HEAT TRANSFER – EXERCISES

CHRIS LONG & NASER SAYMA

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ISBN 978-87-7681-433-5

Contents

Preface

Worked examples are a necessary element to any t
theory (i.e. the principles, concepts and metho
examples can be used, with modification, as a te

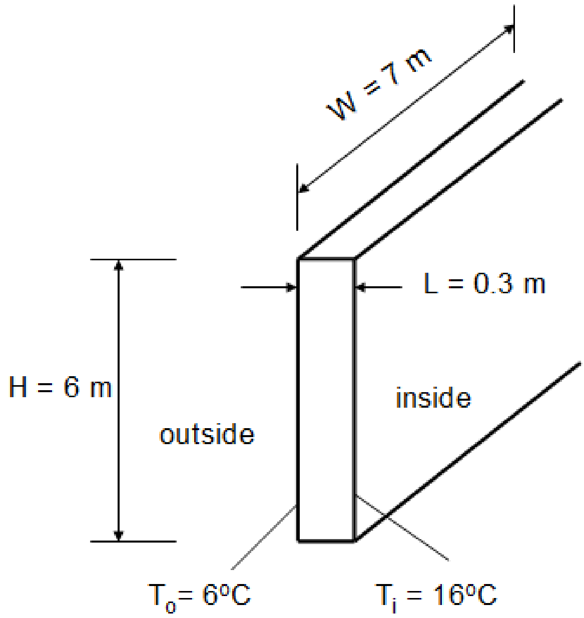
This work book contains examples and full soluti
by Long and Sayma). The subject matter correspon
Heat Transfer, Conduction, Convection, Heat Exch
chosen with the above statement in mind. Whilst
of the need to make them relevant to mechanical
problems are taken from questions that have or r
difficulty ranges from the very simple to challe
which will hopefully allow the reader to occasio
from following the solutions and would welcome y

Christopher Long
Naser Sayma
Brighton, UK, February 2010

1. Introduction

Example 1.1

The wall of a house, 7 m wide and 6 m high … $k = 0.6\ W/m\,K$ …
The surface temperature … 6°C and … 16°C … inside … Find the … heat
flux through the wall and the total heat loss th…



Solution:

For one-dimensional steady state conduction:

$$q = -k\frac{dT}{dx} = -\frac{k}{L}(T_i - T_o)$$

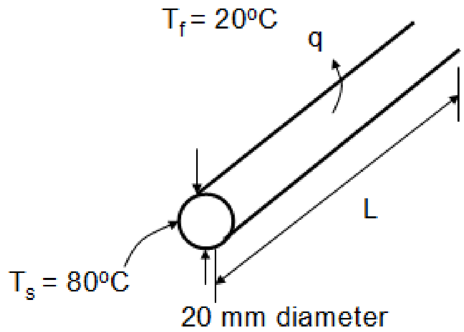
$$q = -\frac{0.6}{0.3}(16 - 6) = -20\ W/m^2$$

$$Q = qA = -20 \times 6 \times 7 = -840\ W$$

The minus sign indicates heat flux from inside t…

Example 1.2

A 20 mm diameter copper pipe is used to carry heated water, the external surface of which is subjected to a convective heat transfer coefficient of $h = 6\ W/m^2K$. Find the heat loss by convection per metre length of the pipe when the external surface temperature is 80°C and the surroundings are at 20°C. Assuming black body radiation what is the heat loss by radiation?



Solution

$$q_{conv} = h(T_s - T_f) = 6(80 - 20) = 360\ W/m^2$$

For 1 metre length of the pipe:

$$Q_{conv} = q_{conv} A_{conv} = q_{conv} \times 2\pi r = 360 \times 2 \times \pi \times 0.01 = 22.6\ W/m$$

For radiation, assuming black body behaviour:

$$q_{rad} = \sigma\left(T_s^4 - T_f^4\right)$$

$$q_{rad} = 5.67 \times 10^{-8}\left(353^4 - 293^4\right)$$

$$q_{rad} = 462\ W/m^2$$

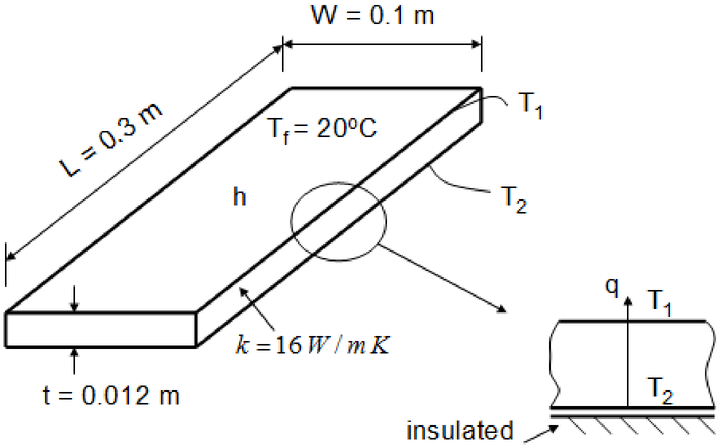
For 1 metre length of the pipe

$$Q_{rad} = q_{rad} A = 462 \times 2 \times \pi \times 0.01 = 29.1\ W/m$$

A value of $h = 6\ W/m^2K$ is representative of free convection ... loss by (black-body) radiation is seen to be co...

Example 1.3

A plate 0.3 m long and 0.1 m wide, with a thickn
($k = 16\ W/m\,K$), the top surface is exposed to an ... °C. In an experi
the plate is heated by an electrical heater (als
the temperature of the plate ... 0°C. A voltmeter and amm
connected to the heater and these read 200 V and
perfectly insulated on all sides except the top



Solution

Heat flux equals power supplied to electric heat

$$q = \frac{V\times I}{A} = \frac{V\times I}{W\times L} = \frac{200\times 0.25}{0.1\times 0.3} = 1666.7\ W/m^2$$

This will equal the conducted heat through the p

$$q = \frac{k}{t}\left(T_2 - T_1\right)$$

$$T_1 = T_2 - \frac{q\,t}{k} = 100 - \frac{1666.7\times 0.012}{16} = 98.75\ ^{\circ}C \quad (371.75\ K)$$

The conducted heat will be transferred by convec

$$q = h\left(T_1 - T_f\right) + \sigma\left(T_1^4 - T_f^4\right)$$

$$h = \frac{q - \sigma\left(T_1^4 - T_f^4\right)}{\left(T_1 - T_f\right)} = \frac{1666.7 - 5.67\times 10^{-8}\left(371.75^4 - 293^4\right)}{371.75 - 293} = \ldots\ W/m^2 K$$

Example 1.4

An electronic component dissipates 0.38 Watts th[…] body) into surroundings at 20 °C. What is the surface temperature of […] transfer coefficient is 6 W/m²K, and the heat sink has an effecti[…]

Solution

$$q = \frac{Q}{A} = h\left(T_s - T_\infty\right) + \sigma\left(T_s^4 - T_\infty^4\right)$$

$$\frac{0.38}{0.001} = 6\left(T_s - 293\right) + 5.67\times10^{-8}\left(T_s^4 - 293^4\right)$$

$$5.67\times10^{-8}\,T_s^4 + 6\,T_s - 2555.9 = 0$$

This equation needs to be solved numerically. Ne[…]

$$f = 5.67\times10^{-8}\,T_s^4 + 6\,T_s - 2555.9$$

$$\frac{df}{dT_s} = 22.68\times10^{-8}\,T_s^3 + 6$$

$$T_s^{n+1} = T_s^n - \frac{f}{\left(\frac{df}{dT_s}\right)} = T_s - \frac{5.67\times10^{-8}\,T_s^4 + 6\,T_s - 2555.9}{22.68\times10^{-8}\,T_s^3 + 6}$$

Start iterations with $T_s^0 = 300\ K$

$$T_s^1 = 300 - \frac{5.67\times10^{-8}\times300^4 + 6\times300 - 2555.9}{22.68\times10^{-8}\times300^3 + 6} = 324.46\ K$$

$$T_s^2 = 324.46 - \frac{5.67\times10^{-8}\times324.46^4 + 6\times324.46 - 2555.9}{22.68\times10^{-8}\times324.46^3 + 6} = 323\ K$$

The difference between the last two iterations i

$$T_s^0 = 323\ K = 50\ °C$$

The value of 300 K as a temperature to begin the
being above the ambient temperature.

2. Conduction

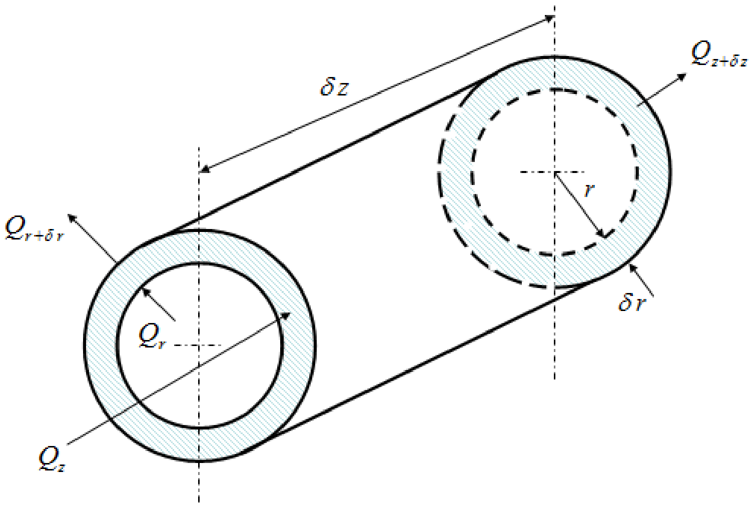
Example 2.1

Using an appropriate control volume show that th
coordinates for a material with constant thermal

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where $\alpha = \frac{k}{\rho c}$ is the thermal diffusivity.

Solution



Consider a heat balance on an annular control vo
the control volume is given by:

Heat in + Heat out = rate of change of internal

$$Q_r + Q_z - Q_{r+\delta r} - Q_{z+\delta z} = \frac{\partial u}{\partial t} \qquad (2.1)$$

$$Q_{r+\delta r} = Q_r + \frac{\partial Q_r}{\partial r}\delta r$$

$$Q_{z+\delta z} = Q_z + \frac{\partial Q_z}{\partial z}\delta z$$

$u = mcT$

Substituting in equation 2.1:

$$-\frac{\partial Q}{\partial r}\delta r - \frac{\partial Q}{\partial z}\delta z = \frac{\partial (mcT)}{\partial t}$$

Fourier's law in the normal direction of the out

$$\frac{Q}{A} = -k\frac{\partial T}{\partial n}$$

$$Q_r = -kA\frac{\partial T}{\partial r} = -k \times 2\pi r\,\delta z\,\frac{\partial T}{\partial r} \qquad A = 2\pi r\,\delta z$$

$$Q_z = -kA\frac{\partial T}{\partial z} = -k \times 2\pi r\,\delta r\,\frac{\partial T}{\partial z} \qquad A = 2\pi r\,\delta r$$

Equation 2.1 becomes

$$-\frac{\partial}{\partial r}\left\{-k \times 2\pi r\,\delta z\,\frac{\partial T}{\partial r}\right\}\delta r - \frac{\partial}{\partial z}\left\{-k \times 2\pi r\,\delta r\,\frac{\partial T}{\partial z}\right\}\delta z = mc\frac{\partial T}{\partial t} \qquad (2.3)$$

Noting that the mass of the control volume is gi

$m = \rho 2\pi r\,\delta r\,\delta z$ Equation 2.3 becomes

$$\frac{\partial}{\partial r}\left\{kr\frac{\partial T}{\partial r}\right\} + \frac{\partial}{\partial z}\left\{kr\frac{\partial T}{\partial z}\right\} = \rho c r\frac{\partial T}{\partial t}$$

Dividing by r, noting that r can be taken outsic
function of z. Also dividing by k since the ther

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial T}{\partial r}\right\} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho c}{k}\frac{\partial T}{\partial t}$$

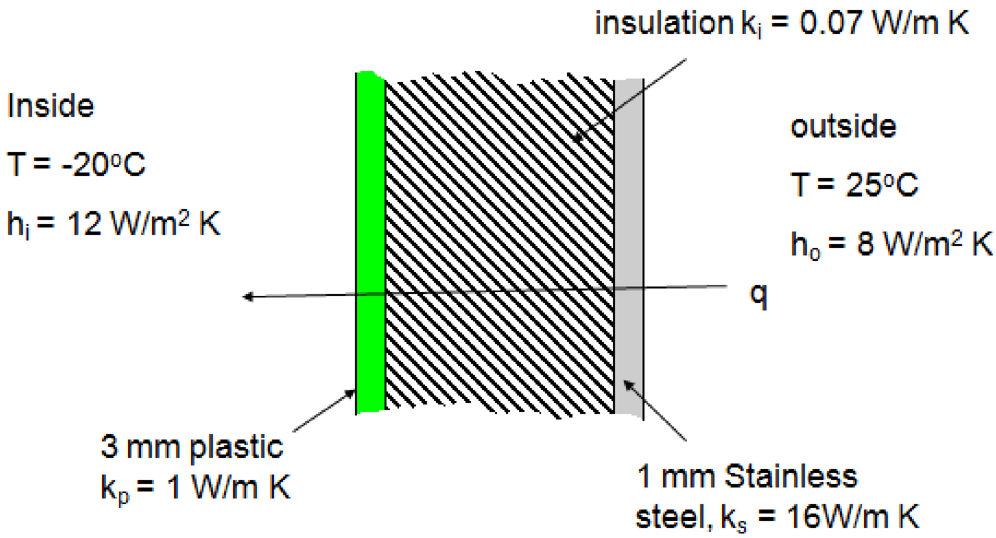
Using the definition $\alpha = \frac{k}{\rho c}$ and the thermal diffusivity expanding the

$$\frac{1}{r}\left\{r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\frac{\partial r}{\partial r}\right\} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ which gives the required

