7.35\text{ kN} \uparrow$

$\uparrow \Sigma F_y = 0: \quad A_y - (2.5\text{ kN/m})(4.2\text{ m}) + 7.35\text{ kN} = 0$

$\mathbf{A}_y = 3.15\text{ kN} \uparrow$

Shear Diag:

V has slope $\dfrac{dV}{dx} = -2.5\text{ kN/m}$ throughout, and jumps at A and B equal to the forces there.

$$V_{B^-} = 3.15\text{ kN} - (2.5\text{ kN/m})(3\text{ m}) = -4.35$$

$$V_{B^+} = -4.35\text{ kN} + 7.35\text{ kN} = 3\text{ kN}$$

$$V_C = 3\text{ kN} - (2.5\text{ kN/m})(1.2\text{ m}) = 0$$

Note, $V = 0$ where $3.15\text{ kN} - (2.5\text{ kN/m})x = 0$, $x = 1.26\text{ m}$.

Moment Diag:

At A, $M = 0$ and $\dfrac{dM}{dx} = 3.15\text{ kN}$. The slope decreases to zero at $x = 1.26\text{ m}$ and to -4.35 kN at B, jumps to 3.0 kN and decreases to 0 at C.

$$M_D = \frac{1}{2}(3.15\text{ kN})(1.26\text{ m}) = 1.9845\text{ kN}\cdot\text{m}$$

$$M_B = 1.9845\text{ kN}\cdot\text{m} - \frac{1}{2}(4.35\text{ kN})(3\text{ m} - 1.26\text{ m}) = -1.80\text{ kN}\cdot\text{m}$$

$$M_C = -1.80\text{ kN}\cdot\text{m} + \frac{1}{2}(3\text{ kN})(1.2\text{ m}) = 0$$

From the diagrams, $|V|_{\max} = 4.35\text{ kN}$ at B

(*b*) $|M|_{\max} = 1.985\text{ kN}$ at D $(1.26\text{ m from } A)$ ◀

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Chapter 7, Solution 73.

(*a*)

Note: $\mathbf{C}_x$ omitted to avoid clutter.

$$\circlearrowleft \Sigma M_C = 0: \quad -(0.5\text{ m})(2\text{ kN/m})(2\text{ m}) + (1.5\text{ m})(1\text{ kN}) - (3\text{ m})(4.5\text{ kN/m})(1\text{ m}) + (3.5\text{ m})B_y = 0$$

$$\mathbf{B}_y = 4.0\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y - (2\text{ kN/m})(2\text{ m}) + 1\text{ kN} - (4.5\text{ kN/m})(1\text{ m}) + 4.0\text{ kN} = 0 \qquad C_y = 3.5\text{ kN} \uparrow$$

Shear Diag:

At A, $V = 0$ and $\dfrac{dV}{dm} = -2$ kN/m, so V decreases to -1 at C, then jumps 3.5 kN to $+2.5$ kN, and continues to decrease, with the same slope, to -0.5 kN at D, jumps 1 kN to $+0.5$ kN from D to E. Then V decreases at rate 4.5 kN/m, to -4.0 kN at B. Note that $V = 0$ where $(-4.5\text{ kN/m})(x) = -4$ kN, $x = \dfrac{8}{9}$m, and where $(2\text{ kN/m})(y) = 2.5$, $y = \dfrac{5}{4}$m.

Moment Diag:

At A, M and $\dfrac{dM}{dx} = 0$, with the slope decreasing to -1 kN at C, where $M = -\dfrac{1}{2}(1\text{ kN})(0.5\text{ m}) = -0.25\text{ kN}\cdot\text{m}$. The slope jumps to 2.5 kN and decreases to zero at F and to -0.5 kN at D.

$$M_F = -0.25\text{ kN}\cdot\text{m} + \frac{1}{2}(2.5\text{ kN})\left(\frac{5}{4}\text{m}\right) = 1.3125\text{ kN}\cdot\text{m}$$

$$M_D = 1.3125\text{ kN}\cdot\text{m} - \frac{1}{2}(0.5\text{ kN})\left(\frac{1}{4}\text{m}\right) = 1.25\text{ kN}\cdot\text{m}.$$

From D, M increases by $(0.5\text{ kN})(1\text{ m})$ to $1.75\text{ kN}\cdot\text{m}$ at G. M continues to increase to $1.75\text{ kN}\cdot\text{m} + \dfrac{1}{2}(0.5\text{ kN})\left(\dfrac{1}{9}\text{m}\right) = 1.7778\text{ kN}\cdot\text{m}$ at G and then decreases by $\dfrac{1}{2}(4\text{ kN})\left(\dfrac{8}{9}\text{m}\right) = 1.7778\text{ kN}\cdot\text{m}$, to zero at B.

(*b*) From the diagrams, $|V|_{\max} = 4.00$ kN at B

$|M|_{\max} = 1.778\text{ kN}\cdot\text{m}$ at G $\left(\dfrac{4}{9}\text{m from } B\right)$ ◀

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Chapter 7, Solution 74.

(*a*)

$\uparrow \Sigma F_y = 0: \quad A_y - (2 \text{ kips/ft})(8 \text{ ft}) - 3 \text{ kips} + 7 \text{ kips} = 0$

$$\mathbf{A}_y = 12 \text{ kips} \uparrow$$

$\circlearrowleft \Sigma M_A = 0: \quad M_A + (4 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) - (14 \text{ ft})(3 \text{ kips})$

$$- (20 \text{ ft})(7 \text{ kips}) = 0 \qquad \mathbf{M}_A = 34 \text{ kip}\cdot\text{ft} \circlearrowright$$

Shear Diag:

V jumps to 12 kips at A, then decreases at 2 kips/ft to -4 kips at C to D. V drops 3 kips to -7 kips from D to B and jumps 7 kips to zero. Note: $V = 0$ where $12 \text{ kips} - (2 \text{ kips/ft})x = 0, \ x = 6 \text{ ft}$.

Moment Diag:

M jumps to $34 \text{ kip}\cdot\text{ft}$ at A and then increases with decreasing slope to $34 \text{ kip}\cdot\text{ft} + \frac{1}{2}(12 \text{ kips})(6 \text{ ft}) = 70 \text{ kip}\cdot\text{ft}$ at E, and decreases by $\frac{1}{2}(4 \text{ kips})(2 \text{ ft}) = 4 \text{ kip}\cdot\text{ft}$, to $66 \text{ kip}\cdot\text{ft}$ at C. M then decreases by $(4 \text{ kips})(6 \text{ ft})$ to $42 \text{ kip}\cdot\text{ft}$ at D, and by $(7 \text{ kips})(6 \text{ ft})$ to zero at B.

$$|M|_{\max} = 70 \text{ kip}\cdot\text{ft} \text{ at } E \blacktriangleleft$$

(*b*)

$\uparrow \Sigma F_y = 0: \quad A_y - (2 \text{ kips/ft})(8 \text{ ft}) - 3 \text{ kips} + 10 \text{ kips} = 0$

$$\mathbf{A}_y = 9 \text{ kips} \uparrow$$

$\circlearrowleft \Sigma M_A = 0: \quad M_A + (4 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) + (14 \text{ ft})(3 \text{ kips})$

$$- (20 \text{ ft})(10 \text{ kips}) = 0 \qquad \mathbf{M}_A = 94 \text{ kip}\cdot\text{ft} \circlearrowright$$

Shear Diag:

V jumps to 9 kips at A, then decreases, at 2 kips/ft, to -7 kips at C to D, drops 3 kips to -10 kips from D to B and jumps 10 kips to 0.

Note: $V = 0$ where $9 \text{ kips} - (2 \text{ kips/ft})x = 0, \ x = 4.5 \text{ ft}$.

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Moment Diag:

M jumps to $94\ \text{kip}\cdot\text{ft}$ at A and increases to $94\ \text{kip}\cdot\text{ft} + \frac{1}{2}(9\ \text{kips})(4.5\ \text{ft}) = 114.25\ \text{kip}\cdot\text{ft}$, then decreases by $\frac{1}{2}(7\ \text{kips})(3.5\ \text{ft})$ to $102\ \text{kip}\cdot\text{ft}$ at C. M decreases linearly by $(7\ \text{kips})(6\ \text{ft})$ to $60\ \text{kip}\cdot\text{ft}$ at D, then by $(10\ \text{kips})(6\ \text{ft})$ to zero at B.

$|M|_{\max} = 114.3\ \text{kip}\cdot\text{ft}$ at E $(4.5\ \text{ft from } A)$ ◀

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Chapter 7, Solution 75.

$\circlearrowleft \Sigma M_A = 0: \quad (15\text{ ft})C_y + (9\text{ ft})(600\text{ lb}) - (4.5\text{ ft})(800\text{ lb/ft})(9\text{ ft}) = 0$

$\mathbf{C}_y = 1800\text{ lb} = 1.8\text{ kips} \uparrow$

$\uparrow \Sigma F_y = 0: \quad A_y + 1800\text{ lb} - (800\text{ lb})(9\text{ ft}) + 600\text{ lb} = 0$

$\mathbf{A}_y = 4800\text{ lb} = 4.8\text{ kips} \uparrow$

Shear Diag:

V jumps to 4.8 kips at A then decreases linearly, at 0.8 kips/ft, to -2.4 kips at B, jumps 0.6 kips to -1.8 kips, is constant to C, and jumps 1.8 kips to zero.

Note: $V = 0$ at D, where $4.8\text{ kips} - (800\text{ kip/ft})x = 0$, $x = 6.0$ ft.

Moment Diag:

M starts at zero and increases with decreasing slope to $\frac{1}{2}(4.8\text{ kips})(6\text{ ft}) = 14.4\text{ kip}\cdot\text{ft}$ at D, then decreases by $\frac{1}{2}(2.4\text{ kips})(3\text{ ft})$ to $10.8\text{ kip}\cdot\text{ft}$ at B. M then decreases with slope -1.8 kips to zero at C.

$|M|_{\max} = 14.40$ kips at D (6 ft from A) ◀

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Chapter 7, Solution 76.

(*a*)

$$\circlearrowleft \Sigma M_A = 0: \quad (6\text{ m})B_y - (1\text{ m})(20\text{ kN/m})(2\text{ m}) - 24\text{ kN}\cdot\text{m}$$

$$-(5\text{ m})(20\text{ kN/m})(2\text{ m}) = 0 \qquad \mathbf{B}_y = 44\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 2(20\text{ kN/m})(2\text{ m}) + 44\text{ kN} = 0$$

$$\mathbf{A}_y = 36\text{ kN}\uparrow$$

Shear Diag:

V jumps to 36 kN at A, then decreases with slope –20 kN/m to -4 kN at C, is constant to E, then decreases with slope –20 kN/m to -44 kN at B.

Note: $V = 0$ at F where $36\text{ kN} - (20\text{ kN/m})x = 0$, $x = 1.8$ m.

Moment Diag:

Starting at zero M increases with decreasing slope to $\frac{1}{2}(36\text{ kN})(1.8\text{ m})$ $= 32.4\text{ kN}\cdot\text{m}$ at F, decreases by $\frac{1}{2}(4\text{ kN})(0.2\text{ m})$ to $32\text{ kN}\cdot\text{m}$ at C, then with slope -4 kN to $28\text{ kN}\cdot\text{m}$ at D, where it jumps to $52\text{ kN}\cdot\text{m}$, M decreases with slope -4 kN to $48\text{ kN}\cdot\text{m}$ at E, then with increasingly negative slope by $\left(\frac{4+44}{2}\text{ kN}\right)(2\text{ m})$ to zero at B.

(*b*)

$$|M|_{\max} = 52\text{ kN}\cdot\text{m (at } D) \blacktriangleleft$$

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Chapter 7, Solution 77.

(*a*)

$$\circlearrowleft \Sigma M_A = 0: \quad (6\text{ m})B_y - (1\text{ m})(20\text{ kN/m})(2\text{ m}) + 24\text{ kN}\cdot\text{m}$$

$$-(5\text{ m})(20\text{ kN/m})(2\text{ m}) = 0 \qquad \mathbf{B}_y = 36\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 2(20\text{ kN/m})(2\text{ m}) + 36\text{ kN} = 0$$

$$\mathbf{A}_y = 44\text{ kN}\uparrow$$

Shear Diag:

V jumps to 44 kN at A, then decreases with slope -20 kN/m to 4 kN at C, is constant to E, then decreases with slope -20 kN/m to -36 kN at B. $V = 0$ at F where $-36\text{ kN} + (20\text{ kN/m})x = 0$, $x = 1.8$ m.

Moment Diag:

M starts at zero, increases with decreasing slope to $\left(\frac{44+4}{2}\text{kN}\right)(2\text{ m})$ $= 48\text{ kN}\cdot\text{m}$ at C, increases with slope 4 kN to $52\text{ kN}\cdot\text{m}$ at D, drops $24\text{ kN}\cdot\text{m}$ to $28\text{ kN}\cdot\text{m}$ then increases with slope 4 kN to $32\text{ kN}\cdot\text{m}$ at E. Then M increases with decreasing slope, by $\frac{1}{2}(4\text{ kN})(0.2\text{ m})$ to $32.4\text{ kN}\cdot\text{m}$ at F and decreases with increasingly negative slope to zero at B.

(*b*) $|M|_{\max} = 52.0\text{ kN}\cdot\text{m}\ (\text{at } D)$ ◀

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Chapter 7, Solution 78.

(*a*)

Note: The 2 kip force at *E* has been replaced by the equivalent force and couple at *C*.

$$\circlearrowleft \Sigma M_A = 0: \; -(6\text{ ft})(1\text{ kip/ft})(12\text{ ft}) + 8\text{ kip}\cdot\text{ft} - (12\text{ ft})(2\text{ kips}) + (24\text{ ft})D_y - (32\text{ ft})(1\text{ kip}) = 0 \qquad \mathbf{D}_y = 5\text{ kips}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - (1\text{ kip/ft})(12\text{ ft}) - 2\text{ kips} + 5\text{ kips} - 1\text{ kip} = 0$$

$$\mathbf{A}_y = 10\text{ kips}\uparrow$$

Shear Diag:

From 10 kips at *A*, *V* decreases with slope -1 kip/ft to -2 kips at *C*, drops 2 kips, is constant at -4 kips to *D*, jumps 5 kips, and is constant at 1 kip to *B*.
$V = 0$ at *E*, where $10\text{ kips} - (1\text{ kip/ft})x = 0, \;\; x = 10\text{ ft}$.

Moment Diag:

From zero at *A*, *M* increases with decreasing slope to $\frac{1}{2}(10\text{ kips})(10\text{ ft}) = 50\text{ kip}\cdot\text{ft}$ at F, decreases by $\frac{1}{2}(2\text{ kips})(2\text{ ft})$ to $48\text{ kip}\cdot\text{ft}$ at C, drops $8\text{ kip}\cdot\text{ft}$ to $40\text{ kip}\cdot\text{ft}$, then decreases with slope -4 kips to $-8\text{ kip}\cdot\text{ft}$ at *D*. Finally *M* increases with slope 1 kip to zero at *B*.

