سوال اول-

A copper sphere initially at a uniform temperature of 132°C is suddenly released at the bottom of a large bath of bismuth at 500°C. The sphere diameter is 1 cm, and it rises through the bath at 1 m/s. How far will the sphere rise before its center temperature is 300°C? What is its surface temperature at that point? (The sphere has a thin nickel plating to protect the copper from the bismuth.)

Copper sphere, 1-cm diameter
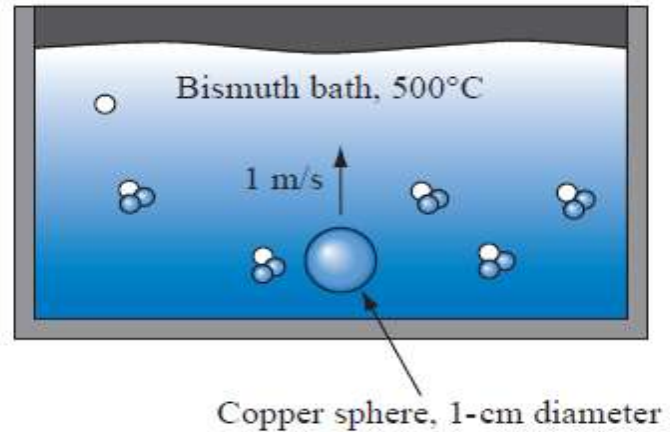
جواب سوال اول

(a) The Biot number for the sphere is

$$Bi = \frac{\bar{h}_c r}{k_s} = \frac{(2.22 \times 10^4 \, \text{W}/(\text{m}^2\text{K}))(0.005\,\text{m})}{(388 \, \text{W}/(\text{mK}))} = 0.287 > 0.1$$

Therefore, internal thermal resistance is significant and the chart solutions of Figure 2.39 must be used.

$$\frac{T(0,t) - T_\infty}{T_o - T_\infty} = \frac{300°C - 500°C}{132°C - 500°C} = 0.543$$

and

$$\frac{1}{Bi} = \frac{1}{0.287} = 3.48$$

From Figure 2.39

$$Fo = \frac{\alpha t}{r_o^2} = 0.75$$

$$\therefore \quad t = 0.75 \frac{r_o^2}{\alpha} = 0.75 \frac{(0.005\,\text{m})^2}{(116.6 \times 10^{-6} \, \text{m}^2/\text{s})} = 0.16 \, \text{s}$$

The distance (x) the sphere will rise during this time is

$$x = U_\infty t = 1 \, \text{m/s} \, (0.16\text{s}) = 0.16 \, \text{m} = 16 \, \text{cm}$$

(b) The surface temperature can be determined from Figure 2.39

$$\frac{1}{Bi} = 3.48 \text{ and } \frac{r}{r_o} = 1 \quad \Rightarrow \quad \frac{T(r_o,t) - T_\infty}{T(0,t) - T_\infty} = 0.84$$
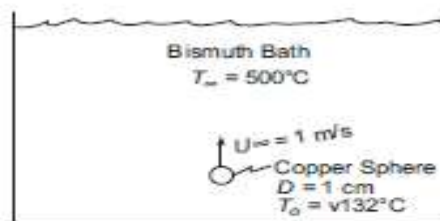
$$T(r_o,t) = 0.86 \, (300°C - 500°C) + 500°C = 332°C$$

ASSUMPTIONS

- Thermal resistance of the nickel plating is negligible
- Thermal properties of the copper can be considered uniform and constant

SKETCH

Bismuth Bath
$T_\infty = 500°C$

$U_\infty = 1$ m/s
Copper Sphere
$D = 1$ cm
$T_0 = v132°C$

PROPERTIES AND CONSTANTS

From Appendix 2, Table 24, for Bismuth at the initial film temperature of 316°C

Thermal conductivity $(k_b) = 16.44$ W/(m K)

Kinematic viscosity $(\nu) = 1.57 \times 10^{-7}$ m²/s

Prandtl number $(Pr) = 0.014$

From Appendix 2, Table 12, for copper

Thermal conductivity $(k_c) = 388$ W/(m K) at its mean temperature of 216°C

Specific heat $(c) = 383$ J/(kg K) at 20°C

Density $(\rho) = 8933$ kg/m³ at 20°C

Thermal diffusivity $(\alpha) = 116.6 \times 10^{-6}$ m²/s

SOLUTION

The Reynolds number is

$$Re_D = \frac{U_\infty D}{\nu} = \frac{(1\,\text{m/s})(0.01\,\text{m})}{(1.57 \times 10^{-7}\,\text{m}^2/\text{s})} = 6.37 \times 10^4$$