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Example 2.2

An industrial freezer is designed to operate with ... °C when the ...
air temperature is 25 °C and the internal and external heat transfer ... 12 ... 8 ...
W/m^{2}K, respectively. The walls of the freezer ar...
layer of plastic (k = 1 W/m K, and thickness of ...
W/m K, and thickness of 1 mm). Sandwiched betwee...
with k = 0.07 W/m K. Find the width of the insul...
loss to 15 W/m^2



Solution

$q = U\Delta T$ where U is the overall heat transfer coefficien

$$U = \frac{q}{\Delta T} = \frac{15}{25-(-20)} = 0.333\ W/m^2 K$$

$$U = \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_i}{k_i} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]^{-1} = 0.333$$

$$\left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_i}{k_i} + \frac{L_s}{k_s} + \frac{1}{h_o}\right] = \frac{1}{0.333}$$

$$L_i = k_i\left\{\frac{1}{0.333} - \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]\right\} = 0.07\left\{\frac{1}{0.333} - \left[\frac{1}{12} + \frac{0.003}{1} + \frac{0.001}{16} + \frac{1}{8}\right]\right\}$$

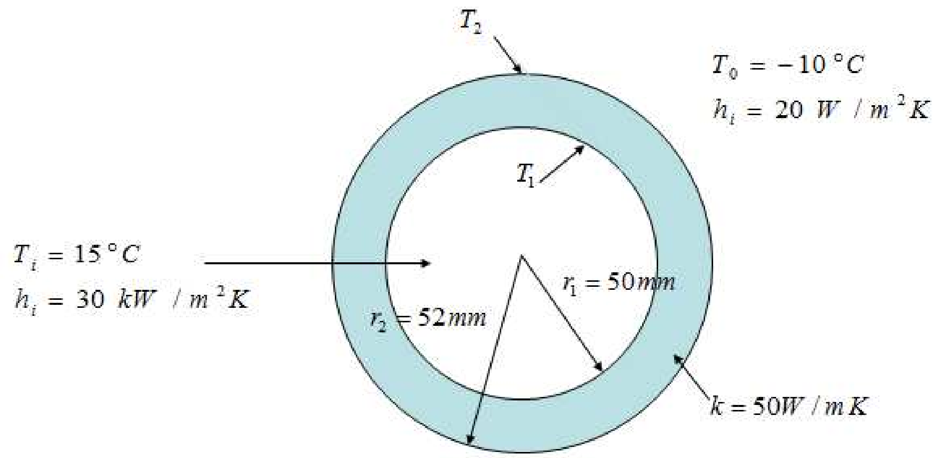
$$L_i = 0.195\ m \qquad (195\ mm)$$

Example 2.3

Water flows through a cast steel pipe (k = 50 W/
wall thickness.

i. Calculate the heat loss by convection and c
when the water temperature is 15°C, the outside air temperature is −10°C, ... the ... heat transfer coefficient on the inside is 30 kW/m²K ... heat transf
ii. Calculate the corresponding heat loss when ...
outer diameter of 300 mm, and thermal condu

Solution



Plain pipe

$$Q = 2\pi r_1 L h_i (T_i - T_1) \quad \rightarrow \quad T_i - T_1 = \frac{Q}{2\pi r_1 L h_i}$$

$$Q = \frac{2\pi L k (T_1 - T_2)}{\ln(r_2/r_1)} \quad \rightarrow \quad T_1 - T_2 = \frac{Q}{2\pi L k / \ln(r_2/r_1)}$$

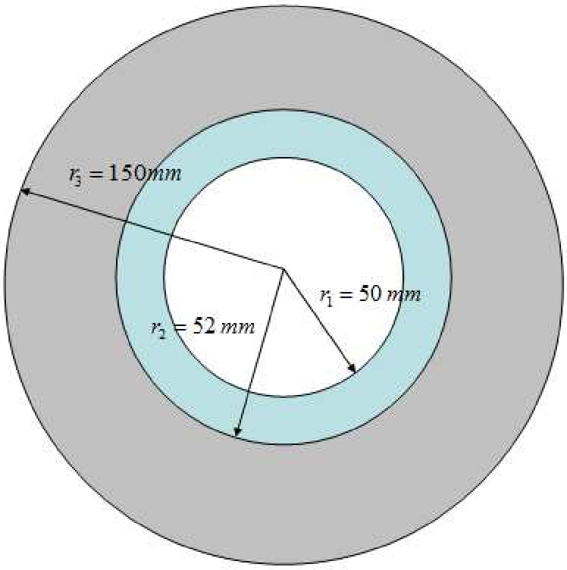
$$Q = 2\pi r_2 L h_o (T_2 - T_o) \quad \rightarrow \quad T_2 - T_o = \frac{Q}{2\pi r_2 L h_o}$$

Adding the three equations on the right column w

$$Q = \frac{2\pi L (T_i - T_o)}{\frac{1}{r_1 h_i} + \frac{\ln(r_2/r_1)}{k} + \frac{1}{r_2 h_o}}$$

$$\frac{Q}{L} = \frac{2\pi(15 - (-10))}{\frac{1}{0.05 \times 30000} + \frac{\ln(0.052/0.05)}{50} + \frac{1}{0.052 \times 20}} = 163\ W/m$$

Insulated pipe



$$\frac{Q}{L} = \frac{2\pi\left(T_i - T_o\right)}{\frac{1}{r_1 h_i} + \frac{\ln\left(r_2 / r_1\right)}{k_s} + \frac{\ln\left(r_3 / r_2\right)}{k_i} + \frac{1}{r_3 h_o}}$$

$$\frac{Q}{L}=\frac{2\pi(152-10)}{\frac{1}{0.05\times3500}+\frac{\ln(0.052/0.05)}{50}+\frac{\ln(0.152/0.052)}{0.05}+\frac{1}{0.152\times20}}=73\ W/m$$

For the plain pipe, the heat loss is governed by
which provides the highest thermal resistance. F
higher thermal resistance and this layer governs

Example 2.4

Water at 80°C is pumped through 100 m of stainless stee
47 mm and 50 mm respectively. The heat transfer
surface of the pipe loses heat by convection
K. Calculate the heat flow through the pipe. Als
of insulation, k = 0.1 W/m K and 50 mm radial th

Solution

The equation for heat flow through a pipe per un

$$Q=\frac{2\pi L(T_i-T_o)}{\frac{1}{r_1h_i}+\frac{\ln(r_2/r_1)}{k}+\frac{1}{r_2h_o}}$$

Hence substituting into this equation:

$$Q=\frac{2\pi\times100(80-20)}{\frac{1}{0.047\times2000}+\frac{\ln(50/47)}{16}+\frac{1}{0.05\times20}}=0.329\times10^{6}\ W$$

For the case with insulation, we also use the eq

$$Q=\frac{2\pi L(T_i-T_o)}{\frac{1}{r_1h_i}+\frac{\ln(r_2/r_1)}{k}+\frac{\ln(r_3/r_2)}{k_{ins}}+\frac{1}{r_3h_o}}$$

$$Q=\frac{2\pi\times100(80-20)}{\frac{1}{0.047\times2000}+\frac{\ln(50/47)}{16}+\frac{\ln(100/50)}{0.1}+\frac{1}{0.1\times20}}=5.9\times10^{3}\ W$$

Notice that with insulation, the thermal resista
above, if we retain the thermal resistance for t

$$Q = \frac{2\pi L (T_i - T_o)}{\dfrac{\ln(r_2/r_1)}{k_{ins}}} = \frac{2\pi \times 1 \times (80 - 20)}{\dfrac{\ln(100/50)}{1.0}} \times 10^{-4} = \ldots 4.5\ W$$

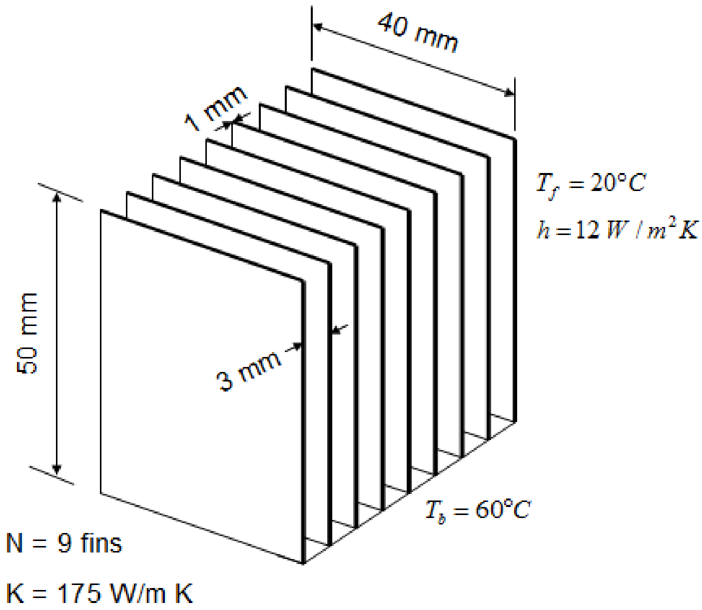
This has less than 1% error compared with the fu

Example 2.5

A diagram of a heat sink to be used in an electr aluminium fins (k = 175 W/m K, ρ = 2700 kg/m³, c = 900 J/kg K), ro each 60 mm long, 40 mm wide and 1 mm thick. The temperature of the base of the heat sink has T_b = 60 °C when the extr air temperature T_f = 20 °C. Under these conditions, the external [h = 12 W/m² K]. The fin may be assumed to be sufficiently thi neglected. The surface temperature T, at a dista

$$\frac{T - T_f}{T_b - T_f} = \frac{\cosh m(L - x)}{\cosh mL} \quad \text{where } m^2 = \frac{hP}{kA_c} \text{ and } A_c \text{ is the cross sectional}$$

Determine the total convective heat transfer fro fin efficiency.



Solution

Total heat fluxed is that from the un-finned sur

$$Q = Q_u + Q_f$$

$$Q_u = h A_u (T_b - T_f) = h (w - N \times s)(T_b - T_f)$$

$$Q_u = (0.4 - 19 \times 0.003)(12)(60 - 20) = 164.6\ W$$

For a single fin:

$$Q_f = -k A_c \left(\frac{dT}{dx} \right)_{x=0}$$

Where A_c is the cross sectional area of each fin

Since

$$T - T_f = \frac{(T_b - T_f) \cosh m(L - x)}{\sinh mL}$$

Then

$$\frac{dT}{dx} = -\frac{m \sinh m(L-x)}{\cosh mL}\left(T_b - T_f\right)$$

Thus

$$Q_f = -kA_c\left(\frac{dT}{dx}\right)_{x=0} = -kA_c\left(\frac{-m\sinh mL}{\cosh mL}\right)\left(T_b - T_f\right)$$

$$Q_f = kA_c m\left(T_b - T_f\right)\tanh(mL) = \left(hPkA_c\right)^{1/2}\left(T_b - T_f\right)\tanh(mL)$$

Since

$$m = \left(\frac{hP}{kA_c}\right)^{\frac{1}{2}}$$

$$P = 2(w+t) = 2(0.04 + 0.001) = 0.082\ \text{m}$$

$$A_c = wt = 0.04 \times 0.001 = 40 \times 10^{-6}\ \text{m}^2$$

$$m = \left(\frac{12 \times 0.082}{175 \times 40 \times 10^{-6}}\right)^{\frac{1}{2}} = 11.856\ \text{m}^{-1}$$

$$mL = 11.856 \times 0.06 = 0.7113$$

$$\tanh(mL) = \tanh(0.7113) = 0.6115$$

$$Q_f = \left(12 \times 0.082 \times 175 \times 40 \times 10^{-6}\right)^{1/2}(60 - 20) \times 0.6115 = 2.03\ \text{W}$$

So total heat flow:

$$Q = Q_f + Q_u = 2.03 + 0.1649 \ldots\ \text{W}$$

Finn effectiveness

$$\varepsilon_{fin} = \frac{\text{Fin heat transfer rate}}{\text{heat transfer rate in fin absence}} = \frac{Q_f}{hA_c\left(T_b - T_f\right)}$$

$$\varepsilon_{fin} = \frac{2.03}{12 \times 40 \times 10^{-6}(60 - 20)} = 106$$

Fin efficiency:

$$\eta_{fin} = \frac{\text{fin heat transfer through fin}}{\text{heat transfer if entire fin were at base temperature}}$$

$$\eta_{fin} = \frac{Q_f}{h A_s (T_b - T_f)}$$

$$A_s = 2(wL + Lt) + wt$$

$$A_s = 2(0.06 \times 0.04 + 0.04 \times 0.001) + \ldots = 4.92 \times 10^{-3}\ m^2$$

$$\eta_{fin} = \frac{2.30}{12 \times 4.92 \times 10^{-3} (60 - 20)} = 0.68$$

Example 2.6

For the fin of example 4.5, a fan was used to increase the heat transfer coefficient to 40 W/m²K. Justify the use of the lumped mass ... to predict the rate of change of temperature wit... below, calculate the time τ taken for the fin to cool from 6...

