Solutions Manual for

Thermodynamics: An Engineering Approach

Seventh Edition

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Chapter 17
COMPRESSIBLE FLOW

Stagnation Properties

17-1C No, there is not significant error, because the velocities encountered in air-conditioning applications are very low, and thus the static and the stagnation temperatures are practically identical.

Discussion If the air stream were supersonic, however, the error would indeed be significant.

17-2C *Stagnation enthalpy* **combines the ordinary enthalpy and the kinetic energy of a fluid**, and offers convenience when analyzing high-speed flows. **It differs from the ordinary enthalpy by the kinetic energy term**.

Discussion Most of the time, we mean *specific enthalpy*, i.e., enthalpy per unit mass, when we use the term *enthalpy*.

17-3C *Dynamic temperature* is **the temperature rise of a fluid during a stagnation process**.

Discussion When a gas decelerates from high speed to zero speed at a stagnation point, the temperature of the gas rises.

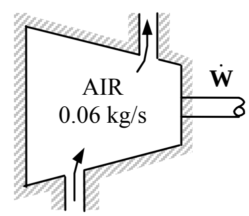
17-4C The temperature of the air **rises as it approaches the nose because of the stagnation process**.

Discussion In the frame of reference moving with the aircraft, the air decelerates from high speed to zero at the nose (stagnation point), and this causes the air temperature to rise.

17-5 The inlet stagnation temperature and pressure and the exit stagnation pressure of air flowing through a compressor are specified. The power input to the compressor is to be determined.

Assumptions **1** The compressor is isentropic. **2** Air is an ideal gas.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4.



Analysis The exit stagnation temperature of air T_{02} is determined from

$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (300.2\text{ K})\left(\frac{900}{100}\right)^{(1.4-1)/1.4} = 562.4\text{ K}$$

From the energy balance on the compressor,

$$\dot{W}_{\text{in}} = \dot{m}(h_{20} - h_{01})$$

or,

$$\dot{W}_{\text{in}} = \dot{m}c_p(T_{02} - T_{01}) = (0.06\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(562.4 - 300.2)\text{K} = \mathbf{15.8\ kW}$$

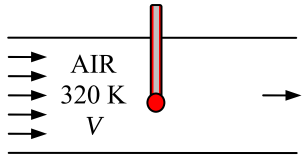
Discussion Note that the stagnation properties can be used conveniently in the energy equation.

17-6 Air at 320 K is flowing in a duct. The temperature that a stationary probe inserted into the duct will read is to be determined for different air velocities.

Assumptions The stagnation process is isentropic.

Properties The specific heat of air at room temperature is c_p = 1.005 kJ/kg·K.

Analysis The air which strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature, T_0. It is determined from $T_0 = T + \dfrac{V^2}{2c_p}$. The results for each case are calculated below:



(*a*) $$T_0 = 320\text{ K} + \frac{(1\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{320.0\ K}$$

(*b*) $$T_0 = 320\text{ K} + \frac{(10\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{320.1 K}$$

(*c*) $$T_0 = 320\text{ K} + \frac{(100\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{325.0\ K}$$

(*d*) $$T_0 = 320\text{ K} + \frac{(1000\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{817.5\ K}$$

Discussion Note that the stagnation temperature is nearly identical to the thermodynamic temperature at low velocities, but the difference between the two is significant at high velocities.

17-7 The states of different substances and their velocities are specified. The stagnation temperature and stagnation pressures are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Helium and nitrogen are ideal gases.

Analysis (*a*) Helium can be treated as an ideal gas with c_p = 5.1926 kJ/kg·K and k = 1.667. Then the stagnation temperature and pressure of helium are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50°\text{C} + \frac{(240\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{55.5°C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.25\text{ MPa})\left(\frac{328.7\text{ K}}{323.2\text{ K}}\right)^{1.667/(1.667-1)} = \mathbf{0.261\ MPa}$$

(*b*) Nitrogen can be treated as an ideal gas with c_p = 1.039 kJ/kg·K and k =1.400. Then the stagnation temperature and pressure of nitrogen are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50°\text{C} + \frac{(300\text{ m/s})^2}{2\times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{93.3°C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.15\text{ MPa})\left(\frac{366.5\text{ K}}{323.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{0.233\ MPa}$$

(*c*) Steam can be treated as an ideal gas with c_p = 1.865 kJ/kg·K and k =1.329. Then the stagnation temperature and pressure of steam are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 350°\text{C} + \frac{(480\text{ m/s})^2}{2\times 1.865\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{411.8°C = 685\ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.1\text{ MPa})\left(\frac{685\text{ K}}{623.2\text{ K}}\right)^{1.329/(1.329-1)} = \mathbf{0.147\ MPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

17-8 The state of air and its velocity are specified. The stagnation temperature and stagnation pressure of air are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Air is an ideal gas.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4.

Analysis The stagnation temperature of air is determined from

$$T_0 = T + \frac{V^2}{2c_p} = 238\text{ K} + \frac{(470\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 347.9\text{ K} \cong \mathbf{348\ K}$$

Other stagnation properties at the specified state are determined by considering an isentropic process between the specified state and the stagnation state,

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (36\text{ kPa})\left(\frac{347.9\text{ K}}{238\text{ K}}\right)^{1.4/(1.4-1)} = 135.9\text{ kPa} \cong \mathbf{136\ kPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

17-9E Steam flows through a device. The stagnation temperature and pressure of steam and its velocity are specified. The static pressure and temperature of the steam are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Steam is an ideal gas.

Properties Steam can be treated as an ideal gas with c_p = 0.4455 Btu/lbm·R and k =1.329.

Analysis The static temperature and pressure of steam are determined from

$$T = T_0 - \frac{V^2}{2c_p} = 700°\text{F} - \frac{(900\text{ ft/s})^2}{2\times 0.4455\text{ Btu/lbm}\cdot°\text{F}}\left(\frac{1\text{ Btu/lbm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = \mathbf{663.7°F}$$

$$P = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (120\text{ psia})\left(\frac{1123.7\text{ R}}{1160\text{ R}}\right)^{1.329/(1.329-1)} = \mathbf{105.5\ psia}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

17-10 Air flows through a device. The stagnation temperature and pressure of air and its velocity are specified. The static pressure and temperature of air are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Air is an ideal gas.

Properties The properties of air at an anticipated average temperature of 600 K are c_p = 1.051 kJ/kg·K and k = 1.376.

Analysis The static temperature and pressure of air are determined from

$$T = T_0 - \frac{V^2}{2c_p} = 673.2 - \frac{(570\text{ m/s})^2}{2\times 1.051\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{518.6\ K}$$

and

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{k/(k-1)} = (0.6\text{ MPa})\left(\frac{518.6\text{ K}}{673.2\text{ K}}\right)^{1.376/(1.376-1)} = \mathbf{0.23\ MPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

17-11 The inlet stagnation temperature and pressure and the exit stagnation pressure of products of combustion flowing through a gas turbine are specified. The power output of the turbine is to be determined.

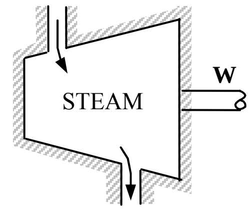
Assumptions **1** The expansion process is isentropic. **2** Products of combustion are ideal gases.

Properties The properties of products of combustion are c_p = 1.157 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.33.

Analysis The exit stagnation temperature T_{02} is determined to be

$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (1023.2\text{ K})\left(\frac{0.1}{1}\right)^{(1.33-1)/1.33} = 577.9\text{ K}$$

1 MPa
750°C



100 kPa

Also,

$$c_p = kc_v = k\left(c_p - R\right) \longrightarrow c_p = \frac{kR}{k-1} = \frac{1.33\left(0.287\text{ kJ/kg}\cdot\text{K}\right)}{1.33-1} = 1.157\text{ kJ/kg}\cdot\text{K}$$

From the energy balance on the turbine,

$$-w_{\text{out}} = (h_{20} - h_{01})$$

or, $$w_{\text{out}} = c_p\left(T_{01} - T_{02}\right) = \left(1.157\text{ kJ/kg}\cdot\text{K}\right)(1023.2 - 577.9)\text{ K} = 515.2\text{ kJ/kg} \cong \mathbf{515\ kJ/kg}$$

Discussion Note that the stagnation properties can be used conveniently in the energy equation.

Speed of Sound and Mach Number

17-12C *Sound* is an **infinitesimally small pressure wave**. It is **generated by a small disturbance in a medium**. It **travels by wave propagation**. **Sound waves cannot travel in a vacuum**.

Discussion Electromagnetic waves, like light and radio waves, can travel in a vacuum, but sound cannot.

17-13C **Yes, the propagation of sound waves is nearly isentropic**. Because the amplitude of an ordinary sound wave is very small, and it does not cause any significant change in temperature and pressure.

Discussion No process is truly isentropic, but the increase of entropy due to sound propagation is negligibly small.

17-14C The *sonic speed* in a medium **depends on the properties of the medium, and it changes as the properties of the medium change**.

Discussion The most common example is the change in speed of sound due to temperature change.

17-15C Sound travels faster in **warm (higher temperature) air** since $c = \sqrt{kRT}$.

Discussion On the microscopic scale, we can imagine the air molecules moving around at higher speed in warmer air, leading to higher propagation of disturbances.

17-16C **Sound travels fastest in helium**, since $c = \sqrt{kRT}$ and helium has the highest kR value. It is about 0.40 for air, 0.35 for argon, and 3.46 for helium.

Discussion We are assuming, of course, that these gases behave as ideal gases – a good approximation at room temperature.

17-17C Air at specified conditions will behave like an ideal gas, and *the speed of sound in an ideal gas depends on temperature only*. Therefore, **the speed of sound is the same in both mediums**.

Discussion If the temperature were different, however, the speed of sound would be different.

17-18C In general, **no**, because the Mach number also depends on the speed of sound in gas, which depends on the temperature of the gas. **The Mach number remains constant only if the temperature *and* the velocity are constant**.

Discussion It turns out that the speed of sound is not a strong function of pressure. In fact, it is not a function of pressure at all for an ideal gas.

17-19 The Mach number of an aircraft and the speed of sound in air are to be determined at two specified temperatures.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis From the definitions of the speed of sound and the Mach number,

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(300\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{347\ m/s}$$

and $$\text{Ma} = \frac{V}{c} = \frac{240\text{ m/s}}{347\text{ m/s}} = \mathbf{0.692}$$

(*b*) At 1000 K,

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(1000\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{634\ m/s}$$

and $$\text{Ma} = \frac{V}{c} = \frac{240\text{ m/s}}{634\text{ m/s}} = \mathbf{0.379}$$

Discussion Note that a constant Mach number does not necessarily indicate constant speed. The Mach number of a rocket, for example, will be increasing even when it ascends at constant speed. Also, the specific heat ratio k changes with temperature, and the accuracy of the result at 1000 K can be improved by using the k value at that temperature (it would give k = 1.386, c = 619 m/s, and Ma = 0.388).

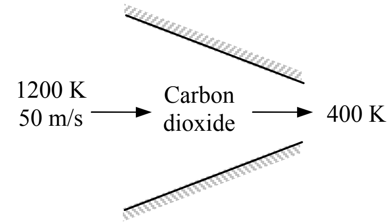
17-20 Carbon dioxide flows through a nozzle. The inlet temperature and velocity and the exit temperature of CO_2 are specified. The Mach number is to be determined at the inlet and exit of the nozzle.

Assumptions **1** CO_2 is an ideal gas with constant specific heats at room temperature. **2** This is a steady-flow process.

Properties The gas constant of carbon dioxide is R = 0.1889 kJ/kg·K. Its constant pressure specific heat and specific heat ratio at room temperature are c_p = 0.8439 kJ/kg·K and k = 1.288.

Analysis (*a*) At the inlet

$$c_1 = \sqrt{k_1 R T_1} = \sqrt{(1.288)(0.1889\ \text{kJ/kg}\cdot\text{K})(1200\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 540.3\ \text{m/s}$$



Thus,

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{50\ \text{m/s}}{540.3\ \text{m/s}} = \mathbf{0.0925}$$

(*b*) At the exit,

$$c_2 = \sqrt{k_2 R T_2} = \sqrt{(1.288)(0.1889\ \text{kJ/kg}\cdot\text{K})(400\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 312.0\ \text{m/s}$$

The nozzle exit velocity is determined from the steady-flow energy balance relation,

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \quad \rightarrow \quad 0 = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$0 = (0.8439\ \text{kJ/kg}\cdot\text{K})(400 - 1200\ \text{K}) + \frac{V_2^2 - (50\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) \longrightarrow V_2 = 1163\ \text{m/s}$$

Thus,

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{1163\ \text{m/s}}{312\ \text{m/s}} = \mathbf{3.73}$$

Discussion The specific heats and their ratio k change with temperature, and the accuracy of the results can be improved by accounting for this variation. Using EES (or another property database):

At 1200 K: c_p = 1.278 kJ/kg·K, k = 1.173 $\rightarrow$ c_1 = 516 m/s, V_1 = 50 m/s, Ma_1 = 0.0969

At 400 K: c_p = 0.9383 kJ/kg·K, k = 1.252 $\rightarrow$ c_2 = 308 m/s, V_2 = 1356 m/s, Ma_2 = 4.41

Therefore, the constant specific heat assumption results in an error of **4.5%** at the inlet and **15.5%** at the exit in the Mach number, which are significant.

17-21 Nitrogen flows through a heat exchanger. The inlet temperature, pressure, and velocity and the exit pressure and velocity are specified. The Mach number is to be determined at the inlet and exit of the heat exchanger.

Assumptions **1** N_2 is an ideal gas. **2** This is a steady-flow process. **3** The potential energy change is negligible.

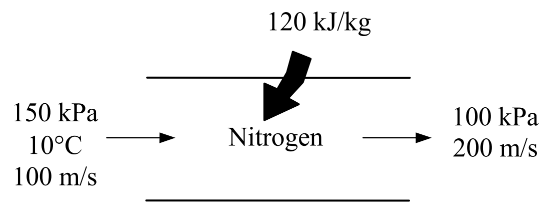
Properties The gas constant of N_2 is R = 0.2968 kJ/kg·K. Its constant pressure specific heat and specific heat ratio at room temperature are c_p = 1.040 kJ/kg·K and k = 1.4.

Analysis

$$c_1 = \sqrt{k_1 R T_1} = \sqrt{(1.400)(0.2968\text{ kJ/kg}\cdot\text{K})(283\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 342.9\text{ m/s}$$

Thus,

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{100\text{ m/s}}{342.9\text{ m/s}} = \mathbf{0.292}$$



From the energy balance on the heat exchanger,

$$q_{\text{in}} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$120\text{ kJ/kg} = (1.040\text{ kJ/kg.°C})(T_2 - 10\text{°C}) + \frac{(200\text{ m/s})^2 - (100\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$

It yields

$$T_2 = 111\text{°C} = 384\text{ K}$$

$$c_2 = \sqrt{k_2 R T_2} = \sqrt{(1.4)(0.2968\text{ kJ/kg}\cdot\text{K})(384\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 399\text{ m/s}$$

Thus,

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{200\text{ m/s}}{399\text{ m/s}} = \mathbf{0.501}$$

Discussion The specific heats and their ratio k change with temperature, and the accuracy of the results can be improved by accounting for this variation. Using EES (or another property database):

At 10°C : c_p = 1.038 kJ/kg·K, k = 1.400 → c_1 = 343 m/s, V_1 = 100 m/s, Ma_1 = 0.292

At 111°C c_p = 1.041 kJ/kg·K, k = 1.399 → c_2 = 399 m/s, V_2 = 200 m/s, Ma_2 = 0.501

Therefore, the constant specific heat assumption results in an error of **4.5%** at the inlet and **15.5%** at the exit in the Mach number, which are almost identical to the values obtained assuming constant specific heats.

17-22 The speed of sound in refrigerant-134a at a specified state is to be determined.

Assumptions R-134a is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of R-134a is R = 0.08149 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.108.

Analysis From the ideal-gas speed of sound relation,

$$c = \sqrt{kRT} = \sqrt{(1.108)(0.08149\text{ kJ/kg}\cdot\text{K})(60+273\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{173\ m/s}$$

Discusion Note that the speed of sound is independent of pressure for ideal gases.

17-23 The Mach number of a passenger plane for specified limiting operating conditions is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis From the speed of sound relation

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(-60+273\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 293\ \text{m/s}$$

Thus, the Mach number corresponding to the maximum cruising speed of the plane is

$$\text{Ma} = \frac{V_{\max}}{c} = \frac{(945/3.6)\ \text{m/s}}{293\ \text{m/s}} = \mathbf{0.897}$$

Discussion Note that this is a subsonic flight since Ma < 1. Also, using a k value at -60°C would give practically the same result.

17-24E Steam flows through a device at a specified state and velocity. The Mach number of steam is to be determined assuming ideal gas behavior.

Assumptions Steam is an ideal gas with constant specific heats.

Properties The gas constant of steam is R = 0.1102 Btu/lbm·R. Its specific heat ratio is given to be k = 1.3.

Analysis From the ideal-gas speed of sound relation,

$$c = \sqrt{kRT} = \sqrt{(1.3)(0.1102\ \text{Btu/lbm}\cdot\text{R})(1160\ \text{R})\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/lbm}}\right)} = 2040\ \text{ft/s}$$

Thus,

$$\text{Ma} = \frac{V}{c} = \frac{900\ \text{ft/s}}{2040\ \text{ft/s}} = \mathbf{0.441}$$

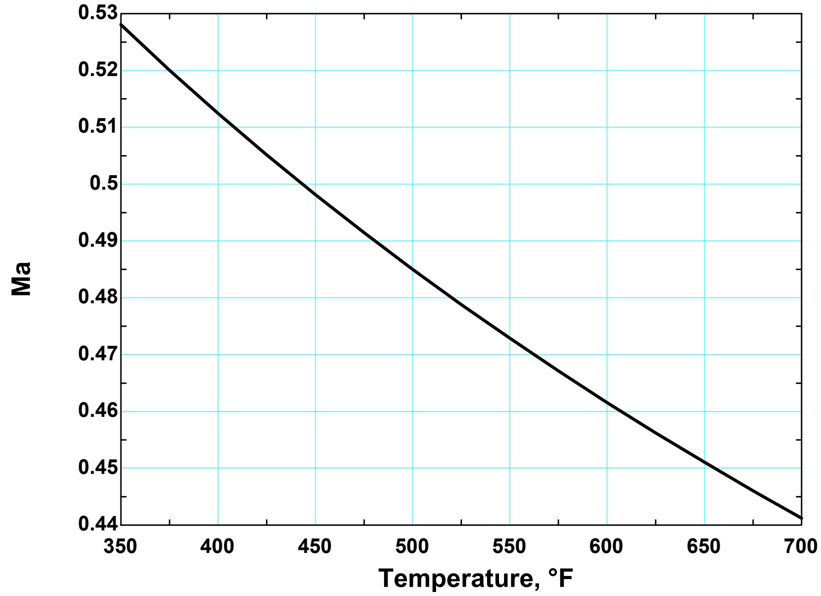
Discussion Using property data from steam tables and not assuming ideal gas behavior, it can be shown that the Mach number in steam at the specified state is 0.446, which is sufficiently close to the ideal-gas value of 0.441. Therefore, the ideal gas approximation is a reasonable one in this case.

17-25E Problem 17-24E is reconsidered. The variation of Mach number with temperature as the temperature changes between 350° and 700°F is to be investigated, and the results are to be plotted.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
T=Temperature+460
R=0.1102
V=900
k=1.3
c=SQRT(k*R*T*25037)
Ma=V/c
```

Temperature, *T*, °F	Mach number Ma
350	0.528
375	0.520
400	0.512
425	0.505
450	0.498
475	0.491
500	0.485
525	0.479
550	0.473
575	0.467
600	0.462
625	0.456
650	0.451
675	0.446
700	0.441



Discussion Note that for a specified flow speed, the Mach number decreases with increasing temperature, as expected.

17-26 The expression for the speed of sound for an ideal gas is to be obtained using the isentropic process equation and the definition of the speed of sound.

Analysis The isentropic relation $Pv^k = A$ where A is a constant can also be expressed as

$$P = A\left(\frac{1}{v}\right)^k = A\rho^k$$

Substituting it into the relation for the speed of sound,

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = \left(\frac{\partial (A\rho)^k}{\partial \rho}\right)_s = kA\rho^{k-1} = k(A\rho^k)/\rho = k(P/\rho) = kRT$$

since for an ideal gas $P = \rho RT$ or $RT = P/\rho$. Therefore, $\boxed{c = \sqrt{kRT}}$, which is the desired relation.

Discussion Notice that pressure has dropped out; the speed of sound in an ideal gas is *not* a function of pressure.

17-27 The inlet state and the exit pressure of air are given for an isentropic expansion process. The ratio of the initial to the final speed of sound is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.287 kJ/kg·K and k = 1.4. The specific heat ratio k varies with temperature, but in our case this change is very small and can be disregarded.

Analysis The final temperature of air is determined from the isentropic relation of ideal gases,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (333.2\text{ K})\left(\frac{0.4\text{ MPa}}{1.5\text{ MPa}}\right)^{(1.4-1)/1.4} = 228.4\text{ K}$$

Treating k as a constant, the ratio of the initial to the final speed of sound can be expressed as

$$\text{Ratio} = \frac{c_2}{c_1} = \frac{\sqrt{k_1 RT_1}}{\sqrt{k_2 RT_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{\sqrt{333.2}}{\sqrt{228.4}} = \mathbf{1.21}$$

Discussion Note that the speed of sound is proportional to the square root of thermodynamic temperature.

17-28 The inlet state and the exit pressure of helium are given for an isentropic expansion process. The ratio of the initial to the final speed of sound is to be determined.

Assumptions Helium is an ideal gas with constant specific heats at room temperature.

Properties The properties of helium are R = 2.0769 kJ/kg·K and k = 1.667.

Analysis The final temperature of helium is determined from the isentropic relation of ideal gases,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (333.2\text{ K})\left(\frac{0.4}{1.5}\right)^{(1.667-1)/1.667} = 196.3\text{ K}$$

The ratio of the initial to the final speed of sound can be expressed as

$$\text{Ratio} = \frac{c_2}{c_1} = \frac{\sqrt{k_1 RT_1}}{\sqrt{k_2 RT_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{\sqrt{333.2}}{\sqrt{196.3}} = \mathbf{1.30}$$

Discussion Note that the speed of sound is proportional to the square root of thermodynamic temperature.

17-29E The inlet state and the exit pressure of air are given for an isentropic expansion process. The ratio of the initial to the final speed of sound is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.06855 Btu/lbm·R and k = 1.4. The specific heat ratio k varies with temperature, but in our case this change is very small and can be disregarded.

Analysis The final temperature of air is determined from the isentropic relation of ideal gases,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (659.7\ \text{R})\left(\frac{60}{170}\right)^{(1.4-1)/1.4} = 489.9\ \text{R}$$

Treating k as a constant, the ratio of the initial to the final speed of sound can be expressed as

$$\text{Ratio} = \frac{c_2}{c_1} = \frac{\sqrt{k_1RT_1}}{\sqrt{k_2RT_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{\sqrt{659.7}}{\sqrt{489.9}} = \mathbf{1.16}$$

Discussion Note that the speed of sound is proportional to the square root of thermodynamic temperature.

One Dimensional Isentropic Flow

17-30C (*a*) The **velocity increases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid decrease**.

Discussion The velocity increase is opposite to what happens in supersonic flow.

17-31C (*a*) The **velocity decreases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid increase**.

Discussion The velocity decrease is opposite to what happens in supersonic flow.

17-32C (*a*) The **exit velocity remains constant at sonic speed**, (*b*) the **mass flow rate through the nozzle decreases because of the reduced flow area**.

Discussion Without a diverging portion of the nozzle, a converging nozzle is limited to sonic velocity at the exit.

17-33C (*a*) The **velocity decreases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid increase**.

Discussion The velocity decrease is opposite to what happens in subsonic flow.

17-34C (*a*) The **velocity increases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid decrease**.

Discussion The velocity increase is opposite to what happens in subsonic flow.

17-35C **The pressures at the two throats are identical**.

Discussion Since the gas has the same stagnation conditions, it also has the same sonic conditions at the throat.

17-36C **No**, it is not possible.

Discussion The only way to do it is to have first a converging nozzle, and then a diverging nozzle.

17-37 The Mach number of scramjet and the air temperature are given. The speed of the engine is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis The temperature is -20 + 273.15 = 253.15 K. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(253.15\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 318.93\text{ m/s}$$

and

$$V = c\text{Ma} = (318.93\text{ m/s})(7)\left(\frac{3.6\text{ km/h}}{1\text{ m/s}}\right) = 8037\text{ km/h} \cong \mathbf{8040\text{ km/h}}$$

Discussion Note that extremely high speed can be achieved with scramjet engines. We cannot justify more than three significant digits in a problem like this.

17-38E The Mach number of scramjet and the air temperature are given. The speed of the engine is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.06855 Btu/lbm·R. Its specific heat ratio at room temperature is k = 1.4 .

Analysis The temperature is 0 + 459.67 = 459.67 R. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.06855\text{ Btu/lbm}\cdot\text{R})(459.67\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)} = 1050.95\text{ ft/s}$$

and

$$V = c\text{Ma} = (1050.95\text{ ft/s})(7)\left(\frac{1\text{ mi/h}}{1.46667\text{ ft/s}}\right) = 5015.9\text{ mi/h} \cong \mathbf{5020\text{ mi/h}}$$

Discussion Note that extremely high speed can be achieved with scramjet engines. We cannot justify more than three significant digits in a problem like this.

17-39 The speed of an airplane and the air temperature are give. It is to be determined if the speed of this airplane is subsonic or supersonic.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis The temperature is -50 + 273.15 = 223.15 K. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(223.15 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}\left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}}\right) = 1077.97 \text{ km/h}$$

and

$$\text{Ma} = \frac{V}{c} = \frac{920 \text{ km/h}}{1077.97 \text{ km/h}} = 0.85346 \cong \mathbf{0.853}$$

The speed of the airplane is **subsonic** since the Mach number is less than 1.

Discussion Subsonic airplanes stay sufficiently far from the Mach number of 1 to avoid the instabilities associated with transonic flights.

17-40 The critical temperature, pressure, and density of air and helium are to be determined at specified conditions.

Assumptions Air and Helium are ideal gases with constant specific heats at room temperature.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K, k = 1.4, and c_p = 1.005 kJ/kg·K. The properties of helium at room temperature are R = 2.0769 kJ/kg·K, k = 1.667, and c_p = 5.1926 kJ/kg·K.

Analysis (*a*) Before we calculate the critical temperature T^*, pressure P^*, and density ρ^*, we need to determine the stagnation temperature T_0, pressure P_0, and density ρ_0.

$$T_0 = 100°\text{C} + \frac{V^2}{2c_p} = 100 + \frac{(250\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 131.1°\text{C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\text{ kPa})\left(\frac{404.3\text{ K}}{373.2\text{ K}}\right)^{1.4/(1.4-1)} = 264.7\text{ kPa}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{264.7\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(404.3\text{ K})} = 2.281\text{ kg/m}^3$$

Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (404.3\text{ K})\left(\frac{2}{1.4+1}\right) = \mathbf{337\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (264.7\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{140\ kPa}$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (2.281\text{ kg/m}^3)\left(\frac{2}{1.4+1}\right)^{1/(1.4-1)} = \mathbf{1.45\ kg/m^3}$$

(*b*) For helium, $T_0 = T + \frac{V^2}{2c_p} = 40 + \frac{(300\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 48.7°\text{C}$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\text{ kPa})\left(\frac{321.9\text{ K}}{313.2\text{ K}}\right)^{1.667/(1.667-1)} = 214.2\text{ kPa}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{214.2\text{ kPa}}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(321.9\text{ K})} = 0.320\text{ kg/m}^3$$

Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (321.9\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{241\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (214.2\text{ kPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{104.3\ kPa}$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (0.320\text{ kg/m}^3)\left(\frac{2}{1.667+1}\right)^{1/(1.667-1)} = \mathbf{0.208\ kg/m^3}$$

Discussion These are the temperature, pressure, and density values that will occur at the throat when the flow past the throat is supersonic.

17-41 Quiescent carbon dioxide at a given state is accelerated isentropically to a specified Mach number. The temperature and pressure of the carbon dioxide after acceleration are to be determined.

Assumptions Carbon dioxide is an ideal gas with constant specific heats at room temperature.

Properties The specific heat ratio of the carbon dioxide at room temperature is $k = 1.288$.

Analysis The inlet temperature and pressure in this case is equivalent to the stagnation temperature and pressure since the inlet velocity of the carbon dioxide is said to be negligible. That is, $T_0 = T_i = 400$ K and $P_0 = P_i = 1200$ kPa. Then,

$$T = T_0\left(\frac{2}{2+(k-1)\text{Ma}^2}\right) = (600\text{ K})\left(\frac{2}{2+(1.288\text{-}1)(0.6)^2}\right) = 570.43\text{ K} \cong \mathbf{570\ K}$$

and

$$P = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (1200\text{ kPa})\left(\frac{570.43\text{ K}}{600\text{ K}}\right)^{1.288/(1.288-1)} = 957.23\text{ K} \cong \mathbf{957\ \ kPa}$$

Discussion Note that both the pressure and temperature drop as the gas is accelerated as part of the internal energy of the gas is converted to kinetic energy.

17-42 Air enters a converging-diverging nozzle at specified conditions. The lowest pressure that can be obtained at the throat of the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air at room temperature is $k = 1.4$.

Analysis The lowest pressure that can be obtained at the throat is the critical pressure P^*, which is determined from

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (800\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{423\ kPa}$$

Discussion This is the pressure that occurs at the throat when the flow past the throat is supersonic.

17-43 Helium enters a converging-diverging nozzle at specified conditions. The lowest temperature and pressure that can be obtained at the throat of the nozzle are to be determined.

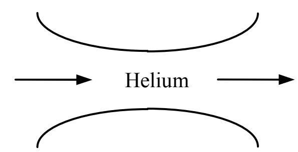
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of helium are k = 1.667 and c_p = 5.1926 kJ/kg·K.

Analysis The lowest temperature and pressure that can be obtained at the throat are the critical temperature T^* and critical pressure P^*. First we determine the stagnation temperature T_0 and stagnation pressure P_0,

$$T_0 = T + \frac{V^2}{2c_p} = 800\text{ K} + \frac{(100\text{ m/s})^2}{2 \times 5.1926\text{ kJ/kg}\cdot{}^\circ\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 801\text{ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.7\text{ MPa})\left(\frac{801\text{ K}}{800\text{ K}}\right)^{1.667/(1.667-1)} = 0.702\text{ MPa}$$



Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (801\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{601\ K}$$

and

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.702\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.342\ MPa}$$

Discussion These are the temperature and pressure that will occur at the throat when the flow past the throat is supersonic.

17-44 Air flows through a duct. The state of the air and its Mach number are specified. The velocity and the stagnation pressure, temperature, and density of the air are to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air at room temperature are $R = 0.287$ kPa.m^3/kg.K and $k = 1.4$.

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(373.2\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 387.2\text{ m/s}$$

Thus,

$$V = \text{Ma}\times c = (0.8)(387.2\text{ m/s}) = \mathbf{310\ m/s}$$

Also,

$$\rho = \frac{P}{RT} = \frac{200\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(373.2\text{ K})} = 1.867\text{ kg/m}^3$$

Then the stagnation properties are determined from

$$T_0 = T\left(1+\frac{(k-1)\text{Ma}^2}{2}\right) = (373.2\text{ K})\left(1+\frac{(1.4\text{-}1)(0.8)^2}{2}\right) = \mathbf{421\ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\text{ kPa})\left(\frac{421.0\text{ K}}{373.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{305\ kPa}$$

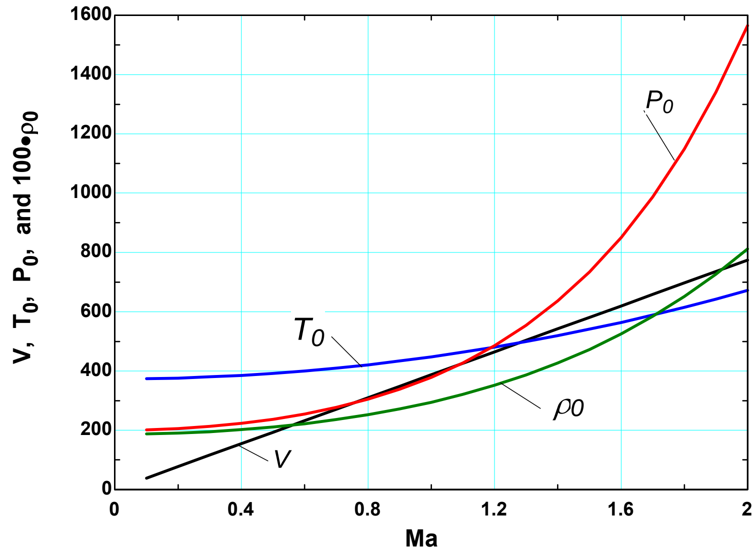
$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{1/(k-1)} = (1.867\text{ kg/m}^3)\left(\frac{421.0\text{ K}}{373.2\text{ K}}\right)^{1/(1.4-1)} = \mathbf{2.52\ kg/m^3}$$

Discussion Note that both the pressure and temperature drop as the gas is accelerated as part of the internal energy of the gas is converted to kinetic energy.

17-45 Problem 17-44 is reconsidered. The effect of Mach number on the velocity and stagnation properties as the Ma is varied from 0.1 to 2 are to be investigated, and the results are to be plotted.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
P=200
T=100+273.15
R=0.287
k=1.4
c=SQRT(k*R*T*1000)
Ma=V/c
rho=P/(R*T)

"Stagnation properties"
T0=T*(1+(k-1)*Ma^2/2)
P0=P*(T0/T)^(k/(k-1))
rho0=rho*(T0/T)^(1/(k-1))
```



Mach num. Ma	Velocity, V, m/s	Stag. Temp, T_0, K	Stag. Press, P_0, kPa	Stag. Density, ρ_0, kg/m^3
0.1	38.7	373.9	201.4	1.877
0.2	77.4	376.1	205.7	1.905
0.3	116.2	379.9	212.9	1.953
0.4	154.9	385.1	223.3	2.021
0.5	193.6	391.8	237.2	2.110
0.6	232.3	400.0	255.1	2.222
0.7	271.0	409.7	277.4	2.359
0.8	309.8	420.9	304.9	2.524
0.9	348.5	433.6	338.3	2.718
1.0	387.2	447.8	378.6	2.946
1.1	425.9	463.5	427.0	3.210
1.2	464.7	480.6	485.0	3.516
1.3	503.4	499.3	554.1	3.867
1.4	542.1	519.4	636.5	4.269
1.5	580.8	541.1	734.2	4.728
1.6	619.5	564.2	850.1	5.250
1.7	658.3	588.8	987.2	5.842
1.8	697.0	615.0	1149.2	6.511
1.9	735.7	642.6	1340.1	7.267
2.0	774.4	671.7	1564.9	8.118

Discussion Note that as Mach number increases, so does the flow velocity and stagnation temperature, pressure, and density.

