Chapter 10, Solution 1.

Link *ABC*:

Assume $\delta\theta$ clockwise

Then, for point *C*

$$\delta x_C = (125\text{ mm})\delta\theta \longrightarrow$$

and for point *D*

$$\delta x_D = \delta x_C = (125\text{ mm})\delta\theta \longrightarrow$$

and for point *E*

$$\delta x_E = \left(\frac{250\text{ mm}}{375\text{ mm}}\right)\delta x_D = \frac{2}{3}\delta x_D \longrightarrow$$

Link *DEFG*: $\delta x_D = (375\text{ mm})\delta\phi$

Thus $(125\text{ mm})\delta\theta = (375\text{ mm})\delta\phi$

$$\delta\phi = \frac{1}{3}\delta\theta$$

Then $\delta_G = \left(100\sqrt{2}\text{ mm}\right)\delta\phi = \left(\frac{100}{3}\sqrt{2}\text{ mm}\right)\delta\theta$

$$\delta y_G = \delta_G \cos 45° = \left(\frac{100}{3}\sqrt{2}\text{ mm}\right)\delta\theta\cos 45° = \left(\frac{100}{3}\text{ mm}\right)\delta\theta$$

Virtual Work: Assume **P** acts downward at *G*

$\delta U = 0$: $(9000\text{ N}\cdot\text{mm})\delta\theta - (180\text{ N})(\delta x_E\text{ mm}) + P(\delta y_G\text{ mm}) = 0$

$$9000\delta\theta - 180\left(\frac{2}{3}\times 125\delta\theta\right) + P\left(\frac{100}{3}\delta\theta\right) = 0$$

or $\mathbf{P} = 180.0\text{ N}\downarrow$ ◀

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Chapter 10, Solution 2.

Have $\mathbf{F}_A = 20 \text{ lb} \uparrow$ at A

$\mathbf{F}_D = 30 \text{ lb} \rightarrow$ at D

Link ABC: $\delta y_A = (16 \text{ in.})\delta\theta$

$\delta y_B = (10 \text{ in.})\delta\theta$

Link BF: $\delta y_F = \delta y_B$

Link $DEFG$: $\delta y_F = (6 \text{ in.})\delta\phi = (10 \text{ in.})\delta\theta$

or $\delta\phi = \dfrac{5}{3}\delta\theta$

$\delta y_G = (12 \text{ in.})\delta\phi = (20 \text{ in.})\delta\theta$

$d_{ED} = \sqrt{(5.5 \text{ in.})^2 + (4.8 \text{ in.})^2} = 7.3 \text{ in.}$

$\delta_D = (7.3 \text{ in.})\delta\phi = \left(\dfrac{5}{3} \times 7.3 \text{ in.}\right)\delta\theta$

$\delta x_D = \dfrac{4.8}{7.3}\,\delta_D = \dfrac{4.8}{7.3}\left(\dfrac{5}{3} \times 7.3 \text{ in.}\right)\delta\theta = (8 \text{ in.})\delta\theta$

Virtual Work: Assume **P** acts upward at G

$\delta U = 0$: $F_A\delta y_A + F_D\delta x_D + P\delta y_G = 0$

or $(20 \text{ lb})\left[(16 \text{ in.})\delta\theta\right] + (30 \text{ lb})\left[(8 \text{ in.})\delta\theta\right] + P\left[(20 \text{ in.})\delta\theta\right] = 0$

or $P = -28.0 \text{ lb}$

$\mathbf{P} = 28.0 \text{ lb} \downarrow$ ◀

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Chapter 10, Solution 3.

Link *ABC*:

Assume $\delta\theta$ clockwise

Then, for point *C*

$$\delta x_C = (125\text{ mm})\,\delta\theta \longrightarrow$$

and for point *D*

$$\delta x_D = \delta x_C = (125\text{ mm})\,\delta\theta \longrightarrow$$

and for point *E*

$$\delta x_E = \left(\frac{250\text{ mm}}{375\text{ mm}}\right)\delta x_D = \frac{2}{3}\delta x_D \longrightarrow$$

Link *DEFG*: $\delta x_D = (375\text{ mm})\,\delta\phi$

Thus $(125\text{ mm})\,\delta\theta = (375\text{ mm})\,\delta\phi$

or $\delta\phi = \frac{1}{3}\delta\theta$

Virtual Work: Assume **M** acts clockwise on link *DEFG*

$\delta U = 0$: $(9000\text{ N}\cdot\text{mm})\,\delta\theta - (180\text{ N})(\delta x_E\text{ mm}) + M\,\delta\phi = 0$

$$9000\,\delta\theta - 180\left(\frac{2}{3}\cdot 125\,\delta\theta\right) + M\left(\frac{1}{3}\delta\theta\right) = 0$$

or $M = 18000\text{ N}\cdot\text{mm}$

or $\mathbf{M} = 18.00\text{ N}\cdot\text{m}$ ◀

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Chapter 10, Solution 4.

Have $\mathbf{F}_A = 20 \text{ lb} \uparrow$ at A

$\mathbf{F}_D = 30 \text{ lb} \rightarrow$ at D

Link ABC: $\delta y_A = (16 \text{ in.})\delta\theta$

$\delta y_B = (10 \text{ in.})\delta\theta$

Link BF: $\delta y_F = \delta y_B$

Link $DEFG$: $\delta y_F = (6 \text{ in.})\delta\phi = (10 \text{ in.})\delta\theta$

or $\delta\phi = \dfrac{5}{3}\delta\theta$

$$d_{ED} = \sqrt{(5.5 \text{ in.})^2 + (4.8 \text{ in.})^2} = 7.3 \text{ in.}$$

$$\delta_D = (7.3 \text{ in.})\delta\phi = \left(\frac{5}{3} \times 7.3 \text{ in.}\right)\delta\theta$$

$$\delta x_D = \frac{4.8}{7.3}\delta_D = \frac{4.8}{7.3}\left(\frac{5}{3} \times 7.3 \text{ in.}\right)\delta\theta = (8 \text{ in.})\delta\theta$$

Virtual Work: Assume $\mathbf{M}$ acts $\circlearrowleft$ on $DEFG$

$\delta U = 0$: $F_A\delta y_A + F_D\delta x_D + M\delta\phi = 0$

or $(20 \text{ lb})\left[(16 \text{ in.})\delta\theta\right] + (30 \text{ lb})\left[(8 \text{ in.})\delta\theta\right] + M\left(\dfrac{5}{3}\delta\theta\right) = 0$

or $M = -336.0 \text{ lb}\cdot\text{in.}$

$\mathbf{M} = 28.0 \text{ lb}\cdot\text{ft} \circlearrowright$ ◀

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Chapter 10, Solution 5.

Assume $\delta\theta$

$$\delta x_A = 10\,\delta\theta \text{ in.} \leftarrow$$

$$\delta y_C = 4\,\delta\theta \text{ in.} \downarrow$$

$$\delta y_D = \delta y_C = 4\,\delta\theta \text{ in.} \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta$$

$$\delta x_G = 15\,\delta\phi = 15\left(\frac{2}{3}\delta\theta\right) = 10\,\delta\theta \text{ in.} \rightarrow$$

Virtual Work: Assume that force **P** is applied at A.

$\delta U = 0$:

$$\delta U = -P\delta x_A + 30\,\delta y_C + 60\,\delta y_D + 240\,\delta\phi + 80\,\delta x_G = 0$$

$$-P(10\ \delta\theta \text{ in.}) + (30 \text{ lb})(4\,\delta\theta \text{ in.}) + (60 \text{ lb})(4\ \delta\theta) + (240 \text{ lb}\cdot\text{in.})\left(\frac{2}{3}\delta\theta\right)$$

$$+(80 \text{ lb})(10\ \delta\theta \text{ in.}) = 0$$

$$-10P + 120 + 240 + 160 + 800 = 0$$

$$10P = 1320$$

$$\mathbf{P} = 132.0 \text{ lb} \rightarrow \blacktriangleleft$$

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Chapter 10, Solution 6.

Note:

$$x_E = 2x_D \qquad \delta x_E = 2\delta x_D$$

$$x_G = 3x_D \qquad \delta x_G = 3\delta x_D$$

$$x_H = 4x_D \qquad \delta x_H = 4\delta x_D$$

$$x_I = 5x_D \qquad \delta x_I = 5\delta x_D$$

(*a*) Virtual Work: $\delta U = 0$: $F_G\,\delta x_G - F_{SP}\delta x_I = 0$

$$(90\text{ N})(3\delta x_D) - F_{SP}(5\delta x_D) = 0$$

or $F_{SP} = 54.0\text{ N}$ ◀

Now $F_{SP} = k\Delta x_I$

$$54\text{ N} = (720\text{ N/m})\Delta x_I$$

$$\Delta x_I = 0.075\text{ m}$$

and $\delta x_D = \frac{1}{4}\delta x_H = \frac{1}{5}\delta x_I$

$$\Delta x_H = \frac{4}{5}\Delta x_I = \frac{4}{5}(0.075\text{ m}) = 0.06\text{ m}$$

or $\Delta x_H = 60.0\text{ mm} \longrightarrow$ ◀

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(*b*) Virtual Work: $\delta U = 0$: $F_G\,\delta x_G + F_H\,\delta x_H - F_{SP}\left(\delta x_I\right) = 0$

$$\left(90\text{ N}\right)\left(3\,\delta x_D\right) + \left(90\text{ N}\right)\left(4\,\delta x_D\right) - F_{SP}\left(5\,\delta x_D\right) = 0$$

or $F_{SP} = 126.0\text{ N}$ ◀

Now $F_{SP} = k\Delta x_I$

$$126.0\text{ N} = \left(720\text{ N/m}\right)\Delta x_I$$

$$\Delta x_I = 0.175\text{ m}$$

From Part (*a*) $\Delta x_H = \frac{4}{5}\Delta x_I = \frac{4}{5}\left(0.175\text{ m}\right) = 0.140\text{ m}$

or $\Delta x_H = 140.0\text{ mm} \rightarrow$ ◀

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Chapter 10, Solution 7.

Note:

$$x_E = 2x_D \qquad \delta x_E = 2\,\delta x_D$$

$$x_G = 3x_D \qquad \delta x_G = 3\,\delta x_D$$

$$x_H = 4x_D \qquad \delta x_H = 4\,\delta x_D$$

$$x_I = 5x_D \qquad \delta x_I = 5\,\delta x_D$$

(a) Virtual Work: $\delta U = 0$: $F_E\,\delta x_E - F_{SP}\,\delta x_I = 0$

$$(90\text{ N})(2\,\delta x_D) - F_{SP}(5\,\delta x_D) = 0$$

or $F_{SP} = 36.0\text{ N}$ ◀

Now $F_{SP} = k\Delta x_I$

$$36\text{ N} = (720\text{ N/m})\Delta x_I$$

$$\Delta x_I = 0.050\text{ m}$$

and $\delta x_D = \frac{1}{4}\delta x_H = \frac{1}{5}\delta x_I$

$$\Delta x_H = \frac{4}{5}\Delta x_I = \frac{4}{5}(0.050\text{ m}) = 0.04\text{ m}$$

or $\Delta x_H = 40.0\text{ mm} \longrightarrow$ ◀

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(*b*) Virtual Work: $\delta U = 0$: $F_D\,\delta x_D + F_E \delta x_E - F_{SP}\delta x_I = 0$

$$(90\text{ N})\delta x_D + (90\text{ N})(2\,\delta x_D) - F_{SP}(5\,\delta x_D) = 0$$

or $F_{SP} = 54.0\text{ N}$ ◀

Now $F_{SP} = k\Delta x_I$

$$54\text{ N} = (720\text{ N/m})\Delta x_I$$

$$\Delta x_I = 0.075\text{ m}$$

From Part (*a*) $\Delta x_H = \frac{4}{5}\Delta x_I = \frac{4}{5}(0.075) = 0.06\text{ m}$

or $\Delta x_H = 60.0\text{ mm} \longrightarrow$ ◀

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Chapter 10, Solution 8.

Assume $\delta y_A \uparrow$

$$\frac{\delta y_A}{16 \text{ in.}} = \frac{\delta y_C}{8 \text{ in.}}; \qquad \delta y_C = \frac{1}{2}\delta y_A \downarrow$$

Bar *CFDE* moves in translation

$$\delta y_E = \delta y_F = \delta y_C = \frac{1}{2}\delta y_A$$

Virtual Work:

$$\delta U = 0: \quad -P(\delta y_A \text{ in.}) + (100 \text{ lb})(\delta y_E \text{ in.}) + (150 \text{ lb})(\delta y_F \text{ in.}) = 0$$

$$-P\delta y_A + 100\left(\frac{1}{2}\delta y_A\right) + 150\left(\frac{1}{2}\delta y_A\right) = 0$$

$$P = 125 \text{ lb}$$

$\mathbf{P} = 125 \text{ lb} \downarrow$ ◀

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Chapter 10, Solution 9.

Have $\quad y_A = 2l\cos\theta; \qquad \delta y_A = -2l\sin\theta\,\delta\theta$

$$CD = 2l\sin\frac{\theta}{2}; \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$Q = 2P\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)} \blacktriangleleft$$

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Chapter 10, Solution 10.

Virtual Work:

Have $\quad x_A = 2l\sin\theta$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

and $\quad y_F = 3l\cos\theta$

$$\delta y_F = -3l\sin\theta\,\delta\theta$$

Virtual Work: $\quad \delta U = 0: \quad Q\delta x_A + P\delta y_F = 0$

$$Q\left(2l\cos\theta\,\delta\theta\right) + P\left(-3l\sin\theta\,\delta\theta\right) = 0$$

$$Q = \frac{3}{2}P\tan\theta \blacktriangleleft$$

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Chapter 10, Solution 11.

Virtual Work:

We note that the virtual work of A_x, A_y and C is zero, since A is fixed and C is $\perp$ to δx_C.

$$\delta U = 0: \qquad P\delta x_D + Q\delta y_D = 0$$

$$x_D = 3l\cos\theta \qquad \delta x_D = -3l\sin\theta\,\delta\theta$$

$$y_D = l\sin\theta \qquad \delta y_D = l\cos\theta\,\delta\theta$$

Thus: $P\left(-3l\,\sin\theta\,\delta\theta\right) + Q\left(l\cos\theta\,\delta\theta\right) = 0$

$$-3Pl\sin\theta + Ql\cos\theta = 0$$

$$Q = \frac{3P\sin\theta}{\cos\theta} = 3P\tan\theta \qquad Q = 3P\tan\theta \blacktriangleleft$$

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Chapter 10, Solution 12.

$$x_A = (a+b)\cos\theta \qquad \delta x_A = -(a+b)\sin\theta\,\delta\theta$$

$$y_G = a\sin\theta \qquad \delta y_G = a\cos\theta\,\delta\theta$$

Virtual Work:

The reactions at A and B are perpendicular to the displacements of A and B hence do no work.

$\delta U = 0$: $\quad T\delta x_A + W\delta y_G = 0$

$$T\left[-(a+b)\sin\theta\,\delta\theta\right] + W\left(a\cos\theta\,\delta\theta\right) = 0$$

$$-T(a+b)\sin\theta + Wa\cos\theta = 0$$

$$T = \frac{a}{a+b}W\cot\theta \qquad\qquad T = \frac{a}{a+b}W\cot\theta \blacktriangleleft$$

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Chapter 10, Solution 13.

Note: $y_H = 2l\sin\theta$

Where $l = 600 \text{ mm}$ (length of a link)

Then $\delta y_H = 2l\cos\theta\,\delta\theta$

Also $\frac{1}{2}W = \frac{1}{2}mg = \frac{1}{2}(450 \text{ kg})(9.81 \text{ m/s}^2)$

$= 2207.3 \text{ N}$

$$d_{AF} = \sqrt{\left(\frac{3}{4}l\cos\theta\right)^2 + \left(\frac{5}{4}l\sin\theta\right)^2}$$

$$= \frac{l}{4}\sqrt{9 + 16\sin^2\theta}$$

$$\delta d_{AF} = \frac{l}{4}\frac{16\sin\theta\cos\theta}{\sqrt{9+16\sin^2\theta}}\delta\theta$$

$$= 4l\frac{\sin\theta\cos\theta}{\sqrt{9+16\sin^2\theta}}\delta\theta$$

Virtual Work: $\delta U = 0$: $F_{\text{cyl}}\,\delta d_{AF} - \left(\frac{1}{2}W\right)\delta y_H = 0$

$$F_{\text{cyl}}\left(4l\frac{\sin\theta\cos\theta}{\sqrt{9+16\sin^2\theta}}\delta\theta\right) - (2.2073 \text{ kN})(2l\cos\theta\,\delta\theta) = 0$$

$$F_{\text{cyl}}\frac{\sin\theta}{\sqrt{9+16\sin^2\theta}} = 1.10365 \text{ kN}$$

For $\theta = 30°$ $F_{\text{cyl}}\frac{\sin 30°}{\sqrt{9+16\sin^2 30°}} = 1.10365 \text{ kN}$

or $F_{\text{cyl}} = 7.96 \text{ kN}$ ◀

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Chapter 10, Solution 14.

From solution of Problem 10.13:

$$F_{\text{cyl}} \frac{\sin\theta}{\sqrt{9 + 16\sin^2\theta}} = 1.10365 \text{ kN}$$

Then for $F_{\text{cyl}} = 35 \text{ kN}$

$$(35 \text{ kN}) \frac{\sin\theta}{\sqrt{9 + 16\sin^2\theta}} = 1.10365 \text{ kN}$$

$$(31.713 \sin\theta)^2 = 9 + 16\sin^2\theta$$

$$\sin^2\theta = \frac{9}{989.71}$$

or $\theta = 5.47°$ ◀

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Chapter 10, Solution 15.

ABC: $y_B = a\sin\theta \Rightarrow \quad \delta y_B = a\cos\theta\delta\theta$

$y_C = 2a\sin\theta \Rightarrow \delta y_C = 2a\cos\theta\delta\theta$

CDE: Note that as *ABC* rotates counterclockwise, *CDE* rotates clockwise while it moves to the left.

Then $\delta y_C = a\delta\phi$

or $2a\cos\theta\delta\theta = a\delta\phi$

or $\delta\phi = 2\cos\theta\delta\theta$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_B - P\delta y_C + M\delta\phi = 0$$

$$-P\left(a\cos\theta\delta\theta\right) - P\left(2a\cos\theta\delta\theta\right) + M\left(2\cos\theta\delta\theta\right) = 0$$

or $M = \dfrac{3}{2}Pa$ ◀

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Chapter 10, Solution 16.

First note $l\sin\theta + \frac{3}{2}l\sin\phi = l$

or $\sin\phi = \frac{2}{3}(1-\sin\theta)$

Then $\cos\phi\,\delta\phi = -\frac{2}{3}\cos\theta\,\delta\theta$

or $\delta\phi = -\frac{2}{3}\frac{\cos\theta}{\cos\phi}\delta\theta$

$= \sqrt{5 + 8\sin\theta - 4\sin^2\theta}$

Now $x_C = -l\cos\theta + \frac{3}{2}l\cos\phi$

Then
$$\delta x_C = l\sin\theta\,\delta\theta - \frac{3}{2}l\sin\phi\,\delta\phi$$
$$= l\left[\sin\theta - \frac{3}{2}\sin\phi\left(\frac{-2}{3}\frac{\cos\theta}{\cos\phi}\right)\right]\delta\theta$$
$$= l\left(\sin\theta + \cos\theta\tan\phi\right)\delta\theta$$
$$= l\left[\sin\theta + \frac{2\cos\theta(1-\sin\theta)}{\sqrt{5+8\sin\theta-4\sin^2\theta}}\right]\delta\theta$$

Virtual Work:

$$\delta U = 0:\ M\delta\theta - P\delta x_C = 0$$
$$M\delta\theta - Pl\left[\sin\theta + \frac{2\cos\theta(1-\sin\theta)}{\sqrt{5+8\sin\theta-4\sin^2\theta}}\right]\delta\theta = 0$$

or $M = Pl\left[\sin\theta + \frac{2\cos\theta(1-\sin\theta)}{\sqrt{5+8\sin\theta-4\sin^2\theta}}\right]$ ◀

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Chapter 10, Solution 17.