(*b*) $|M|_{\max} = 50\text{ kip}\cdot\text{ft}$ at F $(10.00\text{ ft from } A)$ ◀

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Chapter 7, Solution 79.

(*a*)

Note: The 2 kip force at E has been replaced by the equivalent force and couple at C.

$$\circlearrowleft \Sigma M_A = 0: \quad -(6\text{ ft})(1\text{ kip/ft})(12\text{ ft}) + (12\text{ ft})(2\text{ kips}) - 8\text{ kip}\cdot\text{ft}$$

$$+ (24\text{ ft})D_y - (32\text{ ft})(1\text{ kip}) = 0 \qquad \mathbf{D}_y = \frac{11}{3}\text{ kips}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - (1\text{ kip/ft})(12\text{ ft}) + 2\text{ kips} - \frac{11}{3}\text{ kips} - 1\text{ kip} = 0$$

$$\mathbf{A}_y = \frac{22}{3}\text{ kips}\uparrow$$

Shear Diag:

Starting at $\frac{22}{3}$ kips at A, V decreases with slope -1 kip/ft to $-\frac{14}{3}$ kips at C, jumps 2 kips and remains constant at $-\frac{8}{3}$ kips to D, jumps $\frac{11}{3}$ kips and remains constant at 1 kip to B, drops to zero.

$V = 0$ at F, where $\frac{22}{3}\text{kip} - (1\text{ kip/ft})x = 0, \quad x = \frac{22}{3}\text{ft}.$

Moment Diag:

Starting from zero, M increases with decreasing slope to $\frac{1}{2}\left(\frac{22}{3}\text{ kips}\right)\left(\frac{22}{3}\text{ ft}\right) = 26.889\text{ kip}\cdot\text{ft}$ at F. M then decreases by $\frac{1}{2}\left(\frac{14}{3}\text{ kips}\right)\left(\frac{14}{3}\text{ ft}\right)$ to $16\text{ kip}\cdot\text{ft}$ at C, jumps to $24\text{ kip}\cdot\text{ft}$, decreases with slope $-\frac{8}{3}$ kips to $-8\text{ kip}\cdot\text{ft}$ at D, and finally increases with slope 1 kip to zero at B.

(*b*)

$$|M|_{\max} = 26.9\text{ kip}\cdot\text{ft at } F\ (7.33\text{ ft from } A) \blacktriangleleft$$

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Chapter 7, Solution 80.

(*b*)

(*a*) Distributed load $\qquad w = w_0\left[4\left(\frac{x}{L}\right) - 1\right]$

Shear: $\qquad \frac{dV}{dx} = -w,$ and $V(0) = 0,$ so

$$V = \int_0^x -w\,dx = -\int_0^{x/L} Lw\,d\left(\frac{x}{L}\right)$$

$$V = \int_0^{x/L} w_o L\left[1 - 4\left(\frac{x}{L}\right)\right]d\left(\frac{x}{L}\right) = w_0 L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]$$

Notes: At $x = L, V = -w_0 L$

And $V = 0$ at $\qquad \left(\frac{x}{L}\right) = 2\left(\frac{x}{L}\right)^2 \quad$ or $\quad \frac{x}{L} = \frac{1}{2}$

Also V is max where $w = 0 \ \left(\frac{x}{L} = \frac{1}{4}\right)$

$$V_{\max} = \frac{1}{8} w_0 L$$

Moment: $\qquad M(0) = 0, \frac{dM}{dx} = V$

$$M = \int_0^x v\,dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0 L^2 \int_0^{x/L}\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

$$V = w_0 L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right] \blacktriangleleft$$

$$M = w_0 L^2\left[\frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{2}{3}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

$$M_{\max} = \frac{1}{24} w_0 L^2 \text{ at } x = \frac{L}{2}$$

$$M_{\min} = -\frac{1}{6} w_0 L^2 \text{ at } x = L$$

$$M_{\max} = \frac{w_0 L^2}{24} \text{ at } x = \frac{L}{2}$$

(*c*) $\qquad |M|_{\max} = -M_{\min} = \frac{w_0 L^2}{6}$ at $B \blacktriangleleft$

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Chapter 7, Solution 81.

(*b*)

$$\uparrow \Sigma F_y = 0: \quad A_y - \frac{1}{2}(2w_0)(2a) + \frac{1}{2}w_0 a = 0 \qquad A_y = \frac{3}{2}w_0 a$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A + \frac{2}{3}(2a)\frac{1}{2}(2w_0)(2a) - \left(\frac{7}{3}a\right)\frac{1}{2}(w_0)(a) = 0$$

$$M_A = -\frac{3}{2}w_0 a^2$$

(*a*) **for $0 \le x \le 2a$:** $\quad w = \dfrac{w_0}{a}x$

$$V = \frac{3}{2}w_0 a - \int_0^x \frac{w_0}{a}x\,dx = \frac{w_0}{2a}\left(3a^2 - x^2\right) \blacktriangleleft$$

$$M = -\frac{3}{2}w_0 a^2 + \int_0^x \frac{w_0}{2a}\left(3a^2 - x^2\right)dx$$

$$M = \frac{w_0}{6a}\left(-9a^3 + 9a^2 x - x^3\right) \blacktriangleleft$$

Note: $V = 0$ at $x = \sqrt{3}a$, where $M = 0.232\,w_0 a^2$

At $x = 2a$,

$$M = \frac{1}{6}w_0 a^2$$

for $2a \le x \le 3a$: $\quad w = -3w_0 + \dfrac{w_0}{a}x$

$$V = -\frac{1}{2}w_0 a + \int_{2a}^x \frac{w_0}{a}(3a - x)\,dx$$

$$V = \frac{w_0}{2a}\left(-9a^2 + 6ax - x^2\right) \blacktriangleleft$$

$$M = \frac{1}{6}w_0 a + \int_{2a}^x \frac{w_0}{2a}\left(-9a^2 + 6ax - x^2\right)dx$$

$$M = \frac{w_0}{6a}\left(27a^3 - 27a^2 x + 9ax^2 - x^3\right) \blacktriangleleft$$

(*c*)

$$|M|_{\max} = \frac{3}{2}w_0 a^2 \text{ at } A\ (x = 0) \blacktriangleleft$$

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Chapter 7, Solution 82.

(*a*)

FBD Beam:

$$\circlearrowleft \Sigma M_B = 0:\ (3a)\left[\frac{1}{2}w_0(3a)\right] - 5aA_y = 0 \qquad \mathbf{A}_y = 0.9w_0a \uparrow$$

$$\uparrow \Sigma F_y = 0:\ 0.9w_0a - \frac{1}{2}w_0(3a) + B = 0$$

$$\mathbf{B} = 0.6w_0a \uparrow$$

Shear Diag:

$V = A_y = 0.9w_0a$ from A to C and $V = B = -0.6w_0a$ from B to D.

Then from D to C, $w = w_0\dfrac{x_1}{3a}$. If x_1 is measured right to left, $\dfrac{dV}{dx_1} = +w$ and $\dfrac{dM}{dx_1} = -V$. So, from D, $V = -0.6w_0a + \int_0^{x_1}\dfrac{w_0}{3a}x_1dx_1$,

$$V = w_0a\left[-0.6 + \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]$$

Note: $V = 0$ at $\left(\dfrac{x_1}{a}\right)^2 = 3.6$, $x_1 = \sqrt{3.6}a$

Moment Diag:

$M = 0$ at A, increasing linearly $\left(\dfrac{dM}{dx_1} = 0.9w_0a\right)$ to $M_C = 0.9w_0a^2$.

Similarly $M = 0$ at B increasing linearly $\left(\dfrac{dM}{dx} = 0.6w_0a\right)$ to

$M_D = 0.6w_0a^2$. Between C and D

$$M = 0.6w_0a^2 + w_0a\int_0^{x_1}\left[0.6 - \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]dx_1,$$

$$M = w_0a^2\left[0.6 + 0.6\left(\frac{x_1}{a}\right) - \frac{1}{18}\left(\frac{x_1}{a}\right)^3\right]$$

(*b*)

At $\dfrac{x_1}{a} = \sqrt{3.6}$, $M = M_{\max} = 1.359w_0a^2$ ◀

$x_1 = 1.897a$ left of D ◀

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Chapter 7, Solution 83.

$$\circlearrowleft \Sigma M_A = 0:\ LB_y - \frac{L}{5}\left(\frac{1}{4}w_0L\right) = 0 \qquad \mathbf{B}_y = \frac{w_0L}{20}\uparrow$$

$$V = -\frac{w_0L}{20} + \int_0^{x_1} \frac{w_0}{L^3}x_1^3\,dx_1 = \frac{w_0}{20L^3}\left(5x_1^4 - L^4\right)$$

or $$V = \frac{w_0}{20\,L^3}\left[5\left(L-x\right)^4 - L^4\right] \blacktriangleleft$$

Note: $V = 0$ at $x_1 = 5^{-1/4}L = 0.6687\ L$

or $x = 0.3313\ L$

$$M = 0 + \int_0^{x_1} \frac{w_0}{20L^3}\left(5x_1^4 - L^4\right)dx_1$$

$$= \frac{w_0}{20L^3}\left(x_1^5 - L^4x_1\right)$$

or $$M = \frac{w_0}{20L^3}\left[\left(L-x\right)^5 - L^4\left(L-x\right)\right] \blacktriangleleft$$

$$M_{\max} = M \text{ at } x_1 = 0.6687\,L, \quad x = 0.331\,L \blacktriangleleft$$

$$M_{\max} = 0.0267\ w_0L^2 \blacktriangleleft$$

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Chapter 7, Solution 84.

(*a*)

FBD section AC:

$\circlearrowleft \Sigma M_C = 0$: $(395 - 215)\,\text{N}\cdot\text{m} + (0.1\text{ m})(1000\text{ N/m})(0.2\text{ m})$

$$-(0.2\text{ m})V_A = 0, \qquad V_A = 1000\text{ N}$$

$\circlearrowleft \Sigma M_B = 0$: $395\text{ N}\cdot\text{m} + (0.45\text{ m})(1\text{ kN/m})(0.4\text{ m})$

$$-(0.65\text{ m})(1\text{ kN}) + (0.25\text{ m})P = 0$$

$$P = 0.3\text{ kN}, \qquad \mathbf{P} = 300\text{ N}\downarrow \blacktriangleleft$$

FBD whole:

(*b*)

$\uparrow \Sigma F_y = 0$: $1000\text{ N} - 400\text{ N} - 300\text{ N} - Q = 0$

$$\mathbf{Q} = 300\text{ N}\downarrow \blacktriangleleft$$

Shear Diag:

Starting at 1000 N, V decreases with slope -1000 N/m to 600 N at D, drops 300 N and is constant to B where it drops 300 N to zero.

Moment Diag:

Starting at $-395\text{ N}\cdot\text{m}$, M increases with decreasing slope to $-395\text{ N}\cdot\text{m}$ $+\left(\dfrac{1000+600}{2}\text{N}\right)(0.4\text{ m}) = -75\text{ N}\cdot\text{m}$ at D, then increases with slope 300 N to zero at B.

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Chapter 7, Solution 85.

(*a*)

FBD section AC:

$$\circlearrowleft \Sigma M_C = 0: \quad (395 - 215)\,\text{N}\cdot\text{m} - (0.2\ \text{m})A_y + (0.1\ \text{m})(1000\ \text{N/m})(0.2\ \text{m}) = 0$$

$$\mathbf{A}_y = 1000\ \text{N} \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: \quad 395\ \text{N}\cdot\text{m} + (0.325\ \text{m})(1000\ \text{N/m})(0.65\ \text{m}) + (0.25\ \text{m})P = 0 - (0.65\ \text{m})(1000\ \text{N}) = 0$$

$$\mathbf{P} = 175.0\ \text{N} \downarrow \blacktriangleleft$$

(*b*)

FBD Whole:

$$\uparrow \Sigma F_y = 0: \quad 1000\ \text{N} - 175\ \text{N} - Q - (1000\ \text{N/m})(0.65\ \text{m}) = 0$$

$$\mathbf{Q} = 175.0\ \text{N} \downarrow \blacktriangleleft$$

Shear Diag:

V starts at 1000 N, and has slope -1000 N/m throughout, but with drops of 175 N at D and B.

Moment Diag:

M starts at $-395\ \text{N}\cdot\text{m}$ and increases with decreasing slope to $-295\ \text{N}\cdot\text{m} + \left(\dfrac{1000 + 600}{2}\text{N}\right)(0.4\ \text{m}) = -75\ \text{N}\cdot\text{m}$ at D. There is a discontinuity in slope, and M increases by $\left(\dfrac{425 + 175}{2}\text{N}\right)(0.25\ \text{m})$ to zero at B.

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Chapter 7, Solution 86.

(*a*)

FBD AC:

$\circlearrowleft \Sigma M_C = 0: \quad 2.7 \text{ kN}\cdot\text{m} - (0.2 \text{ m})A_y = 0, \qquad \mathbf{A}_y = 1.35 \text{ kN}\uparrow$

$\circlearrowleft \Sigma M_D = 0: \quad (0.2 \text{ m})D_y - 2.5 \text{ kN}\cdot\text{m} = 0, \qquad \mathbf{D}_y = 1.25 \text{ kN}\uparrow$

$\circlearrowleft \Sigma M_C = 0: \quad -(2 \text{ m})(1.35 \text{ kN}) - (2 \text{ m})(0.4 \text{ kN/m})(4 \text{ m}) - (4 \text{ m})Q$

$+ (6 \text{ m})(1.25 \text{ kN}) = 0, \quad Q = 0.4 \text{ kN}$

$\mathbf{Q} = 400 \text{ N}\downarrow \blacktriangleleft$

FBD AB:

$\uparrow \Sigma F_y = 0: \quad 1.35 \text{ kN} - P - (0.4 \text{ kN/m})(4 \text{ m}) - 0.4 \text{ kN} + 1.25 \text{ kN} = 0$

$P = 0.6 \text{ kN}$

$\mathbf{P} = 600 \text{ N}\downarrow \blacktriangleleft$

(*b*)

FBD Whole:

Shear Diag:

V is constant at 1.35 kN from A to C, drops 0.6 kN, then decreases with slope –0.4 kN/m (-1.6 kN) to -0.85 kN at D, drops 0.4 kN to -1.25 kN, and is constant to B.

$V = 0$ where $0.75 \text{ kN} - (0.4 \text{ kN/m})x = 0, \qquad x = 1.875 \text{ m}.$

Moment Diag:

From zero at A, M increases with slope 1.35 kN to $2.70 \text{ kN}\cdot\text{m}$ at C, the slope drops to 0.75 kN and then decreases to zero at E, where $M = 2.7 \text{ kN}\cdot\text{m} + \frac{1}{2}(0.75 \text{ kN})(1.875 \text{ m}) = 3.403 \text{ kN}\cdot\text{m}.$ This curve continues to D where $M = 3.403 \text{ kN}\cdot\text{m} - \frac{1}{2}(0.85 \text{ kN})(2.125 \text{ m})$ $= 2.50 \text{ kN}\cdot\text{m},$ then M decreases with slope -1.25 kN to zero at B.

