Chapter 5, Solution 1.

	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$200 \times 150 = 30\,000$	-100	250	$-30\,000\,000$	$6\,750\,000$
2	$400 \times 300 = 120\,000$	200	150	$24\,000\,000$	$18\,000\,000$
Σ	$150\,000$			$21\,000\,000$	$24\,750\,000$

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{21\,000\,000}{150\,000}\text{mm}$ or $\bar{X} = 140.0 \text{ mm}$ ◀

$\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{24\,750\,000}{150\,000}\text{mm}$ or $\bar{Y} = 165.0 \text{ mm}$ ◀

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Chapter 5, Solution 2.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$10 \times 8 = 80$	5	4	400	320
2	$\frac{1}{2} \times 9 \times 12 = 54$	13	4	702	216
Σ	134			1102	536

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{1102}{134}$ or $X = 8.22$ in. ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{1102}{134}$ or $\bar{Y} = 4.00$ in. ◀

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Chapter 5, Solution 3.

	A, mm^2	$\bar{x}, \text{mm}$	$\bar{x}A, \text{mm}^3$
1	$\frac{1}{2} \times 90 \times 270 = 12\,150$	$\frac{2}{3}(90) = 60$	729 000
2	$\frac{1}{2} \times 135 \times 270 = 18\,225$	$90 + \frac{1}{3}(135) = 135$	2 460 375
Σ	30375		3 189 375

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{3189375}{30375}\text{mm}$ or $\bar{X} = 105.0 \text{ mm}$ ◀

For the whole triangular area by observation:

$\bar{Y} = \frac{1}{3}(270 \text{ mm})$ or $\bar{Y} = 90.0 \text{ mm}$ ◀

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Chapter 5, Solution 4.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\frac{1}{2}(21)(24) = 252$	$\frac{2}{3}(21) = 14$	$40 - \frac{1}{3}(24) = 32$	3528	8064
2	$(13)(40) = 520$	$21 + \frac{1}{2}(13) = 27.5$	20	14 300	10 400
Σ	772			17 828	18 464

Then $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{17828}{772}$ in. or $\bar{X} = 23.1$ in. ◀

$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{18464}{772}$ in. or $\bar{Y} = 23.9$ in. ◀

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Chapter 5, Solution 5.

	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{\pi(225)^2}{4} = 39\ 761$	$-\frac{4(225)}{3\pi} = -95.493$	95.493	−3 796 900	3 796 900
2	$\frac{1}{2}(375)(225) = 42\ 188$	125	75	5 273 500	3 164 100
Σ	81 949			1 476 600	6 961 000

Then $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1476600}{81949}\text{mm}$ or $\bar{X} = 18.02$ mm ◀

$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{6961000}{81949}\text{mm}$ or $\bar{Y} = 84.9$ mm ◀

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Chapter 5, Solution 6.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$17 \times 9 = 153$	8.5	4.5	1300.5	688.5
2	$-\frac{\pi}{4} \times (4.5)^2 = -15.9043$	$8 - \frac{4 \times 4.5}{3\pi} = 6.0901$	$9 - \frac{4 \times 4.5}{3\pi} = 7.0901$	-96.857	-112.761
3	$-\frac{\pi}{4}(6)^2 = -28.274$	10.5465	6.4535	-298.19	-182.466
Σ	108.822			905.45	393.27

Then $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{905.45}{108.822}$ or $\bar{X} = 8.32$ in. ◀

and $\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{393.27}{108.22}$ or $\bar{Y} = 3.61$ in. ◀

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Chapter 5, Solution 7.

	A, in^2	$\bar{x}, \text{in.}$	$\bar{x}A, \text{in}^3$
1	$\dfrac{\pi(16)^2}{4} = 201.06$	$\dfrac{4(16)}{3\pi} = 6.7906$	1365.32
2	$-(8)(8) = -64$	4	-256
Σ	137.06		1109.32

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{1109.32}{137.06}\text{in.}$ or $\bar{X} = 8.09$ in. ◀

and $\bar{Y} = \bar{X}$ by symmetry or $\bar{Y} = 8.09$ in. ◀

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Chapter 5, Solution 8.

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	35 343	63.662	0	2 250 006	0
2	−4417.9	31.831	−31.831	−140 626	140 626.2
Σ	30925.1			2 109 380	140 626.2

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{2\,109\,380}{30\,925.1}$ or $\bar{X} = 68.2$ mm ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{140\,625}{30\,925.1}$ or $\bar{Y} = 4.55$ mm ◀

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Chapter 5, Solution 9.

	A	$\bar{x}$	$\bar{x}A$
1	$-\frac{\pi^2}{4}r_1^2$	$\frac{4r_1}{3\pi}$	$\left(-\frac{\pi}{4}r_1^2\right)\left(\frac{4r_1}{3\pi}\right) = -\frac{r_1^3}{3}$
2	$\frac{\pi}{2}r_2^2$	$\frac{4r_2}{3\pi}$	$\left(\frac{\pi}{2}r_2^2\right)\left(\frac{4r_2}{3\pi}\right) = \frac{2r_2^3}{3}$
Σ	$\frac{\pi}{4}\left(2r_2^2 - r_1^2\right)$		$\frac{1}{3}\left(2r_2^3 - r_1^3\right)$

Therefore, for $\bar{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{4r_1}{3\pi}$:

$$\frac{4r_1}{3\pi} = \frac{4\left(2r_2^3 - r_1^3\right)}{3\pi\left(2r_2^2 - r_1^2\right)} = \frac{4}{3\pi}\frac{r_1^3\left[2\left(\frac{r_2}{r_1}\right)^3 - 1\right]}{r_1^2\left[2\left(\frac{r_2}{r_1}\right)^2 - 1\right]}$$

or $$\pi = \frac{2\rho^3 - 1}{2\rho^2 - 1}, \text{ where } \rho = \frac{r_2}{r_1}$$

or $$2\rho^3 - 2\pi\rho^2 + (\pi - 1) = 0.$$

Solving numerically for ρ and noting that $\rho > 1$: $\frac{r_2}{r_1} = 3.02$ ◀

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Chapter 5, Solution 10.

First, determine the location of the centroid.

From Fig. 5.8A:

$$\bar{y}_2 = \frac{2}{3} r_2 \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)} \qquad A_2 = \left(\tfrac{\pi}{2} - \alpha\right) r_2^2$$

$$= \frac{2}{3} r_2 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)}$$

Similarly

$$\bar{y}_1 = \frac{2}{3} r_1 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \qquad A_1 = \left(\tfrac{\pi}{2} - \alpha\right) r_1^2$$

Then

$$\Sigma \bar{y} A = \frac{2}{3} r_2 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\tfrac{\pi}{2} - \alpha\right) r_2^2\right] - \frac{2}{3} r_1 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\tfrac{\pi}{2} - \alpha\right) r_1^2\right]$$

$$= \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

and

$$\Sigma A = \left(\frac{\pi}{2} - \alpha\right) r_2^2 - \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

$$= \left(\frac{\pi}{2} - \alpha\right)\left(r_2^2 - r_1^2\right)$$

Now

$$\bar{Y} \Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}\left[\left(\frac{\pi}{2} - \alpha\right)\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

$$\bar{Y} = \frac{2}{3} \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \frac{\cos\alpha}{\frac{\pi}{2} - \alpha}$$

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Using Figure 5.8B, $\bar{Y}$ of an arc of radius $\frac{1}{2}(r_1 + r_2)$ is

$$\bar{Y} = \frac{1}{2}(r_1 + r_2)\frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)}$$

$$= \frac{1}{2}(r_1 + r_2)\frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \qquad (1)$$

Now
$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{(r_2 - r_1)\left(r_2^2 + r_1 r_2 + r_1^2\right)}{(r_2 - r_1)(r_2 + r_1)}$$

$$= \frac{r_2^2 + r_1 r_2 + r_1^2}{r_2 + r_1}$$

Let
$$r_2 = r + \Delta$$

$$r_1 = r - \Delta$$

Then
$$r = \frac{1}{2}(r_1 + r_2)$$

and
$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{(r + \Delta)^2 + (r + \Delta)(r - \Delta) + (r - \Delta)^2}{(r + \Delta) + (r - \Delta)}$$

$$= \frac{3r^2 + \Delta^2}{2r}$$

In the limit as $\Delta \to 0$ (i.e., $r_1 = r_2$), then

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{3}{2}r$$

$$= \frac{3}{2} \times \frac{1}{2}(r_1 + r_2)$$

so that
$$\bar{Y} = \frac{2}{3} \times \frac{3}{4}(r_1 + r_2)\frac{\cos\alpha}{\frac{\pi}{2} - \alpha} \qquad \text{or} \quad \bar{Y} = \frac{1}{2}(r_1 + r_2)\frac{\cos\alpha}{\frac{\pi}{2} - \alpha} \blacktriangleleft$$

Which agrees with Eq. (1).

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Chapter 5, Solution 11.

	A, in^2	$\bar{x}$, in.	$\bar{x}A$, in^3
1	27	8.1962	221.30
2	15.5885	3.4641	54.000
3	−18.8495	3.8197	−71.999
Σ	23.739		203.30

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{203.30}{23.739}$ or $\bar{X} = 8.56$ in. ◀

and by symmetry $\bar{Y} = 0$ ◀

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Chapter 5, Solution 12.

	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{1}{2}(240)(150) = 18\,000$	160	50	2 880 000	900 000
2	$-\frac{1}{3}(240)(150) = 12\,000$	$\frac{3}{4}(240) = 180$	$\frac{3}{10}(150) = 45$	−2 160 000	−540 000
Σ	6000			720 000	360 000

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{720\,000}{6000}\text{mm}$ or $\bar{X} = 120.0 \text{ mm}$ ◀

$\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{360\,000}{6000}\text{mm}$ or $\bar{Y} = 60.0 \text{ mm}$ ◀

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Chapter 5, Solution 13.

	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$(18)(8) = 144$	-3	4	-432	576
2	$\frac{1}{2}(6)(9) = 27$	2	-3	54	-81
3	$\frac{\pi}{4}(12)(9) = 84.823$	-5.0930	-3.8197	-432.00	-324.00
Σ	255.82			-810.00	171.00

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{-810.00}{255.82}\text{in.}$ or $\bar{X} = -3.17 \text{ in.}$ ◀

$\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{171.00}{255.82}\text{in.}$ or $\bar{Y} = 0.668 \text{ in.}$ ◀

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Chapter 5, Solution 14.

First, by symmetry $\bar{X} = 90$ mm ◀

	A, mm^2	$\bar{y}, \text{mm}$	$\bar{y}A, \text{mm}^3$
1	$(180)(120) = 21\,600$	60	1 296 000
2	$-\frac{\pi}{4}(90)(120) = -8482.3$	$120 - \frac{4 \times 120}{3\pi} = 69.070$	−585 870
3	$-\frac{\pi}{4}(90)(120) = -8482.3$	$120 - \frac{4 \times 120}{3\pi} = 69.070$	−585 870
Σ	4635.4		124 260

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{124\,260}{4635.4}$$ or $\bar{Y} = 26.8$ mm ◀

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Chapter 5, Solution 15.

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	18 240	-4	12	72 960	218 880
2	-1920	-56	54	107 520	$-103\,680$
3	-4071.5	-41.441	-41.441	168 731	186 731
Σ	12 248.5			$-134\,171$	$-53\,531.1$

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{-134\,171}{12\,248.5}$ or $\bar{X} = -10.95$ mm ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{-53\,531}{12\,248.5}$ or $\bar{Y} = -43.7$ mm ◀

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Chapter 5, Solution 16.

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(200)(200) = 26\ 667$	75	70	2 000 000	1 866 690
2	$-\frac{2}{3}(100)(50) = -3333.3$	37.5	-20	$-125\ 000$	66 666
Σ	23 334			1 875 000	1 933 360

Then $\bar{X} = \dfrac{\Sigma\bar{x}A}{\Sigma A} = \dfrac{1\,875\,000}{23\,334}\text{mm}$ or $\bar{X} = 80.4$ mm ◀

$\bar{Y} = \dfrac{\Sigma\bar{y}A}{\Sigma A} = \dfrac{1\,933\,360}{23\,334}\text{mm}$ or $\bar{Y} = 82.9$ mm ◀

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Chapter 5, Solution 17.

Locate first $\bar{Y}$:

Note that the origin of the X axis is at the bottom of the whole area.

	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$8 \times 15 = 120$	7.5	900
2	$-4 \times 10 = -40$	8	-320
Σ	80		580

Then $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{580}{80} = 7.2500 \text{ in.}$$

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Now, to find the first moment of each area about the x-axis:

Area I:

$$Q_I = \Sigma \bar{y}A = \frac{7.75}{2}(8 \times 7.75) + \frac{5.75}{2}\left[-(4 \times 5.75)\right], \qquad \text{or } Q_I = 174.125 \text{ in}^3 \blacktriangleleft$$

Area II:

$$Q_{II} = \Sigma \bar{y}A = -\frac{7.75}{2}(8 \times 7.25) - \frac{4.25}{2}\left[-(4 \times 4.25)\right], \qquad \text{or } Q_{II} = -174.125 \text{ in}^3 \blacktriangleleft$$

Note that $Q_{(\text{area})} = Q_I + Q_{II} = 0$ which is expected as $\bar{y} = 0$ and $Q_{(\text{area})} = \bar{y}A$ since x is a centroidal axis.

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Chapter 5, Solution 18.

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$(80)(20) = 1600$	90	144 000
2	$(20)(80) = 1600$	40	64 000
Σ	3200		208 000

Then $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{208\,000}{3200} = 65.000 \text{ mm}$$

Now, for the first moments about the x-axis:

Area I

$Q_I = \Sigma \bar{y}A = 25(80 \times 20) + 7.5(20 \times 15) = 42\ 250 \text{ mm}^3$, or $Q_I = 42.3 \times 10^3 \text{ mm}^3$ ◀

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Area II

$Q_{II} = \Sigma \bar{y} A = -32.5(20 \times 65) = 42\ 250\ \text{mm}^3,$ or $Q_{II} = 42.3 \times 10^3\ \text{mm}^3$ ◀

Note that $Q_{(\text{area})} = Q_I + Q_{II} = 0$ which is expected as $\bar{y} = 0$ and $Q_{(\text{area})} = \bar{y}A$ since x is a centroidal axis.

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Chapter 5, Solution 19.

(a) With $Q_x = \Sigma \bar{y} A$ and using Fig. 5.8 A,

$$Q_x = \left[\frac{\frac{2}{3} r \sin\left(\frac{\pi}{2} - \theta\right)}{\frac{\pi}{2} - \theta}\right]\left[r^2\left(\frac{\pi}{2} - \theta\right)\right] - \left(\frac{2}{3} r \sin\theta\right)\left(\frac{1}{2} \times 2r\cos\theta \times r\sin\theta\right)$$

$$= \frac{2}{3} r^3 \left(\cos\theta - \cos\theta \sin^2\theta\right)$$

or $Q_x = \dfrac{2}{3} r^3 \cos^3\theta$ ◀

(b) By observation, Q_x is maximum when $\theta = 0$ ◀

and then $Q_x = \dfrac{2}{3} r^3$ ◀

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Chapter 5, Solution 20.

From the problem statement: F is proportional to Q_x. Therefore:

$$\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}, \text{ or}$$

$$F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

For the first moments:

$$(Q_x)_A = \left(225 + \frac{12}{2}\right)(300 \times 12) = 831\,600 \text{ mm}^3$$

$$(Q_x)_B = (Q_x)A + 2\left(225 - \frac{12}{2}\right)(48 \times 12) + 2(225 - 30)(12 \times 60) = 1\,364\,688 \text{ mm}^3$$

Then $\quad F_B = \dfrac{1364688}{831600}(280\,\text{N}),$ $\quad$ or $F_B = 459$ N ◀

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Chapter 5, Solution 21.

Because the wire is homogeneous, its center of gravity will coincide with the centroid for the corresponding line.

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	400	200	0	80 000	0
2	300	400	150	120 000	45 000
3	600	100	300	60 000	180 000
4	150	−200	225	−30 000	33 750
5	200	−100	150	−20 000	30 000
6	150	0	75	0	11 250
Σ	1800			210 000	300 000

Then $$\bar{X} = \frac{\Sigma \bar{x}L}{\Sigma L} = \frac{210\,000}{1800} = 116.667 \text{ mm}$$ or $\bar{X} = 116.7$ mm ◀

and $$\bar{Y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{300\,000}{1800} = 166.667 \text{ mm}$$ or $\bar{Y} = 166.7$ mm ◀

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Chapter 5, Solution 22.

	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}$, in^2
1	19	9.5	0	180.5	0
2	15	14.5	6	217.5	90
3	4	10	10	40	40
4	10	5	8	50	80
5	8	0	4	0	32
Σ	56			488	242

Then $\bar{X} = \dfrac{\Sigma \bar{x}L}{\Sigma L} = \dfrac{488}{56}$ or $\bar{X} = 8.71$ in. ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{242}{56}$ or $\bar{Y} = 4.32$ in. ◀

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Chapter 5, Solution 23.

Because the wire is homogeneous, its center of gravity will coincide with the centroid for the corresponding line.

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	600	75	0	45 000	0
2	$\sqrt{375^2 + 225^2} = 437.32$	187.5	112.5	81 998	49 199
3	$\frac{\pi}{2}(225)$	$-\frac{2}{\pi}(225)$	$\frac{2}{\pi}(225)$	$-50\,625$	50 625
Σ	1390.75			76 373	99 824

Then $\bar{X} = \dfrac{\Sigma \bar{x}L}{\Sigma L} = \dfrac{76\,373}{1390.75}$ or $\bar{X} = 54.9$ mm ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}L}{\Sigma L} = \dfrac{99\,824}{1390.75}$ or $\bar{Y} = 71.8$ mm ◀

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Chapter 5, Solution 24.

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	75	37.5	0	2812.5	0
2	150	0	75	0	11 250
3	$(150)\pi = 471.24$	95.492	0	45 000	0
4	75	0	-112.5	0	-8437.5
5	$(75)\dfrac{\pi}{2} = 117.81$	47.746	-47.746	5625.0	-5625.0
Σ	889.05			53 437	-2812.5

Then $\bar{X} = \dfrac{\Sigma \bar{x}L}{\Sigma L} = \dfrac{53\,437}{889.05}$, or $\bar{X} = 60.1$ mm ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{-2812.5}{889.05}$ or $\bar{Y} = -3.16$ mm ◀

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Chapter 5, Solution 25.

From Figure 5.8 b:

$$\bar{r} = \frac{(20 \text{ in.})\sin 30°}{\frac{\pi}{6}} = \frac{60}{\pi}\text{in.}$$

Note also that triangle *ABO* is equilateral, where *O* is the origin of the coordinate system in the figure.

For equilibrium:

(a) $\Sigma M_A = 0$: $\left[20 \text{ in.} - \left(\frac{60}{\pi}\text{in.}\right)\cos 30°\right](1.75 \text{ lb}) - \left[(20 \text{ in.})\sin 60°\right]T_{BC} = 0$

Solving for T_{BC}:

$T_{BC} = 0.34960 \text{ lb}$ or $T_{BC} = 0.350 \text{ lb}$ ◀

(b) $\Sigma F_x = 0$: $A_x + (0.34960 \text{ lb})\cos 60° = 0$

$A_x = -0.174800 \text{ lb}$

$\Sigma F_x = 0$: $A_y - 1.75 \text{ lb} + (0.34960 \text{ lb})\sin 60° = 0$

$A_y = 1.44724 \text{ lb}$

Therefore:

$\mathbf{A} = 1.458 \text{ lb} \measuredangle 83.1°$ ◀

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Chapter 5, Solution 26.

The wire supported only by the pin at B is a two-force body. For equilibrium the center of gravity of the wire must lie directly under B. Also, because the wire is homogeneous the center of gravity will coincide with the centroid. In other words, $\bar{x} = 0$, or $\Sigma \bar{x} L = 0$.

$$\Sigma \bar{x} L = -\frac{2(150\text{ mm})}{\pi}\left[\pi(150\text{ mm})\right] + \left(\frac{200\text{ mm}}{2}\right)(200\text{ mm}) + \left(200\text{ mm} - \frac{150\text{ mm}}{2}\cos\theta\right)(150\text{ mm})$$

or

$$\cos\theta = \frac{5000}{11250}$$

or $\theta = 63.6°$ ◀

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Chapter 5, Solution 27.

The wire supported only by the pin at B is a two-force body. For equilibrium the center of gravity of the wire must lie directly under B. Also, because the wire is homogeneous the center of gravity will coincide with the centroid. In other words, $\bar{x} = 0$, or $\Sigma \bar{x} L = 0$.

$$\Sigma \bar{x} L = -\frac{2(150\text{ mm})}{\pi}\left[\pi(150\text{ mm})\right] + \left(\frac{200\text{ mm}}{2}\right)(200\text{ mm}) + \left(200\text{ mm} - \frac{150\text{ mm}}{2}\cos\theta\right)(150\text{ mm})$$

or

$l^2 + 300l - 197602 = 0.$

Solving for l: $l = 319.15$, and $l = -619.15$, and discarding the negative root

$l = 319$ mm ◀

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Chapter 5, Solution 28.

The centroid coincides with the center of gravity because the wire is homogeneous.

	L	x	xL
1	r	$-\dfrac{r}{2}$	$-\dfrac{r^2}{2}$
2	$2\theta r$	$-\dfrac{r\sin\theta}{\theta}$	$-2r^2\sin\theta$
3	l	$\dfrac{l}{2}$	$\dfrac{l^2}{2}$

Then $\bar{X} = \dfrac{\Sigma\bar{x}L}{\Sigma L} = 0 \Rightarrow \Sigma\bar{x}L = 0$ and

$$-\frac{r^2}{2} - 2r^2\sin\theta + \frac{l^2}{2} = 0, \text{ or } l = r\sqrt{1 + 4\sin\theta}$$

(a) $\theta = 15°$:

$l = r\sqrt{1 + 4\sin 15°}$ or $l = 1.427r$ ◀

(b) $\theta = 60°$:

$l = r\sqrt{1 + 4\sin 60°}$ or $l = 2.11r$ ◀

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Chapter 5, Solution 29.

Then $$\bar{y} = \frac{\Sigma yA}{\Sigma A}$$

or $$\bar{y} = \frac{\frac{a}{2}(ab) - \left[\frac{(a+h)}{2}\right]\left[kb(a-h)\right]}{ba - kb(a-h)}$$

$$= \frac{1}{2}\frac{a^2(1-k) + kh^2}{a(1-k) + kh}$$

Let $$c = 1 - k \qquad \text{and} \qquad \zeta = \frac{h}{a}$$

Then $$\bar{y} = \frac{a}{2}\frac{c + k\zeta^2}{c + k\zeta} \tag{1}$$

Now find a value of ζ (or h) for which $\bar{y}$ is minimum:

$$\frac{d\bar{y}}{d\zeta} = \frac{a}{2}\frac{2k\zeta(c + k\zeta) - k\left(c + k\zeta^2\right)}{(c + k\zeta)^2} = 0 \qquad \text{or} \qquad 2\zeta(c + k\zeta) - \left(c + k\zeta^2\right) = 0 \tag{2}$$

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Expanding (2) $\quad 2c\zeta + 2\zeta^2 - c - k\zeta^2 = 0 \quad$ or $\quad k\zeta^2 + 2c\zeta - c = 0$

Then
$$\zeta = \frac{-2c \pm \sqrt{(2c)^2 - 4(k)(c)}}{2k}$$

Taking the positive root, since $h > 0$ (hence $\zeta > 0$)

$$h = a\frac{-2(1-k) + \sqrt{4(1-k)^2 + 4k(1-k)}}{2k}$$

(a) $k = 0.2$:
$$h = a\frac{-2(1-0.2) + \sqrt{4(1-0.2)^2 + 4(0.2)(1-0.2)}}{2(0.2)}$$
or $h = 0.472a$ ◀

(b) $k = 0.6$:
$$h = a\frac{-2(1-0.6) + \sqrt{4(1-0.6)^2 + 4(0.6)(1-0.6)}}{2(0.6)}$$
or $h = 0.387a$ ◀

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Chapter 5, Solution 30.

From Problem 5.29, note that Eq. (2) yields the value of ζ that minimizes h.

Then from Eq. (2)

We see $$2\zeta = \frac{c + k\zeta^2}{c + k\zeta} \qquad (3)$$

Then, replacing the right-hand side of (1) by 2ζ, from Eq. (3)

We obtain $$\bar{y} = \frac{a}{2}(2\zeta)$$

But $$\zeta = \frac{h}{a}$$

So $$\bar{y} = h$$ Q.E.D. ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 5, Solution 31.

Note that $y_1 = -\dfrac{h}{a}x + h$

$$= \frac{h}{a}(a - x)$$

Choose the area element (EL) as

$$dA = (h - y_1)dx = \frac{h}{a}xdx$$

Then $$A = \frac{h}{a}\int_0^a xdx = \frac{h}{a}\left[\frac{1}{2}x^2\right]_0^a = \frac{1}{2}ah$$

Now, noting that $\bar{x}_{EL} = x$, and $\bar{y}_{EL} = \dfrac{1}{2}(h + y_1)$

$$\bar{x} = \frac{1}{A}\int xdA = \frac{2}{ah}\int_0^a x\left(\frac{h}{a}xdx\right) = \frac{2}{a^2}\left[\frac{1}{3}x^3\right]_0^a = \frac{2}{3}a$$

$$\bar{y} = \frac{1}{A}\int\frac{1}{2}(h + y_1)dA = \frac{2}{ah}\int_0^h\left[\frac{1}{2}(h + y_1)\right]\left[(h - y_1)dx\right] = \frac{2}{ah}\left(\frac{1}{2}\right)\int_0^a\left(h^2 - y_1^2\right)dx$$

$$= \frac{1}{ah}\int_0^a\left[h^2 - \frac{h^2}{a^2}(a - x)^2\right]dx = \frac{h}{a}\left[x + \left(\frac{1}{3}\right)\left(\frac{1}{a^2}\right)(a - x)^3\right]_0^A = \frac{h}{a}\left(a - \frac{1}{3}a\right) = \frac{2}{3}h$$

Therefore: $\bar{x} = \dfrac{2}{3}a$ ◀

$\bar{y} = \dfrac{2}{3}h$ ◀

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Chapter 5, Solution 32.