$$(T - T_f) = (T_i - T_f) \exp\left(-\frac{h A_s \tau}{m C}\right)$$

Solution

Consider a single fin (the length scale L for th...

$$B_i = \frac{h L}{k} = \frac{h \times t/2}{k} = \frac{40 \times 0.0005}{175} \approx 10^{-4}$$

Since $B_i << 1$, we can use he "lumped mass" model approx...

$$\frac{(T - T_f)}{(T_i - T_f)} = \exp\left(-\frac{h A_s \tau}{m C}\right)$$

$$\tau = -\frac{m C}{h A_s} \ln\left(\frac{T - T_f}{T_i - T_f}\right)$$

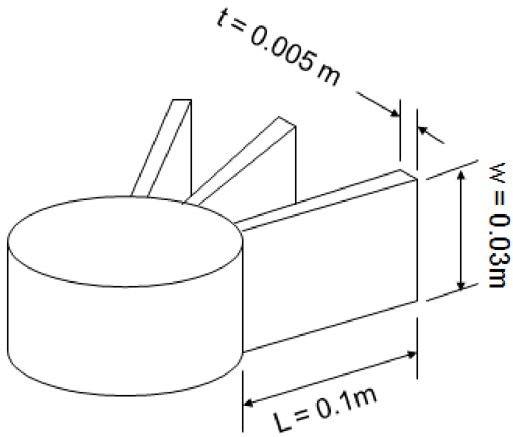
$$m = \rho A_s t/2$$

$$\tau = -\frac{\rho C t}{2h}\ln\left(\frac{T - T_f}{T_i - T_f}\right) = -\frac{2700 \times 900 \times 0.001}{2 \times 40}\ln\left(\frac{30 - 20}{60 - 20}\right) = 42 \text{ seconds}$$

Example 2.7

The figure below shows part of a set of radial a
a small air compressor. The device dissipates 1
20°C. Each fin is 100 mm long, 30 mm high and 5 m
adiabatic and a heat transfer coefficient … ^{2}K … h =

Estimate the number of fins required … °C to ensure t



Solution

Consider a single fin:

$$P = 2(w + t) = 2(0.03 + 0.005) = 0.07\ m$$

$$A_c = wt = 0.005 \times 0.03 = 1.5 \times 10^{-4} = 150 \times 10^{-6}$$

$$m = \left(\frac{hP}{kA_c}\right)^{\frac{1}{2}} = \left(\frac{15 \times 0.07}{180 \times 150 \times 10^{-6}}\right)^{\frac{1}{2}} = 6.2361$$

$$mL = 6.2361 \times 0.1 = 0.62361$$

$$\tanh(mL) = 0.5536$$

$$Q_f = (hPkA_c)^{1/2}(T_b - T_f)\tanh(mL) \quad \text{(From example}$$

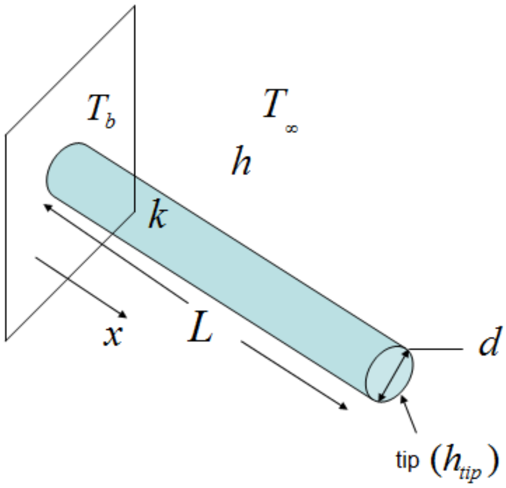
$$Q_f = (15 \times 0.07 \times 180 \times 150 \times 10^{-6})^{1/2}(\ldots) = 32.9\ W$$

So for 1 kW, the total number of fins required:

$$\frac{1000}{32.9} = 30.4$$

Example 2.8

An air temperature probe may be analysed as a fi
L = 20 mm, k = 19 W/m K, D = 3 mm, when h … W/m^2K, there is
an actual air temperature of 50 °C and the surface temperature … °C at the



Solution

The error should be zero when $T_{tip} = T_\infty$. The temperature distribution a
(from the full fin equation) is given by:

$$\frac{T_x - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x) + \frac{h_{tip}}{mk}\sinh m(L-x)}{\cosh mL + \frac{h_{tip}}{mk}\sinh mL}$$

$$m = \left(\frac{hP}{kA}\right)^{1/2} \qquad P = \pi D, \quad A = \pi D^2/4$$

At the tip, $x = L$, the temperature is given by ($\cosh(0) = 1$, $\sinh(0) = 0$)

$$\frac{T_{tip} - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL + \frac{h_{tip}}{mk}\sinh mL} = \phi$$

Where ϕ is the dimensionless error:,

$\phi = 0$, $T_{tip} = T_\infty$ (no error)

$\phi = 1$, $T_L = T_b$ (large error)

For $h_{tip} = 0$, $h = 50\ W/m^2K$, $k = 19\ W/mK$, $D = 3\ mm$, $L = 20\ mm$

$T_b = 60\,°C$, $T_\infty = 5\,°C$

$A = \pi D^2/4$, $P = \pi D$

$$m = \left(\frac{hP}{kA}\right)^{1/2} = \left(\frac{\pi D h}{\pi D^2 k/4}\right)^{1/2} = \left(\frac{4h}{kD}\right)^{1/2} = \left(\frac{4\times 50}{19 \times 0.003}\right)^{1/2} = 59.235\ m^{-1}$$

$$mL = 59.235 \times 0.02 = 1.185$$

$$\frac{h}{mk} = \frac{50}{59.235 \times 19} = 0.0444$$

$$\frac{T_x - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(1.185) + 0.0444 \times \sinh(1.185)} = 0.539$$

$$T_{tip} = T_\infty + (T_b - T_\infty)\,0.539$$

$$T_{tip} = -5 + (50)\,0.539 = 39.55\ °C$$

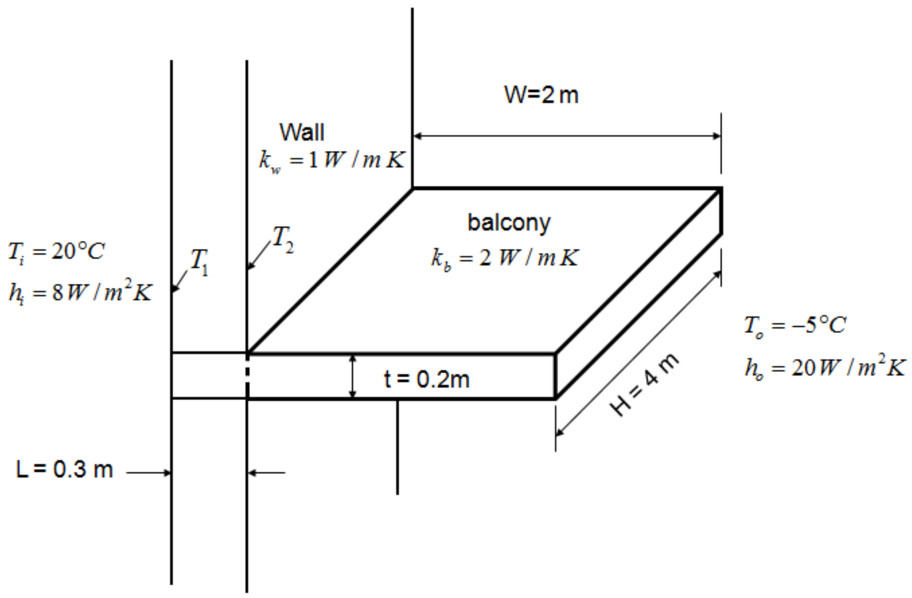
Hence error = 39.5 °C

Example 2.9

A design of an apartment block at a ski resort r
separate apartments. The walls of the building a
W/m K. Use the fin approximation to examine the
suggestions for this design. In each case, the b
(parallel to the wall) of 2 m. Assume the inside temp
5°C; the overall (convective + radiative) heat tr
W/m²K and on that on the outside of the building

a) A balcony constructed from solid concrete and
b) A balcony suspended from 3 steel beams, k = 40
m each of effective cross sectional area A_e = 0.01 m², perimeter P = 0.6 m. The actual flo
the balcony in this case may be considered to
c) No balcony.

Solution



a) For the concrete balcony

Treat the solid balcony as a fin

$$Bi = \frac{h_o\, t/2}{k_b}$$

$$B_i = \frac{20 \times 0.01}{2} = 0.1$$

Not that Bi is not << 1, thus 2D analysis would
give an acceptable result for the purpose of a q

$$P = 2(H + t) = 2(2 + 0.04) = 4.08\ m$$

$$A_c = H \times t = 2 \times 0.04 = 0.08\ m^2$$

To decide if the fin is infinite, we need to eva

$$mW = \left(\frac{hP}{kA}\right)^{1/2} W = \left(\frac{\times 4.08}{\times 0.08}\right)^{1/2} 20 = \times 5.202$$

This is large enough to justify the use of the f

$$Q = (h\,P\,k\,A_c)^{1/2}(T_b - T_o)$$

$$q_b = \frac{1}{A_c}(h\,P\,k\,A_c)^{1/2}(T_b - T_o) = \left(\frac{h_o\,P\,k}{A_c}\right)^{1/2}(T_2 - T_o) \qquad (1)$$

Also assuming 1-D conduction through the wall:

$$q_b = h_i(T_i - T_1) \qquad (2)$$

$$q_b = \frac{k_b}{L}(T_1 - T_2) \qquad (3)$$

Adding equations 1, 2 and 3 and rearranging:

$$q_b = \frac{(T_i - T_o)}{\frac{1}{h_i} + \frac{L}{k_b} + \left(\frac{A_c}{P\,k\,h_o}\right)^{1/2}} \qquad (4)$$

This assumes 1D heat flow through the wall, the
introduce some 2-D effects.

From (4)

$$q_b = \frac{20-(-5)}{\frac{1}{8} + \frac{0.3}{2} + \left(\frac{0.8}{0.8\times 2\times 4.820}\right)^{1/2}} = 77\ W/m^2$$

Compared with no balcony:

$$q_b = \frac{(T_i - T_o)}{\frac{1}{h_o} + \frac{L}{k_w} + \frac{1}{h_i}} = \frac{(20-(-5))}{\frac{1}{8} + \frac{0.3}{1} + \frac{1}{20}} = 52.6\ W/m^2$$

The difference for one balcony is $(22.6 - 52.6)0.84 \cdot A_c = 19.7\ W$

For 350 apartments, the difference is *6891 W*.

For the steel support of each balcony $A_c = 0.01\ m^2$ and $P = 0.6\ m$ where

As before, however, in this case $Bi << 1$ because $k_s >> k_b$

$$mW = \left(\frac{hP}{kA}\right)^{1/2} W = \left(\frac{20 \times 6}{1 \times 0.040}\right)^{1/2} \approx 1 \times 12$$

$mW > 2$, so we can use the infinite fin approximat

$$q_b = \frac{(T_i - T_o)}{\frac{1}{h} + \frac{L}{k_{si}} + \left(\frac{A_c}{P k_s h}\right)^{1/2}} = \frac{(20 - 5)}{\frac{1}{8} + \frac{3}{40} + \left(\frac{0.01}{0 \times 4 \times 620}\right)^{1/2}} = 8 \; W/m^2$$

$$Q_b = q_b A_{cb} = 8 \times 0.01 = 0.8 \; W \; / \; \text{bar}$$

For 350 apartments, $Q_b = 10 \; kW$

Example 2.10

In free convection, the heat transfer coefficien $(T_s - T_f)$. Using the low Biot number approximation an $(T_s - T_f)^n$ where G and n are constants, show that th temperature ratio with time will be given by

$$\theta^{-n} = 1 + n h_{init} \lambda t$$

Where

$$\theta = \frac{(T - T_f)}{T_{init} - T_f}, \quad \lambda = \frac{\text{Area}}{\text{Mass} \times \text{Specific Heat Capacity}}$$

and h_{init} = the heat transfer coefficient at t = 0. U aluminium motorcycle fin (k … ρ … kg/m^3 … C … J/kg K) … a … thickness 2 mm … °C … surroundings at 20 °C … initial … transfer coefficient due … Compare this … from the equation $\theta = e^{-h\lambda t}$ which assumes a constant value of

Solution

Low Biot number approximation $Bi < 1$ for free convectio

Heat transfer by convection = rate of change of

$$-mC\frac{d(T_s-T_f)}{dt} = hA(T_s-T_f) \qquad (1)$$

We know that $h = G(T_s-T_f)^n$

Where G is a constant.