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Chapter 7, Solution 87.

(*a*)

FBD AC:

$\circlearrowleft \Sigma M_C = 0$: $2.7 \text{ kN}\cdot\text{m} - (1.35 \text{ m})A_y = 0, \quad \mathbf{A}_y = 2 \text{ kN}\uparrow$

$\circlearrowleft \Sigma M_D = 0$: $2.5 \text{ kN}\cdot\text{m} - (2 \text{ m})B_y = 0, \quad \mathbf{B}_y = 1.25 \text{ kN}\uparrow$

$\circlearrowleft \Sigma M_C = 0$: $(6.65 \text{ m})(1.25 \text{ kN}) - (4.65 \text{ m})Q$

$$- (2.325 \text{ m})(0.4 \text{ kN/m})(4.65 \text{ m}) - (1.35 \text{ m})(2 \text{ kN}) = 0$$

$$Q = 0.27699 \text{ kN}$$

$$\mathbf{Q} = 277 \text{ N}\downarrow \blacktriangleleft$$

FBD DB:

$\uparrow \Sigma F_y = 0$: $2 \text{ kN} - P - 0.277 \text{ kN} - (0.4 \text{ kN/m})(4.65 \text{ m}) + 1.25 \text{ kN} = 0$

$$\mathbf{P} = 1.113 \text{ kN}\downarrow \blacktriangleleft$$

(*b*)

FBD Whole:

Shear Diag:

V is constant at 2 kN from A to C, drops 1.113 kN to 0.887 kN, then decreases with slope -0.4 kN/m to $0.887 \text{ kN} - (0.4 \text{ kN/m})(4.65 \text{ m}) = -0.973 \text{ kN}$, drops 0.277 kN to -1.25 kN and is constant to B.

$V = 0$ where $0.887 \text{ kN} - (0.4 \text{ kN/m})x = 2.2175 \text{ m}$ at E.

Moment Diag:

Starting from zero, M increases with slope 2 kN to $2.7 \text{ kN}\cdot\text{m}$ at C. The slope drops to 0.887 kN and decreases to zero at E where $M = 2.7 \text{ kN} + \frac{1}{2}(0.887 \text{ kN})(2.2175 \text{ m}) = 3.68 \text{ kN}\cdot\text{m}$. This curve continues to D where $M = 2.5 \text{ kN}\cdot\text{m}$, then M decreases linearly to zero at B.

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Chapter 7, Solution 88.

FBD Cable:

$$\circlearrowleft \Sigma M_A = 0: \quad (5\text{ m})D_y - (4\text{ m})(2.25\text{ kN}) - (2\text{ m})(3\text{ kN}) = 0$$

$$\mathbf{D}_y = 3\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 3\text{ kN} - 2.25\text{ kN} + 3\text{ kN} = 0$$

$$\mathbf{A}_y = 2.25\text{ kN}\uparrow$$

Point D:

$$\rightarrow \Sigma F_x = 0: \quad -A_x + D_x = 0, \qquad A_x = D_x \qquad (1)$$

Since $A_x = D_x$ and $D_y > A_y$, T_{CD} is T_{max}

$$\uparrow \Sigma F_y = 0: \quad 3\text{ kN} - \frac{3}{5}T_{CD} = 0 \qquad T_{CD} = 5\text{ kN}$$

$$\rightarrow \Sigma F_x = 0: \quad -\frac{4}{5}(5\text{ kN}) + D_y = 0 \qquad \mathbf{D}_x = 4\text{ kN}\rightarrow$$

From(1), $\mathbf{A}_x = 4\text{ kN}\leftarrow$

Point A:

$$\frac{d_B}{2m} = \frac{2.25\text{ kN}}{4\text{ kN}}$$

(*a*) $d_B = 1.125\text{ m}$ ◀

(*b*) $\mathbf{A}_x = 4.00\text{ kN}\leftarrow$ ◀

$\mathbf{A}_y = 2.25\text{ kN}\uparrow$ ◀

(*c*) $T_{\max} = 5.00\text{ kN}$ ◀

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Chapter 7, Solution 89.

FBD Cable:

$\circlearrowleft \Sigma M_A = 0: \quad (5\text{ m})D_y - (4\text{ m})(2.25\text{ kN}) - (2\text{ m})(3\text{ kN}) = 0$

$\mathbf{D}_y = 3\text{ kN}\uparrow$

$\uparrow \Sigma F_y = 0: \quad A_y - 3\text{ kN} - 2.25\text{ kN} + 3\text{ kN} = 0$

$\mathbf{A}_y = 2.25\text{ kN}\uparrow$

$\rightarrow \Sigma F_x = 0: \quad -A_x + D_x = 0 \qquad A_x = D_x$

Since $A_x = D_x$, and $D_y > A_y$, $\quad T_{\max}$ is $T_{CD} = 3.6$ kN

Point D:

$$\frac{d_C}{3\text{ kN}} = \frac{1\text{ m}}{D_x} = \frac{\sqrt{1 + d_C^2}}{3.6\text{ kN}}$$

$$1.2\,d_C = \sqrt{1 + d_C^2},$$

$$1.44\,d_C^2 = 1 + d_C^2$$

$$d_C = 1.50756\text{ m}$$

Point A:

Also $D_x = 3\text{ kN}\left(\dfrac{1\text{ m}}{d_C}\right) = 1.98997\text{ kN} = A_x$

$$\frac{d_B}{2.25\text{ kN}} = \frac{2\text{ m}}{1.9900\text{ kN}}$$

(*a*) $d_B = 2.26$ m ◀

(*b*) $d_C = 1.508$ m ◀

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Chapter 7, Solution 90.

FBD Cable:

$$\circlearrowleft \Sigma M_A = 0: \quad 4aE_y - 3a(1.2 \text{ kips}) - 2a(0.8 \text{ kips}) - a(0.4 \text{ kips}) = 0$$

$$\mathbf{E}_y = 1.4 \text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_Y - (0.4 + 0.8 + 1.2 \text{ kips}) + 1.4 \text{ kips} = 0$$

$$\mathbf{A}_y = 1.0 \text{ kips} \uparrow$$

$$\rightarrow \Sigma F_x = 0: \quad -A_x + E_x = 0, \qquad A_x = E_x$$

Since $A_x = E_x$ and $E_y > A_y$, $\qquad T_{\max} = T_{DE}$

FBD CDE:

$$\circlearrowleft \Sigma M_C = 0: \quad (30 \text{ ft})(1.4 \text{ kips}) - (12 \text{ ft})E_x - (15 \text{ ft})(1.2 \text{ kips}) = 0$$

(*a*) $\qquad \mathbf{E}_x = 2.00 \text{ kips} \blacktriangleleft$

$\mathbf{E}_y = 1.400 \text{ kips} \blacktriangleleft$

(*b*) $\qquad T_{\max} = T_{DE} = 2.44 \text{ kips} \blacktriangleleft$

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Chapter 7, Solution 91.

FBD Cable:

$$\circlearrowleft \Sigma M_A = 0: \quad 4aE_y - 3a(1.2 \text{ kips}) - 2a(0.8 \text{ kips}) - a(0.4 \text{ kips}) = 0$$

$$\mathbf{E}_y = 1.4 \text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - (0.4 + 0.8 + 1.2 \text{ kips}) + 1.4 \text{ kips} = 0$$

$$\mathbf{A}_y = 1.0 \text{ kips} \uparrow$$

$$\rightarrow \Sigma F_x = 0: \quad -A_x + E_x = 0, \qquad A_x = E_x$$

Point E:

Since $A_x = E_x$, and $E_y > A_y$, $\quad T_{\max} = T_{DE}$

$$E_x = \sqrt{(5 \text{ kips})^2 + (1.4 \text{ kips})^2} = 4.8 \text{ kips}$$

FBD CDE:

$$\circlearrowleft \Sigma M_C = 0: \quad (30 \text{ ft})(1.4 \text{ kips}) - d_C(4.8 \text{ kips}) - (15 \text{ ft})(1.2 \text{ kips}) = 0$$

$$d_C = 5.00 \text{ ft} \blacktriangleleft$$

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Chapter 7, Solution 92.

(*a*) **FBD Whole:**

$$\circlearrowleft \Sigma M_A = 0: \quad (1.2\text{ m})E_x + (4\text{ m})E_y - (1\text{ m})(1.8\text{ kN}) - (2\text{ m})(3.6\text{ kN}) - (3\text{ m})(1.2\text{ m}) = 0$$

$$1.2E_x + 4E_y = 12.6\text{ kN} \qquad (1)$$

FBD CDE:

$$\circlearrowleft \Sigma M_C = 0: \quad (2\text{ m})E_y - (0.6\text{ m})E_x - (1\text{ m})(1.2\text{ kN}) = 0$$

$$-0.6E_x + 2E_y = 1.2\text{ kN} \qquad (2)$$

Solving (1) and (2) $\quad \mathbf{E}_x = 4.25\text{ kN} \rightarrow$

$\mathbf{E}_y = 1.875\text{ kN} \uparrow$

(*a*) $\quad \mathbf{E} = 4.65\text{ kN} \angle 23.8° \blacktriangleleft$

(*b*) **From FBD whole:**

AB:

$\rightarrow \Sigma F_x = 0 \qquad -A_x + 4.25\text{ kN} = 0 \qquad \mathbf{A}_x = 4.25\text{ kN} \leftarrow$

$\uparrow \Sigma F_y = 0 \qquad A_y - 1.8\text{ kN} - 3.6\text{ kN} - 1.2\text{ kN} + 1.875\text{ kN} = 0$

$\mathbf{A}_y = 4.725\text{ kN} \uparrow$

$$\circlearrowleft \Sigma M_B = 0: \quad d_B(4.25\text{ kN}) - (1\text{ m})(4.725\text{ kN}) = 0$$

$$d_B = 1.112\text{ m} \blacktriangleleft$$

DE:

$$\circlearrowleft \Sigma M_D = 0: \quad (1\text{ m})(1.875\text{ kN}) - (d_D - 1.2\text{ m})(4.25\text{ kN}) = 0$$

$$d_D = 1.641\text{ m} \blacktriangleleft$$

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Chapter 7, Solution 93.

FBD DE:

$\uparrow \Sigma F_y = 0: \quad E_y - 1.2 \text{ kN} = 0 \qquad \mathbf{E}_y = 1.2 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_D = 0: \quad (1 \text{ m})(1.2 \text{ kN}) - (d_C - 1.2 \text{ m})E_x = 0 \qquad (1)$

FBD Whole:

$\circlearrowleft \Sigma M_A = 0: \quad (1.2 \text{ m})E_x + (4 \text{ m})(1.2 \text{ kN}) - (3 \text{ m})(1.2 \text{ kN})$

$- (2 \text{ m})(3.6 \text{ kN}) - (1 \text{ m})(1.8 \text{ kN}) = 0$

$\mathbf{E}_x = 6.5 \text{ kN} \rightarrow$

(*a*) then, from (1) $\qquad d_C = 1.385 \text{ m}$ ◀

$\rightarrow \Sigma F_x = 0: \quad -A_x + 6.5 \text{ kN} = 0 \qquad \mathbf{A}_x = 6.5 \text{ kN} \leftarrow$

$\uparrow \Sigma F_y = 0: \quad A_y - 1.8 \text{ kN} - 3.6 \text{ kN} - 1.2 \text{ kN} + 1.2 \text{ kN} = 0$

$\mathbf{A}_y = 5.4 \text{ kN} \uparrow$

So (*b*) $\qquad \mathbf{A} = 8.45 \text{ kN} \angle 39.7°$ ◀

$\mathbf{E} = 6.61 \text{ kN} \angle 10.46°$ ◀

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Chapter 7, Solution 94.

FBD Cable:

Hanger forces at A and F act on the supports, so $\mathbf{A}_y$ and $\mathbf{F}_y$ act on the cable.

$$\circlearrowleft \Sigma M_F = 0: \quad (6\text{ ft} + 12\text{ ft} + 18\text{ ft} + 24\text{ ft})(400\text{ lb}) - (30\text{ ft})A_y - (5\text{ ft})A_x = 0$$

$$A_x + 6A_y = 4800\text{ lb} \quad (1)$$

FBD ABC:

$$\circlearrowleft \Sigma M_C = 0: \quad (7\text{ ft})A_x - (12\text{ ft})A_y + (6\text{ ft})(400\text{ lb}) = 0 \quad (2)$$

Solving (1) and (2) $\mathbf{A}_x = 800\text{ lb} \leftarrow$

$$\mathbf{A}_y = \frac{2000}{3}\text{ lb} \uparrow$$

From FBD Cable: $\rightarrow \Sigma F_x = 0: \quad -800\text{ lb} + F_x = 0$

$$\mathbf{F}_x = 800\text{ lb} \rightarrow$$

FBD DEF:

$$\uparrow \Sigma F_y = 0: \quad \frac{2000}{3}\text{ lb} - 4(400\text{ lb}) + F_y = 0$$

$$\mathbf{F}_y = \frac{2800}{3}\text{ lb} \uparrow$$

Since $A_x = F_x$ and $F_y > A_y$, $\quad T_{\max} = T_{EF} = \sqrt{(800\text{ lb})^2 + \left(\frac{2800}{3}\text{lb}\right)^2}$

(*a*) $T_{\max} = 1229.27$ lb, $\qquad T_{\max} = 1.229$ kips ◀

$$\circlearrowleft \Sigma M_D = 0: \quad (12\text{ ft})\left(\frac{2800}{3}\text{ lb}\right) - d_D(800\text{ lb}) - (6\text{ ft})(400\text{ lb}) = 0$$

(*b*) $\qquad d_D = 11.00$ ft ◀

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Chapter 7, Solution 95.

FBD CDEF:

$$\circlearrowleft \Sigma M_C = 0: \quad (18\text{ ft})F_y - (9\text{ ft})F_y - (6\text{ ft} + 12\text{ ft})(400\text{ lb}) = 0$$

$$F_x - 2F_y = -800\text{ lb} \qquad (1)$$

FBD Cable:

$$\circlearrowleft \Sigma M_A = 0: \quad (30\text{ ft})F_y - (5\text{ ft})F_x - (6\text{ ft})(1 + 2 + 3 + 4)(400\text{ lb}) = 0$$

$$F_x - 6F_y = -4800\text{ lb} \qquad (2)$$

Solving (1) and (2), $\mathbf{F}_x = 1200\text{ lb} \rightarrow$, $\mathbf{F}_y = 1000\text{ lb} \uparrow$

$\rightarrow \Sigma F_x = 0: \quad -A_x + 1200\text{ lb} = 0, \qquad \mathbf{A}_x = 1200\text{ lb} \rightarrow$

$\uparrow \Sigma F_y = 0: \quad A_y + 1000\text{ lb} - 4(400\text{ lb}) = 0, \qquad \mathbf{A}_y = 600\text{ lb} \uparrow$

Point F:

Since $A_x = A_y$ and $F_y > A_y$, $T_{\max} = T_{EF}$

$$T_{\max} = \sqrt{(1\text{ kip})^2 + (1.2\text{ kips})^2}$$

(*a*) $\qquad T_{\max} = 1.562\text{ kips}$ ◀

FBD DEF:

$$\circlearrowleft \Sigma M_D = 0: \quad (12\text{ ft})(1000\text{ lb}) - d_D(1200\text{ lb}) - (6\text{ ft})(400\text{ lb}) = 0$$

(*b*) $\qquad d_D = 8.00\text{ ft}$ ◀

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Chapter 7, Solution 96.