Have

$$x_B = l\sin\theta$$

$$\delta x_B = l\cos\theta\delta\theta$$

$$y_A = l\cos\theta$$

$$\delta y_A = -l\sin\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad M\delta\theta - P\delta x_B + P\delta y_A = 0$$

$$M\delta\theta - P\left(l\cos\theta\delta\theta\right) + P\left(-l\sin\theta\delta\theta\right) = 0$$

$$M = Pl\left(\sin\theta + \cos\theta\right) \blacktriangleleft$$

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Chapter 10, Solution 18.

Have

$$x_D = l\cos\theta$$

$$\delta x_D = -l\sin\theta\delta\theta$$

$$y_D = 3l\sin\theta$$

$$\delta y_D = 3l\cos\theta\delta\theta$$

Virtual Work: $\delta U = 0$: $M\delta\theta - (P\cos\beta)\delta x_D - (P\sin\beta)\delta y_D = 0$

$$M\delta\theta - (P\cos\beta)(-l\sin\theta\delta\theta) - (P\sin\beta)(3l\cos\theta\delta\theta) = 0$$

$$M = Pl(3\sin\beta\cos\theta - \cos\beta\sin\theta) \qquad (1)$$

(*a*) For **P** directed along BCD, $\beta = \theta$

Equation (1): $M = Pl(3\sin\theta\cos\theta - \cos\theta\sin\theta)$

$M = Pl(2\sin\theta\cos\theta)$ $\qquad M = Pl\sin 2\theta$ ◀

(*b*) For **P** directed ↓, $\beta = 90°$

Equation (1): $M = Pl(3\sin 90°\cos\theta - \cos 90°\sin\theta)$

$M = 3Pl\cos\theta$ ◀

(*c*) For **P** directed →, $\beta = 180°$

Equation (1): $M = Pl(3\sin 180°\cos\theta - \cos 180°\sin\theta)$

$M = Pl\sin\theta$ ◀

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Chapter 10, Solution 19.

Analysis of the geometry:

Law of Sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1) $\qquad \cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$

or $\qquad \delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or $\qquad \delta x_C = -\frac{AB}{\cos\phi}\left(\sin\theta\cos\phi + \sin\phi\cos\theta\right)\delta\theta$

Then $\qquad \delta x_C = -\frac{AB\sin\left(\theta+\phi\right)}{\cos\phi}\delta\theta$

continued

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Virtual Work: $\delta U = 0: \quad -P\delta x_C - M\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

For the given conditions: $P = 1.0$ kip $= 1000$ lb, $AB = 2.5$ in., and $BC = 10$ in.:

(a) When $\theta = 30°: \quad \sin\phi = \frac{2.5}{10}\sin 30°, \qquad \phi = 7.181°$

$$M = (2.5 \text{ in.})\frac{\sin(30° + 7.181°)}{\cos 7.181°}(1.0 \text{ kip}) = 1.5228 \text{ kip}\cdot\text{in.}$$

$$= 0.1269 \text{ kip}\cdot\text{ft}$$

or $\mathbf{M} = 126.9 \text{ lb}\cdot\text{ft}$ ◀

(b) When $\theta = 150°: \quad \sin\phi = \frac{2.5}{10}\sin 150°, \qquad \phi = 7.181°$

$$M = (2.5 \text{ in.})\frac{\sin(150° + 7.181°)}{\cos 7.181°}(1.0 \text{ kip}) = 0.97722 \text{ kip}\cdot\text{in.}$$

or $\mathbf{M} = 81.4 \text{ lb}\cdot\text{ft}$ ◀

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Chapter 10, Solution 20.

From the analysis of Problem 10.19, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P$$

Now, with $M = 75\text{ lb}\cdot\text{ft} = 900\text{ lb}\cdot\text{in.}$

(a) For $\theta = 60°$

$$\sin\phi = \frac{2.5}{10}\sin 60°, \qquad \phi = 12.504°$$

$$(900\text{ lb}\cdot\text{in.}) = (2.5\text{ in.})\frac{\sin(60° + 12.504°)}{\cos 12.504°}(P)$$

or $$P = 368.5\text{ lb}$$

$\mathbf{P} = 369\text{ lb} \longrightarrow$ ◀

(b) For $\theta = 120°$

$$\sin\phi = \frac{2.5}{10}\sin 120°, \qquad \phi = 12.504°$$

$$(900\text{ lb}\cdot\text{in.}) = (2.5\text{ in.})\frac{\sin(120° + 12.504°)}{\cos 12.504°}(P)$$

or $$P = 476.7\text{ lb}$$

$\mathbf{P} = 477\text{ lb} \longrightarrow$ ◀

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Chapter 10, Solution 21.

Consider a virtual rotation $\delta\phi$ of link AB.

Then $\delta_B = a\delta\phi$

Note that $\delta y_B = \delta_B \cos\theta = a\cos\theta\delta\phi$

Disregarding the second-order rotation of link BC,

$$\delta y_C = \delta y_B = a\cos\theta\delta\phi$$

Then $\delta_C = \dfrac{\delta y_C}{\sin\theta} = \dfrac{a\cos\theta\delta\phi}{\sin\theta} = \dfrac{a}{\tan\theta}\delta\phi$

Virtual Work: $\delta U = 0$: $\quad M\delta\phi - P\delta_C = 0$

$$M\delta\phi - P\left(\frac{a}{\tan\theta}\delta\phi\right) = 0$$

or $M\tan\theta = Pa$

Thus $(27\text{ N}\cdot\text{m})\tan 30° = P(0.45\text{ m})$

$$P = 34.6\text{ N}$$

P = 34.6 N ◺ 30.0° ◀

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Chapter 10, Solution 22.

Consider a virtual rotation $\delta\phi$ ↻ of link AB.

Then $\delta_B = a\,\delta\phi$

Note that $\delta y_B = \delta_B \cos\theta = a\cos\theta\,\delta\phi$

Disregarding the second-order rotation of link BC,

$$\delta y_C = \delta y_B = a\cos\theta\,\delta\phi$$

Then $\delta_C = \dfrac{\delta y_C}{\sin\theta} = \dfrac{a\cos\theta\,\delta\phi}{\sin\theta} = \dfrac{a}{\tan\theta}\delta\phi$

Virtual Work: $\delta U = 0$: $\quad M\,\delta\phi - P\delta_C = 0$

$$M\,\delta\phi - P\left(\frac{a}{\tan\theta}\delta\phi\right) = 0$$

or $M\tan\theta = Pa$

Thus $M\tan 40° = (135\text{ N})(0.60\text{ m})$

$$M = 96.53\text{ N}\cdot\text{m}$$

$\mathbf{M} = 96.5\text{ N}\cdot\text{m}$ ↻ ◀

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Chapter 10, Solution 23.

From geometry

$$y_A = 2l\cos\theta, \qquad \delta y_A = -2l\sin\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}, \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or

$$Q = 2P\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

With

$$P = 60 \text{ lb}, \qquad Q = 75 \text{ lb}$$

$$(75 \text{ lb}) = 2(60 \text{ lb})\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

$$\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

or

$$\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

$$\theta = 36.42°$$

$\theta = 36.4°$ ◀

(Additional solutions discarded as not applicable are $\theta = \pm 180°$)

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Chapter 10, Solution 24.

From the solution to Problem 10.16

$$M = Pl\left[\sin\theta + \frac{2\cos\theta(1-\sin\theta)}{\sqrt{5+8\sin\theta-4\sin^2\theta}}\right]$$

Substituting

$$13.5\ \text{N}\cdot\text{m} = (60\ \text{N})(0.25\ \text{m})\left[\sin\theta + \frac{2\cos\theta(1-\sin\theta)}{\sqrt{5+8\sin\theta-4\sin^2\theta}}\right]$$

$$\text{or}\quad \sin\theta + \frac{2\cos\theta(1-\sin\theta)}{\sqrt{5+8\sin\theta-4\sin^2\theta}} = 0.90$$

Solving numerically $\theta = 57.5°$ ◀

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Chapter 10, Solution 25.

Geometry

$$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta; \qquad \delta y_A = -l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\left(-\delta y_A\right) - Q\delta x_B = 0$$

$$Pl\sin\theta\,\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \qquad (1)$$

Then, with $l = 15$ in., $r = 4.5$ in., $P = 15$ lb, and $Q = 30$ lb

$$\cos^2\theta = \frac{(30\text{ lb})(4.5\text{ in.})}{(15\text{ lb})(15\text{ in.})} = 0.6$$

or $\theta = 39.231°$ $\qquad \theta = 39.2°$ ◀

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Chapter 10, Solution 26.

Geometry

$$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta; \qquad \delta y_A = -l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\,\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \qquad (1)$$

Then, with $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb

$$\cos^2\theta = \frac{(150\text{ lb})(5\text{ in.})}{(75\text{ lb})(14\text{ in.})} = 0.7143$$

or $\theta = 32.3115°$ $\qquad \theta = 32.3°$ ◀

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Chapter 10, Solution 27.

We have $\quad x_A = (a+b)\cos\theta \qquad \delta x_A = -(a+b)\sin\theta\,\delta\theta$

$\quad y_G = a\sin\theta \qquad \delta y_A = a\cos\theta\,\delta\theta$

Virtual Work:

The reactions at A and B are perpendicular to the displacements of A and B hence do no work.

$$\delta U = 0:\ T\delta x_A + W\delta y_G = 0$$

$$T\left[-(a+b)\sin\theta\,\delta\theta\right] + W\left(a\cos\theta\,\delta\theta\right) = 0$$

$$-T(a+b)\sin\theta + Wa\cos\theta = 0$$

or $\quad T = \dfrac{a}{a+b}W\cot\theta$

We have $\quad \sin\theta = \dfrac{BD}{AB} = \dfrac{BD}{a+b} = \dfrac{(42\text{ in.})}{(42\text{ in.}) + (28\text{ in.})}$

$$\sin\theta = 0.600$$

$$\theta = 36.87°$$

Thus $\quad T = \dfrac{(42\text{ in.})}{(42\text{ in.}) + (28\text{ in.})}(160\text{ lb})\cot 36.87°$

$$= 127.99\text{ lb}$$

$T = 128.0$ lb ◀

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Chapter 10, Solution 28.

Note that

$$y_A = -(0.15\text{ m})\tan\theta$$

$$y_B = -(0.15\text{ m})\tan\theta + (0.9\text{ m})\sin\theta$$

Then

$$\delta y_A = -(0.15\text{ m})\sec^2\theta\,\delta\theta$$

$$\delta y_B = -(0.15\text{ m})\sec^2\theta\,\delta\theta + (0.9\text{ m})\cos\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad Q\delta y_A + P\delta y_B = 0$$

or $(135\text{ N})\left[-(0.15\text{ m})\sec^2\theta\,\delta\theta\right]$

$$+(75\text{ N})\left[-(0.15\text{ m})\sec^2\theta\,\delta\theta + (0.9\text{ m})\cos\theta\,\delta\theta\right] = 0$$

or $-20.25\sec^2\theta - 11.25\sec^2\theta + 67.5\cos\theta = 0$

or $-31.5 + 67.5\cos^3\theta = 0$

$$\cos^3\theta = 0.4667$$

or $\theta = 39.1°$ ◀

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Chapter 10, Solution 29.

First note

$$BC = BD \therefore \triangle BCD \text{ is isosceles}$$

Then $\angle BCD = \angle BDC = \theta$

and $BE = l\sin\theta = 2l\sin\phi$

or $\sin\phi = \dfrac{1}{2}\sin\theta$

Then $\cos\phi\,\delta\phi = \dfrac{1}{2}\cos\theta\,\delta\theta$

or $\delta\phi = \dfrac{1}{2}\dfrac{\cos\theta}{\cos\phi}\delta\theta$

Now

$$x_C = 2l\cos\phi - l\cos\theta = l\left(2\cos\phi - \cos\theta\right)$$

$$x_D = 2l\cos\phi + l\cos\theta = l\left(2\cos\phi + \cos\theta\right)$$

Then

$$\delta x_C = l\left(-2\sin\phi\,\delta\phi + \sin\theta\,\delta\theta\right) = l\left(\sin\theta - \cos\theta\tan\phi\right)\delta\theta$$

$$\delta x_D = l\left(-2\sin\phi\,\delta\phi - \sin\theta\,\delta\theta\right) = -l\left(\sin\theta + \cos\theta\tan\phi\right)\delta\theta$$

Also

$$F_{SP} = kx_{SP} = k\left(3l - x_D\right) = kl\left[3 - \left(2\cos\phi + \cos\theta\right)\right]$$

Virtual Work: $\delta U = 0$: $P\delta x_C - F_{SP}\left|\delta x_D\right| = 0$

Then $P\left[l\left(\sin\theta - \cos\theta\tan\phi\right)\delta\theta\right] - kl\left[3 - \left(2\cos\phi + \cos\theta\right)\right]l\left(\sin\theta + \cos\theta\tan\phi\right)\delta\theta = 0$

or $P = kl\left[3 - \left(2\cos\phi + \cos\theta\right)\right]\dfrac{\sin\theta + \cos\theta\tan\phi}{\sin\theta - \cos\theta\tan\phi}$

$$= kl\left[3 - \left(2\cos\phi + \cos\theta\right)\right]\frac{\tan\theta + \tan\phi}{\tan\theta - \tan\phi}$$

Now $\sin\phi = \dfrac{1}{2}\sin\theta = \dfrac{1}{2}\sin 25°$

or $\phi = 12.1991°$

$$\therefore P = \left(1600\frac{\text{N}}{\text{m}}\right)\left(0.150\text{ m}\right)\left(3 - 2\cos 12.1991° - \cos 25°\right)\frac{\tan 25° + \tan 12.1991°}{\tan 25° - \tan 12.1991°}$$

or $P = 90.9$ N ◀

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Chapter 10, Solution 30.

$$y_E = \frac{x}{3} + \frac{x}{6} = \frac{x}{2} \qquad \therefore \delta y_E = \frac{1}{2}\delta x$$

Linear Spring: $F_{SP} = ks = (5000 \text{ N/m})(x - 0.30 \text{ m})$

Virtual Work: $\delta U = 0$

$$\delta U = -F_{SP}\delta x + P\delta_E = 0$$

$$-(5000 \text{ N/m})(x - 0.30 \text{ m})\delta x + (900 \text{ N})\left(\frac{1}{2}\delta x\right) = 0$$

$$-5000x + 1500 + 450 = 0$$

$x = 0.390$ m or $x = 390$ mm ◀

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Chapter 10, Solution 31.

$$y_C = \frac{x}{6} \qquad \therefore \delta y_C = \frac{1}{6}\delta x$$

Linear Spring: $F_{SP} = ks = (5000 \text{ N/m})(x - 0.30 \text{ m})$

Virtual Work: $\delta U = 0$

$$\delta U = -F_{SP}\delta x + P\delta y_C = 0$$

$$-(5000 \text{ N/m})(x - 0.30 \text{ m})\delta x + (900 \text{ N})\left(\frac{1}{6}\delta x\right) = 0$$

$$-5000x + 1500 + 150 = 0$$

$x = 0.330$ m or $x = 330$ mm ◀

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Chapter 10, Solution 32.

First note: $y_D = (250\text{ mm})\sin\theta$ $\qquad \delta y_D = (250\text{ mm})\cos\theta\,\delta\theta$

$x_A = 2(300\text{ mm})\cos\theta$ $\qquad \delta x_A = -(600\text{ mm})\sin\theta(-\delta\theta)$

Also, the spring force $\quad F_{SP} = k\left[x_A - (x_A)_0\right]$

$$= (2.5\text{ N/mm})(600\cos\theta - 600\cos 45°)(\text{mm})$$

$$= (1500\text{ N})(\cos\theta - \cos 45°)$$

Virtual Work:

$\delta U = 0$: $\quad (250\text{ N})\delta y_D - F_{SP}\,\delta x_A = 0$

$$(250\text{ N})(250\text{ mm})\cos\theta\,\delta\theta - (1500\text{ N})(\cos\theta - \cos 45°)(600\text{ mm})\sin\theta\,\delta\theta = 0$$

or $\quad 5 - 72\tan\theta(\cos\theta - \cos 45°) = 0$

Solving numerically

$\theta = 15.03°$ and $36.9°$ ◀

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Chapter 10, Solution 33.

From geometry:

$$x_C = 2(15 \text{ in.})\cos\theta \qquad \delta x_C = -(30 \text{ in.})\sin\theta\,\delta\theta$$

$$y_B = (15 \text{ in.})\sin\theta \qquad \delta y_B = (15 \text{ in.})\cos\theta(\delta\theta)$$

$$s = (30 - 30\cos\theta)\text{in.}$$

$$= 30(1 - \cos\theta)\text{in.}$$

Then $\quad F_{SP} = ks = (12.5 \text{ lb/in.})\left[30(1 - \cos\theta)\text{in.}\right]$

$$= (375 \text{ lb})(1 - \cos\theta)$$

Virtual Work:

$\delta U = 0$: $\quad P\delta y_B + F_{SP}\,\delta x_C = 0$

or $\quad (150 \text{ lb})(15 \text{ in.})\cos\theta\,\delta\theta + \left[(375 \text{ lb})(1 - \cos\theta)\right]\left[-(30\text{in.})\sin\theta\,\delta\theta\right] = 0$

or $\quad 2250\cos\theta - 11250(1 - \cos\theta)\sin\theta = 0$

or $\quad (1 - \cos\theta)\tan\theta = 0.200$

Solving numerically, $\quad \theta = 40.22°$

$\theta = 40.2°$ ◀

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Chapter 10, Solution 34.

From geometry:

$$x_C = 2(15 \text{ in.})\cos\theta \qquad \delta x_C = -(30 \text{ in.})\sin\theta\,\delta\theta$$

$$y_B = (15 \text{ in.})\sin\theta \qquad \delta y_B = (15 \text{ in.})\cos\theta\,\delta\theta$$

$$s = (30 - 30\cos\theta)\text{in.}$$

$$= 30(1 - \cos\theta)\text{in.}$$

Then $\quad F_{SP} = ks = (12.5 \text{ lb/in.})\left[30(1 - \cos\theta)\text{in.}\right]$

$$= (375 \text{ lb})(1 - \cos\theta)$$

Virtual Work:

$\delta U = 0$: $\quad P\delta y_B + F_{SP}\,\delta x_C = 0$

or $\quad P\left[(15 \text{ in.})(\cos 25°)\delta\theta\right] + \left[(375 \text{ lb})(1 - \cos 25°)\right]\left[-(30 \text{ in.})(\sin 25°)\delta\theta\right] = 0$

or $\quad P = 32.8 \text{ lb}$ ◀

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Chapter 10, Solution 35.

$$s = r\theta$$

$$\delta s = r\,\delta\theta$$

Spring is unstretched at $\theta = 0°$

$$F_{SP} = ks = k\,r\,\theta$$

$$x_C = l\sin\theta$$

$$\delta x_C = l\cos\theta\,\delta\theta$$

Virtual Work:

$\delta U = 0$: $\quad P\delta x_C - F_{SP}\delta s = 0$

$$P\left(l\cos\theta\,\delta\theta\right) - k\,r\,\theta\left(r\,\delta\theta\right) = 0$$

or $\quad \dfrac{Pl}{kr^2} = \dfrac{\theta}{\cos\theta}$

Thus $\quad \dfrac{(40\text{ lb})(12\text{ in.})}{(9\text{ lb/in.})(5\text{ in.})^2} = \dfrac{\theta}{\cos\theta}$

or $\quad \dfrac{\theta}{\cos\theta} = 2.1333$

$$\theta = 1.054\text{ rad} = 60.39°$$

$\theta = 60.4°$ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 10, Solution 36.