(Note that this relation arises from the usual N
example: $Nu = 0.1(Gr\,Pr)^{1/3}$ turbulent flow, $Nu = 0.54(Gr\,Pr)^{1/4}$ for laminar flow)

Equation 1 then becomes:

$$(T_s-T_f)^n(T_s-T_f) = -\frac{mC}{GA}\frac{d(T_s-T_f)}{dt}$$

$$\int_{t=0}^{t} -\frac{GA}{mC}\,dt = \int_0^t \frac{d(T_s-T_f)}{(T_s-T_f)^{n+1}}$$

$$\frac{GnAt}{mC} = (T_s-T_f)^{-n} - (T_s-T_f)^{-n}_{t=0} \qquad (2)$$

At $t = 0$, $(T_s-T_f) = (T_s-T_f)_i$

If we divide equation 2 by $(T_s-T_f)_i^{-n}$

And use the definition $\theta = \frac{(T_s-T_f)}{(T_s-T_f)_i}$

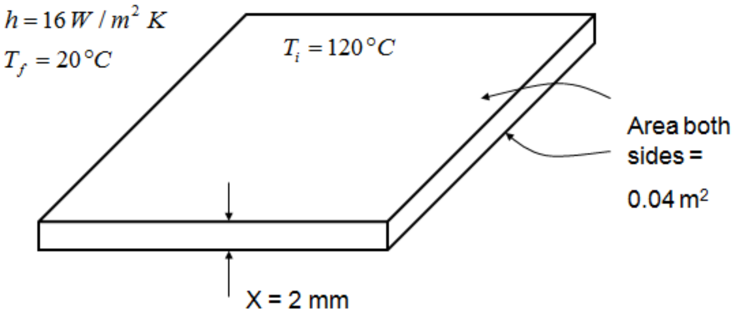
We obtain

$$\frac{GnAt}{(T_s-T_f)_i^{-n}\,mC} = \frac{GnAt}{mC}(T_s-T_f)_i^{n} = \theta^{-n} - 1$$

Since $h_i = G(T_s-T_f)_i^n$ = heat transfer coefficient at tim

$$\theta^{-n} = 1 + \frac{h_i n A t}{mC}$$

Or $\theta^{-n} = \theta_i^{-n} + n\lambda t$



For aluminium $\rho = 2750\ kg/m^3,\ C = 870\ J/kg\,K$

For laminar free convection, n = ¼

$$m = \rho A X = 2750 \times 0.04 \times 0.002 = 0.22\ kg$$

$$\lambda = \frac{A}{mC} = \frac{0.04}{0.22 \times 870} = 2.1 \times 10^{-4}\ m^2K/J$$

$\theta^{-n} = \theta_i^{-n} + n\lambda t$ which gives

$$t = \frac{(\theta)^{-n} - 1}{n\,h\lambda}$$

When $T = 40^\circ C$, $\theta = \frac{40 - 20}{120 - 20} = 0.2$

Then

$$t = \frac{(0.2)^{-1/4} - 1}{(1/4)\,(\;) \times 1.216 \times 10^{-4}} = 590\ s$$

For the equation $\theta = e^{-\lambda h t}$

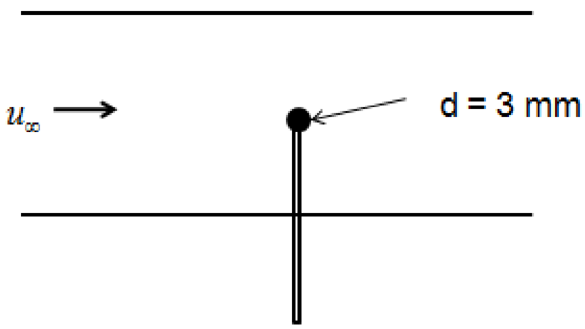
which assumes that the heat transfer coefficient … difference.

$$t = \frac{\ln \theta}{-h\lambda} = \frac{\ln 0.2}{-1.216 \times 10^{-4}} = 479\ s$$

$$\text{Percentage error} = \frac{590 - 479}{590} \times 100\% = 19\%$$

Example 2.11

A 1 mm diameter spherical thermocouple bead (C = …) is required to respond to 99.5% change of … $\mu = 1.8 \times 10^{-5}$ kg/m s, … air, … 0.0262 W/m K and Pr = 0.77) temperature in 10 ms. Wh… will occur?

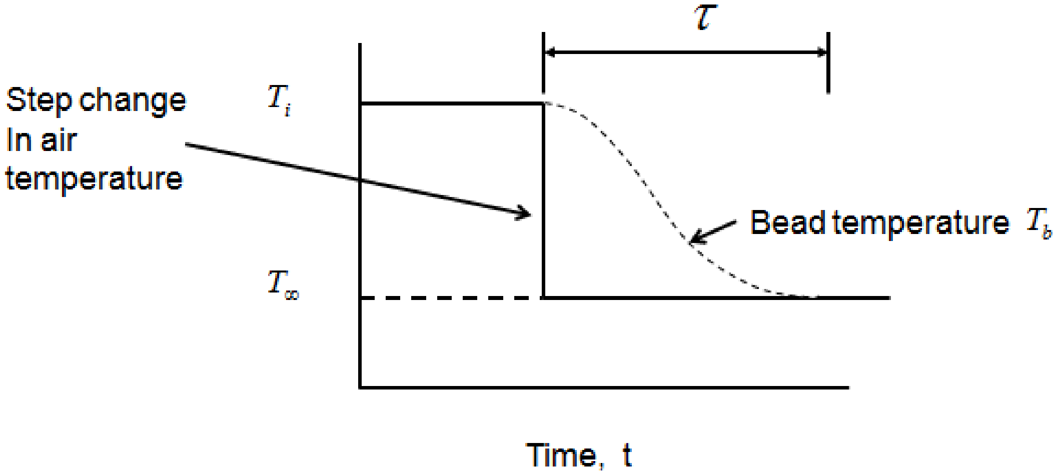


Solution

Spherical bead: $area = \pi d^2$

$volume = \pi d^3/6$

Assume this behaves as a lumped mass, then



$$\frac{T_b - T_\infty}{T_\infty - T_i} = 0.995$$

(given)

For lumped mass on cooling from temperature T_i

$$\frac{T_b - T_\infty}{T_\infty - T_i} = \exp(-\lambda t) = 0.995$$

$$\lambda = \frac{hA}{mC}, \qquad t = 0.01 \ s$$

$$\lambda t = 0.005$$

$$\lambda = 0.5$$

Which gives the required value of heat transfer

$$\frac{hA}{\rho VC} = 0.5$$

So

$$h = 0.5\frac{\pi d^3}{6}\frac{C\rho}{\pi d^2} = \frac{0.5\, d\, C\, \rho}{6}$$

$$h = \frac{0.5\times 10^{-3}\times 400\times 7800}{6} = 260\ W/m^2K$$

$$Nu_D = \frac{hD}{k} = \frac{260\times 10^{-3}}{0.0262} = 9.9$$

For a sphere

$$Nu_D = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Pr^{0.4}$$

From which with $Pr = 0.707$

$$f = 0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3} - 9.4 = 0$$

$$f' = 0.2\, Re_D^{-1/2} + 0.04\, Re_D^{-1/3}$$

Using Newton iteration

$$x^{(n+1)} = x^n - \frac{f(x)}{f'(x)}$$

Starting with $Re_D = 300$

$$Re_D^{(1)} = 300 - \frac{\left[0.4\sqrt{300} + 0.06(300)^{2/3} - 9.4\right]}{\left[\frac{0.2}{\sqrt{300}} + \frac{0.04}{300^{1/3}}\right]} = 300 - \frac{0.222}{0.01782}$$

Which is close enough to 300

From which

$$u_\infty = \frac{Re\,\mu}{D\rho} = 4.5\ m/s$$

3. Convection

Example 3.1

Calculate the Prandtl number ($Pr = \mu C_p / k$) for the following

a) Water at 20°C: $\mu = 1.002 \times 10^{-3}$ kg/m s, $C_p = 4.183$ kJ/kg K and $k = 0.$

b) Water at 90°C: $\rho = 965$ kg/m³, $\nu = 3.22 \times 10^{-7}$ m²/s, $C_p = 4208$ J/kg K and $k =$

c) Air at 20°C and 1 bar: $R = 287$ J/kg K, $C_p = 1005$ J/kg K, and $\nu = 1.5 \times 10^{-5}$ m²/s, $k = 0.02624$ W/m K

d) Air at 100°C: $\mu = \frac{1.46 \times 10^{-6}\, T^{3/2}}{(110 + T)}$ kg/m s

$C_p = 0.917 + 2.58 \times 10^{-4}\, T - 3.98 \times 10^{-8}\, T^2$ kJ/kg K where T is the absolute (in K) and $k = 0.03186$ W/m K.

e) Mercury at 20°C: $\mu = 1520 \times 10^{-6}$ kg/m s, $C_p = 0.139$ kJ/kg K and $k = 0.$

f) Liquid Sodium at 400°C: $\mu = 420 \times 10^{-6}$ kg/m s, $C_p = 1369$ J/kg K and $k = 86$

g) Engine Oil at 60°C: $\mu = 8.36 \times 10^{-2}$ kg/m s, $C_p = 2035$ J/kg K and $k = 0.14$

Solution

a) $\Pr = \dfrac{\mu C_p}{k} = \dfrac{1.002\times10^{-3}\times 4183}{0.603} = 6.95$

b) $\Pr = \dfrac{\mu C_p}{k} = \dfrac{\rho \nu C_p}{k} = \dfrac{965\times 4208\times 3.22\times10^{-7}}{0.676} = 1.93$

c) $\Pr = \dfrac{\rho \nu C_p}{k}$

$\rho = \dfrac{P}{RT} = \dfrac{100000}{287\times 293} = 1.19\ \text{kg/m}^3$

$\Pr = \dfrac{1.19\times 1.563\times10^{-5}\times 1005}{0.02624} = 0.712$

d) $\mu = \dfrac{1.46\times10^{-6}\,T^{3/2}}{(T+110)} = \dfrac{1.46\times10^{-6}\times 373^{3/2}}{373+110} = 2.18\times10^{-5}\ \text{kg/m s}$

$C_p = \ldots = 1.007\ \text{kJ/kg K}$

$\Pr = \dfrac{2.18\times10^{-5}\times 1007}{0.03186} = 0.689$

e) $\Pr = \dfrac{\mu C_p}{k} = \dfrac{1520\times10^{-6}\times 139}{0.0081\times10^{3}} = 0.0261$

f) $Pr = \frac{\mu C_p}{k} = \frac{420 \times 1369 \times 10^{-6}}{86} = 0.0067$

g) $Pr = \frac{\mu C_p}{k} = \frac{8.36 \times 10^{-2} \times 2035}{0.141} = 1207$

Comments:

- Large temperature dependence for water as
- small temperature dependence for air as i
- use of Sutherland's law for viscosity as
- difference between liquid metal and oil a
- units of kW/m K for thermal conductivity;
- use of temperature dependence of c_p (part a).

Example 3.2

Calculate the appropriate Reynolds numbers a
the following:

a) A 10 m (water line length) long ... ($\rho = 1000$ kg/m^3 and $\mu = 1.3 \times 10^{-3}$ kg/m s),

b) A compressor disc of radius 0.3 m rotating

$\mu = \frac{1.46 \times 10^{-6}\, T^{3/2}}{(T + 110)}$ kg/m s

c) 0.05 kg/s of carbon dioxide gas at 400 K f

take $\mu = \frac{1.56 \times 10^{-6}\, T^{3/2}}{(T + 233)}$ kg/m s

d) The roof of a coach 6 m long, ... ($\rho = 1.2$ kg/m^3 and $\mu = 1.8 \times 10^{-5}$ kg/m s)

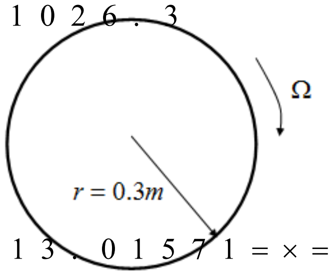
e) The flow of exhaust gas (p = 1.1 bar, T = ..., $\mu = 3.56 \times 10^{-5}$ kg/m s) ... a valve guide of diameter 10 mm in a 1.6 l ... 3000 rev/min (assume 100% volumetric efficiency and ... port diameter of 25 mm)

Solution

a) $\mathrm{Re} = \frac{\rho u L}{\mu} = \frac{10^3 \times \frac{13 \times 10^3}{3600} \times 10}{1.3 \times 10^{-3}} = 2.78 \times 10^7$

b) $T = 400 + 273 = 673\ K$

$\mu = \frac{1.46 \times 10^{-6} \times 673^{3/2}}{(110 + 673)} = 3.26 \times 10^{-5}\ kg/m\,s$



$\Omega = \frac{15000}{60} \times 2\pi = 1571\ rad/s$

$u = \Omega \times r = 1571 \times 0.3 = 471.3\ m/s$

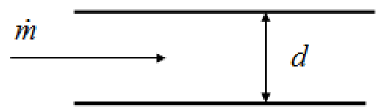
$\rho = \frac{P}{RT} = \frac{100000}{287 \times 673} = 2.59\ kg/m^3$

Characteristic length is r not D

$$Re = \frac{\rho u D}{\mu} = \frac{2.95174 \times 3.3 \times 3}{3.62 \times 10^{-5}} = 1.21 \times 10^{7}$$ (turbulen

c) $\dot{m} = \rho u A = \rho u \frac{\pi D^2}{4}$

$$u = \frac{4\dot{m}}{\rho \pi D^2}$$



$$Re = \frac{\rho u D}{\mu} = \frac{\rho \times 4\dot{m} D}{\rho \pi D^2 \mu} = \frac{4\dot{m}}{\pi D \mu}$$

$$\mu = \frac{1.56 \times 10^{-6} \times 400^{3/2}}{(400 + 233)} = 1.97 \times 10^{-5}\ kg/ms$$