FBD BC:

$\circlearrowleft \Sigma M_A = 0: \quad (9\text{ ft})P - (6\text{ ft})(30\text{ lb}) - a(20\text{ lb}) = 0$

$$P = \left(20 + \frac{20a}{9}\right)\text{ lb with } a \text{ in ft.} \qquad (1)$$

$\rightarrow \Sigma F_x = 0: \quad -T_{ABx} + P = 0 \qquad T_{ABx} = P$

$\uparrow \Sigma F_y = 0: \quad T_{ABy} - 20\text{ lb} - 30\text{ lb} = 0 \qquad T_{ABy} = 50\text{ lb}$

But $\quad \dfrac{T_{ABx}}{T_{ABy}} = \dfrac{a}{7} \quad$ so $\quad P = \dfrac{50a}{7} \qquad (2)$

Solving (1) and (2), $\quad a = 4.0645\text{ ft}, \quad P = 29.032\text{ lb}$

(*a*) $P = 29.0\text{ lb}$ ◀

(*b*) $a = 4.06\text{ ft}$ ◀

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Chapter 7, Solution 97.

FBD C:

$$\circlearrowleft \Sigma M_B = 0: \quad (2\text{ ft})(25\text{ lb}) - (6\text{ ft} - a)(30\text{ lb}) = 0$$

$$a = \frac{13}{3}\text{ ft}$$

$$a = 4.33\text{ ft} \blacktriangleleft$$

FBD BC:

$$\circlearrowleft \Sigma M_A = 0: \quad (b + 2\text{ ft})(25\text{ lb}) - (6\text{ ft})(30\text{ lb}) - \left(\frac{13}{3}\text{ ft}\right)(20\text{ lb}) = 0$$

$$b = \frac{32}{3}\text{ ft}$$

$$b = 10.67\text{ ft} \blacktriangleleft$$

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Chapter 7, Solution 98.

FBD B:

$\rightarrow \Sigma F_x = 0: \quad -\frac{12}{13}T_{AB} + \frac{3}{5}T_{BC} + 1.32 \text{ kN} = 0$

$\uparrow \Sigma F_y = 0: \quad \frac{5}{13}T_{AB} - \frac{4}{5}T_{BC} = 0$

Solving: $T_{AB} = 2.08$ kN, $T_{BC} = 1$ kN

By inspection of A,

(a) $\mathbf{A} = 2.08$ kN $\measuredangle$ 22.6° ◀

FBD C:

$\rightarrow \Sigma F_x = 0: \quad \frac{12}{13}T_{CD} - \frac{3}{5}(1 \text{ kN}) = 0, \qquad T_{CD} = 0.65$ kN

$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}(1 \text{ kN}) + \frac{5}{13}(0.65 \text{ kN}) - w = 0$

$w = 1.05$ kN

(b) $m = \frac{w}{g} = \frac{1050 \text{ N}}{9.81 \text{ m/s}^2} = 107.03$ kg $\qquad m = 107.0$ kg ◀

(c) From above $\qquad T_{AB} = 2.08$ kN ◀

$T_{BC} = 1.000$ kN ◀

$T_{CD} = 650$ N ◀

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Chapter 7, Solution 99.

FBD C:

$$W = mg = (150\text{ kg})(9.81\text{ m/s}^2) = 1471.5\text{ N} = 1.4715\text{ kN}$$

$$\rightarrow \Sigma F_x = 0: \quad \frac{12}{13}T_{CD} - \frac{3}{5}T_{BC} = 0$$

$$\uparrow \Sigma F_y = 0: \quad \frac{5}{13}T_{CD} + \frac{4}{5}T_{BC} - 1.4715\text{ kN} = 0$$

Solving: $T_{CD} = 0.91093\text{ kN} = 911\text{ N}$, $T_{BC} = 1.40143\text{ kN}$

FBD B:

By inspection of D, (*a*) $\mathbf{D} = 911\text{ N} \angle 22.6°$ ◀

$$\rightarrow \Sigma F_x = 0: \quad P - \frac{12}{13}T_{AB} + \frac{3}{5}(1.40143\text{ kN}) = 0$$

$$\uparrow \Sigma F_y = 0: \quad \frac{5}{13}T_{AB} - \frac{4}{5}(1.40143\text{ kN}) = 0, \qquad T_{AB} = 2.91497\text{ kN}$$

(*b*) $\mathbf{P} = 1.850\text{ kN} \rightarrow$ ◀

From above (*c*) $T_{AB} = 2.91\text{ kN}$ ◀

$T_{BC} = 1.401\text{ kN}$ ◀

$T_{CD} = 911\text{ N}$ ◀

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Chapter 7, Solution 100.

FBD E:

$$\frac{E_x}{1} = \frac{E_y}{1} = \frac{T_{DE}}{\sqrt{2}} \qquad E_x = E_y \qquad (1)$$

FBD CDE:

$$\circlearrowleft \Sigma M_C = 0: \quad (6.5\text{ m})E_y - (4.2\text{ m})E_x - (3.5\text{ m})w_D = 0$$

$$2.3E_x = 3.5w_D, \qquad E_x = E_y = \frac{35}{23}w_D$$

$$\circlearrowleft \Sigma M_B = 0: \quad (8.1\text{ m} - 3\text{ m})\left(\frac{35}{23}w_D\right) - (5.1\text{ m})w_D - (1.6\text{ m})w_C = 0$$

$$w_C = 1.66304w_D, \qquad m_C = 1.66304\,m_D$$

$$(a) \qquad m_C = 56.5\text{ kg} \blacktriangleleft$$

FBD B:

$$\rightarrow \Sigma F_x = 0: \quad -\frac{4}{5}T_{BC} + E_x = 0, \qquad T_{BC} = \frac{5}{4}E_x$$

$$\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}\left(\frac{5}{4}E_x\right) - \frac{1}{\sqrt{2}}T_{AB} - \frac{24}{25}T_{BF} = 0$$

$$\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}T_{AB} - \frac{7}{25}T_{BF} - \frac{3}{5}\left(\frac{5}{4}E_x\right) = 0$$

Solving: $\dfrac{E_x}{4} = \dfrac{31}{25}T_{BF}, \qquad T_{BF} = \dfrac{25}{124}E_x$

$$T_{BF} = \frac{25}{124}\left(\frac{35}{23}\right)w_D = 0.30680w_D$$

$$w_D = (34\text{ kg})(9.81\text{ N/kg}) \qquad (b) \qquad T_{BF} = 102.3\text{ N} \blacktriangleleft$$

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Chapter 7, Solution 101.

FBD E:

$$\frac{E_x}{1} = \frac{E_y}{1} \qquad E_x = E_y$$

$$\circlearrowleft \Sigma M_C = 0: \quad (6.5\text{ m})E_y - (4.2\text{ m})E_x - (3.5\text{ m})w_D = 0$$

$$2.3E_x = 3.5w_D, \qquad E_x = E_y = \frac{35}{23}w_D$$

FBD CDE:

$$\circlearrowleft \Sigma M_B = 0: \quad (8.1\text{ m} - 3\text{ m})\frac{35}{23}w_D - (5.1\text{ m})w_D - (1.6\text{ m})w_C = 0$$

$$w_D = 0.60131\,w_C, \qquad m_D = 0.60131m_C = 33.072\text{ kg}$$

$$(a) \qquad m_D = 33.1\text{ kg} \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad -\frac{4}{5}T_{BC} = E_x, \qquad T_{BC} = \frac{5}{4}E_x$$

Point B:

$$\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}\left(\frac{5}{4}E_x\right) - \frac{1}{\sqrt{2}}T_{AB} - \frac{24}{25}T_{BF} = 0$$

$$\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}T_{AB} - \frac{7}{25}T_{BF} + \frac{3}{5}\left(\frac{5}{4}E_x\right) = 0$$

Solving: $\frac{31}{25}T_{BF} = \frac{1}{4}E_x, \qquad T_{BF} = \frac{25}{124}E_x$

$$T_{BF} = \frac{25}{124}\left(\frac{35}{23}w_D\right) = \frac{25}{124}\frac{35}{23}(33.072\text{ kg})(9.81\text{ m/s}^2)$$

$$(b) \qquad T_{BF} = 99.5\text{ N} \blacktriangleleft$$

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Chapter 7, Solution 102.

(*a*) **FBD half wire:**

Since $h = 1.5 \text{ m} \ll L = 30 \text{ m}$ we can approximate the load as evenly distributed horizontally, and the length $S = L$.

$$W = \left(0.6 \frac{\text{kg}}{\text{m}}\right)\left(9.81 \frac{\text{N}}{\text{kg}}\right)(30 \text{ m}) = 176.58 \text{ N}$$

$$\circlearrowleft \Sigma M_B = 0: \quad (15 \text{ m})(176.58 \text{ N}) - (1.5 \text{ m})T_C = 0$$

$$T_C = 1765.8 \text{ N}$$

$$T_{\max} = T_B = \sqrt{T_C^2 + W^2}$$

$$T_{\max} = \sqrt{(1765.8 \text{ N})^2 + (176.58 \text{ N})^2}, \qquad T_{\max} = 177.5 \text{ kN} \blacktriangleleft$$

(*b*)

$$S_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \ldots\right]$$

$$= (30 \text{ m})\left[1 + \frac{2}{3}\left(\frac{1.5}{30}\right)^2 - \frac{2}{5}\left(\frac{1.5}{30}\right)^4 + \ldots\right] = 30.05 \text{ m} \qquad S_{\text{tot}} = 60.1 \text{ m} \blacktriangleleft$$

Note: the third term in the brackets is unnecessary

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Chapter 7, Solution 103.

Half-cable FBDs:

$T_{1x} = T_{2x}$ to create zero horizontal force on tower $\rightarrow$ thus $T_{01} = T_{02}$

FBD I:

$$\circlearrowleft \Sigma M_B = 0: \quad (15\text{ m})\left[w(30\text{ m})\right] - h_1 T_0 = 0$$

$$h_1 = \frac{(450\text{ m}^2)w}{T_0}$$

FBD II:

$$\circlearrowleft \Sigma M_B = 0: \quad (2\text{ m})T_0 - (10\text{ m})\left[w(20\text{ m})\right] = 0$$

$$T_0 = (100\text{ m})w$$

$$(a) \qquad h_1 = \frac{(450\text{ m}^2)w}{(100\text{ m})w} = 4.50\text{ m} \blacktriangleleft$$

FBD I:

$$\rightarrow \Sigma F_x = 0: \quad T_{1x} - T_0 = 0$$

$$T_{1x} = (100\text{ m})w$$

$$\uparrow \Sigma F_y = 0: \quad T_{1y} - (30\text{ m})w = 0$$

$$T_{1y} = (30\text{ m})w$$

$$T_1 = \sqrt{(100\text{ m})^2 + (30\text{ m})^2}\,w$$

$$= (104.4\text{ m})(0.4\text{ kg/m})(9.81\text{ m/s}^2)$$

$$= 409.7\text{ N}$$

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FBD II:

$$\uparrow \Sigma F_y = 0: \quad T_{2y} - (20\text{ m})w = 0$$

$$T_{2y} = (20\text{ m})w$$

$$T_{2x} = T_{1x} = (100\text{ m})w$$

$$T_2 = \sqrt{(100\text{ m})^2 + (20\text{ m})^2}\,w = 400.17\text{ N}$$

(*b*) $T_1 = 410\text{ N}$ ◀

$T_2 = 400\text{ N}$ ◀

*Since $h \quad L$ it is reasonable to approximate the cable weight as being distributed uniformly along the horizontal. The methods of section 7.10 are more accurate for cables sagging under their own weight.

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Chapter 7, Solution 104.

FBD half-span:

(*a*) $\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{2075 \text{ ft}}{2}\right)(23032.5 \text{ kips}) - (464 \text{ ft})T_0 = 0$

$$T_0 = 47{,}246 \text{ kips}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47{,}246 \text{ kips})^2 + (23{,}033 \text{ kips})^2} = 56{,}400 \text{ kips} \blacktriangleleft$$

(*b*) $$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (2075 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^4 + \cdots\right]$$

$s_B = 2142 \text{ ft} \qquad l = 2s_B \qquad\qquad l = 4284 \text{ ft} \blacktriangleleft$

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Chapter 7, Solution 105.

FBD half-span:

$$W = (9.75 \text{ kips/ft})(1750 \text{ ft}) = 17{,}062.5 \text{ kips}$$

$$\circlearrowleft \Sigma M_B = 0: \ (875 \text{ ft})(17{,}065 \text{ kips}) - (316 \text{ ft})T_0 = 0$$

$$T_0 = 47{,}246 \text{ kips}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47{,}246 \text{ kips})^2 + (17{,}063 \text{ kips})^2}$$

$$(a) \qquad T_{\max} = 50{,}200 \text{ kips} \blacktriangleleft$$

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (1750 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^4 + \cdots\right]$$

$$= 1787.3 \text{ ft}$$

$$(b) \qquad l = 2s_B = 3575 \text{ ft} \blacktriangleleft$$

* To get 3-digit accuracy, only two terms are needed.

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Chapter 7, Solution 106.

FBD pipe:

neglecting friction

$\circlearrowleft \Sigma M_B = 0: \quad r\left(T_{\max} - W_B\right) = 0, \quad T_{\max} = W_B = 60 \text{ lb}$

FBD half-cord:

Assuming the weight to be evenly distributed horizontally, and $S = L$

$W = \left(0.02 \text{ lb/ft}\right)\left(75 \text{ ft}\right) = 1.5 \text{ lb}$

$T_0 = \sqrt{\left(60 \text{ lb}\right)^2 - \left(1.5 \text{ lb}\right)^2} = 59.981 \text{ lb}$

$\Sigma M_B = \left(37.5 \text{ ft}\right)\left(1.5 \text{ lb}\right) - h\left(59.981 \text{ lb}\right) = 0$

(*a*) $\quad h = 0.93780 \text{ ft}, \quad h = 11.25 \text{ in.}$ ◀

(*b*) $\quad \theta_B = \sin^{-1}\dfrac{1.5 \text{ lb}}{60 \text{ lb}} = 1.43254°, \qquad \theta_B = 1.433°$ ◀

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Chapter 7, Solution 107.