17-46 An aircraft is designed to cruise at a given Mach number, elevation, and the atmospheric temperature. The stagnation temperature on the leading edge of the wing is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.287 kPa.m^3/kg.K, c_p = 1.005 kJ/kg·K, and k = 1.4 .

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(236.15\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 308.0\ \text{m/s}$$

Thus,

$$V = \text{Ma}\times c = (1.4)(308.0\ \text{m/s}) = 431.2\ \text{m/s}$$

Then,

$$T_0 = T + \frac{V^2}{2c_p} = 236.15 + \frac{(431.2\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{329\ K}$$

Discussion Note that the temperature of a gas increases during a stagnation process as the kinetic energy is converted to enthalpy.

17-47E Air flows through a duct at a specified state and Mach number. The velocity and the stagnation pressure, temperature, and density of the air are to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R and k = 1.4.

Analysis First, T = 320 + 459.67 = 779.67 K. The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.06855\ \text{Btu/1bm}\cdot\text{R})(779.67\ \text{R})\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/1bm}}\right)} = 1368.72\ \text{ft/s}$$

Thus,

$$V = \text{Ma}\times c = (0.7)(1368.72\ \text{ft/s}) = 958.10 \cong \mathbf{958\ ft/s}$$

Also,

$$\rho = \frac{P}{RT} = \frac{25\ \text{psia}}{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(779.67\ \text{R})} = 0.086568\ \text{1bm/ft}^3$$

Then the stagnation properties are determined from

$$T_0 = T\left(1+\frac{(k-1)\text{Ma}^2}{2}\right) = (779.67\ \text{R})\left(1+\frac{(1.4\text{-}1)(0.7)^2}{2}\right) = 856.08\ \text{R} \cong \mathbf{856\ R}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (25\ \text{psia})\left(\frac{856.08\ \text{R}}{779.67\ \text{R}}\right)^{1.4/(1.4-1)} = 34.678\ \text{psia} \cong \mathbf{34.7\ psia}$$

$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{1/(k-1)} = (0.08656\ \ \text{1bm/ft}^3)\left(\frac{856.08\ \text{R}}{779.67\ \text{R}}\right)^{1/(1.4-1)} = 0.10936\ \text{lbm/ft}^3 \cong \mathbf{0.109\ lbm/ft^3}$$

Discussion Note that the temperature, pressure, and density of a gas increases during a stagnation process.

Isentropic Flow Through Nozzles

17-48C The fluid would **accelerate even further** instead of decelerating.

Discussion This is the opposite of what would happen in subsonic flow.

17-49C The fluid would **accelerate even further**, as desired.

Discussion This is the opposite of what would happen in subsonic flow.

17-50C (*a*) The **exit velocity reaches the sonic speed**, (*b*) the **exit pressure equals the critical pressure**, and (*c*) the **mass flow rate reaches the maximum value**.

Discussion In such a case, we say that the flow is *choked*.

17-51C (*a*) **No effect on velocity**. (*b*) **No effect on pressure**. (*c*) **No effect on mass flow rate**.

Discussion In this situation, the flow is already choked initially, so further lowering of the back pressure does not change anything upstream of the nozzle exit plane.

17-52C **If the back pressure is low enough so that sonic conditions exist at the throats, the mass flow rates in the two nozzles would be identical**. However, **if the flow is not sonic at the throat, the mass flow rate through the nozzle with the diverging section would be greater**, because it acts like a subsonic diffuser.

Discussion Once the flow is choked at the throat, whatever happens downstream is irrelevant to the flow upstream of the throat.

17-53C Maximum flow rate through a converging nozzle is achieved when Ma = 1 at the exit of a nozzle. For all other Ma values the mass flow rate decreases. Therefore, **the mass flow rate would decrease if hypersonic velocities were achieved at the throat of a converging nozzle**.

Discussion Note that this is not possible unless the flow upstream of the converging nozzle is already hypersonic.

17-54C Ma* is the **local velocity non-dimensionalized with respect to the sonic speed at the throat**, whereas Ma is the **local velocity non-dimensionalized with respect to the local sonic speed**.

Discussion The two are identical at the throat when the flow is choked.

17-55C (*a*) The **velocity decreases**, (*b*) the **pressure increases**, and (*c*) the **mass flow rate remains the same**.

Discussion Qualitatively, this is the same as what we are used to (in previous chapters) for incompressible flow.

17-56C **No, if the flow in the throat is subsonic**. If the velocity at the throat is subsonic, the diverging section would act like a diffuser and decelerate the flow. **Yes, if the flow in the throat is already supersonic**, the diverging section would accelerate the flow to even higher Mach number.

Discussion In duct flow, the latter situation is not possible unless a second converging-diverging portion of the duct is located upstream, and there is sufficient pressure difference to choke the flow in the upstream throat.

17-57 It is to be explained why the maximum flow rate per unit area for a given ideal gas depends only on $P_0/\sqrt{T_0}$. Also for an ideal gas, a relation is to be obtained for the constant a in $\dot{m}_{\max}/A^* = a\left(P_0/\sqrt{T_0}\right)$.

Properties The properties of the ideal gas considered are $R = 0.287$ kPa.m^3/kg·K and $k = 1.4$.

Analysis The maximum flow rate is given by

$$\dot{m}_{\max} = A^* P_0 \sqrt{k/RT_0}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \quad \text{or} \quad \dot{m}_{\max}/A^* = \left(P_0/\sqrt{T_0}\right)\sqrt{k/R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

For a given gas, k and R are fixed, and thus the mass flow rate depends on the parameter $P_0/\sqrt{T_0}$. Thus, $\dot{m}_{\max}/A^*$ can be expressed as $\dot{m}_{\max}/A^* = a\left(P_0/\sqrt{T_0}\right)$ where

$$a = \sqrt{k/R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} = \sqrt{\frac{1.4}{(0.287\text{ kJ/kg.K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}}\left(\frac{2}{1.4+1}\right)^{2.4/0.8} = \mathbf{0.0404\ (m/s)\sqrt{K}}$$

Discussion Note that when sonic conditions exist at a throat of known cross-sectional area, the mass flow rate is fixed by the stagnation conditions.

17-58 For an ideal gas, an expression is to be obtained for the ratio of the speed of sound where Ma = 1 to the speed of sound based on the stagnation temperature, c^*/c_0.

Analysis For an ideal gas the speed of sound is expressed as $c = \sqrt{kRT}$. Thus,

$$\frac{c^*}{c_0} = \frac{\sqrt{kRT^*}}{\sqrt{kRT_0}} = \left(\frac{T^*}{T_0}\right)^{1/2} = \left(\frac{\mathbf{2}}{\boldsymbol{k}+\mathbf{1}}\right)^{\mathbf{1/2}}$$

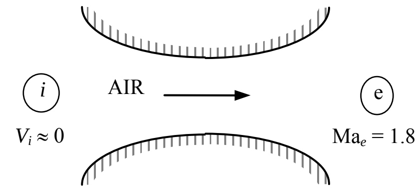
Discussion Note that a speed of sound changes the flow as the temperature changes.

17-59 Air enters a converging-diverging nozzle at a specified pressure. The back pressure that will result in a specified exit Mach number is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis The stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. It remains constant throughout the nozzle since the flow is isentropic,



$$P_0 = P_i = 1.2 \text{ MPa}$$

From Table A-32 at $Ma_e = 1.8$, we read $P_e/P_0 = 0.1740$.

Thus,

$$P = 0.1740P_0 = 0.1740(1.2 \text{ MPa}) = \mathbf{0.209\ MPa}$$

Discussion If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

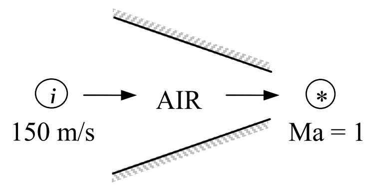
17-60 Air enters a nozzle at specified temperature, pressure, and velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are $k = 1.4$ and $c_p = 1.005$ kJ/kg·K.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 420\text{ K} + \frac{(150\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 431.194\text{ K}$$



and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (0.6\text{ MPa})\left(\frac{431.194\text{ K}}{420\text{ K}}\right)^{1.4/(1.4-1)} = 0.65786\text{ MPa}$$

From Table A-32 (or from Eqs. 17-18 and 17-19) at Ma = 1, we read $T/T_0 = 0.8333$, $P/P_0 = 0.5283$. Thus,

$$T = 0.8333T_0 = 0.8333(431.194\text{ K}) = 359.31\text{ K} \approx \mathbf{359\text{ K}}$$

and

$$P = 0.5283P_0 = 0.5283(0.65786\text{ MPa}) = 0.34754\text{ MPa} \approx \mathbf{0.348\text{ MPa} = 348\text{ kPa}}$$

Also,

$$c_i = \sqrt{kRT}_i = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(420\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 410.799\text{ m/s}$$

and

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{150\text{ m/s}}{410.799\text{ m/s}} = 0.3651$$

From Table A-32 at this Mach number we read $A_i/A^* = 1.7452$. Thus the ratio of the throat area to the nozzle inlet area is

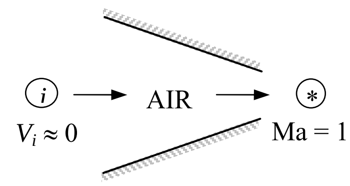
$$\frac{A^*}{A_i} = \frac{1}{1.7452} = 0.57300 \cong \mathbf{0.573}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

17-61 Air enters a nozzle at specified temperature and pressure with low velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is $k = 1.4$.



Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i = 350 \text{ K} \quad \text{and} \quad P_0 = P_i = 0.2 \text{ MPa}$$

From Table A-32 (or from Eqs. 17-18 and 17-19) at Ma =1, we read T/T_0 =0.8333, P/P_0 = 0.5283.

Thus,

$$T = 0.8333T_0 = 0.8333(350 \text{ K}) = \mathbf{292\ K}$$

and

$$P = 0.5283P_0 = 0.5283(0.2 \text{ MPa}) = \mathbf{0.106\ MPa}$$

The Mach number at the nozzle inlet is Ma = 0 since $V_i \cong 0$. From Table A-32 at this Mach number we read $A_i/A^* = \infty$.

Thus the ratio of the throat area to the nozzle inlet area is $\dfrac{A^*}{A_i} = \dfrac{1}{\infty} = \mathbf{0}$.

Discussion If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

17-62E Air enters a nozzle at specified temperature, pressure, and velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

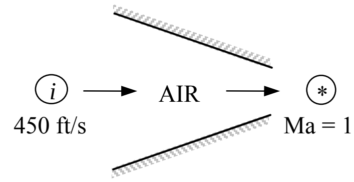
Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are $k = 1.4$ and $c_p = 0.240$ Btu/lbm·R (Table A-2Ea).

Analysis The properties of the fluid at the location where Ma =1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T + \frac{V_i^2}{2c_p} = 630\text{ R} + \frac{(450\text{ ft/s})^2}{2\times 0.240\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/1bm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 646.9\text{ R}$$

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (30\text{ psia})\left(\frac{646.9\text{ K}}{630\text{ K}}\right)^{1.4/(1.4-1)} = 32.9\text{ psia}$$



From Table A-32 (or from Eqs. 17-18 and 17-19) at Ma =1, we read T/T_0 =0.8333, P/P_0 = 0.5283.

Thus,

$$T = 0.8333T_0 = 0.8333(646.9\text{ R}) = \mathbf{539\ R}$$

and

$$P = 0.5283P_0 = 0.5283(32.9\text{ psia}) = \mathbf{17.4\ psia}$$

Also,

$$c_i = \sqrt{kRT_i} = \sqrt{(1.4)(0.06855\text{ Btu/1bm}\cdot\text{R})(630\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 1230\text{ ft/s}$$

and

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{450\text{ ft/s}}{1230\text{ ft/s}} = 0.3657$$

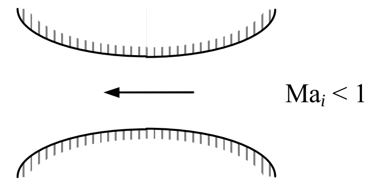
From Table A-32 at this Mach number we read $A_i/A^* = 1.7426$. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{1.7426} = \mathbf{0.574}$$

Discussion If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

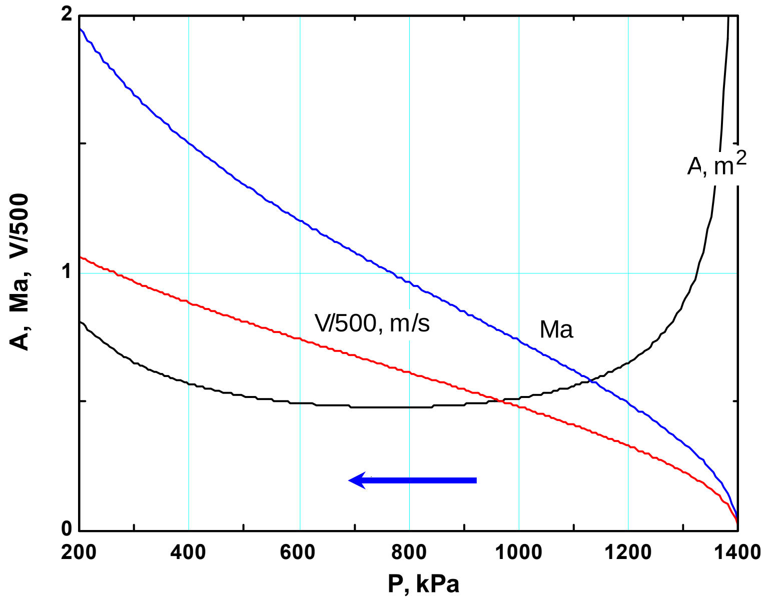
17-63 For subsonic flow at the inlet, the variation of pressure, velocity, and Mach number along the length of the nozzle are to be sketched for an ideal gas under specified conditions.

Assumptions **1** The gas is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The flow is choked at the throat.



Analysis Using EES and CO_2 as the gas, we calculate and plot flow area *A*, velocity *V*, and Mach number Ma as the pressure drops from a stagnation value of 1400 kPa to 200 kPa. Note that the curve for *A* is related to the shape of the nozzle, with horizontal axis serving as the centerline. The EES equation window and the plot are shown below.

```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
P0=1400 "kPa"

T0=473 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
rho_norm=rho/rho_0 "Normalized density"
T=T0*(P/P0)^((k-1)/k)
Tnorm=T/T0  "Normalized temperature"
V=SQRT(2*Cp*(T0-T)*1000)
V_norm=V/500
A=m/(rho*V)*500
C=SQRT(k*R*T*1000)
Ma=V/C
```

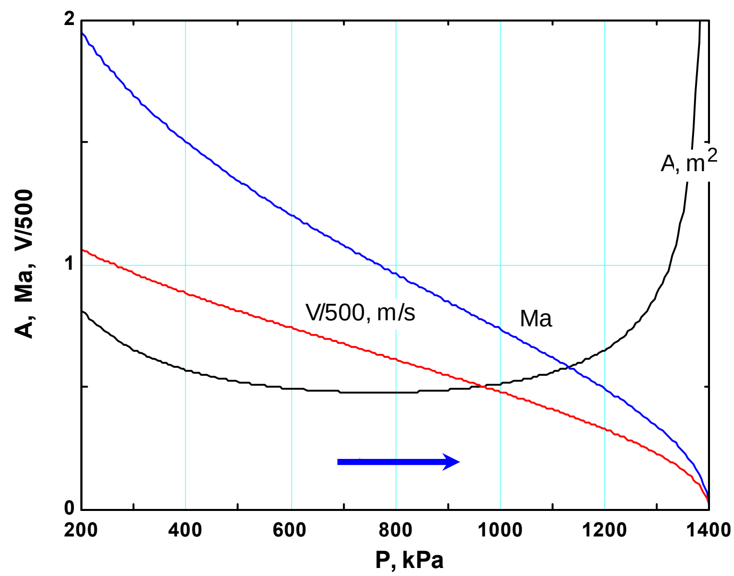


Discussion We are assuming that the back pressure is sufficiently low that the flow is choked at the throat, and the flow downstream of the throat is supersonic without any shock waves. Mach number and velocity continue to rise right through the throat into the diverging portion of the nozzle, since the flow becomes supersonic.

17-64 We repeat the previous problem, but for supersonic flow at the inlet. The variation of pressure, velocity, and Mach number along the length of the nozzle are to be sketched for an ideal gas under specified conditions.

Analysis Using EES and CO_2 as the gas, we calculate and plot flow area *A*, velocity *V*, and Mach number Ma as the pressure rises from 200 kPa at a very high velocity to the stagnation value of 1400 kPa. Note that the curve for *A* is related to the shape of the nozzle, with horizontal axis serving as the centerline.

```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
P0=1400 "kPa"

T0=473 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
rho_norm=rho/rho_0 "Normalized density"
T=T0*(P/P0)^((k-1)/k)
Tnorm=T/T0  "Normalized temperature"
V=SQRT(2*Cp*(T0-T)*1000)
V_norm=V/500
A=m/(rho*V)*500
C=SQRT(k*R*T*1000)
Ma=V/C
```



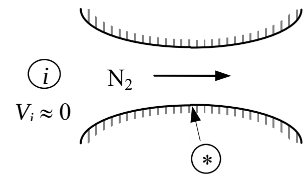
Discussion Note that this problem is identical to the proceeding one, except the flow direction is reversed. In fact, when plotted like this, the plots are identical.

17-65 Nitrogen enters a converging-diverging nozzle at a given pressure. The critical velocity, pressure, temperature, and density in the nozzle are to be determined.

Assumptions **1** Nitrogen is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of nitrogen are $k = 1.4$ and $R = 0.2968$ kJ/kg·K.

Analysis The stagnation pressure in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle,



$$P_0 = P_i = 700 \text{ kPa}$$

$$T_0 = T_i = 400 \text{ K}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{700 \text{ kPa}}{(0.2968 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(400 \text{ K})} = 5.896 \text{ kg/m}^3$$

Critical properties are those at a location where the Mach number is Ma = 1. From Table A-32 at Ma =1, we read T/T_0 =0.8333, $P/P_0 = 0.5283$, and $\rho/\rho_0 = 0.6339$.

Then the critical properties become

$$T^* = 0.8333T_0 = 0.8333(400 \text{ K}) = \mathbf{333\ K}$$

$$P^* = 0.5283P_0 = 0.5283(700 \text{ kPa}) = \mathbf{370\ MPa}$$

$$\rho^* = 0.6339\rho_0 = 0.6339(5.896 \text{ kg/m}^3) = \mathbf{3.74\ kg/m^3}$$

Also,

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.4)(0.2968 \text{ kJ/kg} \cdot \text{K})(333 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{372\ m/s}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

17-66 An ideal gas is flowing through a nozzle. The flow area at a location where Ma = 2.4 is specified. The flow area where Ma = 1.2 is to be determined.

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio is given to be $k = 1.4$.

Analysis The flow is assumed to be isentropic, and thus the stagnation and critical properties remain constant throughout the nozzle. The flow area at a location where $\text{Ma}_2 = 1.2$ is determined using A/A^* data from Table A-32 to be

$$\text{Ma}_1 = 2.4: \frac{A_1}{A^*} = 2.4031 \longrightarrow A^* = \frac{A_1}{2.4031} = \frac{36 \text{ cm}^2}{2.4031} = 14.98 \text{ cm}^2$$

$$\text{Ma}_2 = 1.2: \frac{A_2}{A^*} = 1.0304 \longrightarrow A_2 = (1.0304)A^* = (1.0304)(14.98 \text{ cm}^2) = \mathbf{15.4\ cm^2}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

17-67 An ideal gas is flowing through a nozzle. The flow area at a location where Ma = 2.4 is specified. The flow area where Ma = 1.2 is to be determined.

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The flow is assumed to be isentropic, and thus the stagnation and critical properties remain constant throughout the nozzle. The flow area at a location where Ma_2 = 1.2 is determined using the A/A^* relation,

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left\{\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right\}^{(k+1)/2(k-1)}$$

For k = 1.33 and Ma_1 = 2.4:

$$\frac{A_1}{A^*} = \frac{1}{2.4}\left\{\left(\frac{2}{1.33+1}\right)\left(1+\frac{1.33-1}{2}2.4^2\right)\right\}^{2.33/2\times0.33} = 2.570$$

and

$$A^* = \frac{A_1}{2.570} = \frac{36\ \mathrm{cm}^2}{2.570} = 14.01\ \mathrm{cm}^2$$

For k = 1.33 and Ma_2 = 1.2:

$$\frac{A_2}{A^*} = \frac{1}{1.2}\left\{\left(\frac{2}{1.33+1}\right)\left(1+\frac{1.33-1}{2}1.2^2\right)\right\}^{2.33/2\times0.33} = 1.0316$$

and

$$A_2 = (1.0316)A^* = (1.0316)(14.01\ \mathrm{cm}^2) = \mathbf{14.45\ cm^2}$$

Discussion Note that the compressible flow functions in Table A-32 are prepared for k = 1.4, and thus they cannot be used to solve this problem.

17-68E Air enters a converging-diverging nozzle at a specified temperature and pressure with low velocity. The pressure, temperature, velocity, and mass flow rate are to be calculated in the specified test section.

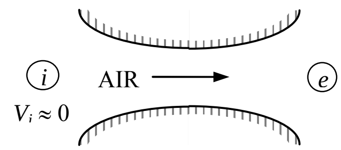
Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are $k = 1.4$ and $R = 0.06855$ Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$$P_0 = P_i = 150 \text{ psia}$$

and



$$T_0 = T_i = 100°\text{F} \approx 560 \text{ R}$$

Then,

$$T_e = T_0\left(\frac{2}{2+(k-1)\text{Ma}^2}\right) = (560 \text{ R})\left(\frac{2}{2+(1.4\text{-}1)2^2}\right) = \mathbf{311 R}$$

$$P_e = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (150 \text{ psia})\left(\frac{311}{560}\right)^{1.4/0.4} = \mathbf{19.1 \text{ psia}}$$

$$\rho_e = \frac{P_e}{RT_e} = \frac{19.1 \text{ psia}}{(0.3704 \text{ psia.ft}^3/\text{1bm}\cdot\text{R})(311 \text{ R})} = 0.166 \text{ 1bm/ft}^3$$

The nozzle exit velocity can be determined from $V_e = \text{Ma}_e c_e$, where c_e is the speed of sound at the exit conditions,

$$V_e = \text{Ma}_e c_e = \text{Ma}_e\sqrt{kRT_e} = (2)\sqrt{(1.4)(0.06855 \text{ Btu/1bm}\cdot\text{R})(311 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = 1729 \text{ ft/s} \cong \mathbf{1730 \text{ ft/s}}$$

Finally,

$$\dot{m} = \rho_e A_e V_e = (0.166 \text{ 1bm/ft}^3)(5 \text{ ft}^2)(1729 \text{ ft/s}) = 1435 \text{ lbm/s} \cong \mathbf{1440 \text{ lbm/s}}$$

Discussion Air must be very dry in this application because the exit temperature of air is extremely low, and any moisture in the air will turn to ice particles.

17-69 Air enters a converging nozzle at a specified temperature and pressure with low velocity. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure are to be calculated and plotted.

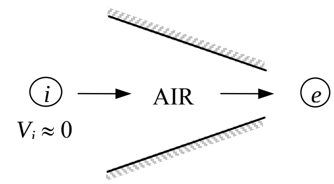
Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are $k = 1.4$, $R = 0.287$ kJ/kg·K, and $c_p = 1.005$ kJ/kg·K.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.,

$$P_0 = P_i = 900 \text{ kPa}$$

$$T_0 = T_i = 400 \text{ K}$$



The critical pressure is determined to be

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (900 \text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 475.5 \text{ kPa}$$

Then the pressure at the exit plane (throat) will be

$$P_e = P_b \qquad \text{for} \qquad P_b \geq 475.5 \text{ kPa}$$

$$P_e = P^* = 475.5 \text{ kPa} \qquad \text{for} \qquad P_b < 475.5 \text{ kPa (choked flow)}$$

Thus the back pressure will not affect the flow when $100 < P_b < 475.5$ kPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from

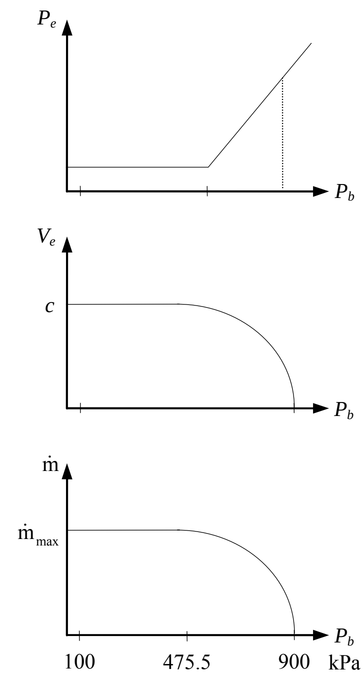
Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (400 \text{ K})\left(\frac{P_e}{900}\right)^{0.4/1.4}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005 \text{ kJ/kg}\cdot\text{K})(400 - T_e)\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.287 \text{ kPa}\cdot\text{m}^3/kg\cdot K)T_e}$$

Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001 \text{ m}^2)$$

The results of the calculations are tabulated as



P_b, kPa	P_e, kPa	T_e, K	V_e, m/s	ρ_e, kg/m^3	$\dot{m}$ kg/s
900	900	400	0	7.840	0
800	800	386.8	162.9	7.206	1.174
700	700	372.3	236.0	6.551	1.546
600	600	356.2	296.7	5.869	1.741
500	500	338.2	352.4	5.151	1.815
475.5	475.5	333.3	366.2	4.971	1.820
400	475.5	333.3	366.2	4.971	1.820
300	475.5	333.3	366.2	4.971	1.820
200	475.5	333.3	366.2	4.971	1.820
100	475.5	333.3	366.2	4.971	1.820

Discussion We see from the plots that once the flow is choked at a back pressure of 475.5 kPa, the mass flow rate remains constant regardless of how low the back pressure gets.

17-70 We are to reconsider the previous problem. Using EES (or other) software, we are to solve the problem for the inlet conditions of 0.8 MPa and 1200 K.

Analysis Air at 800 kPa, 1200 K enters a converging nozzle with a negligible velocity. The throat area of the nozzle is 10 cm2. Assuming isentropic flow, calculate and plot the exit pressure, the exit velocity, and the mass flow rate versus the back pressure P_b for 0.8>= P_b >=0.1 MPa.

