Chapter 17 **COMPRESSIBLE FLOW**

17–1 ▪STAGNATION PROPERTIES

Stagnation (or **total**) **enthalpy** h_0:

$$h_0 = h + \frac{V^2}{2} \qquad (\text{kJ/kg}) \qquad \textbf{(17–1)}$$

When the potential energy of the fluid is negligible, the stagnation enthalpy represents the *total energy of a flowing fluid stream* per unit mass. Thus it simplifies the thermodynamic analysis of high-speed flows.

Throughout this chapter the ordinary enthalpy *h* is referred to as the **static enthalpy,** whenever necessary, to distinguish it from the stagnation enthalpy. Notice that the stagnation enthalpy is a combination property of a fluid, just like the static enthalpy, and these two enthalpies become identical when the kinetic energy of the fluid is negligible.

For flows through nozzles and diffusers any increase in fluid velocity in these devices creates an equivalent decrease in the static enthalpy of the fluid.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \qquad \textbf{(17–2)}$$

$$h_{01} = h_{02} \qquad \textbf{(17–3)}$$

The *stagnation enthalpy* represents the *enthalpy of a fluid when it is brought to rest adiabatically.*

The properties of a fluid at the stagnation state are called **stagnation properties** (stagnation temperature, stagnation pressure, stagnation density, etc.)

The stagnation state is called the **isentropic stagnation state** when the stagnation process is reversible as well as adiabatic (i.e., isentropic).

The stagnation processes are often approximated to be isentropic, and the isentropic stagnation properties are simply referred to as stagnation properties.

When the fluid is approximated as an *ideal gas* with constant specific heats, its enthalpy can be replaced by c_pT and Eq. 17–1 can be expressed as

$$c_pT_0 = c_pT + \frac{V^2}{2}$$

$$T_0 = T + \frac{V^2}{2c_p}$$

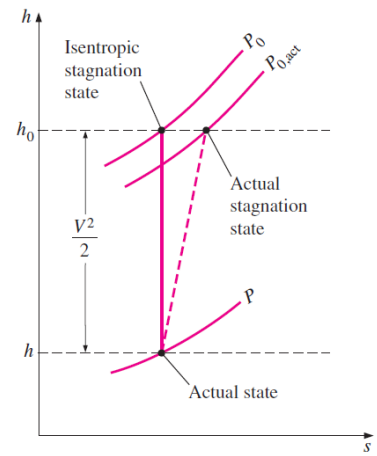


FIGURE 17–4

The actual state, actual stagnation state, and isentropic stagnation state of a fluid on an *h-s* diagram.

Here T_0 is called the **stagnation** (or **total**) **temperature,** and it represents *the temperature an ideal gas attains when it is brought to rest adiabatically.*

The term $V^2/2c_p$ corresponds to the temperature rise during such a process and is called the **dynamic temperature.**

The pressure a fluid attains when brought to rest isentropically is called the **stagnation pressure** P_0. For ideal gases with constant specific heats, P_0 is related to the static pressure of the fluid by

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)}$$

For $\rho = 1/v$

$$Pv^k = P_0v_0^k$$

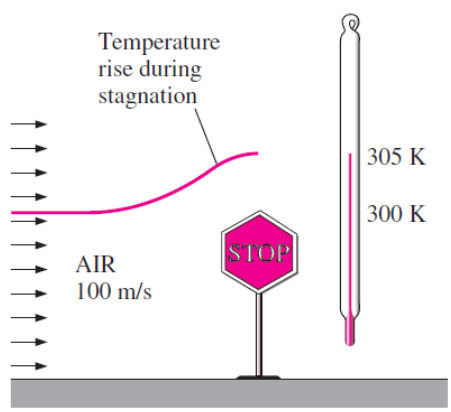


FIGURE 17–5

The temperature of an ideal gas flowing at a velocity V rises by $V^2/2c_p$ when it is brought to a complete stop.

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)} \qquad \textbf{(17–6)}$$

$$q_{\text{in}} + w_{\text{in}} + (h_{01} + gz_1) = q_{\text{out}} + w_{\text{out}} + (h_{02} + gz_2) \qquad \textbf{(17–7)}$$

where h_{01} and h_{02} are the stagnation enthalpies at states 1 and 2, respectively.

When the fluid is an ideal gas with constant specific heats, Eq. 17–7 becomes

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = c_p(T_{02} - T_{01}) + g(z_2 - z_1) \qquad \textbf{(17–8)}$$

17–2 ▪SPEED OF SOUND AND MACH NUMBER

The **speed of sound** (or the **sonic speed**), is the speed at which an infinitesimally small pressure wave travels through a medium.

The pressure wave may be caused by a small disturbance, which creates a slight rise in local pressure.

The amplitude of the ordinary sonic wave is very small and does not cause any appreciable change in the pressure and temperature of the fluid. Therefore, the propagation of a sonic wave is not only adiabatic but also very nearly isentropic.

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s \qquad \textbf{(17–9)}$$

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \textbf{(17–10)}$$

Note that the speed of sound in a fluid is a function of the thermodynamic properties of that fluid.