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\,\delta\theta$$

Spring: $v = CD$

Unstretched when $\theta = 0$

so that $v_0 = \sqrt{2}l$

For θ:

$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$\delta v = l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta$$

Stretched length:

$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

Then $$F = ks = kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\,\delta\theta - kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta = 0$$

or $$\frac{P}{kl} = \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right)\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

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Now, with $P = 150$ lb, $l = 30$ in., and $k = 40$ lb/in.

$$\frac{(150\text{ lb})}{(40\text{ lb/in.})(30\text{ in.})} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta} = 0.61872$$

Solving numerically, $\theta = 17.825°$ $\theta = 17.83°$ ◀

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Chapter 10, Solution 37.

From geometry:

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\,\delta\theta$$

$$x_C = l\cos\theta + l\sin\theta$$

$$= l\left(\cos\theta + \sin\theta\right)$$

$$y_C = l\sin\theta - l\cos\theta$$

$$= l\left(\sin\theta - \cos\theta\right)$$

$$l_{CD} = l\sqrt{\left(\cos\theta + \sin\theta\right)^2 + \left[\left(\sin\theta - \cos\theta\right) - \left(-1\right)\right]^2}$$

$$= l\sqrt{3 + 2\sin\theta - 2\cos\theta}$$

$$\delta l_{CD} = l\frac{\cos\theta + \sin\theta}{\sqrt{3 + 2\sin\theta - 2\cos\theta}}\delta\theta$$

and

$$F_{SP} = k\left(l_{CD} - l\right)$$

$$= kl\left(\sqrt{3 + 2\sin\theta - 2\cos\theta} - 1\right)$$

Virtual Work: $\dfrac{\delta U}{\delta\theta} = 0$: $\quad P\delta y_A - F_{SP}\delta l_{CD} = 0$

or

$$P\left(l\cos\theta\,\delta\theta\right) - kl\left(\sqrt{3 + 2\sin\theta - 2\cos\theta} - 1\right)\left[l\frac{\cos\theta + \sin\theta}{\sqrt{3 + 2\sin\theta - 2\cos\theta}}\delta\theta\right] = 0$$

or

$$\left(1 - \frac{1}{\sqrt{3 + 2\sin\theta - 2\cos\theta}}\right)\left(1 + \tan\theta\right) = \frac{P}{kl}$$

$$= \frac{600\text{ N}}{\left(4000\text{ N/m}\right)\left(0.8\text{ m}\right)}$$

$$= 0.1875$$

Solving numerically $\qquad \theta = 10.77° \blacktriangleleft$

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Chapter 10, Solution 38.

Have $y_C = (375\text{ mm})\tan\theta$ $\qquad \delta y_C = (375\text{ mm})\sec^2\theta\,\delta\theta$

$$\delta_S = (75\text{ mm})\delta\theta$$

$$F_{SP} = ky_C = (0.8\text{ N/mm})(375\text{ mm})\tan\theta$$
$$= (300\text{ N})\tan\theta$$

Virtual Work:

$\delta U = 0$: $\quad P\delta_S - F_{SP}\delta y_C = 0$

$$(480\text{ N})(75\text{ mm})\delta\theta - \left[(300\text{ N})\tan\theta\right](375\text{ mm})\sec^2\theta\,\delta\theta = 0$$

or $\quad 3.125\tan\theta\sec^2\theta = 1$

Solving numerically, $\qquad \theta = 16.41°$ ◀

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Chapter 10, Solution 39.

Have $\quad y_A = l\sin\theta \qquad\qquad \delta y_A = l\cos\theta\,\delta\theta$

Virtual Work:

$\delta U = 0$: $\quad P\delta y_A - M\delta\theta = 0$

$$Pl\cos\theta\,\delta\theta - K\theta\,\delta\theta = 0$$

or $\quad \dfrac{\theta}{\cos\theta} = \dfrac{Pl}{K}$

Thus $\quad \dfrac{\theta}{\cos\theta} = \dfrac{(2000\text{ N})(0.25\text{ m})}{(225\text{ N}\cdot\text{m/rad})}$

$$\frac{\theta}{\cos\theta} = 2.2222\text{ rad}$$

Solving numerically $\qquad \theta = 61.2°$ ◀

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Chapter 10, Solution 40.

Have $\quad y_A = l\sin\theta \qquad \delta y_A = l\cos\theta\,\delta\theta$

Virtual Work:

$$\delta U = 0: \qquad P\delta y_A - M\delta\theta = 0$$

$$Pl\cos\theta\,\delta\theta - K\theta\,\delta\theta = 0$$

or $$\frac{\theta}{\cos\theta} = \frac{Pl}{K}$$

Then $$\frac{\theta}{\cos\theta} = \frac{(6300\text{ N})(0.25\text{ m})}{(225\text{ N}\cdot\text{m/rad})} = 7\text{ rad}$$

or $$\theta = 7\cos\theta$$

Plotting $y = \theta$ and $y = 7\cos\theta$ in the range $0 \le \theta \le \frac{5\pi}{2}$ reveals three points of intersection, and thus three roots:

Then, for each range

$0 < \theta < 90°\left(0 < \theta < \frac{\pi}{2}\right)$: $\quad \theta = 1.37333$ rad $\quad$ or $\quad \theta = 78.7°$ ◀

$270° < \theta < 360°\left(\frac{3\pi}{2} < \theta < 2\pi\right)$: $\quad \theta = 5.6522$ rad $\quad$ or $\quad \theta = 324°$ ◀

$360° < \theta < 450°\left(2\pi < \theta < \frac{5\pi}{2}\right)$: $\quad \theta = 6.6160$ rad $\quad$ or $\quad \theta = 379°$ ◀

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Chapter 10, Solution 41.

Have $\quad d_{AC} = \sqrt{21^2 + 7.2^2} = 22.2 \text{ in.}$

$$\tan\phi = \frac{7.2}{21} \qquad \text{or} \qquad \phi = 18.9246°$$

Now $\quad \alpha = \theta + \phi$

By Law of Cosines:

$$d_{AB}^2 = 22.2^2 + 10^2 - 2(22.2)(10)\cos\alpha$$

$$d_{AB} = \sqrt{592.84 - 444\cos\alpha}\ (\text{in.})$$

and

$$\delta d_{AB} = \frac{222\sin\alpha}{\sqrt{592.84 - 444\cos\alpha}}\delta\alpha(\text{in.})$$

By Virtual Work:

$\delta U = 0$: $\quad P\delta y_D - F_{\text{cyl}}\,\delta d_{AB} = 0$

$$(120 \text{ lb})(8 \text{ in.})\delta\alpha - F_{\text{cyl}}\left[\left(\frac{222\sin\alpha}{\sqrt{592.84 - 444\cos\alpha}}\right)\text{in.}\right]\delta\alpha = 0$$

$$F_{\text{cyl}} = \frac{4.3243}{\sin\alpha}\sqrt{592.84 - 444\cos\alpha}\ (\text{lb})$$

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Given data: $\theta = 60°$, $\alpha = 60° + 18.9246° = 78.9246°$

Thus $$F_{\text{cyl}} = \frac{4.3243}{\sin 78.9246°}\sqrt{592.84 - 444\cos 78.9246°}\,(\text{lb})$$

$$= 99.270 \text{ lb}$$

$$d_{AB} = \sqrt{592.84 - 444\cos 78.9246°} = 22.529 \text{ in.}$$

By Law of Sines:

$$\frac{10}{\sin\beta} = \frac{22.529}{\sin 78.9246°}$$

or $\beta = 25.824°$

$\mathbf{F}_{\text{cyl}} = 99.3$ lb $\measuredangle$ $44.7°$ ◀

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Chapter 10, Solution 42.

Have $\quad d_{AC} = \sqrt{21^2 + 7.2^2} = 22.2 \text{ in.}$

$$\tan\phi = \frac{7.2}{21} \qquad \text{or} \qquad \phi = 18.9246°$$

Now $\quad \alpha = \theta + \phi$

By Law of Cosines:

$$d_{AB}^2 = 22.2^2 + 10^2 - 2(22.2)(10)\cos\alpha$$

$$d_{AB} = \sqrt{592.84 - 444\cos\alpha}\ (\text{in.})$$

and

$$\delta d_{AB} = \frac{222\sin\alpha}{\sqrt{592.84 - 444\cos\alpha}}\delta\alpha(\text{in.})$$

By Virtual Work:

$\delta U = 0$: $\quad P\delta y_D - F_{\text{cyl}}\,\delta d_{AB} = 0$

$$(120 \text{ lb})(8 \text{ in.})\delta\alpha - F_{\text{cyl}}\left[\left(\frac{222\sin\alpha}{\sqrt{592.84 - 444\cos\alpha}}\right)\text{in.}\right]\delta\alpha = 0$$

$$F_{\text{cyl}} = \frac{4.3243}{\sin\alpha}\sqrt{592.84 - 444\cos\alpha}\ (\text{lb})$$

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Thus $$105 \text{ lb} = \frac{4.3243}{\sin\alpha}\sqrt{592.84 - 444\cos\alpha}\ (\text{lb})$$

$$24.28125 \sin\alpha = \sqrt{592.84 - 444\cos\alpha}$$

$$589.5791\sin^2\alpha = 592.84 - 444 \cos\alpha$$

$$589.5791\left(1 - \cos^2\alpha\right) = 592.84 - 444 \cos\alpha$$

$$\cos^2\alpha - 0.75307\cos\alpha + 0.0055309 = 0$$

Solving $\alpha = 41.785°$ and $\alpha = 89.575°$

$$\theta = \alpha - \phi = \alpha - 18.9246°$$

Thus $\theta = 22.9°$ and $\theta = 70.7°$ ◀

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Chapter 10, Solution 43.

First note $\theta + \beta = 90°$

$\alpha + \beta = 90° \quad \therefore \alpha = \theta$

and $s = a\theta$ (length of cord unwound for rotation θ)

Now $y_O = a(1 - \cos\theta)$ (Distance O moves down for rotation θ)

and $y_P = s + y$

or $y_P = a\theta + a(1 - \cos\theta)$ (Distance P moves down for rotation θ)

Then $\delta y_P = (a + a\sin\theta)\,\delta\theta$

By the Law of Cosines

$$l_{SP}^2 = (4a)^2 + (2a)^2 - 2(4a)(2a)\cos\theta$$

or $l_{SP} = 2a\sqrt{5 - 4\cos\theta}$

and $\delta l_{SP} = \dfrac{4a\sin\theta}{\sqrt{5 - 4\cos\theta}}\,\delta\theta$

$$F_{SP} = k\left[l_{SP} - (l_{SP})_0\right] = k\left(2a\sqrt{5 - 4\cos\theta} - 2a\right)$$

$$= 2ka\left(\sqrt{5 - 4\cos\theta} - 1\right)$$

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By Virtual Work:

$\delta U = 0$: $\quad P\delta y_P - F_{SP}\delta l_{SP} = 0$

$$P\left(a + a\sin\theta\right)\delta\theta - 2ka\left(\sqrt{5 - 4\cos\theta} - 1\right)\left(\frac{4a\sin\theta}{\sqrt{5 - 4\cos\theta}}\right)\delta\theta = 0$$

or
$$\left[\frac{P}{8ka}\left(1 + \sin\theta\right) - \sin\theta\right]\sqrt{5 - 4\cos\theta} + \sin\theta = 0$$

Then
$$\left[\frac{12\text{ lb}}{8\left(15\text{ lb/in.}\right)\left(7.5\text{ in.}\right)}\left(1 + \sin\theta\right) - \sin\theta\right]\sqrt{5 - 4\cos\theta} + \sin\theta = 0$$

$$\left[\frac{1}{75}\left(1 + \sin\theta\right) - \sin\theta\right]\sqrt{5 - 4\cos\theta} + \sin\theta = 0$$

Solving numerically, $\theta = 15.27°$ ◀

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Chapter 10, Solution 44.

For Bar ABC: $\delta\alpha = \dfrac{\delta y_C}{2a}$ (where $a = 15$ in.)

For Bar CD, using Law of Cosines

$$a^2 = l_C^2 + l_D^2 - 2l_C l_D \cos 55°$$

Then with a = constant, we have

$$0 = 2l_C\,\delta l_C + 2l_D\delta l_D - 2(\delta l_C)l_D\cos 55° - 2l_C(\delta l_D)\cos 55°$$

Since $\delta l_C = -\delta y_C$:

$$(l_C - l_D\cos 55°)\delta y_C = (l_D - l_C\cos 55°)\delta l_D$$

For the given position of member CD, ΔCDE is isosceles.

Thus $l_D = a$ and $l_C = 2a\cos 55°$

Then $(2a\cos 55° - a\cos 55°)\delta y_C = (a - 2a\cos^2 55°)\delta y_D$

or $$\delta l_D = \frac{\cos 55°}{1 - 2\cos^2 55°}\delta y_C$$

By Virtual Work:

$\delta U = 0$: $M\delta\alpha - P\delta l_D = 0$

$$M\left(\frac{\delta y_C}{2a}\right) - P\left(\frac{\cos 55°}{1 - 2\cos^2 55°}\right)\delta y_C = 0$$

or $$P = \frac{M}{2a}\frac{1 - 2\cos^2 55°}{\cos 55°}$$

Thus for given data

$$P = \frac{(320\text{ lb}\cdot\text{in.})}{2(15\text{ in.})}\;\frac{1 - 2\cos^2 55°}{\cos 55°} = 6.3605\text{ lb}$$

P = 6.36 lb ⊽ 35° ◀

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Chapter 10, Solution 45.

We have

$$x_A = (10\text{ in.})\sin\theta + (24\text{ in.})\cos\theta$$

$$\delta x_A = \left[(10\cos\theta - 24\sin\theta)\text{in.}\right]\delta\theta$$

Law of Cosines:

$$(CD)^2 = (BC)^2 + (BD)^2 - 2(BC)(BD)\cos\theta$$

$$= (20\text{ in.})^2 + (60\text{ in.})^2 - 2(20\text{ in.})(60\text{ in.})\cos\theta$$

$$= \left(4000\text{ in}^2\right) - \left(2400\text{ in}^2\right)\cos\theta$$

$$= \left[400(10 - 6\cos\theta)\text{in}^2\right]$$

Differentiating:

$$2(CD)\delta(CD) = \left(2400\text{ in}^2\right)\sin\theta\,\delta\theta$$

$$\delta(CD) = \frac{\left(1200\text{ in}^2\right)}{\left[20\sqrt{10 - 6\cos\theta}\text{ in.}\right]}\sin\theta\,\delta\theta$$

or $$\delta(CD) = \left[\frac{60\sin\theta}{\sqrt{10 - 6\cos\theta}}\text{in.}\right]\delta\theta$$

Virtual Work:

$$\delta U = P\delta x_A + F_{CD}\,\delta(CD) = 0$$

$$(4000\text{ lb})\left[(10\cos 60° - 24\sin 60°)\text{in.}\right]\delta\theta + F_{CD}\left[\frac{60\sin 60°}{\sqrt{10 - 6\cos 60°}}\text{in.}\right]\delta\theta = 0$$

or $$F_{CD} = 3214.9\text{ lb}$$

$\mathbf{F}_{CD} = 3.21$ kips ◀

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Chapter 10, Solution 46.

Triangle ADE:

$$\tan\alpha = \frac{(2.7\text{ ft})}{(1.5\text{ ft})} = 1.800$$

$$\alpha = 60.945°$$

$$AD = \frac{(2.7\text{ ft})}{\sin 60.945°} = 3.0887\text{ ft}$$

$$y_C = (15\text{ ft})\sin\theta \qquad \delta y_C = (15\text{ ft})\cos\theta\,\delta\theta$$

Law of Cosines: $BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(\alpha+\theta)$

$$= (7.2\text{ ft})^2 + (3.0887\text{ ft})^2 - 2(7.2\text{ ft})(3.0887\text{ ft})\cos(\alpha+\theta)$$

$$= \left[61.38 - 44.4773\cos(\alpha+\theta)\right]\text{ft}^2$$

$$2(BD)(\delta_{BD}) = 44.4773\sin(\alpha+\theta)\,\delta\theta$$

$$\delta_{BD} = \left[\frac{44.4773\sin(\alpha+\theta)}{2(BD)}\delta\theta\right]\text{ft}$$

Virtual Work:

$$\delta U = 0: -P\,\delta y_C + F_{BD}\,\delta_{BD} = 0$$

$$-(500\text{ lb})(15\text{ ft})\cos\theta\,\delta\theta + F_{BD}\left(\frac{44.4773\sin(\alpha+\theta)}{2(BD)}\delta\theta\right)\text{ft} = 0$$

$$F_{BD} = \left[337.25\frac{\cos\theta}{\sin(\alpha+\theta)}BD\right]\text{lb}$$

We have $\theta = 20°$ $\qquad BD^2 = 61.38 - 44.4773\cos(60.945° + 20°)$

$$BD = 7.3743\text{ ft}$$

Thus $\qquad F_{BD} = 337.25\dfrac{\cos 20°}{\sin(60.945° + 20°)}(7.3743)\text{lb}$

$$= 2366.5\text{ lb}$$

$\mathbf{F}_{BD} = 2370$ ↖ lb ◀

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Chapter 10, Solution 47.

$$\text{Input work} = P\,\delta x$$

$$\text{Output work} = (W\sin\alpha)\,\delta x$$

Efficiency:

$$\eta = \frac{W\sin\alpha\,\delta x}{P\,\delta x} \qquad \text{or} \qquad \eta = \frac{W\sin\alpha}{P} \qquad (1)$$

$$+\nearrow\ \Sigma F_x = 0: \quad P - F - W\sin\alpha = 0 \qquad \text{or} \qquad P = W\sin\alpha + F \qquad (2)$$

$$+\nwarrow\ \Sigma F_y = 0: \quad N - W\cos\alpha = 0 \qquad \text{or} \qquad N = W\cos\alpha$$

$$F = \mu N = \mu W\cos\alpha$$

Equation (2): $P = W\sin\alpha + \mu W\cos\alpha = W(\sin\alpha + \mu\cos\alpha)$

Equation (1): $\eta = \dfrac{W\sin\alpha}{W(\sin\alpha + \mu\cos\alpha)}$ or $\eta = \dfrac{1}{1 + \mu\cot\alpha}$ ◀

If block is to remain in place when $P = 0$, we know (see page 416) that $\phi_s \geq \alpha$ or, since

$$\mu = \tan\phi_s, \qquad \mu \geq \tan\alpha$$

Multiply by $\cot\alpha$: $\mu\cot\alpha \geq \tan\alpha\cot\alpha = 1$

Add 1 to each side: $1 + \mu\cot\alpha \geq 2$

Recalling the expression for η, we find $\eta \leq \dfrac{1}{2}$ ◀

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Chapter 10, Solution 48.