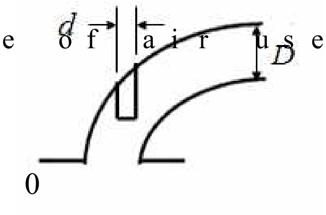
$$Re = \frac{4 \times 0.05}{\pi \times 0.02 \times 1.97 \times 10^{-5}} = 1.6 \times 10^{5}$$ (turbulent)

d) $u = \frac{100 \times 10^{3}}{3600} = 27.8\ m/s$

$$Re = \frac{\rho u L}{\mu} = \frac{1.2 \times 27.8 \times 6}{1.8 \times 10^{-5}} = 1.11 \times 10^{7}$$ (turbulent)

e) Let $\dot{m}$ be the mass flow through the exhaust port

$\dot{m}$ = inlet density X volume of air used in e second



$$\dot{m} = 1.2 \times \frac{1.6 \times 10^{-3}}{4} \times \frac{3600}{60} \times \frac{1}{2} = 0.012\ kg/s$$

$$u = \frac{4\dot{m}}{\rho \pi D^2}$$

$$Re_d = \frac{\rho u d}{\mu}$$

$$Re = \frac{4 \times 0.01 \times 0.012}{\pi \times 0.025 \times 6.3 \times 10^{-5}} = 9686 \quad \text{(laminar)}$$

Comments:

- Note the use of D to obtain the mass flow rat… characteristic length
- Note the different criteria for transition fr… $Re \approx 2300$ … $Re \approx 3 \times 10^5$)

Example 3.3

Calculate the appropriate Grashof numbers and st…

a) A central heating radiator, 0.6 m high with a … $\rho = 1.2\ kg/m^3$, $Pr = 0.72$, $\mu = 1.8 \times 10^{-5}\ kg/m\,s$)]

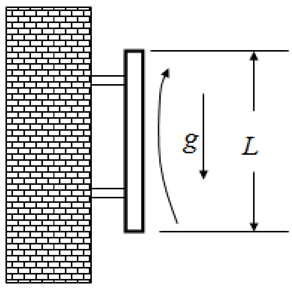
b) A horizontal oil sump, with a surface temperat… oil at 75°C ($\rho = 854\ kg/m^3$, $\beta = 0.7 \times 10^{-3}\ K^{-1}$, $Pr = 546$, $\mu = 3.56 \times 10^{-2}\ kg/m\,s$)

c) The external surface of a heating coil, 30 mm … water at 20°C ($\rho = 1000\ kg/m^3$, $\beta = 0.2 \times 10^{-3}\ K^{-1}$, $Pr = 6.95$, $\mu = 1.0 \times 10^{-3}\ kg/m\,s$)

d) Air at 20°C ($\rho = 1.2\ kg/m^3$, $Pr = 0.72$, $\mu = 1.8 \times 10^{-5}\ kg/m\,s$) adjacent to a … vertical, light bulb with a surface temperatur…

Solution

a) $$Gr = \frac{g \beta \rho^2 L^3 \Delta T}{\mu^2}$$



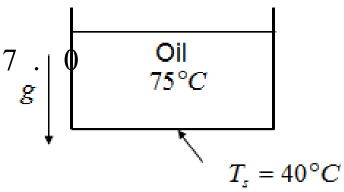
$$\Delta T = 75 - 18 = 57\ K$$

$$\beta = \frac{1}{T} = \frac{1}{(18 + 273)} = \frac{1}{291}\ K^{-1}$$

$$Gr = \frac{9.81 \times 1.2^2 \times 0.6^3 \times 57}{291 \times (1.8 \times 10^{-5})^2} = 1.84 \times 10^{9}$$

$$Gr\,Pr = 1.84 \times 10^{9} \times 0.72 = 1.3 \times 10^{9} \quad \text{(lamina…}$$

b) $$L = \frac{Area}{Perimeter} = \frac{0.4 \times 0.2}{2 \times (0.4 + 0.2)} = 0.0667\ m$$



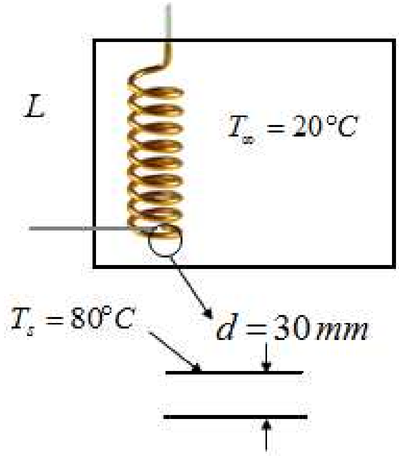
$K T 3 5 4 0 7 5 = - = \Delta$

$Gr = \frac{\beta \Delta \rho\ L^3 T g}{\mu^2} = \frac{\times \times 0 \times 6 \times 6 \times 7 . 0 3 5 1 0 7 . 0 8 1 . 9 8 5 4}{( )\ \times 1 0^2 5 6 . 3} \times 1 = 0\ 1 . 4$

$1 0^7 2 4 . 2 5 4 6 1 0^1 . 4\ Pr \times = \times \times = Gr$

Heated surface facing downward $Pr\ Gr$ results in stab

c)

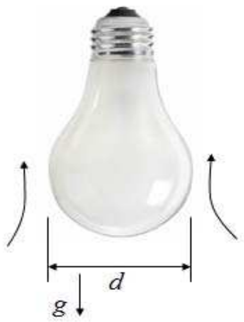


$$\Delta T = 80 - 20 = 60\ K$$

$$Gr = \frac{\beta g \Delta T L^3 \rho^2}{\mu^2} = \frac{\ldots \times 9.81 \times 0.72^2 \times 60 \times \ldots}{(\ldots \times 10^{-3})^2} = 3.6 \times 10^6$$

$$Gr\,Pr = \ldots \times 10^6 \quad (\text{laminar})$$

d) $L = \frac{\text{Area}}{\text{Perimeter}} = \frac{\pi D^2 / 4}{\pi D} = \frac{D}{4}$



$$\Delta T = 90 - 20 = 70\ K$$

$$\beta = \frac{1}{T} = \frac{1}{(20 + 273)} = \frac{1}{293}\ K^{-1}$$

$$Gr = \frac{\beta g \Delta T L^3 \rho^2}{\mu^2} = \frac{9.81 \times 70 \times 0.0125^3 \times 1.2^2}{293 \times (1.8 \times 10^{-5})^2} = 3.5 \times 10^4$$

$$Gr\,Pr = 3.5 \times 10^4 \times 0.72 = 2.5 \times 10^4 \quad (\text{la}$$

Comments:

- Note evaluation of β at film temperature, for gas β is given by $1/T$
- For a horizontal surface $L = A/p$

Example 3.4

Calculate the Nusselt numbers for the following:

a) A flow of gas ($Pr = 0.71$, $\mu = 4.0 \times 10^{-5}$ kg/m s, $c_p = 1.175$ kJ/kg K) over a t… chord length 20 mm, where the average heat tra… W/m^2K.

b) A horizontal electronics component with a surf… long, dissipating 0.1 W by free convection fro… and k = 0.026 W/m K.

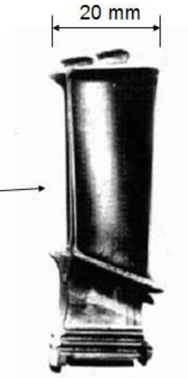
c) A 1 kW central heating radiator 1.5 m long and… dissipating heat by radiation and convection i… body radiation ($\sigma = 56.7 \times 10^{-9}\ W/m^2K^4$)

d) Air at 4°C (k = 0.024 W/m K) adjacent to a wal… 0.3 W/m K, the inside temperature of the wall…

Solution

a) $\Pr = \frac{\mu C_p}{k}$

$$k = \frac{\mu C_p}{\Pr} = \frac{4.63\times10^{-5}\times1175}{0.71} = 0.0766\ W/mK$$



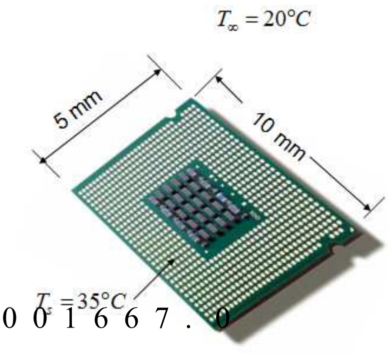
$$Nu = \frac{hL}{k} = \frac{1000\times0.02}{0.0766} = 261$$

b) $Nu = \frac{hL}{k} = \frac{q}{\Delta T}\frac{L}{k}$

$$q = \frac{Q}{A} = \frac{0.1}{0.010\times0.005} = 2000\ W/m^2$$

$$\Delta T = 35 - 20 = 15\ ^\circ C$$

$$L = \frac{\text{Area}}{\text{Perimeter}} = \frac{50}{30} = \frac{5}{3}\ mm = 0.001667\ m$$



$$Nu = \frac{hL}{k} = \frac{2000\times0.001667}{15\times0.026} = 8.5$$

c) $Nu = \frac{q_c}{\Delta T}\frac{L}{k}$

In this case, q must be the convective

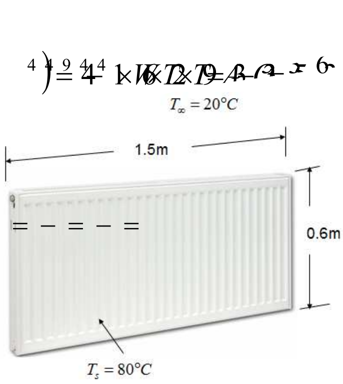
$$T_s = 80 + 273 = 353\ K$$

$$T_\infty = 20 + 273 = 293\ K$$

$$Q_R = \sigma A\left(T_s^4 - T_\infty^4\right) = 5.67\times10^{-8}\times0.9\times\left(353^4 - 293^4\right) = 416\ W$$

$$\Delta T = 80 - 20 = 60\ K$$

$$Q_c = Q - Q_R = 1000 - 416 = 584\ W$$



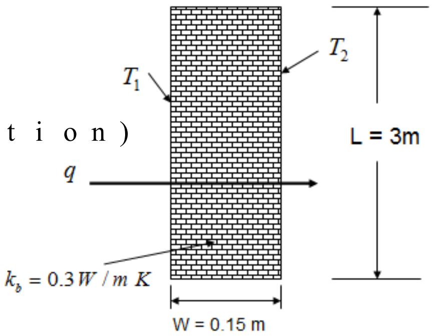
$$q_c = \frac{Q_c}{A} = \frac{584}{1.5 \times 0.6} = 649\ W/m^2$$

$$Nu = \frac{q_c}{\Delta T}\frac{L}{k} = \frac{649}{60}\,\frac{0.6}{0.026} = 249$$

d) $\Delta T = 12 - 4 = 8\ K$

$$q = \frac{k_b (T_1 - T_2)}{W}$$

(assuming 1-D conduction)



$$q = \frac{0.3\,(18 - 12)}{0.15} = 12\ W/m^2$$

$$Nu = \frac{q_c}{\Delta T}\frac{L}{k} = \frac{12}{8}\,\frac{3}{0.024} = 188$$

Comments:

- Nu is based on convective heat flux; sometime
  and must be allowed for.
- The value of k in Nu is that of the fluid (not of solid surf
- Use of appropriate boundary layer growth that

Example 3.5

In forced convection for flow over a flat plate, general expression $Nu_x = C_1 Re_x^n$. In free convection from a vertic
is represented by $Nu_x = C_2 Gr_x^m$ where C_1, C_2, n and m are constants

a) Show that the local heat transfer coefficient difference in forced convection, whereas in fr $h \propto (T_s - T_\infty)^m$

b) In turbulent free convection, it is generally transfer coefficient does not vary with coordi

Solution

a) $Nu_x = \dfrac{h\,x}{k}$

$$Re_x = \frac{\rho\, u\, x}{\mu}$$

For forced convection: $Nu_x = C_1 Re_x^n$

Hence $h = \dfrac{k}{x} C_1 \left(\dfrac{\rho\, u\, x}{\mu}\right)^n$

This shows that the heat transfer coefficient

For free convection $Nu_x = C_2 Gr_x^m$

$$Gr_x = \frac{\beta\, g\, \Delta T\, \rho^2 x^3}{\mu^2}$$

Hence $h = \dfrac{k}{x} C_2 \left(\dfrac{\beta\, g\, \Delta T\, \rho^2 x^3}{\mu^2}\right)^m$ (1)

So for free convection, heat transfer coeffici $h \propto \Delta T^m$

b) From (1), with m = 1/3 for turbulent free conv

$$h = \frac{k}{x} C_2 \left(\frac{\rho^2 g \beta \Delta T x^3}{\mu^2} \right)^{1/3}$$

$$h = \frac{k}{x} C_2 \left(\frac{\rho^2 g \beta \Delta T}{\mu^2} \right)^{1/3} x$$

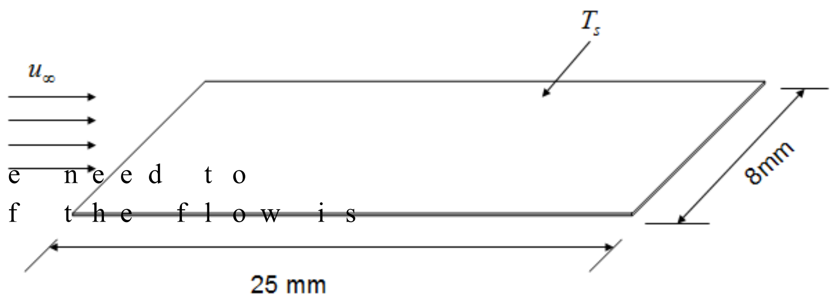
$$= k\, C_2 \left(\frac{\rho^2 g \beta \Delta T}{\mu^2} \right)^{1/3}$$

Hence the convective heat transfer coefficient

Example 3.6

An electrically heated thin foil of length L = 2 ... speed metre. Wind ... T_∞ and velocity U_∞ ... parallel to the lo... is internally heated ($\dot{Q}$ Watts) ... from both sides ... air with ... c_p ... ν ... . The surface temperature ... T_∞ ... is to be measured at the ... be constant. Estimate ... $T_\infty = 20$ °C ... $\dot{Q} = 5$ W ... speed when

Solution



Firstly, we need to estimate if the flow is laminar or turbulent.