When the fluid is an ideal gas ($P = \rho RT$), the differentiation in Eq. 17–10 can easily be performed to yield

$$c = \sqrt{kRT}$$

A second important parameter in the analysis of compressible fluid flow is the **Mach number** Ma and it is the ratio of the actual velocity of the fluid (or an object in still air) to the speed of sound in the same fluid at the same state:

$$\mathrm{Ma} = \frac{V}{c}$$

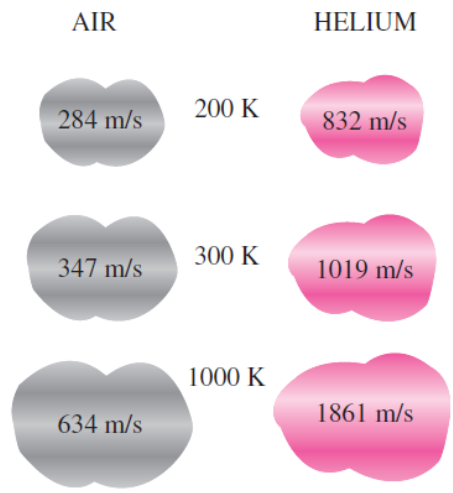


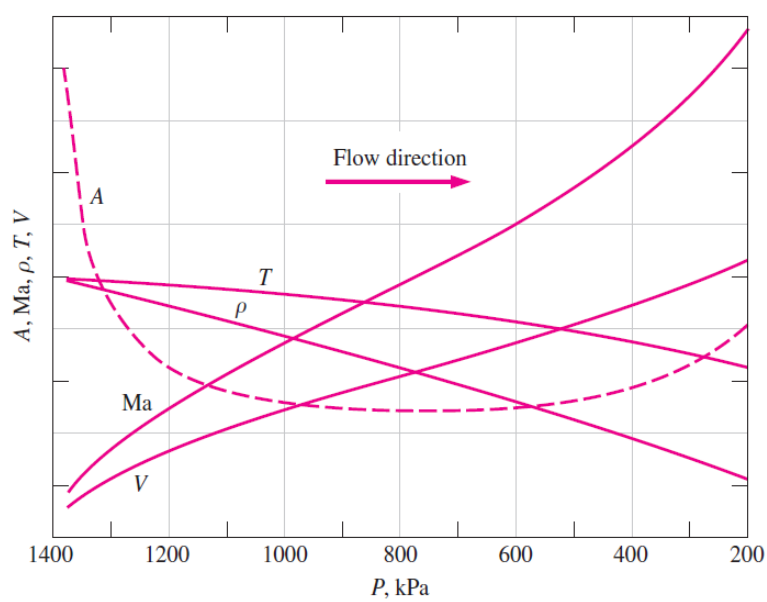
FIGURE 17–9

The speed of sound changes with temperature and varies with the fluid.

Fluid flow regimes are often described in terms of the flow Mach number. The flow is called sonic when Ma = 1, subsonic when Ma< 1, supersonic when Ma > 1, hypersonic when Ma >> 1, and transonic when Ma≅1.

17–3 ▪ONE-DIMENSIONAL ISENTROPIC FLOW

The Mach number is unity at the location of smallest flow area, called the throat.



Variation of Fluid Velocity with Flow Area

$$\dot{m} = \rho A V = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad \textbf{(17–13)}$$

$$\frac{dP}{\rho} + V\,dV = 0 \qquad \textbf{(17–14)}$$

$$\frac{dA}{A} = \frac{dP}{\rho}\left(\frac{1}{V^2} - \frac{d\rho}{dP}\right) \qquad \textbf{(17–15)}$$

Rearranging Eq. 17–9 as $(\partial\rho/\partial P)_s = 1/c^2$ and substituting into Eq. 17–15 yield

$$\frac{dA}{A} = \frac{dP}{\rho V^2}(1 - \mathrm{Ma}^2) \qquad \textbf{(17–16)}$$

For *subsonic* flow (Ma < 1), the term 1 - Ma^2 is positive; and thus *dA* and *dP* must have the same sign. That is, the pressure of the fluid must increase as the flow area of the duct increases and must decrease as the flow area of the duct decreases. Thus, at subsonic velocities, the pressure decreases in converging ducts (subsonic nozzles) and increases in diverging ducts (subsonic diffusers).

Another important relation for the isentropic flow of a fluid is obtained by substituting $\rho V = -dP/dV$ from Eq. 17–14 into Eq. 17–16:

$$\frac{dA}{A} = -\frac{dV}{V}(1 - \mathrm{Ma}^2) \qquad \textbf{(17–17)}$$

For subsonic flow (Ma < 1), $\dfrac{dA}{dV} < 0$

For supersonic flow (Ma > 1), $\dfrac{dA}{dV} > 0$

For sonic flow (Ma = 1), $\dfrac{dA}{dV} = 0$

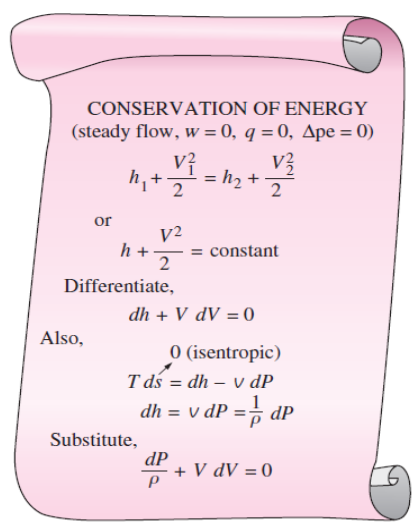
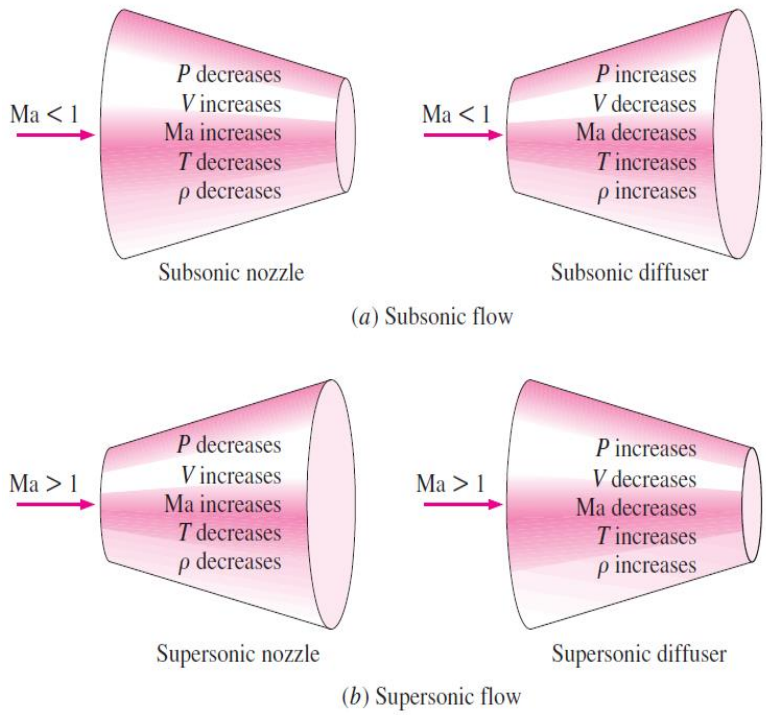


FIGURE 17–15
Derivation of the differential form of the energy equation for steady isentropic flow.



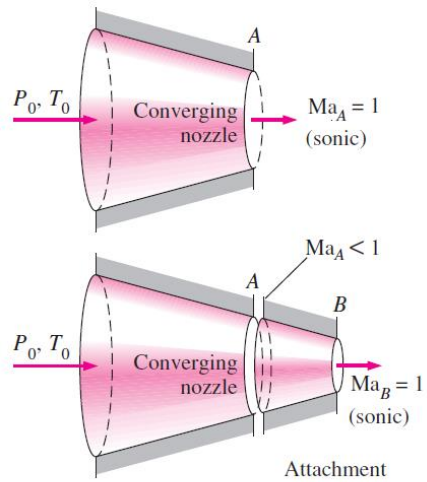


FIGURE 17–16

We cannot obtain supersonic velocities by attaching a converging section to a converging nozzle. Doing so will only move the sonic cross section farther downstream and decrease the mass flow rate.

Property Relations for Isentropic Flow of Ideal Gases

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2 \qquad \textbf{(17–18)}$$

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2\right]^{k/(k-1)} \qquad \textbf{(17–19)}$$

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2\right]^{1/(k-1)} \qquad \textbf{(17–20)}$$

The properties of a fluid at a location where the Mach number is unity (the throat) are called critical properties, and the ratios in Eqs. (17–18) through (17–20) are called critical ratios (Fig. 17–18).

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

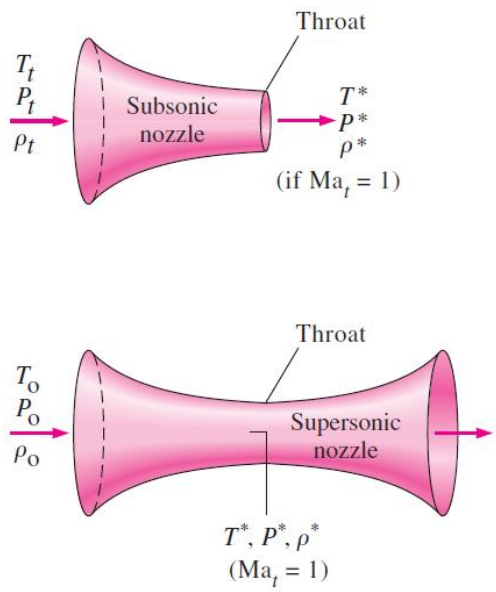


FIGURE 17–18

When $Ma_t = 1$, the properties at the nozzle throat become the critical properties.

17–4 ▪ ISENTROPIC FLOW THROUGH NOZZLES

In this section we consider the effects of **back pressure** **(i.e., the pressure applied at the nozzle discharge region)** on the exit velocity, the mass flow rate, and the pressure distribution along the nozzle.

Converging Nozzles

When the back pressure is reduced to P_3 (= P^*, which is the pressure required to increase the fluid velocity to the speed of sound at the exit plane or throat), the mass flow reaches a maximum value and the flow is said to be **choked.**

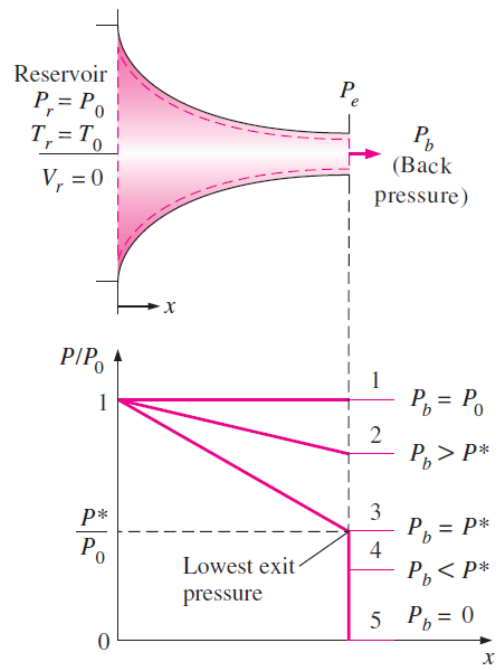


FIGURE 17–20

The effect of back pressure on the pressure distribution along a converging nozzle.