Link BC: $\quad x_B = l\cos\theta$

$$\delta x_B = -l\sin\theta\,\delta\theta$$

$$\text{or } \delta x_B = l\sin\theta\,\delta\theta \longrightarrow$$

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\,\delta\theta \uparrow$$

Link AB: $\quad \delta x_B = \frac{1}{2}l\,\delta\phi \longrightarrow$

Thus $\quad \frac{1}{2}l\,\delta\phi = l\sin\theta\,\delta\theta$

$$\delta\phi = 2\sin\theta\,\delta\theta \circlearrowright$$

Virtual Work:

$\delta U = 0$: $\quad M_{\max}\delta\phi - (P + \mu_s N)\delta y_C = 0$

$$M_{\max}(2\sin\theta\,\delta\theta) - (P + \mu_s N)(l\cos\theta\,\delta\theta) = 0$$

$$M_{\max} = \frac{P + \mu_s N}{2\tan\theta}l$$

Link BC Free-Body Diagram:

$\circlearrowleft + \Sigma M_B = 0$:

$$N(l\sin\theta) - (P + \mu_s N)l\cos\theta = 0$$

$$N\tan\theta - P - \mu_s N = 0$$

$$\text{or } N = \frac{P}{\tan\theta - \mu_s}$$

Substituting N into relationship for $M_{\max}$:

$$M_{\max} = \frac{P + \dfrac{\mu_s P}{\tan\theta - \mu_s}}{2\tan\theta}l = \frac{Pl(\tan\theta - \mu_s + \mu_s)}{2\tan\theta(\tan\theta - \mu_s)}$$

$$M_{\max} = \frac{Pl}{\left[2(\tan\theta - \mu_s)\right]} \blacktriangleleft$$

For $\mu_s \geq \tan\theta$, we have $M_{\max} = \infty$; the system becomes self-locking.

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Chapter 10, Solution 49.

Largest value of M is obtained from the solution of Problem 10.48

$$M_{\max} = \frac{Pl}{\left[2\left(\tan\theta - \mu_s\right)\right]}$$

Thus $M_{\max} = \dfrac{(400\text{ N})(0.500\text{ m})}{2(\tan 35° - 0.30)} = 249.87\text{ N}\cdot\text{m}$

$M_{\max} = 250\text{ N}\cdot\text{m}$ ◀

Smallest value of M occurs when the friction force in Problem 10.48 is directed upward instead of downward. The equations obtained in Problem 10.48 may be used if we replace μ_s by $-\mu_s$. Thus

$$M_{\min} = \frac{P - \mu_s N}{2\tan\theta} l$$

$$N = \frac{P}{\tan\theta + \mu_s}$$

and $$M_{\min} = \frac{Pl}{\left[2\left(\tan\theta + \mu_s\right)\right]}$$

Thus $$M_{\min} = \frac{(400\text{ N})(0.500\text{ m})}{2(\tan 35° + 0.30)} = 99.98\text{ N}\cdot\text{m}$$

$M_{\min} = 100.0\text{ N}\cdot\text{m}$ ◀

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Chapter 10, Solution 50.

For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \qquad \text{or} \qquad \mathbf{A} = \frac{P}{2}\uparrow$$

Then: $$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now $$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

and $$y_F = 3l\cos\theta$$

$$\delta y_F = -3l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\,\delta\theta) + P(-3l\sin\theta\,\delta\theta) = 0$$

or $$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of A impends to the right and **F** acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$

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Chapter 10, Solution 51.

Using the results of Problem 10.50 with

$$\theta = 30°,\ l = 0.2 \text{ m},\ P = 40 \text{ N},\ \text{and}\ \mu_s = 0.15$$

We have

$$Q_{\max} = \frac{P}{2}\left(3\tan\theta + \mu_s\right)$$

$$= \frac{(40 \text{ N})}{2}\left(3\tan 30° + 0.15\right)$$

$$= 37.64 \text{ N}$$

$$Q_{\max} = 37.6 \text{ N} \blacktriangleleft$$

and

$$Q_{\min} = \frac{P}{2}\left(3\tan\theta - \mu_s\right)$$

$$= \frac{(40 \text{ N})}{2}\left(3\tan 30° - 0.15\right)$$

$$= 31.64 \text{ N}$$

$$Q_{\min} = 31.6 \text{ N} \blacktriangleleft$$

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Chapter 10, Solution 52.

Recall Figure 8.9a. Draw force triangle

$$Q = W\tan(\theta + \phi_s)$$

$$y = x\tan\theta \text{ so that } \delta y = \delta x\tan\theta$$

$$\text{Input work} = Q\delta x = W\tan(\theta + \phi_s)\delta x$$

$$\text{Output work} = W\delta y = W(\delta x)\tan\theta$$

Efficiency: $\eta = \dfrac{W\tan\theta\delta x}{W\tan(\theta + \phi_s)\delta x}$; $\eta = \dfrac{\tan\theta}{\tan(\theta + \phi_s)}$ ◀

From page 432, we know the jack is self-locking if

$$\phi_s \geq \theta$$

Then $\theta + \phi_s \geq 2\theta$

so that $\tan(\theta + \phi_s) \geq \tan 2\theta$

From above $\eta = \dfrac{\tan\theta}{\tan(\theta + \phi_s)}$

It then follows that $\eta \leq \dfrac{\tan\theta}{\tan 2\theta}$

But $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Then $\eta \leq \dfrac{\tan\theta(1 - \tan^2\theta)}{2\tan\theta} = \dfrac{1 - \tan^2\theta}{2}$ $\quad \therefore \quad \eta \leq \dfrac{1}{2}$ ◀

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Chapter 10, Solution 53.

To determine Ay, consider a vertical displacement δy_A:

Note that $\delta y_A = \delta y_B = \delta y_C$

$$\frac{\delta y_C}{300 \text{ mm}} = \frac{\delta y_E}{750 \text{ mm}} \text{ or } \delta y_E = 2.5\,\delta y_A$$

$$\frac{\delta y_E}{240 \text{ mm}} = \frac{\delta y_G}{360 \text{ mm}} \text{ or } \delta y_G = \frac{360}{240}(2.5\,\delta y_A) = 3.75\,\delta y_A$$

Virtual Work:

$$\delta U = 0: \; A_y\,\delta y_A + (960 \text{ N})\,\delta y_B - (240 \text{ N})\,\delta y_G = 0$$

$$A_y\,\delta y_A + (960 \text{ N})\,\delta y_A - (240 \text{ N})(3.75\,\delta y_A) = 0$$

$$A_y = -60 \text{ N} \quad \text{or} \quad \mathbf{A}_y = 60 \text{ N} \downarrow$$

To determine $\mathbf{A}_x$, consider a horizontal displacement δx_A:

Virtual Work: $\delta U = 0$: $\quad A_x \delta x_A = 0$, or $A_x = 0$

Thus the total force reaction is: $\quad \bar{\mathbf{A}} = 60.0 \text{ N} \downarrow$ ◀

To determine M_A, consider a counterclockwise rotation $\delta\theta_A$:

Note that $\quad \delta y_B = 600\,\delta\theta_A \qquad \delta y_C = 900\,\delta\theta_A$

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$$\frac{\delta y_C}{300\text{ mm}} = \frac{\delta y_E}{750\text{ mm}} \quad \text{or} \quad \delta y_E = \frac{750}{300}\left(900\,\delta\theta_A\right) = 2250\,\delta\theta_A$$

$$\frac{\delta y_E}{240\text{ mm}} = \frac{\delta y_G}{360\text{ mm}} \quad \text{or} \quad \delta y_G = \frac{360}{240}\left(2250\,\delta\theta_A\right) = 3375\,\delta\theta_A$$

Virtual Work:

$\delta U = 0$:

$$M_A\,\delta\theta_A + \left(960\text{ N}\right)\delta y_B - \left(240\text{ N}\right)\delta y_G = 0$$

$$M_A\,\delta\theta_A + \left(960\text{ N}\right)\left(600\,\delta\theta_A\text{ mm}\right) - \left(240\text{ N}\right)\left(3375\,\delta\theta_A\text{ mm}\right) = 0$$

$$M_A = 234000\text{ N}\cdot\text{mm},$$

or $\bar{\mathbf{M}} = 234\text{ N}\cdot\text{m}$ ◀

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Chapter 10, Solution 54.

To determine D_y, consider a vertical displacement δy_D:

Note that $\dfrac{\delta y_D}{300 \text{ mm}} = \dfrac{\delta y_E}{1050 \text{ mm}}$ or $\delta y_E = 3.5\,\delta y_D$

$\dfrac{\delta y_E}{240 \text{ mm}} = \dfrac{\delta y_G}{360 \text{ mm}}$ or $\delta y_G = \dfrac{360}{240}(3.5\,\delta y_D) = 5.25\,\delta y_D$

Virtual Work:

$$\delta U = 0: \; D_y\,\delta y_D + (240 \text{ N})\,\delta y_G = 0$$

$$D_y\,\delta y_D + (240 \text{ N})(5.25\,\delta y_D) = 0$$

$$D_y = -1260 \text{ N}$$

or $\bar{\mathbf{D}}_y = 1.260 \text{ kN} \downarrow$ ◀

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Chapter 10, Solution 55.

From the solution of Problem 10.41

$$F_{\text{cyl}} = 99.270 \text{ lb}$$

By Virtual Work:

$$\delta U = 0: \qquad F_{\text{cyl}}\left(-\delta d_{AB}\right) - P\,\delta y_D = 0$$

$$\left(99.270 \text{ lb}\right)\left(-\delta d_{AB}\right) - \left(120 \text{ lb}\right)\left(1.2 \text{ in.}\right) = 0$$

$$\delta d_{AB} = -1.45059 \text{ in.}$$

or $\delta d_{AB} = 1.451 \text{ in. (shorter)}$ ◀

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Chapter 10, Solution 56.

From Problem 10.46

$F_{BD} = 2370 \text{ lb}$ as shown.

By Virtual Work: Assume both δy_C and δ_{BD} increase.

$$\delta U = 0: \qquad -(500 \text{ lb})\delta y_C + F_{BD}\,\delta_{BD} = 0$$

$$-(500 \text{ lb})(2.5 \text{ in.}) + (2370 \text{ lb})\delta_{BD} = 0$$

$$\delta_{BD} = 0.5274 \text{ in.}$$

Thus $\delta_{BD} = 0.527$ in. Longer ◀

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Chapter 10, Solution 57.

Apply vertical load P at C:

$$+\circlearrowleft \Sigma M_B = 0: \ (12 \text{ m})J_y - (3 \text{ m})P = 0$$

$$J_y = \frac{P}{4}$$

$$\xrightarrow{+} \Sigma F_x = 0: \ J_x = 0$$

$$+\uparrow \Sigma F_y = 0: \ \frac{P}{4} - \frac{3}{5}F_{FG} = 0$$

$$F_{FG} = \frac{5}{12}P \ \text{(T)}$$

Virtual Work:

Remove member FG and replace it with forces $\bar{F}_{FG}$ and $-\bar{F}_{FG}$ at pins F and G, respectively. Denoting the virtual displacements of F and G as $\delta\bar{r}_F$ and $\delta\bar{r}_G$, respectively, and noting that P and δy_C have the same direction, have by virtual work.

$$\delta U = 0: \ P\delta y_C + \bar{F}_{FG}\cdot\delta\bar{r}_F + \left(-\bar{F}_{FG}\right)\cdot\delta\bar{r}_G = 0$$

$$P\delta y_C + F_{FG}\,\delta r_F\cos\theta_F - F_{FG}\,\delta r_G\cos\theta_G = 0$$

$$P\delta y_C - F_{FG}\left(\delta r_G\cos\theta_G - \delta r_F\cos\theta_F\right) = 0$$

Where $\left(\delta r_G\cos\theta_G - \delta r_F\cos\theta_F\right) = \delta_{FG}$, which is the change in length of member FG. Thus

$$P\delta y_C - F_{FG}\,\delta_{FG} = 0$$

$$P\delta y_C - \left(\frac{5}{12}P\right)(30 \text{ mm}) = 0$$

$$\delta y_C = 12.50 \text{ mm}$$

or $\delta y_C = 12.50 \text{ mm} \downarrow$ ◀

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Chapter 10, Solution 58.

Apply a horizontal load P at C:

$$\xrightarrow{+} \Sigma F_x = 0: \; P - J_x = 0 \;\; \therefore J_x = P \leftarrow$$

$$+\circlearrowleft \Sigma M_B = 0: \; (12 \text{ m})J_y - (2.25 \text{ m})P = 0$$

$$J_y = \frac{3}{16}P \uparrow$$

$$+\uparrow \Sigma F_y = 0: \; \frac{3}{16}P - \frac{3}{5}F_{FG} = 0$$

$$F_{FG} = \frac{5}{16}P \; (\text{T})$$

Virtual Work:

Remove member FG and replace it with forces $\bar{F}_{FG}$ and $-\bar{F}_{FG}$ at pins F and G, respectively. Since P and δx_C have the same direction, and since $\bar{F}_{FG}$ tends to decrease the length FG, have by virtual work.

$$\delta U = 0: \; P\,\delta x_C - F_{FG}\,\delta_{FG} = 0$$

$$P\,\delta x_C - \left(\frac{5}{16}P\right)(30 \text{ mm}) = 0$$

$$\delta x_C = 9.375 \text{ mm}$$

or $\delta x_C = 9.38 \text{ mm} \rightarrow$ ◀

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Chapter 10, Solution 59.

Spring: $s = x - 0.3$ m

$$y_E = -\frac{x}{3} - \frac{x}{6} = -\frac{x}{2}$$

Potential Energy:

$$V = \frac{1}{2}ks^2 + Wy_E$$

$$= \frac{1}{2}k\left(x - 0.3\right)^2 - W\left(\frac{x}{2}\right)$$

For equilibrium:

$$\frac{dV}{dx} = k\left(x - 0.3\right) - \frac{1}{2}W = 0$$

$$\left(5000 \text{ N/m}\right)\left(x - 0.3\right)\text{m} - \frac{1}{2}\left(900 \text{ N}\right) = 0$$

Solving $x = 0.390$ m $\qquad x = 390$ mm ◀

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Chapter 10, Solution 60.

Given: $\left(x_{SP}\right)_0 = 300$ mm

$k = 5$ kN/m

From geometry:

$$y_C = -\frac{1}{3}\left(\frac{x}{2}\right)$$

$$= -\frac{x}{6}$$

and $s_{SP} = x - \left(x_{SP}\right)_0$

$$= \left(x - 0.3\right)\text{m}$$

Potential Energy:

$$V = V_{SP} + V_{FC}$$

$$= \frac{1}{2}k\left(x - 0.3\right)^2 + F_C\left(-\frac{1}{6}x\right)$$

For equilibrium:

$$\frac{dV}{dx} = 0: \qquad k\left(x - 0.3\right) - \frac{1}{6}F_C = 0$$

$$\left(5000 \text{ N/m}\right)\left(x - 0.3\right)\text{m} - \frac{1}{6}\left(900 \text{ N}\right) = 0$$

or $x = 0.330$ m

$x = 330$ mm ◀

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Chapter 10, Solution 61.

Given: $k = 2.5$ kN/m

$(S_{SP})_0 = 0$ at $\theta = 45°$

From geometry:

$$y_D = -(0.25\text{ m})\sin\theta$$

$$(x_{SP})_0 = 2(0.3\text{ m})\cos 45°$$

$$S_{SP} = (x_{SP})_0 - |x_A|$$

$$= (0.6\text{ m})\cos 45° - 2(0.3\text{ m})\cos\theta$$

$$= (0.6\text{ m})(\cos 45° - \cos\theta)$$

Potential Energy:

$$V = V_{SP} + V_{FD}$$

$$= \frac{1}{2}k(0.6\text{ m})^2(\cos 45° - \cos\theta)^2 - (250\text{ N})\left[-(0.25\text{ m})\sin\theta\right]$$

For equilibrium: $\dfrac{dV}{d\theta} = 0$:

$$k(0.36)(\sin\theta)(\cos 45° - \cos\theta) + 62.5\cos\theta = 0$$

or

$$\tan\theta(\cos 45° - \cos\theta) = -\frac{62.5\text{ N}\cdot\text{m}}{(2500\text{ N/m})(0.36\text{ m}^2)}$$

or

$$\tan\theta(\cos 45° - \cos\theta) = -0.06944$$

Solving numerically

and $\theta = 15.03°$ ◀

$\theta = 36.9°$ ◀

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Chapter 10, Solution 62.

$k = 12.5 \dfrac{\text{lb}}{\text{in.}}$ $\qquad P = 150 \text{ lb}$

$s = 0 \quad \text{when } \theta = 0$

From geometry:

$$y_B = -(15 \text{ in.})\sin\theta$$

$$s = 30 \text{ in.} - 2(15 \text{ in.})\cos\theta$$

$$= (30 \text{ in.})(1 - \cos\theta)$$

Potential Energy:

$$V = V_{SP} + V_P$$

$$= \frac{1}{2}ks^2 + P\,y_B$$

$$= \frac{1}{2}k(30 \text{ in.})^2(1 - \cos\theta)^2 + P\left[-(15 \text{ in.})\sin\theta\right]$$

For equilibrium

$$\frac{dV}{d\theta} = 0: \qquad k\left(900 \text{ in}^2\right)(1 - \cos\theta)(\sin\theta) - (15 \text{ in.})\cos\theta P = 0$$

or $$\left(12.5 \frac{\text{lb}}{\text{in.}}\right)\left(900 \text{ in}^2\right)(1 - \cos\theta)(\sin\theta) - (15 \text{ in.})(150 \text{ lb})\cos\theta = 0$$

or $$11250(1 - \cos\theta)(\sin\theta) - 2250\cos\theta = 0$$

or $$(1 - \cos\theta)\tan\theta = 0.200$$

Solving numerically, $\qquad \theta = 40.22°$

$\theta = 40.2°$ ◀

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Chapter 10, Solution 63.

$k = 12.5 \dfrac{\text{lb}}{\text{in.}}$ $\qquad \theta = 25°$

$s = 0$ when $\theta = 0°$

From geometry:

$$y_B = -(15 \text{ in.})\sin\theta$$

$$s = 30 \text{ in.} - 2(15 \text{ in.})\cos\theta$$

$$= (30 \text{ in.})(1 - \cos\theta)$$

Potential Energy:

$$V = V_{SP} + V_P$$

$$= \frac{1}{2}ks^2 + P\,y_B$$

$$= \frac{1}{2}k(30 \text{ in.})^2(1 - \cos\theta)^2 + P\left[-(15 \text{ in.})\sin\theta\right]$$

For equilibrium $\dfrac{dV}{d\theta} = 0$:

$$k\left(900 \text{ in}^2\right)(1 - \cos\theta)(\sin\theta) - (15 \text{ in.})\cos\theta P = 0$$

or

$$\left(12.5 \frac{\text{lb}}{\text{in.}}\right)\left(900 \text{ in}^2\right)(1 - \cos 25°)(\sin 25°) - (15 \text{ in.})(\cos 25°)\,P = 0$$

or $\qquad P = 32.8 \text{ lb}$ ◀

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Chapter 10, Solution 64.

Spring
$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$v = 2l\sin\left(45° + \frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$

$$v_0 = 2l\sin 45° = \sqrt{2}l$$

Deflection of spring
$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]\cos\left(45° + \frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by $\cos\theta$

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

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Then with $P = 150$ lb, $l = 30$ in. and $k = 40$ lb/in.

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{150 \text{ lb}}{(40 \text{ lb/in.})(30 \text{ in.})}$$

$$= 0.125$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.618718$$

Solving numerically, $\theta = 17.83°$ ◀

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Chapter 10, Solution 65.