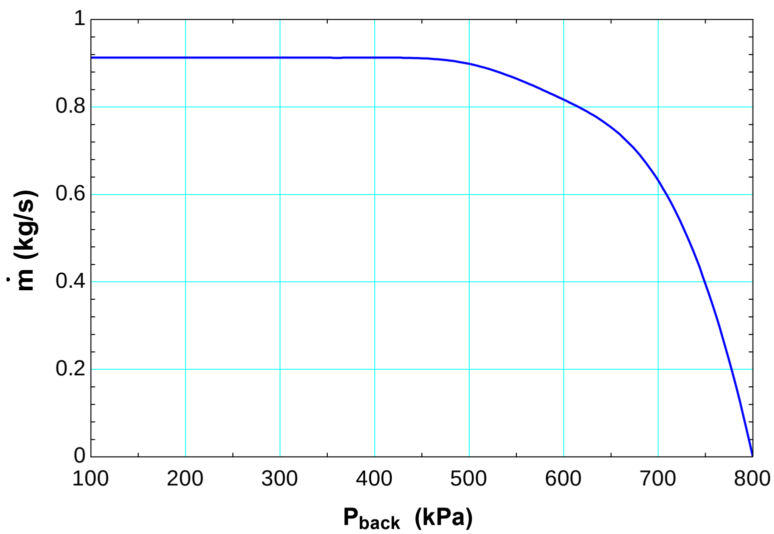
```
Procedure ExitPress(P_back,P_crit : P_exit, Condition$)
If  (P_back>=P_crit)  then
        P_exit:=P_back   "Unchoked Flow Condition"
        Condition$:='unchoked'
else
        P_exit:=P_crit    "Choked Flow Condition"
        Condition$:='choked'
Endif
End

Gas$='Air'
A_cm2=10  "Throat area, cm2"
P_inlet =800"kPa"
T_inlet= 1200"K"
"P_back =422.7"  "kPa"

A_exit = A_cm2*Convert(cm^2,m^2)
C_p=specheat(Gas$,T=T_inlet)
C_p-C_v=R
k=C_p/C_v
M=MOLARMASS(Gas$) "Molar mass of  Gas$"
R= 8.314/M        "Gas constant for Gas$"

"Since the inlet velocity is negligible, the stagnation temperature = T_inlet;
and, since the nozzle is isentropic, the stagnation pressure = P_inlet."

P_o=P_inlet       "Stagnation pressure"
T_o=T_inlet       "Stagnation temperature"
P_crit /P_o=(2/(k+1))^(k/(k-1))       "Critical pressure from Eq. 16-22"
Call ExitPress(P_back,P_crit : P_exit, Condition$)

T_exit /T_o=(P_exit/P_o)^((k-1)/k)"Exit temperature for isentopic flow, K"

V_exit ^2/2=C_p*(T_o-T_exit)*1000          "Exit velocity, m/s"

Rho_exit=P_exit/(R*T_exit)          "Exit density, kg/m3"

m_dot=Rho_exit*V_exit*A_exit    "Nozzle mass flow rate, kg/s"

"If you wish to redo the plots, hide the diagram window and remove the { } from
the first 4 variables just under the procedure.  Next set the desired range of
back pressure in the parametric table.  Finally, solve the table (F3).  "
```

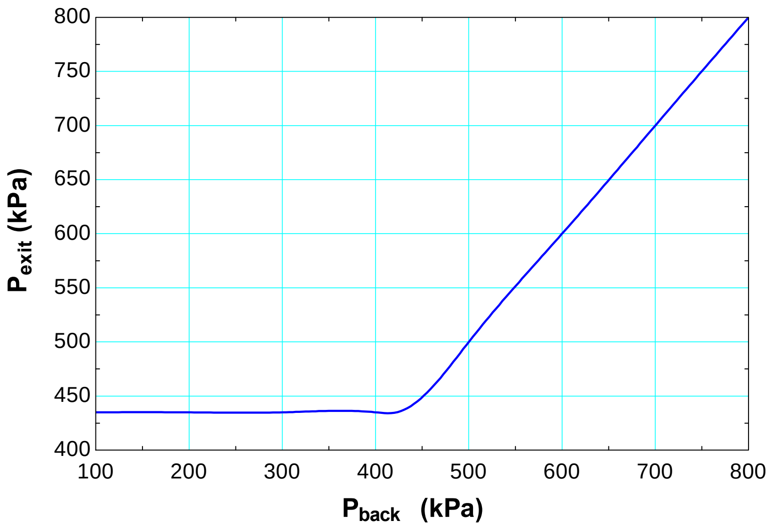
The table of results and the corresponding plot are provided below.

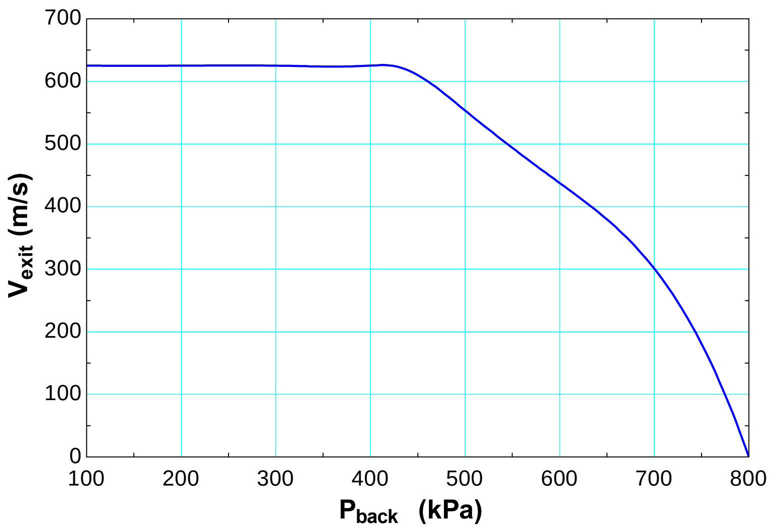
EES SOLUTION

A_cm2=10
A_exit=0.001
Condition$='choked'
C_p=1.208
C_v=0.9211
Gas$='Air'
k=1.312
M=28.97
m_dot=0.9124
P_back=422.7
P_crit=434.9
P_exit=434.9
P_inlet=800
P_o=800
R=0.287
Rho_exit=1.459
T_exit=1038
T_inlet=1200
T_o=1200
V_exit=625.2

P_{back}[kPa]	P_{exit} [kPa]	V_{exit} [m/s]	m [kg/s]	T_{exit} [K]	ρ_{exit} [kg/m^3]
100	434.9	625.2	0.9124	1038	1.459
200	434.9	625.2	0.9124	1038	1.459
300	434.9	625.2	0.9124	1038	1.459
400	434.9	625.2	0.9124	1038	1.459
422.7	434.9	625.2	0.9124	1038	1.459
500	500	553.5	0.8984	1073	1.623
600	600	437.7	0.8164	1121	1.865
700	700	300.9	0.6313	1163	2.098
800	800	0.001523	0.000003538	1200	2.323







Discussion We see from the plot that once the flow is choked at a back pressure of 422.7 kPa, the mass flow rate remains constant regardless of how low the back pressure gets.

Shock Waves and Expansion Waves

17-71C **No**, because the flow must be supersonic before a shock wave can occur. The flow in the converging section of a nozzle is always subsonic.

Discussion A normal shock (if it is to occur) would occur in the supersonic (diverging) section of the nozzle.

17-72C The *Fanno line* represents the **states that satisfy the conservation of mass and energy equations**. The *Rayleigh line* represents the **states that satisfy the conservation of mass and momentum equations**. The *intersections* points of these lines represent the **states that satisfy the conservation of mass, energy, and momentum equations**.

Discussion *T-s* diagrams are quite helpful in understanding these kinds of flows.

17-73C **No**, the second law of thermodynamics requires the flow after the shock to be subsonic.

Discussion A normal shock wave always goes from supersonic to subsonic in the flow direction.

17-74C (*a*) **velocity decreases**, (*b*) **static temperature increases**, (*c*) **stagnation temperature remains the same**, (*d*) **static pressure increases**, and (*e*) **stagnation pressure decreases**.

Discussion In addition, the Mach number goes from supersonic (Ma > 1) to subsonic (Ma < 1).

17-75C *Oblique shocks* occur when **a gas flowing at supersonic speeds strikes a flat or inclined surface**. Normal shock waves are perpendicular to flow whereas inclined shock waves, as the name implies, are typically **inclined relative to the flow direction**. Also, normal shocks form a straight line whereas **oblique shocks can be straight or curved**, depending on the surface geometry.

Discussion In addition, while a normal shock must go from supersonic (Ma > 1) to subsonic (Ma < 1), the Mach number downstream of an oblique shock can be either supersonic or subsonic.

17-76C **Yes**, the *upstream flow* has to be supersonic for an oblique shock to occur. **No**, the flow *downstream* of an oblique shock can be subsonic, sonic, and even supersonic.

Discussion The latter is not true for normal shocks. For a normal shock, the flow must always go from supersonic (Ma > 1) to subsonic (Ma < 1).

17-77C **Yes, the claim is correct**. Conversely, normal shocks can be thought of as special oblique shocks in which the shock angle is $\beta = \pi/2$, or 90°.

Discussion The component of flow in the direction normal to the oblique shock acts exactly like a normal shock. We can think of the flow parallel to the oblique shock as "going along for the ride" – it does not affect anything.

17-78C **When the wedge half-angle δ is greater than the maximum deflection angle θ_{max}**, the shock becomes curved and detaches from the nose of the wedge, forming what is called a *detached oblique shock* or a *bow wave*. The numerical value of the shock angle at the nose is $\beta = 90^o$.

Discussion When δ is less than θ_{max}, the oblique shock is attached to the nose.

17-79C When supersonic flow impinges on a blunt body like the rounded nose of an aircraft, the wedge half-angle δ at the nose is 90^o, and an attached oblique shock cannot exist, regardless of Mach number. Therefore, **a detached oblique shock must occur in front of *all* such blunt-nosed bodies**, whether two-dimensional, axisymmetric, or fully three-dimensional.

Discussion Since $\delta = 90^o$ at the nose, δ is always greater than θ_{max}, regardless of Ma or the shape of the rest of the body.

17-80C The isentropic relations of ideal gases are ***not*** **applicable for flows across (*a*) normal shock waves and (*b*) oblique shock waves**, but they ***are*** **applicable for flows across (*c*) Prandtl-Meyer expansion waves**.

Discussion Flow across any kind of shock wave involves irreversible losses – hence, it cannot be isentropic.

17-81 Air flowing through a nozzle experiences a normal shock. Various properties are to be calculated before and after the shock.

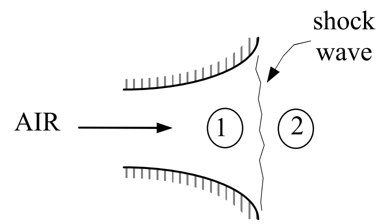
Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air at room temperature are k= 1.4, R = 0.287 kJ/kg·K, and c_p = 1.005 kJ/kg·K.

Analysis The stagnation temperature and pressure before the shock are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 205 + \frac{(740\text{ m/s})^2}{2(1.005\text{ kJ/kg}\cdot\text{K})}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 477.4\text{ K}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (18\text{ kPa})\left(\frac{477.4\text{ K}}{205\text{ K}}\right)^{1.4/(1.4-1)} = 347.0\text{ kPa}$$



The velocity and the Mach number before the shock are determined from

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(205\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{287.0\ m/s}$$

and

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{740\text{ m/s}}{287.0\text{ m/s}} = \mathbf{2.578}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-33. For Ma_1 = 2.578 we read (We obtained the following values using analytical relations in Table A-33.)

$$\text{Ma}_2 = \mathbf{0.5058},\quad \frac{P_{02}}{P_1} = 9.0349,\quad \frac{P_2}{P_1} = 7.5871,\quad \text{and}\quad \frac{T_2}{T_1} = 2.2158$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 9.0349P_1 = (9.0349)(18\text{ kPa}) = \mathbf{162.6\ kPa}$$

$$P_2 = 7.5871P_1 = (7.5871)(18\text{ kPa}) = \mathbf{136.6\ kPa}$$

$$T_2 = 2.2158T_1 = (2.2158)(205\text{ K}) = \mathbf{454.2\ K}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.5058)\sqrt{(1.4)(0.287\text{ kJ/kg.K})(454.2\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{216.1\ m/s}$$

Discussion This problem could also be solved using the relations for compressible flow and normal shock functions. The results would be identical.

17-82 Air flowing through a nozzle experiences a normal shock. The entropy change of air across the normal shock wave is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K and c_p = 1.005 kJ/kg·K.

Analysis The entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$
$$= (1.005 \text{ kJ/kg}\cdot\text{K})\ln(2.2158) - (0.287 \text{ kJ/kg}\cdot\text{K})\ln(7.5871)$$
$$= \mathbf{0.2180\ \ kJ/kg \cdot K}$$

Discussion A shock wave is a highly dissipative process, and the entropy generation is large during shock waves.

17-83 Air flowing through a converging-diverging nozzle experiences a normal shock at the exit. The effect of the shock wave on various properties is to be determined.

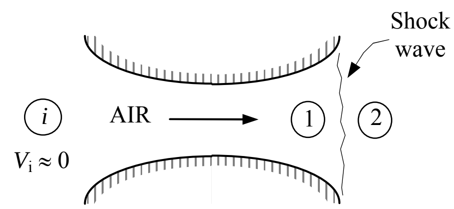
Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Properties The properties of air are $k = 1.4$ and $R = 0.287$ kJ/kg·K.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Then,

$$P_{01} = P_i = 1 \text{ MPa}$$

$$T_{01} = T_i = 300 \text{ K}$$



Then,

$$T_1 = T_{01}\left(\frac{2}{2+(k-1)\text{Ma}_1^2}\right) = (300\text{ K})\left(\frac{2}{2+(1.4-1)2.4^2}\right) = 139.4\text{ K}$$

and

$$P_1 = P_{01}\left(\frac{T_1}{T_0}\right)^{k/(k-1)} = (1\text{ MPa})\left(\frac{139.4}{300}\right)^{1.4/0.4} = 0.06840\text{ MPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-33. For $\text{Ma}_1 = 2.4$ we read

$$\text{Ma}_2 = 0.5231 \cong \mathbf{0.523}, \quad \frac{P_{02}}{P_{01}} = 0.5401, \quad \frac{P_2}{P_1} = 6.5533, \quad \text{and} \quad \frac{T_2}{T_1} = 2.0403$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 0.5401 P_{01} = (0.5401)(1.0\text{ MPa}) = \mathbf{0.540\text{ MPa} = 540\text{ kPa}}$$

$$P_2 = 6.5533 P_1 = (6.5533)(0.06840\text{ MPa}) = \mathbf{0.448\text{ MPa} = 448\text{ kPa}}$$

$$T_2 = 2.0403 T_1 = (2.0403)(139.4\text{ K}) = \mathbf{284\text{ K}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.5231)\sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(284\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{177\text{ m/s}}$$

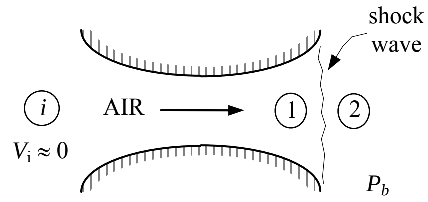
Discussion We can also solve this problem using the relations for normal shock functions. The results would be identical.

17-84 Air enters a converging-diverging nozzle at a specified state. The required back pressure that produces a normal shock at the exit plane is to be determined for the specified nozzle geometry.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Analysis The inlet stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. Since the flow before the shock to be isentropic,

$$P_{01} = P_i = 2 \text{ MPa}$$



It is specified that A/A^* =3.5. From Table A-32, Mach number and the pressure ratio which corresponds to this area ratio are the Ma_1 =2.80 and P_1/P_{01} = 0.0368. The pressure ratio across the shock for this Ma_1 value is, from Table A-33, P_2/P_1 = 8.98. Thus the back pressure, which is equal to the static pressure at the nozzle exit, must be

$$P_2 = 8.98P_1 = 8.98\times0.0368P_{01} = 8.98\times0.0368\times(2 \text{ MPa}) = \mathbf{0.661\ MPa}$$

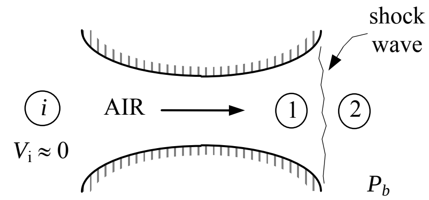
Discussion We can also solve this problem using the relations for compressible flow and normal shock functions. The results would be identical.

17-85 Air enters a converging-diverging nozzle at a specified state. The required back pressure that produces a normal shock at the exit plane is to be determined for the specified nozzle geometry.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Analysis The inlet stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. Since the flow before the shock to be isentropic,

$$P_{0x} = P_i = 2 \text{ MPa}$$



It is specified that A/A^* = 2. From Table A-32, the Mach number and the pressure ratio which corresponds to this area ratio are the Ma_1 =2.20 and P_1/P_{01} = 0.0935. The pressure ratio across the shock for this M_1 value is, from Table A-33, P_2/P_1 = 5.48. Thus the back pressure, which is equal to the static pressure at the nozzle exit, must be

$$P_2 = 5.48P_1 = 5.48\times0.0935P_{01} = 5.48\times0.0935\times(2 \text{ MPa}) = \mathbf{1.02\ MPa}$$

Discussion We can also solve this problem using the relations for compressible flow and normal shock functions. The results would be identical.

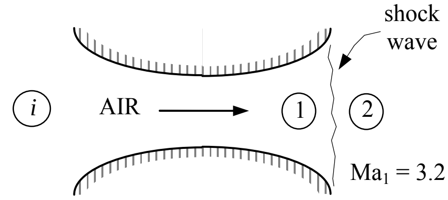
17-86 Air flowing through a nozzle experiences a normal shock. The effect of the shock wave on various properties is to be determined. Analysis is to be repeated for helium under the same conditions.

Assumptions **1** Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are $k = 1.4$ and $R = 0.287$ kJ/kg·K, and the properties of helium are $k = 1.667$ and $R = 2.0769$ kJ/kg·K.

Analysis The air properties upstream the shock are

$$\text{Ma}_1 = 3.2,\ P_1 = 58 \text{ kPa, and } T_1 = 270 \text{ K}$$



Fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions in Table A-33. For $\text{Ma}_1 = 3.2$,

$$\text{Ma}_2 = \mathbf{0.4643},\ \frac{P_{02}}{P_1} = 13.656,\ \frac{P_2}{P_1} = 11.780, \text{ and } \frac{T_2}{T_1} = 2.9220$$

We obtained these values using analytical relations in Table A-33. Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 13.656P_1 = (13.656)(58 \text{ kPa}) = \mathbf{792.0 \text{ kPa}}$$

$$P_2 = 11.780P_1 = (11.780)(58 \text{ kPa}) = \mathbf{683.2 \text{ kPa}}$$

$$T_2 = 2.9220T_1 = (2.9220)(270 \text{ K}) = \mathbf{788.9 \text{ K}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.4643)\sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(788.9 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{261.4 \text{ m/s}}$$

We now repeat the analysis for helium. This time we cannot use the tabulated values in Table A-33 since k is not 1.4. Therefore, we have to calculate the desired quantities using the analytical relations,

$$\text{Ma}_2 = \left(\frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2 k/(k-1) - 1}\right)^{1/2} = \left(\frac{3.2^2 + 2/(1.667-1)}{2\times 3.2^2 \times 1.667/(1.667-1) - 1}\right)^{1/2} = \mathbf{0.5136}$$

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{1 + 1.667\times 3.2^2}{1 + 1.667 \times 0.5136^2} = 12.551$$

$$\frac{T_2}{T_1} = \frac{1 + \text{Ma}_1^2(k-1)/2}{1 + \text{Ma}_2^2(k-1)/2} = \frac{1 + 3.2^2(1.667-1)/2}{1 + 0.5136^2(1.667-1)/2} = 4.0580$$

$$\frac{P_{02}}{P_1} = \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2}\right)\left(1 + (k-1)\text{Ma}_2^2/2\right)^{k/(k-1)}$$

$$= \left(\frac{1 + 1.667\times 3.2^2}{1 + 1.667\times 0.5136^2}\right)\left(1 + (1.667-1)\times 0.5136^2/2\right)^{1.667/0.667} = 15.495$$

Thus, $P_{02} = 15.495P_1 = (15.495)(58 \text{ kPa}) = \mathbf{898.7 \text{ kPa}}$

$$P_2 = 12.551P_1 = (12.551)(58 \text{ kPa}) = \mathbf{728.0 \text{ kPa}}$$

$$T_2 = 4.0580T_1 = (4.0580)(270 \text{ K}) = \mathbf{1096 \text{ K}}$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_y} = (0.5136)\sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(1096 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{1000 \text{ m/s}}$$

Discussion The velocity and Mach number are higher for helium than for air due to the different values of k and R.

17-87 Air flowing through a nozzle experiences a normal shock. The entropy change of air across the normal shock wave is to be determined.

Assumptions **1** Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are R = 0.287 kJ/kg·K and c_p = 1.005 kJ/kg·K, and the properties of helium are R = 2.0769 kJ/kg·K and c_p = 5.1926 kJ/kg·K.

Analysis The entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (1.005\text{ kJ/kg}\cdot\text{K})\ln(2.9220) - (0.287\text{ kJ/kg}\cdot\text{K})\ln(11.780) = \mathbf{0.370\ \text{kJ/kg}\cdot\text{K}}$$

For helium, the entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (5.1926\text{ kJ/kg}\cdot\text{K})\ln(4.0580) - (2.0769\text{ kJ/kg}\cdot\text{K})\ln(12.551) = \mathbf{2.02\ \text{kJ/kg}\cdot\text{K}}$$

Discussion Note that shock wave is a highly dissipative process, and the entropy generation is large during shock waves.

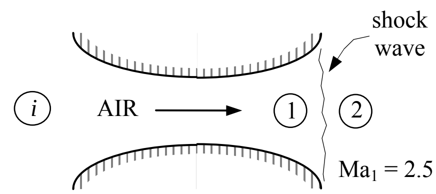
17-88E Air flowing through a nozzle experiences a normal shock. Effect of the shock wave on various properties is to be determined. Analysis is to be repeated for helium

Assumptions **1** Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are k = 1.4 and R = 0.06855 Btu/lbm·R, and the properties of helium are k = 1.667 and R = 0.4961 Btu/lbm·R.

Analysis The air properties upstream the shock are

$Ma_1 = 2.5$, $P_1 = 10$ psia, and $T_1 = 440.5$ R



Fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-33. For $Ma_1 = 2.5$,

$$\mathrm{Ma}_2 = \mathbf{0.513},\ \frac{P_{02}}{P_1} = 8.5262,\ \frac{P_2}{P_1} = 7.125, \text{ and } \frac{T_2}{T_1} = 2.1375$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 8.5262P_1 = (8.5262)(10 \text{ psia}) = \mathbf{85.3\ psia}$$

$$P_2 = 7.125P_1 = (7.125)(10 \text{ psia}) = \mathbf{71.3\ psia}$$

$$T_2 = 2.1375T_1 = (2.1375)(440.5 \text{ R}) = \mathbf{942\ R}$$

The air velocity after the shock can be determined from $V_2 = Ma_2c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \mathrm{Ma}_2 c_2 = \mathrm{Ma}_2\sqrt{kRT_2} = (0.513)\sqrt{(1.4)(0.06855 \text{ Btu/1bm}\cdot\text{R})(941.6 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = \mathbf{772\ ft/s}$$

We now repeat the analysis for helium. This time we cannot use the tabulated values in Table A-33 since k is not 1.4. Therefore, we have to calculate the desired quantities using the analytical relations,

$$\mathrm{Ma}_2 = \left(\frac{\mathrm{Ma}_1^2 + 2/(k-1)}{2\mathrm{Ma}_1^2 k/(k-1) - 1}\right)^{1/2} = \left(\frac{2.5^2 + 2/(1.667-1)}{2\times 2.5^2 \times 1.667/(1.667-1) - 1}\right)^{1/2} = \mathbf{0.553}$$

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{1 + 1.667\times 2.5^2}{1 + 1.667\times 0.553^2} = 7.5632$$

$$\frac{T_2}{T_1} = \frac{1 + \mathrm{Ma}_1^2(k-1)/2}{1 + \mathrm{Ma}_2^2(k-1)/2} = \frac{1 + 2.5^2(1.667-1)/2}{1 + 0.553^2(1.667-1)/2} = 2.7989$$

$$\frac{P_{02}}{P_1} = \left(\frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2}\right)\left(1 + (k-1)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

$$= \left(\frac{1 + 1.667\times 2.5^2}{1 + 1.667\times 0.553^2}\right)\left(1 + (1.667-1)\times 0.553^2/2\right)^{1.667/0.667} = 9.641$$

Thus, $P_{02} = 11.546P_1 = (11.546)(10 \text{ psia}) = \mathbf{115\ psia}$

$$P_2 = 7.5632P_1 = (7.5632)(10 \text{ psia}) = \mathbf{75.6\ psia}$$

$$T_2 = 2.7989T_1 = (2.7989)(440.5 \text{ R}) = \mathbf{1233\ R}$$

$$V_2 = \mathrm{Ma}_2 c_2 = \mathrm{Ma}_2\sqrt{kRT_2} = (0.553)\sqrt{(1.667)(0.4961 \text{ Btu/1bm.R})(1232.9 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = \mathbf{2794\ ft/s}$$

Discussion This problem could also be solved using the relations for compressible flow and normal shock functions. The results would be identical.

17-89E We are to reconsider Prob. 17-88E. Using EES (or other) software, we are to study the effects of both air and helium flowing steadily in a nozzle when there is a normal shock at a Mach number in the range 2 < Mx < 3.5. In addition to the required information, we are to calculate the entropy change of the air and helium across the normal shock, and tabulate the results in a parametric table.

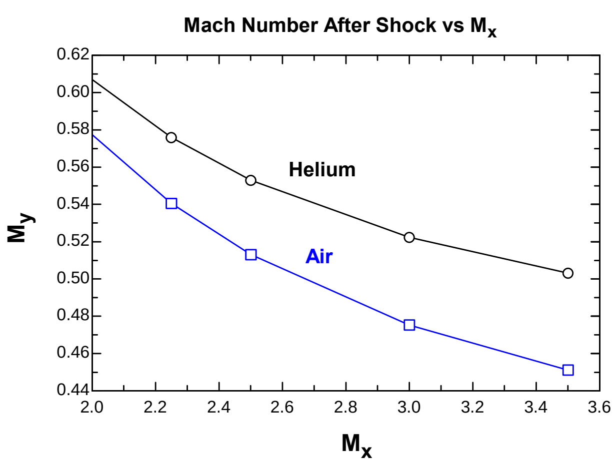
Analysis We use EES to calculate the entropy change of the air and helium across the normal shock. The results are given in the Parametric Table for 2 < M_x < 3.5.

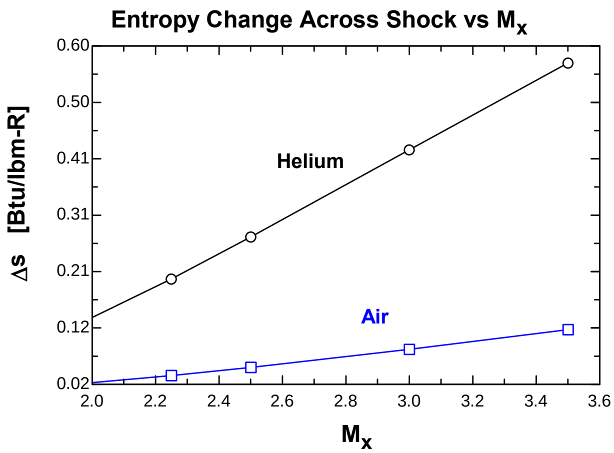
```
Procedure NormalShock(M_x,k:M_y,PyOPx, TyOTx,RhoyORhox, PoyOPox, PoyOPx)
    If M_x < 1 Then
            M_y = -1000;PyOPx=-1000;TyOTx=-1000;RhoyORhox=-1000
            PoyOPox=-1000;PoyOPx=-1000
    else
            M_y=sqrt( (M_x^2+2/(k-1)) / (2*M_x^2*k/(k-1)-1) )
            PyOPx=(1+k*M_x^2)/(1+k*M_y^2)
            TyOTx=( 1+M_x^2*(k-1)/2  )/(1+M_y^2*(k-1)/2  )
            RhoyORhox=PyOPx/TyOTx
            PoyOPox=M_x/M_y*(  (1+M_y^2*(k-1)/2)/ (1+M_x^2*(k-1)/2)   )^((k+1)/(2*(k-1)))
            PoyOPx=(1+k*M_x^2)*(1+M_y^2*(k-1)/2)^(k/(k-1))/(1+k*M_y^2)
    Endif
End

Function ExitPress(P_back,P_crit)
If P_back>=P_crit then ExitPress:=P_back     "Unchoked Flow Condition"
If P_back<P_crit then ExitPress:=P_crit       "Choked Flow Condition"
End

Procedure GetProp(Gas$:Cp,k,R) "Cp and k data are from Text Table A.2E"
    M=MOLARMASS(Gas$) "Molar mass of  Gas$"
    R= 1545/M         "Particular gas constant for Gas$, ft-lbf/lbm-R"
            "k = Ratio of Cp to Cv"
            "Cp = Specific heat at constant pressure"
    if Gas$='Air' then
            Cp=0.24"Btu/lbm-R"; k=1.4
    endif
    if Gas$='CO2' then
            Cp=0.203"Btu/lbm_R"; k=1.289
    endif
    if Gas$='Helium' then
            Cp=1.25"Btu/lbm-R"; k=1.667
    endif
End

"Variable Definitions:"
"M = flow Mach Number"
"P_ratio = P/P_o  for compressible, isentropic flow"
"T_ratio = T/T_o for compressible, isentropic flow"
"Rho_ratio= Rho/Rho_o for compressible, isentropic flow"
"A_ratio=A/A* for compressible, isentropic flow"
"Fluid properties before the shock are denoted with a subscript x"
"Fluid properties after the shock are denoted with a subscript y"
"M_y = Mach Number down stream of normal shock"
"PyOverPx= P_y/P_x  Pressue ratio across normal shock"
"TyOverTx =T_y/T_x  Temperature ratio across normal shock"
"RhoyOverRhox=Rho_y/Rho_x Density ratio across normal shock"
"PoyOverPox = P_oy/P_ox  Stagantion pressure ratio across normal shock"
"PoyOverPx = P_oy/P_x  Stagnation pressure after normal shock ratioed to pressure before shock"
```

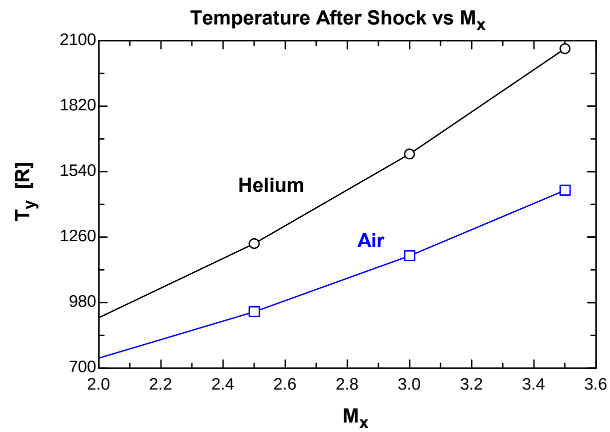
```
"Input Data"
{P_x = 10 "psia"}     "Values of P_x, T_x, and M_x are set in the Parametric Table"
{T_x = 440.5 "R"}
{M_x = 2.5}
Gas$='Air'  "This program has been written for the gases Air, CO2, and Helium"
Call GetProp(Gas$:Cp,k,R)
Call NormalShock(M_x,k:M_y,PyOverPx, TyOverTx,RhoyOverRhox, PoyOverPox, PoyOverPx)
P_oy_air=P_x*PoyOverPx      "Stagnation pressure after the shock"
P_y_air=P_x*PyOverPx        "Pressure after the shock"
T_y_air=T_x*TyOverTx        "Temperature after the shock"
M_y_air=M_y          "Mach number after the shock"

"The velocity after the shock can be found from the product of the Mach number and
speed of sound after the shock."
C_y_air = sqrt(k*R"ft-lbf/lbm_R"*T_y_air"R"*32.2 "lbm-ft/lbf-s^2")
V_y_air=M_y_air*C_y_air
DELTAs_air=entropy(air,T=T_y_air, P=P_y_air) -entropy(air,T=T_x,P=P_x)

Gas2$='Helium'  "Gas2$ can be either Helium or CO2"
Call GetProp(Gas2$:Cp_2,k_2,R_2)
Call NormalShock(M_x,k_2:M_y2,PyOverPx2, TyOverTx2,RhoyOverRhox2, PoyOverPox2, PoyOverPx2)
P_oy_he=P_x*PoyOverPx2      "Stagnation pressure after the shock"
P_y_he=P_x*PyOverPx2        "Pressure after the shock"
T_y_he=T_x*TyOverTx2        "Temperature after the shock"
M_y_he=M_y2          "Mach number after the shock"

"The velocity after the shock can be found from the product of the Mach number and
speed of sound after the shock."
C_y_he = sqrt(k_2*R_2"ft-lbf/lbm_R"*T_y_he"R"*32.2 "lbm-ft/lbf-s^2")
V_y_he=M_y_he*C_y_he
DELTAs_he=entropy(helium,T=T_y_he, P=P_y_he) -entropy(helium,T=T_x,P=P_x)
```

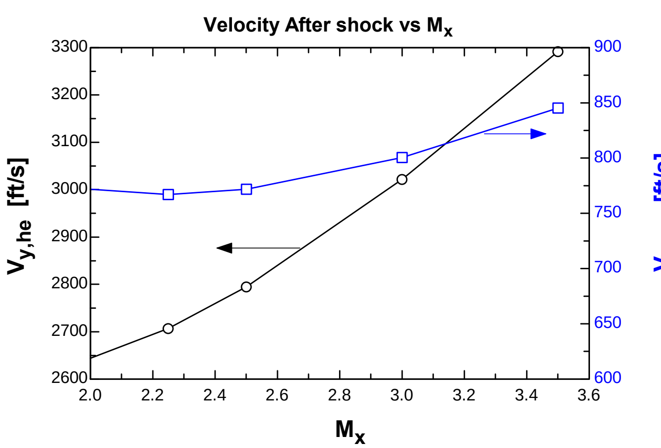
The parametric table and the corresponding plots are shown below.