First determine k:

For $x = a$, $y = 0$ and therefore

$$0 = h\left(1 - ka^3\right) \text{ or } k = a^{-3}, \text{ and therefore}$$

$$y = h\left(1 - \frac{x^3}{a^3}\right)$$

Choosing an area element as in the figure:

$$x_{EL} = x, \qquad y_{EL} = \frac{y}{2}, \qquad \text{and } dA = ydx$$

$$A = \int dA = \int_0^a ydx = \int_0^a h\left(1 - \frac{x^3}{a^3}\right)dx = h\left[x - \frac{x^4}{4a^3}\right]_0^a = \frac{3}{4}ah$$

$$\int x_{EL}dA = \int_0^a xydx = \int_0^a h\left(x - \frac{x^4}{a^3}\right)dx = b\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]_0^a = \frac{3}{10}a^2b$$

$$\int y_{EL}dA = \int_0^a\left(\frac{y}{2}\right)ydx = \frac{1}{2}\int_0^a h^2\left(x - \frac{x^3}{a^3}\right)^2 dx = \frac{b^2}{2}\int_0^a\left(1 - \frac{2x^3}{a^3} + \frac{x^6}{a^6}\right)dx = \frac{b^2}{2}\left[x - \frac{x^4}{2a^3} + \frac{x^7}{7a^6}\right]_0^a = \frac{9}{28}ab^2$$

Now

$$\bar{x} = \frac{1}{A}\int x_{EL}dA = \frac{4}{3ab}\frac{3a^2b}{10} = \frac{2}{5}a$$

and

$$\bar{y} = \frac{1}{A}\int y_{EL}dA = \frac{4}{3ab}\frac{9ab^2}{28} = \frac{3}{7}b$$

Therefore: $\bar{x} = \frac{2}{5}a$ ◀

$\bar{y} = \frac{3}{7}b$ ◀

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Chapter 5, Solution 33.

For the element (EL) shown

At $\quad x = a, y = h: \quad h = k_1 a^3 \quad$ or $\quad k_1 = \dfrac{h}{a^3}$

$$a = k_2 h^3 \qquad \text{or} \qquad k_2 = \frac{a}{h^3}$$

Hence, on line 1

$$y = \frac{h}{a^3} x^3$$

and on line 2

$$y = \frac{h}{a^{1/3}} x^{1/3}$$

Then

$$dA = \left(\frac{h}{a^{1/3}} x^{1/3} - \frac{h}{a^3} x^3\right) dx \qquad \text{and} \qquad \bar{y}_{EL} = \frac{1}{2}\left(\frac{h}{a^{1/3}} x^{1/3} + \frac{h}{a^3} x^3\right)$$

$$\therefore \; A = \int dA = \int_0^a \left(\frac{h}{a^{1/3}} x^{1/3} - \frac{h}{a^3} x^3\right) dx = h\left(\frac{3}{4a^{1/3}} x^{4/3} - \frac{1}{4a^3} x^4\right)\Bigg|_0^a = \frac{1}{2} ah$$

$$\int \bar{x}_{EL} dA = \int_0^a x\left(\frac{h}{a^{1/3}} x^{1/3} - \frac{h}{a^3} x^3\right) dx = h\left(\frac{3}{7a^{1/3}} x^{7/3} - \frac{1}{5a^3} x^5\right)\Bigg|_0^a = \frac{8}{35} a^2 h$$

$$\int \bar{y}_{EL} dA = \int_0^a \frac{1}{2}\left(\frac{h}{a^{1/3}} x^{1/3} + \frac{h}{a^3} x^3\right)\left(\frac{h}{a^{1/3}} x^{1/3} - \frac{h}{a^3} x^3\right) dx$$

$$= \frac{h^2}{2} \int_0^a \left(\frac{x^{2/3}}{a^{2/3}} - \frac{x^6}{a^6}\right) dx = \frac{h^2}{2}\left(\frac{3}{5}\frac{x^{5/3}}{a^{5/3}} - \frac{1}{7}\frac{x^6}{a^6}\right)\Bigg|_0^a = \frac{8}{35} ah^2$$

From $\quad \bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x}\left(\dfrac{ah}{2}\right) = \dfrac{8}{35} a^2 h \quad$ or $\quad \bar{x} = \dfrac{16}{35} a$ ◀

and $\quad \bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y}\left(\dfrac{ah}{2}\right) = \dfrac{8}{35} ah^2 \quad$ or $\quad \bar{y} = \dfrac{16}{35} h$ ◀

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Chapter 5, Solution 34.

Choose as an area element (EL) the shaded area shown:

$$dA = \left(\frac{\pi}{2}r\right)dr$$

$$\bar{x}_{EL} = \frac{2r}{\pi} \text{ and}$$

$$A = \int_{r_1}^{r_2}\left(\frac{\pi}{2}r\right)dr = \frac{\pi}{2}\left[\frac{1}{2}r^2\right]_{r_1}^{r_2} = \frac{\pi}{4}\left(r_2^2 - r_1^2\right)$$

Then

$$\bar{x} = \frac{1}{A}\int \bar{x}_{EL}dA = \frac{4}{\pi\left(r_2^2 - r_1^2\right)}\int_{r_1}^{r_2}\left(\frac{2r}{\pi}\right)\left(\frac{\pi}{2}rdr\right) = \frac{4}{\pi\left(r_2^2 - r_1^2\right)}\left[\frac{1}{3}r^3\right]_{r_1}^{r_2} \qquad \text{or } \bar{x} = \frac{4}{3\pi}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \blacktriangleleft$$

and by symmetry $\qquad \bar{y} = \frac{4}{3\pi}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \blacktriangleleft$

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Chapter 5, Solution 35.

Note that $y_1 = -\frac{b}{a}x + b = \frac{b}{a}(a - x)$, and

$$y_2 = \frac{b}{a}\sqrt{a^2 - x^2}$$

Then for the shaded area element:

$$dA = (y_2 - y_1)dx = \frac{b}{a}\left[\sqrt{a^2 - x^2} - (a - x)\right]dx \text{ and}$$

$$A = \int dA = \int_0^a \frac{b}{a}\left[\sqrt{a^2 - x^2} - (a - x)\right]dx = \frac{b}{a}\left\{\frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)\right] + \frac{1}{2}(a - x)^2\right\}\Bigg|_0^a$$

$$= \frac{b}{a}\left(\frac{1}{2} \times \frac{\pi}{2} - \frac{1}{2}a^2\right) = \frac{ab}{4}(\pi - 2)$$

Noting that $\bar{x}_{EL} = x$, and that $\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)$:

$$\bar{x} = \frac{1}{A}\int \bar{x}_{EL}dA = \frac{4}{ab(\pi - 2)}\int_0^a \frac{b}{a}\left[x\sqrt{x^2 - a^2} - x(a - x)\right]dx$$

$$= \frac{4}{ab(\pi - 2)}\left[-\frac{2}{3}\left(\frac{1}{2}\right)\left(a^2 - x^2\right)^{\frac{3}{2}} + \left(-\frac{1}{2}ax^2 + \frac{1}{3}x^3\right)\right]\Bigg|_0^a = \frac{4}{a^2(\pi - 2)}\left[\left(\frac{1}{3}\right)\left(a^2\right)^{\frac{3}{2}} + \left(-\frac{1}{2}a^3 + \frac{1}{3}a^3\right)\right]$$

$$\text{or } \bar{x} = \frac{2a}{3(\pi - 2)} \blacktriangleleft$$

continued

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$$\bar{y} = \frac{1}{A}\int \bar{y}_{EL}\,dA = \frac{4}{ab(\pi - 2)}\int_0^a \left[\frac{1}{2}(y_2 + y_1)\right]\left[(y_2 - y_1)\,dx\right]$$

$$= \frac{2}{ab(\pi - 2)}\int \left(y_2^2 - y_1^2\right) = \frac{2}{a^2(\pi - 2)}\int_0^a \left[\frac{b^2}{a^2}\left(a^2 - x^2\right) - \frac{b^2}{a^2}(a - x)^2\right]dx$$

$$= \frac{2b}{a^3(\pi - 2)}\int_0^a 2\left(ax - x^2\right)dx = \frac{4b}{a^3(\pi - 2)}\int_0^a \left(ax - x^2\right)dx = \frac{4b}{a^3(\pi - 2)}\left[\frac{1}{2}ax^2 - \frac{1}{3}x^3\right]_0^a$$

$$\text{or } \bar{y} = \frac{2b}{3(\pi - 2)} \blacktriangleleft$$

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Chapter 5, Solution 36.

First note that symmetry implies $\bar{x} = 0$ ◀

For the element (EL) shown

$$y = R\cos\theta, \quad x = R\sin\theta$$

$$dx = R\cos\theta\, d\theta$$

$$dA = y dx = R^2\cos^2\theta\, d\theta$$

Hence

$$A = \int dA = 2\int_0^\alpha R^2\cos^2\theta d\theta = 2R^2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Bigg|_0^\alpha = \frac{1}{2}R^2\left(2\alpha\sin 2\alpha\right)$$

$$\int \bar{y}_{EL} dA = 2\int_0^\alpha \frac{R}{2}\cos\theta\left(R^2\cos^2\theta d\theta\right) = R^3\left(\frac{1}{3}\cos^2\theta\sin\theta + \frac{2}{3}\sin\theta\right)\Bigg|_0^\alpha$$

$$= \frac{R^3}{3}\left(\cos^2\alpha\sin\alpha + 2\sin\alpha\right)$$

But $\bar{y}A = \int \bar{y}_{EL} dA$ so

$$\bar{y} = \frac{\frac{R^3}{3}\left(\cos^2\alpha\sin\alpha + 2\sin\alpha\right)}{\frac{R^2}{2}\left(2\alpha + \sin 2\alpha\right)}$$

or

$$\bar{y} = \frac{2}{3}R\sin\alpha\frac{\left(\cos^2\alpha + 2\right)}{\left(2\alpha + \sin 2\alpha\right)}$$

Alternatively,

$$\bar{y} = \frac{2}{3}R\sin\alpha\frac{3 - \sin^2\alpha}{2\alpha + \sin 2\alpha}$$ ◀

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Chapter 5, Solution 37.

At $\quad x = 0,\ y = b$

$$b = k(0 - a)^2 \qquad \text{or} \qquad k = \frac{b}{a^2}$$

Then $\quad y = \dfrac{b}{a^2}(x - a)^2$

Now $\quad \bar{x}_{EL} = x,\ \bar{y}_{EL} = \dfrac{y}{2} = \dfrac{b}{2a^2}(x - a)^2$

and $\quad dA = ydx = \dfrac{b}{a^2}(x - a)^2\,dx$

Then $\quad A = \int dA = \int_0^a \dfrac{b}{a^2}(x - a)^2\,dx = \dfrac{b}{3a^2}\left[(x - a)^3\right]_0^a = \dfrac{1}{3}ab$

and $\quad \int \bar{x}_{EL}dA = \int_0^a x\left[\dfrac{b}{a^2}(x - a)^2\,dx\right] = \dfrac{b}{a^2}\int_0^a \left(x^3 - 2ax^2 + a^2x\right)dx$

$$= \frac{b}{a^2}\left(\frac{x^4}{4} - \frac{2}{3}ax^3 + \frac{a^2}{2}x^2\right) = \frac{1}{12}a^2b$$

$$\int \bar{y}_{EL}dA = \int_0^a \frac{b}{2a^2}(x - a)^2\left[\frac{b}{a^2}(x - a)^2\,dx\right] = \frac{b^2}{2a^4}\left[\frac{1}{5}(x - a)^5\right]_0^a$$

$$= \frac{1}{10}ab^2$$

Hence $\quad \bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\dfrac{1}{3}ab\right) = \dfrac{1}{12}a^2b \qquad \bar{x} = \dfrac{1}{4}a$ ◀

$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\dfrac{1}{3}ab\right) = \dfrac{1}{10}ab^2 \qquad \bar{y} = \dfrac{3}{10}b$ ◀

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Chapter 5, Solution 38.

For the element (EL) shown on line 1 at

$$x = a,\ b = k_2 a^2 \qquad \text{or} \qquad k_2 = \frac{b}{a^2}$$

$$\therefore\ y = \frac{b}{a^2}x^2$$

On line 2 at $\quad x = a,\ -2b = k_1 a^3 \qquad \text{or} \qquad k_2 = \dfrac{-2b}{a^3}$

$$\therefore\ y = \frac{-2b}{a^3}x^3$$

$$dA = \left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right)dx$$

Then $\quad A = \int dA = \int_0^a \frac{b}{a^2}\left(x^2 + \frac{2x^3}{x}\right)dx = \frac{b}{a^2}\left(\frac{x^3}{3} + \frac{2x^4}{4a}\right)\Bigg|_0^a$

$$= ab\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{5}{6}ab$$

and $\int \bar{x}_{EL}dA = \int_0^a x\left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right)dx = \frac{b}{a^2}\left(\frac{x^4}{4} + \frac{2x^5}{5a}\right)\Bigg|_0^a = a^2b\left(\frac{1}{4} + \frac{2}{5}\right)$

$$= \frac{13}{20}a^2b$$

$$\int \bar{y}_{EL}dA = \int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2 - \frac{2b}{a^3}x^3\right)\left[\left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right)dx\right]$$

$$= \int_0^a \frac{1}{2}\left[\left(\frac{b}{a^2}x^2\right)^2 - \left(\frac{2b}{a^3}x^3\right)^2\right]dx = \frac{b^2}{2a^4}\left(\frac{x^5}{5} - \frac{2}{7a^2}x^7\right)\Bigg|_0^a$$

$$= b^2a^5\left(\frac{1}{10} - \frac{2}{7}\right) = -\frac{13}{70}ab^2$$

Then $\quad \bar{x}A = \int \bar{x}_{EL}dA:\quad \bar{x}\left(\frac{5}{6}ab\right) = \frac{13}{20}a^2b \qquad \text{or} \qquad \bar{x} = \frac{39}{50}a$ ◀

$\bar{y}A = \int \bar{y}_{EL}dA:\quad \bar{y}\left(\frac{5}{6}ab\right) - \frac{13}{70}ab^2 \qquad \text{or} \quad \bar{y} = -\frac{39}{175}b$ ◀

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Chapter 5, Solution 39.

Using the area element shown:

$$\bar{x}_{EL} = x, \qquad \bar{y}_{EL} = \frac{y}{2}, \qquad \text{and } dA = ydx$$

$$A = \int dA = \int_0^L h\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)dx = h\left[x + \frac{x^2}{2L} - \frac{2}{3}\frac{x^3}{L^2}\right]_0^L = \frac{5}{6}hL$$

$$\int \bar{x}_{EL}dA = \int_0^L xh\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)dx = h\int_0^L\left(x + \frac{x^2}{L} - 2\frac{x^3}{L^2}\right)dx = h\left[\frac{x^2}{2} + \frac{1}{3}\frac{x^3}{L} - \frac{2}{4}\frac{x^4}{L^2}\right]_0^L = \frac{1}{3}hL^2$$

$$\int \bar{y}_{EL}dA = \frac{1}{2}\int y^2dx = \frac{h^2}{2}\int_0^L\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)^2 dx = \frac{h^2}{2}\int_0^L\left(1 + \frac{x^2}{L^2} + 4\frac{x^4}{L^4} + 2\frac{x}{L} - 4\frac{x^2}{L^2} - 4\frac{x^3}{L^3}\right)dx$$

$$= \frac{h^2}{2}\left[x + \frac{x^3}{3L^2} + \frac{4x^5}{5L^4} + \frac{x^2}{L} - \frac{4x^3}{3L^2} - \frac{x^4}{L^3}\right]_0^L = \frac{4}{10}h^2L$$

Now

$$\bar{x} = \frac{1}{A}\int x_{EL}dA = \frac{6}{5hL}\left(\frac{1}{3}hL^2\right) = \frac{2}{5}L \text{ and}$$

$$\bar{y} = \frac{1}{A}\int y_{EL}dA = \frac{6}{5hL}\left(\frac{4}{10}h^2L\right) = \frac{12}{25}h$$

Therefore: $\bar{x} = \frac{2}{5}L$ ◀

$\bar{y} = \frac{12}{25}h$ ◀

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Chapter 5, Solution 40.

Note that $y_1 = 0$ at $x = a$, or

$$0 = 2b\left(1 - ka^2\right), \text{ i.e. } k = \frac{1}{a^2}$$

Also, note that the slope of y_2 is $\frac{-3b}{a}$ and $y_2 = 0$ at $x = 2a$. Therefore

$$y_2 = \frac{3b}{a}(2a - x).$$

Pick the area element $dA(EL)$ such that:

for $0 \le x \le a$

$$dA = (3b - y_1)dx, \quad \text{and } \bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2}(3b + y_1)$$

and for $a \le x \le 2a$

$$dA = y_2\,dx, \quad \text{and } \bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2}y_2$$

Then:

$$A = \int dA = \int_0^a (3b - y_1)dx + \int_a^{2a} y_2 dx = \int_0^a \left[3b - \frac{2b}{a^2}\left(a^2 - x^2\right)\right]dx + \int_a^{2a} \frac{3b}{a}(2a - x)dx =$$

$$\int_0^a \left(b + \frac{2b}{a^2}x^2\right)dx + \int_a^{2a} \frac{3b}{a}(2a - x)dx = b\left(x + \frac{2}{3a^2}x^3\right)\Bigg|_0^a + \frac{3b}{a}\left(-\frac{1}{2}\right)(2a - x)^2\Bigg|_a^{2a} =$$

continued

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$$ab\left(1+\frac{2}{3}\right)-\frac{3b}{2a}\left[-(2a-a)^2\right]=\frac{19}{6}ab$$

Now for the centroid:

$$\bar{x}=\frac{1}{A}\int x\,dA=\frac{6}{19ab}\left[b\int_0^a x\left(1+\frac{2}{a^2}x^2\right)dx+\frac{3b}{a}\int_a^{2a}x(2a-x)\,dx\right]=$$

$$=\frac{6}{19a}\left[\left(\frac{1}{2}x^2+\frac{1}{2a^2}x^4\right)\Bigg|_0^a+\frac{3}{a}\left(ax^2-\frac{1}{3}x^3\right)\Bigg|_a^{2a}\right]=\frac{6}{19}\left[\left(\frac{1}{2}+\frac{1}{2}\right)+3\left(4-\frac{8}{3}-1+\frac{1}{3}\right)\right]$$

or $\bar{x}=\frac{18}{19}a$ ◀

$$\bar{y}=\frac{1}{A}\int\bar{y}_{EL}\,dA=\frac{6}{19ab}\left[\int_0^a\frac{1}{2}(3b+y_1)(3b-y_1)\,dx+\int_a^{2a}\frac{1}{2}y_2y_2\,dx\right]$$

$$=\frac{1}{2}\frac{6}{19ab}\left[\int_0^a\frac{1}{2}\left(9b^2-y_1^2\right)dx+\int_a^{2a}\frac{1}{2}y_2^2\,dx\right]=$$

$$=\frac{1}{2}\frac{6}{19ab}\left\{\int_0^a\frac{1}{2}\left[9b^2-\frac{4b^2}{a^4}\left(a^2-x^2\right)^2\right]dx+\int_a^{2a}\frac{9b^2}{a^2}(2a-x)^2\,dx\right\}$$

$$=\frac{3b}{19a}\left[\int_0^a\left(5+\frac{8}{a^2}x^2-\frac{4}{a^4}x^4\right)dx+\int_a^{2a}\frac{9}{a^2}(2a-x)^2\,dx\right]$$

$$=\frac{3b}{19a}\left[\left(5x+\frac{8}{3a^2}x^3-\frac{4}{5a^4}x^5\right)\Bigg|_0^a+\frac{9}{a^2}\left(-\frac{1}{3}\right)(2a-x)^3\Bigg|_a^{2a}\right]$$

$$=\frac{3b}{19}\left[\left(5+\frac{8}{3}-\frac{4}{5}\right)+3(1)^3\right]$$

$\bar{y}=\frac{148}{95}b$ ◀

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Chapter 5, Solution 41.

For y_2 at $x = a,\ y = b$: $a = kb^2$ or $k = \dfrac{a}{b^2}$

Then
$$y_2 = \frac{b}{\sqrt{a}}x^{1/2}$$

Now
$$\bar{x}_{EL} = x$$

and for $0 \le x \le \dfrac{a}{2}$: $\bar{y}_{EL} = \dfrac{y_2}{2} = \dfrac{b}{2}\dfrac{x^{1/2}}{\sqrt{a}},\ dA = y_2 dx = b\dfrac{x^{1/2}}{\sqrt{a}}dx$

For $\dfrac{a}{2} \le x \le a$: $\bar{y}_{EL} = \dfrac{1}{2}(y_1 + y_2) = \dfrac{b}{2}\left(\dfrac{x}{a} - \dfrac{1}{2} + \dfrac{x^{1/2}}{\sqrt{a}}\right)$

$$dA = (y_2 - y_1)dx = b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

Then
$$A = \int dA = \int_0^{a/2} b\frac{x^{1/2}}{\sqrt{a}}dx + \int_{a/2}^{a} b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{3}x^{3/2}\right]_0^{a/2} + b\left[\frac{2}{3}\frac{x^{3/2}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2}x\right]_{a/2}^{a}$$

$$= \frac{2}{3}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{3/2} + (a)^{3/2} - \left(\frac{a}{2}\right)^{3/2}\right]$$

$$+ b\left\{-\frac{1}{2a}\left[(a^2) - \left(\frac{a}{2}\right)^2\right] + \frac{1}{2}\left[(a) - \left(\frac{a}{2}\right)\right]\right\}$$

$$= \frac{13}{24}ab$$

and
$$\int \bar{x}_{EL}dA = \int_0^{a/2} x\left(b\frac{x^{1/2}}{\sqrt{a}}dx\right) + \int_{a/2}^{a} x\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)\right]dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{5}x^{5/2}\right]_0^{a/2} + b\left[\frac{2}{5}\frac{x^{5/2}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^4}{4}\right]_{a/2}^{a}$$

continued

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$$= \frac{2}{5}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{5/2} + (a)^{5/2} - \left(\frac{a}{2}\right)^{5/2}\right]$$

$$+ b\left\{-\frac{1}{3a}\left[(a)^3 - \left(\frac{a}{2}\right)^3\right] + \frac{1}{4}\left[(a)^2 - \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{71}{240}a^2 b$$

$$\int \bar{y}_{EL}dA = \int_0^{a/2}\frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}\left[b\frac{x^{1/2}}{\sqrt{a}}dx\right]$$

$$+ \int_{a/2}^{a}\frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}}\right)\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx\right]$$

$$= \frac{b^2}{2a}\left[\frac{1}{2}x^2\right]_0^{a/2} + \frac{b^2}{2}\left[\left(\frac{x^2}{2a} - \frac{1}{3a}\left(\frac{x}{a} - \frac{1}{2}\right)^3\right)\right]_{a/2}^{a}$$

$$= \frac{b}{4a}\left[\left(\frac{a}{2}\right)^2 + (a)^2 - \left(\frac{a}{2}\right)^2\right] - \frac{b^2}{6a}\left(\frac{a}{2} - \frac{1}{2}\right)^3$$

$$= \frac{11}{48}ab^2$$

Hence $\bar{x}A = \int \bar{x}_{EL}dA$: $\quad \bar{x}\left(\frac{13}{24}ab\right) = \frac{71}{240}a^2 b \quad \bar{x} = \frac{17}{130}a = 0.546a$ ◀

$\bar{y}A = \int \bar{y}_{EL}dA$: $\quad \bar{y}\left(\frac{13}{24}ab\right) = \frac{11}{48}ab^2 \quad \bar{y} = \frac{11}{26}b = 0.423b$ ◀

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Chapter 5, Solution 42.

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Now $\bar{x}_{EL} = r\cos\theta$ and $dL = rd\theta$

Then $L = \int dL = \int_{\pi/4}^{7\pi/4} rd\theta = r\left[\theta\right]_{\pi/4}^{7\pi/4} = \frac{3}{2}\pi r$

and $\int \bar{x}_{EL} dL = \int_{\pi/4}^{7\pi/4} r\cos\theta\left(rd\theta\right)$

$$= r^2\left[\sin\theta\right]_{\pi/4}^{7\pi/4} = r^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = -r^2\sqrt{2}$$

Thus $\bar{x}L = \int \bar{x}dL$: $\bar{x}\left(\frac{3}{2}\pi r\right) = -r^2\sqrt{2}$ $\qquad \bar{x} = -\frac{2\sqrt{2}}{3\pi}r$ ◀

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SOLUTION 5.43 CONTINUED

Then $$\frac{dy}{dx} = \left(a^{2/3} - x^{2/3}\right)^{1/2}\left(-x^{-1/3}\right)$$

Now $$\bar{x}_{EL} = x$$

and $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad dx = \left\{1 + \left[\left(a^{2/3} - x^{2/3}\right)^{1/2}\left(-x^{-1/3}\right)\right]^2\right\}^{1/2} dx$$

Then $$L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}} dx = a^{1/3}\left[\frac{3}{2}x^{2/3}\right]_0^a = \frac{3}{2}a$$

and $$\int \bar{x}_{EL} dL = \int_0^a x\left(\frac{a^{1/3}}{x^{1/3}} dx\right) = a^{1/3}\left[\frac{3}{5}x^{5/3}\right]_0^a = \frac{3}{5}a^2$$

Hence $$\bar{x}L = \int \bar{x}_{EL} dL: \quad \bar{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2 \qquad \bar{x} = \frac{2}{5}a \quad ◀$$

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Chapter 5, Solution 43.