From geometry:

$$y_A = -l\sin\theta$$

$$l_{CD} = l\sqrt{(\cos\theta + \sin\theta)^2 + \left[1 - (\cos\theta - \sin\theta)\right]^2}$$

$$= l\sqrt{3 + 2\sin\theta - 2\cos\theta}$$

$$s_{SP} = (l_{CD} - l)$$

$$= l\left(\sqrt{3 + 2\sin\theta - 2\cos\theta} - 1\right)$$

Potential Energy:

$$V = V_{SP} + V_P$$

$$= \frac{1}{2}ks_{SP}^2 + Py_A$$

$$= \frac{1}{2}kl^2\left(\sqrt{3 + 2\sin\theta - 2\cos\theta} - 1\right)^2 + P(-l\sin\theta)$$

For equilibrium: $\dfrac{dV}{d\theta} = 0$:

$$kl^2\left(\frac{\cos\theta + \sin\theta}{\sqrt{3 + 2\sin\theta - 2\cos\theta}}\right)\left(\sqrt{3 + 2\sin\theta - 2\cos\theta} - 1\right) - Pl\cos\theta = 0$$

or

$$\left(1 - \frac{1}{\sqrt{3 + 2\sin\theta - 2\cos\theta}}\right)(1 + \tan\theta) = \frac{P}{kl}$$

$$= \frac{600\text{ N}}{(4000\text{ N/m})(0.8\text{ m})}$$

$$= 0.1875$$

Solving numerically $\theta = 10.77°$ ◀

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Chapter 10, Solution 66.

From geometry

$$y_C = d_{AC}\tan\theta \qquad d_{AC} = 375 \text{ mm}$$

$$y_P = r\theta \qquad r = 75 \text{ mm}$$

Potential Energy:

$$V = V_{SP} + V_P = \frac{1}{2}ky_C^2 - Py_P$$

$$= \frac{1}{2}kd_{AC}^2\tan^2\theta - Pr\theta$$

For equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad kd_{AC}^2\tan\theta\sec^2\theta - Pr = 0$$

$$(0.8 \text{ N/mm})(375 \text{ mm})^2\tan\theta\sec^2\theta - (480 \text{ N})(75 \text{ mm}) = 0$$

$$3.125\tan\theta\sec^2\theta = 1$$

Solving numerically $\qquad \theta = 16.41° \blacktriangleleft$

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Chapter 10, Solution 67.

Since $\quad x_C = x_D, \qquad 125\theta = 375\phi, \qquad$ or $\qquad \phi = \dfrac{\theta}{3}$

Also $\quad x_E = (250\text{ mm})\phi = \left(\dfrac{250}{3}\theta\right)\text{mm}$

Potential Energy:

$$V = -M\theta + Q\,x_E - Py_G$$

$$= -M\theta + \frac{250}{3}Q\theta - \frac{100}{3}P\theta$$

For Equilibrium:

$$\frac{dV}{d\theta} = \theta: \qquad -M + \frac{250}{3}Q - \frac{100}{3}P = 0$$

Thus at equilibrium, V is constant and the equilibrium is neutral.

Q.E.D. ◀

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Chapter 10, Solution 68.

First note $\quad y_B = y_F$

For small θ, ϕ: $\quad y_B = (10 \text{ in.})\theta$

$$y_F = (6 \text{ in.})\phi$$

Thus $\quad 10\theta = 6\phi$

$$\phi = \frac{5}{3}\theta$$

Also $\quad y_A = (16 \text{ in.})\theta$

$$y_G = (12 \text{ in.})\phi$$

$$= (20 \text{ in.})\theta$$

$$h + x_D \quad (h = \text{constant})$$

$$= h + (8 \text{ in.})\theta$$

$$x_D = (4.8 \text{ in.})\phi$$

$$= (8 \text{ in.})\theta$$

Potential Energy:

$$V = V_{F_A} + V_P + V_W$$

$$= -F_A y_A - P y_G + W y_w$$

$$= -(20 \text{ lb})(16 \text{ in.})\theta - P(20 \text{ in.})\theta + (30 \text{ lb})\left[-(h + 8\theta)\text{in.}\right]$$

$$= \left[(-320 - 20P - 240)\theta - 30h\right](\text{lb}\cdot\text{in.})$$

For equilibrium: $\quad \dfrac{dV}{d\theta} = 0$

$$-320 - 20P - 240 = 0$$

V is a constant therefore equilibrium is neutral.

Q.E.D.

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Chapter 10, Solution 69.

Potential Energy:

$$V = -W\left(\frac{l}{2}\cos\theta\right) - W\left(\frac{l}{2}\sin\theta\right)$$

$$= -\frac{Wl}{2}\left(\cos\theta + \sin\theta\right)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}\left(\sin\theta - \cos\theta\right)$$

For equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad \sin\theta - \cos\theta = 0$$

$$\tan\theta = 1$$

$\theta = 45°$ and $\theta = -135°$ ◀

Stability:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}\left(\cos\theta + \sin\theta\right)$$

$\theta = 45°$: $\dfrac{d^2V}{d\theta^2} = \dfrac{Wl}{2}\left(0.707 + 0.707\right) > 0$ Stable ◀

$\theta = -135°$: $\dfrac{d^2V}{d\theta^2} = \dfrac{Wl}{2}\left(-0.707 - 0.707\right) < 0$ Unstable ◀

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Chapter 10, Solution 70.

Potential Energy:

$$V = -W_{CD}\left(\frac{l}{2}\cos\theta\right) - W_{AB}\left(\frac{l}{2}\sin\theta\right)$$

$$= -W_{CD}\frac{l}{2}\left(\cos\theta + \frac{W_{AB}}{W_{CD}}\sin\theta\right)$$

But $\quad \dfrac{W_{AB}}{W_{CD}} = \dfrac{m_{AB}g}{m_{CD}g} = \dfrac{300}{500} = 0.6$

Thus $\quad V = -W_{CD}\dfrac{l}{2}\left(\cos\theta + 0.6\sin\theta\right)$

$$\frac{dV}{d\theta} = W_{CD}\frac{l}{2}\left(\sin\theta - 0.6\cos\theta\right)$$

For Equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad \sin\theta - 0.6\cos\theta = 0$$

$$\tan\theta = 0.6$$

$\theta = 31.0°$ and $\theta = -149.0°$ ◀

Stability:

$$\frac{d^2V}{d\theta^2} = W_{CD}\frac{l}{2}\left(\cos\theta + 0.6\sin\theta\right)$$

$$\theta = 31.0°: \qquad \frac{d^2V}{d\theta^2} = W_{CD}\frac{l}{2}\left(\cos 31.0° + 0.6\sin 31.0°\right) > 0$$

Stable ◀

$$\theta = -149.0°: \qquad \frac{d^2V}{d\theta^2} = W_{CD}\frac{l}{2}\left[\cos\left(-149.0°\right) + 0.6\sin\left(-149.0°\right)\right]$$

$$= W_{CD}\frac{l}{2}\left[-0.8572 + 0.6\left(-0.5150\right)\right] < 0$$

Unstable ◀

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Chapter 10, Solution 71.

Let each rod be of length L and weight W. Then the potential energy V is

$$V = W\left(\frac{L}{2}\sin\theta\right) + W\left(\frac{L}{2}\cos 2\theta\right)$$

Then

$$\frac{dV}{d\theta} = \frac{W}{2}L\cos\theta - WL\sin 2\theta$$

For equilibrium

$$\frac{dV}{d\theta} = 0: \quad \frac{W}{2}L\cos\theta - WL\sin 2\theta = 0$$

or

$$\cos\theta - 2\sin 2\theta = 0$$

Solving numerically or using a computer algebra system, such as Maple, gives four solutions:

$$\theta = 1.570796327 \text{ rad} = 90.0°$$

$$\theta = -1.570796327 \text{ rad} = 270°$$

$$\theta = 0.2526802551 \text{ rad} = 14.4775°$$

$$\theta = 2.888912399 \text{ rad} = 165.522°$$

Now

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}WL\sin\theta - 2WL\cos 2\theta$$

$$= -WL\left(\frac{1}{2}\sin\theta + 2\cos 2\theta\right)$$

continued

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At $\theta = 14.4775°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 14.4775° + 2\cos\left[2\left(14.4775°\right)\right]\right\}$$

$$= -1.875WL \ (< 0) \qquad \therefore \ \theta = 14.48°, \text{ Unstable} \blacktriangleleft$$

At $\theta = 90°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 90° + 2\cos 180°\right\}$$

$$= 1.5WL \ (> 0) \qquad \therefore \ \theta = 90°, \text{ Stable} \blacktriangleleft$$

At $\theta = 165.522°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 165.522° + 2\cos\left(2 \times 165.522°\right)\right\}$$

$$= -1.875WL \ (< 0) \qquad \therefore \ \theta = 165.5°, \text{ Unstable} \blacktriangleleft$$

At $\theta = 270°$

$$\frac{d^2V}{d\theta^2} = -WL\left(\frac{1}{2}\sin 270° + 2\cos 540°\right)$$

$$= 2.5WL \ (> 0) \qquad \therefore \ \theta = 270°, \text{ Stable} \blacktriangleleft$$

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Chapter 10, Solution 72.

Potential energy $$V = W\left(\frac{l}{2}\cos 1.5\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \qquad W = mg$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}\left(-1.5\sin 1.5\theta\right) + \frac{Wl}{2}\left(-\sin\theta\right)$$

$$= -\frac{Wl}{2}\left(1.5\sin 1.5\theta + \sin\theta\right)$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left(2.25\cos 1.5\theta + \cos\theta\right)$$

For equilibrium $$\frac{dV}{d\theta} = 0: 1.5\sin 1.5\theta + \sin\theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than $180°$ can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8°$$

Now $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left(2.25\cos 1.5\theta + \cos\theta\right)$$

At $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left(2.25\cos 0° + \cos 0°\right)$$

$$= -\frac{Wl}{2}(3.25) \ (< 0) \qquad \therefore\ \theta = 0, \text{ Unstable} ◀$$

At $\theta = 137.8°$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left[2.25\cos\left(1.5 \times 137.8°\right) + \cos 137.8°\right]$$

$$= \frac{Wl}{2}(2.75) \ (> 0) \qquad \therefore\ \theta = 137.8°, \text{ Stable} ◀$$

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Chapter 10, Solution 73.

Potential Energy

$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium: $\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$

For $P = 2$ kN, $l = 250$ mm, $K = 225$ N·m/rad

$$\cos\theta = \frac{225 \text{ N}\cdot\text{m/rad}}{(2000 \text{ N})(0.25 \text{ m})}\theta$$

$$= 0.450\theta$$

Solving numerically, we obtain

$$\theta = 1.06896 \text{ rad} = 61.247°$$

$\theta = 61.2°$ ◀

Stability

$$\frac{d^2V}{d\theta^2} = (225 \text{ N}\cdot\text{m/rad}) + (2000 \text{ N})(0.25 \text{ m})\sin 61.2° > 0$$

$\therefore$ Stable ◀

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Chapter 10, Solution 74.

Potential Energy

$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium $\quad \dfrac{dV}{d\theta} = 0: \quad \cos\theta = \dfrac{K}{Pl}\theta$

For $\quad P = 6.3\text{ kN},\ l = 250\text{ mm, and } K = 225\text{ N}\cdot\text{m/rad}$

$$\cos\theta = \frac{225\text{ N}\cdot\text{m/rad}}{(6300\text{ N})(0.25\text{ m})}\theta$$

or $\quad \cos\theta = \dfrac{\theta}{7}$

Solving numerically, $\quad \theta = 1.37333\text{ rad, } 5.652\text{ rad, and } 6.616\text{ rad}$

or $\quad \theta = 78.7°,\ 323.8°,\ 379.1°$

Stability At $\theta = 78.7°$: $\quad \dfrac{d^2V}{d\theta^2} = (225\text{ N}\cdot\text{m/rad}) + (6300\text{ N})(0.25\text{ m})\sin 78.7°$

$= 1769.5\text{ N}\cdot\text{m} > 0 \qquad \therefore\ \theta = 78.7°,$ Stable ◀

At $\theta = 323.8°$: $\quad \dfrac{d^2V}{d\theta^2} = (225\text{ N}\cdot\text{m/rad}) + (6300\text{ N})(0.25\text{ m})\sin 323.8°$

$= -705.2\text{ N}\cdot\text{m} < 0 \qquad \therefore\ \theta = 324°,$ Unstable ◀

At $\theta = 379.1°$: $\quad \dfrac{d^2V}{d\theta^2} = (225\text{ N}\cdot\text{m/rad}) + (6300\text{ N})(0.25\text{ m})\sin 379.1°$

$= 740.37\text{ N}\cdot\text{m} > 0 \qquad \therefore\ \theta = 379°,$ Stable ◀

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Chapter 10, Solution 75.

Have $\quad y_{SP} = r(\theta - \theta_0), \qquad r = 4 \text{ in.}, \qquad \theta_0 = 20° = \dfrac{\pi}{9}\text{rad}$

$$y_B = l_{AB}\cos\theta, \qquad l_{AB} = 18 \text{ in.}$$

Potential Energy:

$$V = \frac{1}{2}ky_{SP}^2 + Wy_B$$

$$= \frac{1}{2}kr^2(\theta - \theta_0)^2 + Wl_{AB}\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - Wl_{AB}\sin\theta$$

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl_{AB}\cos\theta$$

For equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad (4.5 \text{ lb/in.})(4 \text{ in.})^2\left(\frac{\pi}{4} - \frac{\pi}{9}\right) - W(18 \text{ in.})\sin\frac{\pi}{4} = 0$$

$$W = 2.4683 \text{ lb} \qquad\qquad W = 2.47 \text{ lb} ◀$$

Stability:

$$\frac{d^2V}{d\theta^2} = (4.5 \text{ lb/in.})(4 \text{ in.})^2 - (2.4683 \text{ lb})(18 \text{ in.})\cos\frac{\pi}{4}$$

$$= 40.6 \text{ lb}\cdot\text{in.} > 0 \qquad\qquad \therefore \text{ Stable} ◀$$

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Chapter 10, Solution 76.

Have $\quad y_{SP} = r(\theta - \theta_0), \qquad r = 4 \text{ in.}, \qquad \theta_0 = 20° = \dfrac{\pi}{9} \text{ rad}$

$$y_B = l_{AB}\cos\theta, \qquad l_{AB} = 18 \text{ in.}$$

Potential Energy:

$$V = \frac{1}{2}ky_{SP}^2 + Wy_B$$

$$= \frac{1}{2}kr^2(\theta - \theta_0)^2 + Wl_{AB}\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - Wl_{AB}\sin\theta$$

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl_{AB}\cos\theta$$

For equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad (4.5 \text{ lb/in.})(4 \text{ in.})^2\left(\theta - \frac{\pi}{9}\right) - (6.6 \text{ lb})(18 \text{ in.})\sin\theta = 0$$

$$\left(\theta - \frac{\pi}{9}\right) - 1.65\sin\theta = 0$$

Solving numerically: $\quad \theta = 1.90680 \text{ rad}$

$$= 109.252° \qquad \theta = 109.3° \blacktriangleleft$$

Stability:

$$\frac{d^2V}{d\theta^2} = (4.5 \text{ lb/in.})(4 \text{ in.})^2 - (6.6 \text{ lb})(18 \text{ in.})\cos 109.252°$$

$$= 111.171 \text{ lb}\cdot\text{in.} > 0$$

$\therefore$ Stable ◀

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Chapter 10, Solution 77.

Note:

$$y = \left[20 - \sqrt{(11)^2 - (20 - x)^2}\right] \text{ in.}$$

$$= \left(20 - \sqrt{-x^2 + 40x - 279}\right) \text{ in.}$$

Potential Energy:

$$V = \frac{1}{2}k(x - 7.5)^2 + \frac{1}{2}k(y - 7.5)^2 + W_A(20) + W_B y$$

$$= \frac{1}{2}k(x - 7.5)^2 + \frac{1}{2}k\left(12.5 - \sqrt{-x^2 + 40x - 279}\right)^2 + 20W_A + W_B\left(20 - \sqrt{-x^2 + 40x - 279}\right)$$

Equilibrium Condition:

$$\frac{dV}{dx} = 0: \qquad k(x - 7.5) + k\left(12.5 - \sqrt{-x^2 + 40x - 279}\right)\left(-\frac{40 - 2x}{2\sqrt{-x^2 + 40x - 279}}\right)$$

$$-W_B \frac{40 - 2x}{2\sqrt{-x^2 + 40x - 279}} = 0$$

Simplifying,

$$12.5k(x - 20) + 12.5k\sqrt{-x^2 + 40x - 279} + W_B(x - 20) = 0$$

Substituting $\quad k = 1 \text{ lb/in.}, \quad W_B = 10 \text{ lb}:$

$$12.5(1 \text{ lb/in.})\left[(x - 20) \text{ in.}\right] + 12.5(1 \text{ lb/in.})\left(\sqrt{-x^2 + 40x - 279} \text{ in.}\right) + (10 \text{ lb})\left[(x - 20) \text{ in.}\right] = 0$$

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or $\quad (x - 20) + \sqrt{-x^2 + 40x - 279} + 0.8(x - 20) = 0$

or $\quad \sqrt{-x^2 + 40x - 279} = 1.8(20 - x)$

or $\quad -x^2 + 40x - 279 = (36 - 1.8x)^2$

or $\quad 4.24x^2 - 169.6x + 1575 = 0$

Then $\quad x = \dfrac{169.6 \pm \sqrt{(-169.6)^2 - 4(4.24)(1575)}}{2(4.24)}$

or $\quad x = 14.6579$ in. $\quad$ and $\quad x = 25.342$ in.

Since $\quad x \le 20$ in. $\qquad\qquad$ $x = 14.66$ in. ◀

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Chapter 10, Solution 78.

Deflection of spring = s, where $$s = \sqrt{l^2 + y^2} - l$$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$

Potential Energy: $$V = \frac{1}{2}ks^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

Equilibrium $$\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$$

Now $W = mg = (12\text{ kg})(9.81\text{ m/s}^2) = 117.72\text{ N}$, $l = 0.75\text{ m}$, and $k = 900\text{ N/m}$

Then $$\left(1 - \frac{0.75\text{ m}}{\sqrt{(0.75\text{ m})^2 + y^2}}\right)y = \frac{1}{2}\frac{(117.72\text{ N})}{(900\text{ N/m})}$$

or $$\left(1 - \frac{0.75}{\sqrt{0.5625 + y^2}}\right)y = 0.0654$$

Solving numerically, $$y = 0.45342\text{ m}$$

$y = 453\text{ mm}$ ◀

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Chapter 10, Solution 79.