Assuming a critical (transition) Reynolds number $Re = 5 \times 10^5$ ...

$$u_{turb} = \frac{Re \times \mu}{\rho L} = \frac{Re \times \nu}{L} = \frac{5 \times 10^5 \times 15.3 \times 10^{-6}}{25 \times 10^{-3}} = 306 \ m/s$$

Wind speed is very unlikely to reach this critic...

$$Nu_x = 0.331 Re_x^{1/2} Pr^{1/3}$$

$$Nu_{L\,av} = 0.662 Re^{1/2} Pr^{1/3} = \frac{q L}{k (T_s - T_\infty)}$$

$$Re_L^{1/2} = \frac{q_{av} L}{(T_s - T_\infty) \times k \times 0.662\, Pr^{1/3}}$$

$$q_{av} = \frac{0.5/2}{0.025 \times 0.008} = 1250\ W/m^2$$

$$Re_L^{1/2} = \frac{1250 \times 0.025}{(32 - 20) \times 0.0253 \times 0.662 \times (0.72)^{1/3}} = 173.5$$

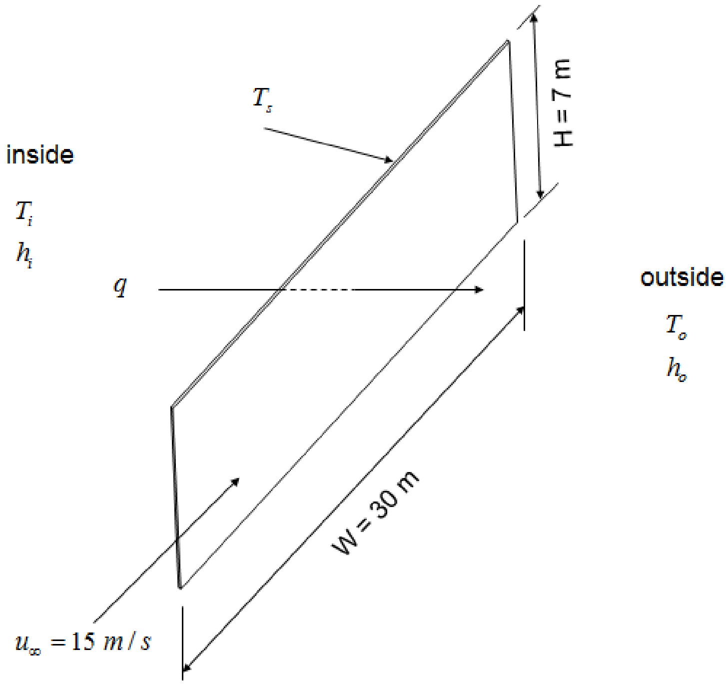
$$Re_L = 3 \times 10^4$$

$$u_\infty = \frac{Re_L\, \nu}{L} = \frac{3 \times 10^4 \times 1.522 \times 10^{-5}}{25 \times 10^{-3}} = 18.3\ m/s$$

Example 3.7

The side of a building of height $H = 7\ m$ and len[gth] ...
heat loss through this glass (ignore the thermal ...
inside the building is 20°C, the outside air te[mperature] ...
the side of the building. Select the appropriat[e] ...
numbers to estimate the *average* heat transfer ... $\mu = 1.8\times10^{-5}$
kg/m s, $C_p = 1$ kJ/kg K and Pr = 0.7.

- Free convection: $Nu_x = 0.59 (Gr_x Pr)^{1/4}$ laminar ($Gr_x Pr < 10^9$)
- Free convection: $Nu_x = 0.1 (Gr_x Pr)^{1/3}$ turbulent ($Gr_x Pr > 10^9$)
- Forced convection: $Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$ ($Re < 5\times10^5$)
- Forced convection: $Nu_x = 0.0288 Re_x^{0.8} Pr^{1/3}$ ($Re > 5\times10^5$)



Solution

$$Pr = \frac{\mu C_p}{k} \quad \text{gives:} \quad k = \frac{\mu C_p}{Pr} = \frac{1.8\times10^{-5}\times1000}{0.7} = 0.026\ W/m\,K$$

First we need to determine if these flows are la[minar] ...

For the inside (Free convection):

$$\beta = \frac{1}{T} = \frac{1}{(20+273)\,K} = \frac{1}{293}\ K^{-1}$$

$$Gr = \frac{g\beta\Delta T L^3 \rho^2}{\mu^2} = \frac{9.81 \times 1.29^2 \times 3^3 \times \Delta T}{293 \times \left(1.8 \times 10^{-5}\right)^2}$$

$$Gr = 5.1 \times 10^{10}\,\Delta T$$

(Flow will be turbulent over most of the surface)

For the outside (Forced convection)

$$Re_L = \frac{\rho u_\infty L}{\mu} = \frac{1.25 \times 10 \times 3}{1.8 \times 10^{-5}} = 1.3 \times 10^{7}$$

(Flow will be turbulent for most of the surface)

Hence we use the following correlations:

On the inside surface $Nu_x = 0.09\, Gr^{1/3}$

On the outside surface $Nu_x = 0.029\, Re_x^{0.8}\, Pr^{1/3}$

For the inside:

$$Nu_x = \frac{h_x x}{k} = 0.09\left(\frac{g\beta(T_s - T_i)x^3\rho^2}{\mu^2}\right)^{1/3}$$

$$h \propto \frac{\left(x^3\right)^{1/3}}{x} = \text{constant}$$

Hence heat transfer coefficient is not a function of x

$$h_{av} = h_{x=L}$$

For the outside:

$$Nu_x = \frac{h_x x}{k} = 0.029\left(\frac{\rho u x}{\mu}\right)^{0.8} Pr^{1/3}$$

$$h = \text{constant} \times \frac{(x)^{0.8}}{x} = C x^{-0.2}$$

$$h_{av} = \frac{1}{L}\int_{x=0}^{x=L} h\,dx = \frac{C}{L}\int_{x=0}^{x=L} x^{-0.2}\,dx = \frac{h_{x=L}}{0.8} \qquad (2)$$

Write a heat balance:

Assuming one-dimensional heat flow and neglecting the thermal resistance of the glass

$$q = h_i\left(T_i - T_s\right)$$

$$q = h_o\left(T_s - T_o\right)$$

$$h_i\left(T_i - T_s\right) = h_o\left(T_s - T_o\right) \qquad (3)$$

From equation 1

$$\frac{h_i\ H}{k} = 0.09\left(\frac{\rho^2 g\left(T_i - T_s\right)H^3}{\mu^2 \times T_i}\right)^{1/3}$$

$$h_i = 0.09\left(\frac{9.81 \times (T_s - T_i) \times 1^3}{\left(1.8 \times 10^{-5}\right)^2 \times 293}\right)^{1/3} \times 0.026$$

$$h_i = 1.24\,(T_s - T_i)^{1/3}$$

From equation 2:

$$\frac{h_o W}{k} = 0.028\left(\frac{\rho u W}{\mu}\right)^{0.8} \Pr^{1/3}$$

$$h_o = \frac{0.026}{0.3} \times 0.028\left(\frac{1.25 \times 0.9 \times 0.3}{1.8 \times 10^{-5}}\right)^{0.8} \times 0.7^{1/3}$$

$$h_o = 26.7\ W/m^2K$$

From (3) with (4) and (5)

$$1.24\,(T_s - T_i)^{4/3} = 26.7\,(T_o - T_s)$$

$$1.24\,(T_s - -20)^{4/3} = 26.7\,(15 - T_s)$$

$$T_s = 15 - 0.0464\,(T_s - -20)^{4/3} \qquad (6)$$

To solve this equation an iterative approach can be used

First guess: $T_s = 10\ °C$

Substitute this on the right hand side of equation (6):

$$T_s = 15 - 0.0464\,(10 - -20)^{4/3} = 10.7\ °C$$

For the second iteration we use the result of the first:

$$T_s = 15 - 0.0464\,(10.7 - -20)^{4/3} = 10.6\ °C$$

The difference between the two iterations is 0.1 °C, so the solution is:

$$T_s \approx 10.6\ °C$$

From which:

$$(\;) \; (\;)_{o \; s \; o} \qquad = + - = / - 1^{2} = 1\,7\,1\,5\,6\,.\,1\,0\,7\,.\,2\,6 \; m \, W \, T \, T \, h \, q$$

$$k \; W \, W \, q \; A \; Q \, 6\,.\,2\,4\,2\,4\,6\,0\,0\,7\,3\,0\,1\,1\,7 = = \times \times = =$$

Example 3.8

The figure below shows part of a heat exchanger
tube and is cooled by fins which are positioned
by convection to the surrounds that are at 27°C.

Estimate the convective heat loss per fin for th
and effect of the cut-out for the tube on the fl

a) natural convection, with an average fin surfac
b) forced convection with an air flow of 15 m/s
with an average fin surface temperature of 37°

The following correlations may be used without p[illegible]
your choice in the answer.

$Nu_x = 0.3\, Re_x^{1/2} Pr^{1/3}$ $Re_x < 3 \times 10^5$

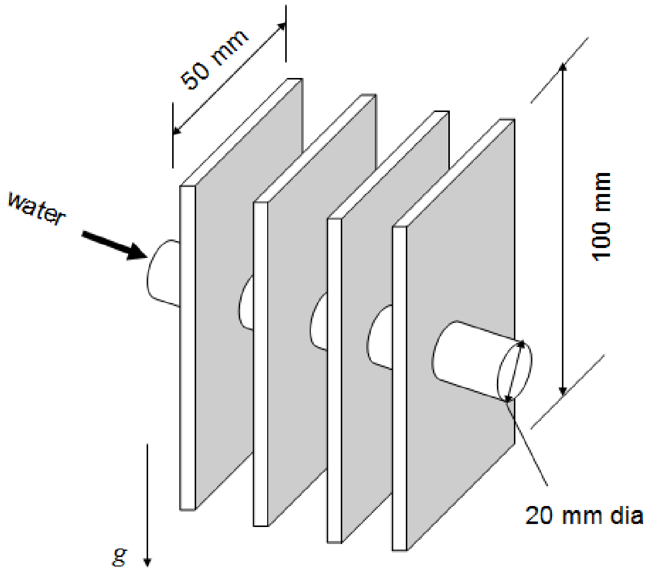
$Nu_x = 0.02\, Re_x^{0.8} Pr^{1/3}$ $Re_x \geq 3 \times 10^5$

$Nu_x = 0.5\, Gr_x^{1/4} Pr^{1/4}$ $Gr_x < 10^9$

$Nu_x = 0.1\, Gr_x^{1/3} Pr^{1/3}$ $Gr_x \geq 10^9$

For air at these conditions, take: [illegible] $Pr = 0.7$ and [illegible]

Solution



On the outside of the water tube, natural convec[illegible]
if flow is laminar ot turbulent

$$Gr = \frac{\beta \rho^2 g \Delta T L^3}{\mu^2}$$

$$\Delta T = 47 - 20 = 27\ K$$

$$\beta = \frac{1}{273 + 20} = \frac{1}{293}\ K^{-1}$$

$$Gr = \frac{[illegible]}{(1.8 \times 10^{-5})^2} = [illegible]$$ (Laminar)

(L here is height because it is in the directio

So we use:

$$Nu_x = 0.5\,(Gr_x Pr)^{1/4}$$

$$h_{av} = \frac{1}{L}\int_0^L h\,dx = \text{const}\int_0^L x^{-1/4}\,dx$$

$$h_{av} = \frac{(h)_{x=L}}{3/4}$$

$$Nu_{av} = \frac{2}{3}(Gr_L Pr)^{1/4}$$

$$Nu_{av} = \frac{2}{3}\left(\;\right)^{1/4} \quad 6 \quad = 2 \times 3 \times 7 = .\;0102$$