$$\dot{m} = \rho A V = \left(\frac{P}{RT}\right) A\left(\mathrm{Ma}\sqrt{kRT}\right) = PA\mathrm{Ma}\sqrt{\frac{k}{RT}}$$

Solving for T from Eq. 17–18 and for P from Eq. 17–19 and substituting,

$$\dot{m} = \frac{A\mathrm{Ma}P_0\sqrt{k/(RT_0)}}{[1 + (k-1)\mathrm{Ma}^2/2]^{(k+1)/[2(k-1)]}} \qquad \textbf{(17–24)}$$

$$\dot{m}_{\max} = A^* P_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

For all back pressures lower than the critical pressure P*, the pressure at the exit plane of the converging nozzle Pe is equal to P*, the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate. Because the velocity of the flow is sonic at the throat for the maximum flow rate, a back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

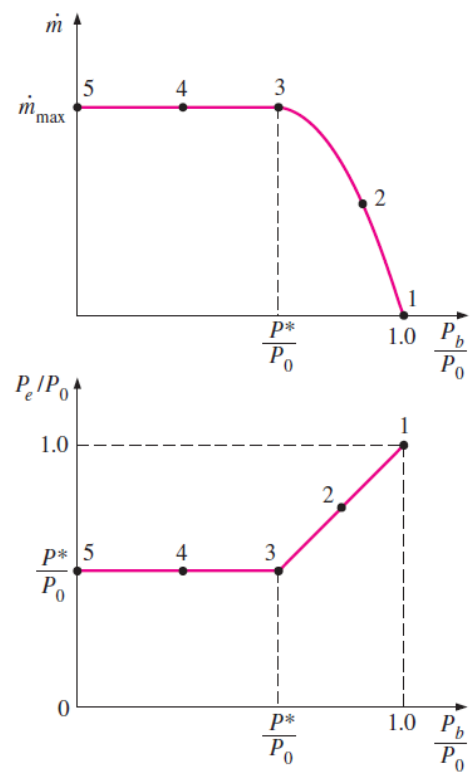


FIGURE 17–21

The effect of back pressure P_b on the mass flow rate $\dot{m}$ and the exit pressure P_e of a converging nozzle.

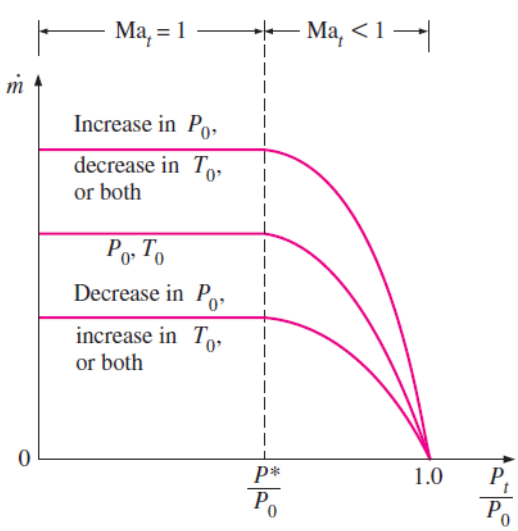


FIGURE 17–22

The variation of the mass flow rate through a nozzle with inlet stagnation properties.

Also illustrated on this figure is the effect of back pressure on the nozzle exit pressure Pe. We observe that

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$

A relation for the variation of flow area A through the nozzle relative to throat area A* can be obtained by combining Eqs 17–24 and 17–25 for the same mass flow rate and stagnation properties of a particular fluid. This yields

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{(k+1)/[2(k-1)]} \qquad \textbf{(17–26)}$$

Another parameter sometimes used in the analysis of one-dimensional isentropic flow of ideal gases is Ma*, which is the ratio of the local velocity to the speed of sound at the throat:

$$\mathrm{Ma}^* = \frac{V}{c^*} \qquad \textbf{(17–27)}$$

or

$$\mathrm{Ma}^* = \frac{V}{c}\frac{c}{c^*} = \frac{\mathrm{Ma}c}{c^*} = \frac{\mathrm{Ma}\sqrt{kRT}}{\sqrt{kRT^*}} = \mathrm{Ma}\sqrt{\frac{T}{T^*}}$$

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2 + (k-1)\mathrm{Ma}^2}} \qquad \textbf{(17–28)}$$

Note that the parameter Ma* differs from the Mach number Ma in that Ma* is the local velocity nondimensionalized with respect to the sonic velocity at the throat, whereas Ma is the local velocity nondimensionalized with respect to the local sonic velocity. (Recall that the sonic velocity in a nozzle varies with temperature and thus with location.)