(*a*) We note that in $\triangle ABC$,

$$\angle A = \angle B = \frac{1}{2}\left[180° - \left(90° + \theta\right)\right]$$

$$= 45° - \frac{\theta}{2}$$

Thus $AB = 2a\cos\left(45° - \frac{\theta}{2}\right)$

$$= 2a\left(\cos 45° \cos\frac{\theta}{2} + \sin 45° \sin\frac{\theta}{2}\right)$$

$$= 2a\frac{\sqrt{2}}{2}\left(cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

$$= \sqrt{2}a\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

Elongation of Spring: $s = AB - \sqrt{2}a$

$$= \sqrt{2}a\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2} - 1\right)$$

Potential Energy: $V = \frac{1}{2}ks^2 - W\left(l\sin\theta\right)$

$$V = \frac{1}{2}k\left(2a^2\right)\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2} - 1\right)^2 - Wl\sin\theta$$

$$= ka^2\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + 1 + 2\cos\frac{\theta}{2}\sin\frac{\theta}{2} - 2\cos\frac{\theta}{2} - 2\sin\frac{\theta}{2}\right) - Wl\sin\theta$$

$$= ka^2\left(1 + 1 + \sin\theta - 2\cos\frac{\theta}{2} - 2\sin\frac{\theta}{2}\right) - Wl\sin\theta$$

$$= 2ka^2\left(1 - \cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) + ka^2\left(1 - \frac{Wl}{ka^2}\right)\sin\theta$$

continued

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$$\frac{dV}{d\theta} = 2ka^2\left(\frac{1}{2}\sin\frac{\theta}{2} - \frac{1}{2}\cos\frac{\theta}{2}\right) + ka^2\left(1 - \frac{Wl}{ka^2}\right)\cos\theta$$

$$= ka^2\left[-\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right) + \left(1 - \frac{Wl}{ka^2}\right)\cos\theta\right]$$

For Equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad \cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \left(1 - \frac{Wl}{ka^2}\right)\cos\theta \qquad (1) \quad ◀$$

(*b*) Given data:

$k = 75$ lb/in., $a = 10$ in., $l = 15$ in., and $W = 100$ lb

Using equation (1): $$\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \left(1 - \frac{(100\text{ lb})(15\text{ in.})}{(75\text{ lb/in.})(10\text{ in.})^2}\right)\cos\theta$$

$$\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = 0.8\cos\theta$$

Letting $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$:

Then $$\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = 0.8\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

Which yields

$$\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = 0 \qquad \text{and} \qquad \cos\frac{\theta}{2} + \sin\frac{\theta}{2} = 1.25$$

The first equation yields $\frac{\theta}{2} = 45°$, $\theta = 90.0°$ ◀

In the second equation let $x = \cos\frac{\theta}{2}$ and the equation becomes $x + \sqrt{1 - x^2} = 1.25$

or $x^2 - 1.25x + 0.28125 = 0$

Solving $x = 0.95572$ and $x = 0.29428$

$$\frac{\theta}{2} = \cos^{-1}0.95572 \text{ and } \frac{\theta}{2} = \cos^{-1}0.29428$$

$\theta = 34.2°$ and $\theta = 145.8°$ ◀

Stability:

$$\frac{d^2V}{d\theta^2} = \frac{1}{2}ka^2\left[\sin\frac{\theta}{2} + \cos\frac{\theta}{2} - 2\left(1 - \frac{Wl}{ka^2}\right)\sin\theta\right]$$

$$= \frac{1}{2}ka^2\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2} - 1.6\sin\theta\right)$$

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$\theta = 34.2°$: $$\frac{d^2V}{d\theta^2} = \frac{1}{2}ka^2\left(0.2940 + 0.9558 - 0.8993\right) > 0$$ Stable ◀

$\theta = 90.0°$: $$\frac{d^2V}{d\theta^2} = \frac{1}{2}ka^2\left(0.707 + 0.707 - 1.6\right) < 0$$ Unstable ◀

$\theta = 145.8°$: $$\frac{d^2V}{d\theta^2} = \frac{1}{2}ka^2\left(0.9558 + 0.2940 - 0.8993\right) > 0$$ Stable ◀

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Chapter 10, Solution 80.

Note:

$$x_{SP_A} = r_A \sin\theta$$

$$x_{SP_B} = x_{SP_A}$$

$$x_{SP} = x_{SP_A} + x_{SP_B}$$

$$= 2r_A \sin\theta, \qquad r_A = 150 \text{ mm}$$

$$y_{\text{BLOCK}} = r\theta, \qquad r = 200 \text{ mm}$$

Potential Energy: $$V = \frac{1}{2}kx_{SP}^2 - mgy_{\text{BLOCK}}$$

$$= \frac{1}{2}k\left(2r_A \sin\theta\right)^2 - mgr\theta$$

$$= 2kr_A^2 \sin^2\theta - mgr\theta$$

$$\frac{dV}{d\theta} = 2kr_A^2\left(2\sin\theta\cos\theta\right) - mgr = 2kr_A^2 \sin 2\theta - mgr$$

$$\frac{d^2V}{d\theta^2} = 4kr_A^2 \cos 2\theta \qquad (1)$$

Equilibrium Condition:

$$\frac{dV}{d\theta} = 0: \qquad 2kr_A^2 \sin 2\theta - mgr = 0$$

Thus $$2\left(2000 \text{ N/m}\right)\left(0.15 \text{ m}\right)^2 \sin 2\theta - m\left(9.81 \text{ m/s}^2\right)\left(0.2 \text{ m}\right) = 0$$

or $$m = 45.872 \sin 2\theta \left(\text{kg}\right) \qquad (2)$$

(*a*) From Eq. (2), with $m \geq 0$: $0 \leq m \leq 45.9$ kg ◀

(*b*) For Stable equilibrium: $$\frac{d^2V}{d\theta^2} > 0$$

Then from Eq. (1) $\cos 2\theta > 0$ or $0 \leq \theta \leq 45.0°$ ◀

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Chapter 10, Solution 81.

Note:

$$x_{SP_A} = r_A \sin\theta$$

$$x_{SP_B} = x_{SP_A}$$

$$x_{SP} = x_{SP_A} + x_{SP_B}$$

$$= 2r_A \sin\theta, \qquad r_A = 150 \text{ mm}$$

$$y_{\text{BLOCK}} = r\theta, \qquad r = 200 \text{ mm}$$

Potential Energy: $V = \frac{1}{2}kx_{SP}^2 - mgy_{\text{BLOCK}}$

$$= \frac{1}{2}k\left(2r_A \sin\theta\right)^2 - mgr\theta$$

$$= 2kr_A^2 \sin^2\theta - mgr\theta$$

$$\frac{dV}{d\theta} = 2kr_A^2\left(2\sin\theta\cos\theta\right) - mgr = 2kr_A^2 \sin 2\theta - mgr$$

$$\frac{d^2V}{d\theta^2} = 4kr_A^2 \cos 2\theta \qquad (1)$$

Equilibrium Condition:

$\frac{dV}{d\theta} = 0$: $\quad 2kr_A^2 \sin 2\theta - mgr = 0$

Thus $\quad 2\left(2000 \text{ N/m}\right)\left(0.15 \text{ m}\right)^2 \sin 2\theta - m\left(9.81 \text{ m/s}^2\right)\left(0.2 \text{ m}\right) = 0$

or $\quad m = 45.872 \sin 2\theta \left(\text{kg}\right) \qquad (2)$

$$20 = 45.872 \sin 2\theta$$

Solving $\quad \theta = 12.9243° \quad$ and $\quad \theta = 77.076°$

Stability: Using Eq. (1)

$\theta = 12.9243°$: $\quad \frac{d^2V}{d\theta^2} = 4kr_A^2 \cos\left(2 \times 12.9243°\right) > 0 \qquad \therefore\ \theta = 12.92°$, Stable ◀

$\theta = 77.076°$: $\quad \frac{d^2V}{d\theta^2} = 4kr_A^2 \cos\left(2 \times 77.076°\right) < 0 \qquad \therefore\ \theta = 77.1°$, Unstable ◀

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Chapter 10, Solution 82.

(*a*) $s_{SP} = l\cos\theta_0 - l\cos\theta$

$= l\left(\cos\theta_0 - \cos\theta\right)$

Potential Energy:

$$V = \frac{1}{2}ks_{SP}^2 - mg\left(2l\sin\theta\right)$$

$$= \frac{1}{2}kl^2\left(\cos\theta_0 - \cos\theta\right)^2 - 2mgl\sin\theta$$

$$\frac{dV}{d\theta} = kl^2\left(\cos\theta_0 - \cos\theta\right)\sin\theta - 2mgl\cos\theta \qquad (1)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: \qquad kl^2\left(\cos\theta_0 - \cos\theta\right)\sin\theta - 2mgl\cos\theta = 0$$

Since $\cos\theta = 0$ is not a solution of the equation, we can divide all terms by $kl^2\cos\theta$ and write

$$\left(\cos\theta_0 - \cos\theta\right)\tan\theta = \frac{2mg}{kl} \qquad (2)$$

The spring is unstretched for $\theta = 0$ thus $\theta_0 = 0$ in Eq. (2) and we have

$$\left(1 - \cos\theta\right)\tan\theta = \frac{2mg}{kl} \blacktriangleleft$$

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(*b*) For the given data

$$(1-\cos\theta)\tan\theta = \frac{2(5\text{ kg})(9.81\text{ m/s}^2)}{(800\text{ N/m})(0.250\text{ m})} = 0.4905$$

Solving by trial and error: $\theta = 51.96°$, $\theta = 52.0°$ ◀

Stability: Differentiating Eq. (1):

$$\frac{d^2V}{d\theta^2} = kl^2\left(\sin^2\theta + \cos\theta_0\cos\theta - \cos^2\theta\right) + 2mgl\sin\theta$$

$$= kl^2\left(\cos\theta_0\cos\theta - \cos 2\theta + \frac{2mg}{kl}\sin\theta\right) \qquad (3)$$

For $\theta_0 = 0$, $\theta = 51.96°$, and the given data

$$\frac{d^2V}{d\theta^2} = kl^2\left(\cos 51.96° - \cos 103.92° + 0.4905\sin 51.96°\right)$$

$$= 1.2431\,kl^2 > 0$$

$\theta = 52.0°$, Stable ◀

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Chapter 10, Solution 83.

Law of Sines: $$\frac{y_A}{\sin(90° + \beta - \theta)} = \frac{l}{\sin(90° - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{l}{\cos\beta}$$

$$y_A = l\frac{\cos(\theta - \beta)}{\cos\beta}$$

From Eq. (1): $$y_B = l\frac{\cos(\theta - \beta)}{\cos\beta} - l\cos\theta$$

Potential Energy:

$$V = -Py_B - Qy_A = -P\left[\frac{l\cos(\theta - \beta)}{\cos\beta} - l\cos\theta\right] - Ql\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -Pl\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + l\sin\theta\right] + Ql\frac{\sin(\theta - \beta)}{\cos\beta}$$

Equilibrium: $$\frac{dV}{d\theta} = 0: \qquad (P + Q)\sin(\theta - \beta) = P\sin\theta\cos\beta$$

or $$(P + Q)(\sin\theta\cos\beta - \cos\theta\sin\beta) = P\sin\theta\cos\beta$$

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Substracting $P(\sin\theta\cos\beta)$ from each member yields

$$-(P+Q)\cos\theta\sin\beta + Q\sin\theta\cos\beta = 0$$

or $$\tan\theta = \frac{P+Q}{Q}\tan\beta$$

Given data: $\beta = 30°,\ P = Q = 400\text{ N}$

$$\tan\theta = \frac{(400\text{ N}) + (400\text{ N})}{(400\text{ N})}\tan 30°$$

$$\tan\theta = 2(0.57735) = 1.1547$$

$\theta = 49.1°$ ◀

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Chapter 10, Solution 84.

(1)

Law of Sines: $$\frac{y_A}{\sin(90° + \beta - \theta)} = \frac{l}{\sin(90° - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{l}{\cos\beta}$$

$$y_A = l\frac{\cos(\theta - \beta)}{\cos\beta}$$

From Eq. (1): $$y_B = l\frac{\cos(\theta - \beta)}{\cos\beta} - l\cos\theta$$

Potential Energy:

$$V = -Py_B - Qy_A = -P\left[l\frac{\cos(\theta - \beta)}{\cos\beta} - l\cos\theta\right] - Ql\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -Pl\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + l\sin\theta\right] + Ql\frac{\sin(\theta - \beta)}{\cos\beta}$$

Equilibrium: $$\frac{dV}{d\theta} = 0: \qquad (P + Q)\sin(\theta - \beta) = P\sin\theta\cos\beta$$

or $$(P + Q)(\sin\theta\cos\beta - \cos\theta\sin\beta) = P\sin\theta\cos\beta$$

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Substracting $P(\sin\theta\cos\beta)$ from each member yields

$$-(P+Q)\cos\theta\sin\beta + Q\sin\theta\cos\beta = 0$$

or $$\tan\theta = \frac{P+Q}{Q}\tan\beta$$

Given data: $P = 100\text{ N},\ Q = 25\text{ N},\ \beta = 30°$

$$\tan\theta = \frac{(100\text{ N}) + (25\text{ N})}{(25\text{ N})}\tan 30°$$

$$= 5(0.57735) = 2.8868$$

$\theta = 70.9°$ ◀

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Chapter 10, Solution 85.

First note, by Law of Cosines:

$$d^2 = (16)^2 + \left(16\sin\frac{\theta}{2}\right)^2 - 2(16)\left(16\sin\frac{\theta}{2}\right)\cos\frac{\theta}{2}$$

$$d = 16\sqrt{1+\sin^2\frac{\theta}{2} - \sin\theta}\ \text{in.}$$

Also note $\qquad x_{SP} = r_A\theta$

Potential Energy:

$$V = \frac{1}{2}kx_{SP}^2 + W_D y_D$$

$$= \frac{1}{2}k\left(r_A\theta\right)^2 + W_D\left[\left(y_D\right)_0 - \left(16-d\right)\sin 60°\right]$$

$$= \frac{1}{2}kr_A^2\theta^2 + W_D\left[\left(y_D\right)_0 - \left(16 - 16\sqrt{1+\sin^2\frac{\theta}{2} - \sin\theta}\right)\sin 60°\right]$$

Equilibrium condition:

$$\frac{dV}{d\theta} = 0: \qquad kr_A^2\theta + 16W_D\sin 60°\frac{\frac{1}{2}\left(\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \cos\theta\right)}{\sqrt{1+\sin^2\frac{\theta}{2} - \sin\theta}} = 0$$

$$kr_A^2\theta + 4W_D\sin 60°\frac{\sin\theta - 2\cos\theta}{\sqrt{1+\sin^2\frac{\theta}{2} - \sin\theta}} = 0$$

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Substituting,

$$(2.5 \text{ lb/in.})(2 \text{ in.})^2 \theta \sqrt{1 + \sin^2 \frac{\theta}{2} - \sin\theta} + 4(25 \text{ lb}) \sin 60° (\sin\theta - 2\cos\theta) = 0$$

or $$\theta \sqrt{1 + \sin^2 \frac{\theta}{2} - \sin\theta} + 8.6603(\sin\theta - 2\cos\theta) = 0$$

Solving numerically, $\theta = 1.08572$ rad

or $\theta = 62.2°$ ◀

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Chapter 10, Solution 86.

First note that cable tension is uniform throughout, hence

$$F_{SP_1} = F_{SP_2}$$

$$k_1 x_1 = k_2 x_2$$

$$x_2 = \frac{k_1}{k_2} x_1 = \frac{6 \text{ lb/in.}}{3 \text{ lb/in.}} x_1$$

$$x_2 = 2x_1$$

Now, with C midway between the pulleys,

$$2d = 16 \text{ in.} + x_1 + x_2$$

$$d = 8 + \frac{1}{2}(x_1 + x_2)$$

Then $\quad y^2 = d^2 - 8^2$

$$= \left[8 + \frac{1}{2}(x_1 + x_2)\right]^2 - 8^2$$

$$= 8(x_1 + x_2) + \frac{1}{4}(x_1 + x_2)^2$$

$$= 8(x_1 + 2x_1) + \frac{1}{4}(x_1 + 2x_1)^2$$

$$= 24x_1 + \frac{9}{4}x_1^2 \left(\text{in}^2\right)$$

$$y = \frac{1}{2}\sqrt{96x_1 + 9x_1^2}$$

Potential Energy:

$$V = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2 - Wy$$

$$= \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 (2x_1)^2 - W\left(\frac{1}{2}\sqrt{96x_1 + 9x_1^2}\right)$$

$$= \frac{1}{2}(k_1 + 4k_2)x_1^2 - W\left(\frac{1}{2}\sqrt{96x_1 + 9x_1^2}\right)$$

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Equilibrium condition:

$$\frac{dV}{dx_1} = 0: \qquad (k_1 + 4k_2)x_1 - W\left(\frac{96 + 18x_1}{4\sqrt{96x_1 + 9x_1^2}}\right) = 0$$

or $$\left[6 + 4(3)\right] \text{lb/in.} \times \left[(x_1) \text{ in.}\right]\left[\left(\sqrt{96x_1 + 9x_1^2}\right) \text{ in.}\right] - \frac{1}{4}(25 \text{ lb})\left[(96 + 18x_1) \text{ in.}\right] = 0$$

or $$18x_1\sqrt{96x_1 + 9x_1^2} - 600 - 112.5x_1 = 0$$

Solving, $x_1 = 2.7677$ in.

Then $$y = \frac{1}{2}\sqrt{96(2.7677) + 9(2.7677)^2}$$

$$= 9.1466 \text{ in.}$$

$y = 9.15$ in. ◀

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Chapter 10, Solution 87.

Stretch of Spring

$$s = AB - r$$

$$s = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

Potential Energy: $V = \frac{1}{2}ks^2 - Wr\sin 2\theta \qquad W = mg$

$$V = \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta - 2Wr\cos 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \qquad -kr^2(2\cos\theta - 1)\sin\theta - Wr\cos 2\theta = 0$$

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -\frac{W}{kr}$$

Now $\frac{W}{kr} = \frac{(20\text{ kg})(9.81\text{ m/s}^2)}{(3000\text{ N/m})(0.180\text{ m})} = 0.36333$

Then $\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -0.36333$

Solving numerically, $\theta = 0.9580\text{ rad} = 54.9°$ $\qquad \theta = 54.9°$ ◀

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Chapter 10, Solution 88.

Have $l_{AD} = 2r\sin\theta$

Then $y_A = -L_{AD}\sin\left[\left(90° - \theta\right) - 45°\right]$

$$= -2r\sin\theta\sin\left(45° - \theta\right)$$

Also for spring

$$s = l_{AB} - r$$

$$= 2r\cos\theta - r$$

$$= r\left(2\cos\theta - 1\right)$$

Potential Energy: $V = V_{SP} + V_m$

$$= \frac{1}{2}ks^2 + mgy_A$$

$$= \frac{1}{2}kr^2\left(2\cos\theta - 1\right)^2 - 2mgr\sin\theta\sin\left(45° - \theta\right)$$

For Equilibrium:

$$\frac{dV}{d\theta} = 0:$$

$$kr^2\left(-2\sin\theta\right)\left(2\cos\theta - 1\right) - 2mgr\left[\cos\theta\sin\left(45° - \theta\right) - \sin\theta\cos\left(45° - \theta\right)\right] = 0$$

or $$-kr\sin\theta\left(2\cos\theta - 1\right) - mg\sin\left(45° - 2\theta\right) = 0$$

or $$\frac{\sin\theta\left(2\cos\theta - 1\right)}{\sin\left(45° - 2\theta\right)} = -\frac{mg}{kr} = -\frac{\left(20\text{ kg}\right)\left(9.81\text{ m/s}^2\right)}{\left(3000\text{ N/m}\right)\left(0.18\text{ m}\right)}$$

$$= -0.36333$$

Solving numerically $\theta = 46.6°$ ◀

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Chapter 10, Solution 89.