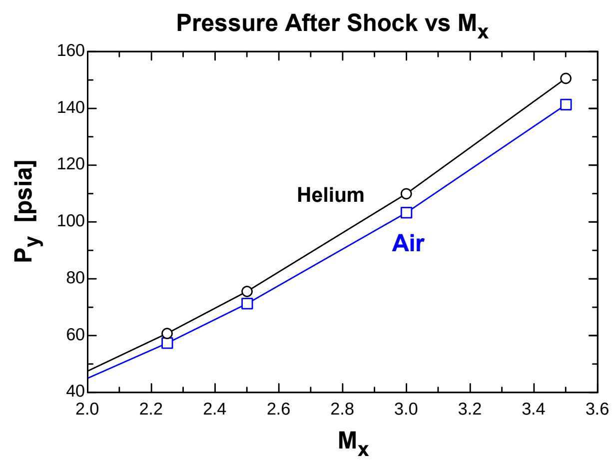
$V_{y,he}$ [ft/s]	$V_{y,air}$ [ft/s]	$T_{y,he}$ [R]	$T_{y,air}$ [R]	T_x [R]	$P_{y,he}$ [psia]	$P_{y,air}$ [psia]	P_x [psia]	$P_{oy,he}$ [psia]	$P_{oy,air}$ [psia]	$M_{y,he}$	$M_{y,air}$	M_x	Δs_{he} [Btu/lbm-R]	Δs_{air} [Btu/lbm-R]
2644	771.9	915.6	743.3	440.5	47.5	45	10	63.46	56.4	0.607	0.5774	2	0.1345	0.0228
2707	767.1	1066	837.6	440.5	60.79	57.4	10	79.01	70.02	0.5759	0.5406	2.25	0.2011	0.0351
2795	771.9	1233	941.6	440.5	75.63	71.25	10	96.41	85.26	0.553	0.513	2.5	0.2728	0.04899
3022	800.4	1616	1180	440.5	110	103.3	10	136.7	120.6	0.5223	0.4752	3	0.4223	0.08
3292	845.4	2066	1460	440.5	150.6	141.3	10	184.5	162.4	0.5032	0.4512	3.5	0.5711	0.1136











Discussion In all cases, regardless of the fluid or the Mach number, entropy increases across a shock wave. This is because a shock wave involves irreversibilities.

17-90 For an ideal gas flowing through a normal shock, a relation for V_2/V_1 in terms of k, Ma_1, and Ma_2 is to be developed.

Analysis The conservation of mass relation across the shock is $\rho_1 V_1 = \rho_2 V_2$ and it can be expressed as

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{P_1 / RT_1}{P_2 / RT_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)$$

From Eqs. 17-35 and 17-38,

$$\frac{V_2}{V_1} = \left(\frac{1 + k\mathrm{Ma}_2^2}{1 + k\mathrm{Ma}_1^2}\right)\left(\frac{1 + \mathrm{Ma}_1^2 (k-1)/2}{1 + \mathrm{Ma}_2^2 (k-1)/2}\right)$$

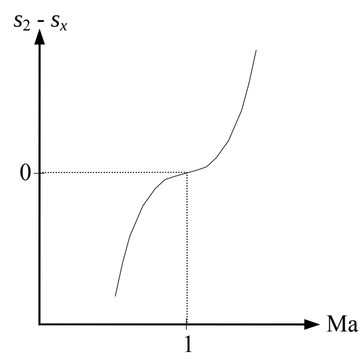
Discussion This is an important relation as it enables us to determine the velocity ratio across a normal shock when the Mach numbers before and after the shock are known.

17-91 The entropy change of air across the shock for upstream Mach numbers between 0.5 and 1.5 is to be determined and plotted.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are $k = 1.4$, $R = 0.287$ kJ/kg·K, and $c_p = 1.005$ kJ/kg·K.

Analysis The entropy change across the shock is determined to be



$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

where

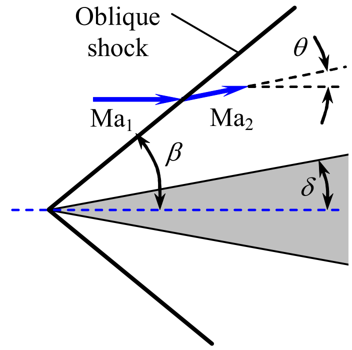
$$\mathrm{Ma}_2 = \left(\frac{\mathrm{Ma}_1^2 + 2/(k-1)}{2\mathrm{Ma}_1^2 k/(k-1) - 1}\right)^{1/2}, \quad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2}, \text{ and } \frac{T_2}{T_1} = \frac{1 + \mathrm{Ma}_1^2 (k-1)/2}{1 + \mathrm{Ma}_2^2 (k-1)/2}$$

The results of the calculations can be tabulated as

Ma_1	Ma_2	T_2/T_1	P_2/P_1	s_2 - s_1
0.5	2.6458	0.1250	0.4375	-1.853
0.6	1.8778	0.2533	0.6287	-1.247
0.7	1.5031	0.4050	0.7563	-0.828
0.8	1.2731	0.5800	0.8519	-0.501
0.9	1.1154	0.7783	0.9305	-0.231
1.0	1.0000	1.0000	1.0000	0.0
1.1	0.9118	1.0649	1.2450	0.0003
1.2	0.8422	1.1280	1.5133	0.0021
1.3	0.7860	1.1909	1.8050	0.0061
1.4	0.7397	1.2547	2.1200	0.0124
1.5	0.7011	1.3202	2.4583	0.0210

Discussion The total entropy change is negative for upstream Mach numbers Ma_1 less than unity. Therefore, normal shocks cannot occur when $Ma_1 < 1$.

17-92 Supersonic airflow approaches the nose of a two-dimensional wedge and undergoes a straight oblique shock. For a specified Mach number, the minimum shock angle and the maximum deflection angle are to be determined.



Assumptions Air is an ideal gas with a constant specific heat ratio of k = 1.4 (so that Fig. 17-41 is applicable).

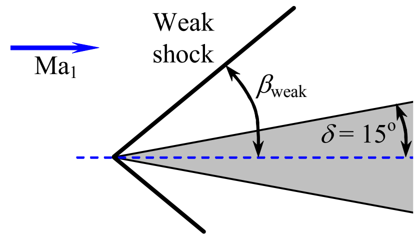
Analysis For Ma = 5, we read from Fig. 17-41

Minimum shock (or wave) angle: $\beta_{min} = \mathbf{12°}$

Maximum deflection (or turning) angle: $\theta_{max} = \mathbf{41.5°}$

Discussion Note that the minimum shock angle decreases and the maximum deflection angle increases with increasing Mach number Ma_1.

17-93E Air flowing at a specified supersonic Mach number is forced to undergo a compression turn (an oblique shock)., The Mach number, pressure, and temperature downstream of the oblique shock are to be determined.



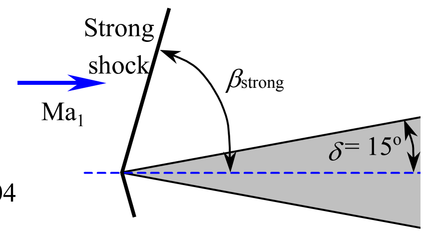
Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15^o$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2} \quad \rightarrow \quad \tan 15^\circ = \frac{2(2^2 \sin^2 \quad - 1)/\tan\beta}{2^2(1.4 + \cos 2\beta) + 2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{weak} = 45.34^o$ and $\beta_{strong} = 79.83^o$. Then the upstream "normal" Mach number $Ma_{1,n}$ becomes



Weak shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 45.34^\circ = 1.423$

Strong shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 79.83^\circ = 1.969$

Also, the downstream normal Mach numbers $Ma_{2,n}$ become

Weak shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.423)^2 + 2}{2(1.4)(1.423)^2 - 1.4 + 1}} = 0.7304$$

Strong shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.969)^2 + 2}{2(1.4)(1.969)^2 - 1.4 + 1}} = 0.5828$$

The downstream pressure and temperature for each case are determined to be

Weak shock: $$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,n}^2 - k + 1}{k+1} = (6\text{ psia})\frac{2(1.4)(1.423)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{13.2\ psia}$$

$$T_2 = T_1 \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1}\frac{2 + (k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = (480\text{ R})\frac{13.2\text{ psia}}{6\text{ psia}}\frac{2 + (1.4-1)(1.423)^2}{(1.4+1)(1.423)^2} = \mathbf{609\ R}$$

Strong shock: $$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,n}^2 - k + 1}{k+1} = (6\text{ psia})\frac{2(1.4)(1.969)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{26.1\ psia}$$

$$T_2 = T_1 \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1}\frac{2 + (k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = (480\text{ R})\frac{26.1\text{ psia}}{6\text{ psia}}\frac{2 + (1.4-1)(1.969)^2}{(1.4+1)(1.969)^2} = \mathbf{798\ R}$$

The downstream Mach number is determined to be

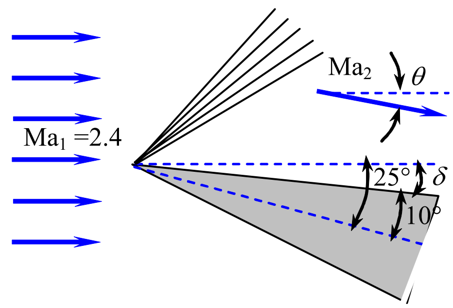
Weak shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.7304}{\sin(45.34^\circ - 15^\circ)} = \mathbf{1.45}$$

Strong shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.5828}{\sin(79.83^\circ - 15^\circ)} = \mathbf{0.644}$$

Discussion Note that the change in Mach number, pressure, temperature across the *strong shock* are much greater than the changes across the *weak shock*, as expected. For both the weak and strong oblique shock cases, $Ma_{1,n}$ is supersonic and $Ma_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock.

17-94 Air flowing at a specified supersonic Mach number undergoes an expansion turn over a tilted wedge. The Mach number, pressure, and temperature downstream of the sudden expansion above the wedge are to be determined.

Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.



Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, the deflection angle is determined to be $\theta \approx \delta = 25° - 10° = 15°$. Then the upstream and downstream Prandtl-Meyer functions are determined to be

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}^2-1}\right)$$

Upstream:

$$\nu(\mathrm{Ma}_1) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}(2.4^2-1)}\right) - \tan^{-1}\left(\sqrt{2.4^2-1}\right) = 36.75°$$

Then the downstream Prandtl-Meyer function becomes

$$\nu(\mathrm{Ma}_2) = \theta + \nu(\mathrm{Ma}_1) = 15° + 36.75° = 51.75°$$

Now Ma_2 is found from the Prandtl-Meyer relation, which is now implicit:

Downstream: $\nu(\mathrm{Ma}_2) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}(\mathrm{Ma}_2^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}_2^2-1}\right) = 51.75°$

It gives $\mathrm{Ma}_2 =$ **3.105**. Then the downstream pressure and temperature are determined from the isentropic flow relations

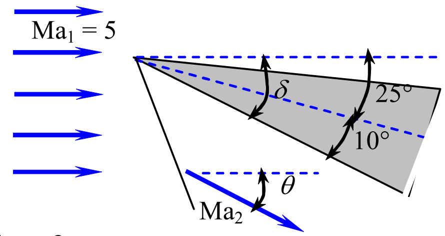
$$P_2 = \frac{P_2/P_0}{P_1/P_0}P_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-k/(k-1)}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-k/(k-1)}}P_1 = \frac{[1+3.105^2(1.4-1)/2]^{-1.4/0.4}}{[1+2.4^2(1.4-1)/2]^{-1.4/0.4}}(70\text{ kPa}) = \mathbf{23.8\ kPa}$$

$$T_2 = \frac{T_2/T_0}{T_1/T_0}T_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-1}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-1}}T_1 = \frac{[1+3.105^2(1.4-1)/2]^{-1}}{[1+2.4^2(1.4-1)/2]^{-1}}(260\text{ K}) = \mathbf{191\ K}$$

Note that this is an expansion, and Mach number increases while pressure and temperature decrease, as expected.

Discussion There are compressible flow calculators on the Internet that solve these implicit equations that arise in the analysis of compressible flow, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html .

17-95 Air flowing at a specified supersonic Mach number undergoes a compression turn (an oblique shock) over a tilted wedge. The Mach number, pressure, and temperature downstream of the shock below the wedge are to be determined.



Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, the deflection angle is determined to be $\theta \approx \delta = 25° + 10° = 35°$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2} \quad \rightarrow \quad \tan 12° = \frac{2(3.4^2 \sin^2 \quad - 1)/\tan\beta}{3.4^2(1.4 + \cos 2\beta) + 2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\text{weak}} = 49.86^{\text{o}}$ and $\beta_{\text{strong}} = 77.66^{\text{o}}$. Then for the case of strong oblique shock, the upstream "normal" Mach number $\mathrm{Ma}_{1,n}$ becomes

$$\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 5\sin 77.66° = 4.884$$

Also, the downstream normal Mach numbers $\mathrm{Ma}_{2,n}$ become

$$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(4.884)^2 + 2}{2(1.4)(4.884)^2 - 1.4 + 1}} = 0.4169$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,n}^2 - k + 1}{k+1} = (70\text{ kPa})\frac{2(1.4)(4.884)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{1940\ kPa}$$

$$T_2 = T_1 \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1}\frac{2 + (k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = (260\text{ K})\frac{1940\text{ kPia}}{70\text{ kPa}}\frac{2 + (1.4-1)(4.884)^2}{(1.4+1)(4.884)^2} = \mathbf{1450\ K}$$

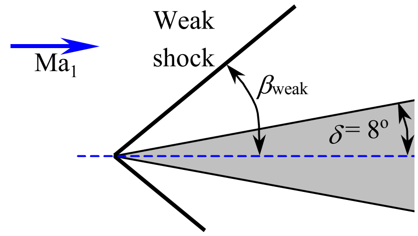
The downstream Mach number is determined to be

$$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.4169}{\sin(77.66° - 35°)} = \mathbf{0.615}$$

Discussion Note that $\mathrm{Ma}_{1,n}$ is supersonic and $\mathrm{Ma}_{2,n}$ and Ma_2 are subsonic. Also note the huge rise in temperature and pressure across the strong oblique shock, and the challenges they present for spacecraft during reentering the earth's atmosphere.

17-96E Air flowing at a specified supersonic Mach number is forced to turn upward by a ramp, and weak oblique shock forms. The wave angle, Mach number, pressure, and temperature after the shock are to be determined.

Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.



Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the ramp, i.e., $\theta \approx \delta = 8°$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\text{Ma}_1^2\sin^2\beta - 1)/\tan\beta}{\text{Ma}_1^2(k+\cos 2\beta)+2} \quad \rightarrow \quad \tan 8° = \frac{2(2^2\sin^2\beta - 1)/\tan\beta}{2^2(1.4+\cos 2\beta)+2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives β_{weak} **= 37.21°** and $\beta_{\text{strong}} = 85.05°$. Then for the case of weak oblique shock, the upstream "normal" Mach number $\text{Ma}_{1,n}$ becomes

$$\text{Ma}_{1,n} = \text{Ma}_1 \sin\beta = 2\sin 37.21° = 1.209$$

Also, the downstream normal Mach numbers $\text{Ma}_{2,n}$ become

$$\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.209)^2+2}{2(1.4)(1.209)^2 - 1.4 + 1}} = 0.8363$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1\frac{2k\text{Ma}_{1,n}^2 - k + 1}{k+1} = (12\text{ psia})\frac{2(1.4)(1.209)^2 - 1.4 + 1}{1.4+1} = \mathbf{18.5\ psia}$$

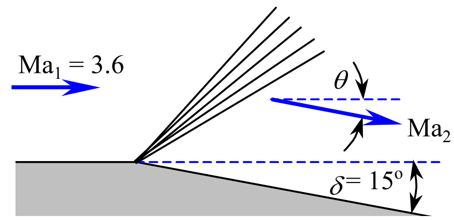
$$T_2 = T_1\frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1\frac{P_2}{P_1}\frac{2+(k-1)\text{Ma}_{1,n}^2}{(k+1)\text{Ma}_{1,n}^2} = (490\text{ R})\frac{18.5\text{ psia}}{12\text{ psia}}\frac{2+(1.4-1)(1.209)^2}{(1.4+1)(1.209)^2} = \mathbf{556\ R}$$

The downstream Mach number is determined to be

$$\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta-\theta)} = \frac{0.8363}{\sin(37.21° - 8°)} = \mathbf{1.71}$$

Discussion Note that $\text{Ma}_{1,n}$ is supersonic and $\text{Ma}_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock (it is *subsonic* across the strong oblique shock).

17-97 Air flowing at a specified supersonic Mach number undergoes an expansion turn. The Mach number, pressure, and temperature downstream of the sudden expansion along a wall are to be determined.



Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15°$. Then the upstream and downstream Prandtl-Meyer functions are determined to be

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}^2-1}\right)$$

Upstream:

$$\nu(\mathrm{Ma}_1) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}(3.6^2-1)}\right) - \tan^{-1}\left(\sqrt{3.6^2-1}\right) = 60.09°$$

Then the downstream Prandtl-Meyer function becomes

$$\nu(\mathrm{Ma}_2) = \theta + \nu(\mathrm{Ma}_1) = 15° + 60.09° = 75.09°$$

Ma_2 is found from the Prandtl-Meyer relation, which is now implicit:

Downstream: $$\nu(\mathrm{Ma}_2) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}\mathrm{Ma}_2^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}_2^2-1}\right) = 75.09°$$

Solution of this implicit equation gives Ma_2 = **4.81**. Then the downstream pressure and temperature are determined from the isentropic flow relations:

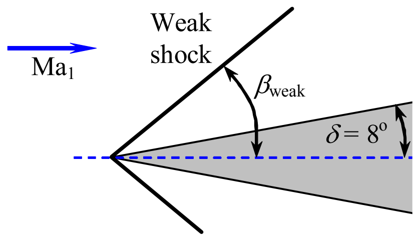
$$P_2 = \frac{P_2/P_0}{P_1/P_0}P_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-k/(k-1)}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-k/(k-1)}}P_1 = \frac{[1+4.81^2(1.4-1)/2]^{-1.4/0.4}}{[1+3.6^2(1.4-1)/2]^{-1.4/0.4}}(40\text{ kPa}) = \mathbf{8.31\,kPa}$$

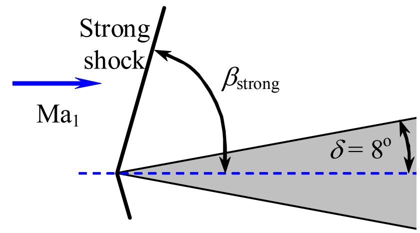
$$T_2 = \frac{T_2/T_0}{T_1/T_0}T_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-1}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-1}}T_1 = \frac{[1+4.81^2(1.4-1)/2]^{-1}}{[1+3.6^2(1.4-1)/2]^{-1}}(280\text{ K}) = \mathbf{179\,K}$$

Note that this is an expansion, and Mach number increases while pressure and temperature decrease, as expected.

Discussion There are compressible flow calculators on the Internet that solve these implicit equations that arise in the analysis of compressible flow, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html .

17-98 Air flowing at a specified supersonic Mach number impinges on a two-dimensional wedge, The shock angle, Mach number, and pressure downstream of the weak and strong oblique shock formed by a wedge are to be determined.





Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is k = 1.4.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 8^o$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k+\cos 2\beta)+2} \quad \rightarrow \quad \tan 8° = \frac{2(3.4^2\sin^2 \quad -1)/\tan\beta}{3.4^2(1.4+\cos 2\beta)+2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\boldsymbol{\beta_{weak}}$ **= 23.15°** and $\boldsymbol{\beta_{strong}}$ **= 87.45°**. Then the upstream "normal" Mach number $\mathrm{Ma}_{1,n}$ becomes

Weak shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 3.4\sin 23.15° = 1.336$

Strong shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 3.4\sin 87.45° = 3.397$

Also, the downstream normal Mach numbers $\mathrm{Ma}_{2,n}$ become

Weak shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2+2}{2k\mathrm{Ma}_{1,n}^2-k+1}} = \sqrt{\frac{(1.4-1)(1.336)^2+2}{2(1.4)(1.336)^2-1.4+1}} = 0.7681$$

Strong shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2+2}{2k\mathrm{Ma}_{1,n}^2-k+1}} = \sqrt{\frac{(1.4-1)(3.397)^2+2}{2(1.4)(3.397)^2-1.4+1}} = 0.4553$$

The downstream pressure for each case is determined to be

Weak shock: $$P_2 = P_1\frac{2k\mathrm{Ma}_{1,n}^2-k+1}{k+1} = (60\text{ kPa})\frac{2(1.4)(1.336)^2-1.4+1}{1.4+1} = \mathbf{115.0\ kPa}$$

Strong shock: $$P_2 = P_1\frac{2k\mathrm{Ma}_{1,n}^2-k+1}{k+1} = (60\text{ kPa})\frac{2(1.4)(3.397)^2-1.4+1}{1.4+1} = \mathbf{797.6\ kPa}$$

The downstream Mach number is determined to be

Weak shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta-\theta)} = \frac{0.7681}{\sin(23.15°-8°)} = \mathbf{2.94}$$

Strong shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta-\theta)} = \frac{0.4553}{\sin(87.45°-8°)} = \mathbf{0.463}$$

Discussion Note that the change in Mach number and pressure across the *strong shock* are much greater than the changes across the *weak shock*, as expected. For both the weak and strong oblique shock cases, $\mathrm{Ma}_{1,n}$ is supersonic and $\mathrm{Ma}_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock.

Duct Flow with Heat Transfer and Negligible Friction (Rayleigh Flow)

17-99C The characteristic aspect of Rayleigh flow is **its involvement of heat transfer**. The main assumptions associated with Rayleigh flow are: the flow is **steady**, **one-dimensional**, and **frictionless** through a constant-area duct, and the fluid is an **ideal gas with constant specific heats**.

Discussion Of course, there is no such thing as frictionless flow. It is better to say that frictional effects are negligible compared to the heating effects.

17-100C The points on the Rayleigh line represent the **states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations for a given state**. Therefore, for a given inlet state, the fluid cannot exist at any downstream state outside the Rayleigh line on a *T-s* diagram.

Discussion The *T-s* diagram is quite useful, since any downstream state must lie on the Rayleigh line.

17-101C In Rayleigh flow, **the effect of heat gain is to increase the entropy** of the fluid, and **the effect of heat loss is to decrease the entropy**.

Discussion You should recall from thermodynamics that the entropy of a system can be lowered by removing heat.

17-102C In Rayleigh flow, the **stagnation temperature T_0 always increases with heat transfer to the fluid**, but the temperature *T* decreases with heat transfer in the Mach number range of 0.845 < Ma < 1 for air. Therefore, the **temperature in this case will decrease**.

Discussion This at first seems counterintuitive, but if heat were *not* added, the temperature would drop even *more* if the air were accelerated isentropically from Ma = 0.92 to 0.95.

17-103C Heating the fluid **increases the flow velocity in subsonic Rayleigh flow**, but **decreases the flow velocity in supersonic Rayleigh flow**.

Discussion These results are not necessarily intuitive, but must be true in order to satisfy the conservation laws.

17-104C The flow is choked, and thus the flow at the duct exit **remains sonic**.

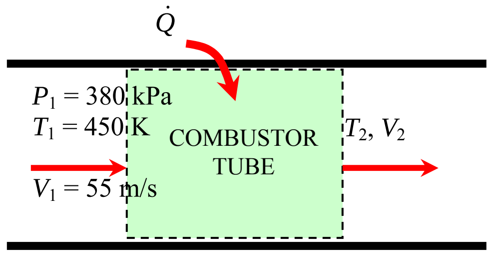
Discussion There is no mechanism for the flow to become supersonic in this case.

17-105 Fuel is burned in a tubular combustion chamber with compressed air. For a specified exit Mach number, the exit temperature and the rate of fuel consumption are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Combustion is complete, and it is treated as a heat addition process, with no change in the chemical composition of flow. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The inlet density and mass flow rate of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{380 \text{ kPa}}{(0.287 \text{ kJ/kgK})(450 \text{ K})} = 2.942 \text{ kg/m}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.942 \text{ kg/m}^3)[\pi(0.16 \text{ m})^2/4](55 \text{ m/s}) = 3.254 \text{ kg/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 450 \text{ K} + \frac{(55 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg}\cdot\text{K}}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 451.5 \text{ K}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(450 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 425.2 \text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{55 \text{ m/s}}{425.2 \text{ m/s}} = 0.1293$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34) (We used analytical functions):

$\text{Ma}_1 = 0.1293$: $\quad T_1/T^* = 0.09201$, $\quad T_{01}/T^* = 0.07693$, $\quad V_1/V^* = 0.03923$

$\text{Ma}_2 = 0.8$: $\quad T_2/T^* = 1.0255$, $\quad T_{02}/T^* = 0.9639$, $\quad V_2/V^* = 0.8101$

The exit temperature, stagnation temperature, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0255}{0.09201} = 11.146 \quad \rightarrow \quad T_2 = 11.146T_1 = 11.146(450 \text{ K}) = \mathbf{5016 \text{ K}}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.07693} = 12.530 \rightarrow \quad T_{02} = 12.530T_{01} = 12.530(451.5 \text{ K}) = 5658 \text{ K}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.8101}{0.03923} = 20.650 \quad \rightarrow \quad V_2 = 20.650V_1 = 20.650(55 \text{ m/s}) = 1136 \text{ m/s}$$

Then the mass flow rate of the fuel is determined to be

$$q = c_p(T_{02} - T_{01}) = (1.005 \text{ kJ/kg}\cdot\text{K})(5658 - 451.5) \text{ K} = 5232 \text{ kJ/kg}$$

$$\dot{Q} = \dot{m}_{\text{air}} q = (3.254 \text{ kg/s})(5232 \text{ kJ/kg}) = 17{,}024 \text{ kW}$$

$$\dot{m}_{\text{fuel}} = \frac{\dot{Q}}{HV} = \frac{17{,}024 \text{ kJ/s}}{39{,}000 \text{ kJ/kg}} = \mathbf{0.4365 \text{ kg/s}}$$

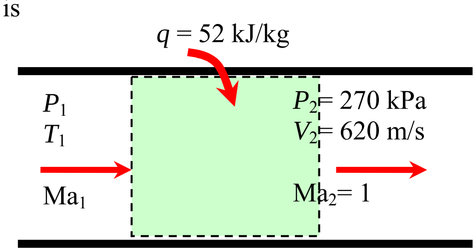
Discussion Note that both the temperature and velocity increase during this subsonic Rayleigh flow with heating, as expected. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

17-106 Air is heated in a duct during subsonic flow until it is choked. For specified pressure and velocity at the exit, the temperature, pressure, and velocity at the inlet are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis Noting that sonic conditions exist at the exit, the exit temperature is



$$c_2 = V_2/\mathrm{Ma}_2 = (620\ \mathrm{m/s})/1 = 620\ \mathrm{m/s}$$

$$c_2 = \sqrt{kRT_2} \rightarrow \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 620\ \mathrm{m/s}$$

It gives T_2 = 956.7 K. Then the exit stagnation temperature becomes

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 956.7\ \mathrm{K} + \frac{(620\ \mathrm{m/s})^2}{2\times 1.005\ \mathrm{kJ/kg\cdot K}}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 1148\ \mathrm{K}$$

The inlet stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{01} = T_{02} - \frac{q}{c_p} = 1148\ \mathrm{K} - \frac{52\ \mathrm{kJ/kg}}{1.005\ \mathrm{kJ/kg\cdot K}} = 1096\ \mathrm{K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value in this case is T_{02} since the flow is choked. Therefore, $T_0^* = T_{02}$ = 1148 K. Then the stagnation temperature ratio at the inlet, and the Mach number corresponding to it are, from Table A-34,

$$\frac{T_{01}}{T_0^*} = \frac{1096\ \mathrm{K}}{1148\ \mathrm{K}} = 0.9547 \quad \rightarrow \quad \mathrm{Ma}_1 = 0.7792 \cong \mathbf{0.779}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34):

$\mathrm{Ma}_1 = 0.7792$: $T_1/T^* = 1.022$, $P_1/P^* = 1.297$, $V_1/V^* = 0.7877$

$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $V_2/V^* = 1$

Then the inlet temperature, pressure, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{1.017} \quad \rightarrow \quad T_1 = 1.022T_2 = 1.022(956.7\ \mathrm{K}) = \mathbf{977.5\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.319} \quad \rightarrow \quad P_1 = 1.319P_2 = 1.297(270\ \mathrm{kPa}) = \mathbf{350.3\ kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{1}{0.7719} \quad \rightarrow \quad V_1 = 0.7877V_2 = 0.7877(620\ \mathrm{m/s}) = \mathbf{488.4\ m/s}$$

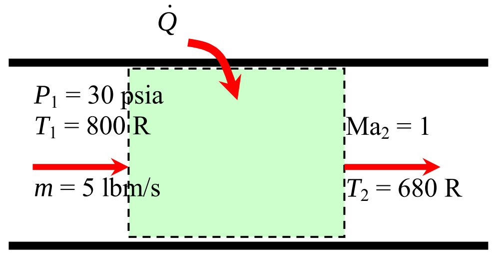
Discussion Note that the temperature and pressure decreases with heating during this subsonic Rayleigh flow while velocity increases. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

17-107E Air flowing with a subsonic velocity in a round duct is accelerated by heating until the flow is choked at the exit. The rate of heat transfer and the pressure drop are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The flow is choked at the duct exit. **3** Mass flow rate remains constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 0.2400$ Btu/lbm·R, and $R = 0.06855$ Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

Analysis The inlet density and velocity of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{30\text{ psia}}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(800\text{ R})} = 0.1012\text{ lbm/ft}^3$$

$$V_1 = \frac{\dot{m}_{air}}{\rho_1 A_{cl}} = \frac{5\text{ lbm/s}}{(0.1012\text{ lbm/ft}^3)[\pi(4/12\text{ ft})^2/4]} = 565.9\text{ ft/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 800\text{ R} + \frac{(565.9\text{ ft/s})^2}{2\times 0.2400\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/lbm}}{25,037\text{ ft}^2/\text{s}^2}\right) = 826.7\text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855\text{ Btu/lbm}\cdot\text{R})(800\text{ R})\left(\frac{25,037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)} = 1386\text{ ft/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{565.9\text{ ft/s}}{1386\text{ ft/s}} = 0.4082$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34):

Ma$_1$ = 0.4082: $T_1/T^* = 0.6310$, $P_1/P^* = 1.946$, $T_{01}/T_0^* = 0.5434$

Ma$_2$ = 1: $T_2/T^* = 1$, $P_2/P^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6310} \quad\rightarrow\quad T_2 = T_1/0.6310 = (800\text{ R})/0.6310 = 1268\text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.946} \quad\rightarrow\quad P_2 = P_1/2.272 = (30\text{ psia})/1.946 = 15.4\text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.5434} \quad\rightarrow\quad T_{02} = T_{01}/0.1743 = (826.7\text{ R})/0.5434 = 1521\text{ R}$$

Then the rate of heat transfer and the pressure drop become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (5\text{ lbm/s})(0.2400\text{ Btu/lbm}\cdot\text{R})(1521 - 826.7)\text{ R} = \mathbf{834\ Btu/s}$$

$$\Delta P = P_1 - P_2 = 30 - 15.4 = \mathbf{14.6\ psia}$$

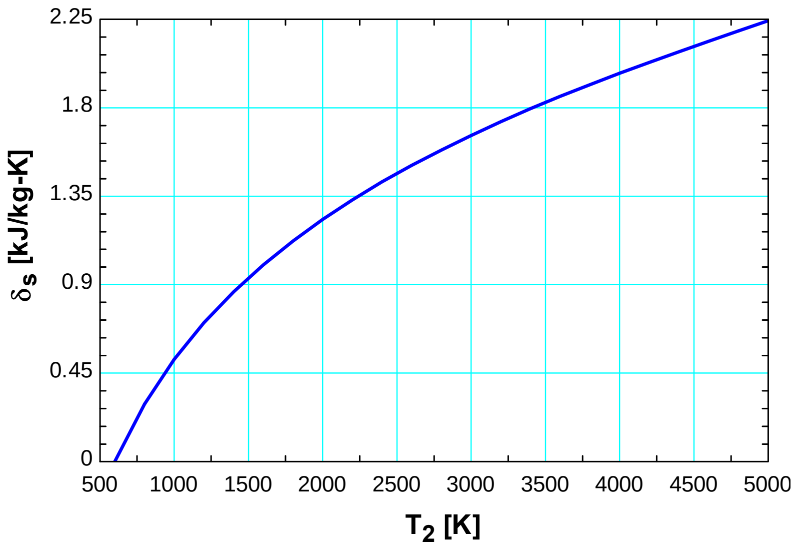
Discussion Note that the entropy of air increases during this heating process, as expected.