Converging–Diverging Nozzles

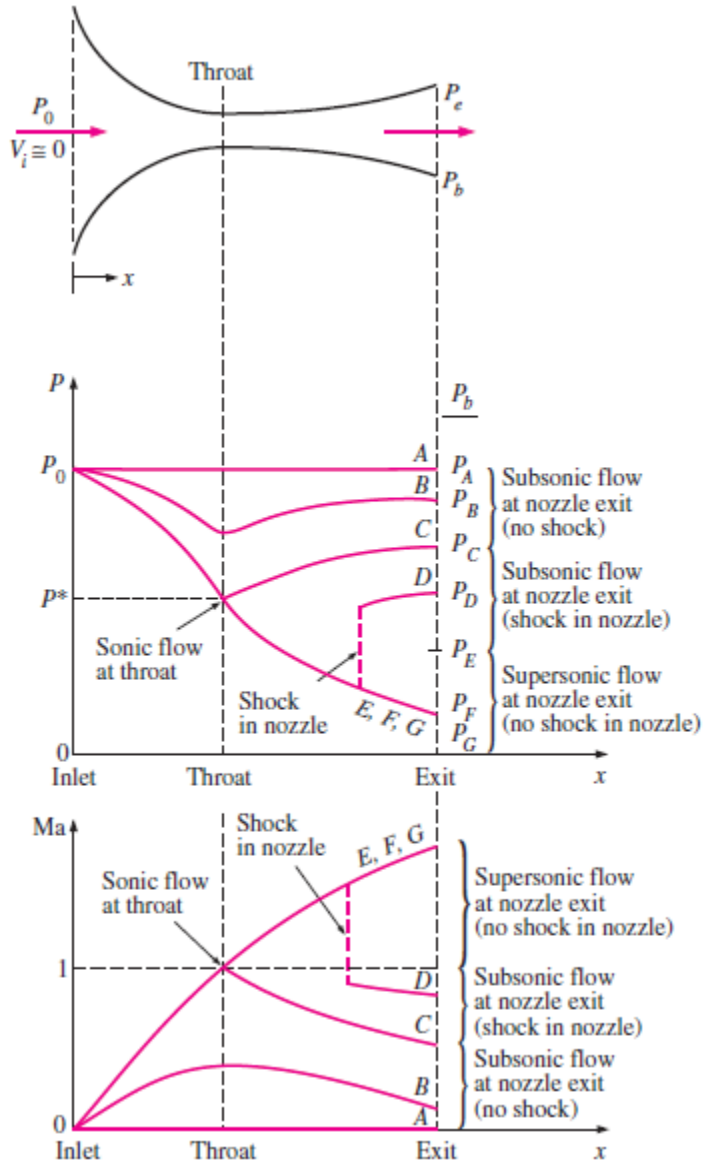


FIGURE 17–27

The effects of back pressure on the flow through a converging–diverging nozzle.

17–5 ▪ SHOCK WAVES AND EXPANSION WAVES

Conservation of mass:

$$\rho_1 A V_1 = \rho_2 A V_2 \tag{17–29}$$

Conservation of energy:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \tag{17–30}$$

or

$$h_{01} = h_{02} \tag{17–31}$$

Conservation of momentum: Rearranging Eq. 17–14 and integrating yield

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1) \tag{17–32}$$

Increase of entropy:

$$s_2 - s_1 \geq 0 \tag{17–33}$$

We can combine the conservation of mass and energy relations into a single equation and plot it on an *h-s* diagram, using property relations. The resultant curve is called the **Fanno line,** and it is the locus of states that have the same value of stagnation enthalpy and mass flux (mass flow per unit flow area). Likewise, combining the conservation of mass and momentum equations into a single equation and plotting it on the *h-s* diagram yield a curve called the **Rayleigh line.**

The Fanno and Rayleigh lines intersect at two points (points 1 and 2), which represent the two states at which all three conservation equations are satisfied.

The larger the Mach number before the shock, the stronger the shock will be. In the limiting case of Ma _ 1, the shock wave simply becomes a sound wave.

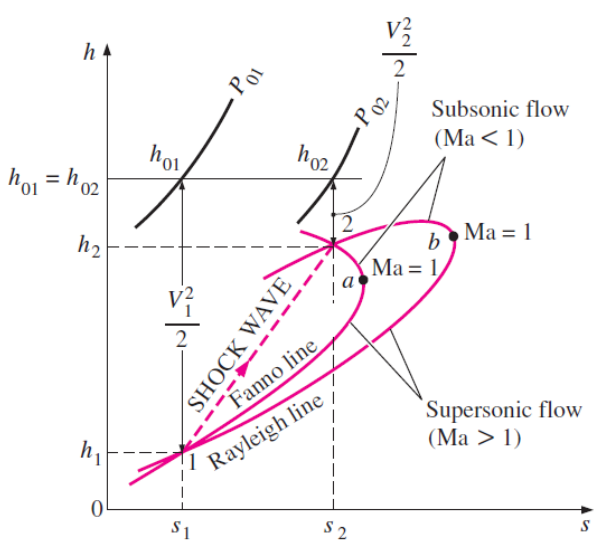


FIGURE 17–31

The *h-s* diagram for flow across a normal shock.

The conservation of energy principle (Eq. 17–31) requires that the stagnation enthalpy remain constant across the shock. That is, the stagnation temperature of an ideal gas also remains constant across the shock. Note, however, that the stagnation pressure decreases across the shock because of the irreversibilities,

while the thermodynamic temperature rises drastically because of the conversion of kinetic energy into enthalpy due to a large drop in fluid velocity (see Fig. 17–32).