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Now $\quad \bar{x}_{EL} = a\cos^3\theta \quad$ and $\quad dL = \sqrt{dx^2 + dy^2}$

Where $\quad x = a\cos^3\theta: \quad dx = -3a\cos^2\theta\sin\theta d\theta$

$$y = a\sin^3\theta: \quad dy = 3a\sin^2\theta\cos\theta d\theta$$

Then
$$dL = \left[\left(-3a\cos^2\theta\sin\theta d\theta\right)^2 + \left(3a\sin^2\theta\cos\theta d\theta\right)^2\right]^{1/2}$$
$$= 3a\cos\theta\sin\theta\left(\cos^2\theta + \sin^2\theta\right)^{1/2} d\theta$$
$$= 3a\cos\theta\sin\theta d\theta$$
$$\therefore \quad L = \int dL = \int_0^{\pi/2} 3a\cos\theta\sin\theta d\theta = 3a\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2}$$
$$= \frac{3}{2}a$$

and
$$\int \bar{x}_{EL} dL = \int_0^{\pi/2} a\cos^3\theta\left(3a\cos\theta\sin\theta d\theta\right)$$
$$= 3a^2\left[-\frac{1}{5}\cos^5\theta\right]_0^{\pi/2} = \frac{3}{5}a^2$$

Hence $\quad \bar{x}L = \int \bar{x}_{EL} dL: \quad \bar{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2 \qquad \bar{x} = \frac{2}{5}a$ ◀

Alternative solution

$$x = a\cos^3\theta \Rightarrow \cos^2\theta = \left(\frac{x}{a}\right)^{2/3}$$
$$y = a\sin^3\theta \Rightarrow \sin^2\theta = \left(\frac{y}{a}\right)^{2/3}$$
$$\therefore \quad \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1 \qquad \text{or} \qquad y = \left(a^{2/3} - x^{2/3}\right)^{3/2}$$

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Then $$\frac{dy}{dx} = \left(a^{2/3} - x^{2/3}\right)^{1/2}\left(-x^{-1/3}\right)$$

Now $$\bar{x}_{EL} = x$$

and $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad dx = \left\{1 + \left[\left(a^{2/3} - x^{2/3}\right)^{1/2}\left(-x^{-1/3}\right)\right]^2\right\}^{1/2} dx$$

Then $$L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}} dx = a^{1/3}\left[\frac{3}{2}x^{2/3}\right]_0^a = \frac{3}{2}a$$

and $$\int \bar{x}_{EL} dL = \int_0^a x\left(\frac{a^{1/3}}{x^{1/3}} dx\right) = a^{1/3}\left[\frac{3}{5}x^{5/3}\right]_0^a = \frac{3}{5}a^2$$

Hence $$\bar{x}L = \int \bar{x}_{EL} dL: \quad \bar{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2 \qquad \bar{x} = \frac{2}{5}a \blacktriangleleft$$

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Chapter 5, Solution 44.

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Have at $\quad x = a,\ y = a:\ a = ka^2 \quad$ or $\quad k = \dfrac{1}{a}$

Thus $\quad y = \dfrac{1}{a}x^2 \quad$ and $\quad dy = \dfrac{2}{a}xdx$

Then $\quad dL = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\,dx = \sqrt{1 + \left(\dfrac{2}{a}x\right)^2}\,dx$

$$\therefore\quad L = \int dL = \int_0^a \sqrt{1 + \frac{4}{a^2}x^2}\,dx = \left[\frac{x}{2}\sqrt{1 + \frac{4x^2}{a^2}} + \frac{a}{4}\ln\left(\frac{2}{a}x + \sqrt{1 + \frac{4x^2}{a^2}}\right)\right]_0^a$$

$$= \frac{a}{2}\sqrt{5} + \frac{a}{4}\ln\left(2 + \sqrt{5}\right) = 1.4789a$$

$$\int \bar{x}_{EL}dL = \int_0^a x\left(\sqrt{1 + \frac{4x^2}{a^2}}dx\right) = \left[\frac{2}{3}\left(\frac{a^2}{8}\right)\left(1 + \frac{4}{a^2}x^2\right)^{3/2}\right]_0^a$$

$$= \frac{a^2}{12}\left(5^{3/2} - 1\right) = 0.8484a^2$$

Then $\quad \bar{x}L = \int \bar{x}_{EL}dL:\ \ \bar{x}\left(1.4789a\right) = 0.8484a^2 \qquad \bar{x} = 0.574a$ ◀

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Chapter 5, Solution 45.

Have $$\bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2} x \sin \frac{\pi x}{L}$$

and $$dA = y dx$$

$$A = \int dA = \int_0^{L/2} x \sin \frac{\pi x}{L} dx = \left[\frac{L^2}{\pi^2} \sin \frac{\pi x}{L} - \frac{L}{\pi} x \cos \frac{\pi x}{L} \right]_0^{L/2} = \frac{L^2}{\pi^2}$$

and $$\bar{x} = \int \bar{x}_{EL} dA = \int_0^{L/2} x \left(x \sin \frac{\pi x}{L} dx \right)$$

$$= \left[\frac{2L^2}{\pi^2} x \sin \left(\frac{\pi x}{L} \right) + \frac{2L^3}{\pi^3} \cos \left(\frac{\pi x}{L} \right) - \frac{L}{\pi} x^2 \sin \left(\frac{\pi x}{L} \right) \right]_0^{L/2} = \frac{L^3}{\pi^2} - 2 \frac{L^3}{\pi^3}$$

Also $$\bar{y} = \int \bar{y}_{EL} dA = \int_0^{L/2} \frac{1}{2} x \sin \frac{\pi x}{L} \left(x \sin \frac{\pi x}{L} dx \right)$$

$$= \frac{1}{2} \left[\frac{2L^2}{\pi^2} x \sin \frac{\pi x}{L} - \left(\frac{L}{\pi} x - \frac{2L^3}{\pi^3} \right) \cos \frac{\pi x}{L} \right]_0^{L/2}$$

$$= \frac{1}{2} \left[\frac{1}{6} \left(\frac{L^3}{8} \right) - \frac{L^2}{4\pi^2} \left(\frac{L}{2} \right) (-1) \right] = \frac{L^3}{96\pi^2} \left(6 + \pi^2 \right)$$

Hence $$\bar{x} A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{L^2}{\pi^2} \right) = L^3 \left(\frac{1}{\pi^2} - \frac{z}{\pi^3} \right)$$

or $\bar{x} = 0.363L$ ◀

$$\bar{y} A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{L^2}{\pi^2} \right) = \frac{L^3}{96\pi^2} \left(\frac{1}{\pi^2} - \frac{2}{\pi^3} \right)$$

or $\bar{y} = 0.1653L$ ◀

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Chapter 5, Solution 46.

First note that by symmetry $\bar{y} = 0$.

Using the area element shown in the figure,

$$\bar{x}_{EL} = \frac{2}{3} r\cos\theta = \frac{2}{3} R\cos 2\theta\cos\theta$$

$$dA = \frac{1}{2} r^2 d\theta = \frac{1}{2} R^2 \cos^2 2\theta\, d\theta$$

$$A = \int dA = \frac{1}{2} R^2 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2\theta\, d\theta = R^2 \int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta = \frac{1}{2} R^2 \int_0^{\frac{\pi}{4}} \left(1 + \cos 4\theta\right) d\theta$$

$$= \frac{1}{2} R^2 \left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{4}} = \frac{1}{8}\pi R^2$$

$$\int \bar{x}_{EL} dA = \int_{-\frac{\pi}{\rho}}^{\frac{\pi}{4}} \left(\frac{2}{3} R\cos 2\theta\cos\theta\right)\left(\frac{1}{2} R^2 \cos^2 2\theta d\theta\right) = \frac{2}{3} R^2 \int_0^{\frac{\pi}{4}} \cos^3 2\theta\cos\theta d\theta$$

$$= \frac{2}{3} R^3 \int_0^{\frac{\pi}{4}} \left(1 - 2\sin^2\theta\right)^3 \cos\theta d\theta = \frac{2}{3} R^3 \int_0^{\frac{\pi}{4}} \left(1 - 6\sin^2\theta + 12\sin^4\theta - 8\sin^6\theta\right)\cos\theta d\theta$$

$$= \frac{2}{3} R^3 \left[\sin\theta - 2\sin^3\theta + \frac{12}{5}\sin^5\theta - \frac{8}{7}\sin^7\theta\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3} R^3 \frac{\sqrt{2}}{2}\left[1 - \frac{2}{2} + \frac{12}{5}\frac{1}{4} - \frac{8}{7}\frac{1}{8}\right] = \frac{16\sqrt{2}}{105} R^3$$

Now:

$$\bar{x} = \frac{1}{A}\int x_{EL} dA = \frac{8}{\pi R^2}\left(\frac{16\sqrt{2}}{105} R^3\right) = \frac{128\sqrt{2}}{105\pi} R$$

or $\bar{x} = 0.549\, R$ ◀

$\bar{y} = 0$ ◀

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Chapter 5, Solution 47.

From the solution to problem 5.2:

$$A = 134 \text{ in}^2, \qquad \Sigma\bar{x}A = 1102 \text{ in}^3, \qquad \Sigma\bar{y}A = 536 \text{ in}^3$$

and from the solution of problem 5.22

$$L = 56 \text{ in., and } \Sigma\bar{x}L = 488 \text{ in}^2$$

Applying the theorems of Pappus-Guldinus, we have

(a) Rotation about the x-axis:

$$\text{Volume} = 2\pi y_{area}A = 2\pi\Sigma\bar{y}A = 2\pi\left(536 \text{ in}^3\right) = 3367.8 \text{ in}^3 \qquad \text{or} \qquad V = 1.949 \text{ ft}^3 \blacktriangleleft$$

$$\text{Area} = 2\pi\bar{y}_{\text{length}}L = 2\pi\Sigma\bar{y}L = 2\pi\left[6(15) + 10(4) + 8(10) + 4(18)\right] = 1520.53 \text{ in}^2$$

$$\text{or} \qquad A = 10.56 \text{ ft}^2 \blacktriangleleft$$

(b) Rotation about $x = 19$ in.:

$$\text{Volume} = 2\pi\left(19 - \bar{x}_{\text{area}}\right)A = 2\pi\left(19A - \Sigma\bar{x}A\right) = 2\pi\left[(19 \text{ in})\left(134 \text{ in}^2\right) - 1102 \text{ in}^3\right]$$

$$= 9072.9 \text{ in}^3 \qquad \text{or} \qquad V = 5.25 \text{ ft}^3 \blacktriangleleft$$

$$\text{Area} = 2\pi\left(19 - \bar{x}_{\text{line}}\right)L = 2\pi\left(19L - \Sigma\bar{x}L\right) = 2\pi\left[(19 \text{ in.})(56 \text{ in.}) - 488 \text{ in}^2\right]$$

$$= 3619.1 \text{ in}^2$$

$$\text{or} \qquad A = 25.1 \text{ ft}^2 \blacktriangleleft$$

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Chapter 5, Solution 48.

From the solution to problem 5.4:

$A = 772 \text{ in}^2, \qquad \Sigma\bar{x}A = 17828 \text{ in}^3, \qquad \Sigma\bar{y}A = 18464 \text{ in}^3 \qquad (\text{Area})$

and for the line

	L	$\bar{x}$	$\bar{y}$	$\bar{x}L$	$\bar{y}L$
1	13	27.5	0	357.5	0
2	40	34	20	1360	800
3	34	17	40	578	1360
4	$\sqrt{21^2 + 24^2} = 31.890$	10.5	28	334.85	892.92
5	16	21	8	336	128
Σ	134.89			2966.4	3180.9

(*a*) $V = 2\pi\bar{x}_{\text{area}}A = 2\pi\Sigma\bar{x}A = 2\pi\left(17828 \text{ in}^3\right) = 112\ 017 \text{ in}^3$ or $V = 64.8 \text{ ft}^3$ ◀

$A = 2\pi\bar{x}_{\text{line}}L = 2\pi\Sigma\bar{x}L = 2\pi\left(2966.4 \text{ in}^2\right) = 18\ 638.1 \text{ in}^2$ or $A = 129.4 \text{ ft}^2$ ◀

continued

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(*b*)

$$V = 2\pi\left(40 - \bar{y}_{\text{area}}\right)A = 2\pi\left(40A - \Sigma\bar{y}A\right)$$

$$= 2\pi\left[\left(40\text{ in.}\right)\left(772\text{ in}^2\right) - 18\,464\text{ in}^3\right] = 78\,012\text{ in}^3$$

or $V = 45.1\text{ ft}^3$ ◀

$$A = 2\pi\left|\left(\bar{y}_{\text{line}}\right)_{y=40}\right|L = -2\pi\left[\Sigma L\left(\bar{y} - 40\right)\right] = -2\pi\left(\Sigma L\bar{y} - 40\Sigma L\right)$$

$$= -2\pi\left(3180.9 - 40\times 134.89\right) = 13\,915.3\text{ in}^2$$

or $A = 96.6\text{ ft}^2$ ◀

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Chapter 5, Solution 49.

From the solution of Problem 5.1:

$$A = 150000 \text{ mm}^2, \quad \bar{x}_A = 140 \text{ mm}, \quad \bar{y}_A = 165 \text{ mm}$$

From the solution of Problem 5.21:

$$L = 1800 \text{ mm}, \quad \bar{x}_L = 116.667 \text{ mm}, \quad \bar{y}_L = 166.667 \text{ mm}$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about the *x*-axis:

$$A_x = 2\pi \bar{y}_L L = 2\pi(166.667 \text{ mm})(1800 \text{ mm}) = 1\,884\,960 \text{ mm}^2$$

or $\quad A = 1.885 \times 10^6 \text{ mm}^2$ ◀

$$V_x = 2\pi \bar{y}_A = 2\pi(165 \text{ mm})(150\,000 \text{ mm}^2) = 155\,509\,000 \text{ mm}^3$$

or $\quad V = 155.5 \times 10^6 \text{ mm}^3$ ◀

(*b*) Rotation about $x = 400$ mm:

$$A_{x=400 \text{ mm}} = 2\pi(400 \text{ mm} - \bar{x}_L)L = 2\pi\left[(400 - 116.667)\text{mm}\right](1800 \text{ mm}) = 3\,204\,420 \text{ mm}^2$$

or $\quad A = 3.20 \times 10^6 \text{mm}^2$ ◀

$$V_x = 2\pi(400 \text{ mm} - \bar{x}_A)A = 2\pi\left[(400 - 140)\text{mm}\right](150\,000 \text{ mm}^2) = 245\,040\,000 \text{ mm}^3$$

or $\quad V = 245 \times 10^6 \text{ mm}^3$ ◀

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Chapter 5, Solution 50.

Applying the second theorem of Pappus-Guldinus, we have

(a) Rotation about axis AA':

$$\text{Volume} = 2\pi\bar{y}A = 2\pi(a)\left(\frac{\pi ab}{2}\right) = \pi^2 a^2 b \qquad V = \pi^2 a^2 b \blacktriangleleft$$

(b) Rotation about axis BB':

$$\text{Volume} = 2\pi\bar{y}A = 2\pi(2a)\left(\frac{\pi ab}{2}\right) = 2\pi^2 a^2 b \qquad V = 2\pi^2 a^2 b \blacktriangleleft$$

(c) Rotation about y-axis:

$$\text{Volume} = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi ab}{2}\right) = \frac{2}{3}\pi a^2 b \qquad V = \frac{2}{3}\pi a^2 b \blacktriangleleft$$

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Chapter 5, Solution 51.

The area A and circumference C of the cross section of the bar are

$$A = \frac{\pi}{4}d^2 \text{ and } C = \pi d.$$

Also, the semicircular ends of the link can be obtained by rotating the cross section through a horizontal semicircular arc of radius R. Now, applying the theorems of Pappus-Guldinus, we have for the volume V:

$$V = 2\left(V_{\text{side}}\right) + 2\left(V_{\text{end}}\right) = 2\left(AL\right) + 2\left(\pi RA\right) = 2\left(L + \pi R\right)A$$

$$\text{or } V = 2\left[3 \text{ in.} + \pi\left(0.75 \text{ in.}\right)\right]\left[\frac{\pi}{4}\left(0.5 \text{ in.}\right)^2\right] = 2.1034 \text{ in}^3 \qquad \text{or } V = 2.10 \text{ in}^3 \blacktriangleleft$$

For the area A:

$$A = 2\left(A_{\text{side}}\right) + 2\left(A_{\text{end}}\right) = 2\left(CL\right) + 2\left(\pi RC\right) = 2\left(L + \pi R\right)C$$

$$\text{or } A = 2\left[3 \text{ in.} + \pi\left(0.75 \text{ in.}\right)\right]\left[\pi\left(0.5 \text{ in.}\right)\right] = 16.8270 \text{ in}^2 \qquad \text{or } A = 16.83 \text{ in}^2 \blacktriangleleft$$

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Chapter 5, Solution 52.

Following the second theorem of Pappus-Guldinus, in each case a specific generating area A will be rotated about the x axis to produce the given shape. Values of $\bar{y}$ are from Fig. 5.8A.

(1) Hemisphere: the generating area is a quarter circle

Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}a^2\right)$$

or $V = \frac{2}{3}\pi a^3$ ◀

(2) Semiellipsoid of revolution: the generating area is a quarter ellipse

Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}ha\right)$$

or $V = \frac{2}{3}\pi a^2 h$ ◀

(3) Paraboloid of revolution: the generating area is a quarter parabola

Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{3}{8}a\right)\left(\frac{2}{3}ah\right)$$

or $V = \frac{1}{2}\pi a^2 h$ ◀

(4) Cone: the generating area is a triangle

Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{a}{3}\right)\left(\frac{1}{2}ha\right)$$

or $V = \frac{1}{3}\pi a^2 h$ ◀

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Chapter 5, Solution 53.

The required volume can be generated by rotating the area shown about the y axis. Applying the second theorem of Pappus-Guldinus, we have

$$V = 2\pi\bar{x}A = 2\pi\left[\left(\frac{5}{3} + 7.5\right)\text{mm}\right] \times \left[\frac{1}{2} \times 5\text{ mm} \times 5\text{ mm}\right]$$

or $V = 720\text{ mm}^3$ ◀

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Chapter 5, Solution 54.

Applying the first theorem of Pappus-Guldinus, the contact area A_C of a belt is given by:

$$A_C = \pi \bar{y} L = \pi \Sigma \bar{y} L$$

where the individual lengths are the lengths of the belt cross section that are in contact with the pulley.

(a)

$$A_C = \pi\left[2\left(\bar{y}_1 L_1\right) + \bar{y}_2 L_2\right] = \pi\left\{2\left[\left(3 - \frac{0.125}{2}\right)\text{in.}\right]\left[\frac{0.125\text{ in.}}{\cos 20°}\right] + \left[\left(3 - 0.125\right)\text{in.}\right]\left(0.625\text{ in.}\right)\right\}$$

or $A_C = 8.10\text{ in}^2$ ◀

(b)

$$A_C = \pi\left[2\left(\bar{y}_1 L_1\right)\right] = 2\pi\left[\left(3 - 0.08 - \frac{0.375}{2}\right)\text{in.}\right]\left(\frac{0.375\text{ in.}}{\cos 20°}\right)$$

or $A_C = 6.85\text{ in}^2$ ◀

(c)

$$A_C = \pi\left[2\left(\bar{y}_1 L_1\right)\right] = \pi\left[\left(3 - \frac{2\left(0.25\right)}{\pi}\right)\text{in.}\right]\left[\pi\left(0.25\text{ in.}\right)\right]$$

or $A_C = 7.01\text{ in}^2$ ◀

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Chapter 5, Solution 55.

Volume:

The volume can be obtained by rotating the triangular area shown through π radians about the y axis.

The area of the triangle is:

$$A = \frac{1}{2}(52)(60) = 1560 \text{ mm}^2$$

Applying the theorems of Pappus-Guldinus, we have

$$V = \pi \bar{x} A = \pi(52 \text{ mm})(1560 \text{ mm}^2) \qquad \text{or} \quad V = 255 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

The surface area can be obtained by rotating the triangle shown through an angle of π radians about the y axis.

Considering each line BD, DE, and BE separately:

Line BD: $L_1 = \sqrt{22^2 + 60^2} = 63.906$ mm $\qquad \bar{x}_1 = 20 + \dfrac{22}{2} = 31$ mm

Line DE: $L_2 = 52$ mm $\qquad \bar{x}_2 = 20 + 22 + 26 = 68$ mm

Line BE: $L_3 = \sqrt{74^2 + 60^2} = 95.268$ mm $\qquad \bar{x}_1 = 20 + \dfrac{74}{2} = 57$ mm

continued

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Then applying the theorems of Pappus-Guldinus for the part of the surface area generated by the lines:

$$A_L = \pi\Sigma\bar{x}A = \pi\left[(31)(63.906) + (68)(52) + (57)(95.268)\right] = \pi[10947.6] = 34.392 \times 10^3 \text{mm}^2$$

The area of the "end triangles":

$$A_E = 2\left[\frac{1}{2}(52)(60)\right] = 3.12 \times 10^3 \text{ mm}^2$$

Total surface area is therefore:

$$A = A_L + A_E = (34.392 + 3.12) \times 10^3 \text{ mm}^2 \qquad \text{or } A = 37.5 \times 10^3 \text{ mm}^2 \blacktriangleleft$$

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Chapter 5, Solution 56.

The mass of the escutcheon is given by $m = (\text{density})V$, where V is the volume. V can be generated by rotating the area A about the x-axis.

From the figure:

$$L_1 = \sqrt{75^2 - 12.5^2} = 73.9510 \text{ m}$$

$$L_2 = \frac{37.5}{\tan 26°} = 76.8864 \text{ mm}$$

$$a = L_2 - L_1 = 2.9354 \text{ mm}$$

$$\phi = \sin^{-1}\frac{12.5}{75} = 9.5941°$$

$$\alpha = \frac{26° - 9.5941°}{2} = 8.2030° = 0.143168 \text{ rad}$$

Area A can be obtained by combining the following four areas:

Applying the second theorem of Pappus-Guldinus and using Figure 5.8 a, we have

$$V = 2\pi \bar{y}A = 2\pi \Sigma \bar{y}A$$

continued

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Seg.	A, mm^2	$\bar{y}$, mm	$\bar{y}_A$, mm^3
1	$\frac{1}{2}(76.886)(37.5) = 1441.61$	$\frac{1}{3}(37.5) = 12.5$	18 020.1
2	$-\alpha(75)^2 = -805.32$	$\frac{2(75)\sin\alpha}{3\alpha}\sin(\alpha+\phi) = 15.2303$	−12 265.3
3	$-\frac{1}{2}(73.951)(12.5) = -462.19$	$\frac{1}{3}(12.5) = 4.1667$	−1925.81
4	$-(2.9354)(12.5) = -36.693$	$\frac{1}{2}(12.5) = 6.25$	−229.33
Σ			3599.7

Then

$$V = 2\pi\Sigma\bar{y}A = 2\pi\left(3599.7\text{ mm}^3\right) = 22618\text{ mm}^3$$

$$m = (\text{density})V$$

$$= \left(8470\text{ kg/m}^3\right)\left(22.618\times10^{-6}\text{m}^3\right)$$

$$= 0.191574\text{ kg}$$

or $m = 191.6$ g ◀

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Chapter 5, Solution 57.

The volume of the waste wood is:

$$V_{\text{waste}} = V_{\text{blank}} - V_{\text{top}}, \text{ where}$$

$$V_{\text{blank}} = \pi(22 \text{ in.})^2(1.25 \text{ in.}) = 1900.664 \text{ in}^3$$

$$V_{\text{top}} = V_1 + V_2 + V_3 + V_4$$

The volumes V_i can be obtained through the use of the theorem of Pappus-Guldinus:

$$V_{\text{top}} = \left[\pi(21.15 \text{ in.})^2(0.75 \text{ in.})\right] + \left[\pi(21.4 \text{ in.})^2(0.5 \text{ in.})\right]$$

$$+ 2\pi\left\{\left[\left(21.4 + \frac{(4)(0.5)}{3\pi}\right)\text{in.}\right] \times \frac{\pi}{4}(0.5 \text{ in.})^2\right\} + 2\pi\left\{\left[\left(21.15 + \frac{(4)(0.75)}{3\pi}\right)\text{in.}\right] \times \frac{\pi}{4}(0.75 \text{ in.})^2\right\}$$

$$= (1053.979 + 719.362 + 26.663 + 59.592)\text{in}^3 = 1859.596 \text{ in}^3$$

Therefore

$$V_{\text{waste}} = 1900.664 \text{ in}^3 - 1859.596 \text{ in}^3$$

$$= 41.068 \text{ in}^3$$

Then

$$W_{\text{waste}} = \gamma_{\text{wood}} V_{\text{waste}} N_{\text{tops}}$$

$$= (0.025 \text{ lb/in}^3)(41.068 \text{ in}^3)(5000 \text{ tops})$$

$$= 5133.5 \text{ lb, or}$$

$$V_{\text{waste}} = 5.13 \text{ kips} \blacktriangleleft$$

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Chapter 5, Solution 58.

The total surface area can be divided up into the top circle, bottom circle, and the edge.

$$A_{\text{Total}} = A_{\text{Top circle}} + A_{\text{Bottom circle}} + A_{\text{Edge}}, \text{or}$$

$$A_{\text{Total}} = \left[\pi(21.4\text{ in.})^2\right] + \left[\pi(21.15\text{ in.})^2\right]$$

$$+\left\{2\pi\left[\left(21.4 + \frac{2(0.5)}{\pi}\right)\text{in.}\right] \times \frac{\pi}{2}(0.5\text{ in.})\right\} + \left\{2\pi\left[\left(21.15 + \frac{2(0.75)}{\pi}\right)\text{in.}\right] \times \frac{\pi}{2}(0.75\text{ in.})\right\}$$

$$= (1438.72 + 1405.31 + 107.176 + 160.091)$$

$$= 3111.3\text{ in}^2$$

Now, knowing that 1 gallon of lacquer covers 500 ft^2, the number of gallons needed, N_{Gallons} is

$$N_{\text{Gallons}} = A_{\text{Surface}} \times \text{coverage} \times (\text{number of tops}) \times (\text{number of coats})$$

$$N_{\text{Gallons}} = 3111.3\text{ in}^2 \times \frac{1\text{ Gallon}}{(500)(144\text{ in}^2)} \times 5000 \times 3$$

$$= 648.19\text{ gal}$$

or $N_{\text{Gallons}} = 648$

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Chapter 5, Solution 59.