Have $\quad x_C = d\sin\theta \qquad y_B = h\cos\theta$

Potential Energy: $\quad V = 2\left(\frac{1}{2}kx_C^2\right) + Wy_B$

$$= kd^2\sin^2\theta + Wh\cos\theta$$

Then $\quad \dfrac{dV}{d\theta} = 2kd^2\sin\theta\cos\theta - Wh\sin\theta$

$$= kd^2\sin 2\theta - Wh\sin\theta$$

and $\quad \dfrac{d^2V}{d\theta^2} = 2kd^2\cos 2\theta - Wh\cos\theta \qquad (1)$

For equilibrium position $\theta = 0$ to be stable, we must have

$$\frac{d^2V}{d\theta^2} = 2kd^2 - Wh > 0$$

or

$$kd^2 > \frac{1}{2}Wh \qquad (2)$$

Note: For $kd^2 = \frac{1}{2}Wh$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write

$$\frac{d^3V}{d\theta^3} = -4kd^2\sin 2\theta + Wh\sin\theta = 0 \qquad \text{for } \theta = 0$$

$$\frac{d^4V}{d\theta^4} = -8kd^2\cos 2\theta + Wh\cos\theta$$

For $\theta = 0$: $\quad \dfrac{d^4V}{d\theta^4} = -8kd^2 + Wh$

Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

With $\quad W = 160$ lb, $h = 50$ in., and $d = 24$ in.

Equation (2) gives $\quad k(24\text{ in.})^2 > \frac{1}{2}(160\text{ lb})(50\text{ in.})$

or $\quad k > 6.944$ lb/in.

$k > 6.94$ lb/in. ◀

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Chapter 10, Solution 90.

Using Equation (2) of problem 10.89 with

$h = 30$ in., $k = 4$ lb/in., and $W = 40$ lb

$$kd^2 > \frac{1}{2}Wh$$

or $$(4 \text{ lb/in.})d^2 > \frac{1}{2}(40 \text{ lb})(30 \text{ in.})$$

$$d^2 > 150 \text{ in}^2$$

or $$d > 12.247 \text{ in.}$$

Smallest $d = 12.25$ in. ◀

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Chapter 10, Solution 91.

Consider a small clockwise rotation θ of the plate about its center.

Then $$V = 2V_P + 4V_{SP}$$

where $$V_P = P\left(\frac{a}{2}\cos\theta\right)$$

$$= \frac{1}{2}\left(Pa\cos\theta\right)$$

and $$V_{SP} = \frac{1}{2}ky_{SP}^2$$

Now $$d = \sqrt{\left(\frac{a}{2}\right)^2 + a^2}$$

$$= \frac{a}{2}\sqrt{5}$$

and $$\alpha = 180° - \left[\phi + \left(90° - \frac{\theta}{2}\right)\right]$$

$$= 90° - \left(\phi - \frac{\theta}{2}\right)$$

Then $$y_{SP} = \left[\left(\frac{a}{2}\sqrt{5}\right)\theta\right]\sin\alpha$$

$$= \frac{a}{2}\theta\sqrt{5}\sin\left[90° - \left(\phi - \frac{\theta}{2}\right)\right]$$

$$= \frac{a}{2}\theta\sqrt{5}\cos\left(\phi - \frac{\theta}{2}\right)$$

continued

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and
$$V_{SP} = \frac{1}{2}k\left[\frac{a}{2}\theta\sqrt{5}\cos\left(\phi - \frac{\theta}{2}\right)\right]^2$$
$$= \frac{5}{8}ka^2\theta^2\cos^2\left(\phi - \frac{\theta}{2}\right)$$
$$\therefore\ V = Pa\cos\theta + \frac{5}{2}ka^2\theta^2\cos^2\left(\phi - \frac{\theta}{2}\right)$$

Then

$$\frac{dV}{d\theta} = -Pa\sin\theta + \frac{5}{8}ka^2\left[2\theta\cos^2\left(\phi - \frac{\theta}{2}\right)\right.$$
$$\left. + \theta^2\left(-\frac{1}{2}\right)\cos\left(\phi - \frac{\theta}{2}\right)\sin\left(\phi - \frac{\theta}{2}\right)\right]$$
$$= -Pa\sin\theta + \frac{5}{2}ka^2\left[2\theta\cos^2\left(\phi - \frac{\theta}{2}\right) + \frac{1}{2}\theta^2\sin(2\phi - \theta)\right]$$

$$\frac{d^2V}{d\theta^2} = -Pa\cos\theta + \frac{5}{2}ka^2\left[2\cos^2\left(\phi - \frac{\theta}{2}\right)\right.$$
$$- 2\theta\left(-\frac{1}{2}\right)\cos\left(\phi - \frac{\theta}{2}\right)\sin\left(\phi - \frac{\theta}{2}\right) + \theta\sin(2\phi - \theta)$$
$$\left. - \frac{1}{2}\theta^2\cos(2\phi - \theta)\right]$$
$$= -Pa\cos\theta + \frac{5}{2}ka^2\left[2\cos^2\left(\phi - \frac{\theta}{2}\right) + \frac{3}{2}\theta\sin(2\phi - \theta)\right.$$
$$\left. - \frac{1}{2}\theta^2\cos(2\phi - \theta)\right]$$

$$\frac{d^3V}{d\theta^3} = Pa\sin\theta + \frac{5}{2}ka^2\left[4\left(-\frac{1}{2}\right)\cos\left(\phi - \frac{\theta}{2}\right)\sin\left(\phi - \frac{\theta}{2}\right) + \frac{3}{2}\sin(2\phi - \theta)\right.$$
$$\left. - \frac{3}{2}\theta\cos(2\phi - \theta) - \theta\cos(2\phi - \theta) + \frac{1}{2}\theta^2\sin(2\phi - \theta)\right]$$
$$= Pa\sin\theta + \frac{5}{2}ka^2\left[\frac{1}{2}\sin(2\phi - \theta) - \frac{5}{2}\theta\cos(2\phi - \theta)\right.$$
$$\left. + \frac{1}{2}\theta^2\sin(2\phi - \theta)\right]$$

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When $\theta = 0$, $\dfrac{dV}{d\theta} = 0$ for all values of P.

For stable equilibrium when $\theta = 0$, require

$$\frac{d^2V}{d\theta^2} > 0: \quad -Pa + \frac{5}{2}ka^2\left(2\cos^2\phi\right) > 0$$

Now, when $\theta = 0$,

$$\cos\phi = \frac{\frac{a}{2}}{\frac{a}{2}\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\therefore \quad -Pa + 5ka^2\left(\frac{1}{5}\right) > 0$$

or

$$P < ka$$

When $P = ka$ $\left(\text{for } \theta = 0\right)$:

$$\frac{dV}{d\theta} = 0$$

$$\frac{d^2V}{d\theta^2} = 0$$

$$\frac{d^3V}{d\theta^3} = \frac{5}{4}ka^2\sin 2\phi > 0 \Rightarrow \text{unstable}$$

$\therefore$ Stable equilibrium for $0 \le P < ka$ ◀

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Chapter 10, Solution 92.

Spring: $$s = \frac{L}{3}\sin\phi = \frac{2L}{3}\sin\theta$$

For small values of ϕ and θ: $\phi = 2\theta$

$$V = P\left(\frac{L}{3}\cos\phi + \frac{2L}{3}\cos\theta\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}\left(\cos 2\theta + 2\cos\theta\right) + \frac{1}{2}k\left(\frac{2L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}\left(-2\sin 2\theta - 2\sin\theta\right) + \frac{2}{9}kL^2\sin\theta\cos\theta$$

$$= -\frac{PL}{3}\left(2\sin 2\theta + 2\sin\theta\right) + \frac{2}{9}kL^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{PL}{3}\left(4\cos 2\theta + 2\cos\theta\right) + \frac{4}{9}kL^2\cos 2\theta$$

When $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -\frac{6PL}{3} + \frac{4}{9}kL^2$$

For stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -2PL + \frac{4}{9}kL^2 > 0$$

$$0 \le P < \frac{2}{9}kL \quad ◀$$

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Chapter 10, Solution 93.

From geometry:

$$x_C = -a\sin\theta = -2a\sin\phi$$

For small values of θ, ϕ

$$\theta = 2\phi$$

$$\text{or} \quad \phi = \frac{1}{2}\theta$$

$$y_A = a\cos\theta + 3a\cos\phi$$

$$= a\left(\cos\theta + 3\cos\frac{\theta}{2}\right)$$

For spring:

$$s = x_C = -a\sin\theta$$

Potential Energy:

$$V = V_{SP} + V_P$$

$$= \frac{1}{2}k\left(-a\sin\theta\right)^2 + Pa\left(\cos\theta + 3\cos\frac{\theta}{2}\right)$$

$$\frac{dV}{d\theta} = ka^2\cos\theta\sin\theta - Pa\left(\sin\theta + \frac{3}{2}\sin\frac{\theta}{2}\right)$$

$$\frac{d^2V}{d\theta^2} = ka^2\left(-\sin^2\theta + \cos^2\theta\right) - Pa\left(\cos\theta + \frac{3}{4}\cos\frac{\theta}{2}\right)$$

For stable equilibrium: $\dfrac{d^2V}{d\theta^2} > 0$

Then, with $\theta = 0$

$$ka^2 - Pa\left(1 + \frac{3}{4}\right) > 0$$

or $\quad P < \dfrac{4}{7}ka$ ◀

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Chapter 10, Solution 94.

Consider a small disturbance of the system defined by the angle θ. Have

$$x_C = 2a\sin\theta = a\sin\phi$$

For small θ: $\qquad 2\theta = \phi$

Now, the Potential Energy is

$$V = \frac{1}{2}kx_B^2 + Py_E$$

where $\qquad x_B = a\sin\theta$

and

$$\begin{aligned} y_E &= y_C + y_{E/C} \\ &= 2a\cos\theta + 2a\cos\phi \\ &= 2a\left(\cos\theta + \cos 2\theta\right) \end{aligned}$$

Then

$$V = \frac{1}{2}ka^2\sin^2\theta + 2Pa\left(\cos\theta + \cos 2\theta\right)$$

and

$$\begin{aligned} \frac{dV}{d\theta} &= \frac{1}{2}ka^2\left(2\sin\theta\cos\theta\right) - 2Pa\left(\sin\theta + 2\sin 2\theta\right) \\ &= \frac{1}{2}ka^2\sin 2\theta - 2Pa\left(\sin\theta + 2\sin 2\theta\right) \end{aligned}$$

$$\frac{d^2V}{d\theta^2} = ka^2\cos 2\theta - 2Pa\left(\cos\theta + 4\cos 2\theta\right)$$

For $\theta = 0$ and for stable equilibrium:

$$\frac{d^2V}{d\theta^2} > 0$$

or

$$ka^2 - 2Pa\left(1 + 4\right) > 0$$

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or $$P < \frac{1}{10}ka$$

$$\therefore\ 0 \le P < \frac{1}{10}ka \blacktriangleleft$$

Check stability for $$P = \frac{ka}{10}$$

$$\frac{d^3V}{d\theta^3} = -2ka^2 \sin 2\theta + 2Pa\left(\sin\theta + 8\sin 2\theta\right)$$

$$\frac{d^4V}{d\theta^4} = -4ka^2 \cos 2\theta + 2Pa\left(\cos\theta + 16\cos 2\theta\right)$$

Then, with $\theta = 0$ and $P = \frac{ka}{10}$

$$\frac{dV}{d\theta} = 0$$

$$\frac{d^2V}{d\theta^2} = 0$$

$$\frac{d^3V}{d\theta^3} = 0$$

$$\frac{d^4V}{d\theta^4} = -4ka^2 + 2\left(\frac{1}{10}ka\right)(a)(1+16)$$

$$= -0.6ka^2 < 0 \Rightarrow \text{Unstable}$$

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Chapter 10, Solution 95.

Displacements:

$$x_G = x_C = a\sin\theta$$

$$\tan\phi = \frac{x_G}{a + c - a\cos\theta} = \frac{a\sin\theta}{a + c - a\cos\theta} = \frac{\sin\theta}{1 + \dfrac{c}{a} - \cos\theta}$$

Differentiating both sides with respect to θ:

$$\left(\frac{1}{\cos^2\phi}\right)\frac{d\phi}{d\theta} = \frac{\cos\theta\left(1 + \dfrac{c}{a} - \cos\theta\right) - \sin\theta(\sin\theta)}{\left(1 + \dfrac{c}{a} - \cos\theta\right)^2}$$

For $\theta = \phi = 0$:

$$\frac{d\phi}{d\theta} = \frac{\dfrac{c}{a}}{\left(\dfrac{c}{a}\right)^2} = \frac{a}{c} \qquad (1)$$

Potential Energy:

$$V = W_1 y_1 + W_2 y_2 = m_1 gb(\cos\phi - 1) + m_2 ga(1 - \cos\theta)$$

$$\frac{dV}{d\theta} = -m_1 gb\sin\phi\frac{d\phi}{d\theta} + m_2 ga\sin\theta$$

$$\frac{d^2V}{d\theta^2} = -m_1 gb\cos\phi\left(\frac{d\phi}{d\theta}\right)^2 - m_1 gb\sin\phi\left(\frac{d^2\phi}{d\theta^2}\right) + m_2 ga\cos\theta$$

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For $\theta = \phi = 0$, and recalling Eq. (1),

$$\frac{d^2V}{d\theta^2} = -m_1 gb\left(\frac{a}{c}\right)^2 - 0 + m_2 ga$$

For stability we need $\quad \dfrac{d^2V}{d\theta^2} > 0 \quad$ or $\quad m_1 g\dfrac{ba^2}{c^2} < m_2 ga$

Thus $\quad m_1 < m_2\dfrac{c^2}{ab}$

The smallest value of m_1 for stable equilibrium is thus

$$m_1 = m_2\frac{c^2}{ab} \quad ◀$$

Note: To determine whether the equilibrium is stable when m_1 has the exact value we found, we should determine the values of the derivatives $\dfrac{d^3V}{d\theta^3}$ and $\dfrac{d^4V}{d\theta^4}$ for $m_1 = \dfrac{m_2c^2}{(ab)}$. In practice, however we shall want to keep m_1 below this value.

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Chapter 10, Solution 96.

First note $\qquad A = a\sin\theta = b\sin\phi$

For small values of θ and ϕ: $\qquad a\theta = b\phi$

$$\phi = \frac{a}{b}\theta$$

$$V = P(a+b)\cos\phi - 2Q(a+b)\cos\theta$$

$$= (a+b)\left[P\cos\left(\frac{a}{b}\theta\right) - 2Q\cos\theta\right]$$

$$\frac{dV}{d\theta} = (a+b)\left[-\frac{a}{b}P\sin\left(\frac{a}{b}\theta\right) + 2Q\sin\theta\right]$$

$$\frac{d^2V}{d\theta^2} = (a+b)\left[-\frac{a^2}{b^2}P\cos\left(\frac{a}{b}\theta\right) + 2Q\cos\theta\right]$$

When $\theta = 0$: $\qquad \dfrac{d^2V}{d\theta^2} = (a+b)\left(-\dfrac{a^2}{b^2}P + 2Q\right)$

Stability: $\qquad \dfrac{d^2V}{d\theta^2} > 0: \; -\dfrac{a^2}{b^2}P + 2Q > 0$

$$P < 2\frac{b^2}{a^2}Q \qquad (1)$$

or

$$Q > \frac{a^2}{2b^2}P \qquad (2)$$

With $\qquad P = 600$ N, $\quad a = 480$ mm and $b = 400$ mm

$$Q > \frac{1}{2}\frac{(480\text{ mm})^2}{(400\text{ mm})^2}(600\text{ N}) = 432\text{ N}$$

$Q > 432$ N ◀

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Chapter 10, Solution 97.

Have $\quad x_B = l\sin\theta$

$$x_C = l\sin\theta_1 + l\sin\theta_2$$

$$y_C = l\cos\theta_1 + l\cos\theta_2$$

$$V = Py_C + \frac{1}{2}kx_B^2 + \frac{1}{2}kx_C^2$$

or $\quad V = Pl\left(\cos\theta_1 + \cos\theta_2\right) + \frac{1}{2}kl^2\left[\sin^2\theta_1 + \left(\sin\theta_1 + \sin\theta_2\right)^2\right]$

For small values of θ_1 and θ_2:

$$\sin\theta_1 \approx \theta_1, \qquad \sin\theta_2 \approx \theta_2, \qquad \cos\theta_1 \approx 1 - \frac{1}{2}\theta_1^2, \qquad \cos\theta_2 \approx 1 - \frac{1}{2}\theta_2^2$$

Then $\quad V = Pl\left(1 - \frac{\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right) + \frac{1}{2}kl^2\left[\theta_1^2 + \left(\theta_1 + \theta_2\right)^2\right]$

and $\quad \frac{\partial V}{\partial\theta_1} = -Pl\theta_1 + kl^2\left[\theta_1 + \left(\theta_1 + \theta_2\right)\right]$

$$\frac{\partial V}{\partial\theta_2} = -Pl\theta_2 + kl^2\left(\theta_1 + \theta_2\right)$$

$$\frac{\partial^2 V}{\partial\theta_1^2} = -Pl + 2kl^2 \qquad \frac{\partial^2 V}{\partial\theta_2^2} = -Pl + kl^2$$

$$\frac{\partial^2 V}{\partial\theta_1\partial\theta_2} = kl^2$$

continued

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Stability Conditions for stability (see page 583).

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial V}{\partial \theta_1} = \dfrac{\partial V}{\partial \theta_2} = 0$ (condition satisfied)

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0$$

Substituting,

$$\left(kl^2\right)^2 - \left(-Pl + 2kl^2\right)\left(-Pl + kl\right) < 0$$

$$k^2l^4 - P^2l^2 + 3Pkl^3 - 2k^2l^4 < 0$$

$$P^2 - 3klP + k^2l^2 > 0$$

Solving,

$$P < \frac{3-\sqrt{5}}{2}kl \quad \text{or} \quad P > \frac{3+\sqrt{5}}{2}kl$$

or

$$P < 0.382kl \quad \text{or} \quad P > 2.62kl$$

$$\frac{\partial^2 V}{\partial \theta_1^2} > 0: \quad -Pl + 2kl^2 > 0$$

or

$$P < \frac{1}{2}kl$$

$$\frac{\partial^2 V}{\partial \theta_2^2} > 0: \quad -Pl + kl^2 > 0$$

or

$$P < kl$$

Therefore, all conditions for stable equilibrium are satisfied when

$0 \le P < 0.382kl$ ◀

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Chapter 10, Solution 98.

From the analysis of Problem 10.97 with

$$l = 400 \text{ mm} \qquad \text{and} \qquad k = 1.25 \text{ kN/m}$$

$$P < 0.382kl = 0.382(1250 \text{ N/m})(0.4 \text{ m}) = 191 \text{ N}$$

$$0 \le P < 191.0 \text{ N} \blacktriangleleft$$

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Chapter 10, Solution 99.