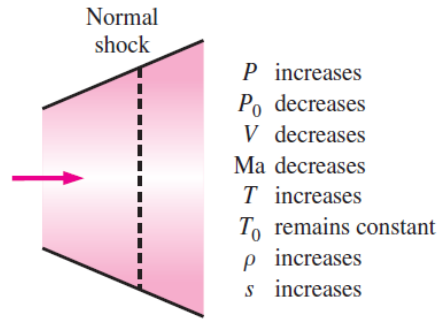


FIGURE 17–32
Variation of flow properties across a normal shock.

$$\frac{T_2}{T_1} = \frac{1 + \mathrm{Ma}_1^2(k-1)/2}{1 + \mathrm{Ma}_2^2(k-1)/2}$$

$$\frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1} = \frac{P_2\mathrm{Ma}_2c_2}{P_1\mathrm{Ma}_1c_1} = \frac{P_2\mathrm{Ma}_2\sqrt{T_2}}{P_1\mathrm{Ma}_1\sqrt{T_1}} = \left(\frac{P_2}{P_1}\right)^2\left(\frac{\mathrm{Ma}_2}{\mathrm{Ma}_1}\right)^2 \qquad \textbf{(17–36)}$$

$$\frac{P_2}{P_1} = \frac{\mathrm{Ma}_1\sqrt{1 + \mathrm{Ma}_1^2(k-1)/2}}{\mathrm{Ma}_2\sqrt{1 + \mathrm{Ma}_2^2(k-1)/2}} \qquad \textbf{(17–37)}$$

$$\frac{\rho_1}{\rho_2} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} \qquad \textbf{(17–38)}$$

$$\mathrm{Ma}_2^2 = \frac{\mathrm{Ma}_1^2 + 2/(k-1)}{2\mathrm{Ma}_1^2k/(k-1) - 1} \qquad \textbf{(17–39)}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad \textbf{(17–40)}$$

For adiabatic flows, shock waves can exist only for supersonic flows, Ma1 > 1.

Oblique Shocks

Like normal shocks, the Mach number decreases across an oblique shock, and oblique shocks are possible only if the upstream flow is supersonic. However, unlike normal shocks, in which the downstream Mach number is always subsonic, Ma2 downstream of an oblique shock can be subsonic, sonic, or supersonic, depending on the upstream Mach number Ma1 and the turning angle.

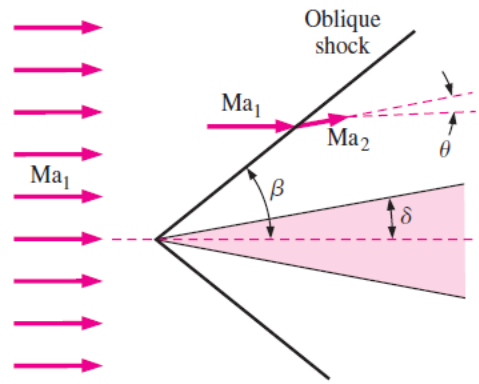


FIGURE 17–37

An oblique shock of *shock angle* β formed by a slender, two-dimensional wedge of half-angle δ. The flow is

Conservation of mass reduces to

$$\rho_1 V_{1,n} A = \rho_2 V_{2,n} A \quad \rightarrow \quad \rho_1 V_{1,n} = \rho_2 V_{2,n}$$

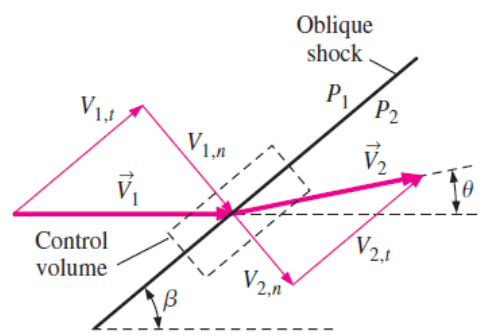


FIGURE 17–38

Velocity vectors through an oblique shock of shock angle β and deflection angle θ.

When we apply conservation of momentum in the direction *normal* to the oblique shock, the only forces are pressure forces, and we get

$$P_1 A - P_2 A = \rho V_{2,n} A V_{2,n} - \rho V_{1,n} A V_{1,n} \rightarrow P_1 - P_2 = \rho_2 V_{2,n}^2 - \rho_1 V_{1,n}^2 \quad \textbf{(17–42)}$$

$$h_1 + \frac{1}{2} V_{1,n}^2 = h_2 + \frac{1}{2} V_{2,n}^2 \quad \textbf{(17–43)}$$

$$\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta \quad \text{and} \quad \mathrm{Ma}_{2,n} = \mathrm{Ma}_2 \sin(\beta - \theta)$$

All the equations, shock tables, etc., for normal shocks apply to oblique shocks as well, provided that we use only the **normal** components of the Mach number.

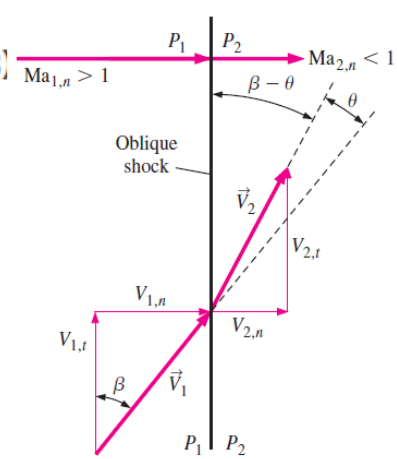


FIGURE 17–39

The same velocity vectors of Fig. 17–38, but rotated by angle $\pi/2 - \beta$, so that the oblique shock is vertical. Normal Mach numbers $\mathrm{Ma}_{1,n}$ and $\mathrm{Ma}_{2,n}$ are also defined.