17-108 Air flowing with a subsonic velocity in a duct. The variation of entropy with temperature is to be investigated as the exit temperature varies from 600 K to 5000 K in increments of 200 K. The results are to be tabulated and plotted.

Analysis We solve this problem using EES making use of Rayleigh functions. The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
k=1.4
cp=1.005
R=0.287
P1=350
T1=600
V1=70
C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T1*(1+0.5*(k-1)*Ma1^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))
F1=1+0.5*(k-1)*Ma1^2
T01Ts=2*(k+1)*Ma1^2*F1/(1+k*Ma1^2)^2
P01Ps=((1+k)/(1+k*Ma1^2))*(2*F1/(k+1))^(k/(k-1))
T1Ts=(Ma1*((1+k)/(1+k*Ma1^2)))^2
P1Ps=(1+k)/(1+k*Ma1^2)
V1Vs=Ma1^2*(1+k)/(1+k*Ma1^2)
F2=1+0.5*(k-1)*Ma2^2
T02Ts=2*(k+1)*Ma2^2*F2/(1+k*Ma2^2)^2
P02Ps=((1+k)/(1+k*Ma2^2))*(2*F2/(k+1))^(k/(k-1))
T2Ts=(Ma2*((1+k)/(1+k*Ma2^2)))^2
P2Ps=(1+k)/(1+k*Ma2^2)
V2Vs=Ma2^2*(1+k)/(1+k*Ma2^2)
T02=T02Ts/T01Ts*T01
P02=P02Ps/P01Ps*P01
T2=T2Ts/T1Ts*T1
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
Delta_s=cp*ln(T2/T1)-R*ln(P2/P1)
```

Exit temperature T_2, K	Exit Mach number, Ma_2	Exit entropy relative to inlet, s_2, kJ/kg·K
600.1	0.143	0.000
800	0.166	0.292
1000	0.188	0.519
1200	0.208	0.705
1400	0.227	0.863
1600	0.245	1.001
1800	0.263	1.123
2000	0.281	1.232
2200	0.299	1.331
2400	0.316	1.423
2600	0.333	1.507
2800	0.351	1.586
3000	0.369	1.660
3200	0.387	1.729
3400	0.406	1.795
3600	0.426	1.858
3800	0.446	1.918
4000	0.467	1.975
4200	0.490	2.031
4400	0.515	2.085
4600	0.541	2.138
4800	0.571	2.190
5000	0.606	2.242



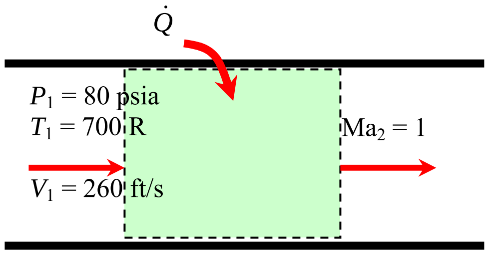
Discussion Note that the entropy of air increases during this heating process, as expected.

17-109E Air flowing with a subsonic velocity in a square duct is accelerated by heating until the flow is choked at the exit. The rate of heat transfer and the entropy change are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The flow is choked at the duct exit. **3** Mass flow rate remains constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 0.2400$ Btu/lbm·R, and $R = 0.06855$ Btu/lbm·R = 0.3704 psia·ft³/lbm·R.

Analysis The inlet density and mass flow rate of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{80 \text{ psia}}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(700 \text{ R})} = 0.3085 \text{ lbm/ft}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.3085 \text{ lbm/ft}^3)(6\times 6/144 \text{ ft}^2)(260 \text{ ft/s}) = 20.06 \text{ lbm/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 700 \text{ R} + \frac{(260 \text{ ft/s})^2}{2\times 0.2400 \text{ Btu/lbm}\cdot\text{R}}\left(\frac{1 \text{ Btu/lbm}}{25{,}037 \text{ ft}^2/\text{s}^2}\right) = 705.6 \text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855 \text{ Btu/lbm}\cdot\text{R})(700 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = 1297 \text{ ft/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{260 \text{ ft/s}}{1297 \text{ ft/s}} = 0.2005$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34):

$\text{Ma}_1 = 0.2005$: $T_1/T^* = 0.2075$, $P_1/P^* = 2.272$, $T_{01}/T_0^* = 0.1743$

$\text{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.2075} \quad\rightarrow\quad T_2 = T_1/0.2075 = (700 \text{ R})/0.2075 = 3374 \text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{2.272} \quad\rightarrow\quad P_2 = P_1/2.272 = (80 \text{ psia})/2.272 = 35.2 \text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.1743} \quad\rightarrow\quad T_{02} = T_{01}/0.1743 = (705.6 \text{ R})/0.1743 = 4048 \text{ R}$$

Then the rate of heat transfer and entropy change become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (20.06 \text{ lbm/s})(0.2400 \text{ Btu/lbm}\cdot\text{R})(4048 - 705.6) \text{ R} = \mathbf{16{,}090 \text{ Btu/s}}$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= (0.2400 \text{ Btu/lbm}\cdot\text{R})\ln\frac{3374 \text{ R}}{700 \text{ R}} - (0.06855 \text{ Btu/lbm}\cdot\text{R})\ln\frac{35.2 \text{ psia}}{80 \text{ psia}} = \mathbf{0.434 \text{ Btu/lbm}\cdot\text{R}}$$

Discussion Note that the entropy of air increases during this heating process, as expected.

17-110 Air enters the combustion chamber of a gas turbine at a subsonic velocity. For a specified rate of heat transfer, the Mach number at the exit and the loss in stagnation pressure to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The cross-sectional area of the combustion chamber is constant. **3** The increase in mass flow rate due to fuel injection is disregarded.

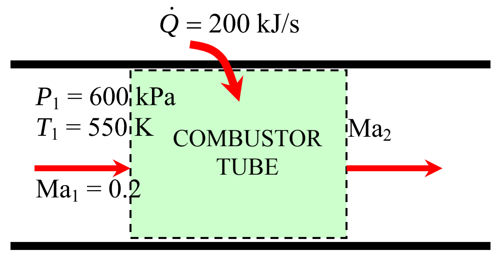
Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The inlet stagnation temperature and pressure are

$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (550\text{ K})\left(1+\frac{1.4-1}{2}0.2^2\right) = 554.4\text{ K}$$

$$P_{01} = P_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\text{ kPa})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right)^{1.4/0.4}$$

$$= 617.0\text{ kPa}$$



The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) \quad\rightarrow\quad 200\text{ kJ/s} = (0.3\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(T_{02} - 554.4)\text{ K}$$

It gives

$$T_{02} = 1218\text{ K}.$$

At Ma_1 = 0.2 we read from T_{01}/T_0^* = 0.1736 (Table A-34). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{554.4\text{ K}}{0.1736} = 3193.5\text{ K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-34)

$$\frac{T_{02}}{T_0^*} = \frac{1218\text{ K}}{3193.5\text{ K}} = 0.3814 \quad\rightarrow\quad \mathrm{Ma}_2 = 0.3187 \cong \mathbf{0.319}$$

Also,

$$\mathrm{Ma}_1 = 0.2 \quad\rightarrow\quad P_{01}/P_0^* = 1.2346$$

$$\mathrm{Ma}_2 = 0.3187 \quad\rightarrow\quad P_{02}/P_0^* = 1.191$$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.191}{1.2346} = 0.9647 \quad\rightarrow\quad P_{02} = 0.9647P_{01} = 0.9647(617\text{ kPa}) = 595.2\text{ kPa}$$

and

$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 595.2 = \mathbf{21.8\text{ kPa}}$$

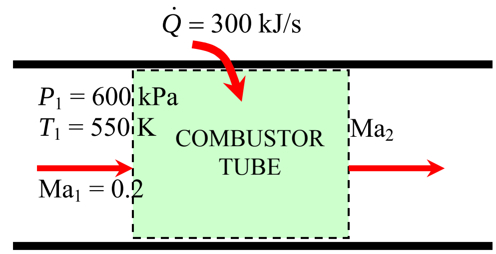
Discussion This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

17-111 Air enters the combustion chamber of a gas turbine at a subsonic velocity. For a specified rate of heat transfer, the Mach number at the exit and the loss in stagnation pressure to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The cross-sectional area of the combustion chamber is constant. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The inlet stagnation temperature and pressure are



$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (550\ \mathrm{K})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right) = 554.4\ \mathrm{K}$$

$$P_{01} = P_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\ \mathrm{kPa})\left(1+\frac{1.4\text{-}1}{2}0.2^2\right)^{1.4/0.4}$$
$$= 617.0\ \mathrm{kPa}$$

The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\mathrm{air}} c_p (T_{02} - T_{01}) \quad\rightarrow\quad 300\ \mathrm{kJ/s} = (0.3\ \mathrm{kg/s})(1.005\ \mathrm{kJ/kg\cdot K})(T_{02} - 554.4)\ \mathrm{K}$$

It gives

$$T_{02} = 1549\ \mathrm{K}.$$

At Ma_1 = 0.2 we read from T_{01}/T_0^* = 0.1736 (Table A-34). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{554.4\ \mathrm{K}}{0.1736} = 3193.5\ \mathrm{K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-34)

$$\frac{T_{02}}{T_0^*} = \frac{1549\ \mathrm{K}}{3193.5\ \mathrm{K}} = 0.4850 \quad\rightarrow\quad \mathrm{Ma}_2 = 0.3753 \cong \mathbf{0.375}$$

Also,

$$\mathrm{Ma}_1 = 0.2 \quad\rightarrow\quad P_{01}/P_0^* = 1.2346$$

$$\mathrm{Ma}_2 = 0.3753 \quad\rightarrow\quad P_{02}/P_0^* = 1.167$$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.167}{1.2346} = 0.9452 \quad\rightarrow\quad P_{02} = 0.9452 P_{01} = 0.9452(617\ \mathrm{kPa}) = 583.3\ \mathrm{kPa}$$

and

$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 583.3 = \mathbf{33.7\ kPa}$$

Discussion This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

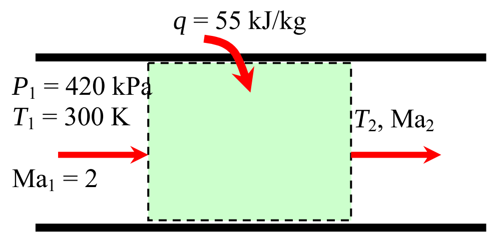
17-112 Fuel is burned in a rectangular duct with compressed air. For specified heat transfer, the exit temperature and Mach number are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The stagnation temperature and Mach number at the inlet are

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(300\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 347.2\text{ m/s}$$



$$V_1 = \text{Ma}_1 c_1 = 2(347.2\text{ m/s}) = 694.4\text{ m/s}$$

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 300\text{ K} + \frac{(694.4\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 539.9\text{ K}$$

The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 539.9\text{ K} + \frac{55\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 594.6\text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A-34 or from the appropriate relation. At Ma_1 = 2 we read T_{01}/T_0^* = 0.7934. Therefore,

$$T_0^* = \frac{T_{01}}{0.7934} = \frac{539.9\text{ K}}{0.7934} = 680.5\text{ K}$$

The stagnation temperature ratio at the exit and the Mach number corresponding to it are, from Table A-34,

$$\frac{T_{02}}{T_0^*} = \frac{594.6\text{ K}}{680.5\text{ K}} = 0.8738 \quad\to\quad \text{Ma}_2 = 1.642 \cong \mathbf{1.64}$$

Also,

$$\text{Ma}_1 = 2 \quad\to\quad T_1/T^* = 0.5289$$

$$\text{Ma}_2 = 1.642 \quad\to\quad T_2/T^* = 0.6812$$

Then the exit temperature becomes

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.6812}{0.5289} = 1.288 \quad\to\quad T_2 = 1.288T_1 = 1.288(300\text{ K}) = \mathbf{386\text{ K}}$$

Discussion Note that the temperature increases during this supersonic Rayleigh flow with heating. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

17-113 Compressed air is cooled as it flows in a rectangular duct. For specified heat rejection, the exit temperature and Mach number are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

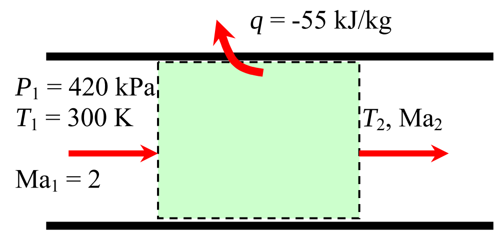
Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The stagnation temperature and Mach number at the inlet are

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(300\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 347.2\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(347.2\text{ m/s}) = 694.4\text{ m/s}$$

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 300\text{ K} + \frac{(694.4\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 539.9\text{ K}$$



The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 539.9\text{ K} + \frac{-55\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 485.2\text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A-34 or from the appropriate relation. At Ma_1 = 2 we read T_{01}/T_0^* = 0.7934. Therefore,

$$T_0^* = \frac{T_{01}}{0.7934} = \frac{539.9\text{ K}}{0.7934} = 680.5\text{ K}$$

The stagnation temperature ratio at the exit and the Mach number corresponding to it are, from Table A-34,

$$\frac{T_{02}}{T_0^*} = \frac{485.2\text{ K}}{680.5\text{ K}} = 0.7130 \quad\rightarrow\quad \text{Ma}_2 = 2.479 \cong \mathbf{2.48}$$

Also,

$$\text{Ma}_1 = 2 \quad\rightarrow\quad T_1/T^* = 0.5289$$

$$\text{Ma}_2 = 2.479 \quad\rightarrow\quad T_2/T^* = 0.3838$$

Then the exit temperature becomes

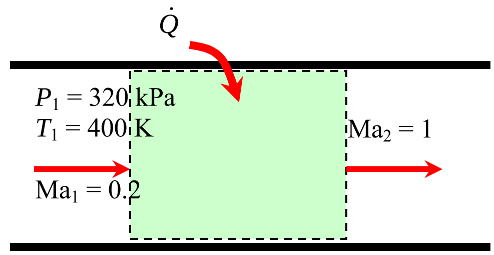
$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.3838}{0.5289} = 0.7257 \quad\rightarrow\quad T_2 = 0.7257T_1 = 0.7257(300\text{ K}) = \mathbf{218\text{ K}}$$

Discussion Note that the temperature decreases and Mach number increases during this supersonic Rayleigh flow with cooling. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

17-114 Argon flowing at subsonic velocity in a constant-diameter duct is accelerated by heating. The highest rate of heat transfer without reducing the mass flow rate is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Mass flow rate remains constant.

Properties We take the properties of argon to be $k = 1.667$, $c_p = 0.5203$ kJ/kg·K, and $R = 0.2081$ kJ/kg·K.



Analysis Heat transfer stops when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. The inlet stagnation temperature is

$$T_{01} = T_1\left(1+\frac{k-1}{2}Ma_1^2\right) = (400\text{ K})\left(1+\frac{1.667\text{-}1}{2}0.2^2\right) = 405.3\text{ K}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $Ma_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)Ma_1^2[2+(k-1)Ma_1^2]}{(1+kMa_1^2)^2} = \frac{(1.667+1)0.2^2[2+(1.667-1)0.2^2]}{(1+1.667\times 0.2^2)^2} = 0.1900$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.1900} \quad \rightarrow \quad T_{02} = T_{01}/0.1900 = (405.3\text{ K})/0.1900 = 2133\text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{air}c_p(T_{02} - T_{01}) = (1.2\text{ kg/s})(0.5203\text{ kJ/kg}\cdot\text{K})(2133 - 400)\text{ K} = \mathbf{1080\ kW}$$

Discussion It can also be shown that $T_2 = 1600$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-34 since they are based on $k = 1.4$.

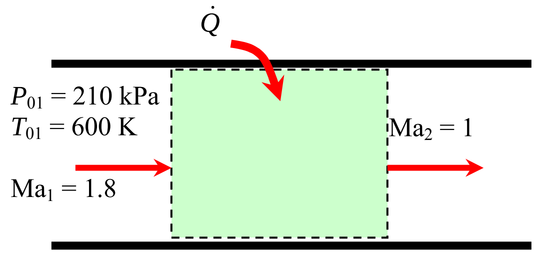
17-115 Air flowing at a supersonic velocity in a duct is decelerated by heating. The highest temperature air can be heated by heat addition and the rate of heat transfer are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Mass flow rate remains constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\mathrm{Ma}_2 = V_2/c_2 = 1$. Knowing stagnation properties, the static properties are determined to be

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (600\text{ K})\left(1+\frac{1.4-1}{2}1.8^2\right)^{-1} = 364.1\text{ K}$$



$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (210\text{ kPa})\left(1+\frac{1.4-1}{2}1.8^2\right)^{-1.4/0.4} = 36.55\text{ kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{36.55\text{ kPa}}{(0.287\text{ kJ/kgK})(364.1\text{ K})} = 0.3498\text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(364.1\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 382.5\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.8(382.5\text{ m/s}) = 688.5\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.3498\text{ kg/m}^3)[\pi(0.10\text{ m})^2/4](688.5\text{ m/s}) = 1.891\text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34):

$\mathrm{Ma}_1 = 1.8$: $\quad T_1/T^* = 0.6089$, $\quad T_{01}/T_0^* = 0.8363$

$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $\quad T_{02}/T_0^* = 1$

Then the exit temperature and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6089} \quad\rightarrow\quad T_2 = T_1/0.6089 = (364.1\text{ K})/0.6089 = \mathbf{598\ K}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.8363} \quad\rightarrow\quad T_{02} = T_{01}/0.8363 = (600\text{ K})/0.8363 = 717.4\text{ K} \cong \mathbf{717\ K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (1.891\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(717.4 - 600)\text{ K} = \mathbf{223\ kW}$$

Discussion Note that this is the highest temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature will cause the mass flow rate to decrease. Also, once the sonic conditions are reached, the thermodynamic temperature can be increased further by cooling the fluid and reducing the velocity (see the *T*-*s* diagram for Rayleigh flow).

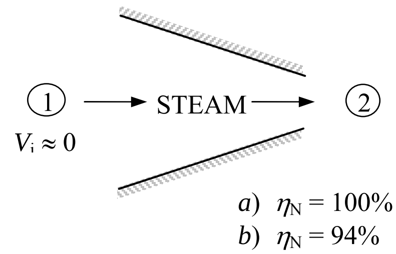
Steam Nozzles

17-116C The delay in the condensation of the steam is called supersaturation. It occurs in high-speed flows where there isn't sufficient time for the necessary heat transfer and the formation of liquid droplets.

17-117 Steam enters a converging nozzle with a low velocity. The exit velocity, mass flow rate, and exit Mach number are to be determined for isentropic and 90 percent efficient nozzle cases.

Assumptions **1** Flow through the nozzle is steady and one-dimensional. **2** The nozzle is adiabatic.

Analysis (*a*) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{01} = h_1$.



At the inlet,

$$\left.\begin{array}{l} P_1 = P_{01} = 4\text{ MPa} \\ T_1 = T_{01} = 400°\text{C} \end{array}\right\} \begin{array}{l} h_1 = h_{01} = 3214.5\text{ kJ/kg} \\ s_1 = s_{2s} = 6.7714\text{ kJ/kg}\cdot\text{K} \end{array}$$

At the exit,

$$\left.\begin{array}{l} P_2 = 2.5\text{ MPa} \\ s_2 = 6.7714\text{ kJ/kg}\cdot\text{K} \end{array}\right\} \begin{array}{l} h_2 = 3083.4\text{ kJ/kg} \\ \boldsymbol{v}_2 = 0.1058\text{ m}^3\text{/kg} \end{array}$$

Then the exit velocity is determined from the steady-flow energy balance $\dot{E}_{in} = \dot{E}_{out}$ with $q = w = 0$,

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \longrightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^{2\nearrow 0}}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(3214.5 - 3083.4)\text{ kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{512.0\ m/s}$$

The mass flow rate is determined from

$$\dot{m} = \frac{1}{\boldsymbol{v}_2} A_2 V_2 = \frac{1}{0.1058\text{ m}^3\text{/kg}}(32\times 10^{-4}\text{ m}^2)(512.0\text{ m/s}) = \mathbf{15.49\ kg/s}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 6.7714 kJ/kg·K and at pressures just below and just above the specified pressure (2.0 and 3.0 MPa) are determined to be 0.1257 and 0.09183 m^3/kg. Substituting,

$$c_2 = \sqrt{\frac{(3000 - 2000)\text{ kPa}}{\left(\dfrac{1}{0.09183} - \dfrac{1}{0.1257}\right)\text{kg/m}^3}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kPa.m}^3}\right)} = 583.7\text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{512.0\text{ m/s}}{583.7\text{ m/s}} = \mathbf{0.877}$$

(*b*) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{01} = h_1$.

At the inlet,

$$\left.\begin{array}{l} P_1 = P_{01} = 4\text{ MPa} \\ T_1 = T_{01} = 400°\text{C} \end{array}\right\} \begin{array}{l} h_1 = h_{01} = 3214.5\text{ kJ/kg} \\ s_1 = s_{2s} = 6.7714\text{ kJ/kg}\cdot\text{K} \end{array}$$

At state 2s,

$$\left.\begin{array}{l} P_{2s} = 2.5\text{ MPa} \\ s_{2s} = 6.7714\text{ kJ/kg}\cdot\text{K} \end{array}\right\} h_{2s} = 3083.4\text{ kJ/kg}$$

The enthalpy of steam at the actual exit state is determined from

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \longrightarrow 0.94 = \frac{3214.5 - h_2}{3214.5 - 3288.7} \longrightarrow h_2 = 3091.3\text{ kJ/kg}$$

Therefore,

$$\left.\begin{array}{l} P_2 = 2.5\text{ MPa} \\ h_2 = 3091.3\text{ kJ/kg} \end{array}\right\} \begin{array}{l} \boldsymbol{v}_2 = 0.1065\text{ m}^3\text{/kg} \\ s_2 = 6.7844\text{ kJ/kg}\cdot\text{K} \end{array}$$

Then the exit velocity is determined from the steady-flow energy balance $\dot{E}_{in} = \dot{E}_{out}$ with $q = w = 0$,

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \longrightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2 {}^{\nearrow 0}}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(3214.5 - 3091.3)\text{ kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{496.4\ m/s}$$

The mass flow rate is determined from

$$\dot{m} = \frac{1}{v_2}A_2V_2 = \frac{1}{0.1065\text{ m}^3/\text{kg}}(32\times10^{-4}\text{ m}^2)(496.4\text{ m/s}) = \mathbf{14.92\ kg/s}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 6.7844 kJ/kg·K and at pressures just below and just above the specified pressure (2.0 and 3.0 MPa) are determined to be 0.1266 and 0.09246 m^3/kg. Substituting,

$$c_2 = \sqrt{\frac{(3000 - 2000)\text{ kPa}}{\left(\frac{1}{0.09246} - \frac{1}{0.1266}\right)\text{kg/m}^3}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kPa.m}^3}\right)} = 585.7\text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{496.4\text{ m/s}}{585.7\text{ m/s}} = \mathbf{0.848}$$

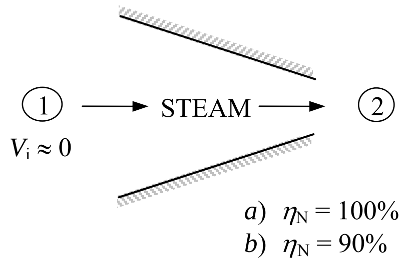
17-118E Steam enters a converging nozzle with a low velocity. The exit velocity, mass flow rate, and exit Mach number are to be determined for isentropic and 90 percent efficient nozzle cases.

Assumptions **1** Flow through the nozzle is steady and one-dimensional. **2** The nozzle is adiabatic.

Analysis (*a*) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{01} = h_1$.

At the inlet,

$$\left.\begin{array}{l} P_1 = P_{01} = 450 \text{ psia} \\ T_1 = T_{01} = 900°\text{F} \end{array}\right\} \begin{array}{l} h_1 = h_{01} = 1468.6 \text{ Btu/lbm} \\ s_1 = s_{2s} = 1.7117 \text{ Btu/lbm}\cdot\text{R} \end{array}$$



At the exit,

$$\left.\begin{array}{l} P_2 = 275 \text{ psia} \\ s_{2s} = 1.7117 \text{ Btu/lbm}\cdot\text{R} \end{array}\right\} \begin{array}{l} h_2 = 1400.5 \text{ Btu/lbm} \\ v_2 = 2.5732 \text{ ft}^3/\text{lbm} \end{array}$$

Then the exit velocity is determined from the steady-flow energy balance $\dot{E}_{in} = \dot{E}_{out}$ with $q = w = 0$,

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \longrightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2 {}^{\nearrow 0}}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(1468.6 - 1400.5) \text{ Btu/lbm}\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = \mathbf{1847 \text{ ft/s}}$$

Then,

$$\dot{m} = \frac{1}{v_2} A_2 V_2 = \frac{1}{2.5732 \text{ ft}^3/\text{lbm}}(3.75/144 \text{ ft}^2)(1847 \text{ ft/s}) = \mathbf{18.7 \text{ lbm/s}}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/v)}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 1.7117 Btu/lbm·R and at pressures just below and just above the specified pressure (250 and 300 psia) are determined to be 2.7709 and 2.4048 ft^3/lbm. Substituting,

$$c_2 = \sqrt{\frac{(300 - 250) \text{ psia}}{\left(\frac{1}{2.4048} - \frac{1}{2.7709}\right) \text{lbm/ft}^3}\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)\left(\frac{1 \text{ Btu}}{5.4039 \text{ ft}^3\cdot\text{psia}}\right)} = 2053 \text{ ft/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{1847 \text{ ft/s}}{2053 \text{ ft/s}} = \mathbf{0.900}$$

(*b*) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{01} = h_1$.

At the inlet,

$$\left.\begin{array}{l} P_1 = P_{01} = 450 \text{ psia} \\ T_1 = T_{01} = 900°\text{F} \end{array}\right\} \begin{array}{l} h_1 = h_{01} = 1468.6 \text{ Btu/lbm} \\ s_1 = s_{2s} = 1.7117 \text{ Btu/lbm}\cdot\text{R} \end{array}$$

At state 2s,

$$\left.\begin{array}{l} P_{2s} = 275 \text{ psia} \\ s_{2s} = 1.7117 \text{ Btu/lbm}\cdot\text{R} \end{array}\right\} h_{2s} = 1400.5 \text{ Btu/lbm}$$

The enthalpy of steam at the actual exit state is determined from

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \longrightarrow 0.90 = \frac{1468.6 - h_2}{1468.6 - 1400.5} \longrightarrow h_2 = 1407.3 \text{ Btu/1bm}$$

Therefore,

$$\left.\begin{array}{l} P_2 = 275 \text{ psia} \\ h_2 = 1407.3 \text{ Btu/lbm} \end{array}\right\} \begin{array}{l} \boldsymbol{v}_2 = 2.6034 \text{ ft}^3\text{/1bm} \\ s_2 = 1.7173 \text{ Btu/1bm}\cdot\text{R} \end{array}$$

Then the exit velocity is determined from the steady-flow energy balance $\dot{E}_{in} = \dot{E}_{out}$ with $q = w = 0$,

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \longrightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^{2 \nearrow 0}}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(1468.6 - 1407.3) \text{ Btu/1bm}\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = \mathbf{1752 \text{ ft/s}}$$

Then,

$$\dot{m} = \frac{1}{\boldsymbol{v}_2} A_2 V_2 = \frac{1}{2.6034 \text{ ft}^3/\text{1bm}}(3.75/144 \text{ ft}^2)(1752 \text{ ft/s}) = \mathbf{17.53 \text{ 1bm/s}}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 1.7173 Btu/lbm·R and at pressures just below and just above the specified pressure (250 and 300 psia) are determined to be 2.8036 and 2.4329 ft^3/lbm. Substituting,

$$c_2 = \sqrt{\frac{(300 - 250) \text{ psia}}{\left(\frac{1}{2.4329} - \frac{1}{2.8036}\right) \text{lbm/ft}^3}\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)\left(\frac{1 \text{ Btu}}{5.4039 \text{ ft}^3\cdot\text{psia}}\right)} = 2065 \text{ ft/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{1752 \text{ ft/s}}{2065 \text{ ft/s}} = \mathbf{0.849}$$

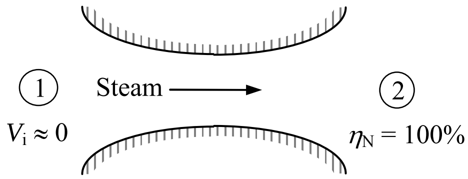
17-119 Steam enters a converging-diverging nozzle with a low velocity. The exit area and the exit Mach number are to be determined.

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{01} = h_1$.

At the inlet,

$$\left. \begin{array}{l} P_1 = P_{01} = 1\text{ MPa} \\ T_1 = T_{01} = 500°\text{C} \end{array} \right\} \begin{array}{l} h_1 = h_{01} = 3479.1\text{ kJ/kg} \\ s_1 = s_{2s} = 7.7642\text{ kJ/kg}\cdot\text{K} \end{array}$$