(*a*) **FBD ship:**

$$\rightarrow \Sigma F_x = 0: \qquad T_0 - 300 \text{ N} = 0, \qquad T_0 = 300 \text{ N}$$

FBD half-span:*

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(300 \text{ N})^2 = (54 \text{ N})^2} = 305 \text{ N} \blacktriangleleft$$

(*b*) $$\circlearrowleft \Sigma M_A = 0: \; hT_x - \frac{L}{4}W = 0, \qquad h = \frac{LW}{4T_x}$$

$$s = x\left[1 + \frac{2}{3}\left(\frac{4}{x}\right)^2 + \cdots\right] \qquad \text{but} \qquad y_A = h = \frac{LW}{4T_x} \qquad \text{so} \qquad \frac{y_A}{x_A} = \frac{W}{2T_x}$$

$$(2.5 \text{ m}) = \frac{L}{2}\left[1 + \frac{2}{3}\left(\frac{53.955 \text{ N}}{600 \text{ N}}\right)^2 - \cdots\right] \rightarrow L = 4.9732 \text{ m}$$

$$\text{So } h = \frac{LW}{4T_x} = 0.2236 \text{ m} \qquad h = 224 \text{ mm} \blacktriangleleft$$

*See note Prob. 7.103

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Chapter 7, Solution 108.

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

Knowing

$$l = 2s_{\text{TOT}} = L\left[1 + \frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \frac{2}{5}\left(\frac{h}{L/2}\right)^2 + \cdots\right]$$

Winter:

$$l_w = (4260\text{ ft})\left[1 + \frac{2}{3}\left(\frac{386\text{ ft}}{2130\text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{386\text{ ft}}{2130\text{ ft}}\right)^4 + \cdots\right] = 4351.43\text{ ft}$$

Summer:

$$l_s = (4260\text{ ft})\left[1 + \frac{2}{3}\left(\frac{394\text{ ft}}{2130\text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{394\text{ ft}}{2130\text{ ft}}\right)^4 + \cdots\right] = 4355.18\text{ ft}$$

$$\Delta l = l_s - l_w = 3.75\text{ ft} \blacktriangleleft$$

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Chapter 7, Solution 109.

FBD whole:

$$\circlearrowleft \Sigma M_D = 0: \quad (2\text{m})T_{Ax} - (5\text{ m})(98.1 + 147.15)\text{ N} = 0$$

$$T_{Ax} = 1103.6\text{ N}$$

FBD half-cable:

$$\rightarrow \Sigma F_x = 0: \quad T_0 - T_{Ax} = 0, \qquad T_0 = 1103.6\text{ N}$$

$$\uparrow \Sigma F_y = 0: \quad T_{Ay} = 49.05\text{ N} = 0, \qquad T_{Ay} = 49.05\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad h(1103.6\text{ N}) - (2.5\text{ m})(49.05\text{ N}) = 0$$

$$h = 0.11111\text{ m}$$

(*a*) $h = 111.1\text{ mm}$ ◀

(*b*) $\theta = \tan^{-1}\dfrac{T_{Ay}}{T_{Ax}} = \tan^{-1}\dfrac{49.05}{1103.6} = 2.5449°, \qquad \theta = 2.54°$ ◀

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Chapter 7, Solution 110.

FBD AB:

$$\circlearrowleft \Sigma M_A = 0: \quad (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:

$$\circlearrowleft \Sigma M_C = 0: \quad (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

Solving (1) and (2) $\quad T_{By} = 28{,}798 \text{ kips}$

$T_{Bx} = 51{,}425 \text{ kips}$

$$T_{\max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2}, \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that $\quad (a) \quad T_{\max} = 58{,}900 \text{ kips}$ ◀

$(b) \quad \theta_B = 29.2°$ ◀

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Chapter 7, Solution 111.

Cable profile:

Eqn: $y = \dfrac{w}{2T_0}x^2$

$x_B - x_A = 45$ in.

at A: $\quad 4 \text{ in.} = \dfrac{w}{2T_0}\left(x_B - 45 \text{ in.}\right)^2 \qquad (1)$

at B: $\quad 1 \text{ in.} = \dfrac{w}{2T_0}x_B^2 \qquad (2)$

$$\therefore \quad \frac{\left(x_B - 45 \text{ in.}\right)^2}{x_B^2} = 4$$

or $\quad x_B^2 + 30x_B - 675 = 0$

$x_B = \left(-15 \pm 30\right)\text{in.} \qquad x_B = 15$ in.

$x_A = x_B - 45$ in. $\qquad x_A = -30$ in.

(a) lowest point ($x = 0$) is 30 in. from A ◀

From (2), $\quad T_0 = \dfrac{wx_B^2}{2} = \dfrac{1}{2}\left(\dfrac{0.18}{12}\dfrac{\text{lb}}{\text{in.}}\right)\left(15 \text{ in.}\right)^2$

$T_0 = 1.6875$ lb

T_{max} occurs at A where slope is maximum

$$T_{\max} = \sqrt{T_0^2 + \left(wx_A\right)^2} = \sqrt{\left(1.6875 \text{ lb}\right)^2 + \left(\frac{0.18}{12}\frac{\text{lb}}{\text{in.}}\right)\left(-30 \text{ in.}\right)^2} = 1.74647 \text{ lb}$$

(b) $\quad T_{\max} = 1.747$ lb ◀

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Chapter 7, Solution 112.

$$\circlearrowleft \Sigma M_A = 0: \quad y_A T_0 - \frac{a}{2} wa = 0 \qquad \circlearrowleft \Sigma M_B = 0: \quad -y_B T_0 + \frac{b}{2} wb = 0$$

$$y_A = \frac{wa^2}{2T_0} \qquad\qquad y_B = \frac{wb^2}{2T_0}$$

$$d = (y_B - y_B) = \frac{w}{2T_0}\left(b^2 - a^2\right)$$

But $$T_0 = \sqrt{T_B^2 - (wb)^2} = \sqrt{T_{\max}^2 - (wb)^2}$$

$$\therefore \quad (2d)^2\left[T_{\max}^2 - (wb)^2\right] = w^2\left(b^2 - a^2\right)^2 = L^2 w^2\left(4b^2 - 4Lb + L^2\right)$$

or $$4\left(L^2 + d^2\right)b^2 - 4L^3 b + \left(L^4 - 4d^2 \frac{T_{\max}^2}{w^2}\right) = 0$$

Using $L = 6$ m, $d = 0.9$ m, $T_{\max} = 8$ kN, $w = (85 \text{ kg/m})\left(9.81 \text{ m/s}^2\right) = 833.85$ N/m

yields $b = (2.934 \pm 1.353)$m, $b = 4.287$ m $(\text{since } b > 3 \text{ m})$

(*a*) $a = 6 \text{ m} - b = 1.713 \text{ m}$ ◀

$$T_0 = \sqrt{T_{\max}^2 - (wb)^2} = 7156.9 \text{ N}$$

$$\frac{y_A}{x_A} = \frac{wa}{2T_0} = 0.09979, \qquad \frac{y_B}{x_B} = \frac{wb}{2T_0} = 0.24974$$

$$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 + \cdots\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

$$= (1.713 \text{ m})\left[1 + \frac{2}{3}(0.09979)^2\right] + (4.287 \text{ m})\left[1 + \frac{2}{3}(0.24974)^2\right] = 6.19 \text{ m}$$

(*b*) $l = 6.19$ m ◀

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Chapter 7, Solution 113.

Geometry:

FBD Segment:

$$\circlearrowleft \Sigma M_P = 0: \quad \frac{x}{2} wx - yT_0 = 0$$

$$y = \frac{wx^2}{2T_0} \qquad \text{so} \qquad \frac{y}{x} = \frac{wx}{2T_0}$$

$$\text{and } d = y_B - y_A = \frac{w}{2T_0}\left(b^2 - a^2\right)$$

Also

$$l = s_A + s_B \doteq a\left[1 + \frac{2}{3}\left(\frac{y_A}{a}\right)^2\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{b}\right)^2\right]$$

$$l - L \doteq \frac{2}{3}\left[\left(\frac{y_A}{a}\right)^2 + \left(\frac{y_B}{b}\right)^2\right] = \frac{w^2}{6T_0^2}\left(a^3 + b^3\right)$$

$$= \frac{1}{6}\frac{4d^2}{\left(b^2 - a^2\right)^2}\left(a^3 + b^3\right) = \frac{2}{3}\frac{d^2\left(a^3 + b^3\right)}{\left(b^2 - a^2\right)^2}$$

Using $l = 6.4$ m, $L = 6$ m, $d = 0.9$ m, $b = 6$ m $- a$, and solving for a, knowing that $a < 3$ ft

$$a = 2.2196 \text{ m} \qquad (a) \qquad a = 2.22 \text{ m} \blacktriangleleft$$

Then $$T_0 = \frac{w}{2d}\left(b^2 - a^2\right)$$

And with $$w = (85 \text{ kg/m})\left(9.81 \text{ m/s}^2\right) = 833.85 \text{ N/m}$$

And

$$b = 6 \text{ m} - a = 3.7804 \text{ m} \qquad T_0 = 4338 \text{ N}$$

$$T_{\max} = T_B = \sqrt{T_0^2 + (wb)^2}$$

$$= \sqrt{(4338 \text{ N})^2 + (833.85 \text{ N/m})^2 (3.7804 \text{ m})^2}$$

$$T_{\max} = 5362 \text{ N} \qquad (b) \qquad T_{\max} = 5.36 \text{ kN} \blacktriangleleft$$

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Chapter 7, Solution 114.

FBD Cable:

$$\circlearrowleft \Sigma M_B = 0: \quad LA_{Cy} + aT_0 - \Sigma M_{B\ \text{loads}} = 0 \qquad (1)$$

(Where $\Sigma M_{B\ \text{loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0: \quad xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\ \text{left}} = 0 \qquad (2)$$

(Where $\Sigma M_{C\ \text{left}}$ includes all loads left of C)

FBD AC:

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma M_{B\ \text{loads}} + \Sigma M_{C\ \text{left}} = 0 \qquad (3)$$

$$\circlearrowleft \Sigma M_B = 0: \quad LA_{By} - \Sigma M_{B\ \text{loads}} = 0 \qquad (4)$$

FBD Beam:

$$\circlearrowleft \Sigma M_C = 0: \quad xA_{By} - \Sigma M_{C\ \text{left}} - M_C = 0 \qquad (5)$$

$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma M_{B\ \text{loads}} + \Sigma M_{C\ \text{left}} + M_C = 0 \qquad (6)$$

FBD AC:

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.

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Chapter 7, Solution 115.

FBD Beam:

$$\circlearrowleft \Sigma M_D = 0: \quad (1\text{ m})(2.25\text{ kN}) + (3\text{ m})(3\text{ kN}) - (5\text{ m})A_{By} = 0$$

$$\mathbf{A}_{By} = 2.25\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad 2.25\text{ kN} - 3\text{ kN} - 2.25\text{ kN} + D_{By} = 0$$

$$\mathbf{D}_{By} = 3\text{ kN}\uparrow$$

FBD AB:

$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (2\text{ m})(2.25\text{ kN}) = 0, \qquad \mathbf{M}_B = 4.5\text{ kN}\cdot\text{m}\circlearrowright$$

Note, since A and D are in line horizontally, $\mathbf{A}_{Cy} = \mathbf{A}_{By}$ and $\mathbf{D}_{Cy} = \mathbf{D}_{By}$.

Also, since $D_y > A_y$, $\quad T_{\max} = T_{CD} = 3.6\text{ kN}$

$$T_0 = \sqrt{T_{CD}^2 - D_{Cy}^2} = \sqrt{(3.6\text{ kN})^2 - (3\text{ kN})^2}$$

$$T_0 = \sqrt{3.96}\text{ kN}$$

Cable:

$$d_B = \frac{M_B}{T_0} = \frac{4.5\text{ kN}\cdot\text{m}}{\sqrt{3.96}\text{ kN}} = 2.2613\text{ m}$$

$$d_B = 2.26\text{ m} \blacktriangleleft$$

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Chapter 7, Solution 116.

FBD Beam:

$$\circlearrowleft \Sigma M_E = 0: \quad (1\text{ m})(1.2\text{ kN}) + (2\text{ m})(3.6\text{ kN}) + (3\text{ m})(1.8\text{ kN}) - (4\text{ m})A_{By} = 0 \qquad \mathbf{A}_{By} = 3.45\text{ kN}\uparrow$$

$$\rightarrow \Sigma F_x = 0: \quad A_{Bx} = 0$$

FBD AB:

$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (1\text{ m})(3.45\text{ kN}) = 0, \qquad M_B = 3.45\text{ kN}\cdot\text{m}$$

FBD AC:

$$\circlearrowleft \Sigma M_C = 0: \quad M_C + (1\text{ m})(1.8\text{ kN}) - (2\text{ m})(3.45\text{ kN}) = 0$$

$$M_C = 5.1\text{ kN}\cdot\text{m}$$

$$\circlearrowleft \Sigma M_D = 0: \quad M_D + (1\text{ m})(3.6\text{ kN}) + (2\text{ m})(1.8\text{ kN}) - (3\text{ m})(3.45\text{ kN}) = 0$$

$$M_D = 3.15\text{ kN}\cdot\text{m}$$

FBD AD:

$$h_C = d_C - 0.6\text{ m} = 1.8\text{ m} - 0.6\text{ m} = 1.2\text{ m}$$

$$T_0 = \frac{M_C}{h_C} = \frac{5.1\text{ kN}\cdot\text{m}}{1.2\text{ m}} = 4.25\text{ kN}$$

$$h_B = \frac{M_B}{T_0} = \frac{3.45\text{ kN}\cdot\text{m}}{4.25\text{ kN}} = 0.81176\text{ m}$$

$$d_B = h_B + 0.3\text{ m} = 1.11176\text{ m}, \qquad d_B = 1.112\text{ m} \blacktriangleleft$$

Cable:

$$h_D = \frac{M_D}{T_0} = \frac{3.15\text{ kN}\cdot\text{m}}{4.25\text{ kN}} = 0.74118\text{ m}$$

$$d_D = h_D + 0.9\text{ m} = 1.64118\text{ m}, \qquad d_D = 1.641\text{ m} \blacktriangleleft$$

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Chapter 7, Solution 117.

FBD Beam:

$\rightarrow \Sigma F_x = 0: \qquad A_{Bx} = 0$

By symmetry $\mathbf{A}_{By} = \mathbf{F}_{By} = 800 \text{ lb}\uparrow$

FBD AC:

$\circlearrowleft \Sigma M_C = 0: \quad M_C + (6 \text{ ft})(400 \text{ lb}) - (12 \text{ ft})(800 \text{ lb}) = 0$

$$M_C = 7200 \text{ lb}\cdot\text{ft}$$

By symmetry, $M_D = M_C = 7200 \text{ lb}\cdot\text{ft}$

$\therefore \quad h_C = h_D$

Cable:

$h_C = d_C - 3 \text{ ft} = 12 \text{ ft} - 3 \text{ ft} = 9 \text{ ft}$

$d_D = h_D + 2 \text{ ft} = 9 \text{ ft} + 2 \text{ ft} = 11 \text{ ft}$

$d_D = 11.00 \text{ ft}$ ◀

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Chapter 7, Solution 118.