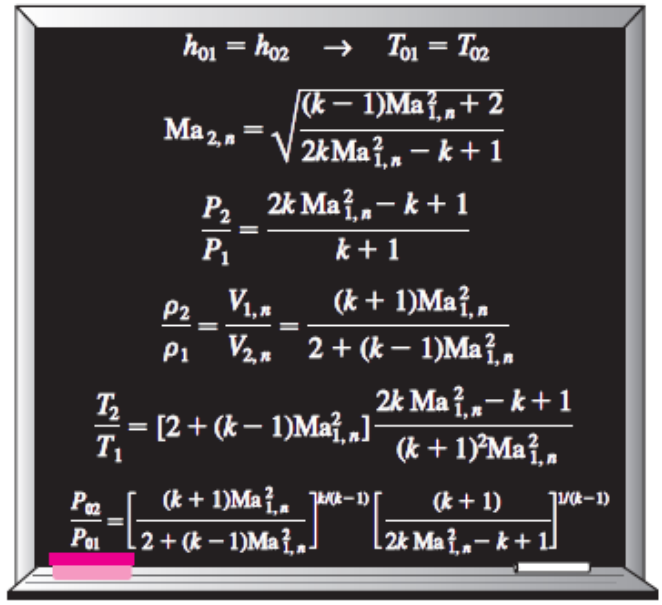


FIGURE 17–40

Relationships across an oblique shock for an ideal gas in terms of the normal component of upstream Mach number $\mathrm{Ma}_{1,n}$.

$$\frac{V_{2,n}}{V_{1,n}} = \frac{\tan(\beta - \theta)}{\tan\beta} = \frac{2 + (k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = \frac{2 + (k-1)\mathrm{Ma}_1^2 \sin^2\beta}{(k+1)\mathrm{Ma}_1^2 \sin^2\beta} \quad \textbf{(17–45)}$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta \quad \text{and} \quad \tan(\beta - \theta) = \frac{\tan\beta - \tan\theta}{1 + \tan\beta\tan\theta}$$

The θ-β-Ma relationship:

$$\tan\theta = \frac{2\cot\beta(\mathrm{Ma}_1^2 \sin^2\beta - 1)}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2} \quad \textbf{(17–46)}$$

For any value

of Mach number Ma_1 greater than 1, the possible values of θ range from θ = 0° at some value of β between 0 and 90°, to a maximum value u u_{max} at an intermediate value of b, and then back to θ = 0° at β= 90°. Straight oblique shocks for θ or β outside of this range cannot and do not exist. At Ma_1 = 1.5, for example, straight oblique shocks cannot exist in air with shock angle β less than about 42°, nor with deflection angle θ greater than about 12°. If the wedge half-angle is greater than θ_{max}, the shock becomes curved and detaches from the nose of the wedge, forming what is called a detached oblique shock or a bow wave (Fig. 17–42).

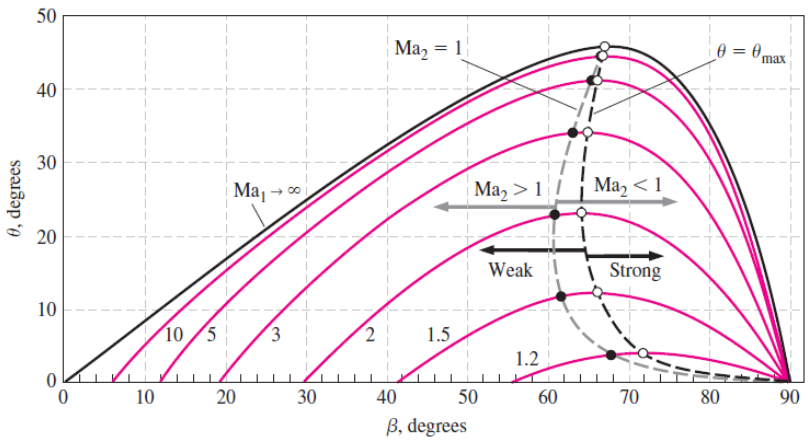


FIGURE 17–41

The dependence of straight oblique shock deflection angle θ on shock angle β for several values of upstream Mach number Ma_1. Calculations are for an ideal gas with $k = 1.4$. The dashed black line connects points of maximum deflection angle ($\theta = \theta_{max}$). *Weak oblique shocks* are to the left of this line, while *strong oblique shocks* are to the right of this line. The dashed gray line connects points where the downstream Mach number is *sonic* ($Ma_2 = 1$). *Supersonic downstream flow* ($Ma_2 > 1$) is to the left of this line, while *subsonic downstream flow* ($Ma_2 < 1$) is to the right of this line.

Mach angle: $\mu = \sin^{-1}(1/Ma_1)$

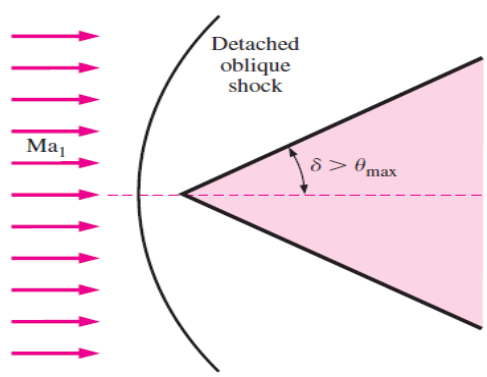


FIGURE 17–42

A *detached oblique shock* occurs upstream of a two-dimensional wedge of half-angle δ when δ is greater than the maximum possible deflection angle θ. A shock of this kind is called a *bow wave* because of its resemblance to the water wave that forms at the bow of a ship.