Applying Equation (7.14)

$$\overline{Nu}_D = 2 + 0.386\,(RePr)^{\frac{1}{2}} = 2 + 0.386\left[6.37 \times 10^4\,(0.014)\right]^{\frac{1}{2}} = 13.52$$

$$\overline{h}_c = \overline{Nu}_D \frac{k}{D} = 13.53\,\frac{(16.44\,\text{W/(m K)})}{0.01\,\text{m}} = 2.22 \times 10^4\,\text{W/(m}^2\,\text{K)}$$

سوال دوم

A turbine blade 6.3 cm long, with cross-sectional area $A = 4.6 \times 10^{-4}$ m^2 and perimeter $P = 0.12$ m, is made of stainless steel ($k = 18$ W/m K). The temperature of the root, T_s, is 482°C. The blade is exposed to a hot gas at 871°C, and the heat transfer coefficient $\bar{h}$ is 454 W/m^2 K. Determine the temperature of the blade tip and the rate of heat flow at the root of the blade. Assume that the tip is insulated.
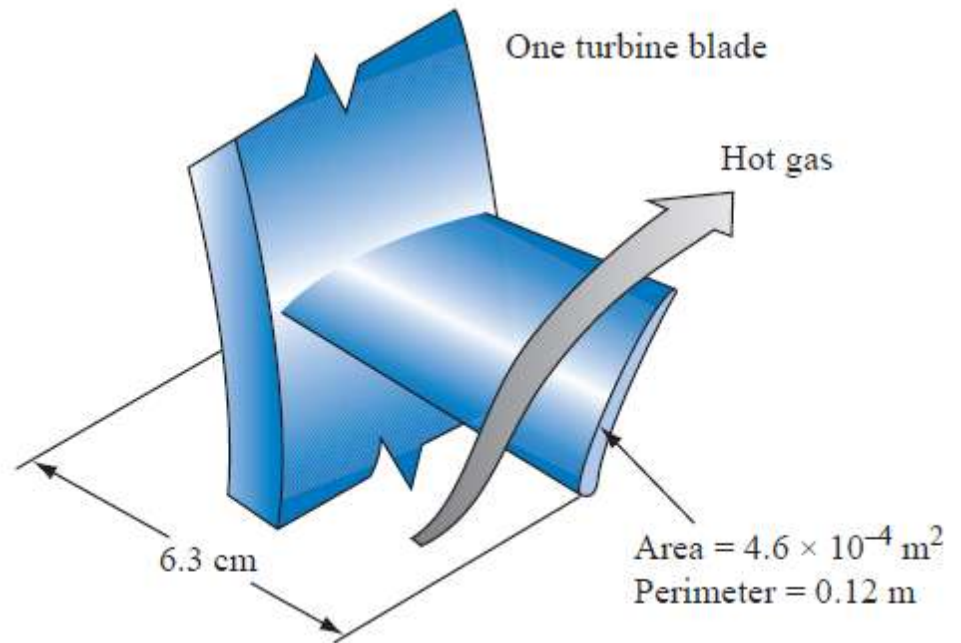
One turbine blade

Hot gas

6.3 cm

Area $= 4.6 \times 10^{-4}$ m^2
Perimeter $= 0.12$ m

جواب سوال دوم

GIVEN

- Stainless steel turbine blade
- Length of blade $(L) = 6.3$ cm $= 0.063$ m
- Cross-sectional area $(A) = 4.6 \times 10^{-4}$ m^2
- Perimeter $(P) = 0.12$ m
- Thermal conductivity $(k) = 18$ W/(m K)
- Temperature of the root $(T_s) = 482°C$
- Temperature of the hot gas $(T_\infty) = 871°C$
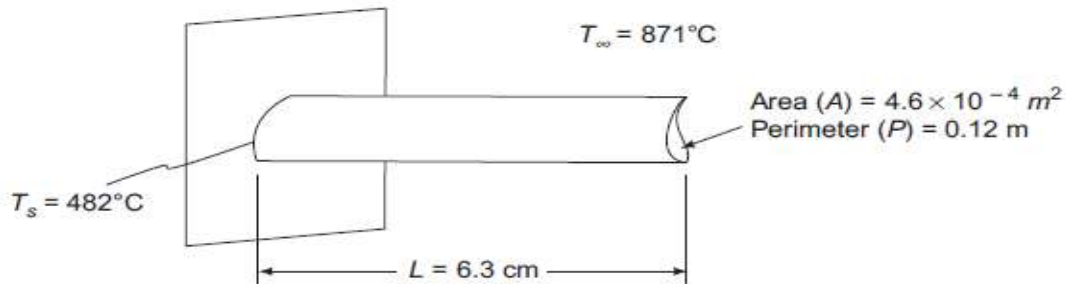- Heat transfer coefficient $(\bar{h}_c) = 454$ W/(m^2 K)