The mass of the lamp shade is given by

$$m = \rho V = \rho A t$$

where A is the surface area and t is the thickness of the shade. The area can be generated by rotating the line shown about the x axis. Applying the first theorem of Pappus-Guldinus we have

$$A = 2\pi \bar{y} L = 2\pi \Sigma \bar{y} L = 2\pi\left(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4\right)$$

$$\text{or} \quad A = 2\pi\left[\frac{13\text{ mm}}{2}(13\text{ mm}) + \left(\frac{13+16}{2}\right)\text{mm} \times \sqrt{(32\text{ mm})^2 + (3\text{ mm})^2}\right.$$

$$\left. + \left(\frac{16+28}{2}\right)\text{mm} \times \sqrt{(8\text{ mm})^2 + (12\text{ mm})^2} + \left(\frac{28+33}{2}\right)\text{mm} \times \sqrt{(28\text{ mm})^2 + (5\text{ mm})^2}\right]$$

$$= 2\pi(84.5 + 466.03 + 317.29 + 867.51) = 10903.4\text{ mm}^2$$

Then $\quad m = \rho A t = \left(2800\text{ kg/m}^3\right)\left(10.9034 \times 10^{-3}\text{m}^2\right)(0.001\text{ m})$

or $\quad m = 30.5\ g$ ◀

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Chapter 5, Solution 60.

Free-Body Diagram:

First note that the required surface area A can be generated by rotating the parabolic cross section through 2π radians about the x axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \bar{y} L$$

Now, since $\quad x = ky^2, \quad$ at $\quad x = a: a = k(7.5)^2$

or

$$a = 56.25\,k \tag{1}$$

At $\quad x = (a + 15)\text{mm}: a + 15 = k(12.5)^2$

or

$$a + 15 = 156.25k \tag{2}$$

Then $\quad \dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \dfrac{a + 15}{a} = \dfrac{156.25k}{56.25k} \quad$ or $\quad a = 8.4375 \text{ mm}$

$$\text{Eq. (1)} \Rightarrow k = 0.15 \frac{1}{\text{mm}}$$

$$\therefore\ x = 0.15\,y^2 \qquad \text{and} \qquad \frac{dx}{dy} = 0.3y$$

Now $\quad dL = \sqrt{1 + \left(\dfrac{dx}{dy}\right)^2}\,dy = \sqrt{1 + 0.09y^2}\,dy$

So $\quad A = 2\pi \bar{y} L \quad$ and $\quad \bar{y} L = \int y\,dL$

$$\therefore\ A = 2\pi \int_{7.5}^{12.5} y\sqrt{1 + 0.09y^2}\,dy$$

$$= 2\pi \left[\frac{2}{3}\left(\frac{1}{0.18}\right)\left(1 + 0.09y^2\right)^{3/2} \right]_{7.5}^{12.5}$$

$$= 1013 \text{ mm}^2 \qquad \text{or } A = 1013 \text{ mm}^2 \blacktriangleleft$$

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Chapter 5, Solution 61.

(*a*) Note that in the free-body diagram:

$$R_1 = \frac{1}{2}(4.2\text{ m})(600\text{ N/m}) = 1260\text{ N}, \quad \text{and} \quad R_2 = \frac{1}{2}(4.2\text{ m})(240\text{ N/m}) = 504\text{ N}$$

Then for the equivalence of the systems of forces:

ΣF_y: $\quad R = R_1 + R_2 = 1260 + 504 = 1764\text{ N}$ $\qquad R = 1764\text{ N}$ ◀

$$\Sigma M_A: \quad -\bar{x}(1764\text{ N}) = \left[\left(2 + \frac{1}{3}4.2\right)\text{m}\right](1260\text{ N}) + \left[\left(2 + \frac{2}{3}4.2\right)\text{m}\right](504\text{ N}) = 3.8000\text{ m}$$

or $\bar{x} = 3.80\text{ m}$ ◀

(*b*) Equilibrium:

$\Sigma F_x = 0$: $\quad A_x = 0$

$\Sigma F_y = 0$: $\quad A_y - 1764 = 0$

$\mathbf{A} = 1764\text{ N}\uparrow$ ◀

$\Sigma M_A = 0$: $\quad M_A - (3.80\text{ m})(1764\text{ N}) = 0$

$\mathbf{M}_A = 6.70\text{ kN}\cdot\text{m}\circlearrowleft$ ◀

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Chapter 5, Solution 62.

With $R_1 = (20 \text{ lb/ft})(18 \text{ ft}) = 360 \text{ lb}$, and $R_2 = \frac{1}{3}(60 \text{ lb/ft})(18 \text{ ft}) = 360 \text{ lb}$:

$+\uparrow \ \Sigma F_y: \quad -R = -R_1 - R_2$

or $\quad R = 360 \text{ lb} + 360 \text{ lb} = 720 \text{ lb}$

$$R = 720 \text{ lb} \blacktriangleleft$$

$+\Sigma M_A: \quad -\bar{x}(720 \text{ lb}) = -(9 \text{ ft})(360 \text{ lb}) - (13.5 \text{ ft})(360 \text{ ft})$

$$\bar{x} = 11.25 \text{ ft} \blacktriangleleft$$

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Chapter 5, Solution 63.

$R = (1800 \text{ N/m})(3.2 \text{ m}) = 5.76 \text{ kN}$

$+\Sigma M_A = 0: \quad -(5.76 \text{ kN})(1.2 \text{ m} + 1.6 \text{ m}) + B_y(3.6 \text{ m}) = 0$, or

$B_y = 4.48 \text{ kN}$

$\mathbf{B} = 4.48 \text{ kN} \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad A_y + 4.48 - 5.76 = 0$, or

$A_y = 1.28 \text{ kN}$

$+\Sigma F_x = 0: \quad A_x = 0$

Therefore:

$\mathbf{A} = 1.28 \text{ kN} \uparrow$ ◀

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Chapter 5, Solution 64.

$$R_1 = \left(1.5\frac{\text{kN}}{\text{m}}\right)(1.6\text{ m}) = 2.4\text{ kN}$$

$$R_2 = \frac{1}{2}\left(3\frac{\text{kN}}{\text{m}}\right)(2.4\text{ m}) = 3.6\text{ kN}$$

$$R_3 = \left(3\frac{\text{kN}}{\text{m}}\right)(1.6\text{ m}) = 4.8\text{ kN}$$

Equilibrium:

$+\Sigma F_x = 0$: $\quad A_x = 0$

$+\Sigma M_B = 0$: $\quad (4.8\text{ m})(2.4\text{ kN}) + (2.4\text{ m})(3.6\text{ kN}) + (0.8\text{ m})(4.8\text{ kN}) - (4.0\text{ m})A_y = 0$

$A_y = 6.0000\text{ kN}$ $\qquad$ **A** = 6.00 kN ↑ ◀

$+\Sigma F_y = 0$: $\quad 6\text{ kN} - 2.4\text{ kN} - 3.6\text{ kN} - 4.8\text{ kN} + B_y = 0$

$B_y = 4.8000\text{ kN}$ $\qquad$ **B** = 4.80 kN ↑ ◀

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Chapter 5, Solution 65.

$$R_1 = \left(240\frac{\text{lb}}{\text{ft}}\right)(4.8\text{ ft}) = 1152\text{ lb}$$

$$R_2 = \frac{1}{2}\left(180\frac{\text{lb}}{\text{ft}}\right)(3.6\text{ ft}) = 324\text{ lb}$$

Equilibrium:

$+\Sigma F_x = 0: \quad A_x = 0$

$+\Sigma F_y = 0: \quad A_y - 1152\text{ lb} + 324\text{ lb} = 0$

$A_y = 828.00\text{ lb}$ $\qquad$ $\mathbf{A} = 828\text{ lb}\uparrow$ ◀

$+\Sigma M_A = 0: \quad M_A - (2.4\text{ ft})(1152\text{ lb}) + (6\text{ ft})(324\text{ lb}) = 0$

$M_A = 820.80\text{ lb}\cdot\text{ft}$ $\qquad$ $\mathbf{M}_A = 821\text{ lb}\cdot\text{ft}$ ◀

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Chapter 5, Solution 66.

The distributed load given can be simplified as in the diagram below with the resultants R_1 and R_2.

The resultants are:

$$R_1 = (6\text{ ft})(30\text{ lb/ft}) = 180\text{ lb, and } R_2 = \frac{1}{2}(4.5\text{ ft})(120\text{ lb/ft}) = 270\text{ lb}$$

Now, for equilibrium:

$\Sigma F_x = 0$: $\quad A_x = 0$

$\Sigma F_y = 0$: $\quad A_y + 180 - 270 = 0$

$\quad A_y = 90.0\text{ lb}$

Therefore: $\mathbf{A} = 90.0\text{ lb} \uparrow$ ◀

$\Sigma M_A = 0$: $\quad M_A + (3\text{ ft})(180\text{ lb}) - \left(1.5 + \frac{2}{3} \times 4.5\right)\text{ft} \times (270\text{ lb}) = 0$

or $\mathbf{M} = 675\text{ lb}\cdot\text{ft}$ ◀

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Chapter 5, Solution 67.

$$R_1 = \left(1.5\frac{\text{kN}}{\text{m}}\right)(2.4\text{ m}) = 3.6\text{ kN}$$

$$R_2 = \frac{2}{3}\left(9\frac{\text{kN}}{\text{m}}\right)(2.4\text{ m}) = 14.4\text{ kN}$$

Equilibrium:

$+\Sigma F_x = 0$: $\quad A_x = 0$

$+\Sigma M_B = 0$: $\quad -(3.3\text{ m})A_y - (1.8\text{ m})(3.6\text{ kN}) + (2.1\text{ m})(14.4\text{ kN}) = 0$

$A_y = 7.2000\text{ kN}$ $\qquad$ $\mathbf{A} = 7.20\text{ kN}\uparrow$ ◀

$+\Sigma F_y = 0$: $\quad 7.2\text{ kN} + 3.6\text{ kN} - 14.4\text{ kN} + B_y = 0$

$B_y = 3.6000\text{ kN}$ $\qquad$ $\mathbf{B} = 3.60\text{ kN}\uparrow$ ◀

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Chapter 5, Solution 68.

The resultants:

$$R_1 = \frac{2}{3}(3.2\text{ ft})(120\text{ lb/ft}) = 256\text{ lb}$$

$$R_2 = \frac{1}{3}(2.4\text{ ft})(120\text{ lb/ft}) = 96\text{ lb}$$

$$R_3 = \frac{1}{3}(1.6\text{ ft})(45\text{ lb/ft}) = 24\text{ lb}$$

Then for equilibrium:

$\Sigma F_x = 0$: $\quad A_x = 0$

$\Sigma M_B = 0$: $\quad -(7.2\text{ ft})A_y + \left(4 + \frac{3}{8}\times 3.2\right)\text{ft}\times(256\text{ lb})$

$$+\left(1.6 + \frac{3}{4}\times 2.4\right)\text{ft}\times(96\text{ lb}) + \left(\frac{1}{4}\times 1.6\text{ ft}\right)(24\text{ lb} = 0)$$

$$A_y = 231.56\text{ lb}$$

$\mathbf{A} = 232\text{ lb}\uparrow$ ◀

$\Sigma F_y = 0$: $\quad 23.56 - 256 - 96 - 24 + B_y = 0$

$\mathbf{B} = 144.4\text{ lb}\uparrow$ ◀

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Chapter 5, Solution 69.

Have $$R_{\mathrm{I}} = \frac{1}{2}(9\text{ m})(2\text{ kN/m}) = 9\text{ kN}$$

$$R_{\mathrm{II}} = (9\text{ m})(1.5\text{ kN/m}) = 13.5\text{ kN}$$

Then $\xrightarrow{+}\ \Sigma F_x = 0:\quad C_x = 0$

$+\circlearrowleft\ \Sigma M_B = 0:\quad -50\text{ kN}\cdot\text{m} - (1\text{ m})(9\text{ kN}) - (2.5\text{ m})(13.5\text{ kN}) + (6\text{ m})C_y = 0$

or $C_y = 15.4583\text{ kN}$ $\qquad$ $\mathbf{C} = 15.46\text{ kN}\uparrow$ ◀

$+\uparrow\ \Sigma F_y = 0:\quad B_y - 9\text{ kN} - 13.5\text{ kN} + 15.4583 = 0$

or $B_y = 7.0417\text{ kN}$ $\qquad$ $\mathbf{B} = 7.04\text{ kN}\uparrow$ ◀

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Chapter 5, Solution70.

Have $$R_{\mathrm{I}} = \frac{1}{2}(9\text{ m})\left[(3.5 - w_0)\text{ kN/m}\right] = 4.5(3.5 - w_0)\text{ kN}$$

$$R_{\mathrm{II}} = (9\text{ m})(w_0\text{ kN/m}) = 9w_0\text{ kN}$$

(*a*) Then $$+\circlearrowleft \Sigma M_C = 0:\quad -50\text{ kN}\cdot\text{m} + (5\text{ m})\left[4.5(3.5 - w_0)\text{ kN}\right] + (3.5\text{ m})(9w_0\text{ kN}) = 0$$

or $$9w_0 + 28.75 = 0$$

so $$w_0 = -3.1944\text{ kN/m} \qquad w_0 = 3.19\text{ kN/m} \uparrow \blacktriangleleft$$

Note: the negative sign means that the distributed force w_0 is upward.

(*b*) $$\xrightarrow{+} \Sigma F_x = 0:\quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0:\quad -4.5(3.5 + 3.19)\text{ kN} + 9(3.19)\text{ kN} + C_y = 0$$

or $$C_y = 1.375\text{ kN} \qquad \mathbf{C} = 1.375\text{ kN} \uparrow \blacktriangleleft$$

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Chapter 5, Solution 71.

The distributed load can be represented in terms of resultants:

$$R_1 = (8\text{ m})(300\text{ N/m}) = 2400\text{ N}$$

$$R_2 = \frac{1}{2}\left[(8-a)\text{m}\right](2400\text{ N/m}) = 1200(8-a)\text{N}$$

For equilibrium:

$$\Sigma M_B = 0: \qquad -8A_y + 4(2400) + \left[\frac{1}{3}(8-a)\right]\left[1200(8-a)\right] = 0$$

$$A_y = 1200 + 50(8-a)^2 \qquad (1)$$

$$\Sigma M_A = 0: \qquad 8B_y - 4(2400) - \left[a + \frac{2}{3}(8-a)\right]\left[1200(8-a)\right] = 0$$

$$B_y = 1200 + 50(16+a)(8-a) \qquad (2)$$

$$(a)\ \Sigma F_y = 0: \qquad A_y + B_y - 2400 - 1200(8-a) = 0 \qquad (3)$$

Using the requirement $B_y = 2A_y$ and (1)

$$3\left[1200 + 50(8-a)^2\right] - 2400 - 1200(8-a) = 0$$

or $(8-a)^2 - 8(8-a) + 8 = 0,$ which gives

continued

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$$(8 - a) = \frac{8 \pm \sqrt{(-8)^2 - 4(8)}}{2} = 6.82843 \text{ m or } 1.17157 \text{ m}$$

$a = 1.17157$ m or $a = 6.82843$ m, and therefore

$a_{\min} = 1.17157$ m

or $a_{\min} = 1.172$ m ◀

(*b*) $\Sigma F_x = 0$: $A_x = 0$

Equation (1) gives:

$$A_y = 1200 + 50(6.82843)^2$$

$$= 3531.4 \text{ N}$$

or $\mathbf{A} = 3.53$ kN ↑ ◀

$B_y = 2A_y$ gives $B_y = 2(3531.4 \text{ N})$, and

$\mathbf{B} = 7.06$ kN ↑ ◀

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Chapter 5, Solution 72.

The distributed load can be represented in terms of resultants:

$$R_1 = (8\text{ m})(300\text{ N/m}) = 2400\text{ N}$$

$$R_2 = \frac{1}{2}\left[(8-a)\text{m}\right](2400\text{ N/m}) = 1200(8-a)\text{N}$$

For equilibrium:

$$\Sigma M_B = 0: \qquad -8A_y + 4(2400) + \left[\frac{1}{3}(8-a)\right]\left[1200(8-a)\right] = 0$$

$$A_y = 1200 + 50(8-a)^2 \tag{1}$$

$$\Sigma M_A = 0: \qquad 8B_y - 4(2400) - \left[a + \frac{2}{3}(8-a)\right]\left[1200(8-a)\right] = 0$$

$$B_y = 1200 + 50(16+a)(8-a) \tag{2}$$

(a) Dividing Equation (1) by Equation (2):

$$\frac{B_y}{A_y} = \frac{1200 + 50(16+a)(8-a)}{1200 + 50(8-a)^2}$$

$$= \frac{24 + \left(128 - 8a - a^2\right)}{24 + \left(64 - 16a + a^2\right)}$$

$$= \frac{152 - 8a - a^2}{88 - 16a + a^2}$$

continued

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Differentiating $\dfrac{B_y}{A_y}$:

$$\frac{d}{da}\left(\frac{B_y}{A_y}\right) = \frac{(-8-2a)\left(88-16a+a^2\right)-\left(152-8a-a^2\right)(-16+2a)}{\left(88-16a+a^2\right)^2} = 0$$

or $\quad a^2 - 20a + 72 = 0$

or $\quad a = \dfrac{20 \pm \sqrt{(-20)^2 - 4(72)}}{2}$

Knowing that $a \le 8$ m: $a = 4.7085$ m

or $\quad a = 4.71$ m ◀

(b) For equilibrium:

$\Sigma F_x = 0: \quad A_x = 0$

and from (1):

$$A_y = 1200 + 50(8 - 4.7085)^2$$

$$= 1741.70 \text{ N}$$

$\mathbf{A} = 1.742$ kN ↑ ◀

Also,

$\Sigma F_y = 0: \quad 1741.70 - 2400 - 1200(8 - 4.7085) + B_y = 0$

$B_y = 4608.1$ N

or $\quad \mathbf{B} = 4.61$ kN ↑ ◀

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Chapter 5, Solution 73.

$$R_1 = (3.6 \text{ ft})(w_A \text{ kips/ft}) = 3.6 w_A \text{ kips}$$

$$R_2 = \frac{1}{2}(5.4 \text{ ft})\left(\frac{1}{2} w_A \text{ kips/ft}\right) = 1.35 w_A \text{ kips}$$

$$R_3 = (5.4 \text{ ft})\left(\frac{1}{2} w_A \text{ kips/ft}\right) = 2.7 w_A \text{ kips}$$

Equilibrium:

$$+\Sigma M_C = 0: \quad -(1.8 \text{ ft})\left[(3.6 w_A)\text{kips}\right] + (1.8 \text{ ft})\left[(1.35 w_A)\text{kips}\right]$$

$$+(2.7 \text{ ft})\left[(2.7 w_A)\text{kips}\right] + (2.1 \text{ ft})(6 \text{ kips})$$

$$-(2.4 \text{ ft})(4.5 \text{ kips}) - (3.6 \text{ ft})(1 \text{ kip}) = 0$$

$$w_A = 0.55556 \text{ kips/ft} \qquad \text{or} \qquad w_A = 556 \text{ lb/ft} \blacktriangleleft$$

$$+\Sigma F_{yA} = 0: \quad R_R - (3.6)(0.55556)\text{kips} + 1.35(0.55556)\text{kips}$$

$$+2.7(0.55556)\text{kips} - 6 \text{ kips} - 4.5 \text{ kips} - 1 \text{ kip} = 0$$

Solving for R_R:

$$R_R = 7.2500 \text{ kips} \qquad \text{or} \qquad R_R = 7.25 \text{ kips} \blacktriangleleft$$

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Chapter 5, Solution 74.

$R_1 = (3.6 \text{ ft})(w_A \text{ kips/ft}) = 3.6w_A \text{ kips}$

$R_2 = \frac{1}{2}(5.4 \text{ ft})(0.6w_A \text{ kips/ft}) = 1.62w_A \text{ kips}$

$R_3 = (5.4 \text{ ft})(0.4w_A \text{ kips/ft}) = 2.16w_A \text{ kips}$

Equilibrium:

$+\Sigma M_A = 0$: $\quad -(1.8 \text{ ft})\left[(3.6w_A)\text{kips}\right] + (3.6 \text{ ft})R_R$

$+(5.4 \text{ ft})\left[(1.62w_A)\text{kips}\right] + (6.3 \text{ ft})\left[(2.16\, w_A)\text{kips}\right] - (1.5 \text{ ft})(6 \text{ kips})$

$-(6 \text{ ft})(4.5 \text{ kips}) - (7.2 \text{ ft})P = 0$

or $\quad 28.836w_A + 3.6R_R - 7.2P - 36 = 0 \qquad (1)$

$+\Sigma F_{y_y} = 0$: $\quad R_R + 3.6w_A + 1.62w_A + 2.16w_A - 6 - 4.5 - P = 0$

or $\quad 7.38w_A + R_R - P - 10.5 = 0 \qquad (2)$

$(28.836)\text{Eq. }(2) - (7.38)\text{Eq. }(1) = 0$ gives

$2.268R_R - 37.098 + 24.3P = 0$

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Since $R_R \geq 0$, the maximum acceptable value of P is that for which $R_R = 0$, and

$P = 1.52667$ kips or $P = 1.527$ kips ◀

(*b*) Now, from (2):

$$7.38w_A - 1.52667 - 10.5 = 0$$

or $w_A = 1.630$ kips/ft ◀

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Chapter 5, Solution 75.

Noting that the weight of a section of the dam is $W_i = \gamma V_i$ (V_i being the volume of that section):

$$W_1 = \left(150\frac{\text{lb}}{\text{ft}^3}\right)\left[(10.5\text{ ft})(9\text{ ft})(1\text{ ft})\right] = 14175\text{ lb}$$

$$W_2 = \left(150\frac{\text{lb}}{\text{ft}^3}\right)\left[\frac{1}{2}(10.5\text{ ft})(21\text{ ft})(1\text{ ft})\right] = 16537.5\text{ lb}$$

$$W_3 = \left(150\frac{\text{lb}}{\text{ft}^3}\right)\left[(18\text{ ft})(30\text{ ft})(1\text{ ft})\right] = 81000\text{ lb}$$

$$W_4 = \left(150\frac{\text{lb}}{\text{ft}^3}\right)\left[\frac{1}{2}(3\text{ ft})(30\text{ ft})(1\text{ ft})\right] = 6750\text{ lb}$$

From the free-body diagram:

$\bar{x}_1 = 5.25$ ft, $\bar{x}_2 = \frac{2}{3}(10.5\text{ ft}) = 7$ ft, $\bar{x}_3 = 19.5$ ft, and $\bar{x}_4 = 29.5$ ft

For the distance a:

$\frac{a}{24} = \frac{3}{30}$, or $a = 2.4$ ft

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Therefore:

$$W_s = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left[\frac{1}{2}(2.4\text{ ft})(24\text{ ft})(1\text{ ft})\right] = 1797.12\text{ lb, and}$$

$$\bar{x}_s = 31.5 - \frac{1}{3}(2.4) = 30.7\text{ ft}$$

Now, for the pressure force P:

$$P = \frac{1}{2}P_B A = \frac{1}{2}(\gamma_W h_B)A$$

$$= \frac{1}{2}\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(24\text{ ft})\left[(24\text{ ft})(1\text{ ft})\right]$$

$$= 17971.2\text{ lb}$$

Then, for equilibrium:

(a) $\Sigma F_x = 0$: $\quad H - P = 0$

$$H = 17971.2\text{ lb}$$

or $\mathbf{H} = 17.97\text{ kips} \longrightarrow$ ◀

$\Sigma F_y = 0$: $\quad V - 14175 - 16537.5 - 81000 - 6750 - 1797.12 = 0$

$$V = 120259.62\text{ lb}$$

or $\mathbf{V} = 120.3\text{ kips} \uparrow$ ◀

(b) From moment equilibrium:

$\Sigma M_A = 0$: $\quad \bar{x}(120259.62\text{ lb}) + \left(\frac{1}{3}\times 24\text{ ft}\right)(17971.2\text{ lb}) - (5.25\text{ ft})(14175\text{ lb}) - (7\text{ ft})(16537.5\text{ lb})$

$$(19.5\text{ ft})(81000\text{ lb}) - (29.5\text{ ft})(6750\text{ lb}) - (30.7\text{ ft})(1797.12\text{ lb}) = 0$$

or $\bar{x} = 15.63\text{ ft}$ ◀

(c) free-body diagram for section of water:

continued

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For equilibrium:

$$\Sigma \mathbf{F} = 0: \qquad \mathbf{W}_s + \mathbf{P} + (-\mathbf{R}) = 0$$

where **R** is the force of the water on the face *BD* of the dam, and

$$P = 17971.2 \text{ lb, and } W_s = 1797.12 \text{ lb}$$

Then from the force triangle:

$$R = \sqrt{(17971.2 \text{ lb})^2 + (1797.12 \text{ lb})^2} = 18.06 \text{ kips}$$

$$\theta = \tan^{-1}\left(\frac{1797.12}{17971.2}\right) = 5.71°$$

Therefore: $\mathbf{R} = 18.06 \text{ kips} \measuredangle 5.71°$ ◀

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Chapter 5, Solution 76.