At the exit,

$$\left. \begin{array}{l} P_2 = 0.2\text{ MPa} \\ s_2 = 7.7642\text{ kJ/kg}\cdot\text{K} \end{array} \right\} \begin{array}{l} h_2 = 3000.0\text{ kJ/kg} \\ \boldsymbol{v}_2 = 1.2325\text{ m}^3\text{/kg} \end{array}$$

Then the exit velocity is determined from the steady-flow energy balance $\dot{E}_{in} = \dot{E}_{out}$ with $q = w = 0$,

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \longrightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2 {}^{\nearrow 0}}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(3479.1 - 3000.0)\text{ kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 978.9\text{ m/s}$$

The exit area is determined from

$$A_2 = \frac{\dot{m}\boldsymbol{v}_2}{V_2} = \frac{(2.5\text{ kg/s})(1.2325\text{ m}^3/\text{kg})}{(978.9\text{ m/s})} = 31.5\times10^{-4}\text{ m}^2 = \mathbf{31.5\text{ cm}^2}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 7.7642 kJ/kg·K and at pressures just below and just above the specified pressure (0.1 and 0.3 MPa) are determined to be 2.0935 and 0.9024 m^3/kg. Substituting,

$$c_2 = \sqrt{\frac{(300-100)\text{ kPa}}{\left(\frac{1}{0.9024} - \frac{1}{2.0935}\right)\text{kg/m}^3}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kPa}\cdot\text{m}^3}\right)} = 563.2\text{ m/s}$$

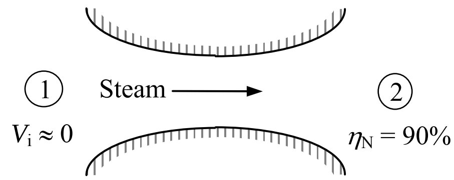
Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{978.9\text{ m/s}}{563.2\text{ m/s}} = \mathbf{1.738}$$

17-120 Steam enters a converging-diverging nozzle with a low velocity. The exit area and the exit Mach number are to be determined.

Assumptions Flow through the nozzle is steady and one-dimensional.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{01} = h_1$.



At the inlet, $\left.\begin{array}{l} P_1 = P_{01} = 1\text{ MPa} \\ T_1 = T_{01} = 500°\text{C} \end{array}\right\} \begin{array}{l} h_1 = h_{01} = 3479.1\text{ kJ/kg} \\ s_1 = s_{2s} = 7.7642\text{ kJ/kg}\cdot\text{K} \end{array}$

At state 2s, $\left.\begin{array}{l} P_2 = 0.2\text{ MPa} \\ s_2 = 7.7642\text{ kJ/kg}\cdot\text{K} \end{array}\right\} h_{2s} = 3000.0\text{ kJ/kg}$

The enthalpy of steam at the actual exit state is determined from

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \longrightarrow 0.90 = \frac{3479.1 - h_2}{3479.1 - 3000.0} \longrightarrow h_2 = 3047.9\text{ kJ/kg}$$

Therefore,

$$\left.\begin{array}{l} P_2 = 0.2\text{ MPa} \\ h_2 = 3047.9\text{ kJ/kg} \end{array}\right\} \begin{array}{l} \boldsymbol{v}_2 = 1.2882\text{ m}^3\text{/kg} \\ s_2 = 7.7642\text{ kJ/kg}\cdot\text{K} \end{array}$$

Then the exit velocity is determined from the steady-flow energy balance $\dot{E}_{in} = \dot{E}_{out}$ with $q = w = 0$,

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \longrightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^{2\nearrow 0}}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(3479.1 - 3047.9)\text{ kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 928.7\text{ m/s}$$

The exit area is determined from

$$A_2 = \frac{\dot{m}\boldsymbol{v}_2}{V_2} = \frac{(2.5\text{ kg/s})(1.2882\text{ m}^3\text{/kg})}{928.7\text{ m/s}} = 34.7\times 10^{-4}\text{ m}^2 = \mathbf{34.7\text{ cm}^2}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 7.7642 kJ/kg·K and at pressures just below and just above the specified pressure (0.1 and 0.3 MPa) are determined to be 2.1903 and 0.9425 m^3/kg. Substituting,

$$c_2 = \sqrt{\frac{(300 - 100)\text{ kPa}}{\left(\frac{1}{0.9425} - \frac{1}{2.1903}\right)\text{kg/m}^3}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kPa}\cdot\text{m}^3}\right)} = 575.2\text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{928.7\text{ m/s}}{575.2\text{ m/s}} = \mathbf{1.61}$$

Review Problems

17-121 A leak develops in an automobile tire as a result of an accident. The initial mass flow rate of air through the leak is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow of air through the hole is isentropic.

Properties The gas constant of air is $R = 0.287$ kPa·m^3/kg·K. The specific heat ratio of air at room temperature is $k = 1.4$ (Table A-2a).

Analysis The absolute pressure in the tire is

$$P = P_{\text{gage}} + P_{\text{atm}} = 220 + 94 = 314 \text{ kPa}$$

The critical pressure is, from Table 17-2,

$$P^* = 0.5283 P_0 = (0.5283)(314 \text{ kPa}) = 166 \text{ kPa} > 94 \text{ kPa}$$

Therefore, the flow is choked, and the velocity at the exit of the hole is the sonic speed. Then the flow properties at the exit becomes

$$\rho_0 = \frac{P_0}{RT_0} = \frac{314 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298 \text{ K})} = 3.671 \text{ kg/m}^3$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (3.671 \text{ kg/m}^3)\left(\frac{2}{1.4+1}\right)^{1/(1.4-1)} = 2.327 \text{ kg/m}^3$$

$$T^* = \frac{2}{k+1}T_0 = \frac{2}{1.4+1}(298 \text{ K}) = 248.3 \text{ K}$$

$$V = c = \sqrt{kRT^*} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)(248.3 \text{ K})} = 315.9 \text{ m/s}$$

Then the initial mass flow rate through the hole becomes

$$\dot{m} = \rho AV = (2.327 \text{ kg/m}^3)[\pi(0.004 \text{ m})^2/4](315.9 \text{ m/s}) = 0.00924 \text{ kg/s} = \mathbf{0.554 \text{ kg/min}}$$

Discussion The mass flow rate will decrease with time as the pressure inside the tire drops.

17-122 The thrust developed by the engine of a Boeing 777 is about 380 kN. The mass flow rate of air through the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific properties. **2** Flow of combustion gases through the nozzle is isentropic. **3** Choked flow conditions exist at the nozzle exit. **4** The velocity of gases at the nozzle inlet is negligible.

Properties The gas constant of air is R = 0.287 kPa.m^3/kg.K (Table A-1), and it can also be used for combustion gases. The specific heat ratio of combustion gases is k = 1.33 (Table 17-2).

Analysis The velocity at the nozzle exit is the sonic velocity, which is determined to be

$$V = c = \sqrt{kRT} = \sqrt{(1.33)(0.287\text{ kJ/kg}\cdot\text{K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)(265\text{ K})} = 318.0\text{ m/s}$$

Noting that thrust F is related to velocity by $F = \dot{m}V$, the mass flow rate of combustion gases is determined to be

$$\dot{m} = \frac{F}{V} = \frac{380{,}000\text{ N}}{318.0\text{ m/s}}\left(\frac{1\text{ kg.m/s}^2}{1\text{ N}}\right) = \mathbf{1194.8\ kg/s}$$

Discussion The combustion gases are mostly nitrogen (due to the 78% of N_2 in air), and thus they can be treated as air with a good degree of approximation.

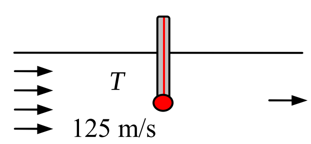
17-123 A stationary temperature probe is inserted into an air duct reads 50°C. The actual temperature of air is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** The stagnation process is isentropic.

Properties The specific heat of air at room temperature is c_p = 1.005 kJ/kg·K (Table A-2a).

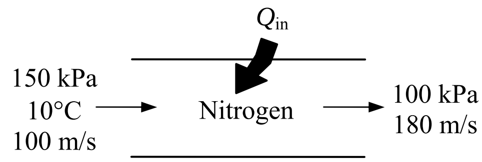
Analysis The air that strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature. The actual air temperature is determined from

$$T = T_0 - \frac{V^2}{2c_p} = 50°\text{C} - \frac{(125\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{42.2°C}$$



Discussion Temperature rise due to stagnation is very significant in high-speed flows, and should always be considered when compressibility effects are not negligible.

17-124 Nitrogen flows through a heat exchanger. The stagnation pressure and temperature of the nitrogen at the inlet and the exit states are to be determined.



Assumptions **1** Nitrogen is an ideal gas with constant specific properties. **2** Flow of nitrogen through the heat exchanger is isentropic.

Properties The properties of nitrogen are c_p = 1.039 kJ/kg.K and k = 1.4 (Table A-2a).

Analysis The stagnation temperature and pressure of nitrogen at the inlet and the exit states are determined from

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 10°\text{C} + \frac{(100\text{ m/s})^2}{2 \times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{14.8°C}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (150\text{ kPa})\left(\frac{288.0\text{ K}}{283.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{159.1\,kPa}$$

From the energy balance relation $E_{in} - E_{out} = \Delta E_{system}$ with $w = 0$

$$q_{in} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + \Delta\text{pe}^{\nearrow 0}$$

$$125\text{ kJ/kg} = (1.039\text{ kJ/kg}\cdot°\text{C})(T_2 - 10°\text{C}) + \frac{(180\text{ m/s})^2 - (100\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$

$$T_2 = 119.5°\text{C}$$

and

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 119.5°\text{C} + \frac{(180\text{ m/s})^2}{2 \times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{135.1°C}$$

$$P_{02} = P_2\left(\frac{T_{02}}{T_2}\right)^{k/(k-1)} = (100\text{ kPa})\left(\frac{408.3\text{ K}}{392.7\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{114.6\,kPa}$$

Discussion Note that the stagnation temperature and pressure can be very different than their thermodynamic counterparts when dealing with compressible flow.

17-125 An expression for the speed of sound based on van der Waals equation of state is to be derived. Using this relation, the speed of sound in carbon dioxide is to be determined and compared to that obtained by ideal gas behavior.

Properties The properties of CO_2 are R = 0.1889 kJ/kg·K and k = 1.279 at T = 50°C = 323.2 K (Table A-2b).

Analysis Van der Waals equation of state can be expressed as

$$P = \frac{RT}{v - b} - \frac{a}{v^2}$$

Differentiating,

$$\left(\frac{\partial P}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Noting that $\rho = 1/v \longrightarrow d\rho = -dv/v^2$, the speed of sound relation becomes

$$c^2 = k\left(\frac{\partial P}{\partial r}\right)_T = -v^2 k\left(\frac{\partial P}{\partial v}\right)_T$$

Substituting,

$$c^2 = \frac{v^2 kRT}{(v-b)^2} - \frac{2ak}{v}$$

Using the molar mass of CO_2 (M = 44 kg/kmol), the constant a and b can be expressed per unit mass as

$$a = 0.1882\,\text{kPa}\cdot\text{m}^6/\text{kg}^2 \quad \text{and} \qquad b = 9.705\times10^{-4}\ \text{m}^3/\text{kg}$$

The specific volume of CO_2 is determined to be

$$200\ \text{kPa} = \frac{(0.1889\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323.2\ \text{K})}{v - 0.0009705\ \text{m}^3/\text{kg}} - \frac{2\times 0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^2}{v^2} \to v = 0.3031\,\text{m}^3/\text{kg}$$

Substituting,

$$c = \left(\begin{array}{l}\dfrac{(0.3031\,\text{m}^3/\text{kg})^2(1.279)(0.1889\ \text{kJ/kg}\cdot\text{K})(323.2\ \text{K})}{(0.3031-0.0009705\,\text{m}^3/\text{kg})^2}\left(\dfrac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right) \\ -\dfrac{2(0.1882\ \text{kPa.m}^6/\text{kg}^3)(1.279)}{(0.3031\,\text{m}^3/\text{kg})^2}\left(\dfrac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kPa}\cdot\text{m}^3/\text{kg}}\right)\end{array}\right)^{1/2} = \mathbf{277.5\ m/s}$$

If we treat CO_2 as an ideal gas, the speed of sound becomes

$$c = \sqrt{kRT} = \sqrt{(1.279)(0.1889\ \text{kJ/kg}\cdot\text{K})(323.2\ \text{K})\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right)} = \mathbf{279.4\ m/s}$$

Discussion Note that the ideal gas relation is the simplest equation of state, and it is very accurate for most gases encountered in practice. At high pressures and/or low temperatures, however, the gases deviate from ideal gas behavior, and it becomes necessary to use more complicated equations of state.

17-126 The equivalent relation for the speed of sound is to be verified using thermodynamic relations.

Analysis The two relations are $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$ and $c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$

From $r = 1/v \longrightarrow dr = -dv/v^2$. Thus,

$$c^2 = \left(\frac{\partial P}{\partial r}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = -v^2\left(\frac{\partial P}{\partial T}\frac{\partial T}{\partial v}\right)_s = -v^2\left(\frac{\partial P}{\partial T}\right)_s\left(\frac{\partial T}{\partial v}\right)_s$$

From the cyclic rule,

$$(P,T,s): \left(\frac{\partial P}{\partial T}\right)_s\left(\frac{\partial T}{\partial s}\right)_P\left(\frac{\partial s}{\partial P}\right)_T = -1 \longrightarrow \left(\frac{\partial P}{\partial T}\right)_s = -\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial P}{\partial s}\right)_T$$

$$(T,v,s): \left(\frac{\partial T}{\partial v}\right)_s\left(\frac{\partial v}{\partial s}\right)_T\left(\frac{\partial s}{\partial T}\right)_v = -1 \longrightarrow \left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial T}{\partial s}\right)_v$$

Substituting,

$$c^2 = -v^2\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial P}{\partial s}\right)_T\left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial T}{\partial s}\right)_v = -v^2\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial T}{\partial s}\right)_v\left(\frac{\partial P}{\partial s}\right)_T$$

Recall that

$$\frac{c_p}{T} = \left(\frac{\partial s}{\partial T}\right)_P \quad \text{and} \quad \frac{c_v}{T} = \left(\frac{\partial s}{\partial T}\right)_v$$

Substituting,

$$c^2 = -v^2\left(\frac{c_p}{T}\right)\left(\frac{T}{c_v}\right)\left(\frac{\partial P}{\partial v}\right)_T = -v^2 k\left(\frac{\partial P}{\partial v}\right)_T$$

Replacing $-dv/v^2$ by $d\rho$,

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

Discussion Note that the differential thermodynamic property relations are very useful in the derivation of other property relations in differential form.

17-127 For ideal gases undergoing isentropic flows, expressions for P/P^*, T/T^*, and ρ/ρ^* as functions of k and Ma are to be obtained.

Analysis Equations 17-18 and 17-21 are given to be

$$\frac{T_0}{T} = \frac{2+(k-1)\mathrm{Ma}^2}{2}$$

and

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

Multiplying the two,

$$\left(\frac{T_0}{T}\frac{T^*}{T_0}\right) = \left(\frac{2+(k-1)\mathrm{Ma}^2}{2}\right)\left(\frac{2}{k+1}\right)$$

Simplifying and inverting,

$$\frac{T}{T^*} = \frac{k+1}{2+(k-1)\mathrm{Ma}^2} \qquad (1)$$

From

$$\frac{P}{P^*} = \left(\frac{T}{T^*}\right)^{k/(k-1)} \longrightarrow \frac{P}{P^*} = \left(\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{k/(k-1)} \qquad (2)$$

From

$$\frac{\rho}{\rho^*} = \left(\frac{\rho}{\rho^*}\right)^{k/(k-1)} \longrightarrow \frac{\rho}{\rho^*} = \left(\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{k/(k-1)} \qquad (3)$$

Discussion Note that some very useful relations can be obtained by very simple manipulations.

17-128 It is to be verified that for the steady flow of ideal gases $dT_0/T = dA/A + (1\text{-Ma}^2)\ dV/V$. The effect of heating and area changes on the velocity of an ideal gas in steady flow for subsonic flow and supersonic flow are to be explained.

Analysis We start with the relation $\frac{V^2}{2} = c_p(T_0 - T)$, (1)

Differentiating, $V\,dV = c_p(dT_0 - dT)$ (2)

We also have $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$ (3)

and $\frac{dP}{\rho} + V\,dV = 0$ (4)

Differentiating the ideal gas relation $P = \rho RT$, $\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} = 0$ (5)

From the speed of sound relation, $c^2 = kRT = (k-1)c_pT = kP/\rho$ (6)

Combining Eqs. (3) and (5), $\frac{dP}{P} - \frac{dT}{T} + \frac{dA}{A} + \frac{dV}{V} = 0$ (7)

Combining Eqs. (4) and (6), $\frac{dP}{\rho} = \frac{dP}{kP/c^2} = -VdV$

or, $\frac{dP}{P} = -\frac{k}{c^2}V\,dV = -k\frac{V^2}{c^2}\frac{dV}{V} = -k\text{Ma}^2\frac{dV}{V}$ (8)

Combining Eqs. (2) and (6),

$$dT = dT_0 - V\frac{dV}{c_p}$$

or, $\frac{dT}{T} = \frac{dT_0}{T} - \frac{V^2}{c_pT}\frac{dV}{V} = \frac{dT}{T} = \frac{dT_0}{T} - \frac{V^2}{c^2/(k-1)}\frac{dV}{V} = \frac{dT_0}{T} - (k-1)\text{Ma}^2\frac{dV}{V}$ (9)

Combining Eqs. (7), (8), and (9),

$$-(k-1)\text{Ma}^2\frac{dV}{V} - \frac{dT_0}{T} + (k-1)\text{Ma}^2\frac{dV}{V} + \frac{dA}{A} + \frac{dV}{V} = 0$$

or, $\frac{dT_0}{T} = \frac{dA}{A} + \left[-k\text{Ma}^2 + (k-1)\text{Ma}^2 + 1\right]\frac{dV}{V}$

Thus, $\frac{dT_0}{T} = \frac{dA}{A} + (1 - \text{Ma}^2)\frac{dV}{V}$ (10)

Differentiating the steady-flow energy equation $q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$

$$\delta q = c_p dT_0 \quad (11)$$

Eq. (11) relates the stagnation temperature change dT_0 to the net heat transferred to the fluid. Eq. (10) relates the velocity changes to area changes dA, and the stagnation temperature change dT_0 or the heat transferred.

(*a*) When Ma < 1 (subsonic flow), the fluid will accelerate if the duck converges ($dA < 0$) or the fluid is heated ($dT_0 > 0$ or $\delta q > 0$). The fluid will decelerate if the duck converges ($dA < 0$) or the fluid is cooled ($dT_0 < 0$ or $\delta q < 0$).

(*b*) When Ma > 1 (supersonic flow), the fluid will accelerate if the duck diverges ($dA > 0$) or the fluid is cooled ($dT_0 < 0$ or $\delta q < 0$). The fluid will decelerate if the duck converges ($dA < 0$) or the fluid is heated ($dT_0 > 0$ or $\delta q > 0$).

17-129 A pitot tube measures the difference between the static and stagnation pressures for a subsonic airplane. The speed of the airplane and the flight Mach number are to be determined.

Assumptions **1** Air is an ideal gas with constant specific heat ratio. **2** The stagnation process is isentropic.

Properties The properties of air are R = 0.287 kJ/kg.K and k = 1.4 (Table A-2a).

Analysis The stagnation pressure of air at the specified conditions is

$$P_0 = P + \Delta P = 30.8 + 20 = 50.8 \text{ kPa}$$

Then,

$$\frac{P_0}{P} = \left(1 + \frac{(k-1)\text{Ma}^2}{2}\right)^{k/k-1} \longrightarrow \frac{50.8}{30.8} = \left(1 + \frac{(1.4-1)\text{Ma}^2}{2}\right)^{1.4/0.4}$$

It yields

$$\text{Ma} = \mathbf{0.877}$$

The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(240 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 310.5 \text{ m/s}$$

Thus, $V = \text{Ma} \times c = (0.877)(310.5 \text{ m/s}) = \mathbf{272\ \ m/s}$

Discussion Note that the flow velocity can be measured in a simple and accurate way by simply measuring pressure.

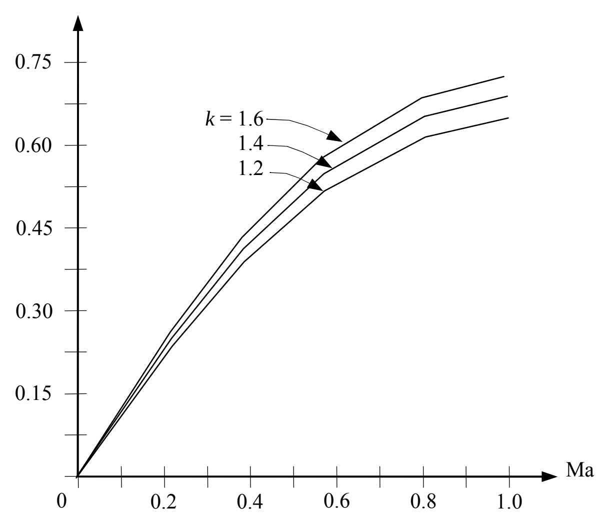
17-130 The mass flow parameter $\dot{m}\sqrt{RT_0}/(AP_0)$ versus the Mach number for k = 1.2, 1.4, and 1.6 in the range of $0 \le \text{Ma} \le 1$ is to be plotted.

Analysis The mass flow rate parameter $(\dot{m}\sqrt{RT_0})/P_0A$ can be expressed as

$$\frac{\dot{m}\sqrt{RT_0}}{P_0A} = \text{Ma}\sqrt{k}\left(\frac{2}{2+(k-1)M^2}\right)^{(k+1)/2(k-1)}$$

Thus,

Ma	**k = 1.2**	**k = 1.4**	**k = 1.6**
0.0	0	0	0
0.1	0.1089	0.1176	0.1257
0.2	0.2143	0.2311	0.2465
0.3	0.3128	0.3365	0.3582
0.4	0.4015	0.4306	0.4571
0.5	0.4782	0.5111	0.5407
0.6	0.5411	0.5763	0.6077
0.7	0.5894	0.6257	0.6578
0.8	0.6230	0.6595	0.6916
0.9	0.6424	0.6787	0.7106
1.0	0.6485	0.6847	0.7164



Discussion Note that the mass flow rate increases with increasing Mach number and specific heat ratio. It levels off at Ma = 1, and remains constant (choked flow).

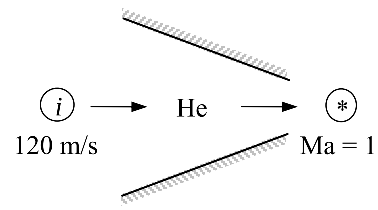
17-131 Helium gas is accelerated in a nozzle. The pressure and temperature of helium at the location where Ma = 1 and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of helium are R = 2.0769 kJ/kg·K, c_p = 5.1926 kJ/kg·K, and k = 1.667 (Table A-2a).

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 500\text{ K} + \frac{(120\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 501.4\text{ K}$$



and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (0.8\text{ MPa})\left(\frac{501.4\text{ K}}{500\text{ K}}\right)^{1.667/(1.667-1)} = 0.806\text{ MPa}$$

The Mach number at the nozzle exit is given to be Ma = 1. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (501.4\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{376\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.806\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.393\ MPa}$$

The speed of sound and the Mach number at the nozzle inlet are

$$c_i = \sqrt{kRT}_i = \sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(500\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1316\text{ m/s}$$

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{120\text{ m/s}}{1316\text{ m/s}} = 0.0912$$

The ratio of the entrance-to-throat area is

$$\frac{A_i}{A^*} = \frac{1}{\text{Ma}_i}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}_i^2\right)\right]^{(k+1)/[2(k-1)]}$$

$$= \frac{1}{0.0912}\left[\left(\frac{2}{1.667+1}\right)\left(1+\frac{1.667-1}{2}(0.0912)^2\right)\right]^{2.667/(2\times 0.667)}$$

$$= 6.20$$

Then the ratio of the throat area to the entrance area becomes

$$\frac{A^*}{A_i} = \frac{1}{6.20} = \mathbf{0.161}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

17-132 Helium gas enters a nozzle with negligible velocity, and is accelerated in a nozzle. The pressure and temperature of helium at the location where Ma = 1 and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The entrance velocity is negligible.

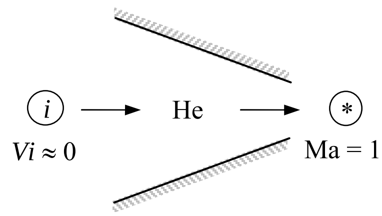
Properties The properties of helium are R = 2.0769 kJ/kg·K, c_p = 5.1926 kJ/kg·K, and k = 1.667 (Table A-2a).

Analysis We treat helium as an ideal gas with k = 1.667. The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *.

The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$T_0 = T_i = 500\text{ K}$

$P_0 = P_i = 0.8\text{ MPa}$



The Mach number at the nozzle exit is given to be Ma = 1. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (500\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{375\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.8\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.390\ MPa}$$

The ratio of the nozzle inlet area to the throat area is determined from

$$\frac{A_i}{A^*} = \frac{1}{\text{Ma}_i}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}_i^2\right)\right]^{(k+1)/[2(k-1)]}$$

But the Mach number at the nozzle inlet is Ma = 0 since $V_i \cong 0$. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{\infty} = \mathbf{0}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

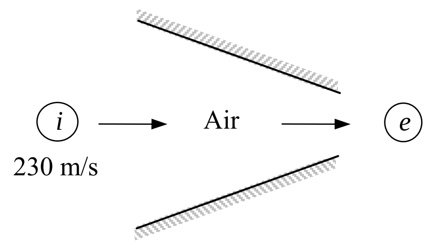
17-133 Air enters a converging nozzle. The mass flow rate, the exit velocity, the exit Mach number, and the exit pressure-stagnation pressure ratio versus the back pressure-stagnation pressure ratio for a specified back pressure range are to be calculated and plotted.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K, c_p = 1.005 kJ/kg·K, and k = 1.4 (Table A-2a).

Analysis The stagnation properties remain constant throughout the nozzle since the flow is isentropic. They are determined from

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 500\text{ K} + \frac{(230\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 526.3\text{ K}$$



and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (900\text{ kPa})\left(\frac{526.3\text{ K}}{500\text{ K}}\right)^{1.4/(1.4-1)} = 1077\text{ kPa}$$

The critical pressure is determined to be

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1077\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 569.0\text{ kPa}$$

Then the pressure at the exit plane (throat) will be

$P_e = P_b$ for $P_b \geq 569.0$ kPa

$P_e = P^* = 569.0$ kPa for $P_b < 569.0$ kPa (choked flow)

Thus the back pressure will not affect the flow when $100 < P_b < 569.0$ kPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from

Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (526.3\text{ K})\left(\frac{P_e}{1077}\right)^{0.4/1.4}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005\text{ kJ/kg}\cdot\text{K})(526.3 - T_e)\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Speed of sound $$c_e = \sqrt{kRT}_e = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Mach number $$\text{Ma}_e = V_e / c_e$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

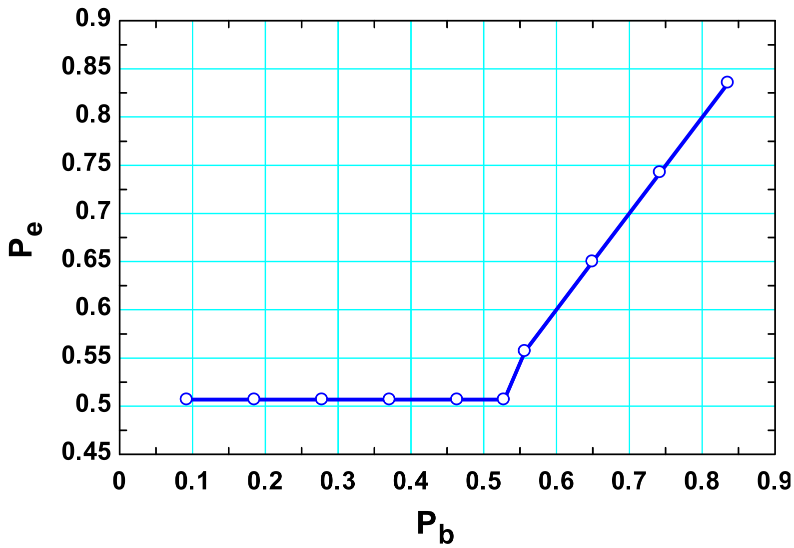
Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001\text{ m}^2)$$

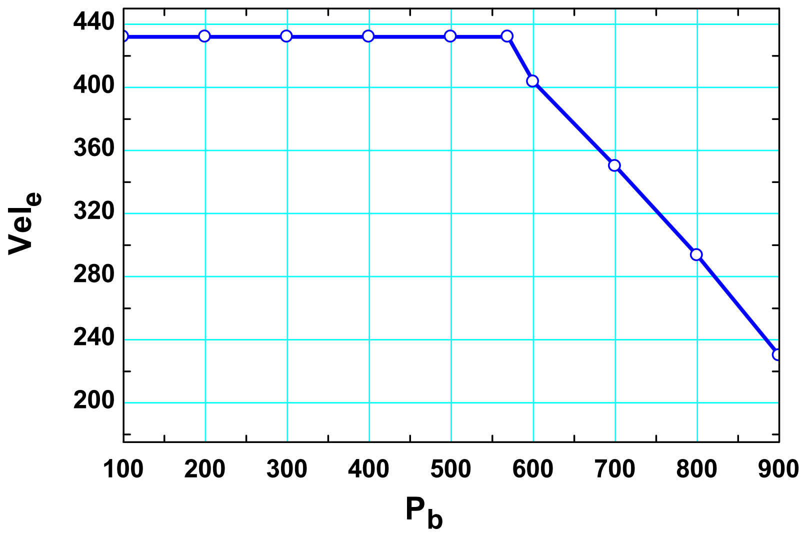
The EES solution and the results are given below

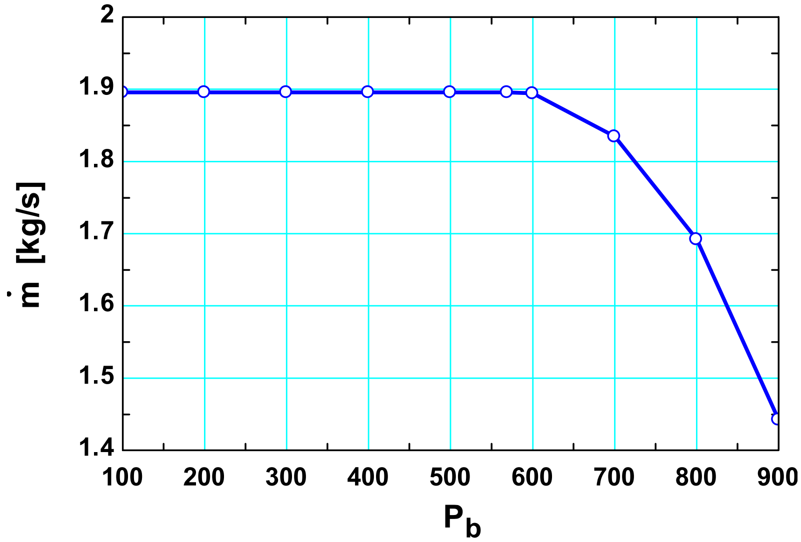
```
"Given"
P_i=900 "[kPa]"
T_i=500 "[K]"
Vel_i=230 "[m/s]"
A_e=10E-4 "[m^2]"
```