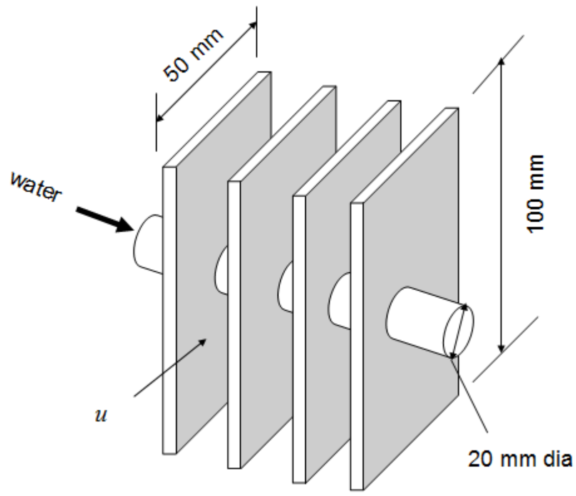
$$h_{av} = \frac{k\,Nu_{av}}{L} = \frac{\times\;02.}{0.1} = 023.6\;W/m^2K$$

$$q_{av} = h_{av}\,\Delta T$$

$$q_{av} = h_{av}\,A\,\Delta T = 2 \times \ldots \; 205.01(0.20 \ldots 6.4 \ldots)$$

$$Q = 0.29\;W$$

For forced convection, we need to evaluate Re to



$$Re = \frac{\rho u L}{\mu} = \frac{1.1 \times 15 \times 0.05}{1.8 \times 10^{-5}} = 4.71 \times 10^{4} \qquad \text{(Laminar)}$$

(L here is the width because flow is along that

$$Nu_x = 0.3\, Re_x^{1/2}\, Pr^{1/3}$$

$$h_{av} = \frac{1}{L}\int_0^L h\, dx = \frac{h_{x=L}}{1/2}$$

$$Nu_{av} = 0.6\, Re_L^{1/2}\, Pr^{1/3} = 0.6 \times \left(4.71 \times 10^{4}\right)^{1/2} \times 0.7^{1/3} = 109$$

$$h_{av} = \frac{k\, Nu_{av}}{L} = \frac{0.02 \times 109}{0.05} = 43.5\ W/m^2K$$

$$Q = h_{av} A \Delta T = 43.5 \times 0.1 \times 0.05 \times (20)\ ^\circ C$$

$$Q = 4.35\ W$$

Example 3.9

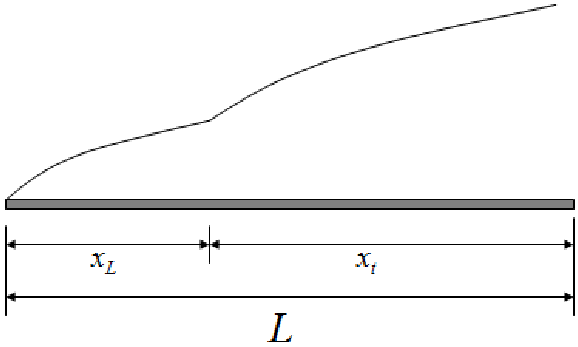
Consider the case of a laminar boundary layer i
turbulent boundary layer. For a constant fluid t
numbers are given by:

$$Nu_x = 0.3\, Re_x^{1/2}\, Pr^{1/3} \quad (Re_x < 10^5)$$
$$Nu_x = 0.04\, Re_x^{0.8}\, Pr^{1/3} \quad (Re_x \geq 10^5)$$

Show that for a plate of length, L, the *average*

$$Nu_{av} = \left(0.05\, Re_L^{0.8} - 310\right) Pr^{1/3}$$

Solution



$$Nu_{av} = \frac{h_{av} L}{k}$$

Where for a constant surface-to-fluid temperatur

$$h_{av} = \frac{1}{L}\left\{\int_0^{x_L} h_{laminar}\,dx + \int_{x_L}^{L} h_{turbulent}\,dx\right\}$$

Since for laminar flow ($Re_x < 10^5$)

$$Nu_x = 0.3\, Re_x^{1/2} Pr^{1/3}$$

$$h_{lam} = 0.3 \frac{k}{x} \times \left(\frac{\rho u_\infty x}{\mu}\right)^{1/2} Pr^{1/3}$$

$$= 0.3\, k \times \left(\frac{\rho u_\infty}{\mu}\right)^{1/2} Pr^{1/3} x^{-1/2} = C_{lam} x^{-1/2}$$

Where C_{lam} does not depend on x

Similarly:

$$h_{turb} = C_{turb} x^{-0.2}$$

Where

$$C_{turb} = 0.04\, k \times \left(\frac{\rho u_\infty}{\mu}\right)^{0.8} Pr^{1/3}$$

Hence

$$h_{av} = \frac{1}{L}\left\{\int_0^{x_L} C_{lam} x^{-1/2} dx + \int_{x_L}^{L} C_{turb} x^{-0.2} dx\right\}$$

$$h_{av} = \frac{1}{L}\left\{C_{lam}\left[\frac{x^{1/2}}{1/2}\right]_0^{x_L} + C_{turb}\left[\frac{x^{0.8}}{0.8}\right]_{x_L}^{L}\right\}$$

$$Nu_{av} = \frac{h_{av} L}{k}$$

$$Nu_{av} = \frac{C_{lam}}{k} 2 x_L^{1/2} + \frac{C_{turb}}{0.8k}\left[L^{0.8} - x_L^{0.8}\right]$$

$$Nu_{av} = 0.6\, Pr^{1/3} \left(\frac{\rho u_\infty x_L}{\mu}\right)^{1/2} + 0.05\, Pr^{1/3}\left[\left(\frac{\rho u_\infty L}{\mu}\right)^{0.8} - \left(\frac{\rho u_\infty x_L}{\mu}\right)^{0.8}\right]$$

But $\frac{\rho_\infty x_L u_\infty}{\mu} = 5\times10^5$ (The transition Reynolds number)

So

$$Nu_{Lav} = Pr^{1/3}\left[0.05\,Re^{0.8} - 0.05\left(5\times10^5\right)^{0.8} + 0.601\left(5\times10^5\right)^{1/2}\right]$$

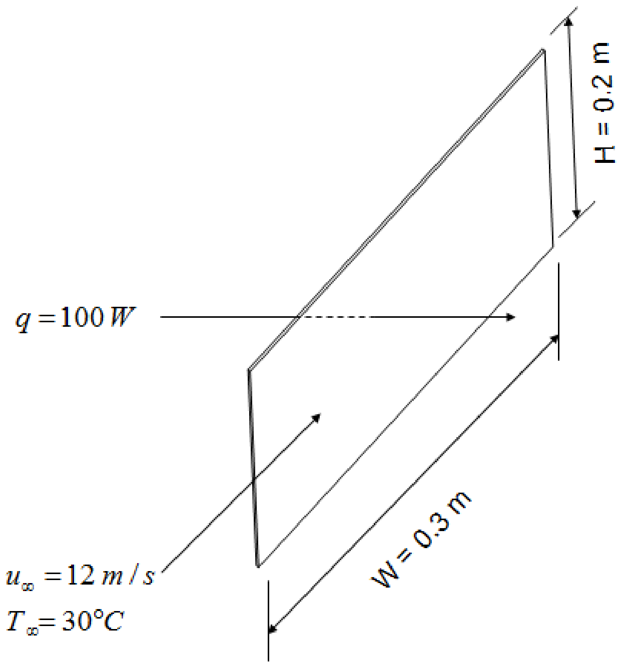
$$Nu_{Lav} = Pr^{1/3}\left(0.05\,Re^{0.8} - 1810\right)$$

Example 3.10

A printed circuit board dissipates 100 W from or
cool this board with a flow speed of 12 m / s pa
average Nusselt number relationship given in Exa
temperature of the board for an air temperature

Take an ambient pressure of 1 bar, R = 287 J / k

Cp = 1 kJ / kg K, k = 0.03 W / m K and μ = 2 x 1$0^{-5}$ kg / m s



Solution

$$q_{av} = \frac{Q}{A} = \frac{100}{0.2\times0.3} = 1666\ W/m^2$$

$$\Pr = \frac{\mu C_P}{k} = \frac{\times 1 \times 0^{-5}\, 1\ 0\ 2^{3}}{0\,3\,.\,0} = 6\,6\,7\,.\,0$$

$$\mathrm{Re}_L = \frac{\rho_\infty L u}{\mu}$$

$$\rho = \frac{P}{R\,T} = \frac{1\,0^{5}}{\times 3\,0\,3\,2\,8\,7} = 1\,5\,g\,.\ 1\ k\,m^3 /$$

$$\mathrm{Re}_L = \frac{\times\ \times 3\,.\,0\,1\,2\,1\,5\,.\,1}{\times 1\,0^{-5}\,2} = 1\,0\,0\,7\,.\,2$$

Using the formula for Nusselt Number obtained in

$$Nu_{L\,av} = \left(\ \right) \Pr^{3/1}\ 3\ 1^{8}\ 0^{0}\ \mathrm{Re}\ 0\,5\,.\,0\ - =$$

$$Nu_{av} = \left(\{\ \}^{-5\quad 8\,.\,0} \right)(\ )^{3/1} = 5 \times 1 - 1 \times 6 \times 6 = 7\,.\,0\,3\,1\,0\,1\,0\,0\,7\,.\,2\,0\,5\,.\,0$$

$$Nu_{av} = \frac{h_{av} L}{k} = \frac{q_a L}{\Delta T\, k}$$

$$\Delta T = \frac{q_{av} L}{k\, Nu_{av}} = \frac{\times\ 3\,.\,0\,7\,1\,6\,6\,6}{\times\ 0\,3\,.\,0\,5\,1\,1\,1} = 6\,.\,3\,2\ ^\circ C$$

$$T_s = T_\infty + \Delta T$$

$$T_s = 3\,2\,3\,0 + 6\,.\,6\,2\,6\,.\ = \ ^\circ C$$

4. Radiation

Example 4.1

In a boiler, heat is radiated from the burning f
The temperatures of the fuel bed and the tubes are T_1 and T_2 respectively

a) Assuming that the side walls (denoted by the s
temperature of the side walls is given by:

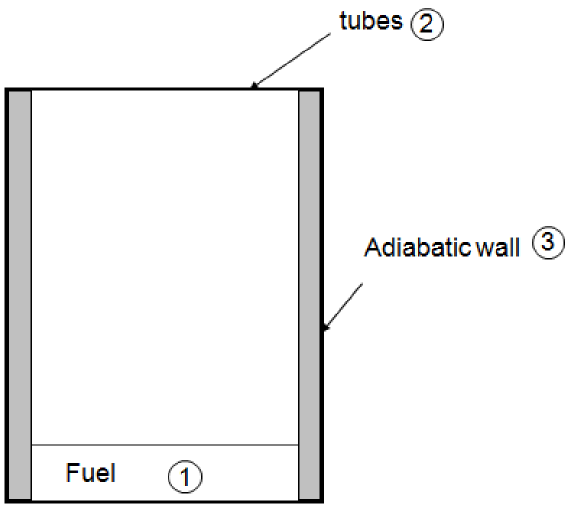
$$T_3 = \left(\frac{F_{13} A_1 T_1^4 + F_{23} A_2 T_2^4}{F_{13} A_1 + F_{23} A_2} \right)^{1/4}$$

where F_{13} and F_{23} are the appropriate view factors.

b) Show that the total radiative heat transfer t $Q_{1,2}$ is given by:

$$\dot{Q}_{1,2} = \left(F_{12} A_1 + \frac{F_{13} A_1 F_{23} A_2}{F_{13} A_1 + F_{23} A_2} \right) \sigma \left(T_1^4 - T_2^4 \right)$$

c) Calculate the radiative heat transfer if $T_1 = 1700\ °C$, $T_2 = 300\ °C$, $A_1 = A_2 = 12\ m^2$ and the the view factors are each 0.5?



Solution

a) $$\dot Q_2 = \dot Q_{1-2} + \dot Q_{3-2} \qquad (1)$$

Since the walls are adiabatic

$$\dot Q_{1-3} = \dot Q_{3-2}$$

From (2)

$$A_1 F_{13}\,\sigma\left(T_1^4 - T_3^4\right) = A_3 F_{32}\,\sigma\left(T_3^4 - T_2^4\right)$$

$$T_3 = \frac{A_1 F_{13} T_1^4 + A_3 F_{32} T_2^4}{A_1 F_{13} + A_3 F_{32}}$$

$$T_3 = \left(\frac{A_1 F_{13} T_1^4 + A_2 F_{23} T_2^4}{A_1 F_{13} + A_2 F_{23}}\right)^{1/4} \qquad \text{since } A_i F_{ij} = A_j F_{ji}$$

b) From (1)

$$\dot Q_2 = A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right) + A_3 F_{32}\,\sigma\left(T_3^4 - T_2^4\right)$$

$$\dot Q_2 = A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right) + A_2 F_{23}\,\sigma\left(T_3^4 - T_2^4\right)$$

$$\dot Q_1 = F_{12}A_1\sigma\left(T_1^4 - T_2^4\right) + \sigma F_{13}A_1\left(T_1^4 - \frac{F_{13}A_1T_1^4 + F_{23}A_2T_2^4}{F_{13}A_1 + F_{23}A_2}\right)$$

$$\dot Q_1 = F_{12}A_1\sigma\left(T_1^4 - T_2^4\right) + \sigma F_{13}A_1\left(\frac{F_{13}A_1T_1^4 + F_{23}A_2T_1^4 - F_{13}A_1T_1^4 - F_{23}A_2T_2^4}{F_{13}A_1 + F_{23}A_2}\right)$$

$$\dot Q_1 = F_{12}A_1\sigma\left(T_1^4 - T_2^4\right) + \sigma F_{13}A_1\left(\frac{F_{23}A_2T_1^4 - F_{23}A_2T_2^4}{F_{13}A_1 + F_{23}A_2}\right)$$

$$\dot Q_1 = \sigma\left(T_1^4 - T_2^4\right)\left(F_{12}A_1 + \frac{F_{13}A_1F_{23}A_2}{F_{13}A_1 + F_{23}A_2}\right)$$

$$\dot Q_1 = \sigma\left(T_1^4 - T_2^4\right)\left(F_{12}A_1 + \frac{F_{13}A_1F_{23}A_2}{F_{13}A_1 + F_{23}A_2}\right)$$

$$\dot Q_1 = \sigma\left(T_1^4 - T_2^4\right)\left(F_{12}A_1 + \frac{F_{13}A_1F_{32}A_3}{F_{13}A_1 + F_{32}A_3}\right)$$

c) $T_3 = \dfrac{F_{13}A_1T_1^4 + F_{23}A_2T_2^4}{F_{13}A_1 + F_{23}A_2} = \dfrac{F_{13}A_1T_1^4 + F_{32}A_3T_2^4}{F_{13}A_1 + F_{32}A_3}$

$$T_3^4 = \frac{0.5\times0.12\times1973^4 + 0.5\times0.12\times573^4}{0.5\times0.12 + 0.5\times0.12} \Rightarrow T_3 = 1662\ K$$

$\dot Q_2$ = 5.67 × 10 ... (1973 ...) (6 + 6 × 6 / 6 + 6) ... = ... 576 ... [illegible]

Example 4.2

Two adjacent compressor discs (Surfaces 1 and 2) ... by a 0.1 wide shroud (Surface 3).

a) Given $F_{12} = 0.6$, calculate all the other view fac...

b) The emissivity and temperature ... $\varepsilon_1 = 0.4$, $T_1 = 800$ K, $\varepsilon_2 = 0.3$, $T_2 = 700$ K ... 1 a... and Surface 3 can be treated as ... $T_3 = 900$ K. Apply ... body radiation analysis to Surface 1 and to Su...