Free-Body Diagram:

Locations of centers of gravity:

$$\bar{x}_1 = \frac{5}{8}(5\text{ m}) = 3.125\text{ m} \qquad \bar{x}_2 = \left[5 + \frac{1}{2}(2)\right]\text{m} = 6\text{ m}$$

$$\bar{x}_3 = \left[7 + \frac{1}{3}(4)\right] = \frac{25}{3}\text{ m} \qquad \bar{x}_4 = \left[7 + \frac{5}{8}(4)\right] = 9.5\text{ m}$$

Weights: $W_i = \rho_i g V_i$

$$W_1 = \left(2400\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[\frac{2}{3}(5\text{ m})(8\text{ m})(1\text{ m})\right] = 627\,840\text{ N}$$

$$W_2 = \left(2400\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[(2\text{ m})(8\text{ m})(1\text{ m})\right] = 376\,700\text{ N}$$

$$W_3 = \left(2400\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[\frac{1}{3}(4\text{ m})(6\text{ m})(1\text{ m})\right] = 188\,352\text{ N}$$

$$W_4 = \left(2400\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[\frac{2}{3}(4\text{ m})(6\text{ m})(1\text{ m})\right] = 156\,960\text{ N}$$

The pressure force P is:

$$P = \frac{1}{2}A\rho gh = \frac{1}{2}\left[(6\text{ m})(1\text{ m})\right]\left[\left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(6\text{ m})\right] = 176\,580\text{ N}$$

Equilibrium:

$(a)\quad +\Sigma F_x = 0: \qquad H - 176.580\text{ kN} = 0$

$H = 176.580\text{ kN}$ or $\mathbf{H} = 176.6\text{ kN} \rightarrow$ ◀

$+\Sigma F_y = 0: \qquad V - 627.84\text{ kN} - 376.70\text{ kN} - 188.352\text{ kN} - 156.960\text{ kN} = 0$

$V = 1349.85\text{ kN}$ or $\mathbf{V} = 1350\text{ kN} \uparrow$ ◀

$(b)\quad +\Sigma M_A = 0: \qquad \bar{x}(1349.85\text{ kN}) - (3.125\text{ m})(627.84\text{ kN}) - (6\text{ m})(376.70\text{ kN})$

$$-\left(\frac{25}{3}\text{ m}\right)(188.352\text{ kN}) - (9.5\text{ m})(156.960\text{ kN}) + (2\text{ m})(176.580\text{ kN}) = 0$$

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$\bar{x} = 5.1337$ m

Thus the point of application of the resultant is: 5.13 m to the right of A. ◀

(c) Free-body diagram and force triangle for the water section BCD

From the force triangle:

$$R = \sqrt{(176.580)^2 + (156.960)^2} = 236.26 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{156.960}{176.580}\right) = 41.634°$$

or on the face BD of the dam **R** = 236 kN ∡ 41.6° ◀

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Chapter 5, Solution 77.

Free-Body Diagram:

Note that valve opens when $\mathbf{B} = 0$.

Pressures p_1 and p_2 at top and bottom of valve:

$$p_1 = \left(10^3\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(d - 0.225\ \text{m}\right) = \left(9810d - 2207.3\right)\text{N/m}^2$$

$$p_2 = \left(10^3\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(d\right) = \left(9810d\right)\text{N/m}^2$$

Force P_1 and P_2:

$$P_1 = \frac{1}{2}p_1 A = \frac{1}{2}\left[\left(9810d - 2207.3\right)\text{N/m}^2\right]\left[\left(0.225\ \text{m}\right)\left(0.225\ \text{m}\right)\right]$$

$$= \left(248.32d - 55.872\right)\text{N}$$

$$P_2 = \frac{1}{2}p_2 A = \frac{1}{2}\left[\left(9810d\right)\text{N/m}^2\right]\left[\left(0.225\ \text{m}\right)\left(0.225\ \text{m}\right)\right]$$

$$= \left(248.32d\right)\text{N}$$

$$+\Sigma M_A = 0: \qquad -\left[\left(0.15 - 0.09\right)\text{m}\right]\left[\left(248.32d - 55.872\right)\text{N}\right] + \left[\left(0.09 - 0.075\right)\text{m}\right]\left[\left(248.32d\right)\text{N}\right] = 0$$

Thus $d = 0.30000$ m, or $d = 300$ mm ◀

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Chapter 5, Solution 78.

Free-Body Diagram:

Note that valve opens when $\mathbf{B} = 0$.

Pressures p_1 and p_2 at top and bottom of valve:

$$p_1 = \left(10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(0.225 \text{ m}\right) = 2207.3 \text{ N/m}^2$$

$$p_2 = \left(10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(0.450 \text{ m}\right) = 4414.5 \text{ N/m}^2$$

Force P_1 and P_2:

$$P_1 = \frac{1}{2}p_1 A = \frac{1}{2}\left(2207.3 \text{ N/m}^2\right)\left[\left(0.225 \text{ m}\right)\left(0.225 \text{ m}\right)\right]$$

$$= 55.872 \text{ N}$$

$$P_2 = \frac{1}{2}p_2 A = \frac{1}{2}\left(4414.5 \text{ N/m}^2\right)\left[\left(0.225 \text{ m}\right)\left(0.225 \text{ m}\right)\right]$$

$$= 111.742 \text{ N}$$

$$+\Sigma M_A = 0: \qquad -\left[\left(0.15 - h\right)\text{m}\right]\left(55.872 \text{ N}\right) + \left[\left(h - 0.075\right)\text{m}\right]\left(111.742 \text{ N}\right) = 0$$

Solving for h: $h = 0.100\,000$ m, or

$h = 100.0$ mm ◀

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Chapter 5, Solution 79.

Since gate is 4 ft wide:

$$p_1 = 4\gamma(h-3)$$
$$p_2 = 4\gamma h$$
$$p_1' = 4\gamma'(d-3)$$
$$p_2' = 4\gamma' d$$

$$P_1' - P_1 = \frac{1}{2}(3\text{ ft})\left(p_1' - p_1\right) = \frac{1}{2}(3\text{ ft})\left[4\gamma'(d-3) - 4\gamma(h-3)\right] = 6\gamma'(d-3) - 6\gamma(h-3)$$

$$P_2' - P_2 = \frac{1}{2}(3\text{ ft})\left(p_2' - p_2\right) = \frac{1}{2}(3\text{ ft})\left[4\gamma' d - 4\gamma h\right] = 6\gamma' d - 6\gamma h$$

This gives the free-body diagram:

$+\Sigma M_A = 0$: $\quad (3\text{ ft})B - (1\text{ ft})\left(P_1' - P_1\right) - (2\text{ ft})\left(P_2' - P_2\right) = 0$

or $\quad B = \frac{1}{3}\left(P_1' - P_1\right) - \frac{2}{3}\left(P_2' - P_2\right)$

$$= \frac{1}{3}\left[6\gamma'(d-3) - 6\gamma(h-3)\right] - \frac{2}{3}\left[6\gamma' d - 6\gamma h\right]$$

$$= 2\gamma'(d-3) - 2\gamma(h-3) + 4\gamma' d - 4\gamma h$$

or $\quad B = 6\gamma'(d-1) - 6\gamma(h-1) \qquad (1)$

$+\Sigma F_x = 0$: $\quad A + B - \left(P_1' - P_1\right) - \left(P_2' - P_2\right) = 0$, or using (1)

$$A + \left[6\gamma'(d-1) - 6\gamma(h-1)\right] - \left[6\gamma'(d-3) - 6\gamma(h-3)\right] - \left[6\gamma' d - 6\gamma h\right] = 0, \text{ or}$$

$$A = 6\gamma'(d-2) - 6\gamma(h-2) \qquad (2)$$

Using the given data in (1) and (2):

$$h = 6\text{ ft},\ \ d = 9\text{ ft},\ \ \gamma = 62.4\text{ lb/ft}^3,\ \ \gamma' = 64\text{ lb/ft}^3$$

$$A = 6(64)(9-2) - 6(62.4)(6-2)$$

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$$= 2688 \text{ lb} - 1497.6 \text{ lb} = 1190.4 \text{ lb}$$

$$B = 6(64)(9-1) - 6(62.4)(6-1)$$

$$= 3072 \text{ lb} - 1872 \text{ lb} = 1200 \text{ lb}$$

$$\mathbf{A} = 1190 \text{ lb} \rightarrow \blacktriangleleft$$

$$\mathbf{B} = 1200 \text{ lb} \rightarrow \blacktriangleleft$$

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Chapter 5, Solution 80.

First, determine the force on the dam face without the silt.

Have $$P_w = \frac{1}{2}Ap_w = \frac{1}{2}A(\rho gh)$$

$$= \frac{1}{2}\left[(6\text{ m})(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(6\text{ m})\right]$$

$$= 176.58\text{ kN}$$

Next, determine the force on the dam face with silt.

Have $$P'_w = \frac{1}{2}\left[(4.5\text{ m})(1\text{m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4.5\text{ m})\right]$$

$$= 99.326\text{ kN}$$

$$(P_s)_{\text{I}} = \left[(1.5\text{ m})(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4.5\text{ m})\right]$$

$$= 66.218\text{ kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}\left[(1.5\text{ m})(1\text{ m})\right]\left[\left(1.76\times10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.5\text{ m})\right]$$

$$= 19.424\text{ kN}$$

Then $$P' = P'_w + (P_s)_{\text{I}} + (P_s)_{\text{II}} = 184.97\text{ kN}$$

The percentage increase, % inc., is then given by

$$\%\text{ inc.} = \frac{P' - P_w}{P_w}\times100\% = \frac{(184.97 - 176.58)}{176.58}\times100\% = 4.7503\%$$

% inc. = 4.75% ◀

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Chapter 5, Solution 81.

From Problem 5.80, the force on the dam face before the silt is deposited, is $P_w = 176.58\text{ kN}$. The maximum allowable force P_{allow} on the dam is then:

$$P_{\text{allow}} = 1.5P_w = (1.5)(176.58\text{ kN}) = 264.87\text{ kN}$$

Next determine the force P' on the dam face after *a* depth *d* of silt has settled.

Have

$$P'_w = \frac{1}{2}\left[(6-d)\text{m}\times(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(6-d)\text{m}\right]$$

$$= 4.905(6-d)^2\text{ kN}$$

$$(P_s)_{\text{I}} = \left[d(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(6-d)\text{m}\right]$$

$$= 9.81\left(6d - d^2\right)\text{kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}\left[d(1\text{ m})\right]\left[\left(1.76\times10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(d)\text{m}\right]$$

$$= 8.6328d^2\text{ kN}$$

$$P' = P'_w + (P_s)_{\text{I}} + (P_s)_{\text{II}} = \left[4.905\left(36 - 12d + d^2\right) + 9.81\left(6d - d^2\right) + 8.6328d^2\right]\text{kN}$$

$$= \left[3.7278d^2 + 176.58\right]\text{kN}$$

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Now required that $P' = P_{\text{allow}}$ to determine the maximum value of d.

$$\therefore \left(3.7278d^2 + 176.58\right)\text{kN} = 264.87\text{ kN}$$

or

$$d = 4.8667\text{ m}$$

Finally

$$4.8667\text{ m} = 20 \times 10^{-3}\,\frac{\text{m}}{\text{year}} \times N$$

or $N = 243$ years ◀

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Chapter 5, Solution 82.

Pressure force from the water on board *AB*:

$P_i = \frac{1}{2} A p_i$ where p_1 and p_2 are the pressures at the top and bottom of the board:

$$P_1 = \frac{1}{2}\left[(0.5\text{ m})(1.5\text{ m})\right]\left[\left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.6\text{ m})\right] = 2207.3\text{ N}$$

$$P_2 = \frac{1}{2}\left[(0.5\text{ m})(1.5\text{ m})\right]\left[\left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(1\text{ m})\right] = 3678.8\text{ N}$$

Free-Body Diagram:

$\mathbf{A}_x$ denotes the force from one piling and is therefore multiplied by two in the free-body diagram.

$+\Sigma M_A = 0$: $\quad -(0.3\text{ m})B + \left[\frac{1}{3}(0.5)\text{ m}\right](2207.3\text{ N}) + \left[\frac{2}{3}(0.5)\text{ m}\right](3678.8\text{ N}) = 0$, or

$B = 5313.8\text{ N}$

$+\Sigma F_x = 0$: $\quad 2A_x + \frac{4}{5}(2207.3\text{ N}) + \frac{4}{5}(3678.8\text{ N}) = 0$, or

$A_x = -2354.4\text{ N}$

$+\Sigma F_y = 0$: $\quad 5318.8\text{ N} - \frac{3}{5}(2207.3\text{ N}) - \frac{3}{5}(3678.8\text{ N}) + A_y = 0$, or

$A_y = -1782.14\text{ N}$

Therefore: (*a*) $\mathbf{A}_x = 2.35$ kN ← ◀

(*b*) $\mathbf{A}_y = 1.782$ kN ↓ ◀

(*c*) $\mathbf{B} = 5.31$ kN ↑ ◀

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Chapter 5, Solution 83.

Pressure force from the water on board AB:

$P_i = \frac{1}{2} A p_i$ where p_1 and p_2 are the pressures at the top and bottom of the board:

$$P_1 = \frac{1}{2}\left[(0.5\text{ m})(1.5\text{ m})\right]\left[\left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.6\text{ m})\right] = 2207.3\text{ N}$$

$$P_2 = \frac{1}{2}\left[(0.5\text{ m})(1.5\text{ m})\right]\left[\left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(1\text{ m})\right] = 3678.8\text{ N}$$

Note that the board can move in two ways: by rotating about A if the rope is pulled upward, and by sliding down at A if the rope is pulled sideways to the left.

Case 1 (rotation about A):

For minimum tension the rope will be perpendicular to the board.

Free-Body Diagram:

$$+\Sigma M_A = 0: \qquad -(0.5\text{ m})T_{BC} + \left[\frac{1}{3}(0.5)\text{ m}\right](2207.3\text{ N}) + \left[\frac{2}{3}(0.5)\text{ m}\right](3678.8\text{ N}) = 0, \text{ or}$$

$$T_{BC} = 3188.3\text{ N}$$

Case 2 (sliding down at A):

When the board is just about to slide down at A, $\mathbf{A}_y = 0$.

continued

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Free-Body Diagram:

$$+\Sigma M_B = 0: \qquad -(0.4\text{ m})(2A_x) - \left[\frac{1}{3}(0.5)\text{ m}\right](3678.8\text{ N}) - \left[\frac{2}{3}(0.5)\text{ m}\right](2207.3\text{ N}) = 0, \text{ or}$$

$$2A_x = -3372.3\text{ N}$$

$$+\Sigma F_x = 0: \qquad -T_{BC} - 3372.3 + \frac{4}{5}(2207.3\text{ N}) + \frac{4}{5}(3678.8\text{ N}) = 0, \text{ or}$$

$$T_{BC} = 1336.58\text{ N}$$

Thus: $(\mathbf{T}_{BC})_{\min} = 1.337\text{ kN} \longleftarrow$ ◀

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Chapter 5, Solution 84.

Free-Body Diagram:

Force from water pressure:

$P - \frac{1}{2}Ap_B$ where A is the rectangular cross sectional area through line BD, and p_B is the pressure at point B. Thus

$$P = \frac{1}{2}A(\gamma h) = \frac{1}{2}\left[(16\text{ ft})(10\text{ ft})\right]\left[\left(62.4\text{ lb/ft}^3\right)(10\text{ ft})\right] = 18720.0\text{ lb} = 18.72\text{ kips}$$

$$W = \gamma V = \left(62.4\text{ lb/ft}^3\right)\left[\frac{1}{2}(3\text{ ft})(6\text{ ft})(6\text{ ft})\right] = 3369.61\text{ lb} = 3.3696\text{ kips}$$

Equilibrium:

$$+\Sigma M_A = 0: \qquad (18.72\text{ kips})\left(\frac{20}{3}\text{ ft}\right) - B(3\text{ ft}) + (3.3696\text{ kips})(2\text{ ft}) = 0.$$

Solving for B:

$B = 43.846$ kips, or $\qquad$ **B** = 43.8 kips ↑ ◀

$+\Sigma F_x = 0$: $\quad 18.72\text{ kips} + A_x = 0$, or

$A_x = -18.7200$ kips

$+\Sigma F_y = 0$: $\quad A_y - 3.3693\text{ kips} + 43.846\text{ kips} = 0$, or

$A_y = -40.476$ kips

$$A = \sqrt{(-18.7200)^2 + (-40.476)^2} = 44.595\text{ kips}$$

$$\theta = \tan^{-1}\left(\frac{40.476}{18.7200}\right) = 65.180°$$

Therefore: **A** = 44.6 kips ↙ 65.2° ◀

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Chapter 5, Solution 85.

Consider a 1-in. thick section of the gate and a triangular section *BDE* of water above the gate

Free-Body Diagram:

Pressure force *P*:

$$P = \frac{1}{2}Ap_B = \frac{1}{2}(d \times 1 \text{ in.})(\gamma d) = \frac{1}{2}\gamma d^2 \text{ lb}$$

Weight of water section above gate:

$$W_W = \gamma V_W = \gamma\left(\frac{1}{2} \times \frac{8}{15}d \times d \times 1 \text{ in.}\right) = \frac{4}{15}\gamma d^2 \text{ lb}$$

For impending motion of gate: $\mathbf{B}_y = 0$, and for equilibrium:

$$+\Sigma M_a = 0: \quad \left[\frac{2}{3}(16) - \frac{1}{3}\left(\frac{8}{15}d\right)\right]\left(\frac{4}{15}\gamma d^2\right) - \left[\frac{d}{3} - 6\right]\left(\frac{1}{2}\gamma d^2\right) = 0, \text{ and}$$

$d = 27.301$ in., or $\qquad d = 27.3$ in. ◀

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Chapter 5, Solution 86.

Consider a 1-in. thick section of the gate and a triangular section *BDE* of water above the gate

Free-Body Diagram:

Pressure force P:

$$P = \frac{1}{2}Ap_B = \frac{1}{2}(d \times 1 \text{ in.})(\gamma d) = \frac{1}{2}\gamma d^2 \text{ lb}$$

Weight of water section above gate:

$$W_W = \gamma V_W = \gamma\left(\frac{1}{2} \times \frac{8}{15}d \times d \times 1 \text{ in.}\right) = \frac{4}{15}\gamma d^2 \text{ lb}$$

For impending motion of gate: $\mathbf{B}_y = 0$, and for equilibrium:

$$+\Sigma M_a = 0: \qquad \left[\frac{2}{3}(16) - \frac{1}{3}\left(\frac{8}{15}d\right)\right]\left(\frac{4}{15}\gamma d^2\right) - \left[\frac{d}{3} - (10 - h)\right]\left(\frac{1}{2}\gamma d^2\right) = 0, \text{ and}$$

with $d = 30$ in.

$$\left[\frac{2}{3}(16) - \frac{1}{3}\left(\frac{8}{15}30\right)\right]\left(\frac{4}{15}\gamma 30^2\right) - \left[\frac{d}{3} - (10 - h)\right]\left(\frac{1}{2}\gamma 30^2\right) = 0, \text{ and}$$

$h = 2.8444$ in., or $\qquad h = 2.84$ in. ◀

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Chapter 5, Solution 87.

Free-Body Diagram:

$W = (125\text{ kg})(9.81\text{ m/s}^2) = 1226.25\text{ N}$

Denoting the water pressure at a depth h by p_h, the forces due to the water pressure $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$ can be obtained as follows:

$$\mathbf{P}_1 = \frac{1}{2}A_{DC}p_{(0.15\text{ m})}, \text{ or with}$$

$$p_{(0.15\text{ m})} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.15\text{ m}) = 1471.50\text{ N/m}^2$$

$$\mathbf{P}_1 = \frac{1}{2}\left[(0.15\text{ m})(1\text{ m})\right](1471.50\text{ N/m}^2) = 110.363\text{ N}$$

$$\mathbf{P}_2 = A_{CB}p_{(0.15\text{ m})}, \text{ or}$$

$$\mathbf{P}_2 = \left[(0.6\text{ m})(1\text{ m})\right](1471.50\text{ N/m}^2) = 882.90\text{ N}$$

$$\mathbf{P}_3 = \frac{1}{2}A_{BA}p_{(0.15\text{ m})}, \text{ or}$$

$$\mathbf{P}_3 = \frac{1}{2}\left[(0.6\text{ m})(1\text{ m})\right](1471.50\text{ N/m}^2) = 441.45\text{ N}$$

$$\mathbf{P}_4 = \frac{1}{2}A_{BA}p_{(0.75\text{ m})}, \text{ or with}$$

$$p_{(0.75\text{ m})} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.75\text{ m}) = 7357.5\text{ N/m}^2$$

$$\mathbf{P}_4 = \frac{1}{2}\left[(0.16\text{ m})(1\text{ m})\right](7357.5\text{ N/m}^2) = 2207.3\text{ N}$$

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Now from the free-body diagram:

$+\Sigma M_A = 0$: $\quad -(1.2\text{ m})D + (0.6\text{ m})(1226.25\text{ N}) + (0.3\text{ m})(1226.25\text{ N})$

$$-\left[(0.6 + 0.05\text{ m})\right](110.363\text{ N}) - (0.3\text{ m})(882.90\text{ N}) - (0.4\text{ m})(441.45\text{ N})$$

$$-(0.2\text{ m})(2207.3\text{ N}) = 0, \text{ or}$$

$D = 124.149$ N, and $\mathbf{D} = 124.1$ N $\longrightarrow$ ◀

$+\Sigma F_x = 0$: $\quad A_x + 110.363\text{ N} + 441.45\text{ N} + 2207.3\text{ N} = 0$, or

$$A_x = -2759.1\text{ N}$$

$+\Sigma F_y = 0$: $\quad A_y - 3(1226.25\text{ N}) + 882.90\text{ N} = 0$, or

$$A_y = 2795.9\text{ N}$$

Then,

$$A = \sqrt{(-2759.1)^2 + (2795.9)^2} = 3930\text{ N, and}$$

$$\theta = \tan^{-1}\frac{2795.9}{2759.1} = 45.4°$$

Therefore: $\mathbf{A} = 3930$ N $\nwarrow$ 45.4° ◀

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Chapter 5, Solution 88.

Free-Body Diagram:

$W = (125\text{ kg})(9.81\text{ m/s}^2) = 1226.25\text{ N}$

Denoting the water pressure at a depth h by p_h, the forces due to the water pressure $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$ can be obtained as follows:

$$\mathbf{P}_1 = \frac{1}{2}A_{DC}p_{(d-0.6\text{ m})}, \text{ or } w$$

$$\mathbf{P}_1 = \frac{1}{2}\left[(d-0.6)\text{m}\times(1\text{ m})\right](\gamma\text{ N/m}^3)(d-0.6)\text{m}$$

$$= \frac{1}{2}\gamma(d-0.6)^2\text{ N}$$

where γ denotes the specific weight of water. In the same way

$$\mathbf{P}_2 = \frac{1}{2}A_{CB}p_{(d-0.6\text{ m})}, \text{ or}$$

$$\mathbf{P}_2 = \left[(0.6\text{ m})\times(1\text{ m})\right](\gamma\text{ N/m}^3)(d-0.6)\text{m}$$

$$= 0.6\gamma(d-0.6)\text{N}$$

$$\mathbf{P}_3 = \frac{1}{2}A_{BA}p_{(d-0.6\text{ m})}, \text{ or}$$

$$\mathbf{P}_3 = \frac{1}{2}\left[(0.6\text{ m})\times(1\text{ m})\right](\gamma\text{ N/m}^3)(d-0.6)\text{m}$$

$$= 0.3\gamma(d-0.6)\text{N}$$

$$\mathbf{P}_4 = \frac{1}{2}A_{BA}p_{(d\text{ m})}, \text{ or}$$

$$\mathbf{P}_4 = \frac{1}{2}\left[(0.6\text{ m})\times(1\text{ m})\right](\gamma\text{ N/m}^3)(d\text{ m})$$

$$= 0.3\gamma d\text{ N}$$

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Now from the free-body diagram:

$+\Sigma M_A = 0$: $(0.6\text{ m})(1226.25\text{ N}) + (0.3\text{ m})(1226.25\text{ N})$

$$-\left[\left(0.6\text{ m} + \frac{1}{3}(d - 0.6)\text{ m}\right)\right]\left[\frac{1}{2}\gamma(d - 0.6)^2\text{ N}\right] - (0.3\text{ m})\left[0.6\gamma(d - 0.6)\text{N}\right]$$

$$-\left[\frac{2}{3}(0.6\text{ m})\right]\left[0.3\gamma(d - 0.6)\text{ N}\right] - \left[\frac{1}{3}(0.6\text{ m})\right]\left[0.3\gamma(d - 0.6)\text{N} + 0.18\gamma\text{ N}\right] = 0\text{, or}$$

$$\frac{1}{6}(d - 0.6)^3 + 0.3(d - 0.6)^2 + 0.36(d - 0.6) = \frac{1103.63}{\gamma} - 0.036$$

With $\gamma = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right) = 9810\text{ N/m}^3$, this gives

$$\frac{1}{6}(d - 0.6)^3 + 0.3(d - 0.6)^2 + 0.36(d - 0.6) = \frac{1103.63}{9810\text{ N/m}^3} - 0.036 = 0.076501$$

Solving for d numerically: $d = 0.782\text{ m}$ ◀

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Chapter 5, Solution 89.