FBD Beam:

$\rightarrow \Sigma F_x = 0: \qquad A_{Bx} = 0$

By symmetry $\mathbf{A}_{By} = \mathbf{F}_{By} = 800 \text{ lb} \uparrow$

FBD AC:

$\circlearrowleft \Sigma M_C = 0: \qquad M_C + (6 \text{ ft})(400 \text{ lb}) - (12 \text{ ft})(800 \text{ lb}) = 0$

$$M_C = 7200 \text{ lb}\cdot\text{ft}$$

By symmetry, $M_D = M_C = 7200 \text{ lb}\cdot\text{ft}$

$\therefore \quad h_C = h_D$

Cable:

$h_C = d_C - 3 \text{ ft} = 9 \text{ ft} - 3 \text{ ft} = 6 \text{ ft}$

$d_D = h_D + 2 \text{ ft} = 6 \text{ ft} + 2 \text{ ft} = 8 \text{ ft}$

$d_D = 8.00 \text{ ft} \blacktriangleleft$

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Chapter 7, Solution 119.

FBD Elemental segment:

$$\uparrow \Sigma F_y = 0: \quad T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

So
$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

In $\lim\limits_{\Delta x \to 0}$: $$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$$ Q.E.D.

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Chapter 7, Solution 120.

$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.119

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So $$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \qquad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad \left[\text{using } y(0) = 0\right] \blacktriangleleft$$

But $$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \qquad \text{so} \qquad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And $$T_0 = T_{\min} \qquad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=\frac{L}{2}} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi}$$

$$T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$

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Chapter 7, Solution 121.

Elemental Segment:

Load on segment* $\quad w(x)\,dx = \dfrac{w_0}{\cos^2\theta}\,ds$

But $\quad dx = \cos\theta\, ds, \quad$ so $\quad w(x) = \dfrac{w_0}{\cos^3\theta}$

From Problem 7.119 $\quad \dfrac{d^2y}{dx^2} = \dfrac{w(x)}{T_0} = \dfrac{w_0}{T_0\cos^3\theta}$

In general $\quad \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}(\tan\theta) = \sec^2\theta\dfrac{d\theta}{dx}$

So $\quad \dfrac{d\theta}{dx} = \dfrac{w_0}{T_0\cos^3\theta\sec^2\theta} = \dfrac{w_0}{T_0\cos\theta}$

or $\quad \dfrac{T_0}{w_0}\cos\theta\, d\theta = dx = r\,d\theta\cos\theta$

Giving $r = \dfrac{T_0}{w_0} =$ constant. So curve is circular arc $\quad$ Q.E.D.

*For large sag, it is not appropriate to approximate *ds* by *dx*.

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Chapter 7, Solution 122.

FBD half-span:

$$w = (0.07\text{ kg/m})(9.81\text{ m/s}^2) = 0.6867\text{ N/m}$$

$$L = 10\text{ m}$$

$$S_{CB} = \frac{12\text{ m}}{2} = 6\text{ m}$$

$$S_{CB} = c\sinh\frac{x_B}{c}, \qquad 6\text{ m} = c\sinh\frac{5\text{ m}}{c}$$

Solving numerically, $c = 4.6954$ m

$$y_B = c\cosh\frac{x_B}{c} = (4.6954\text{ m})\cosh\left(\frac{5\text{ m}}{4.6954\text{ m}}\right) = 7.6188\text{ m}$$

$$h_B = y_B - c = 7.6188\text{ m} - 4.6954\text{ m} = 2.9234\text{ m}$$

(*a*) $h_B = 2.92$ m ◀

$$T_B = wy_B = (0.6867\text{ N/m})(7.6188\text{ m}) = 5.2318\text{ N}$$

(*b*) $T_B = 5.23$ N ◀

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Chapter 7, Solution 123.

$$s_B = 30 \text{ ft}, \qquad w = \frac{120 \text{ lb}}{60 \text{ ft}} = 2 \text{ lb/ft}$$

$$h_B = 24 \text{ ft}, \qquad x_B = \frac{L}{2}$$

$$y_B^2 = c^2 + s_B^2 = (h_B + c)^2$$
$$= h_B^2 + 2ch_B + c^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(30 \text{ ft})^2 - (24 \text{ ft})^2}{2(24 \text{ ft})}$$

$$c = 6.75 \text{ ft}$$

Then $$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c}$$

$$x_B = (6.75 \text{ ft}) \sinh^{-1}\left(\frac{30 \text{ ft}}{6.75 \text{ ft}}\right) = 14.83 \text{ ft}$$

(*a*) $L = 2x_B = 29.7 \text{ ft}$ ◀

$$T_{\max} = T_B = wy_B = w(c + h_B) = (2 \text{ lb/ft})(6.75 \text{ ft} + 24 \text{ ft}) = 61.5 \text{ lb}$$

(*b*) $T_{\max} = 61.5 \text{ lb}$ ◀

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Chapter 7, Solution 124.

FBD half-span:

$$S_B = 250 \text{ ft}$$

$$w = 2.8 \text{ lb/ft}$$

$$h_B = 125 \text{ ft}$$

$$y_B = h_B + c = 125 \text{ ft} + c$$

$$y_B^2 - s_B^2 = c^2 \qquad (125 \text{ ft} - c)^2 - (250 \text{ ft})^2 = c^2$$

$$c = 187.5 \text{ ft}$$

$$s_B = c\sinh\frac{x_B}{c}, \qquad 250 \text{ ft} = (187.5 \text{ ft})\sinh\frac{x_B}{c}$$

$$\frac{x_B}{c} = \sinh^{-1}\frac{4}{3} = 1.0986, \qquad x_c = 205.99 \text{ ft}$$

(*a*) span $L = 2x_B = 411.98$ ft $\qquad L = 412$ ft ◀

(*b*) $T_{\max} = wy_B = (2.8 \text{ lb/ft})(125 \text{ ft} + 187.5 \text{ ft}) = 875$ lb

$T_{\max} = 875$ lb ◀

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Chapter 7, Solution 125.

FBD half-span:

$$s_B = 65\text{ m}, \qquad h_B = 30\text{ m}$$

$$w = (3.4\text{ kg/m})(9.81\text{ m/s}^2) = 33.35\text{ N/m}$$

$$y_B^2 = c^2 + s_B^2$$

$$(c + h_B)^2 = c^2 + s_B^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(65\text{ m})^2 - (30\text{ m})^2}{2(30\text{ m})}$$

$$= 55.417\text{ m}$$

Now $$s_B = c\sinh\frac{x_B}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (55.417\text{ m})\sinh^{-1}\left(\frac{65\text{ m}}{55.417\text{ m}}\right)$$

$$= 55.335\text{ m}$$

$$L = 2x_B = 2(55.335\text{ m}) = 110.7\text{ m} \blacktriangleleft$$

$$T_{\max} = wy_B = w(c + h_B) = (33.35\text{ N/m})(55.417\text{ m} + 30\text{ m}) = 2846\text{ N}$$

$$T_{\max} = 2.85\text{ kN} \blacktriangleleft$$

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Chapter 7, Solution 126.

FBD Cable:

$$s = 30 \text{ m} \qquad \left(\text{so } s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}\right)$$

$$w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$h_B = 12 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So
$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

$$c = \frac{(15 \text{ m})^2 - (12 \text{ m})^2}{2(12 \text{ m})} = 3.375 \text{ m}$$

Now
$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (3.375 \text{ m}) \sinh^{-1}\left(\frac{15 \text{ m}}{3.375 \text{ m}}\right)$$

$$x_B = 7.4156 \text{ m}$$

$$P = T_0 = wc = (2.943 \text{ N/m})(3.375 \text{ m}) \qquad (a) \qquad \mathbf{P} = 9.93 \text{ N} \rightarrow \blacktriangleleft$$

$$L = 2x_B = 2(7.4156 \text{ m}) \qquad (b) \qquad L = 14.83 \text{ m} \blacktriangleleft$$

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Chapter 7, Solution 127.

FBD Cable:

$$s_T = 30 \text{ m}, \qquad w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$P = T_0 = wc, \qquad c = \frac{P}{w}$$

$$c = \frac{30 \text{ N}}{2.943 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0, \qquad s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}$$

$$h^2 + 2(10.1937 \text{ m})h - 225 \text{ m}^2 = 0$$

$$h = 7.9422 \text{ m} \qquad (a) \qquad h = 7.94 \text{ m} \blacktriangleleft$$

$$s_B = c \sinh \frac{x_A}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (10.1937 \text{ m}) \sinh^{-1}\left(\frac{15 \text{ m}}{10.1937 \text{ m}}\right)$$

$$= 12.017 \text{ m}$$

$$L = 2x_B = 2(12.017 \text{ m}) \, (b) \qquad L = 24.0 \text{ m} \blacktriangleleft$$

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Chapter 7, Solution 128.

FBD half-span:

$$x_D = \frac{3.6\,w}{2} = 1.8 \text{ m}$$

$$w = \left(3.72 \frac{\text{kg}}{\text{m}}\right)\left(9.81 \frac{\text{m}}{s^2}\right) = 36.4932 \text{ N/m}$$

$$s_D = \frac{\text{length}}{2} = \frac{3.8 \text{ m}}{2} = 1.9 \text{ m}$$

$$s_D = c \sinh \frac{x_D}{c} \qquad 1.9 \text{ m} = c \sinh \frac{1.8 \text{ m}}{c}$$

Solving numerically, $c = 3.1433$ m

$$y_D^2 - s_D^2 = c^2 \qquad y_D^2 = (1.9 \text{ m})^2 + (3.1433 \text{ m})^2$$

$$y_D = 3.6729$$

$$h = y_D - c = 3.6729 \text{ m} - 3.1433 \text{ m} = 0.5296 \text{ m}$$

$$h = 0.530 \text{ m}$$

(*a*) $h = 530$ mm ◀

$$T_{Dx} = wc = \left(36.4932 \frac{\text{N}}{\text{m}}\right)(3.1433 \text{ m}) = 114.71 \text{ N}$$

(*b*) $T_{Dx} = 114.7$ N ◀

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Chapter 7, Solution 129.

FBD half-span:

$$s_B = \frac{90\text{ m}}{2} = 45\text{ m}$$

$$x_B = \frac{L}{2} = \frac{60\text{ m}}{2} = 30\text{ m}$$

$$T_B = T_{\max} = 300\text{ N}$$

$$s_B = c\sinh\frac{x_B}{c}, \qquad 45\text{ m} = c\sinh\frac{30\text{ m}}{c}$$

Solving numerically, $\quad c = 18.495\text{ m}$

$$y_B = c\cosh\frac{x_B}{c} = (18.495\text{ m})\cosh\left(\frac{30\text{ m}}{18.495\text{ m}}\right)$$

$$= 48.651\text{ m}$$

$$h_B = y_B - c = 48.651\text{ m} - 18.495\text{ m} = 30.156\text{ m}$$

(*a*) $\quad h = 30.2\text{ m}$ ◀

$$T_{\max} = wy_B \qquad 300\text{ N} = w(48.651\text{ m})$$

$$w = 6.1664\ \frac{\text{N}}{\text{m}}$$

$$W = w(\text{length}) = \left(6.1664\ \frac{\text{N}}{\text{m}}\right)(90\text{ m}) = 554.97\text{ N}$$

$$m = \frac{W}{g} = \frac{554.97\text{ N}}{9.81\text{ m/s}^2} = 56.57\text{ kg}$$

(*b*) $\quad m = 56.6\text{ kg}$ ◀

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Chapter 7, Solution 130.

$$s_B = \frac{45 \text{ ft}}{2} = 22.5 \text{ ft} \qquad L = 20 \text{ ft}$$

$$x_B = \frac{L}{2} = 10 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}$$

$$22.5 \text{ ft} = c \sinh \frac{10 \text{ ft}}{c}$$

Solving numerically: $\quad c = 4.2023 \text{ ft}$

$$y_B = c \cosh \frac{x_B}{c}$$

$$= (4.2023 \text{ ft}) \cosh \frac{10 \text{ ft}}{4.2023 \text{ ft}} = 22.889 \text{ ft}$$

$$h_B = y_B - c = 22.889 \text{ ft} - 4.202 \text{ ft}$$

$$h_B = 18.69 \text{ ft} \blacktriangleleft$$

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Chapter 7, Solution 131.

l = total length

$$y_B = c\cosh\frac{x_B}{c}, \qquad L + c = c\cosh\frac{L/2}{c}$$

or $$1 = \cosh\frac{L}{2c} - \frac{L}{c}$$

Solving numerically, $$\frac{L}{c} = 4.933$$

$$s_B = c\sinh\frac{x_B}{c} \qquad \frac{l}{2} = c\sinh\frac{L}{2c}$$

So $$c = \frac{l}{2\sinh\left(\frac{L}{2c}\right)} = \frac{10\text{ ft}}{2\sinh\left(\frac{4.933}{2}\right)} = 0.8550\text{ ft}$$

$$L = 4.933c = (4.933)(0.8550\text{ ft}) = 4.218\text{ ft}$$

(a) $L = 4.22$ ft ◀

from $y = c\cosh\frac{x}{c}$ $\qquad \frac{dy}{dx} = \sinh\frac{x}{c}$

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_B = \sinh\frac{L}{2c} = \sinh\left(\frac{4.933}{2}\right) = 5.848$$

(b) $\theta_B = 80.3°$ ◀

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Chapter 7, Solution 132.

$$w = (3\ \text{kg/m})(9.81\ \text{m/s}^2) = 29.43\ \text{N/m}$$

$$L = 48\ \text{m}, \qquad T_{\max} \le 1800\ \text{N}$$

$$T_{\max} = w y_B \rightarrow y_B = \frac{T_{\max}}{w}$$

$$y_B \le \frac{1800\ \text{N}}{29.43\ \text{N/m}} = 61.162\ \text{m}$$

$$y_B = c \cosh \frac{x_B}{c}, \qquad 61.162\ \text{m} = c \cosh \frac{\frac{48\ \text{m}}{2}}{c} *$$

Solving numerically, $\qquad c = 55.935\ \text{m}$

$$h = y_B - c = 61.162\ \text{m} - 55.935\ \text{m}$$

$h = 5.23\ \text{m}$ ◀

*Note: There is another value of c which will satisfy this equation. It is much smaller, thus corresponding to a much larger h.

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Chapter 7, Solution 133.

Pulley B:

$T_B = wa$

but $\quad T_B = wy_B, \quad$ so $\quad y_B = a$

now $\quad y_B = c\cosh\dfrac{x_B}{c} = a$

So $\quad a = c\cosh\dfrac{3\text{ ft}}{c} \qquad (1)$

also $\quad s_B = c\sinh\dfrac{x_B}{c}$

$$\frac{24\text{ ft} - a}{2} = c\sinh\frac{3\text{ ft}}{c}$$

or $\quad 12\text{ ft} = c\sinh\dfrac{3\text{ ft}}{c} + \dfrac{a}{2} \qquad (2)$

$$= c\sinh\frac{3\text{ ft}}{c} + \frac{c}{2}\cosh\frac{3\text{ ft}}{c}$$

Solving numerically $\quad c = 1.13194\text{ ft}$

or 17.7167 ft

from (1) $\quad a = c\cosh\dfrac{3\text{ ft}}{c} = (1.13194\text{ ft})\cosh\dfrac{3\text{ ft}}{1.13194\text{ ft}}$

$a = 8.05\text{ ft}$ ◀

or $\quad a = (17.7167\text{ ft})\cosh\left(\dfrac{3\text{ ft}}{17.7167\text{ ft}}\right)$

$a = 17.97\text{ ft}$ ◀

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Chapter 7, Solution 134.