```
"Properties"
C_p=1.005
k=1.4
R=0.287

"Analysis"
T0=T_i+Vel_i^2/(2*C_p)*Convert(m^2/s^2, kJ/kg)
P0=P_i*(T0/T_i)^(k/(k-1))
P_star=P0*(2/(k+1))^(k/(k-1))
T_e=T0*(P_e/P0)^((k-1)/k)
Vel_e=sqrt(2*C_p*(T0-T_e)*Convert(kJ/kg, m^2/s^2))
C_e=sqrt(k*R*T_e*Convert(kJ/kg, m^2/s^2))
M_e=Vel_e/C_e
rho_e=P_e/(R*T_e)
m_dot=rho_e*Vel_e*A_e
RatioP_e=P_e/P0
RatioP_b=P_b/P0
"P_e=P_b  for  P_b >= P_star   and   P_e=P_star  for  P_b < P_star
```

P_b, kPa	RatioP$_b$	P_e, kPa	RatioP$_e$	T_e, K	V_e, m/s	M$_e$	ρ_e, kg/m^3	$\dot{m}$, kg / s
900	0.836	900	0.836	500	230	0.51	6.272	1.443
800	0.743	800	0.743	483.5	293.5	0.67	5.766	1.692
700	0.650	700	0.650	465.4	350	0.81	5.241	1.835
600	0.557	600	0.557	445.3	403.5	0.95	4.695	1.894
569	0.528	545.9	0.507	433.4	432.1	1.04	4.388	1.896
500	0.464	545.9	0.507	433.4	432.1	1.04	4.388	1.896
400	0.371	545.9	0.507	433.4	432.1	1.04	4.388	1.896
300	0.279	545.9	0.507	433.4	432.1	1.04	4.388	1.896
200	0.186	545.9	0.507	433.4	432.1	1.04	4.388	1.896
100	0.093	545.9	0.507	433.4	432.1	1.04	4.388	1.896




440
400
360
320
280
240
200
Vel$_e$
100
200
300
400
500
600
700
800
900
P$_b$


2
1.9
1.8
1.7
1.6
1.5
1.4
ṁ [kg/s]
100
200
300
400
500
600
700
800
900
P$_b$

17-134 Steam enters a converging nozzle. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure for a specified back pressure range are to be plotted.

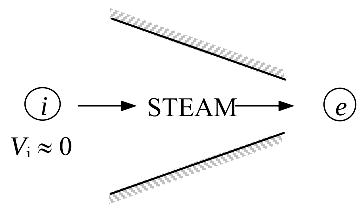
Assumptions **1** Steam is to be treated as an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The ideal gas properties of steam are given to be R = 0.462 kJ/kg.K, c_p = 1.872 kJ/kg.K, and k = 1.3.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Since the flow is isentropic, they remain constant throughout the nozzle,

$$P_0 = P_i = 6 \text{ MPa}$$

$$T_0 = T_i = 700 \text{ K}$$



The critical pressure is determined from to be

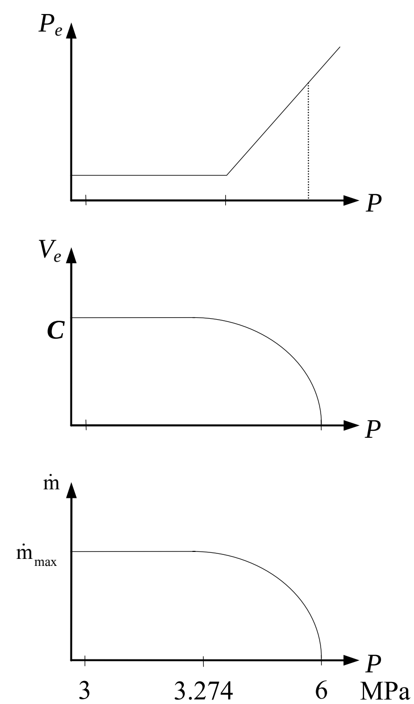
$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (6 \text{ MPa})\left(\frac{2}{1.3+1}\right)^{1.3/0.3} = 3.274 \text{ MPa}$$

Then the pressure at the exit plane (throat) will be

$P_e = P_b$ for $P_b \geq 3.274$ MPa

$P_e = P^* = 3.274$ MPa for $P_b < 3.274$ MPa (choked flow)

Thus the back pressure will not affect the flow when $3 < P_b < 3.274$ MPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from



Temperature

$$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (700 \text{ K})\left(\frac{P_e}{6}\right)^{0.3/1.3}$$

Velocity

$$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.872 \text{ kJ/kg}\cdot\text{K})(700 - T_e)\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}$$

Density

$$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.462 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

Mass flow rate

$$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.0008 \text{ m}^2)$$

The results of the calculations can be tabulated as follows:

P_b, MPa	P_e, MPa	T_e, K	V_e, m/s	ρ_e, kg/m^3	$\dot{m}$, kg/s
6.0	6.0	700	0	18.55	0
5.5	5.5	686.1	228.1	17.35	3.166
5.0	5.0	671.2	328.4	16.12	4.235
4.5	4.5	655.0	410.5	14.87	4.883
4.0	4.0	637.5	483.7	13.58	5.255
3.5	3.5	618.1	553.7	12.26	5.431
3.274	3.274	608.7	584.7	11.64	5.445
3.0	3.274	608.7	584.7	11.64	5.445

17-135 An expression for the ratio of the stagnation pressure after a shock wave to the static pressure before the shock wave as a function of k and the Mach number upstream of the shock wave is to be found.

Analysis The relation between P_1 and P_2 is

$$\frac{P_2}{P_1} = \frac{1+k\mathrm{Ma}_2^2}{1+k\mathrm{Ma}_1^2} \longrightarrow P_2 = P_1\left(\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}\right)$$

Substituting this into the isentropic relation

$$\frac{P_{02}}{P_1} = \left(1+(k-1)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

Then,

$$\frac{P_{02}}{P_1} = \left(\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}\right)\left(1+(k-1)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

where

$$\mathrm{Ma}_2^2 = \frac{\mathrm{Ma}_1^2 + 2/(k-1)}{2k\mathrm{Ma}_2^2/(k-1)-1}$$

Substituting,

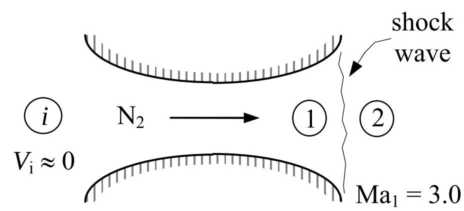
$$\frac{P_{02}}{P_1} = \left(\frac{(1+k\mathrm{Ma}_1^2)(2k\mathrm{Ma}_1^2-k+1)}{k\mathrm{Ma}_1^2(k+1)-k+3}\right)\left(1+\frac{(k-1)\mathrm{Ma}_1^2/2+1}{2k\mathrm{Ma}_1^2/(k-1)-1}\right)^{k/(k-1)}$$

17-136 Nitrogen entering a converging-diverging nozzle experiences a normal shock. The pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock are to be determined. The results are to be compared to those of air under the same conditions.

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of nitrogen are $R = 0.2968$ kJ/kg·K and $k = 1.4$ (Table A-2a).

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Assuming the flow before the shock to be isentropic,



$$P_{01} = P_i = 700\text{ kPa}$$
$$T_{01} = T_i = 300\text{ K}$$

Then,

$$T_1 = T_{01}\left(\frac{2}{2+(k-1)\text{Ma}_1^2}\right) = (300\text{ K})\left(\frac{2}{2+(1.4\text{-}1)3^2}\right) = 107.1\text{ K}$$

and

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{k/(k-1)} = (700\text{ kPa})\left(\frac{107.1}{300}\right)^{1.4/0.4} = 19.06\text{ kPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-33. For $\text{Ma}_1 = 3.0$ we read

$$\text{Ma}_2 = \mathbf{0.4752},\ \frac{P_{02}}{P_{01}} = 0.32834,\ \frac{P_2}{P_1} = 10.333,\text{ and }\frac{T_2}{T_1} = 2.679$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 0.32834P_{01} = (0.32834)(700\text{ kPa}) = \mathbf{230\text{ kPa}}$$
$$P_2 = 10.333P_1 = (10.333)(19.06\text{ kPa}) = \mathbf{197\text{ kPa}}$$
$$T_2 = 2.679T_1 = (2.679)(107.1\text{ K}) = \mathbf{287\text{ K}}$$

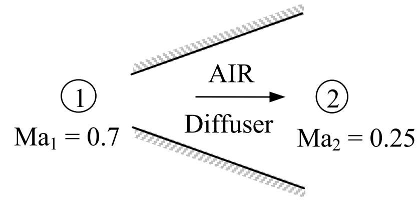
The velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.4752)\sqrt{(1.4)(0.2968\text{ kJ/kg}\cdot\text{K})(287\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{164\ \text{m/s}}$$

Discussion For **air** at specified conditions $k = 1.4$ (same as nitrogen) and $R = 0.287$ kJ/kg·K. Thus the only quantity which will be different in the case of air is the velocity after the normal shock, which happens to be 161.3 m/s.

17-137 The diffuser of an aircraft is considered. The static pressure rise across the diffuser and the exit area are to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the diffuser is steady, one-dimensional, and isentropic. **3** The diffuser is adiabatic.



Properties Air properties at room temperature are $R = 0.287$ kJ/kg·K, $c_p = 1.005$ kJ/kg·K, and $k = 1.4$ (Table A-2a).

Analysis The inlet velocity is

$$V_1 = \mathrm{Ma}_1 c_1 = M_1\sqrt{kRT_1} = (0.7)\sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(242.7\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 218.6\ \mathrm{m/s}$$

Then the stagnation temperature and pressure at the diffuser inlet become

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 242.7 + \frac{(218.6\ \mathrm{m/s})^2}{2(1.005\ \mathrm{kJ/kg\cdot K})}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 266.5\ \mathrm{K}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (41.1\ \mathrm{kPa})\left(\frac{266.5\ \mathrm{K}}{242.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 57.0\ \mathrm{kPa}$$

For an adiabatic diffuser, the energy equation reduces to $h_{01} = h_{02}$. Noting that $h = c_pT$ and the specific heats are assumed to be constant, we have

$$T_{01} = T_{02} = T_0 = 266.5\ \mathrm{K}$$

The isentropic relation between states 1 and 02 gives

$$P_{02} = P_1\left(\frac{T_{02}}{T_1}\right)^{k/(k-1)} = (41.1\ \mathrm{kPa})\left(\frac{266.5\ \mathrm{K}}{242.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 55.59\ \mathrm{kPa}$$

The exit velocity can be expressed as

$$V_2 = \mathrm{Ma}_2 c_2 = \mathrm{Ma}_2\sqrt{kRT_2} = (0.25)\sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})\,T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 5.01\sqrt{T_2}$$

Thus ,
$$T_2 = T_{02} - \frac{V_2^2}{2c_p} = (266.5) - \frac{5.01^2 T_2\ \mathrm{m^2/s^2}}{2(1.005\ \mathrm{kJ/kg\cdot K})}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) \longrightarrow T_2 = 263.2\ \mathrm{K}$$

Then the static exit pressure becomes

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{k/(k-1)} = (55.59\ \mathrm{kPa})\left(\frac{263.2\ \mathrm{K}}{266.5\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 53.23\ \mathrm{kPa}$$

Thus the static pressure rise across the diffuser is

$$\Delta P = P_2 - P_1 = 53.23 - 41.1 = \mathbf{12.13\ kPa}$$

Also,
$$\rho_2 = \frac{P_2}{RT_2} = \frac{53.23\ \mathrm{kPa}}{(0.287\ \mathrm{kPa\cdot m^3/kg\cdot K})(263.2\ \mathrm{K})} = 0.7047\ \mathrm{kg/m^3}$$

$$V_2 = 6.01\sqrt{T_2} = 5.01\sqrt{263.2} = 81.3\ \mathrm{m/s}$$

Thus,
$$A_2 = \frac{\dot m}{\rho_2 V_2} = \frac{30\ \mathrm{kg/s}}{(0.7047\ \mathrm{kg/m^3})(81.3\ \mathrm{m/s})} = \mathbf{0.524\ m^2}$$

Discussion The pressure rise in actual diffusers will be lower because of the irreversibilities. However, flow through well-designed diffusers is very nearly isentropic.

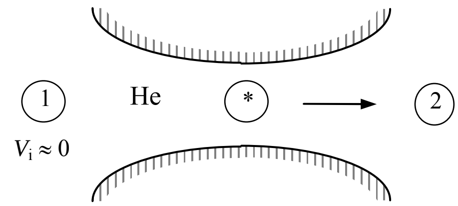
17-138 Helium gas is accelerated in a nozzle isentropically. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of helium are R = 2.0769 kJ/kg.K, c_p = 5.1926 kJ/kg.K, k = 1.667 (Table A-2a).

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

$$T_{01} = T_1 = 500 \text{ K}$$
$$P_{01} = P_1 = 1.0 \text{ MPa}$$



The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 500 \text{ K}$$
$$P_{02} = P_{01} = 1.0 \text{ MPa}$$

The critical pressure and temperature are determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (500\text{ K})\left(\frac{2}{1.667+1}\right) = 375.0\text{ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1.0\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 0.487\text{ MPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{487\text{ kPa}}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(375\text{ K})} = 0.625\text{ kg/m}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(375\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1139.4\text{ m/s}$$

Thus the throat area is

$$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.25\text{ kg/s}}{(0.625\text{ kg/m}^3)(1139.4\text{ m/s})} = 3.51\times10^{-4}\text{ m}^2 = \mathbf{3.51\,cm^2}$$

At the nozzle exit the pressure is P_2 = 0.1 MPa. Then the other properties at the nozzle exit are determined to be

$$\frac{P_0}{P_2} = \left(1+\frac{k-1}{2}\text{Ma}_2^2\right)^{k/(k-1)} \longrightarrow \frac{1.0\text{ MPa}}{0.1\text{ MPa}} = \left(1+\frac{1.667-1}{2}\text{Ma}_2^2\right)^{1.667/0.667}$$

It yields Ma_2 = 2.130, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0\left(\frac{2}{2+(k-1)\text{Ma}_2^2}\right) = (500\text{ K})\left(\frac{2}{2+(1.667-1)\times 2.13^2}\right) = 199.0\text{ K}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{100\text{ kPa}}{(2.0769\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(199\text{ K})} = 0.242\text{ kg/m}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (2.13)\sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(199\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1768.0\text{ m/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.25\text{ kg/s}}{(0.242\text{ kg/m}^3)(1768\text{ m/s})} = 5.84\times10^{-4}\text{ m}^2 = \mathbf{5.84\,cm^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

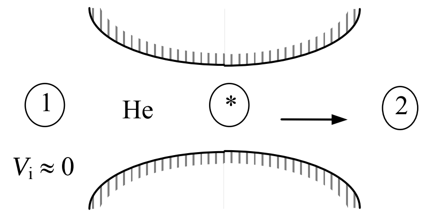
17-139E Helium gas is accelerated in a nozzle. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined for the cases of isentropic and 97% efficient nozzles.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of helium are R = 0.4961 Btu/lbm·R = 2.6809 psia·ft³/lbm·R, c_p = 1.25 Btu/lbm·R, and k = 1.667 (Table A-2Ea).

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

$$T_{01} = T_1 = 900 \text{ R}$$
$$P_{01} = P_1 = 150 \text{ psia}$$



The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 900 \text{ R}$$
$$P_{02} = P_{01} = 150 \text{ psia}$$

The critical pressure and temperature are determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (900 \text{ R})\left(\frac{2}{1.667+1}\right) = 674.9 \text{ R}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (150 \text{ psia})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 73.1 \text{ psia}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{73.1 \text{ psia}}{(2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(674.9 \text{ R})} = 0.0404 \text{ 1bm/ft}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(674.9 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = 3738 \text{ ft/s}$$

and $$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.2 \text{ 1bm/s}}{(0.0404 \text{ 1bm/ft}^3)(3738 \text{ ft/s})} = \mathbf{0.00132 \text{ ft}^2}$$

At the nozzle exit the pressure is P_2 = 15 psia. Then the other properties at the nozzle exit are determined to be

$$\frac{p_0}{p_2} = \left(1 + \frac{k-1}{2}\text{Ma}_2^2\right)^{k/(k-1)} \longrightarrow \frac{150 \text{ psia}}{15 \text{ psia}} = \left(1 + \frac{1.667-1}{2}\text{Ma}_2^2\right)^{1.667/0.667}$$

It yields Ma_2 = 2.130, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0\left(\frac{2}{2+(k-1)\text{Ma}_2^2}\right) = (900 \text{ R})\left(\frac{2}{2+(1.667-1)\times 2.13^2}\right) = 358.1 \text{ R}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{15 \text{ psia}}{(2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(358.1 \text{ R})} = 0.0156 \text{ 1bm/ft}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (2.13)\sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(358.1 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = 5800 \text{ ft/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.2 \text{ lbm/s}}{(0.0156 \text{ lbm/ft}^3)(5800 \text{ ft/s})} = \mathbf{0.00221 \text{ ft}^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

17-140 Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-32 for an ideal gas with $k = 1.667$.

Properties The specific heat ratio of the ideal gas is given to be $k = 1.667$.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma}\sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1}$$

```
k=1.667
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
0.0	0	∞	1.0000	1.0000	1.0000
0.1	0.1153	5.6624	0.9917	0.9950	0.9967
0.2	0.2294	2.8879	0.9674	0.9803	0.9868
0.3	0.3413	1.9891	0.9288	0.9566	0.9709
0.4	0.4501	1.5602	0.8782	0.9250	0.9493
0.5	0.5547	1.3203	0.8186	0.8869	0.9230
0.6	0.6547	1.1760	0.7532	0.8437	0.8928
0.7	0.7494	1.0875	0.6850	0.7970	0.8595
0.8	0.8386	1.0351	0.6166	0.7482	0.8241
0.9	0.9222	1.0081	0.5501	0.6987	0.7873
1.0	1.0000	1.0000	0.4871	0.6495	0.7499
1.2	1.1390	1.0267	0.3752	0.5554	0.6756
1.4	1.2572	1.0983	0.2845	0.4704	0.6047
1.6	1.3570	1.2075	0.2138	0.3964	0.5394
1.8	1.4411	1.3519	0.1603	0.3334	0.4806
2.0	1.5117	1.5311	0.1202	0.2806	0.4284
2.2	1.5713	1.7459	0.0906	0.2368	0.3825
2.4	1.6216	1.9980	0.0686	0.2005	0.3424
2.6	1.6643	2.2893	0.0524	0.1705	0.3073
2.8	1.7007	2.6222	0.0403	0.1457	0.2767
3.0	1.7318	2.9990	0.0313	0.1251	0.2499
5.0	1.8895	9.7920	0.0038	0.0351	0.1071
∝	1.9996	∞	0	0	0

17-141 Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-33 for an ideal gas with $k = 1.667$.

Properties The specific heat ratio of the ideal gas is given to be $k = 1.667$.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k+1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

```
k=1.667
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	2.0530
1.1	0.9131	1.2625	1.1496	1.0982	0.999	2.3308
1.2	0.8462	1.5500	1.2972	1.1949	0.9933	2.6473
1.3	0.7934	1.8626	1.4413	1.2923	0.9813	2.9990
1.4	0.7508	2.2001	1.5805	1.3920	0.9626	3.3838
1.5	0.7157	2.5626	1.7141	1.4950	0.938	3.8007
1.6	0.6864	2.9501	1.8415	1.6020	0.9085	4.2488
1.7	0.6618	3.3627	1.9624	1.7135	0.8752	4.7278
1.8	0.6407	3.8002	2.0766	1.8300	0.8392	5.2371
1.9	0.6227	4.2627	2.1842	1.9516	0.8016	5.7767
2.0	0.6070	4.7503	2.2853	2.0786	0.763	6.3462
2.1	0.5933	5.2628	2.3802	2.2111	0.7243	6.9457
2.2	0.5814	5.8004	2.4689	2.3493	0.6861	7.5749
2.3	0.5708	6.3629	2.5520	2.4933	0.6486	8.2339
2.4	0.5614	6.9504	2.6296	2.6432	0.6124	8.9225
2.5	0.5530	7.5630	2.7021	2.7989	0.5775	9.6407
2.6	0.5455	8.2005	2.7699	2.9606	0.5442	10.3885
2.7	0.5388	8.8631	2.8332	3.1283	0.5125	11.1659
2.8	0.5327	9.5506	2.8923	3.3021	0.4824	11.9728
2.9	0.5273	10.2632	2.9476	3.4819	0.4541	12.8091
3.0	0.5223	11.0007	2.9993	3.6678	0.4274	13.6750
4.0	0.4905	19.7514	3.3674	5.8654	0.2374	23.9530
5.0	0.4753	31.0022	3.5703	8.6834	0.1398	37.1723
∞	0.4473	∞	3.9985	∞	0	∞

17-142 The critical temperature, pressure, and density of an equimolar mixture of oxygen and nitrogen for specified stagnation properties are to be determined.

Assumptions Both oxygen and nitrogen are ideal gases with constant specific heats at room temperature.

Properties The specific heat ratio and molar mass are $k = 1.395$ and $M = 32$ kg/kmol for oxygen, and $k = 1.4$ and $M = 28$ kg/kmol for nitrogen (Tables A-1 and A-2).

Analysis The gas constant of the mixture is

$$M_m = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5 \times 32 + 0.5 \times 28 = 30 \text{ kg/kmol}$$

$$R_m = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol} \cdot \text{K}}{30 \text{ kg/kmol}} = 0.2771 \text{ kJ/kg} \cdot \text{K}$$

The specific heat ratio is 1.4 for nitrogen, and nearly 1.4 for oxygen. Therefore, the specific heat ratio of the mixture is also 1.4. Then the critical temperature, pressure, and density of the mixture become

$$T^* = T_0\left(\frac{2}{k+1}\right) = (600 \text{ K})\left(\frac{2}{1.4+1}\right) = \mathbf{500.0 \ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (300 \text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{158.5 \ kPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{158.5 \text{ kPa}}{(0.2771 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(500 \text{ K})} = \mathbf{1.144 \ kg/m^3}$$

Discussion If the specific heat ratios k of the two gases were different, then we would need to determine the k of the mixture from $k = c_{p,m}/c_{v,m}$ where the specific heats of the mixture are determined from

$$c_{p,m} = \text{mf}_{O_2} c_{p,O_2} + \text{mf}_{N_2} c_{p,N_2} = (y_{O_2} M_{O_2} / M_m) c_{p,O_2} + (y_{N_2} M_{N_2} / M_m) c_{p,N_2}$$

$$c_{v,m} = \text{mf}_{O_2} c_{v,O_2} + \text{mf}_{N_2} c_{v,N_2} = (y_{O_2} M_{O_2} / M_m) c_{v,O_2} + (y_{N_2} M_{N_2} / M_m) c_{v,N_2}$$

where mf is the mass fraction and y is the mole fraction. In this case it would give

$$c_{p,m} = (0.5 \times 32/30) \times 0.918 + (0.5 \times 28/30) \times 1.039 = 0.9745 \text{ kJ/kg.K}$$

$$c_{v,m} = (0.5 \times 32/30) \times 0.658 + (0.5 \times 28/30) \times 0.743 = 0.6977 \text{ kJ/kg.K}$$

and

$$k = 0.9745/0.6977 = 1.397$$

17-143 Using EES (or other) software, the shape of a converging-diverging nozzle is to be determined for specified flow rate and stagnation conditions. The nozzle and the Mach number are to be plotted.

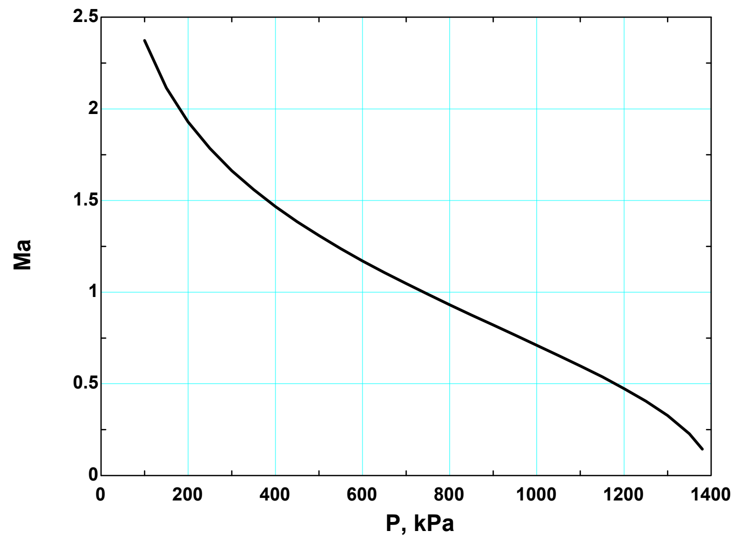
Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

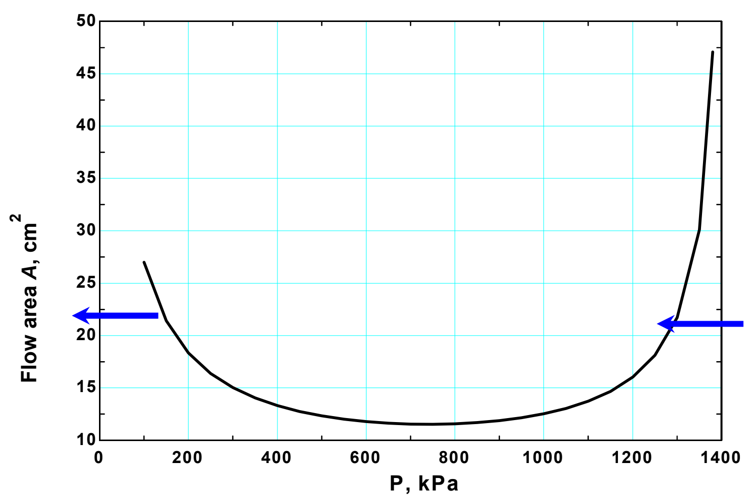
Properties The specific heat ratio of air at room temperature is 1.4 (Table A-2a).

Analysis The problem is solved using EES, and the results are tabulated and plotted below.

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
P0=1400 "kPa"
T0=200+273 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
T=T0*(P/P0)^((k-1)/k)
V=SQRT(2*Cp*(T0-T)*1000)
A=m/(rho*V)*10000 "cm2"
C=SQRT(k*R*T*1000)
Ma=V/C
```

Pressure *P*, kPa	Flow area *A*, cm^2	Mach number Ma
1400	∞	0
1350	30.1	0.229
1300	21.7	0.327
1250	18.1	0.406
1200	16.0	0.475
1150	14.7	0.538
1100	13.7	0.597
1050	13.0	0.655
1000	12.5	0.710
950	12.2	0.766
900	11.9	0.820
850	11.7	0.876
800	11.6	0.931
750	11.5	0.988
700	11.5	1.047
650	11.6	1.107
600	11.8	1.171
550	12.0	1.237
500	12.3	1.308
450	12.8	1.384
400	13.3	1.467
350	14.0	1.559
300	15.0	1.663
250	16.4	1.784
200	18.3	1.929
150	21.4	2.114
100	27.0	2.373





17-144 Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-32 for air.

Properties The specific heat ratio is given to be $k = 1.4$ for air

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma}\sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1}$$

Air:

```
k=1.4
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma^*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.5	1.3646	1.1762	0.2724	0.3950	0.6897
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.5	1.8257	2.6367	0.0585	0.1317	0.4444
3.0	1.9640	4.2346	0.0272	0.0762	0.3571
3.5	2.0642	6.7896	0.0131	0.0452	0.2899
4.0	2.1381	10.7188	0.0066	0.0277	0.2381
4.5	2.1936	16.5622	0.0035	0.0174	0.1980
5.0	2.2361	25.0000	0.0019	0.0113	0.1667
5.5	2.2691	36.8690	0.0011	0.0076	0.1418
6.0	2.2953	53.1798	0.0006	0.0052	0.1220
6.5	2.3163	75.1343	0.0004	0.0036	0.1058
7.0	2.3333	104.1429	0.0002	0.0026	0.0926
7.5	2.3474	141.8415	0.0002	0.0019	0.0816
8.0	2.3591	190.1094	0.0001	0.0014	0.0725
8.5	2.3689	251.0862	0.0001	0.0011	0.0647
9.0	2.3772	327.1893	0.0000	0.0008	0.0581
9.5	2.3843	421.1314	0.0000	0.0006	0.0525
10.0	2.3905	535.9375	0.0000	0.0005	0.0476

17-145 Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-32 for methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma}\sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1}$$

Methane:

```
k=1.3
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma^*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
1.0	1.0000	1.0000	0.5457	0.6276	0.8696
1.5	1.3909	1.1895	0.2836	0.3793	0.7477
2.0	1.6956	1.7732	0.1305	0.2087	0.6250
2.5	1.9261	2.9545	0.0569	0.1103	0.5161
3.0	2.0986	5.1598	0.0247	0.0580	0.4255
3.5	2.2282	9.1098	0.0109	0.0309	0.3524
4.0	2.3263	15.9441	0.0050	0.0169	0.2941
4.5	2.4016	27.3870	0.0024	0.0095	0.2477
5.0	2.4602	45.9565	0.0012	0.0056	0.2105
5.5	2.5064	75.2197	0.0006	0.0033	0.1806
6.0	2.5434	120.0965	0.0003	0.0021	0.1563
6.5	2.5733	187.2173	0.0002	0.0013	0.1363
7.0	2.5978	285.3372	0.0001	0.0008	0.1198
7.5	2.6181	425.8095	0.0001	0.0006	0.1060
8.0	2.6350	623.1235	0.0000	0.0004	0.0943
8.5	2.6493	895.5077	0.0000	0.0003	0.0845
9.0	2.6615	1265.6040	0.0000	0.0002	0.0760
9.5	2.6719	1761.2133	0.0000	0.0001	0.0688
10.0	2.6810	2416.1184	0.0000	0.0001	0.0625

17-146 Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-33 for air.