(*a*)

Free-body diagram for a 24-in. long parabolic section of water:

In the free body diagram force P is:

$$P = \frac{1}{2}AP = \frac{1}{2}A(\gamma h) = \frac{1}{2}\left(\frac{3}{12}\text{ ft}\right)\left(\frac{24}{12}\text{ ft}\right)\left[\left(62.4\text{ lb/ft}^3\right)\left(\frac{3}{12}\text{ ft}\right)\right] = 3.9000\text{ lb}$$

$$W_w = \gamma V$$

$$= \left(62.4\text{ lb/ft}^3\right)\left[\frac{2}{3}\left(\frac{4.5}{12}\text{ ft}\right)\left(\frac{3}{12}\text{ ft}\right)\left(\frac{24}{12}\text{ ft}\right)\right]$$

$$= 7.8000\text{ lb}$$

From the force triangle:

$$R = \sqrt{P^2 + W_w^{\,2}} = \sqrt{(3.9)^2 + (7.8)^2} = 8.7207\text{ lb}$$

$$\theta = \tan^{-1}\frac{W_w}{P} = \tan^{-1}\frac{7.8}{3.9} = 63.435°\text{, or}$$

$$\mathbf{R} = 8.72\text{ lb} \measuredangle 63.4° \blacktriangleleft$$

(*b*)

Free-body diagram for a 24-in. long section of the water:

From (*a*) $W_W = 7.8000$ lb

From the free-body diagram:

$$B_y = 7.8000\text{ lb}$$

$$+\Sigma M_B = 0: \qquad M_B + \left[(2.25 - 1.8)\text{in.}\right](7.8000\text{ lb}) = 0\text{, or}$$

$$M_B = -3.5100\text{ lb}\cdot\text{in.}$$

Therefore, the force-couple system on the gutter is:

$$\mathbf{R} = 7.8\text{ lb}\downarrow\text{; }\mathbf{M} = 3.51\text{ lb}\cdot\text{in.} \blacktriangleleft$$

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Chapter 5, Solution 90.

Note, for the axes shown

	V	$\bar{y}$	$\bar{y}V$
1	$(\pi R^2)(2R) = 2\pi R^3$	$-R$	$-2\pi R^4$
2	$-\frac{2}{3}\pi r^3$	$-\frac{3}{8}r$	$\frac{1}{4}\pi r^4$
Σ	$2\pi\left(R^3 - \frac{r^3}{3}\right)$		$-2\pi\left(R^4 - \frac{r^4}{8}\right)$

Then
$$\bar{Y} = \frac{\Sigma \bar{y} V}{\Sigma V} = -\frac{R^4 - \frac{1}{8}r^4}{R^3 - \frac{1}{3}r^3}$$

$$= \frac{1 - \frac{1}{8}\left(\frac{r}{R}\right)^4}{1 - \frac{1}{3}\left(\frac{r}{R}\right)^3}$$

(a)
$$r = \frac{3}{4}R: \quad \bar{y} = -\frac{1 - \frac{1}{3}\left(\frac{3}{4}\right)^4}{1 - \frac{1}{3}\left(\frac{3}{4}\right)^3}R$$

or $\bar{y} = -1.118R$ ◀

(b)
$$\bar{y} = -1.2R: \quad -1.2R = -\frac{1 - \frac{1}{8}\left(\frac{r}{R}\right)^4}{1 - \frac{1}{3}\left(\frac{r}{R}\right)^3}R$$

or
$$\left(\frac{r}{R}\right)^4 - 3.2\left(\frac{r}{R}\right)^3 + 1.6 = 0$$

Solving numerically $\frac{r}{R} = 0.884$ ◀

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Chapter 5, Solution 91.

Labeling the two parts of the body as follows:

	V	$\bar{y}$	$\bar{y}V$
1	$\frac{1}{2}\pi a^2 h$	$\frac{h}{2}$	$\frac{1}{4}\pi a^2 h^2$
2	$\frac{1}{6}\pi a^2 h$	$\frac{h}{4}$	$\frac{1}{24}\pi a^2 h^2$
Σ	$\frac{2}{3}\pi a^2 h$		$\frac{7}{24}\pi a^2 h^2$

Then $\bar{Y} = \dfrac{\Sigma \bar{y} V}{\Sigma V} = \dfrac{\left(\frac{7}{24}\pi a^2 h^2\right)}{\left(\frac{2}{3}\pi a^2 h\right)}$ or $\bar{Y} = \dfrac{7}{16}h$ ◀

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Chapter 5, Solution 92.

Labeling the two parts of the body as follows:

	V	$\bar{z}$	$\bar{z}V$
1	$\frac{1}{2}\pi a^2 h$	$-\frac{4a}{3\pi}$	$-\frac{2}{3}a^3 h$
2	$\frac{1}{6}\pi a^2 h$	$\frac{a}{\pi}$	$\frac{1}{6}a^3 h$
Σ	$\frac{2}{3}\pi a^2 h$		$-\frac{1}{2}a^3 h$

Then $\bar{Z} = \frac{\Sigma \bar{z}V}{\Sigma V} = \frac{\left(-\frac{1}{2}a^3 h\right)}{\left(\frac{2}{3}\pi a^2 h\right)}$ or $\bar{Z} = -\frac{3a}{4\pi}$ ◀

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Chapter 5, Solution 93.

	V	$\bar{x}$	$\bar{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L+\frac{1}{4}h$	$\frac{1}{6}abh\left(L+\frac{1}{4}h\right)$

Then $\Sigma V = ab\left(L+\frac{1}{6}h\right) \qquad \Sigma\bar{x}V = \frac{1}{6}ab\left[3L^2+h\left(L+\frac{1}{4}h\right)\right]$

Now $\bar{X}\Sigma V = \Sigma\bar{x}V$ so that

$$\bar{X}\left[ab\left(L+\frac{1}{6}h\right)\right] = \frac{1}{6}ab\left(3L^2+hL+\frac{1}{4}h^2\right)$$

or

$$\bar{X}\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right) \qquad (1)$$

(a) $\bar{X} = ?$ when $h = \frac{1}{2}L$

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1)

$$\bar{X}\left[1+\frac{1}{6}\left(\frac{1}{2}\right)\right] = \frac{1}{6}L\left[3+\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

or $\bar{X} = \frac{57}{104}L$ $\qquad \bar{X} = 0.548L$ ◀

(b) $\frac{h}{L} = ?$ when $\bar{X} = L$

Substituting into Eq. (1) $\qquad L\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right)$

or $\qquad 1+\frac{1}{6}\frac{h}{L} = \frac{1}{2}+\frac{1}{6}\frac{h}{L}+\frac{1}{24}\frac{h^2}{L^2}$

or $\qquad \frac{h^2}{L^2} = 12$ $\qquad \therefore \frac{h}{L} = 2\sqrt{3}$ ◀

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Chapter 5, Solution 94.

Assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the volume.

	V, mm^3	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}V$, mm^4	$\bar{z}V$, mm^4
1	$(60)(105)(10) = 63\,000$	-5	52.5	$-315\,000$	3 307 500
2	$\frac{1}{2}\pi(30)^2(10) = 14\,137.2$	-5	$105+\frac{4(30)}{3\pi}=117.732$	$-70\,686$	1 664 400
3	$(15)(30)(60) = 27\,000$	15	30	405 000	810 000
4	$-\pi(19)^2(10) = -11\,341.1$	-5	105	56 706	$-1\,190\,820$
5	$-\frac{1}{2}\pi(19)^2(15) = -8505.9$	$30-\frac{4(19)}{3\pi}=21.936$	30	$-186\,585$	$-255\,180$
Σ	84 290			$-110\,565$	4 335 900

Then $\bar{Y} = \frac{\Sigma\bar{y}V}{\Sigma V} = \frac{-110\,565}{84\,290}$ mm or $\bar{Y} = -1.312$ mm ◀

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Chapter 5, Solution 95.

Assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the volume.

	V, mm^3	$\bar{z}$, mm	$\bar{z}V$, mm^4
1	$(60)(105)(10) = 63\,000$	52.5	3 307 500
2	$\frac{1}{2}\pi(30)^2(10) = 14\,137.2$	$105 + \frac{4(30)}{3\pi} = 117.732$	1 664 400
3	$(15)(30)(60) = 27\,000$	30	810 000
4	$-\pi(19)^2(10) = -11\,341.1$	105	−1 190 820
5	$-\frac{1}{2}\pi(19)^2(15) = -8505.9$	30	−255 180
Σ	84 290		4 335 900

Then $\bar{Z} = \frac{\Sigma \bar{z}V}{\Sigma V} = \frac{4\,335\,900}{84\,290}$ mm or $\bar{Z} = 51.4$ mm ◀

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Chapter 5, Solution 96.

Assume that the bracket is homogeneous so that its center of gravity coincides with the centroid of the volume.

	V, mm^3	$\bar{x}$, mm	$\bar{x}V$, mm^4
1	$(100)(88)(12) = 105\,600$	50	5 280 000
2	$(100)(12)(88) = 105\,600$	50	5 280 000
3	$\frac{1}{2}(62)(51)(10) = 15\,810$	39	616 590
4	$-\frac{1}{2}(66)(45)(12) = -17\,820$	$34 + \frac{2}{3}(66) = 78$	−1 389 960
Σ	209 190		9 786 600

Then $\bar{X} = \dfrac{\Sigma \bar{x}V}{\Sigma V} = \dfrac{9\,786\,600}{209\,190}$ mm or $\bar{X}$ = 46.8 mm ◀

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Chapter 5, Solution 97.

Assume that the bracket is homogeneous so that it center of gravity coincides with the centroid of the volume.

	V, mm^3	$\bar{z}$, mm	$\bar{z}V$, mm^4
1	$(100)(88)(12) = 105\,600$	6	633 600
2	$(100)(12)(88) = 105\,600$	$12 + \frac{1}{2}(88) = 56$	5 913 600
3	$\frac{1}{2}(62)(51)(10) = 15\,810$	$12 + \frac{1}{3}(51) = 29$	458 490
4	$-\frac{1}{2}(66)(45)(12) = -17\,820$	$55 + \frac{2}{3}(45) = 85$	−1 514 700
Σ	209 190		5 491 000

Then $\bar{Z} = \dfrac{\Sigma \bar{z}V}{\Sigma V} = \dfrac{5\,491\,000}{209\,190}$ mm or $\bar{Z} = 26.2$ mm ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 5, Solution 98.

Assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the volume.

	V, in^3	$\bar{x}$, in.	$\bar{x}V$, in^4
1	$(8)(0.9)(2.7) = 19.44$	4	77.76
2	$\frac{1}{2}(2.1)(6)(2.7) = 17.01$	2	34.02
3	$\frac{1}{2}\pi(1.35)^2(0.9) = 2.5765$	$8 + \frac{1.8}{\pi}$	22.088
4	$-\pi(0.8)^2(0.9) = -1.80956$	8	−14.4765
Σ	37.217		119.392

Then $\bar{X} = \frac{\Sigma \bar{x} V}{\Sigma V} = \frac{119.392}{37.217}$ in. or $\bar{X} = 3.21$ in.◀

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Chapter 5, Solution 99.

Assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the volume.

	V, in^3	$\bar{y}$, in.	$\bar{y}V$, in^4
1	$(8)(0.9)(2.7) = 19.44$	0.45	8.748
2	$\frac{1}{2}(2.1)(6)(2.7) = 17.01$	1.6	27.216
3	$\frac{1}{2}\pi(1.35)^2(0.9) = 2.5765$	0.45	1.15943
4	$-\pi(0.8)^2(0.9) = -1.80956$	0.45	-0.81430
Σ	37.217		36.309

Then $\bar{Y} = \dfrac{\Sigma \bar{y}V}{\Sigma V} = \dfrac{36.309}{37.217}$ in. or $\bar{Y} = 0.976$ in. ◀

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Chapter 5, Solution 100.

Labeling the five parts of the body as follows, and noting that the center of gravity coincides with the centroid of the area due to the uniform thickness.

$$\bar{z}_5 = -\left(300 - \frac{4 \times 150}{3\pi}\right) = -236.34, \quad A_5 = -\frac{\pi}{2}(150)^2 = -11\,250\pi = -35\,343$$

	A mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}A$, 10^6 mm^3	$\bar{y}A$, 10^6 mm^3	$\bar{z}A$, 10^6 mm^3
1	$(600)(400) = 240\,000$	300	200	0	72	48	0
2	$(300)(400) = 120\,000$	600	200	-150	72	24	-18
3	$-(120)(280) = -3360$	600	140	-240	-20.160	-4.7040	8.0640
4	$(600)(300) = 180\,000$	300	400	-150	54	72	-27
5	$-35\,343$	240	400	-236.34	-2.7000	-4.5	2.6588
Σ	471 057				169.358	125.159	-28.583

continued

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Therefore:

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{169\,358\,000}{471\,057} = 359.53 \text{ mm} \qquad \text{or } \bar{X} = 360 \text{ mm} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{125\,159\,000}{471\,057} = 265.70 \text{ mm} \qquad \text{or } \bar{Y} = 266 \text{ mm} \blacktriangleleft$$

$$\bar{Z} = \frac{\Sigma \bar{z} A}{\Sigma A} = \frac{-28\,583\,000}{471\,057} = -60.678 \text{ mm} \qquad \text{or } \bar{Z} = -60.7 \text{ mm} \blacktriangleleft$$

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Chapter 5, Solution 101.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3	$\bar{z}A$, in^3
1	$\frac{1}{2}(4.5)(3) = 6.75$	1.5	7	0	10.125	47.25	0
2	$(4.5)(10) = 45$	2.25	3	4	101.25	135	180
3	$-(2.25)(5) = -11.25 - 2.25$	1.125	1.5	6	−12.6563	−16.875	−67.5
4	$\frac{\pi}{2}(2.25)^2 = 7.9522$	2.25	0	$8 + \frac{4(2.25)}{3\pi}$	17.8925	0	71.211
Σ	48.452				116.611	165.375	183.711

Then $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{116.611}{48.452}$ in. or $\bar{X} = 2.41$ in. ◀

$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{165.375}{48.452}$ in. or $\bar{Y} = 3.41$ in. ◀

$\bar{Z} = \frac{\Sigma \bar{z}A}{\Sigma A} = \frac{183.711}{48.452}$ in. or $\bar{Z} = 3.79$ in. ◀

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Chapter 5, Solution 102.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

First note that by symmetry: $\bar{X} = 150$ mm ◀

For 1: $\bar{y} = 180 + 96 + \dfrac{4(150)}{3\pi} = 339.7$ mm

$\bar{z} = 0$

For 2: $\bar{y} = 180 + \dfrac{2(96)}{\pi} = 241.1$ mm

$\bar{z} = \dfrac{2(96)}{\pi} = 61.11$ mm

For 3: Length $DE = \sqrt{(180)^2 + (96)^2} = 204$ mm

$\bar{y} = 90$ mm, $\bar{z} = 48$ mm

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	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
1	$\frac{\pi}{2}(150)^2 = 35.34 \times 10^3$	339.7	0	12.005×10^6	0
2	$\frac{\pi}{2}(96)(300) = 45.24 \times 10^3$	244.1	61.11	10.907×10^6	2.765×10^6
3	$(204)(300) = 61.2 \times 10^3 - 2.25$	90	48	5.508×10^6	2.938×10^6
Σ	141.78×10^3			28.420×10^6	5.702×10^6

Then

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{28.420 \times 10^6}{141.78 \times 10^3} \text{ mm} \qquad \text{or } \bar{Y} = 200 \text{ mm} \blacktriangleleft$$

$$\bar{Z} = \frac{\Sigma \bar{z}A}{\Sigma A} = \frac{5.702 \times 10^6}{141.78 \times 10^3} \text{ mm} \qquad \text{or } \bar{Y} = 40.2 \text{ mm} \blacktriangleleft$$

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Chapter 5, Solution 103.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(360)(270) = 97\,200$	0	135	0	13 122 000
2	$(339)(270) = 91\,530$	168	135	15 377 000	12 356 600
3	$\frac{1}{2}(339)(72) = 12\,204$	224	294	2 733 700	3 588 000
4	$(360)(343.63) = 123\,707$	168	306	20 783 000	37 854 000
5	$\frac{1}{2}(343.63)(45) = 7731.73$	224	318	1 731 900	2 458 700
6	7731.7	224	318	1 731 900	2 458 700
7	12 204	224	294	2 733 700	3 588 000
8	91 530	168	135	15 377 000	12 356 600
Σ	443 838			60 468 200	87 782 600

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Then $\bar{X} = \dfrac{\Sigma \bar{x} A}{\Sigma A} = \dfrac{60\,468\,200}{443\,838}\text{mm}$ or $\bar{X} = 136.2 \text{ mm}$ ◀

$\bar{Y} = \dfrac{\Sigma \bar{y} A}{\Sigma A} = \dfrac{87\,782\,600}{443\,838}\text{mm}$ or $\bar{Y} = 197.8 \text{ mm}$ ◀

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Chapter 5, Solution 104.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

Note that by symmetry $\bar{X} = 9$ in.

	A, in^2	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}A$, in^3	$\bar{z}A$, in^3
1	16.2	1.8	2	29.16	32.4
2	16.2	1.8	2	29.16	32.4
3	97.2	2.7	0	262.44	0
4	1017.876	−15.2789	20.72113	−15552	21091.54
5	1017.876	−15.2789	20.72113	−15552	21091.54
6	−706.858	−12.7324	23.2676	9000	−16446.9
7	−706.858	−12.7324	23.2676	9000	−16446.9
8	1017.876	−22.9183	13.08169	−23328	13315.54
Σ	1769.511			−36111.24	22669.6

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Therefore

$$\bar{X} = 9 \text{ in.} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{-36111.24}{1769.511} \qquad \text{or } \bar{Y} = -20.4 \text{ in.} \blacktriangleleft$$

$$\bar{Z} = \frac{\Sigma \bar{z} A}{\Sigma A} = \frac{22669.6}{1769.511} \qquad \text{or } \bar{Z} = 12.81 \text{ in.} \blacktriangleleft$$

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Chapter 5, Solution 105.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\pi(8)(12) = 96\pi$	0	6	0	576π
2	$-\frac{\pi}{2}(8)(4) = -16\pi$	$\frac{2(4)}{\pi} = \frac{8}{\pi}$	10	-128	-160π
3	$\frac{\pi}{2}(4)^2 = 8\pi$	$-\frac{4(4)}{3\pi} = -\frac{16}{3\pi}$	12	-42.667	96π
4	$(8)(12) = 96$	6	12	576	1152
5	$(8)(12) = 96$	6	8	576	768
6	$-\frac{\pi}{2}(4)^2 = -8\pi$	$\frac{4(4)}{3\pi} = \frac{16}{3\pi}$	8	-42.667	-64π
7	$(4)(12) = 48$	6	10	288	480
8	$(4)(12) = 48$	6	10	288	480
Σ	539.33			1512.6	4287.4

Then $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1514.67}{539.33}$ in. or $\bar{X} = 2.81$ in. ◀

$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{4287.4}{539.33}$ in. or $\bar{Y} = 7.95$ in. ◀

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Chapter 5, Solution 106.

First, assume that the sheet metal is homogeneous so that the center of gravity of the awning coincides with the centroid of the corresponding area.

$$\bar{y}_{\text{II}} = \bar{y}_{\text{VI}} = 80 + \frac{(4)(500)}{3\pi} = 292.2 \text{ mm}$$

$$\bar{z}_{\text{II}} = \bar{z}_{\text{VI}} = \frac{(4)(500)}{3\pi} = 212.2 \text{ mm}$$

$$\bar{y}_{\text{IV}} = 80 + \frac{(2)(500)}{\pi} = 398.3 \text{ mm}$$

$$\bar{z}_{\text{IV}} = \frac{(2)(500)}{\pi} = 318.3 \text{ mm}$$

$$A_{\text{II}} = A_{\text{VI}} = \frac{\pi}{4}(500)^2 = 196\,350 \text{ mm}^2$$

$$A_{\text{IV}} = \frac{\pi}{2}(500)(680) = 534\,071 \text{ mm}^2$$

	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	(80)(500) = 40 000	40	250	1.6×10^6	10×10^6
II	196 350	292.2	212.2	57.4×10^6	41.67×10^6
III	(80)(680) = 54 400	40	500	0.2176×10^6	27.2×10^6
IV	534 071	398.3	318.3	212.7×10^6	170×10^6
V	(80)(500) = 40 000	40	250	1.6×10^6	10×10^6
VI	196 350	292.2	212.2	57.4×10^6	41.67×10^6
Σ	1.061×10^6			332.9×10^6	300.5×10^6

Now, symmetry implies $\bar{X} = 340 \text{ mm}$ ◀

and $\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}\left(1.061 \times 10^6 \text{ mm}^2\right) = 332.9 \times 10^6 \text{ mm}^3$

or $\bar{Y} = 314 \text{ mm}$ ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A$: $\bar{Z}\left(1.061 \times 10^6 \text{ mm}^2\right) = 300.5 \times 10^6 \text{ mm}^3$

or $\bar{Z} = 283 \text{ mm}$ ◀

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Chapter 5, Solution 107.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

	A, in^2	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}A$, in^3	$\bar{z}A$, in^3
1	$(15)(14) = 120$	0	7	0	1470
2	$(14)(2.5) = 35.0$	1.25	7	43.75	245
3	$(15)(2.5) = 37.5$	1.25	0	46.875	0
4	$(14)(2.5) = 35.0$	1.25	7	43.75	245
5	$-\pi(5)^2 = -78.540$	0	6.5	0	-510.51
6	$-\frac{1}{4}\pi(1.5)^2 = -1.76715$	0	$13 - \frac{4(1.5)}{3\pi} = 12.36348$	0	-21.848
7	$(4)(12) = 48$	6	10	288	480
8	$(4)(12) = 48$	6	10	288	480
Σ	235.43			134.375	1405.79

Then $\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{134.375}{235.43}$ in. or $\bar{Y} = 0.571$ in. ◀

$\bar{Z} = \frac{\Sigma \bar{z}A}{\Sigma A} = \frac{1405.79}{235.43}$ in. or $\bar{Z} = 5.97$ in. ◀

and by symmetry $\bar{X} = 7.50$ in. ◀

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Chapter 5, Solution 108.

$AB^2 = (500\text{ mm})^2 + (750\text{ mm})^2 + (300\text{ mm})^2$, or

$AB = 950\text{ mm}$

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2	$\bar{z}L$, mm^2
AB	950	250	375	150	237.5×10^3	356.25×10^3	142.5×10^3
BD	300	500	0	150	150×10^3	0	45×10^3
DO	500	250	0	0	125×10^3	0	0
OA	750	0	375	0	0	281.25×10^3	0
Σ	2500				512.5×10^3	637.5×10^3	187.5×10^3

Then

$$\bar{X} = \frac{\Sigma\bar{x}L}{\Sigma L} = \frac{512.5\times10^3}{2500} \qquad \text{or } \bar{X} = 205\text{ mm} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma\bar{y}L}{\Sigma L} = \frac{637.5\times10^3}{2500} \qquad \text{or } \bar{Y} = 255\text{ mm} \blacktriangleleft$$

$$\bar{Z} = \frac{\Sigma\bar{z}L}{\Sigma L} = \frac{187.5\times10^3}{2500} \qquad \text{or } \bar{Z} = 75\text{ mm} \blacktriangleleft$$

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Chapter 5, Solution 109.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the line

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2	$\bar{z}L$, mm^2
1	300	0	150	0	0	45 000	0
2	280	140	0	0	39 200	0	0
3	260	230	0	120	59 800	0	31 200
4	$\frac{\pi}{2}(300) = 150\pi$	$\frac{3}{5}\left(\frac{2\times 300}{\pi}\right) = \frac{360}{\pi}$	$\frac{600}{\pi}$	$\frac{480}{\pi}$	54 000	90 000	72 000
Σ	1311.24				153 000	135 000	103 200

Then

$$\bar{X} = \frac{\Sigma\bar{x}L}{\Sigma L} = \frac{153\,000}{1311.24} \qquad \text{or } \bar{X} = 116.7 \text{ mm} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma\bar{y}L}{\Sigma L} = \frac{135\,000}{1311.24} \qquad \text{or } \bar{Y} = 103.0 \text{ mm} \blacktriangleleft$$

$$\bar{Z} = \frac{\Sigma\bar{z}L}{\Sigma L} = \frac{103\,200}{1311.24} \qquad \text{or } \bar{Z} = 78.7 \text{ mm} \blacktriangleleft$$

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Chapter 5, Solution 110.

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the line.

	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2
1	10	$4\cos 45° = 2.8284$	5	28.284	50
2	10	$4\cos 45° = 2.8284$	5	28.284	50
3	4π	0	12.5465	0	157.664
4	4π	$\frac{2(4)}{\pi} = \frac{8}{\pi}$	10	32	125.664
5	2π	$\frac{2(4)}{\pi} = \frac{8}{\pi}$	12.5465	16	78.832
Σ	51.416			104.568	462.16

Then

$$\bar{X} = \frac{\Sigma \bar{x}L}{\Sigma L} = \frac{104.568}{51.416} \qquad \text{or} \quad \bar{X} = 2.03 \text{ ft} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{462.16}{51.416} \qquad \text{or} \quad \bar{Y} = 8.99 \text{ ft} \blacktriangleleft$$

and by symmetry:

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Chapter 5, Solution 111.

First note by symmetry:

$\bar{Z} = 3.00$ ft ◀

To simplify the calculations replace:

(a) The two rectangular sides with an element of length

$$\bar{L}_{(a)} = 2\left[2(7\text{ ft}) + 2(5\text{ ft})\right] = 48\text{ ft}$$

and center of gravity at (3.5 ft, 2.5 ft, 3 ft)

(b) The two semicircular members with an element of length

$$L_b = 2\left[\pi(3\text{ ft})\right] = 6\pi\text{ ft}$$

and with center of gravity at $\left(2\text{ ft}, 5 + \dfrac{2\times 3}{\pi}\text{ft}, 3\text{ ft}\right) = (2\text{ ft}, 6.9099\text{ ft}, 3\text{ ft})$

(c) The cross members 1 and 2 with an element of length

$$L_c = 2(6\text{ ft}) = 12\text{ ft}$$

and with center of gravity at $(2\text{ ft}, 5\text{ ft}, 3\text{ ft})$

(d) This leaves a single straight piece of pipe, labeled (d) in the figure.

Now for the centroid of the frame:

	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2
(a)	48	3.5	2.5	168	120
(b)	$6\pi = 18.8496$	2	6.9099	37.699	130.249
(c)	12	2	5	24	60
(d)	6	7	5	42	30
Σ	85.850			271.70	340.25

Then $\bar{X} = \dfrac{\Sigma\bar{x}L}{\Sigma L} = \dfrac{271.70}{84.850}$ or $\bar{X} = 3.20$ ft ◀

$\bar{Y} = \dfrac{\Sigma\bar{y}L}{\Sigma L} = \dfrac{340.25}{84.850}$ or $\bar{Y} = 4.01$ ft ◀

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Chapter 5, Solution 112.