FBD Cable:

s_A = length = 10 ft

$\rightarrow \Sigma F_x = 0: \qquad -F + T_0 = 0, \qquad F = T_0$

$$y_A = c + h = c\cosh\frac{x_A}{c} = c\cosh\frac{a}{c} = c\cosh\frac{h}{c}$$

So $\qquad 1 + \dfrac{h}{c} = \cosh\dfrac{h}{c}$

Solving numerically, $\qquad \dfrac{h}{c} = 1.61614$

$$s_A = c\sinh\frac{h}{c} = c\sinh(1.61614) = 2.41748c = 10 \text{ ft}$$

So $\quad c = 4.1365$ ft, $\qquad h = 6.6852$ ft

$$y_A = c + h = 10.8217 \text{ ft}$$

$$F = T_0 = wc = \left(\frac{20 \text{ lb}}{10 \text{ ft}}\right)(4.1365 \text{ ft}) = 8.2730 \text{ lb}$$

(*a*) $\qquad F = 8.27$ lb ◀

(*b*) $\qquad h = a = 6.69$ ft ◀

$$T_{\max} = wy_A = \left(2\frac{\text{lb}}{\text{ft}}\right)(10.8217 \text{ ft}) = 21.643 \text{ lb}$$

(*c*) $\qquad T_{\max} = 21.6$ lb ◀

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Chapter 7, Solution 135.

FBD Cable:

Pulley

$$T_A = mg = T_{\max}$$

$$mg = (40\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = 392.4\text{ N}$$

Also $L = 15$ m, $\quad h = 5$ m

So $\quad x_A = -\dfrac{15\text{ m}}{2} = -7.5\text{ m}, \qquad x_B = +7.5\text{ m}$

$$y_A = c\cosh\frac{x_A}{c} = c + h$$

So $\quad c\cosh\dfrac{-7.5\text{ m}}{c} = c + 5\text{ m}$

Solving numerically, $\quad c = 6.3175$ m

$$s_B = c\sinh\frac{x_B}{c} = (6.3175\text{ m})\sinh\left(\frac{7.5\text{ m}}{6.3175\text{ m}}\right) = 9.390\text{ m}$$

(*a*) $\qquad$ cable length $= 2s_B = 18.78$ m ◀

$$T_{\max} = wy_B = w(c + h) = w(6.3175\text{ m} + 5\text{ m})$$

$$w = \frac{392.4\text{ N}}{11.3175\text{ m}} = 34.672\text{ N/m}$$

(*b*) $\qquad$ mass/length $= \dfrac{w}{g} = \dfrac{34.672\text{ N/m}}{9.81\text{ N/kg}} = 3.53\dfrac{\text{kg}}{\text{m}}$ ◀

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Chapter 7, Solution 136.

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

$$12\text{ ft} = c\left(\cosh\frac{10.8\text{ ft}}{c} - 1\right)$$

Solving numerically, $c = 6.2136\text{ ft}$

Then $y_B = (6.2136\text{ ft})\cosh\dfrac{10.8\text{ ft}}{6.2136\text{ ft}} = 18.2136\text{ ft}$

$$F = T_{\max} = wy_B = (1.5\text{ lb/ft})(18.2136\text{ ft})$$

$\mathbf{F} = 27.3\text{ lb} \longrightarrow$ ◀

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Chapter 7, Solution 137.

$$y_D = c\cosh\frac{x_D}{c}$$

$$c + h = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

$$12\text{ ft} = c\left(\cosh\frac{18\text{ ft}}{c} - 1\right)$$

Solving numerically $\quad c = 15.162\text{ ft}$

$$y_B = h + c = 12\text{ ft} + 15.162\text{ ft} = 27.162\text{ ft}$$

$$F = T_D = wy_D = (1.5\text{ lb/ft})(27.162\text{ ft}) = 40.74\text{ lb}$$

$\mathbf{F} = 40.7\text{ lb} \longrightarrow$ ◀

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Chapter 7, Solution 138.

$$w = 4\text{ kg/m}\left(9.81\text{ m/s}^2\right) = 39.24\text{ N/m}$$

$$P = T_0 = wc, \qquad c = \frac{P}{w} = \frac{800\text{ N}}{39.24\text{ N/m}}$$

$$c = 20.387\text{ m}$$

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$$

$$a = c\sinh^{-1}(\tan\theta) = (20.387\text{ m})\sinh^{-1}(\tan 60°)$$

$$a = 26.849\text{ m}$$

$$y_A = c\cosh\frac{a}{c} = (20.387\text{ m})\cosh\frac{26.849\text{ m}}{20.387\text{ m}} = 40.774\text{ m}$$

$$b = y_A - c = 40.774\text{ m} - 20.387\text{ m} = 20.387\text{ m}$$

So (*a*) *B* is 26.8 m right and 20.4 m down from *A* ◀

$$s = c\sinh\frac{a}{c} = (20.387\text{ m})\sinh\frac{26.849\text{ m}}{20.387\text{ m}} = 35.31\text{ m}$$

(*b*) $s = 35.3\text{ m}$ ◀

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Chapter 7, Solution 139.

$$w = (4\text{ kg/m})(9.81\text{ m/s}^2) = 39.24\text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{600\text{ N}}{39.24\text{ N/m}}$$

$$c = 15.2905\text{ m}$$

$$y = c\cosh\frac{x}{c}, \qquad \frac{dy}{dx} = \sinh\frac{x}{c}$$

At A: $\quad \tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$

So $\quad a = c\sinh^{-1}(\tan\theta) = (15.2905\text{ m})\sinh^{-1}(\tan 60°) = 20.137\text{ m}$

$$y_B = h + c = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

$$= (15.2905\text{ m})\left(\cosh\frac{20.137\text{ m}}{15.2905\text{ m}} - 1\right)$$

$$= 15.291\text{ m}$$

So $\quad (a) \quad$ B is 20.1 m right and 15.29 m down from A ◀

$$s = c\sinh\frac{a}{c} = (15.291\text{ m})\ \sinh\frac{20.137\text{ m}}{15.291\text{ m}} = 26.49\text{ m}$$

(b) $\quad s = 26.5\text{ m}$ ◀

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Chapter 7, Solution 140.

Cable:

Since $y_A = c\cosh\dfrac{x_A}{c}$, $\quad y_B = c\cosh\dfrac{x_B}{c}$

$$\text{and } x_B - x_A = 8 \text{ m} \qquad (x_A < 0)$$

$$(c + 0.5 \text{ m}) = c\cosh\frac{x_A}{c}$$

$$(c + 1.2 \text{ m}) = c\cosh\left(\frac{x_B}{c}\right)$$

$$\cosh^{-1}\left(1 + \frac{1.2 \text{ m}}{c}\right) + \cosh^{-1}\left(1 + \frac{0.5 \text{ m}}{c}\right) = \frac{8 \text{ m}}{c}$$

Solving numerically, $\quad c = 9.9987$ m

So
$$|x_A| = \left[\cosh^{-1}\left(1 + \frac{0.5 \text{ m}}{9.9987 \text{ m}}\right)\right](9.9987 \text{ m})$$
$$= 3.15 \text{ m}$$

(*a*) C is 3.15 m from house ◀

$$T_{\max} = T_B = wy_B$$
$$= \left(2.1 \frac{\text{kg}}{\text{m}}\right)\left(9.81 \frac{\text{N}}{\text{kg}}\right)(1.2 \text{ m} + 9.9987 \text{ m})$$
$$= 230.7 \text{ N}$$

(*b*) $T_{\max} = 231$ N ◀

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Chapter 7, Solution 141.

$$y_A = c\cosh\frac{-a}{c} = c + 6 \text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{6 \text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 11.4 \text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{c}\right)$$

So $$a + b = c\left[\cosh^{-1}\left(1 + \frac{6 \text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{\text{c}}\right)\right] = 36 \text{ ft}$$

Solving numerically, $$c = 20.446 \text{ ft}$$

$$b = (20.446 \text{ ft})\cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{20.446 \text{ ft}}\right) = 20.696 \text{ ft}$$

(*a*) C is 20.7 ft left of and 11.4 ft below B ◀

$$T_{\max} = wy_B = (0.3 \text{ lb/ft})(20.446 \text{ ft})\cosh\left(\frac{20.696 \text{ ft}}{20.446 \text{ ft}}\right) = 9.554 \text{ lb}$$

(*b*) $T_{\max} = 9.55 \text{ lb}$ ◀

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Chapter 7, Solution 142.

(a) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta, \quad \text{Q.E.D.}$$

(b) Also $$y^2 = s^2 + c^2, \qquad \left(\cosh^2 x = \sinh^2 x + 1\right)$$

So $$y^2 = c^2\left(\tan^2\theta + 1\right) = c^2\sec^2\theta$$

And $$y = c\sec\theta, \quad \text{Q.E.D.}$$

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Chapter 7, Solution 143.

$$T_B = T_{\max} = wy_B$$

$$= wc\cosh\frac{x_B}{c} = w\frac{L}{2}\left(\frac{2c}{L}\right)\cosh\frac{L}{2c}$$

Let $\xi = \dfrac{L}{2c}$ so $T_{\max} = \dfrac{wL}{2\xi}\cosh\xi$

$$\frac{dT_{\max}}{d\xi} = \frac{wL}{2\xi}\left(\sinh\xi - \frac{1}{\xi}\cosh\xi\right)$$

For $\min T_{\max}$, $\tanh\xi - \dfrac{1}{\xi} = 0$

Solving numerically $\xi = 1.1997$

$$\left(T_{\max}\right)_{\min} = \frac{wL}{2(1.1997)}\cosh(1.1997) = 0.75444wL$$

(*a*) $L_{\max} = \dfrac{T_{\max}}{0.75444w} = 1.3255\dfrac{T_{\max}}{w}$ ◀

If $T_{\max} = 32$ kN and $w = (0.34\text{ kg/m})(9.81\text{ m/s}^2) = 3.3354$ N/m

$$L_{\max} = 1.3255\frac{32.000\text{ N}}{3.3354\text{ N/m}} = 12\,717\text{ m}$$

(*b*) $L_{\max} = 12.72$ km ◀

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Chapter 7, Solution 144.

$$y_{\max} = c\cosh\frac{L}{2c} = h + c$$

$$T_{\max} = wy_{\max}, \qquad y_{\max} = \frac{T_{\max}}{w}$$

$$y_{\max} = \frac{80\text{ lb}}{2\text{ lb/ft}} = 40\text{ ft}$$

$$c\cosh\frac{9\text{ ft}}{c} = 40\text{ ft}$$

Solving numerically, $c_1 = 2.6388\text{ ft}$

$$c_2 = 38.958\text{ ft}$$

$$h = y_{\max} - c$$

$$h_1 = 40\text{ ft} - 2.6388\text{ ft} \qquad h_1 = 37.4\text{ ft} \blacktriangleleft$$

$$h_2 = 40\text{ ft} - 38.958\text{ ft} \qquad h_2 = 1.042\text{ ft} \blacktriangleleft$$

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Chapter 7, Solution 145.

$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1$$

$$= 0.154701$$

$$\frac{h_B}{L} = \frac{h_B/c}{2(L/2c)}$$

$$= \frac{0.5(0.154701)}{0.549306} = 0.14081$$

$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$

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Chapter 7, Solution 146.

(*a*)
$$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\min T_{\max}$, $\dfrac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \longrightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375 \quad \frac{h}{L} = 0.338 \blacktriangleleft$$

(*b*)
$$T_0 = wc, \qquad T_{\max} = wc\cosh\frac{L}{2c}, \qquad \frac{T_{\max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

But
$$T_0 = T_{\max}\cos\theta_B, \qquad \frac{T_{\max}}{T_0} = \sec\theta_B$$

So
$$\theta_B = \sec^{-1}\left(\frac{y_B}{c}\right) = \sec^{-1}(1.8102)$$

$$= 56.46° \qquad \theta_B = 56.5° \blacktriangleleft$$

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

$$T_{\max} = 0.755wL \blacktriangleleft$$

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Chapter 7, Solution 147.

FBD AB:

$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70\text{ lb}) = 0$$

$$\mathbf{C} = 100\text{ lb} \nearrow$$

$$\rightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100\text{ lb}) = 0$$

$$\mathbf{A}_x = 80\text{ lb} \leftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100\text{ lb}) - 70\text{ lb} = 0$$

$$\mathbf{A}_y = 10\text{ lb} \uparrow$$

FBD AJ:

$$\searrow \Sigma F_{x'} = 0: F - (80\text{ lb})\sin 30° - (10\text{ lb})\cos 30° = 0$$

$$F = 48.66\text{ lb}$$

$$\mathbf{F} = 48.7\text{ lb} \searrow 60° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: V - (80\text{ lb})\cos 30° + (10\text{ lb})\sin 30° = 0$$

$$V = 64.28\text{ lb}$$

$$\mathbf{V} = 64.3\text{ lb} \measuredangle 30° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: (8\text{ in.})(48.66\text{ lb}) - (8\text{ in.})(10\text{ lb}) - M = 0$$

$$M = 309.28\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 309\text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

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Chapter 7, Solution 148.

FBD Whole:

$$\circlearrowleft \Sigma M_A = 0: (5.4\text{ ft})B_x - (7.8\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{B}_x = 130\text{ lb} \leftarrow$$

FBD BE with pulleys and cord:

$$\circlearrowleft \Sigma M_E = 0: (5.4\text{ ft})(130\text{ lb}) - (7.2\text{ ft})B_y + (4.8\text{ ft})(90\text{ lb}) - (0.6\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{B}_y = 150\text{ lb} \uparrow$$

$$\rightarrow \Sigma F_x = 0: E_x - 130\text{ lb} = 0$$

$$\mathbf{E}_x = 130\text{ lb} \rightarrow$$

$$\uparrow \Sigma F_y = 0: E_y + 150\text{ lb} - 90\text{ lb} - 90\text{ lb} = 0$$

$$\mathbf{E}_y = 30\text{ lb} \uparrow$$

$$\searrow \Sigma F_{x'} = 0: -F - 90\text{ lb} + \frac{4}{5}(130\text{ lb}) + \frac{3}{5}(90\text{ lb} - 30\text{ lb}) = 0$$

$$\mathbf{F} = 50.0\text{ lb} \nwarrow \blacktriangleleft$$

FBD JE and pulley:

$$\nearrow \Sigma F_{y'} = 0: V + \frac{3}{5}(130\text{ lb}) + \frac{4}{5}(30\text{ lb} - 90\text{ lb}) = 0$$

$$V = -30\text{ lb} \qquad \mathbf{V} = 30.0\text{ lb} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: -M + (1.8\text{ ft})(130\text{ lb}) + (2.4\text{ ft})(30\text{ lb}) + (0.6\text{ ft})(90\text{ lb}) - (3.0\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{M} = 90.0\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 149.