Have $$V = \frac{1}{2}k\left(a\theta_2\right)^2 + \frac{1}{2}k\left(a\sin\theta_1 + a\sin\theta_2\right)^2 + P\left(2a\cos\theta_1 + a\cos\theta_2\right)$$

Then $$\frac{\partial V}{\partial \theta_1} = ka^2\left(\sin\theta_1 + \sin\theta_2\right)\cos\theta_1 - 2Pa\sin\theta_1$$

$$= ka^2\left(\frac{1}{2}\sin 2\theta_1 + \cos\theta_1\sin\theta_2\right) - 2Pa\sin\theta_1$$

and $$\frac{\partial^2 V}{\partial \theta_1^2} = ka^2\left(\cos 2\theta_1 - \sin\theta_1\sin\theta_2\right) - 2Pa\cos\theta_1$$

$$\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = ka^2\cos\theta_1\cos\theta_2$$

Also $$\frac{\partial V}{\partial \theta_2} = ka^2\theta_2 + ka^2\left(\sin\theta_1 + \sin\theta_2\right)\cos\theta_2 - Pa\sin\theta_2$$

$$= ka^2\theta_2 + ka^2\left(\sin\theta_1\cos\theta_2 + \frac{1}{2}\sin 2\theta_2\right) - Pa\sin\theta_2$$

and $$\frac{\partial^2 V}{\partial \theta_2^2} = ka^2 + ka^2\left(-\sin\theta_1\sin\theta_2 + \cos 2\theta_2\right) - Pa\cos\theta_2$$

When $$\theta_1 = \theta_2 = 0$$

$$\frac{\partial V}{\partial \theta_1} = 0 \qquad \frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = ka^2 \qquad \frac{\partial V}{\partial \theta_2} = 0$$

$$\frac{\partial^2 V}{\partial \theta_1^2} = ka^2 - 2Pa \qquad \frac{\partial^2 V}{\partial \theta_2^2} = ka^2 + ka^2 - Pa = 2ka^2 - Pa$$

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Apply Equations 10.24 $\dfrac{\partial V}{\partial \theta_1} = 0$: condition satisfied

$\dfrac{\partial V}{\partial \theta_2} = 0$: condition satisfied

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0: \ \left(ka^2\right)^2 - \left(ka^2 - 2Pa\right)\left(2ka^2 - Pa\right) < 0$$

or $$k^2a^2 - \left(ka - 2P\right)\left(2ka - P\right) < 0$$

Expanding $$k^2a^2 - 2k^2a^2 + 5kaP - 2P^2 < 0$$

or $$2P^2 - 5kaP + k^2a^2 > 0$$

or $$P < \frac{5 - \sqrt{17}}{4}ka \quad \text{and} \quad P > \frac{5 + \sqrt{17}}{4}ka$$

or $$P < 0.21922ka \quad \text{and} \quad P > 2.2808ka$$

Also $$\frac{\delta^2 V}{\delta \theta_1^2} > 0: \ ka^2 - 2Pa > 0 \quad \text{or} \quad \frac{\delta^2 V}{\delta \theta_2^2} > 0: \ 2ka^2 - Pa > 0$$

or $$P < \frac{1}{2}ka \quad \text{or} \quad P < 2ka$$

$\therefore$ For stable equilibrium when $\theta_1 = \theta_2 = 0$:

$$0 \le P < 0.219ka \blacktriangleleft$$

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Chapter 10, Solution 100.

Potential Energy:

$$V = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + Py$$

$$= \frac{1}{2}k\left(a\sin\theta_1 + a\sin\theta_2\right)^2 + \frac{1}{2}k\left(a\theta_2\right)^2 + P\left(2a\cos\theta_1 + a\cos\theta_2\right)$$

Then

$$\frac{\partial V}{\partial \theta_1} = ka^2\left(\sin\theta_1 + \sin\theta_2\right)\cos\theta_1 - 2Pa\sin\theta_1$$

$$= ka^2\left(\frac{1}{2}\sin 2\theta_1 + \cos\theta_1\sin\theta_2\right) - 2Pa\sin\theta_1$$

$$\frac{\partial^2 V}{\partial \theta_1^2} = ka^2\left(\cos 2\theta_1 - \sin\theta_1\sin\theta_2\right) - 2Pa\cos\theta_1$$

$$\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = ka^2\cos\theta_1\cos\theta_2$$

$$\frac{\partial V}{\partial \theta_2} = ka^2\left(\sin\theta_1 + \sin\theta_2\right)\cos\theta_2 + ka^2\theta_2 - Pa\sin\theta_2$$

$$= ka^2\left(\sin\theta_1\cos\theta_2 + \frac{1}{2}\sin 2\theta_2\right) + ka^2\theta_2 - Pa\sin\theta_2$$

$$\frac{\partial^2 V}{\partial \theta_2^2} = ka^2\left(-\sin\theta_1\sin\theta_2 + \cos 2\theta_2\right) + ka^2 - Pa\cos\theta_2$$

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When $\theta_1 = \theta_2 = 0$

$$\frac{\partial V}{\partial \theta_1} = 0 \qquad \frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = ka^2 \qquad \frac{\partial V}{\partial \theta_2} = 0$$

$$\frac{\partial^2 V}{\partial \theta_1^2} = ka^2 - 2Pa$$

$$\frac{\partial^2 V}{\partial \theta_2^2} = ka^2 + ka^2 - Pa = 2ka^2 - Pa$$

Apply Eq. 10.24:

$$\frac{\partial V}{\partial \theta_1} = 0: \quad \text{Condition satisfied}$$

$$\frac{\partial V}{\partial \theta_2} = 0: \quad \text{Condition satisfied}$$

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0:$$

$$\left(ka^2\right)^2 - \left(ka^2 - 2Pa\right)\left(2ka^2 - Pa\right) < 0$$

$$k^2a^2 - \left(ka - 2P\right)\left(2ka - P\right) < 0$$

$$k^2a^2 - 2k^2a^2 + 5Pka - 2P^2 < 0$$

$$2P^2 - 5Pka + k^2a^2 > 0$$

$$P < \frac{5 - \sqrt{17}}{4}ka \quad \text{and} \quad P > \frac{5 + \sqrt{17}}{4}ka$$

or $P < 0.21922\,ka$ and $P > 2.2808\,ka$

$$\frac{\partial^2 V}{\partial \theta_1^2} > 0 \quad \text{or} \quad \frac{\partial^2 V}{\partial \theta_2^2} < 0:$$

$$ka^2 - 2Pa > 0 \qquad 2ka^2 - Pa > 0$$

or $P < \frac{1}{2}ka$ or $P < 2ka$

continued

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Thus, for stable equilibrium when $\theta_1 = \theta_2 = 0$:

$$0 \le P < 0.21922\ ka$$

with $k = 2$ kN/m and $a = 350$ mm

$$0 \le P < 0.21922(2000\ \text{N/m})(0.35\ \text{m})$$

or $\quad 0 \le P < 153.5\ \text{N}$ ◀

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Chapter 10, Solution 101.

From sketch

$$y_A = 4y_C$$

Thus, $$\delta y_A = 4\delta y_C$$

(*a*) Virtual Work:

$$\delta U = 0: \quad P\delta y_A - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

$$F = 300 \text{ N}: \quad P = \frac{1}{4}(300 \text{ N}) = 75 \text{ N}$$

$$\mathbf{P} = 75.0 \text{ N} \uparrow \blacktriangleleft$$

(*b*) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0: \quad R + P - F = 0$$

$$R + 75 \text{ N} - 300 \text{ N} = 0$$

$$\mathbf{R} = 225 \text{ N} \downarrow \blacktriangleleft$$

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Chapter 10, Solution 102.

First note, by the Law of Cosines

$$DB^2 = (3\text{ ft})^2 + (2\text{ ft})^2 - 2(3\text{ ft})(2\text{ ft})\cos\theta$$

$$= [13 - 12\cos\theta](\text{ft}^2)$$

$$DB = \sqrt{13 - 12\cos\theta}$$

Then

$$\delta_B = \delta DB = \frac{1}{2}\frac{(12)(\sin\theta)}{\sqrt{13 - 12\cos\theta}}\delta\theta$$

or

$$\delta_B = \frac{6\sin\theta}{\sqrt{13 - 12\cos\theta}}\delta\theta$$

Also $\quad y_A = 4.5\cos\theta$

Then $\quad \delta y_A = -4.5\sin\theta\delta\theta$

Virtual Work

$$\delta U = 0: \quad -(8\text{ kips})\delta y_A - F_{DB}\delta_B = 0$$

Then

$$-8(-4.5\sin\theta)\delta\theta - F_{DB}\left(\frac{6\sin\theta}{\sqrt{13 - 12\cos\theta}}\right)\delta\theta = 0$$

or

$$F_{DB} = \frac{(8)(4.5\sin\theta)}{6\sin\theta}\sqrt{13 - 12\cos\theta}$$

or

$$F_{DB} = 6\sqrt{13 - 12\cos\theta}$$

For

$$\theta = 70°$$

$$F_{DB} = 17.895\text{ kips}$$

$$\mathbf{F}_{DB} = 17.90\text{ kips} \nearrow \blacktriangleleft$$

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Chapter 10, Solution 103.

Given:

$l_{AB} = 3.6$ in.

$l_{BC} = 1.6$ in.

$l_{CD} = 1.2$ in.

$l_{DE} = 1.6$ in.

$l_{EF} = 1.6$ in.

$l_{FG} = 4.8$ in.

Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{(1.6 \text{ in.})}{(3.6 \text{ in.})}\,\delta y_A = \frac{4}{9}\,\delta y_A \leftarrow$$

$$\delta y_D = \delta y_C = \frac{4}{9}\delta y_A \leftarrow$$

$$\delta y_G = \frac{(4.8 \text{ in.})}{(3.2 \text{ in.})}\,\delta y_D = \frac{(4.8 \text{ in.})}{(3.2 \text{ in.})}\left[\frac{4}{9}\delta y_A\right] = \frac{2}{3}\delta y_A \downarrow$$

$$\delta\phi = \frac{\delta y_D}{(3.2 \text{ in.})} = \frac{4}{9(3.2 \text{ in.})}\delta y_A$$

Virtual Work: $\delta U = 0$:

$$(20 \text{ lb})\delta y_A + P\delta y_G + (180 \text{ lb}\cdot\text{in.})\delta\phi = 0$$

$$(20 \text{ lb})\delta y_A + P\left(\frac{2}{3}\delta y_A\right) + (180 \text{ lb}\cdot\text{in.})\left[\frac{4}{9(3.2 \text{ in.})}\delta y_A\right] = 0$$

Solving $P = -67.5$ lb

P = 67.5 lb ↑ ◀

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Chapter 10, Solution 104.

Given:

$l_{AB} = 3.6$ in.

$l_{BC} = 1.6$ in.

$l_{CD} = 1.2$ in.

$l_{DE} = 1.6$ in.

$l_{EF} = 1.6$ in.

$l_{FG} = 4.8$ in.

Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{(1.6 \text{ in.})}{(3.6 \text{ in.})}\delta y_A = \frac{4}{9}\delta y_A \leftarrow$$

$$\delta y_D = \delta y_C = \frac{4}{9}\delta y_A \leftarrow$$

$$\delta\phi = \frac{\delta y_D}{(3.2 \text{ in.})} = \frac{4}{9(3.2 \text{ in.})}\delta y_A = \frac{5}{(36 \text{ in.})}\delta y_A \circlearrowright$$

Virtual Work: $\delta U = 0$:

$$(20 \text{ lb})\delta y_A + (180 \text{ lb}\cdot\text{in.})\delta\phi + M\delta\phi = 0$$

or

$$20\delta y_A + 180\left(\frac{5}{36}\delta y_A\right) + M\left(\frac{5}{36}\delta y_A\right) = 0$$

Solving $M = -324.0$ lb·in.

or 27.0 lb·ft $\circlearrowright$ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell
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Chapter 10, Solution 105.

Have

$$x_B = l\cos\theta$$
$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$
$$y_C = l\sin\theta$$
$$\delta y_C = l\cos\theta\delta\theta$$

Now $$\delta x_B = \frac{1}{2}l\delta\phi$$

Substituting from Equation (1)

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or $$\delta\phi = -2\sin\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad M\delta\varphi + P\delta y_C = 0$$

$$M\left(-2\sin\theta\delta\theta\right) + P\left(l\cos\theta\delta\theta\right) = 0$$

or $$M = \frac{1}{2}Pl\frac{\cos\theta}{\sin\theta}$$

$$M = \frac{Pl}{2\tan\theta} \blacktriangleleft$$

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Chapter 10, Solution 106.

$$y_E = l\cos\theta$$

$$\delta y_E = -l\sin\theta\,\delta\theta$$

Spring: Unstretched length $= 2\left(2l\sin 30°\right) = 2l$

$$x = 2\left(2l\sin\theta\right) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\,\delta\theta$$

$$F_{SP} = k\left(x - 2l\right)$$

$$= k\left(4l\sin\theta - 2l\right)$$

Virtual Work:

$$\delta U = 0: \qquad P\,\delta y_E - F_{SP}\,\delta x = 0$$

$$P\left(-l\sin\theta\,\delta\theta\right) - k\left(4l\sin\theta - 2l\right)\left(4l\cos\theta\,\delta\theta\right) = 0$$

or $$-P\sin\theta - 8kl\left(2\sin\theta - 1\right)\cos\theta = 0$$

or $$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

We have $P = 40$ lb, $l = 10$ in., and $k = 1.5$ lb/in.

Thus $$\frac{\left(40\text{ lb}\right)}{8\left(1.5\text{ lb/in.}\right)\left(10\text{ in.}\right)} = \frac{1 - 2\sin\theta}{\tan\theta}$$

Solving $\theta = 24.98°$

or $\theta = 25.0°$ ◀

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Chapter 10, Solution 107.

$$\delta x_B = \frac{1}{2} l\,\delta\phi$$

$$\delta C = \frac{\delta C_x}{\cos\theta} = \frac{\delta x_B}{\cos\theta} = \frac{l\,\delta\phi}{2\cos\theta}$$

Virtual Work:

$$\delta U = 0: \qquad M\,\delta\phi - Q\,\delta C = 0$$

$$M\,\delta\phi - Q\left(\frac{l}{2\cos\theta}\right)\delta\phi = 0$$

or

$$M = \frac{Ql}{2\cos\theta}$$

Thus

$$M = \frac{1}{2}\frac{(40\text{ lb})(1.8\text{ ft})}{\cos 65°} = 85.18\text{ lb}\cdot\text{ft}$$

or $\quad M = 85.2\text{ lb}\cdot\text{ft}$ ◀

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Chapter 10, Solution 108.

Assuming

$$\delta y_A \downarrow$$

it follows

$$\delta y_C = \frac{120}{80}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{180}{60}\delta y_E = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta y_G = \frac{100}{60}\delta y_E = \frac{100}{60}(1.5\delta y_A) = 2.5\delta y_A \downarrow$$

Then, by Virtual Work

$$\delta U = 0: \quad (300\text{ N})\delta y_A - (100\text{ N})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

$$P = +60\text{ N}$$

$$\mathbf{P} = 60\text{ N} \downarrow \blacktriangleleft$$

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Chapter 10, Solution 109.

Apply vertical load P at D.

$$+\circlearrowleft \Sigma M_H = 0: \quad -P(12\text{ m}) + E(36\text{ m}) = 0$$

$$\mathbf{E} = \frac{P}{3}\downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9}P$$

Virtual Work:

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

Virtual Work: $\quad \delta U = 0: \quad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$

$$P\delta D + F_{BF}\,\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$

where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(75\text{ mm}) = 0$$

$$\delta D = +41.67\text{ mm}$$

$$\delta D = 41.7\text{ mm}\downarrow \blacktriangleleft$$

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Chapter 10, Solution 110.

Apply horizontal load P at D.

$$+\circlearrowleft \Sigma M_H = 0: \quad P(9\text{ m}) - E_y(36\text{ m}) = 0$$

$$E_y = \frac{P}{4} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12}P$$

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

Virtual Work: $\delta U = 0$: $P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$

$$P\delta D + F_{BF}\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$

where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{12}P\right)(75\text{ mm}) = 0$$

$\delta D = 31.25$ mm $\qquad\qquad$ $\delta D = 31.3$ mm $\longrightarrow$ ◀

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Chapter 10, Solution 111.

Potential Energy

$$V = \left(3.5\text{ kg} \times 9.81\text{ m/s}^2\right)\left(-\frac{l}{2}\sin\theta\right) + \left(1.75\text{ kg} \times 9.81\text{ m/s}^2\right)\left(\frac{l}{2}\cos\theta\right)$$

$$= \left(8.5838\text{ N}\right)l\left(-2\sin\theta + \cos\theta\right)$$

$$\frac{dV}{d\theta} = \left(8.5838\text{ N}\right)l\left(-2\cos\theta - \sin\theta\right)$$

$$\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left(2\sin\theta - \cos\theta\right)$$

Equilibrium: $\frac{dV}{d\theta} = 0: \quad -2\cos\theta - \sin\theta = 0$

or $\tan\theta = -2$

Thus $\theta = -63.4°$ and $116.6°$

Stability

At $\theta = -63.4°$: $\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left[2\sin\left(-63.4°\right) - \cos\left(-63.4°\right)\right]$

$$= \left(8.5838\text{ N}\right)l\left(-1.788 - 0.448\right) < 0$$

$\therefore \quad \theta = -63.4°$, Unstable ◀

At $\theta = 116.6°$: $\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left[2\sin\left(116.6°\right) - \cos\left(116.6°\right)\right]$

$$= \left(8.5838\text{ N}\right)l\left(1.788 + 0.447\right) > 0$$

$\therefore \quad \theta = 116.6°$, Stable ◀

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Chapter 10, Solution 112.

Elongation of Spring:

$$s = l\sin\theta + l\cos\theta - l$$

$$= l\left(\sin\theta + \cos\theta - 1\right)$$

Potential Energy:

$$V = \frac{1}{2}ks^2 - W\frac{l}{2}\sin\theta$$

$$= \frac{1}{2}kl^2\left(\sin\theta + \cos\theta - 1\right)^2 - mg\frac{l}{2}\sin\theta$$

$$\frac{dV}{d\theta} = kl^2\left(\sin\theta + \cos\theta - 1\right)\left(\cos\theta - \sin\theta\right) - \frac{1}{2}mgl\cos\theta$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: \quad \left(\sin\theta + \cos\theta - 1\right)\left(\cos\theta - \sin\theta\right) - \frac{mg}{2kl}\cos\theta = 0$$

or

$$\cos\theta\left[\left(\sin\theta + \cos\theta - 1\right)\left(1 - \tan\theta\right) - \frac{mg}{2kl}\right] = 0$$

Now with

$$W = mg = \left(125\text{ kg}\right)\left(9.81\text{ m/s}^2\right) = 1226.25\text{ N}$$

$$l = 320\text{ mm, and } k = 15\text{ kN/m,}$$

$$\cos\theta\left[\left(\sin\theta + \cos\theta - 1\right)\left(1 - \tan\theta\right) - \frac{1226.25\text{ N}}{2\left(15000\text{ N/m}\right)\left(0.32\text{ m}\right)}\right] = 0$$

or

$$\cos\theta\left[\left(\sin\theta + \cos\theta - 1\right)\left(1 - \tan\theta\right) - 0.12773\right] = 0$$

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By inspection, one solution is $\cos\theta = 0$ or $\theta = 90.0°$

Solving numerically: $\theta = 0.38338$ rad $= 9.6883°$ and $\theta = 0.59053$ rad $= 33.8351°$

Stability

$$\frac{d^2V}{d\theta^2} = kl^2\left[(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta - 1)(-\sin\theta - \cos\theta)\right] + \frac{1}{2}mgl\sin\theta$$

$$= kl^2\left[\left(1 + \frac{mg}{2kl}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

$$= (15000\text{ N/m})(0.32\text{ m})^2\left[\left(1 + \frac{(1226.25\text{ N})}{2(15000\text{ N/m})(0.32\text{ m})}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

$$= (1536\text{ N}\cdot\text{m})[1.12773\sin\theta + \cos\theta - 2\sin 2\theta]$$

Thus

At $\theta = 90°$: $\frac{d^2V}{d\theta^2} = 1732.2 > 0$ $\therefore\ \theta = 90.0°$, Stable ◀

At $\theta = 9.6883°$: $\frac{d^2V}{d\theta^2} = 786.4 > 0$ $\therefore\ \theta = 9.69°$, Stable ◀

At $\theta = 33.8351°$: $\frac{d^2V}{d\theta^2} = -600.6 < 0$ $\therefore\ \theta = 33.8°$, Unstable ◀

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