First, note that symmetry implies $\bar{Y} = \bar{Z} = 0$ ◀

$$\bar{x}_{\mathrm{I}} = \frac{5}{8}(0.5\text{ in.}) = 0.3125\text{ in.},\ W_{\mathrm{I}} = \left(0.0374\text{ lb/in}^3\right)\left(\frac{2\pi}{3}\right)(0.5\text{ in})^3 = 0.009791\text{ lb}$$

$$\bar{x}_{\mathrm{II}} = 1.6\text{ in.} + 0.5\text{ in.} = 2.1\text{ in.}\ W_{\mathrm{II}} = \left(0.0374\text{ lb/in}^3\right)(\pi)(0.5\text{ in})^2(3.2\text{ in.}) = 0.093996\text{ lb}$$

$$\bar{x}_{\mathrm{III}} = 3.7\text{ in.} - 1\text{ in.} = 2.7\text{ in.},\ W_{\mathrm{III}} = -\left(0.0374\text{ lb/in}^3\right)\left(\frac{\pi}{4}\right)(0.12\text{ in})^2(2\text{ in.}) = -0.000846\text{ lb}$$

$$\bar{x}_{\mathrm{IV}} = 7.3\text{ in.} - 2.8\text{ in.} = 4.5\text{ in.},\ W_{\mathrm{IV}} = \left(0.284\text{ lb/in}^3\right)\left(\frac{\pi}{4}\right)(0.12\text{ in})^2(5.6\text{ in})^2 = 0.017987\text{ lb}$$

$$\bar{x}_{\mathrm{V}} = 7.3\text{ in.} + \frac{1}{4}(0.4\text{ in.}) = 7.4\text{ in.},\ W_{\mathrm{V}} = \left(0.284\text{ lb/in}^3\right)\left(\frac{\pi}{3}\right)(0.06\text{ in})^2(0.4\text{ in.}) = 0.000428\text{ lb}$$

	W, lb	$\bar{x}$, in.	$\bar{x}W$, in·lb
I	0.009791	0.3125	0.003060
II	0.093996	2.1	0.197393
III	−0.000846	2.7	−0.002284
IV	0.017987	4.5	0.080942
V	0.000428	7.4	0.003169
Σ	0.12136		0.28228

Have $\bar{X}\Sigma W = \Sigma\bar{x}W$: $\bar{X}(0.12136\text{ lb}) = 0.28228\text{ in.}\cdot\text{lb}$ or $\bar{X} = 2.33\text{ in.}$ ◀

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Chapter 5, Solution 113.

Determine first the masses of the component pieces:

$$m_1 = \left(8800\ \text{kg/m}^3\right)\left[\frac{\pi}{4}\left(0.016^2 - 0.012^2\right)\text{m}^2 \times \left(0.014\ \text{m}\right)\right] = 0.0108372\ \text{kg}$$

$$m_2 = \left(1250\ \text{kg/m}^3\right)\left[\frac{\pi}{4}\left(0.036^2 - 0.016^2\right)\text{m}^2 \times \left(0.014\ \text{m}\right)\right] = 0.0142942\ \text{kg}$$

$$m_3 = \left(1250\ \text{kg/m}^3\right)\left[\frac{\pi}{4}\left(0.060^2 - 0.036^2\right)\text{m}^2 \times \left(0.006\ \text{m}\right)\right] = 0.0135717\ \text{kg}$$

$$m_4 = \left(1250\ \text{kg/m}^3\right)\left[\frac{\pi}{4}\left(0.080^2 - 0.060^2\right)\text{m}^2 \times \left(0.010\ \text{m}\right)\right] = 0.027489\ \text{kg}$$

Now, for the center of mass:

	m, kg	$\bar{x}$, mm	$\bar{x}m$, kg·mm
1	0.0108372	7	0.075860
2	0.0142942	7	0.100059
3	0.0135717	3	0.040715
4	0.027489	5	0.137445
Σ	0.066192		0.35408

Then $\bar{X} = \dfrac{\Sigma \bar{x}m}{\Sigma m} = \dfrac{0.35408}{0.066192}$ or $\bar{X} = 5.35\ \text{mm}$ ◀

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Chapter 5, Solution 114.

Assume that the stone is homogeneous so that its center of gravity coincides with the centroid of the volume and locate the center of gravity.

To determine the centroid of the truncated pyramid note that:

$$\bar{y}_1 = \frac{3}{4}(1.4\text{ m}) = 1.05\text{ m, and} \qquad V_1 = \frac{1}{3}(0.3\text{ m})(0.3\text{ m})(1.4\text{ m}) = 0.042\text{ m}^3$$

$$\bar{y}_2 = \frac{3}{4}(0.7\text{ m}) = 0.525\text{ m, and} \qquad V_2 = -\frac{1}{3}(0.15\text{ m})(0.15\text{ m})(0.7\text{ m}) = -0.00525\text{ m}^3$$

Then $V_{stone} = V_1 + V_2 = 0.042\text{ m}^3 - 0.00525\text{ m}^3 = 0.03675\text{ m}^3$, and

$$\bar{y} = \frac{\Sigma \bar{y} V}{\Sigma V} = \frac{(1.05\text{ m})(0.042\text{ m}^3) + (0.525\text{ m})(-0.00525\text{ m}^3)}{0.03675\text{ m}^3}$$

$$= 1.12500\text{ m}$$

The center of gravity of the stone is therefore 0.425 m (i.e. 1.125 m – 0.7m) above the base.

Now to determine the center of gravity of the marker:

$$W_{stone} = (\rho g V)_{stone} = (2570\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.03675\text{ m}^3) = 926.53\text{ N}$$

$$W_{steel} = (\rho g V)_{steel} = (7860\text{ kg/m}^3)(9.81\text{ m/s}^2)[(0.3\text{ m})(0.3\text{ m})h] = (6939.6h)\text{N}$$

continued

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Then

$$\bar{y}_{mar\,\mathrm{ker}} = \frac{\Sigma \bar{y} W}{\Sigma W}, \text{ or}$$

$$0.3 \text{ m} = \frac{(0.425 \text{ m})(926.53 \text{ N}) + \left(-\frac{h}{2} \text{ m}\right)(6939.6\,h)\text{N}}{(926.53 + 6939.6\ h)\text{N}}, \text{ or}$$

$$h^2 + 0.6\,h - 0.033378 = 0.$$

Solving for h and discarding the negative root, this gives $h = 0.051252$ m, or

$h = 50$ mm ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 5, Solution 115.

Since the brass plates are equally spaced and by the symmetry of the cylinder:

$$\bar{X} = \bar{Y} = 0 \blacktriangleleft$$

For the pipe:

Specific weight of steel: $\gamma_s = 0.284 \text{ lb/in}^3$

$\bar{y}_1 = 4 \text{ in.}$

outside diameter: 2.5 in.

Inside diameter: $2.5 \text{ in.} - 2(0.25 \text{ in.}) = 2.00 \text{ in.}$

Volume: $V_1 = \frac{\pi}{4}\left(2.5^2 - 2.0^2\right)8 = 14.137 \text{ in}^3$

Weight: $W_1 = \gamma_s V_1 = \left(0.284 \text{ lb/in}^3\right)\left(14.137 \text{ in}^3\right) = 4.015 \text{ lb}$

For each brass plate:

Specific weight for brass: $\gamma_B = 0.306 \text{ lb/in}^3$

$\bar{y}_2 = \frac{8}{3} 2.667 \text{ in.}$

Volume: $V_2 = \frac{1}{2}(8)(4)(0.2) = 3.2 \text{ in}^3$

Weight: $W_1 = \gamma_s V_1 = \left(0.306 \text{ lb/in}^3\right)\left(3.2 \text{ in}^3\right) = 0.979 \text{ lb}$

continued

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For flagpole base:

$$\Sigma W = (4.015 \text{ lb}) + 3(0.979 \text{ lb}) = 6.952 \text{ lb}$$

$$\Sigma \bar{y} W = (4 \text{ in.})(4.015 \text{ lb}) + 3\left[(2.667 \text{ in.})(0.979 \text{ lb})\right] = 23.892 \text{ in.}\cdot\text{lb, or}$$

$$\bar{Y} = \frac{\Sigma \bar{y} W}{\Sigma W} = \frac{23.892 \text{ in.}\cdot\text{lb}}{6.952 \text{ lb}} = 3.437 \text{ in.}$$

$\bar{Y} = 3.437$ in. ◀

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Chapter 5, Solution 116.

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x^2 + y^2 = a^2$ so that $r^2 = a^2 - x^2$ and then

$$dV = \pi\left(a^2 - x^2\right)dx$$

Component 1

$$V_1 = \int_0^{a/2} \pi\left(a^2 - x^2\right)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_0^{a/2}$$

$$= \frac{11}{24}\pi a^3$$

and

$$\int_1 \bar{x}_{EL} dV = \int_0^{a/2} x\left[\pi\left(a^2 - x^2\right)dx\right]$$

$$= \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{a/2}$$

$$= \frac{7}{64}\pi a^4$$

Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^3\right) = \frac{7}{64}\pi a^4$$

or $\bar{x}_1 = \dfrac{21}{88}a$ ◀

Component 2

$$V_2 = \int_{a/2}^{a} \pi\left(a^2 - x^2\right)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2(a) - \frac{a^3}{3}\right] - \left[a^2\left(\frac{a}{2}\right) - \frac{\left(\frac{a}{2}\right)^3}{3}\right]\right\}$$

continued

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$$= \frac{5}{24}\pi a^3$$

and $$\int_2 \bar{x}_{\text{EL}} dV = \int_{a/2}^{a} x\left[\pi\left(a^2 - x^2\right)dx\right] = \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2\frac{(a)^2}{2} - \frac{(a)^4}{4}\right] - \left[a^2\frac{\left(\frac{a}{2}\right)^2}{2} - \frac{\left(\frac{a}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^4$$

Now $$\bar{x}_2 V_2 = \int_2 \bar{x}_{\text{EL}} dV: \quad \bar{x}_2\left(\frac{5}{24}\pi a^3\right) = \frac{9}{64}\pi a^4$$

or $\bar{x}_2 = \frac{27}{40}a$ ◀

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Chapter 5, Solution 117.

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{\text{EL}} = x$$

The equation of the generating curve is $\dfrac{x^2}{h^2} + \dfrac{y^2}{a^2} = 1$ so that $r^2 = \dfrac{a^2}{h^2}\left(h^2 - x^2\right)$ and then

$$dV = \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx$$

Component 1

$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \frac{11}{24}\pi a^2 h$$

and

$$\int_1 \bar{x}_{EL} dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx\right]$$

$$= \pi \frac{a^2}{h^2}\left[h^2 \frac{x^2}{2} - \frac{x^4}{4}\right]_0^{h/2}$$

$$= \frac{7}{64}\pi a^2 h^2$$

Now $\quad \bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\dfrac{11}{24}\pi a^2 h\right) = \dfrac{7}{64}\pi a^2 h^2$

or $\bar{x}_1 = \dfrac{21}{88}h$ ◀

continued

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Component 2

$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h^2}\left\{\left[h^2(h) - \frac{(h)^3}{3}\right] - \left[h^2\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^2 h$$

and $$\int_2 \bar{x}_{EL} dV = \int_{h/2}^{h} x\left[\pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx\right]$$

$$= \pi \frac{a^2}{h^2}\left[h^2 \frac{x^2}{2} - \frac{x^4}{4}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h^2}\left\{\left[h^2 \frac{(h)^2}{2} - \frac{(h)^4}{4}\right] - \left[h^2 \frac{\left(\frac{h}{2}\right)^2}{2} - \frac{\left(\frac{h}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^2 h^2$$

Now $$\bar{x}_2 V_2 = \int_2 \bar{x}_{EL} dV: \quad \bar{x}_2\left(\frac{5}{24}\pi a^2 h\right) = \frac{9}{64}\pi a^2 h^2$$

or $\bar{x}_2 = \frac{27}{40}h$ ◀

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Chapter 5, Solution 118.

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{\text{EL}} = x$$

The equation of the generating curve is $x = h - \dfrac{h}{a^2}y^2$ so that $r^2 = \dfrac{a^2}{h}(h - x)$ and then

$$dV = \pi \frac{a^2}{h}(h - x)\,dx$$

Component 1

$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h}(h - x)\,dx$$

$$= \pi \frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_0^{h/2}$$

$$= \frac{3}{8}\pi a^2 h$$

and

$$\int_1 \bar{x}_{EL}\,dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h}(h - x)\,dx\right]$$

$$= \pi \frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \frac{1}{12}\pi a^2 h^2$$

Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL}\,dV: \quad \bar{x}_1\left(\frac{3}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$

or $\bar{x}_1 = \dfrac{2}{9}h$ ◀

continued

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Component 2

$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h}(h-x)\,dx = \pi \frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h}\left\{\left[h(h) - \frac{(h)^2}{2}\right] - \left[h\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^2}{2}\right]\right\}$$

$$= \frac{1}{8}\pi a^2 h$$

and $$\int_2 \bar{x}_{EL}\,dV = \int_{h/2}^{h} x\left[\pi \frac{a^2}{h}(h-x)\,dx\right] = \pi \frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h}\left\{\left[h\frac{(h)^2}{2} - \frac{(h)^3}{3}\right] - \left[h\frac{\left(\frac{h}{2}\right)^2}{2} - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{1}{12}\pi a^2 h^2$$

Now $$\bar{x}_2 V_2 = \int_2 \bar{x}_{EL}\,dV: \quad \bar{x}_2\left(\frac{1}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$

or $\bar{x}_2 = \frac{2}{3}h$ ◀

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Chapter 5, Solution 119.

First note that symmetry implies $\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

Now $r = b\left(1 - \dfrac{x^2}{a^2}\right)$ so that

$$dV = \pi b^2 \left(1 - \frac{x^2}{a^2}\right)^2 dx$$

Then
$$V = \int_0^a \pi b^2 \left(1 - \frac{x^2}{a^2}\right)^2 dx = \int_0^a \pi b^2 \left(1 - \frac{2x^2}{a^2} + \frac{x^4}{a^4}\right) dx$$

$$= \pi b^2 \left(x - \frac{2x^3}{3a^2} + \frac{x^5}{5a^4}\right)\Bigg|_0^a$$

$$= \pi a b^2 \left(1 - \frac{2}{3} + \frac{1}{5}\right)$$

$$= \frac{8}{15}\pi a b^2$$

and
$$\int \bar{x}_{EL} dV = \int_0^a \pi b^2 x \left(1 - \frac{2x^2}{a^2} + \frac{x^4}{a^4}\right) dx$$

$$= \pi b^2 \left(\frac{x^2}{2} - \frac{2x^4}{4a^2} + \frac{x^6}{6a^4}\right)\Bigg|_0^a$$

continued

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$$= \pi a^2 b^2 \left(\frac{1}{2} - \frac{1}{2} + \frac{1}{6} \right)$$

$$= \frac{1}{6} \pi a^2 b^2$$

Then $\bar{x}V = \int x_{EL} dV$: $\bar{x}\left(\frac{8}{15} \pi a b^2 \right) = \frac{1}{16} \pi a^2 b^2$

or $\bar{x} = \frac{15}{6} a$ ◀

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Chapter 5, Solution 120.

First, note that symmetry implies $\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx.
Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

Now $r = 1 - \dfrac{1}{x}$ so that

$$dV = \pi\left(1 - \frac{1}{x}\right)^2 dx$$

$$= \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$$

Then
$$V = \int_1^3 \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx = \pi\left[x - 2\ln x - \frac{1}{x}\right]_1^3$$

$$= \pi\left[\left(3 - 2\ln 3 - \frac{1}{3}\right) - \left(1 - 2\ln 1 - \frac{1}{1}\right)\right]$$

$$= (0.46944\pi)\ \text{m}^3$$

and
$$\int \bar{x}_{EL} dV = \int_1^3 x\left[\pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx\right] = \pi\left[\frac{x^2}{2} - 2x + \ln x\right]_1^3$$

$$= \pi\left\{\left[\frac{3^2}{2} - 2(3) + \ln 3\right] - \left[\frac{1^3}{2} - 2(1) + \ln 1\right]\right\}$$

$$= (1.09861\pi)\ \text{m}$$

Now
$$\bar{x}V = \int \bar{x}_{EL} dV: \quad \bar{X}\left(0.46944\pi\ \text{m}^3\right) = 1.09861\pi\ \text{m}^4$$

or $\bar{x} = 2.34\ \text{m}$ ◀

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Chapter 5, Solution 121.

First, by symmetry:

$$\bar{x} = a \blacktriangleleft$$

$$\bar{y} = 0 \blacktriangleleft$$

Next determine the constants k in $y = kx^{1/3}$:

At $\quad x = a, \quad b = ka^{1/3}$ or $k = \dfrac{b}{a^{1/3}}$

Therefore, $y = \dfrac{b}{a^{1/3}} x^{1/3}$, or $x = \dfrac{a}{b^3} y^3$

Choosing horizontal disks of thickness dy for volume elements (dV in the figure above)

$$V = \int_0^b \pi \left[a^2 - (a - x)^2 \right]$$

$$= \pi \int_0^b \left(2ax - x^2 \right) dy$$

$$= \pi \int_0^b \left(2a \times \frac{a}{b^3} y^3 - \frac{a^2}{b^6} y^6 \right) dy$$

$$= \pi \frac{a^2}{b^3} \left(2 \times \frac{1}{4} y^4 - \frac{1}{b^3} \times \frac{1}{7} y^7 \right) \Bigg|_0^b = \frac{5}{14} \pi a^2 b$$

Now $\bar{y} = \dfrac{1}{V} \int \bar{y}_{EL} dV$, or

$$\bar{y} = \frac{14}{5\pi a^2 b} \int_0^b y \left[\pi \left(2 \frac{a^2}{b^3} y^3 - \frac{a^2}{b^6} y^6 \right) dy \right]$$

$$= \frac{14}{5b^4} \left(2 \times \frac{y^5}{5} - \frac{1}{b^3} \frac{y^8}{8} \right) \Bigg|_0^b$$

or $\quad \bar{y} = \dfrac{77}{100} b \blacktriangleleft$

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Chapter 5, Solution 122.

First note that by symmetry:

$\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as a volume element a disk of radius y and thickness dx. Then:

$\bar{x}_{EL} = x$, and

$dV = \pi y^2 dx$, or

$$dV = \pi h^2 \cos^2 \frac{\pi x}{2a} dx$$

Using the identity:

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, this gives

$$dV = \frac{1}{2}\pi h^2 \left(1 + \cos\frac{\pi x}{a}\right) dx.$$

Then:

$$V = \int dV = \frac{\pi h^2}{2}\int_0^a \left(1 + \cos\frac{\pi x}{a}\right) dx = \frac{\pi h^2}{2}\left[x + \frac{a}{\pi}\sin\frac{\pi x}{a}\right]_0^a = \frac{1}{2}\pi h^2 a.$$

continued

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Also,

$$\int \bar{x}_{EL} dV = \frac{\pi h^2}{2}\int_0^a \left(x + x\cos\frac{\pi x}{a} \right) dx. \text{ Integrating by parts,}$$

$$= \frac{\pi h^2}{2}\left\{ \left[\frac{x^2}{2} \right]_0^a + \frac{a}{\pi}\left[x\sin\frac{\pi x}{a} - \int \sin\frac{\pi x}{a} \right]_0^a \right\}$$

$$= \frac{\pi h^2}{2}\left\{ \frac{a^2}{2} + \frac{a}{\pi}\left[x\sin\frac{\pi x}{a} + \frac{a}{\pi}\cos\frac{\pi x}{a} \right]_0^a \right\}$$

$$= \frac{\pi h^2}{2}\left\{ \frac{a^2}{2} + \frac{a}{\pi}\left(0 - 0 - \frac{2a}{\pi} \right) \right\} = \frac{1}{4}\pi a^2 h^2 \left(1 - \frac{4}{\pi^2} \right)$$

Now,

$$\bar{x} = \frac{1}{V}\int x_{EL} dV = \frac{2}{\pi h^2 a}\left[\frac{1}{4}\pi a^2 h^2 \left(1 - \frac{4}{\pi^2} \right) \right], \text{ or}$$

$$\bar{x} = \frac{1}{2}a\left(1 - \frac{4}{\pi^2} \right) \blacktriangleleft$$

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Chapter 5, Solution 123.

First note that by symmetry:

$$\bar{x} = 0 \blacktriangleleft$$

$$\bar{y} = 0 \blacktriangleleft$$

Choosing the volume element shown in the figure, i.e. a cylindrical shell of radius r, height h and thickness dr:

$$\bar{y}_{EL} = \frac{1}{2}y, \qquad \text{and}$$

$$dV = 2\pi r y\, dr = 2\pi r \cos\frac{\pi r}{2a}\, dr, \text{ and}$$

$$V = \int dV = 2\pi h \int_0^a r\cos\frac{\pi r}{2a}\, dr, \text{ or, integrating by parts}$$

$$V = 2\pi h \frac{2a}{\pi}\left[r\sin\frac{\pi r}{2a} - \int \sin\frac{\pi r}{2a}\, dr \right]_0^a$$

$$= 4ah\left[r\sin\frac{\pi r}{2a} + \frac{2a}{\pi}\cos\frac{\pi r}{2a} \right]_0^a$$

$$= 4ah\left(a - \frac{2a}{\pi} \right) = 4a^2 h\left(1 - \frac{2}{\pi} \right)$$

Also,

$$\int \bar{y}_{EL}\, dV = \pi h^2 \int_0^a r\cos^2\frac{\pi r}{2a}\, dr = \frac{\pi h^2}{2}\int_0^a r\left(1 + \cos\frac{\pi r}{a} \right) dr$$

$$= \frac{\pi h^2}{2}\left\{ \left[\frac{1}{2}r^2 \right]_0^a + \frac{a}{\pi}\left[r\sin\frac{\pi r}{a} - \int \sin\frac{\pi r}{a}\, dr \right]_0^a \right\}$$

continued

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$$= \frac{\pi h^2}{2}\left\{\frac{1}{2}a^2 + \frac{a}{\pi}\left[r\sin\frac{\pi r}{a} + \frac{a}{\pi}\cos\frac{\pi r}{a}\right]_0^a\right\}$$

$$= \frac{\pi h^2}{2}\left\{\frac{1}{2}a^2 + \frac{a}{\pi}\left(0 + 0 - \frac{2a}{\pi}\right)\right\} = \frac{1}{4}\pi a^2 h^2\left(1 - \frac{4}{\pi^2}\right)$$

Now,

$$\bar{y} = \frac{1}{V}\int y_{EL}\,dV = \frac{1}{4a^2h\left(1 - \frac{2}{\pi}\right)}\left[\frac{1}{4}\pi a^2 h^2\left(1 - \frac{4}{\pi^2}\right)\right], \text{ or}$$

$$\bar{y} = \frac{(\pi + 2)}{16}h \blacktriangleleft$$

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Chapter 5, Solution 124.

Choose as the element of a horizontal slice of thickness dy. For any number N of sides, the area of the base of the pyramid is given by

$$A_{\text{base}} = kb^2$$

where $k = k(N)$; see note below. Using similar triangles, have

$$\frac{s}{b} = \frac{h-y}{h}$$

or

$$s = \frac{b}{h}(h-y)$$

Then

$$dV = A_{\text{slice}}dy = ks^2dy = k\frac{b^2}{h^2}(h-y)^2\,dy$$

and

$$V = \int_0^h k\frac{b^2}{h^2}(h-y)^2\,dy = k\frac{b^2}{h^2}\left[-\frac{1}{3}(h-y)^3\right]_0^h$$

$$= \frac{1}{3}kb^2h$$

Also

$$\bar{y}_{EL} = y$$

so then

$$\int \bar{y}_{EL}dV = \int_0^h y\left[k\frac{b^2}{h^2}(h-y)^2\,dy\right] = k\frac{b^2}{h^2}\int_0^h\left(h^2y - 2hy^2 + y^3\right)dy$$

$$= k\frac{b^2}{h^2}\left[\frac{1}{2}h^2y^2 - \frac{2}{3}hy^3 + \frac{1}{4}y^4\right]_0^h = \frac{1}{12}kb^2h^2$$

Now

$$\bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{1}{3}kb^2h\right) = \frac{1}{12}kb^2h^2$$

or $y = \frac{1}{4}h$ Q.E.D. ◀

Note:

$$A_{\text{base}} = N\left(\frac{1}{2}\times b\times\frac{\frac{b}{2}}{\tan\frac{\pi}{N}}\right)$$

$$= \frac{N}{4\tan\frac{\pi}{N}}b^2$$

$$= k(N)\,b^2$$

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Chapter 5, Solution 125.

Since the spherical cup is uniform, the center of gravity will coincide with the centroid. Also, because the cup is thin, it can be treated like an area in finding the centroid.

An element of area is obtained by rotating arc *ds* about the *y* axis. With the *y* axis pointing downwards,

$$dA = 2\pi r ds = 2\pi\left(R\sin\theta\right)Rd\theta$$

$$= 2\pi R^2 \sin\theta d\theta$$

$$\bar{y}_{EL} = y = R\cos\theta$$

$$A = \int dA = 2\pi R^2 \int_0^{\phi} \sin\theta d\theta = 2\pi R^2\left[-\cos\theta\right]_0^{\phi} = 2\pi R^2\left(1-\cos\phi\right)$$

$$\int \bar{y}_{EL} dA = \int_0^{\phi}\left(R\cos\theta\right)\left(2\pi R^2 \sin\theta d\theta\right) = 2\pi R^3 \int_0^{\phi} \cos\theta \sin\theta d\theta$$

$$= 2\pi R^3\left[-\frac{1}{2}\cos^2\theta\right]_0^{\phi} = \pi R^3\left(1-\cos^2\phi\right)$$

Then,

$$\bar{y} = \frac{1}{A}\int \bar{y}_{EL} dA = \frac{1}{2\pi R^2\left(1-\cos\phi\right)}\pi R^3\left(1-\cos^2\phi\right), \text{ or}$$

$$\bar{y} = \frac{R}{2}\left(1+\cos\phi\right)$$

Using

$$\cos\phi = \frac{R-h}{R} = 1-\frac{h}{R}:$$

$$\bar{y} = \frac{R}{2}\left[1+\left(1-\frac{h}{R}\right)\right] = R-\frac{h}{2}$$

The center of gravity is therefore located at a distance of

$$R-\bar{y} = R-\left(R-\frac{h}{2}\right) = \frac{h}{2}, \text{ above the base.(Q.E.D)}$$

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Chapter 5, Solution 126.