FIND

(a) The temperature of the blade tip (T_L)
(b) The rate of heat flow (q) at the roof of the blade

ASSUMPTIONS

- Steady state conditions prevail
- The thermal conductivity is uniform
- The tip is insulated
- The cross-section of the blade is uniform
- One dimensional conduction

SKETCH

$T_\infty = 871°C$

Area $(A) = 4.6 \times 10^{-4}$ m^2
Perimeter $(P) = 0.12$ m

$T_s = 482°C$

$L = 6.3$ cm

SOLUTION

(a) The temperature distribution in a fin of uniform cross-section with an insulated tip, from Table 2.1, is

$$\frac{\theta}{\theta_s} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

where $m = \sqrt{\dfrac{\bar{h} P}{k A}} = \sqrt{\dfrac{454 \text{W/(m}^2\text{K)}(0.12 \text{ m})}{18 \text{W/(m K)}(4.6 \times 10^{-4} \text{ m}^2)}} = 81.1 \dfrac{1}{\text{m}}$

$$\theta = T - T_\infty$$

At the blade tip, $x = L$, therefore

$$\frac{\theta_L}{\theta_s} = \frac{T_L - T_\infty}{T_s - T_\infty} = \frac{\cosh[m(0)]}{\cosh(mL)} = \frac{1}{\cosh(mL)}$$

$$T_L = T_\infty + \frac{T_s - T_\infty}{\cosh(mL)} = 871°C + \frac{482°C - 871°C}{\cosh\left[\left(81.1\dfrac{1}{\text{m}}\right)(0.063\,\text{m})\right]} = 866°C$$

(b) The rate of heat transfer from the fin is given by Table 2.1 to be

$$q = M \tanh (m L)$$

where

$$M = \sqrt{h_c P k A} \ \theta_s$$

$$M = \sqrt{454 \text{W/(m}^2\text{K)}(0.12 \text{ m})(18 \text{W/(m K)})(4.6 \times 10^{-4} \text{m}^2)} \ (482°\text{C} - 871°\text{C}) = -261 \text{ W}$$

$$\therefore q = (-261 \text{ W}) \tanh \left[81.1 \frac{1}{\text{m}} (0.063 \text{ m}) \right] = -261 \text{ W (out of the blade)}$$

سوال سوم:

Air at 320 K with a free-stream velocity of 10 m/s is used to cool small electronic devices mounted on a printed circuit board as shown in the sketch below. Each device is 5 mm × 5 mm square in planform and dissipates 60 mW. A turbulator is located at the leading edge to trip the boundary layer so that it will become turbulent. Assuming that the lower surfaces of the electronic devices are insulated, estimate the surface temperature at the center of the fifth device on the circuit board (see next page).