FBD Rod:

$$\rightarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi} \uparrow$$

FBD AJ:

$$\alpha = 15°, \text{ weight of segment } = W\frac{30°}{90°} = \frac{W}{3}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/12}\sin 15° = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \quad \frac{2W}{\pi}\cos 30° - \frac{W}{3}\cos 30° - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right) \swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15°\frac{W}{3} = 0$$

$$\mathbf{M} = 0.0557Wr \circlearrowright \blacktriangleleft$$

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Chapter 7, Solution 150.

(*a*)

Along AC:

$\uparrow \Sigma F_y = 0: \quad -3 \text{ kip} - V = 0 \qquad V = -3 \text{ kips}$

$\circlearrowleft \Sigma M_J = 0: \quad M + x(3 \text{ kips}) = 0 \qquad M = (3 \text{ kips})x$

$M = -9 \text{ kip}\cdot\text{ft at } C$

Along CD:

$\uparrow \Sigma F_y = 0: \quad -3 \text{ kips} - 5 \text{ kips} - V = 0 \qquad V = -8 \text{ kips}$

$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 3 \text{ ft})(5 \text{ kips}) + x(3 \text{ kips}) = 0$

$M = +15 \text{ kip}\cdot\text{ft} - (8 \text{ kips})x$

$M = -16.2 \text{ kip}\cdot\text{ft at } D \ (x = 3.9 \text{ ft})$

Along DE:

$\uparrow \Sigma F_y = 0: \quad -3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - V = 0$

$V = -2 \text{ kips}$

$\circlearrowleft \Sigma M_L = 0: \quad M - x_1(6 \text{ kips}) + (.9 \text{ ft} + x_1)(5 \text{ kips})$

$+ (3.9 \text{ ft} + x_1)(3 \text{ kips}) = 0$

$M = -16.2 \text{ kip}\cdot\text{ft} - (2 \text{ kips})x_1$

$M = -18.6 \text{ kip}\cdot\text{ft at } E \ (x_1 = 1.2 \text{ ft})$

continued

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Along EB:

$\uparrow \Sigma F_y = 0$: $-3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - 4 \text{ kips} - V = 0 \qquad V = -6 \text{ kips}$

$\circlearrowleft \Sigma M_N = 0$: $\quad M + (4 \text{ kips})x_2 + (2.1 \text{ ft} + x_2)(5 \text{ kips})$

$$+ (5.1 \text{ ft} + x_2)(3 \text{ kips}) - (1.2 \text{ ft} + x_2)(6 \text{ kips}) = 0$$

$$M = -18.6 \text{ kip}\cdot\text{ft} - (6 \text{ kips})x_2$$

$$M = -33 \text{ kip}\cdot\text{ft at } B \quad (x_2 = 2.4 \text{ ft})$$

(*b*) From diagrams: $|V|_{\max} = 8.00 \text{ kips on } CD$ ◀

$|M|_{\max} = 33.0 \text{ kip}\cdot\text{ft at } B$ ◀

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Chapter 7, Solution 151.

(*a*)

By symmetry:

$$A_y = B = 8\text{ kN} + \frac{1}{2}(4\text{ kN/m})(5\text{ m}) \qquad \mathbf{A}_y = \mathbf{B} = 18\text{ kN} \uparrow$$

Along AC:

$$\uparrow \Sigma F_y = 0: \quad 18\text{ kN} - V = 0 \qquad V = 18\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - x(18\text{ kN}) \qquad M = (18\text{ kN})x$$

$$M = 36\text{ kN}\cdot\text{m at } C \ (x = 2\text{ m})$$

Along CD:

$$\uparrow \Sigma F_y = 0: \quad 18\text{ kN} - 8\text{ kN} - (4\text{ kN/m})x_1 - V = 0$$

$$V = 10\text{ kN} - (4\text{ kN/m})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ m}(\text{at center})$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + \frac{x_1}{2}(4\text{ kN/m})x_1 + (8\text{ kN})x_1 - (2\text{ m} + x_1)(18\text{ kN}) = 0$$

$$M = 36\text{ kN}\cdot\text{m} + (10\text{ kN/m})x_1 - (2\text{ kN/m})x_1^2$$

$$M = 48.5\text{ kN}\cdot\text{m at } x_1 = 2.5\text{ m}$$

Complete diagram by symmetry

From diagrams: $|V|_{\max} = 18.00\text{ kN on } AC \text{ and } DB$ ◀

(*b*) $|M|_{\max} = 48.5\text{ kN}\cdot\text{m at center}$ ◀

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Chapter 7, Solution 152.

By symmetry: $A_y = B = 60\text{ kN} - P$

Along AC:

$\circlearrowleft \Sigma M_J = 0: \quad M - x(60\text{ kN} - P) = 0 \qquad M = (60\text{ kN} - P)x$

$$M = 120\text{ kN}\cdot\text{m} - (2\text{ m})P \quad \text{at} \quad x = 2\text{ m}$$

Along CD:

$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 2\text{ m})(60\text{ kN}) - x(60\text{ kN} - P) = 0$

$$M = 120\text{ kN}\cdot\text{m} - Px$$

$$M = 120\text{ kN}\cdot\text{m} - (4\text{ m})P \quad \text{at} \quad x = 4\text{ m}$$

Along DE:

$\circlearrowleft \Sigma M_L = 0: \quad M - (x - 4\text{ m})P + (x - 2\text{ m})(60\text{ kN})$

$$- x(60\text{ kN} - P) = 0$$

$$M = 120\text{ kN}\cdot\text{m} - (4\text{ m})P \quad \text{(const)}$$

Complete diagram by symmetry

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

$$120\text{ kN}\cdot\text{m} - (2\text{ m})P = -\left[120\text{ kN}\cdot\text{m} - (4\text{ m})P\right]$$

$$P = 40.0\text{ kN} \blacktriangleleft$$

(*a*) $M_{\min} = 120\text{ kN}\cdot\text{m} - (4\text{ m})P$

(*b*) $|M|_{\max} = 40.0\text{ kN}\cdot\text{m} \blacktriangleleft$

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Chapter 7, Solution 153.

(*a*)

(*b*)

FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: \quad (4\text{ m})B - (1\text{ m})(20\text{ kN/m})(2\text{ m}) - M = 0$$

$$B = 10\text{ kN} + \frac{M}{4\text{ m}}$$

(*a*) $\mathbf{B} = 10\text{ kN} \uparrow$

(*b*) $\mathbf{B} = 13\text{ kN} \uparrow$

$$\uparrow \Sigma F_y = 0: \quad A_y - (20\text{ kN/m})(2\text{ m}) + B = 0$$

$$A_y = 40\text{ kN} - B$$

(*a*) $\mathbf{A}_y = 30\text{ kN} \uparrow$

(*b*) $\mathbf{A}_y = 27\text{ kN} \uparrow$

Shear Diags:

$V_A = A_y$, then V is linear $\left(\dfrac{dV}{dx} = -20\text{ kN/m}\right)$ to C.

$$V_C = A_y - (20\text{ kN/m})(2\text{ m}) = A_y - 40\text{ kN}$$

(*a*) $V_C = -10\text{ kN}$

(*b*) $V_C = -13\text{ kN}$

$$V = 0 = A_y - (20\text{ kN/m})x_1 \text{ at } x_1 = \frac{A_y\text{ m}}{20\text{ kN}}$$

(*a*) $x_1 = 1.5\text{ m}$

(*b*) $x_1 = 1.35\text{ m}$

V is constant from C to B.

continued

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Moment Diags:

M_A = applied M. Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$

M is max where $V = 0$. $M_{\max} = M + \dfrac{1}{2}A_y x_1$.

(*a*) $$|M|_{\max} = \frac{1}{2}(30\text{ kN})(1.5\text{ m}) = 22.5\text{ kN}\cdot\text{m} \blacktriangleleft$$

1.500 m from A ◀

(*b*) $$M_{\max} = 12\text{ kN}\cdot\text{m} + \frac{1}{2}(27\text{ kN})(1.35\text{ m}) = 30.225\text{ kN}\cdot\text{m}$$

$|M|_{\max} = 30.2$ kN, 1.350 m from A ◀

$$M_C = M_{\max} - \frac{1}{2}V_C(2\text{ m} - x_1)$$

(*a*) $M_C = 20\text{ kN}\cdot\text{m}$

(*b*) $M_C = 26\text{ kN}\cdot\text{m}$

Finally, M is linear $\left(\dfrac{dM}{dx} = V_C\right)$ to zero at B.

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Chapter 7, Solution 154.

(*b*)

(*a*) Distributed load $w = w_0\left(1 - \dfrac{x}{L}\right)$ $\left(\text{total} = \dfrac{1}{2}w_0L\right)$

$$\circlearrowleft \Sigma M_A = 0: \quad \frac{L}{3}\left(\frac{1}{2}w_0L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0L}{6} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - \frac{1}{2}w_0L + \frac{w_0L}{6} = 0 \qquad \mathbf{A}_y = \frac{w_0L}{3} \uparrow$$

Shear:

$$V_A = A_y = \frac{w_0L}{3},$$

Then $$\frac{dV}{dx} = -w \rightarrow V = V_A - \int_0^x w_0\left(1 - \frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0L}{3}\right) - w_0x + \frac{1}{2}\frac{w_0}{L}x^2 = w_0L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $x = L$, $V = -\dfrac{w_0L}{6}$;

$$V = 0 \text{ at } \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3} = 0 \rightarrow \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$$

Moment:

$$M_A = 0,$$

Then $$\left(\frac{dM}{dx}\right) = V \rightarrow M = \int_0^x V dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\int_0^{x/L}\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

continued

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$$M_{\max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

(a) $$V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] \blacktriangleleft$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

(c) $$M_{\max} = 0.0642\, w_0 L^2 \blacktriangleleft$$

$$\text{at } x = 0.423L \blacktriangleleft$$

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Chapter 7, Solution 155.

(*a*) $\uparrow \Sigma F_y = 0: \; w_g L - \int_0^L \frac{4w_0}{L^2}\left(Lx - x^2\right)dx = 0$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L \qquad w_g = \frac{2w_0}{3}$$

Define $\xi = \frac{x}{L}$ so $d\xi = \frac{dx}{L} \rightarrow$ net load $w = 4w_0\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{2}{3}w_0$

or $$w = 4w_0\left(-\frac{1}{6} + \xi - \xi^2\right)$$

$$V = V(0) - \int_0^{\xi} 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi$$

$$= 0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right)$$

$$V = \frac{2}{3}w_0 L\left(\xi - 3\xi^2 + 2\xi^3\right) \blacktriangleleft$$

$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^{\xi}\left(\xi - 3\xi^2 + 2\xi^3\right)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2\left(\xi^2 - 2\xi^3 + \xi^4\right) \blacktriangleleft$$

(*b*) Max M occurs where $V = 0 \rightarrow 1 - 3\xi + 2\xi^2 = 0 \rightarrow \xi = \frac{1}{2}$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$

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Chapter 7, Solution 156.

(*a*) **FBD cable:**

$$\circlearrowleft \Sigma M_E = 0: \quad (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN})$$
$$-(3\text{ m})A_x - (16\text{ m})A_y = 0$$
$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) + (1\text{ m})A_x - (8\text{ m})A_y = 0$$
$$A_x - 8A_y = -4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\quad A_x = 3.2\text{ kN} \qquad A_y = 1\text{ kN}$

So $\mathbf{A} = 3.35\text{ kN} \measuredangle 17.35°$ ◀

FBD ABC:

(*b*) cable: $\rightarrow \Sigma F_x = 0: -A_x + E_x = 0$

$$E_x = A_x = 3.2\text{ kN}$$

$$\uparrow \Sigma F_y = 0: A_y - (1.2 + 0.8 + 1.2)\text{ kN} + E_y = 0$$

$$E_y = 3.2\text{ kN} - A_y = (3.2 - 1)\text{ kN} = 2.2\text{ kN}$$

So $\mathbf{E} = 3.88\text{ kN} \measuredangle 34.5°$ ◀

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Chapter 7, Solution 157.

FBD AC: **FBD CB:**

$$\circlearrowleft \Sigma M_A = 0: (13.5 \text{ ft})T_0 - \frac{a}{2}(57.5 \text{ lb/ft})a = 0$$

$$T_0 = \left(2.12963 \text{ lb/ft}^2\right)a^2$$

$$\circlearrowleft \Sigma M_B = 0: \frac{60 \text{ ft} - a}{2}(57.5 \text{ lb/ft})(60 \text{ ft} - a) - (6 \text{ ft})T_0 = 0$$

$$6T_0 = \left(28.75 \text{ lb/ft}^2\right)\left[3600 \text{ ft}^2 - (120 \text{ ft})a + a^2\right] \qquad (2)$$

Using (1) in (2), $\quad 0.55a^2 - (120 \text{ ft})a + 3600 \text{ ft}^2 = 0$

Solving: $\quad a = (108 \pm 72) \text{ ft}, \quad a = 36 \text{ ft}$ (180 ft out of range)

So $\quad C$ is 36 ft from A

(*a*) $\quad C$ is 6 ft below and 24 ft left of B ◀

$$T_0 = 2.1296 \text{ lb/ft}^2(36 \text{ ft})^2 = 2760 \text{ lb}$$

$$W_1 = (57.5 \text{ lb/ft})(36 \text{ ft}) = 2070 \text{ lb}$$

(*b*) $\quad T_{\max} = T_A = \sqrt{T_0^2 + W_1^2} = \sqrt{(2760 \text{ lb})^2 + (2070 \text{ lb})^2} = 3450 \text{ lb}$ ◀

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Chapter 7, Solution 158.

$$s_B = 100 \text{ ft}, \qquad w = \frac{4 \text{ lb}}{200 \text{ ft}} = 0.02 \text{ lb/ft}$$

$$T_{\max} = 16 \text{ lb}$$

$$T_{\max} = T_B = wy_B$$

$$y_B = \frac{T_B}{w} = \frac{16 \text{ lb}}{0.02 \text{ lb/ft}} = 800 \text{ ft}$$

$$c^2 = y_B^2 - s_B^2$$

$$c = \sqrt{(800 \text{ ft})^2 - (100 \text{ ft})^2} = 793.73 \text{ ft}$$

But

$$y_B = x_B \cosh \frac{x_B}{c} \rightarrow x_B = c \cosh^{-1} \frac{y_B}{c}$$

$$= (793.73 \text{ ft}) \cosh^{-1}\left(\frac{800 \text{ ft}}{793.73 \text{ ft}}\right) = 99.74 \text{ ft}$$

$$L = 2x_B = 2(99.74 \text{ ft}) = 199.5 \text{ ft} \blacktriangleleft$$

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