(*a*) Bowl

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

for the coordinate axes shown below. Now assume that the bowl may be treated as a shell; the center of gravity of the bowl will coincide with the centroid of the shell. For the walls of the bowl, an element of area is obtained by rotating the arc *ds* about the *y* axis. Then

$$dA_{\text{wall}} = (2\pi R\sin\theta)(Rd\theta)$$

and $$(\bar{y}_{EL})_{\text{wall}} = -R\cos\theta$$

Then $$A_{\text{wall}} = \int_{\pi/6}^{\pi/2} 2\pi R^2 \sin\theta d\theta = 2\pi R^2\left[-\cos\theta\right]_{\pi/6}^{\pi/2}$$

$$= \pi\sqrt{3}R^2$$

and $$\bar{y}_{\text{wall}}A_{\text{wall}} = \int(\bar{y}_{EL})_{\text{wall}}\,dA$$

$$= \int_{\pi/6}^{\pi/2}(-R\cos\theta)\left(2\pi R^2\sin\theta d\theta\right)$$

$$= \pi R^3\left[\cos^2\theta\right]_{\pi/6}^{\pi/2}$$

$$= -\frac{3}{4}\pi R^3$$

By observation $$A_{\text{base}} = \frac{\pi}{4}R^2, \qquad \bar{y}_{\text{base}} = -\frac{\sqrt{3}}{2}R$$

Now $$\bar{y}\Sigma A = \Sigma\bar{y}A$$

or $$\bar{y}\left(\pi\sqrt{3}R^2 + \frac{\pi}{4}R^2\right) = -\frac{3}{4}\pi R^3 + \frac{\pi}{4}R^2\left(-\frac{\sqrt{3}}{2}R\right)$$

or $$\bar{y} = -0.48763R \qquad R = 350\text{ mm}$$

$\therefore\ \bar{y} = -170.7$ mm ◀

(*b*) Punch

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

and that because the punch is homogeneous, its center of gravity will coincide with the centroid of the corresponding volume. Choose as the element of volume a disk of radius *x* and thickness *dy*. Then

$$dV = \pi x^2 dy,\ \bar{y}_{EL} = y$$

continued

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Now $x^2 + y^2 = R^2$ so that $dV = \pi\left(R^2 - y^2\right)dy$

Then $V = \int_{-\sqrt{3}/2R}^{0} \pi\left(R^2 - y^2\right)dy = \pi\left[R^2 y - \frac{1}{3}y^3\right]_{-\sqrt{3}/2R}^{0}$

$$= -\pi\left[R^2\left(-\frac{\sqrt{3}}{2}R\right) - \frac{1}{3}\left(-\frac{\sqrt{3}}{2}R\right)^3\right] = \frac{3}{8}\pi\sqrt{3}R^3$$

and $\int \bar{y}_{EL}dV = \int_{-\sqrt{3}/2R}^{0}(y)\left[\pi\left(R^2 - y^2\right)dy\right] = \pi\left[\frac{1}{2}R^2y^2 - \frac{1}{4}y^4\right]_{-\sqrt{3}/2R}^{0}$

$$= -\pi\left[\frac{1}{2}R^2\left(-\frac{\sqrt{3}}{2}R\right)^2 - \frac{1}{4}\left(-\frac{\sqrt{3}}{2}R\right)^4\right] = -\frac{15}{64}\pi R^4$$

Now $\bar{y}V = \int \bar{y}_{EL}dV$: $\bar{y}\left(\frac{3}{8}\pi\sqrt{3}R^3\right) = -\frac{15}{64}\pi R^4$

or $\bar{y} = -\frac{5}{8\sqrt{3}}R$ $R = 350$ mm

$\therefore\ \bar{y} = -126.3$ mm ◀

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Chapter 5, Solution 127.

The centroid can be found by integration. The equation for the bottom of the gravel is:

$y = a + bx + cz,$ where the constants a, b, and c can be determined as follows:

For $x = 0,$ and $z = 0$: $y = -3$ in., and therefore

$$-\frac{3}{12}\text{ft} = a, \text{ or } a = -\frac{1}{4}\text{ft}$$

For $x = 30$ ft, and $z = 0$: $y = -5$ in., and therefore

$$-\frac{5}{12}\text{ft} = -\frac{1}{4}\text{ft} + b(30\text{ ft}), \text{ or } b = -\frac{1}{180}$$

For $x = 0$, and $z = 50$ ft: $y = -6$ in., and therefore

$$-\frac{6}{12}\text{ft} = -\frac{1}{4}\text{ft} + c(50\text{ ft}), \text{ or } c = -\frac{1}{200}$$

Therefore:

$$y = -\frac{1}{4}\text{ft} - \frac{1}{180}x - \frac{1}{200}z$$

Now

$$\bar{x} = \frac{\int x_{EL}\,dV}{V}$$

A volume element can be chosen as:

$$dV = |y|\,dxdz,$$

or $$dV = \frac{1}{4}\left(1 + \frac{1}{45}x + \frac{1}{50}z\right)dx\,dz, \text{ and}$$

$$\bar{x}_{EL} = x$$

continued

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Then

$$\int x_{EL}\,dV = \int_0^{50}\int_0^{30}\frac{x}{4}\left(1+\frac{1}{45}x+\frac{1}{50}z\right)dx\,dz$$

$$= \frac{1}{4}\int_0^{50}\left[\frac{x^2}{2}+\frac{1}{135}x^3+\frac{z}{100}x^2\right]_0^{30}dz$$

$$= \frac{1}{4}\int_0^{50}(650+9z)\,dz$$

$$= \frac{1}{4}\left[650z+\frac{9}{2}z^2\right]_0^{50}$$

$$= 10937.5\text{ ft}^4$$

The volume is:

$$V\int dV = \int_0^{50}\int_0^{30}\frac{1}{4}\left(1+\frac{1}{45}x+\frac{1}{50}z\right)dx\,dz$$

$$= \frac{1}{4}\int_0^{50}\left[x+\frac{1}{90}x^2+\frac{z}{50}x\right]_0^{30}dz$$

$$= \frac{1}{4}\int_0^{50}\left(40+\frac{3}{5}z\right)dz$$

$$= \frac{1}{4}\left[40z+\frac{3}{10}z^2\right]_0^{50}$$

$$= 687.50\text{ ft}^3$$

Then

$$\bar{x} = \frac{\int \bar{x}_{EL}\,dV}{V} = \frac{10937.5\text{ ft}^4}{687.5\text{ ft}^3} = 15.9091\text{ ft}$$

Therefore: $V = 688\text{ ft}^3$ ◀

$\bar{x} = 15.91\text{ ft}$ ◀

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Chapter 5, Solution 128.

Choosing the element of volume shown, i.e. a filament of sides, y, dx, and dz:

$dV = y\,dx\,dy$, and

$\bar{z}_{EL} = z$

$$V = \int dV = \int_0^b \int_0^a \left(y_0 - y_1 \frac{x}{a} - y_2 \frac{z}{b} \right) dx\,dz$$

$$= \int_0^b \left[y_0 x - y_1 \frac{x^2}{2a} - y_2 \frac{zx}{b} \right]_0^a dz = \int_0^b \left(y_0 a - \frac{1}{2} y_1 a - y_2 \frac{a}{b} z \right) dz$$

$$= \left[y_0 az - \frac{1}{2} y_1 az - y_2 \frac{a}{b} \frac{z^2}{2} \right]_0^b = \left(y_0 - \frac{1}{2} y_1 - \frac{1}{2} y_2 \right) ab$$

$$\int \bar{z}_{EL} dV = \int_0^b \int_0^a \left(y_0 z - y_1 \frac{xz}{a} - y_2 \frac{z^2}{b} \right) dx\,dz$$

$$= \int_0^b \left[y_0 zx - y_1 \frac{x^2 z}{2a} - y_2 \frac{z^2 x}{b} \right]_0^a dz = \int_0^b \left(y_0 za - y_1 \frac{za}{2} - y_2 \frac{z^2 a}{b} \right) dz$$

$$= \left[y_0 \frac{z^2 a}{2} - y_1 \frac{z^2 a}{4} - y_2 \frac{z^3 a}{3b} \right]_0^b = \left(\frac{1}{2} y_0 - \frac{1}{4} y_1 - \frac{1}{3} y_2 \right) ab^2$$

continued

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Now,

$$\bar{z} = \frac{1}{V}\int \bar{z}_{EL}\,dV = \frac{1}{\left(y_0 - \frac{1}{2}y_1 - \frac{1}{2}y_2\right)ab}\left(\frac{1}{2}y_0 - \frac{1}{4}y_1 - \frac{1}{3}y_2\right)ab^2, \text{ or}$$

$$\bar{z} = \frac{\frac{1}{2}y_0 - \frac{1}{4}y_1 - \frac{1}{3}y_2}{y_0 - \frac{1}{2}y_1 - \frac{1}{2}y_2}b \blacktriangleleft$$

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Chapter 5, Solution 129.

First note that symmetry implies $\bar{x} = 0$ ◀

Choose as the element of volume a vertical slice of width $2x$, thickness dz, and height y. Then

$$dV = 2xy\,dz, \quad \bar{y}_{EL} = \frac{1}{2}y, \quad \bar{z}_{EL} = z$$

Now $$x = \sqrt{a^2 - z^2} \qquad \text{and} \qquad y = \frac{h}{2} - \frac{h}{2a}z = \frac{h}{2}\left(1 - \frac{z}{a}\right)$$

So $$dV = h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz$$

Then $$V = \int_0^a h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz = h\left\{\frac{1}{2}\left[z\sqrt{a^2 - z^2} + a^2\sin^{-1}\left(\frac{z}{a}\right)\right] + \frac{1}{3a}\left(a^2 - z^2\right)^{3/2}\right\}\Bigg|_{-a}^{a}$$

$$= \frac{1}{2}a^2h\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= \frac{\pi}{2}a^2h$$

Then $$\int \bar{y}_{EL}dV = \int_{-a}^{a}\left[\frac{1}{2} \times \frac{h}{2}\left(1 - \frac{z}{a}\right)\right]\left[h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz\right]$$

$$= \frac{h^2}{4}\int_{-a}^{a}\sqrt{a^2 - z^2}\left(1 - 2\frac{z}{a} + \frac{z^2}{a^2}\right)dz$$

continued

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$$= \frac{h^2}{4}\left\{\frac{1}{2}\left[z\sqrt{a^2 - z^2} + a^2 \sin^{-1}\left(\frac{z}{a}\right)\right] + \left[\frac{2}{3a}\left(a^2 - z^2\right)^{\frac{3}{2}}\right]\right.$$

$$\left. + \frac{1}{a^2}\left[-\frac{z}{4}\left(a^2 - z^2\right)^{\frac{3}{2}} + \frac{a^2 z}{8}\sqrt{a^2 - z^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{z}{a}\right)\right]\right\}\Bigg|_{-a}^{a}$$

$$= \frac{5h^2a^2}{32}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

Then $\bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{\pi a^2}{2}h\right) = \frac{5h^2a^2}{32}(\pi)$

or $\bar{y} = \frac{5}{16}h$ ◀

and $\int z_{EL}dV = \int_{-a}^{a} z\left[h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz\right]$

$$= h\left\{-\frac{1}{3}\left(a^2 - z^2\right)^{\frac{3}{2}} - \frac{1}{a}\left[-\frac{z}{4}\left(a^2 - z^2\right)^{\frac{3}{2}} + \frac{a^2 z}{8}\sqrt{a^2 - z^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{z}{a}\right)\right]\right\}\Bigg|_{-a}^{a}$$

$$= -\frac{a^3h}{8}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$\bar{z}V = \int \bar{z}_{EL}dV: \quad \bar{z}\left(\frac{\pi a^2 h}{2}\right) = -\frac{\pi a^3 h}{8}$$

or $\bar{z} = -\frac{a}{4}$ ◀

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Chapter 5, Solution 130.

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$21 \times 22 = 462$	1.5	11	693	5082
2	$-\frac{1}{2}(6)(9) = -27$	-6	2	162	-54
3	$-\frac{1}{2}(6)(12) = -36$	8	2	-288	-72
Σ	399			567	4956

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{567 \text{ mm}^3}{399 \text{ mm}^2}$ or $\bar{X} = 1.421$ mm ◀

and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{4956 \text{ mm}^3}{399 \text{ mm}^2}$ or $\bar{Y} = 12.42$ mm ◀

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Chapter 5, Solution 131.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\frac{1}{3}(10)(15) = 50$	4.5	7.5	225	375
2	$\frac{\pi}{4}(15)^2 = 176.71$	6.366	16.366	1125	2892
Σ	226.71			1350	3267

Then $$\bar{X}\Sigma A = \Sigma \bar{x} A$$

$$\bar{X}\left(226.71 \text{ in}^2\right) = 1350 \text{ in}^3 \qquad \text{or} \quad \bar{X} = 5.95 \text{ in.} \blacktriangleleft$$

and $$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}\left(226.71 \text{ in}^2\right) = 3267 \text{ in}^3 \qquad \text{or} \quad \bar{Y} = 14.41 \text{ in.} \blacktriangleleft$$

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Chapter 5, Solution 132.

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	$\sqrt{12^2 + 6^2} = 13.416$	6	3	80.50	40.25
2	16	12	14	192	224
3	21	1.5	22	31.50	462
4	16	-9	14	-144	224
5	$\sqrt{6^2 + 9^2} = 10.817$	-4.5	3	-48.67	32.45
Σ	77.233			111.32	982.7

Then $$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(77.233 \text{ mm}) = 111.32 \text{ mm}^2 \qquad \text{or} \quad \bar{X} = 1.441 \text{ mm} \blacktriangleleft$$

and $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

$$\bar{Y}(77.233 \text{ mm}) = 982.7 \text{ mm}^2 \qquad \text{or} \quad \bar{Y} = 12.72 \text{ mm} \blacktriangleleft$$

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Chapter 5, Solution 133.

First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through C. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

Thus $\Sigma M_C = 0$, which implies that $\bar{x} = 0$

or $\Sigma \bar{x}_i L_i = 0$

Hence $\frac{L}{2}(L) + (-4 \text{ in.})(8 \text{ in.}) + (-4 \text{ in.})(10 \text{ in.}) = 0$

or $L^2 = 144 \text{ in}^2$ or $L = 12.00 \text{ in.}$ ◀

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Chapter 5, Solution 134.

For the element (EL) shown

At $\quad x = a, y = h: h = ka^3 \quad$ or $\quad k = \dfrac{h}{a^3}$

Then $\quad x = \dfrac{a}{h^{1/3}} y^{1/3}$

Now

$$dA = xdy$$

$$= \frac{a}{h^{1/3}} y^{1/3}\, dy$$

$$\bar{x}_{EL} = \frac{1}{2} x = \frac{1}{2} \frac{a}{h^{1/3}} y^{1/3}, \quad \bar{y}_{EL} = y$$

Then $\quad A = \int dA = \int_0^h \frac{a}{h^{1/3}} y^{1/3} dy = \frac{3}{4} \frac{a}{h^{1/3}} \left(y^{4/3} \right) \Big|_0^h = \frac{3}{4} ah$

and $\quad \int \bar{x}_{EL} dA = \int_0^h \frac{1}{2} \frac{a}{h^{1/3}} y^{1/3} \left(\frac{a}{h^{1/3}} y^{1/3} dy \right) = \frac{1}{2} \frac{a}{h^{2/3}} \left(\frac{3}{5} y^{5/3} \right) \Big|_0^h = \frac{3}{10} a^2 h$

$$\int \bar{y}_{EL} dA = \int_0^h y \left(\frac{a}{h^{1/3}} y^{1/3} dy \right) = \frac{a}{h^{1/3}} \left(\frac{3}{7} y^{7/3} \right) \Big|_0^h = \frac{3}{7} ah^2$$

Hence $\quad \bar{x}A = \int \bar{x}_{EL} dA : \bar{x} \left(\frac{3}{4} ah \right) = \frac{3}{10} a^2 h \qquad \bar{x} = \frac{2}{5} a$ ◀

$\bar{y}A = \int \bar{y}_{EL} dA : \bar{y} \left(\frac{3}{4} ah \right) = \frac{3}{7} ah^2 \qquad \bar{y} = \frac{4}{7} h$ ◀

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Chapter 5, Solution 135.

For y_1 at $x = a,\ y = 2b$ $\quad 2b = ka^2 \quad$ or $k = \dfrac{2b}{a^2}$

Then $$y_1 = \frac{2b}{a^2}x^2$$

By observation $$y_2 = -\frac{b}{a}(x + 2b) = b\left(2 - \frac{x}{a}\right)$$

Now $$\bar{x}_{EL} = x$$

and for $0 \le x \le a$:

$$\bar{y}_{EL} = \frac{1}{2}y_1 = \frac{b}{a^2}x^2 \qquad \text{and} \qquad dA = y_1 dx = \frac{2b}{a^2}x^2 dx$$

For $a \le x \le 2a$:

$$\bar{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}\left(2 - \frac{x}{a}\right) \qquad \text{and} \qquad dA = y_2 dx = b\left(2 - \frac{x}{a}\right)dx$$

Then $$A = \int dA = \int_0^a \frac{2b}{a^2}x^2 dx + \int_a^{2a} b\left(2 - \frac{x}{a}\right)dx$$

$$= \frac{2b}{a^2}\left[\frac{x^3}{3}\right]_0^a + b\left[-\frac{a}{2}\left(2 - \frac{x}{a}\right)^2\right]_0^{2a} = \frac{7}{6}ab$$

and $$\int \bar{x}_{EL} dA = \int_0^a x\left(\frac{2b}{a^2}x^2 dx\right) + \int_a^{2a} x\left[b\left(2 - \frac{x}{a}\right)dx\right]$$

$$= \frac{2b}{a^2}\left[\frac{x^4}{4}\right]_0^a + b\left[x^2 - \frac{x^3}{3a}\right]_0^{2a}$$

$$= \frac{1}{2}a^2 b + b\left\{\left[(2a)^2 - (a)^2\right] + \frac{1}{3a}\left[\left(2a^2\right) - (a)^3\right]\right\}$$

$$= \frac{7}{6}a^2 b$$

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$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{a^2}x^2\left[\frac{2b}{a^2}x^2dx\right] + \int_0^{2a}\frac{b}{2}\left(2-\frac{x}{a}\right)\left[b\left(2-\frac{x}{a}\right)dx\right]$$

$$= \frac{2b^2}{a^4}\left[\frac{x^5}{5}\right]_0^a + \frac{b^2}{2}\left[-\frac{a}{3}\left(2-\frac{x}{a}\right)^3\right]_a^{2a}$$

$$= \frac{17}{30}ab^2$$

Hence $\bar{x}A = \int \bar{x}_{EL}\,dA$: $\bar{x}\left(\frac{7}{6}ab\right) = \frac{7}{6}a^2b$ $\qquad \bar{x} = a$ ◀

$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}\left(\frac{7}{6}ab\right) = \frac{17}{30}ab^2$ $\qquad \bar{y} = \frac{17}{35}b$ ◀

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Chapter 5, Solution 136.

The volume can be generated by rotating the triangle and circular sector shown about the y axis. Applying the second theorem of Pappus-Guldinus and using Fig. 5.8A, we have

$$V = 2\pi\bar{x}A = 2\pi\Sigma\bar{x}A = 2\pi\left(\bar{x}_1A_1 + \bar{x}_2A_2\right)$$

$$= 2\pi\left[\left(\frac{1}{3}\times\frac{1}{2}R\right)\left(\frac{1}{2}\times\frac{1}{2}R\times\frac{\sqrt{3}}{2}R\right) + \left(\frac{2R\sin 30^\circ}{3\times\frac{\pi}{6}}\cos 30^\circ\right)\left(\frac{\pi}{6}R^2\right)\right]$$

$$= 2\pi\left(\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}}\right) = \frac{3\sqrt{3}}{8}\pi R^3$$

$$= \frac{3\sqrt{3}}{8}\pi\left(12\text{ in.}\right)^3 = 3526.03\text{ in}^3$$

Since $\quad 1\text{ gal} = 231\text{ in}^3$

$$V = \frac{3526.03\text{ in}^3}{231\text{ in}^3/\text{gal}} = 15.26\text{ gal}$$

$V = 15.26\text{ gal}$ ◀

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Chapter 5, Solution 137.

Have $R_{\mathrm{I}} = (9\text{ ft})(200\text{ lb/ft}) = 1800\text{ lb}$

$$R_{\mathrm{II}} = \frac{1}{2}(3\text{ ft})(200\text{ lb/ft}) = 300\text{ lb}$$

Then $\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(4.5\text{ ft})(1800\text{ lb}) - (10\text{ ft})(300\text{ lb}) + (9\text{ ft})B_y = 0$$

or $B_y = 1233.3\text{ lb}$ $\qquad$ $\mathbf{B} = 1233\text{ lb}\uparrow$ ◀

$$\uparrow \Sigma F_y = 0: \quad A_y - 1800\text{ lb} - 300\text{ lb} + 1233.3\text{ lb} = 0$$

or $A_y = 866.7\text{ lb}$ $\qquad$ $\mathbf{A} = 867\text{ lb}\uparrow$ ◀

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Chapter 5, Solution 138.

Have $$R_{\mathrm{I}} = \frac{1}{3}(4\text{ m})(2000\text{ kN/m}) = 2667\text{ N}$$

$$R_{\mathrm{II}} = \frac{1}{3}(2\text{ m})(1000\text{ kN/m}) = 666.7\text{ N}$$

Then $\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y - 2667\text{ N} - 666.7\text{ N} = 0$

or $A_y = 3334\text{ N}$ $\qquad \mathbf{A} = 3.33\text{ kN}\uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (1\text{ m})(2667\text{ N}) - (5.5\text{ m})(666.7\text{ N})$

or $M_A = 6334\text{ N}\cdot\text{m}$ $\qquad \mathbf{M}_A = 6.33\text{ kN}\cdot\text{m}\circlearrowleft$ ◀

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Chapter 5, Solution 139.

Consider the free-body diagram of the side.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma d)$$

Now $$+\circlearrowleft \Sigma M_A = 0: \quad (9\text{ ft})T - \frac{d}{3}P = 0$$

Then, for $d_{\max}$:

$$(9\text{ ft})\left[(0.2)\left(40\times10^3\text{ lb}\right)\right] - \frac{d_{\max}}{3}\left\{\frac{1}{2}\left[(12\text{ ft})(d_{\max})\right]\left(62.4\text{ lb/ft}^3\right)d_{\max}\right\} = 0$$

or $$216\times10^3\text{ ft}^3 = 374.4\,d_{\max}^3$$

or $$d_{\max}^3 = 576.92\text{ ft}^3 \qquad d_{\max} = 8.32\text{ ft} \blacktriangleleft$$

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Chapter 5, Solution 140.

First, assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.

	V, in^3	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}V$, in^4	$\bar{y}V$, in^4
I	$(4)(3.6)(0.75) = 10.8$	2.0	0.375	21.6	4.05
II	$(2.4)(2.0)(0.6) = 2.88$	3.7	1.95	10.656	5.616
III	$\pi(0.45)^2(0.4) = 0.2545$	4.2	2.15	1.0688	0.54711
IV	$-\pi(0.5)^2(0.75) = -0.5890$	1.2	0.375	-0.7068	-0.22089
Σ	13.3454			32.618	9.9922

Have

$$\bar{X}\Sigma V = \Sigma \bar{x} V$$

$$\bar{X}\left(13.3454 \text{ in}^3\right) = 32.618 \text{ in}^4 \qquad \text{or } \bar{X} = 2.44 \text{ in.} \blacktriangleleft$$

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Chapter 5, Solution 141.

First, assume that the sheet metal is homogeneous so that the center of gravity of the bracket coincides with the centroid of the corresponding area. Then (see diagram)

$$\bar{z}_V = 22.5 - \frac{4(6.25)}{3\pi}$$
$$= 19.85 \text{ mm}$$
$$A_V = -\frac{\pi}{2}(6.25)^2$$
$$= -61.36 \text{ mm}^2$$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	$(25)(60) = 1500$	12.5	0	30	18 750	0	45 000
II	$(12.5)(60) = 750$	25	−6.25	30	18 750	−4687.5	22 500
III	$(7.5)(60) = 450$	28.75	−12.5	30	12 937.5	−5625	13 500
IV	$-(12.5)(30) = -375$	10	0	37.5	−3750	0	−14 062.5
V	−61.36	10	0	19.85	−613.6	0	−1218.0
Σ	2263.64				46 074	−10 313	65 720

Have

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$
$$\bar{X}\left(2263.64 \text{ mm}^2\right) = 46\ 074 \text{ mm}^3 \qquad \text{or } \bar{X} = 20.4 \text{ mm} ◀$$

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$
$$\bar{Y}\left(2263.64 \text{ mm}^2\right) = -10\ 313 \text{ mm}^3 \qquad \text{or } \bar{Y} = -4.55 \text{ mm} ◀$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A$$
$$\bar{Z}\left(2263.64 \text{ mm}^2\right) = 65\ 720 \text{ mm}^3 \qquad \text{or } \bar{Z} = 29.0 \text{ mm} ◀$$

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