Air
10 m/s
320 K

0.005 m

Electronic
devices

Turbulator
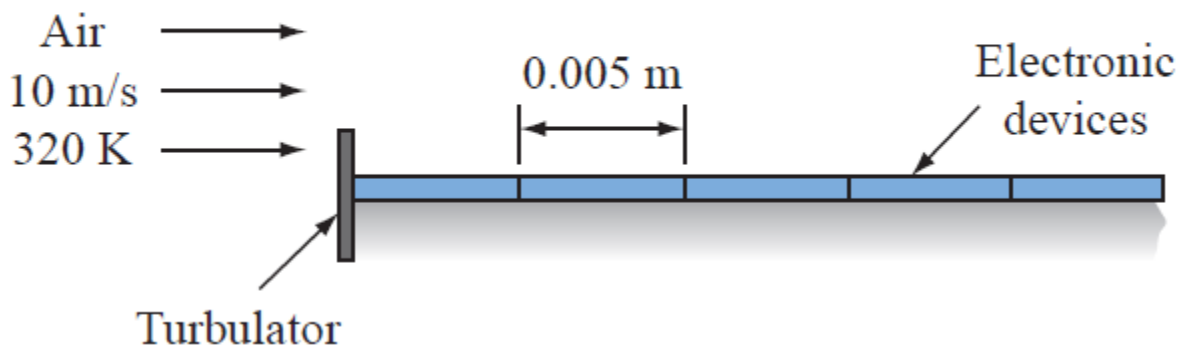
جواب سوال

GIVEN

- Air flows over small electronic devices
- Air temperature (T_∞) = 320 K
- Air velocity (U_∞) = 10 m/s
- Dimensions of each device = 5 mm × 5 mm = .005 m × .005 m
- Power dissipation per device $(\dot{q}_G)$ = 60 milliwatts = 0.06 W
- There is a turbulator at the leading edge
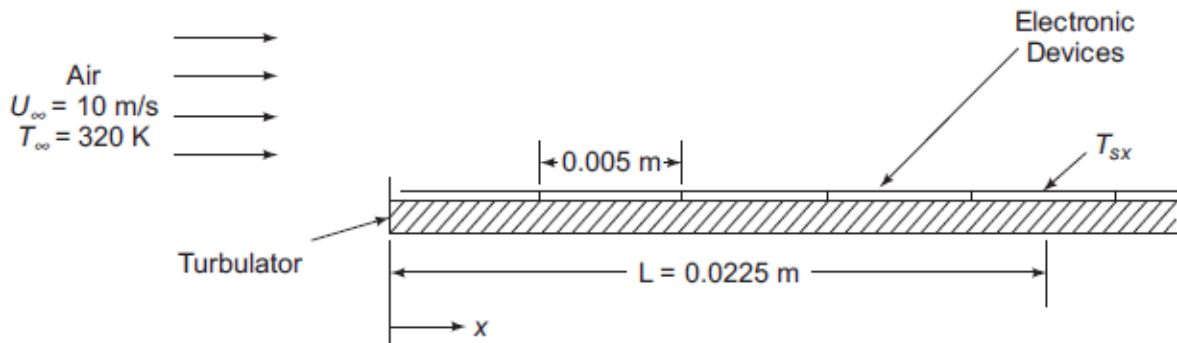
FIND

- The surface temperature (T_{sx}) at the center of the fifth device

ASSUMPTIONS

- Steady state
- Lower surface of the devices is insulated (negligible heat loss)
- The devices are placed edge-to-edge on the board
- The boundary layer is turbulent from the leading edge on
- The bulk fluid temperature is constant

SKETCH

PROPERTIES AND CONSTANTS

From Appendix 2, Table 27, for dry air at the average of 320 K

Kinematic viscosity $(v) = 18.2 \times 10^{-6} \text{ m}^2/\text{s}$

Thermal conductivity $(k) = 0.0270 \text{ W/(m K)}$

Prandtl number $(Pr) = 0.71$

SOLUTION

The center of the fifth chip is 0.0225 m from the leading edge. The Reynolds number at this point is

$$Re_x = \frac{U_\infty x}{v} = \frac{(10\,\text{m/s})(0.0225\,\text{m})}{18.2 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.24 \times 10^4$$

Although this would normally be a laminar boundary layer, in this case, it will be turbulent due to the turbulator at the leading edge. For a turbulent boundary layer, the local heat transfer coefficient is given by Equation (4.81)

$$h_{cx} = \frac{k}{x}\,0.0288\,Re_x^{0.8}\,Pr^{\frac{1}{3}} = \frac{(0.0270\,\text{W/(m K)})}{0.0225\,\text{m}}\,0.0288\,(1.24 \times 10^4)^{0.8}\,(0.71)^{\frac{1}{3}} = 57.9\ \text{W/(m}^2\ \text{K)}$$

For steady state, the rate of convective heat flux at $x = 0.0225$ m must equal the rate of heat generation per unit surface area

$$\frac{q_{cx}}{A} = h_{cx}(T_{sx} - T_\infty) = \frac{q_G}{A}$$

Solving for the surface temperature

$$T_{sx} = T_\infty + \frac{1}{h_{cx}}\frac{q_G}{A} = 320\,\text{K} + \frac{1}{57.9\,\text{W/(m}^2\,\text{K)}}\left(0.06\,\text{W/chip}\frac{1\,\text{chip}}{(0.005\,\text{m})(0.005\,\text{m})}\right)$$

$$= 361\,\text{K} = 88°\text{C}$$

The film temperature is therefore (320 K + 361 K)/2 = 341 K. Performing another iteration using air properties evaluated at 341 K yields the following results

$$v = 20.2 \times 10^{-6}\,\text{m}^2/\text{s}$$

$$k = 0.0285\ \text{W/(m K)}$$

$$Pr = 0.71$$

$$Re_x = 11,117$$

$$h_{cx} = 56.1$$

$$T_{sx} = 363\,\text{K} = 90°\text{C}$$