Prandtl–Meyer Expansion Waves

Turning angle across an expansion fan: $$\theta = \nu(\mathrm{Ma}_2) - \nu(\mathrm{Ma}_1) \qquad \textbf{(17–48)}$$

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left[\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2-1)}\right] - \tan^{-1}\left(\sqrt{\mathrm{Ma}^2-1}\right) \qquad \textbf{(17–49)}$$

Note that ϑ(Ma) is an angle, and can be calculated in either degrees or radians. Physically, ϑ (Ma) is the angle through which the flow must expand, starting with ϑ = 0 at Ma = 1, in order to reach a supersonic Mach number, Ma > 1.

17–6 ▪DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

Mass equation

$$\rho_1 V_1 = \rho_2 V_2 \qquad \textbf{(17–50)}$$

***x*-Momentum equation**

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$$

Noting that the flows are high speed

and turbulent, the momentum flux correction factor is approximately 1

($\beta \cong 1$) and thus can be neglected. Then

$$P_1A_1 - P_2A_2 = \dot{m}V_2 - \dot{m}V_1 \quad \rightarrow \quad P_1 - P_2 = (\rho_2V_2)V_2 - (\rho_1V_1)V_1$$

or

$$P_1 + \rho_1V_1^2 = P_2 + \rho_2V_2^2 \qquad \textbf{(17–51)}$$

Energy equation

$$\dot{Q} + \dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \rightarrow q + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \textbf{(17–52)}$$

For an ideal gas with constant specific heats, $\Delta h = c_p\,\Delta T$, and thus

$$q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} \quad \textbf{(17–53)}$$

or

$$q = h_{02} - h_{01} = c_p(T_{02} - T_{01}) \quad \textbf{(17–54)}$$

Entropy change

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \quad \textbf{(17–55)}$$

Equation of state

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \quad \textbf{(17–5}$$

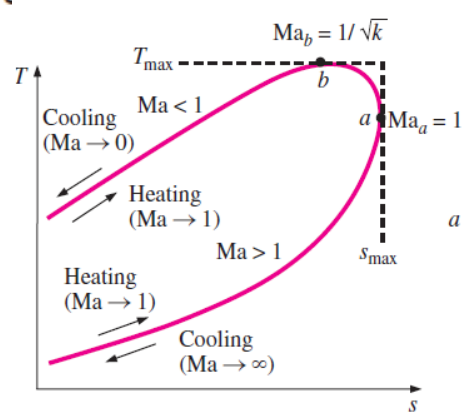


FIGURE 17–52

T-s diagram for flow in a constant-area duct with heat transfer and negligible friction (Rayleigh flow).

Property Relations for Rayleigh Flow

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} \qquad (17\text{–}58)$$

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{\mathrm{Ma}_2\sqrt{kRT_2}}{\mathrm{Ma}_1\sqrt{kRT_1}} = \frac{\mathrm{Ma}_2\sqrt{T_2}}{\mathrm{Ma}_1\sqrt{T_1}} \qquad (17\text{–}59)$$

$$\frac{T_2}{T_1} = \left[\frac{\mathrm{Ma}_2(1 + k\mathrm{Ma}_1^2)}{\mathrm{Ma}_1(1 + k\mathrm{Ma}_2^2)}\right]^2 \qquad (17\text{–}61)$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{\mathrm{Ma}_1^2(1 + k\mathrm{Ma}_2^2)}{\mathrm{Ma}_2^2(1 + k\mathrm{Ma}_1^2)} \qquad (17\text{–}62)$$

$$\frac{P}{P^*} = \frac{1 + k}{1 + k\mathrm{Ma}^2} \qquad \frac{T}{T^*} = \left[\frac{\mathrm{Ma}(1 + k)}{1 + k\mathrm{Ma}^2}\right]^2 \quad \text{and} \quad \frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1 + k)\mathrm{Ma}^2}{1 + k\mathrm{Ma}^2} \qquad (17\text{–}63)$$

$$\frac{T_0}{T_0^*} = \frac{(k + 1)\mathrm{Ma}^2[2 + (k - 1)\mathrm{Ma}^2]}{(1 + k\mathrm{Ma}^2)^2} \qquad (17\text{–}65)$$

$$\frac{P_0}{P_0^*} = \frac{k + 1}{1 + k\mathrm{Ma}^2}\left[\frac{2 + (k - 1)\mathrm{Ma}^2}{k + 1}\right]^{k/(k-1)} \qquad (17\text{–}67)$$

Choked Rayleigh Flow

If we keep heating the fluid, we will simply move the critical state further downstream and reduce the flow rate since fluid density at the critical state will now be lower. Therefore, for a given inlet state, the corresponding critical state fixes the maximum possible heat transfer for steady flow (Fig. 17–57). That is,

$$q_{\max} = h_0^* - h_{01} = c_p(T_0^* - T_{01}) \qquad \textbf{(17–68)}$$

Further heat transfer causes choking and thus the inlet state to change (e.g., inlet velocity will decrease), and the flow no longer follows the same Rayleigh line.

It may seem like supersonic Rayleigh flow can be cooled indefinitely, but it turns out that there is a limit. Taking the limit of Eq. 17–65 as the Mach number approaches infinity gives

$$\mathrm{Lim}_{\mathrm{Ma}\to\infty} \frac{T_0}{T_0^*} = 1 - \frac{1}{k^2} \qquad \textbf{(17–69)}$$

which yields T_0/T^*_0= 0.49 for k = 1.4. Therefore, if the critical stagnation temperature is 1000 K, air cannot be cooled below 490 K in Rayleigh flow. Physically this means that the flow velocity reaches infinity by the time the temperature reaches 490 K—a physical impossibility. When supersonic flow cannot be sustained, the flow undergoes a normal shock wave and becomes subsonic.