Properties The specific heat ratio is given to be $k = 1.4$ for air.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\text{Ma}_2 = \sqrt{\frac{(\;\;-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1}$$

$$\frac{T_2}{T_1} = \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2} = \frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}} = \frac{\text{Ma}_1}{\text{Ma}_2}\left[\frac{1 + \text{Ma}_2^2(k-1)/2}{1 + \text{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\text{Ma}_1^2)[1 + \text{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\text{Ma}_2^2}$$

Air:

```
k=1.4
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8929
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.5	0.5130	7.1250	3.3333	2.1375	0.499	8.5261
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
3.5	0.4512	14.1250	4.2609	3.3151	0.2129	16.2420
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
4.5	0.4236	23.4583	4.8119	4.8751	0.0917	26.5387
5.0	0.4152	29.0000	5.0000	5.8000	0.06172	32.6535
5.5	0.4090	35.1250	5.1489	6.8218	0.04236	39.4124
6.0	0.4042	41.8333	5.2683	7.9406	0.02965	46.8152
6.5	0.4004	49.1250	5.3651	9.1564	0.02115	54.8620
7.0	0.3974	57.0000	5.4444	10.4694	0.01535	63.5526
7.5	0.3949	65.4583	5.5102	11.8795	0.01133	72.8871
8.0	0.3929	74.5000	5.5652	13.3867	0.008488	82.8655
8.5	0.3912	84.1250	5.6117	14.9911	0.006449	93.4876
9.0	0.3898	94.3333	5.6512	16.6927	0.004964	104.7536
9.5	0.3886	105.1250	5.6850	18.4915	0.003866	116.6634
10.0	0.3876	116.5000	5.7143	20.3875	0.003045	129.2170

17-147 Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-33 for methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k+1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

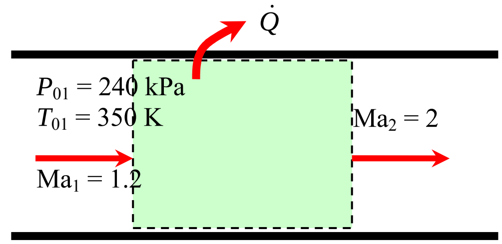
$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

Methane:

```
k=1.3
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8324
1.5	0.6942	2.4130	1.9346	1.2473	0.9261	3.2654
2.0	0.5629	4.3913	2.8750	1.5274	0.7006	5.3700
2.5	0.4929	6.9348	3.7097	1.8694	0.461	8.0983
3.0	0.4511	10.0435	4.4043	2.2804	0.2822	11.4409
3.5	0.4241	13.7174	4.9648	2.7630	0.1677	15.3948
4.0	0.4058	17.9565	5.4118	3.3181	0.09933	19.9589
4.5	0.3927	22.7609	5.7678	3.9462	0.05939	25.1325
5.0	0.3832	28.1304	6.0526	4.6476	0.03613	30.9155
5.5	0.3760	34.0652	6.2822	5.4225	0.02243	37.3076
6.0	0.3704	40.5652	6.4688	6.2710	0.01422	44.3087
6.5	0.3660	47.6304	6.6218	7.1930	0.009218	51.9188
7.0	0.3625	55.2609	6.7485	8.1886	0.006098	60.1379
7.5	0.3596	63.4565	6.8543	9.2579	0.004114	68.9658
8.0	0.3573	72.2174	6.9434	10.4009	0.002827	78.4027
8.5	0.3553	81.5435	7.0190	11.6175	0.001977	88.4485
9.0	0.3536	91.4348	7.0837	12.9079	0.001404	99.1032
9.5	0.3522	101.8913	7.1393	14.2719	0.001012	110.367
10.0	0.3510	112.9130	7.1875	15.7096	0.000740	122.239

17-148 Air flowing at a supersonic velocity in a duct is accelerated by cooling. For a specified exit Mach number, the rate of heat transfer is to be determined.



Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K (Table A-2a).

Analysis Knowing stagnation properties, the static properties are determined to be

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (350\text{ K})\left(1+\frac{1.4-1}{2}1.2^2\right)^{-1} = 271.7\text{ K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (240\text{ kPa})\left(1+\frac{1.4-1}{2}1.2^2\right)^{-1.4/0.4} = 98.97\text{ kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{98.97\text{ kPa}}{(0.287\text{ kJ/kgK})(271.7\text{ K})} = 1.269\text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(271.7\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 330.4\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.2(330.4\text{ m/s}) = 396.5\text{ m/s}$$

$$\dot{m}_{\text{air}} = \rho_1 A_{c1} V_1 = (1.269\text{ kg/m}^3)[\pi(0.20\text{ m})^2/4](330.4\text{ m/s}) = 15.81\text{ kg/s}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-34):

$\mathrm{Ma}_1 = 1.8$: $\quad T_{01}/T_0^* = 0.9787$

$\mathrm{Ma}_2 = 2$: $\quad T_{02}/T_0^* = 0.7934$

Then the exit stagnation temperature is determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.7934}{0.9787} = 0.8107 \quad\rightarrow\quad T_{02} = 0.8107T_{01} = 0.8107(350\text{ K}) = 283.7\text{ K}$$

Finally, the rate of heat transfer is

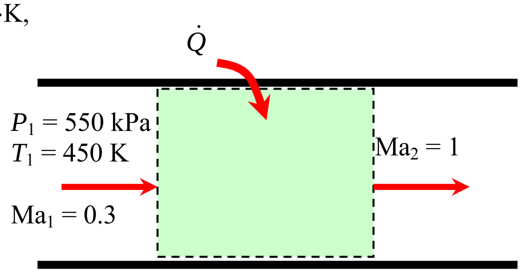
$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (15.81\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(283.7 - 350)\text{ K} = \mathbf{-1053\ kW}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 158 K at the exit, which is extremely low. Therefore, the duct may need to be heavily insulated to maintain indicated flow conditions.

17-149 Air flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K (Table A-2a).



Analysis Heat transfer will stop when the flow is choked, and thus $Ma_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{550 \text{ kPa}}{(0.287 \text{ kJ/kgK})(450 \text{ K})} = 4.259 \text{ kg/m}^3$$

$$T_{01} = T_1\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right) = (450 \text{ K})\left(1 + \frac{1.4\text{-}1}{2}0.3^2\right) = 458.1 \text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(450 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 425.2 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.3(425.2 \text{ m/s}) = 127.6 \text{ m/s}$$

$$\dot{m}_{\text{air}} = \rho_1 A_{c1} V_1 = (4.259 \text{ kg/m}^3)(0.08 \times 0.08 \text{ m}^2)(127.6 \text{ m/s}) = 3.477 \text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $Ma_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\text{Ma}_1^2[2+(k-1)\text{Ma}_1^2]}{(1+k\text{Ma}_1^2)^2} = \frac{(1.4+1)0.3^2[2+(1.4-1)0.3^2]}{(1+1.4\times 0.3^2)^2} = 0.3469$$ Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.3469} \rightarrow T_{02} = T_{01}/0.3469 = (458.1 \text{ K})/0.3469 = 1320.7 \text{ K}$$

Then the rate of heat transfer becomes

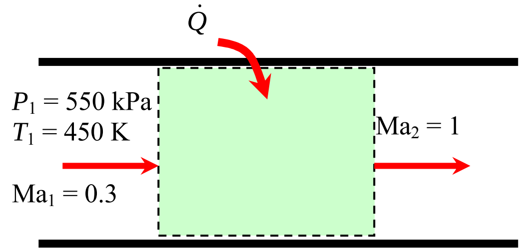
$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (3.477 \text{ kg/s})(1.005 \text{ kJ/kg}\cdot\text{K})(1320.7 - 458.1) \text{ K} = \mathbf{3014 \text{ kW}}$$

Discussion It can also be shown that $T_2 = 1101$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. We can also solve this problem using the Rayleigh function values listed in Table A-34.

17-150 Helium flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of helium to be $k = 1.667$, $c_p = 5.193$ kJ/kg·K, and $R = 2.077$ kJ/kg·K (Table A-2a).



Analysis Heat transfer will stop when the flow is choked, and thus $\text{Ma}_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{550 \text{ kPa}}{(2.077 \text{ kJ/kgK})(450 \text{ K})} = 0.5885 \text{ kg/m}^3$$

$$T_{01} = T_1\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right) = (450 \text{ K})\left(1 + \frac{1.667 - 1}{2}0.3^2\right) = 463.5 \text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.667)(2.077 \text{ kJ/kg}\cdot\text{K})(450 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 1248 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.3(1248 \text{ m/s}) = 374.5 \text{ m/s}$$

$$\dot{m}_{\text{air}} = \rho_1 A_{c1} V_1 = (0.5885 \text{ kg/m}^3)(0.08 \times 0.08 \text{ m}^2)(374.5 \text{ m/s}) = 1.410 \text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since $\text{Ma}_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\text{Ma}_1^2[2 + (k-1)\text{Ma}_1^2]}{(1 + k\text{Ma}_1^2)^2} = \frac{(1.667+1)0.33^2[2 + (1.667-1)0.3^2]}{(1 + 1.667 \times 0.3^2)^2} = 0.3739$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.3739} \quad \rightarrow \quad T_{02} = T_{01}/0.3739 = (463.5 \text{ K})/0.3739 = 1239.8 \text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (1.410 \text{ kg/s})(5.193 \text{ kJ/kg}\cdot\text{K})(1239.8 - 463.5) \text{ K} = \mathbf{5685 \text{ kW}}$$

Discussion It can also be shown that $T_2 = 930$ K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-34 since they are based on $k = 1.4$.

17-151 Air flowing at a subsonic velocity in a duct is accelerated by heating. For a specified exit Mach number, the heat transfer for a specified exit Mach number as well as the maximum heat transfer are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K (Table A-2a).

Analysis The inlet Mach number and stagnation temperature are

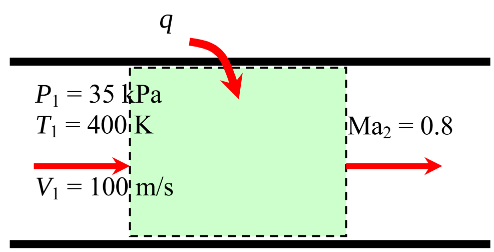
$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(400\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 400.9\ \text{m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{100\ \text{m/s}}{400.9\ \text{m/s}} = 0.2494$$

$$T_{01} = T_1\left(1 + \frac{k-1}{2}\text{Ma}_1^2\right)$$

$$= (400\ \text{K})\left(1 + \frac{1.4-1}{2}0.2494^2\right)$$

$$= 405.0\ \text{K}$$



The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-34):

$\text{Ma}_1 = 0.2494$: $T_{01}/T^* = 0.2559$

$\text{Ma}_2 = 0.8$: $T_{02}/T^* = 0.9639$

Then the exit stagnation temperature and the heat transfer are determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.2559} = 3.7667 \rightarrow \quad T_{02} = 3.7667T_{01} = 3.7667(405.0\ \text{K}) = 1526\ \text{K}$$

$$q = c_p(T_{02} - T_{01}) = (1.005\ \text{kJ/kg}\cdot\text{K})(1526 - 405)\ \text{K} = \mathbf{1126\ kJ/kg}$$

Maximum heat transfer will occur when the flow is choked, and thus Ma_2 = 1 and thus T_{02}/T^* = 1. Then,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.2559} \rightarrow \quad T_{02} = T_{01}/0.2559 = 405.0\ \text{K})/0.2559 = 1583\ \text{K}$$

$$q_{\max} = c_p(T_{02} - T_{01}) = (1.005\ \text{kJ/kg}\cdot\text{K})(1583 - 405)\ \text{K} = \mathbf{1184\ kJ/kg}$$

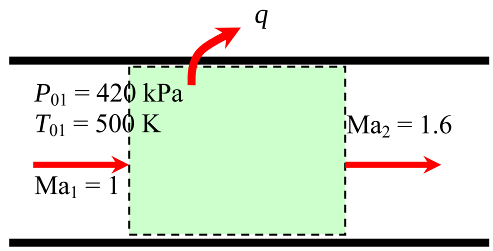
Discussion This is the maximum heat that can be transferred to the gas without affecting the mass flow rate. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease.

17-152 Air flowing at sonic conditions in a duct is accelerated by cooling. For a specified exit Mach number, the amount of heat transfer per unit mass is to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K (Table A-2a).

Analysis Noting that $\mathrm{Ma}_1 = 1$, the inlet stagnation temperature is



$$T_{01} = T_1\left(1 + \frac{k-1}{2}\mathrm{Ma}_1^2\right)$$

$$= (500\ \mathrm{K})\left(1 + \frac{1.4\text{-}1}{2}1^2\right) = 600\ \mathrm{K}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-34):

$\mathrm{Ma}_1 = 1$: $T_{01}/T_0^* = 1$

$\mathrm{Ma}_2 = 1.6$: $T_{02}/T_0^* = 0.8842$

Then the exit stagnation temperature and heat transfer are determined to be

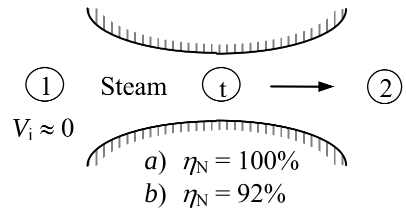
$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.8842}{1} = 0.8842 \qquad \to \quad T_{02} = 0.8842T_{01} = 0.8842(600\ \mathrm{K}) = 530.5\ \mathrm{K}$$

$$q = c_p(T_{02} - T_{01}) = (1.005\ \mathrm{kJ/kg\cdot K})(530.5 - 600)\ \mathrm{K} = \mathbf{-69.8\ kJ/kg}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 351 K at the exit

17-153 Saturated steam enters a converging-diverging nozzle with a low velocity. The throat area, exit velocity, mass flow rate, and exit Mach number are to be determined for isentropic and 90 percent efficient nozzle cases.

Assumptions **1** Flow through the nozzle is steady and one-dimensional. **2** The nozzle is adiabatic.



Analysis (*a*) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{10} = h_1$. At the inlet,

$$h_1 = (h_f + x_1 h_{fg})_{@1.75\text{ MPa}} = 878.16 + 0.90 \times 1917.1 = 2603.5\text{ kJ/kg}$$

$$s_1 = (s_f + x_1 s_{fg})_{@1.75\text{ MPa}} = 2.3844 + 0.90 \times 4.0033 = 5.9874\text{ kJ/kg}\cdot\text{K}$$

At the exit, $P_2 = 1.2$ MPa and $s_2 = s_{2s} = s_1 = 5.9874$ kJ/kg·K. Thus,

$$s_2 = s_f + x_2 s_{fg} \rightarrow 5.9874 = 2.2159 + x_2(4.3058) \rightarrow x_2 = 0.8759$$

$$h_2 = h_f + x_2 h_{fg} = 798.33 + 0.8759 \times 1985.4 = 2537.4\text{ kJ/kg}$$

$$\boldsymbol{v}_2 = \boldsymbol{v}_f + x_2 \boldsymbol{v}_{fg} = 0.001138 + 0.8759 \times (0.16326 - 0.001138) = 0.14314\text{ m}^3/\text{kg}$$

Then the exit velocity is determined from the steady-flow energy balance to be

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \rightarrow \quad 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2603.5 - 2537.4)\text{kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{363.7\ m/s}$$

The mass flow rate is determined from

$$\dot{m} = \frac{1}{\boldsymbol{v}_2} A_2 V_2 = \frac{1}{0.14314\text{ m}^3/\text{kg}}(25 \times 10^{-4}\text{ m}^2)(363.7\text{ m/s}) = \mathbf{6.35\ kg/s}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial r}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at $s_2 = 5.9874$ kJ/kg·K and at pressures just below and just above the specified pressure (1.1 and 1.3 MPa) are determined to be 0.1547 and 0.1333 m^3/kg. Substituting,

$$c_2 = \sqrt{\frac{(1300 - 1100)\text{kPa}}{\left(\frac{1}{0.1333} - \frac{1}{0.1547}\right)\text{kg/m}^3}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kPa}\cdot\text{m}^3}\right)} = 438.9\text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{363.7\text{ m/s}}{438.9\text{ m/s}} = \mathbf{0.829}$$

The steam is saturated, and thus the critical pressure which occurs at the throat is taken to be

$$P_t = P^* = 0.576 \times P_{01} = 0.576 \times 1.75 = 1.008\text{ MPa}$$

Then at the throat,

$$P_t = 1.008\text{ MPa} \quad \text{and} \quad s_t = s_1 = 5.9874\text{ kJ/kg}\cdot\text{K}$$

Thus,

$$h_t = 2507.7 \text{ kJ/kg}$$
$$\boldsymbol{v}_t = 0.1672 \text{ m}^3/\text{kg}$$

Then the throat velocity is determined from the steady-flow energy balance,

$$h_1 + \frac{V_1^2 \nearrow^0}{2} = h_t + \frac{V_t^2}{2} \rightarrow 0 = h_t - h_1 + \frac{V_t^2}{2}$$

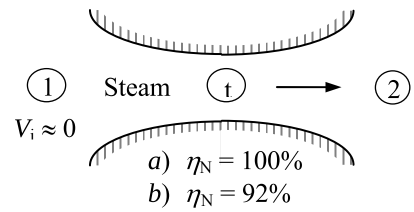
Solving for V_t,

$$V_t = \sqrt{2(h_1 - h_t)} = \sqrt{2(2603.5 - 2507.7)\text{kJ/kg}\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 437.7 \text{ m/s}$$

Thus the throat area is

$$A_t = \frac{\dot{m}\boldsymbol{v}_t}{V_t} = \frac{(6.35 \text{ kg/s})(0.1672 \text{ m}^3/\text{kg})}{437.7 \text{ m/s}} = 24.26\times10^{-4} \text{ m}^2 = \mathbf{24.26\ cm^2}$$

(*b*) The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Thus $h_{10} = h_1$. At the inlet,



$$h_1 = (h_f + x_1 h_{fg})_{@1.75 \text{ MPa}} = 878.16 + 0.90\times1917.1 = 2603.5 \text{ kJ/kg}$$
$$s_1 = (s_f + x_1 s_{fg})_{@1.75 \text{ MPa}} = 2.3844 + 0.90\times4.0033 = 5.9874 \text{ kJ/kg}\cdot\text{K}$$

At state 2s, P_2 = 1.2 MPa and $s_2 = s_{2s} = s_1$ = 5.9874 kJ/kg·K. Thus,

$$s_{2s} = s_f + x_2 s_{fg} \rightarrow 5.9874 = 2.2159 + x_2(4.3058) \rightarrow x_{2s} = 0.8759$$
$$h_{2s} = h_f + x_2 h_{fg} = 798.33 + 0.8759\times1985.4 = 2537.4 \text{ kJ/kg}$$

The enthalpy of steam at the actual exit state is determined from

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \longrightarrow 0.92 = \frac{2603.5 - h_2}{2603.4 - 2537.4} \longrightarrow h_2 = 2542.7 \text{ kJ/kg}$$

Therefore at the exit, P_2 = 1.2 MPa and h_2 = 2542.7 kJ/kg·K. Thus,

$$h_2 = h_f + x_2 h_{fg} \longrightarrow 2542.7 = 798.33 + x_2(1985.4) \longrightarrow x_2 = 0.8786$$
$$s_2 = s_f + x_2 s_{fg} = 2.2159 + 0.8786\times4.3058 = 5.9989$$
$$\boldsymbol{v}_2 = \boldsymbol{v}_f + x_2\boldsymbol{v}_{fg} = 0.001138 + 0.8786\times(0.16326 - 0.001138) = 0.1436 \text{ kJ/kg}$$

Then the exit velocity is determined from the steady-flow energy balance to be

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \rightarrow 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

Solving for V_2,

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2603.5 - 2542.7)\text{kJ/kg}\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{348.9\ m/s}$$

The mass flow rate is determined from

$$\dot{m} = \frac{1}{\boldsymbol{v}_2}A_2V_2 = \frac{1}{0.1436 \text{ m}^3/\text{kg}}(25\times10^{-4} \text{ m}^2)(348.9 \text{ m/s}) = \mathbf{6.07\ kg/s}$$

The velocity of sound at the exit of the nozzle is determined from

$$c = \left(\frac{\partial P}{\partial \rho}\right)_s^{1/2} \cong \left(\frac{\Delta P}{\Delta(1/\boldsymbol{v})}\right)_s^{1/2}$$

The specific volume of steam at s_2 = 5.9989 kJ/kg·K and at pressures just below and just above the specified pressure (1.1 and 1.3 MPa) are determined to be 0.1551 and 0.1337 m^3/kg. Substituting,

$$c_2 = \sqrt{\frac{(1300-1100)\text{kPa}}{\left(\frac{1}{0.1337}-\frac{1}{0.1551}\right)\text{kg/m}^3}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kPa}\cdot\text{m}^3}\right)} = 439.7\text{ m/s}$$

Then the exit Mach number becomes

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{348.9\text{ m/s}}{439.7\text{ m/s}} = \mathbf{0.793}$$

The steam is saturated, and thus the critical pressure which occurs at the throat is taken to be

$$P_t = P^* = 0.576 \times P_{01} = 0.576 \times 1.75 = 1.008\text{ MPa}$$

At state 2ts, P_{ts} = 1.008 MPa and $s_{ts} = s_1$ = 5.9874 kJ/kg·K. Thus, h_{ts} = 2507.7 kJ/kg.

The actual enthalpy of steam at the throat is

$$\eta_N = \frac{h_{01}-h_t}{h_{01}-h_{ts}} \longrightarrow 0.92 = \frac{2603.5-h_t}{2603.5-2507.7} \longrightarrow h_t = 2515.4\text{ kJ/kg}$$

Therefore at the throat, $P_2 = 1.008\text{ MPa}$ and $h_t = 2515.4\text{ kJ/kg}$. Thus, v_t = 0.1679 m^3/kg.

Then the throat velocity is determined from the steady-flow energy balance,

$$h_1 + \frac{V_1^2 \nearrow 0}{2} = h_t + \frac{V_t^2}{2} \rightarrow 0 = h_t - h_1 + \frac{V_t^2}{2}$$

Solving for V_t,

$$V_t = \sqrt{2(h_1-h_t)} = \sqrt{2(2603.5-2515.4)\text{kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 419.9\text{ m/s}$$

Thus the throat area is

$$A_t = \frac{\dot{m}v_t}{V_t} = \frac{(6.07\text{ kg/s})(0.1679\text{ m}^3/\text{kg})}{419.9\text{ m/s}} = 24.30\times10^{-4}\text{ m}^2 = \mathbf{24.30\text{ cm}^2}$$

Fundamentals of Engineering (FE) Exam Problems

17-154 An aircraft is cruising in still air at 5°C at a velocity of 400 m/s. The air temperature at the nose of the aircraft where stagnation occurs is

(a) 5°C (b) 25°C (c) 55°C (d) 80°C (e) 85°C

Answer (e) 85°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
T1=5 "C"
Vel1= 400 "m/s"
T1_stag=T1+Vel1^2/(2*Cp*1000)

"Some Wrong Solutions with Common Mistakes:"
W1_Tstag=T1 "Assuming temperature rise"
W2_Tstag=Vel1^2/(2*Cp*1000) "Using just the dynamic temperature"
W3_Tstag=T1+Vel1^2/(Cp*1000) "Not using the factor 2"
```

17-155 Air is flowing in a wind tunnel at 25°C, 80 kPa, and 250 m/s. The stagnation pressure at a probe inserted into the flow stream is

(a) 87 kPa (b) 93 kPa (c) 113 kPa (d) 119 kPa (e) 125 kPa

Answer (c) 113 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
T1=25 "K"
P1=80 "kPa"
Vel1= 250 "m/s"
T1_stag=(T1+273)+Vel1^2/(2*Cp*1000) "C"
T1_stag/(T1+273)=(P1_stag/P1)^((k-1)/k)

"Some Wrong Solutions with Common Mistakes:"
T11_stag/T1=(W1_P1stag/P1)^((k-1)/k);   T11_stag=T1+Vel1^2/(2*Cp*1000)  "Using deg. C for temperatures"
T12_stag/(T1+273)=(W2_P1stag/P1)^((k-1)/k);   T12_stag=(T1+273)+Vel1^2/(Cp*1000)  "Not using the factor 2"
T13_stag/(T1+273)=(W3_P1stag/P1)^(k-1);   T13_stag=(T1+273)+Vel1^2/(2*Cp*1000)  "Using wrong isentropic relation"
```

17-156 An aircraft is reported to be cruising in still air at -20°C and 40 kPa at a Mach number of 0.86. The velocity of the aircraft is

(a) 91 m/s (b) 220 m/s (c) 186 m/s (d) 280 m/s (e) 378 m/s

Answer (d) 280 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
T1=-20+273 "K"
P1=40 "kPa"
Mach=0.86
VS1=SQRT(k*R*T1*1000)
Mach=Vel1/VS1

"Some Wrong Solutions with Common Mistakes:"
W1_vel=Mach*VS2; VS2=SQRT(k*R*T1) "Not using the factor 1000"
W2_vel=VS1/Mach "Using Mach number relation backwards"
W3_vel=Mach*VS3; VS3=k*R*T1 "Using wrong relation"
```

17-157 Air is flowing in a wind tunnel at 12°C and 66 kPa at a velocity of 230 m/s. The Mach number of the flow is

(a) 0.54 (b) 0.87 (c) 3.3 (d) 0.36 (e) 0.68

Answer (e) 0.68

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
T1=12+273 "K"
P1=66 "kPa"
Vel1=230 "m/s"
VS1=SQRT(k*R*T1*1000)
Mach=Vel1/VS1

"Some Wrong Solutions with Common Mistakes:"
W1_Mach=Vel1/VS2; VS2=SQRT(k*R*(T1-273)*1000) "Using C for temperature"
W2_Mach=VS1/Vel1 "Using Mach number relation backwards"
W3_Mach=Vel1/VS3; VS3=k*R*T1 "Using wrong relation"
```

17-158 Consider a converging nozzle with a low velocity at the inlet and sonic velocity at the exit plane. Now the nozzle exit diameter is reduced by half while the nozzle inlet temperature and pressure are maintained the same. The nozzle exit velocity will

(a) remain the same. (b) double. (c) quadruple. (d) go down by half. (e) go down to one-fourth.

Answer (a) remain the same.

17-159 Air is approaching a converging-diverging nozzle with a low velocity at 12°C and 200 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of air at the throat of the nozzle is

(a) 338 m/s (b) 309 m/s (c) 280 m/s (d) 256 m/s (e) 95 m/s

Answer (b) 309 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
"Properties at the inlet"
T1=12+273 "K"
P1=200 "kPa"
Vel1=0 "m/s"
To=T1 "since velocity is zero"
Po=P1
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)

"Some Wrong Solutions with Common Mistakes:"
W1_Vthroat=SQRT(k*R*T1*1000) "Using T1 for temperature"
W2_Vthroat=SQRT(k*R*T2_throat*1000); T2_throat=2*(To-273)/(k+1) "Using C for temperature"
W3_Vthroat=k*R*T_throat "Using wrong relation"
```

17-160 Argon gas is approaching a converging-diverging nozzle with a low velocity at 20°C and 120 kPa, and it leaves the nozzle at a supersonic velocity. If the cross-sectional area of the throat is 0.015 m^2, the mass flow rate of argon through the nozzle is

(a) 0.41 kg/s (b) 3.4 kg/s (c) 5.3 kg/s (d) 17 kg/s (e) 22 kg/s

Answer (c) 5.3 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.667
Cp=0.5203 "kJ/kg.K"
R=0.2081 "kJ/kg.K"
A=0.015 "m^2"
"Properties at the inlet"
T1=20+273 "K"
P1=120 "kPa"
Vel1=0 "m/s"
To=T1 "since velocity is zero"
Po=P1
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
rho_throat=P_throat/(R*T_throat)
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)
m=rho_throat*A*V_throat

"Some Wrong Solutions with Common Mistakes:"
W1_mass=rho_throat*A*V1_throat;  V1_throat=SQRT(k*R*T1_throat*1000); T1_throat=2*(To-273)/(k+1) "Using C for temp"
W2_mass=rho2_throat*A*V_throat; rho2_throat=P1/(R*T1)  "Using density at inlet"
```

17-161 Carbon dioxide enters a converging-diverging nozzle at 60 m/s, 310°C, and 300 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of carbon dioxide at the throat of the nozzle is

(a) 125 m/s (b) 225 m/s (c) 312 m/s (d) 353 m/s (e) 377 m/s

Answer (d) 353 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
"Properties at the inlet"
T1=310+273 "K"
P1=300 "kPa"
Vel1=60 "m/s"
To=T1+Vel1^2/(2*Cp*1000)
To/T1=(Po/P1)^((k-1)/k)
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)

"Some Wrong Solutions with Common Mistakes:"
W1_Vthroat=SQRT(k*R*T1*1000) "Using T1 for temperature"
W2_Vthroat=SQRT(k*R*T2_throat*1000); T2_throat=2*(T_throat-273)/(k+1) "Using C for temperature"
W3_Vthroat=k*R*T_throat "Using wrong relation"
```

17-162 Consider gas flow through a converging-diverging nozzle. Of the five statements below, select the one that is incorrect:

(a) The fluid velocity at the throat can never exceed the speed of sound.

(b) If the fluid velocity at the throat is below the speed of sound, the diversion section will act like a diffuser.

(c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.

(d) There will be no flow through the nozzle if the back pressure equals the stagnation pressure.

(e) The fluid velocity decreases, the entropy increases, and stagnation enthalpy remains constant during flow through a normal shock.

Answer (c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.

17-163 Combustion gases with k = 1.33 enter a converging nozzle at stagnation temperature and pressure of 350°C and 400 kPa, and are discharged into the atmospheric air at 20°C and 100 kPa. The lowest pressure that will occur within the nozzle is

(a) 13 kPa (b) 100 kPa (c) 216 kPa (d) 290 kPa (e) 315 kPa

Answer (c) 216 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.33
Po=400 "kPa"
"The critical pressure is"
P_throat=Po*(2/(k+1))^(k/(k-1))
"The lowest pressure that will occur in the nozzle is the higher of the critical or atmospheric pressure."

"Some Wrong Solutions with Common Mistakes:"
W2_Pthroat=Po*(1/(k+1))^(k/(k-1)) "Using wrong relation"
W3_Pthroat=100 "Assuming atmospheric pressure"
```

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