$2.5J_1 - J_2 = 9455.45$ W/m²

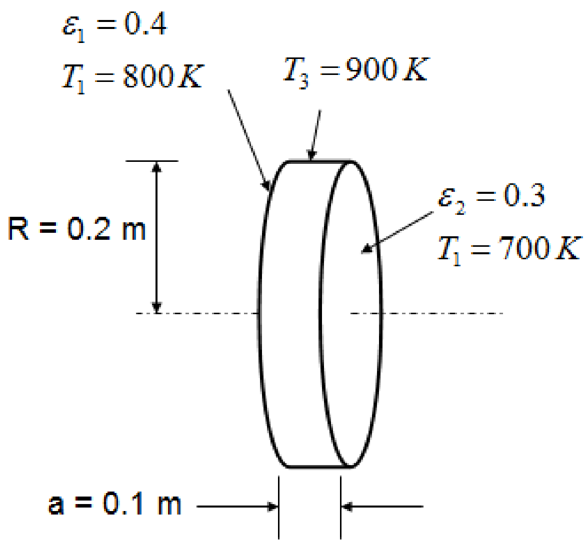
and

$3.333 J_2 - J_1 = 4483.34$ W/m^2

The following equation may be used without pro

$$\frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{N} F_{i,j}(J_i - J_j)$$

c) Determine the radiative heat flux to Surface 2



Solution

a) $r_1 = r_2 = r = 0.2\ m$

$a = 0.1\ m$

$\frac{r_2}{a} = \frac{0.2}{0.1} = 2$

$\frac{a}{r_1} = \frac{0.1}{0.2} = 0.5$

$F_{12} = 0.6$ (Although this is given in the questio
with the above parameters)

$F_{11} = 0$ (As surface 1 is flat, it cannot see its

$F_{13} = 1 - 0.6 = 0.4$ (from $\sum F_{ij} = 1$ in an enclosure)

$F_{21} = 0.6$ (Symmetry)

$F_{22} = 0$

$F_{23} = 0.4$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{\pi \times 0.2^2}{\pi \times 0.2 \times 1} \times 0.4 = 0.04$$

$F_{32} = 0.04$ (Symmetry)

$F_{33} = 1 - 0.04 - 0.04 = 0.92$

b) $\dfrac{E_{b,i} - J_i}{\dfrac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{n} F_{i-j}\left(J_i - J_j\right)$

Apply to surface 1, (i = 1)

Let $\dfrac{1-\varepsilon_1}{\varepsilon_1} = \phi_1$

$\left(E_{b,1} - J_1\right) = \phi_1\left[F_{1-2}\left(J_1 - J_2\right) + F_{1-3}\left(J_1 - J_3\right)\right]$

$J_1\left\{1 + \phi_1 F_{1-2} + \phi_1 F_{1-3}\right\} - \phi_1 F_{1-2} J_2 - \phi_1 F_{1-3} J_3 = E_{b,1}$

$E_{b,1} = \sigma T_1^4$

$J_3 = \sigma T_3^4$ (Radiatively black surface)

$\phi_1 = \dfrac{1-\varepsilon_1}{\varepsilon_1} = \dfrac{1-0.4}{0.4} = 1.5$

$2.50 J_1 - 0.9 J_2 - 0.6\sigma T_3^4 = \sigma T_1^4$

$2.50 J_1 - 0.9 J_2 = 0.6 \; 5.67 \times 10^{-8} \; 900^4 \; 10^{-9}$

$2.5 J_1 - 0.9 J_2 = 54554 \; W/m^2 \quad (1)$

Applying to surface 2 (i = 2)

$J_2\left\{1 + \phi_2 F_{2-1} + \phi_2 F_{2-3}\right\} - \phi_2 F_{2-1} J_1 - \phi_2 F_{2-3} J_3 = E_{b,2}$

$E_{b,2} = \sigma T_2^4$

$\phi_2 = \dfrac{1-\varepsilon_2}{\varepsilon_2} = \dfrac{1-0.3}{0.3} = 2.333$

$3.333 J_2 - 1.40 J_1 - 0.933\sigma T_3^4 = \sigma T_2^4$

$3.333 J_2 - 1.4 J_1 = 43384 \; W/m^2$

c) From (2):

$$J_1 = \frac{J_2 - 48334333.3}{4.1}$$

Substituting in (1)

$$5\times\frac{J_2 - 48334333.3}{4.1} - 2\,mWJ_2 = - \;/\; 4^2\,55459.0$$

$$_2 \quad /\; 2^2\,6099\,mWJ =$$

The net radiative flux to surface 2 is given b

$$q_2 = \frac{E_{b_2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{-\;\;{}^{4\,9}-2\times6\times0997001\,07\;\;56}{\frac{-\;3.01}{3.0}} \times - = \;/m^2W\,0351.5$$

The minus sign indicates a net influx of radia
consideration of surface temperatures.

Example 4.3

The figure below shows a simplified representati
modelled as a cylinder of radius $r_1 = 10$ mm (Surface 1). The burner c
radius $r_2 = 40$ mm and height h = 40 mm), concentric w
of radius $r_3 = 40$ mm. The end of the cylinder, Surface
surrounding environment.

a) Given that $F_{21} = 0.143$ and $F_{22} = 0.445$ use the dimensions indica
all the other relevant view factors.

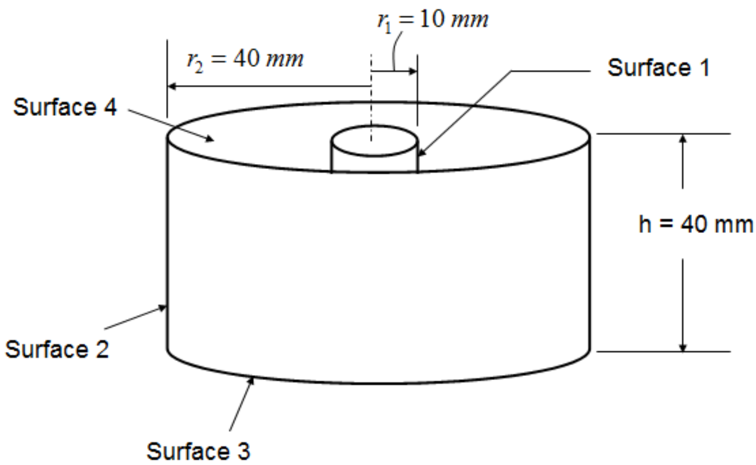
b) The flame, base and surroundings can be represented by T_1, T_3 and T_4, respectively. The emissivity of Surface 2 is $\varepsilon_2 = 0.5$. Applying a gr
radiation analysis to Surface 2 and show that

$$J_2 = \frac{\sigma\left(T_2^4 + F_{21}T_1^4 + F_{23}T_3^4 + F_{24}T_4^4\right)}{1 + F_{21} + F_{23} + F_{24}}$$

The following equation may be used without proof

$$\frac{E_{b,i} - J_i}{(1-\varepsilon_i)/\varepsilon_i} = \sum_{j=1}^{N} F_{ij}\left(J_i - J_j\right)$$

c) The temperatures of the flame and base are found to be $T_1 = 1800$ K and $T_3 = 1200$ K, and the su
500 K. Estimate the temperature T_2, using a radiative heat balance
2, where the emissivity is $\varepsilon_2 = 0.8$



Solution

a) $A_1 = 2\pi r_1 h$

$A_2 = 2\pi r_2 h$

$A_3 = A_4 = \pi\left(r_2^2 - r_1^2\right)$

$F_{11} = 0$

$F_{13} = F_{14}$

$F_{11} + F_{12} + F_{13} + F_{14} = 1$

but

$A_1 F_{12} = A_2 F_{21}$

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{r_2}{r_1} F_{21} = \frac{40}{10} \times 0.14338 = 0.57352$$

Thus

$$F_{13} = F_{14} = \frac{1 - 0.57352}{2} = 0.21324$$

$F_{21} + F_{22} + F_{23} + F_{24} = 1$

$$F_{23} = F_{24} = \frac{1 - F_{21} - F_{22}}{2} = \frac{1 - 0.14338 - 0.44515}{2} = 0.20574$$

$F_{31} + F_{32} + F_{33} + F_{34} = 1$

$F_{33} = 0$

$A_1 F_{13} = A_3 F_{31}$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{2\pi r_1 h}{\pi\left(r_2^2 - r_1^2\right)} F_{13} = \frac{2 \times 0.01 \times 0.04}{0.04^2 - 0.01^2} \times 0.21324 = 0.11373$$

$A_2 F_{23} = A_3 F_{32}$

$$F_{32} = \frac{A_2}{A_3}F_{23} = \frac{2\pi r_2 h}{\pi\left(r_2^2 - r_1^2\right)}F_{23} = \frac{2\times 0.04\times 0.02}{0.04^2 - 0.01^2}\times 0.47502 = 0.19834$$

$$F_{34} = 1 - 0.37311 - 0.19834 = 0.637$$

Similarly (using symmetry)

$$F_{31} = F_{41} = 0.373$$

$$F_{32} = F_{42} = 0.198$$

$$F_{34} = F_{43} = 0.637$$

$$F_{44} = 0$$

b) $\frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{n} F_{ij}\left(J_i - J_j\right)$

For surface 2, i = 2, j = 1, 3, 4

$$\frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2}} = F_{2-1}\left(J_2 - J_1\right) + F_{2-3}\left(J_2 - J_3\right) + F_{2-4}\left(J_2 - J_4\right)$$

$$\varepsilon_2 = 0.5, \quad \frac{1 - 0.5}{0.5} = 1$$

$J_1 = E_{b,1}$ $J_3 = E_{b,3}$ $J_4 = E_{b,4}$ (1, 3, 4 are black)

$$E_{b,2} - J_2 = F_{2-1}\left(J_2 - E_{b,1}\right) + F_{2-3}\left(J_2 - E_{b,3}\right) + F_{2-4}\left(J_2 - E_{b,4}\right)$$

$$J_2\left(1 + F_{2-1} + F_{2-3} + F_{2-4}\right) = \sigma T_2^4 + F_{2-1}\sigma T_1^4 + F_{2-3}\sigma T_3^4 + F_{2-4}\sigma T_4^4$$

$$J_2 = \frac{\sigma\left(T_2^4 + F_{2-1}T_1^4 + F_{2-3}T_3^4 + F_{2-4}T_4^4\right)}{1 + F_{2-1} + F_{2-3} + F_{2-4}}$$

c) $J_2 = \dfrac{{}^{-9}\left(T_2^4 \ldots \times 2 \times 0 + 5 \times 7 + 4 \times . \; 0\,5\,0\,0\,2\,0\,5\,7\,4 \ldots\right)}{1 + 2 + 0\,5\,7\,4\,.\,0\,2\,0\,5\,7\,4\,5\,7\,3\,5\,2\,.\,0 \ldots}$

$J_2 = \ldots 7 \times 10^{-9} T_2^4 + 9\,1\,3\,1\,0\,4\,7\,.\,3\,6$

On the outside of surface 2:

$$q = \varepsilon_{2,0}\,\sigma\left(T_2^4 - T_4^4\right)$$

Also

$$q_2 = \frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2}} = \sigma T_2^4 - \ldots 7 \times 10^{-9} T_2^4 - 9\,1\,3\,1\,0\,4\,7\,.\,3\,6$$

$\ldots 10^{-9} \ldots \left(T_2^4 - 5\,0^4\right) \; 8\,.\,0\,1\,0\,7\,.\,5\,6\,7\,0\,9\,1\,3\,1\,0\,2\,3\,. \ldots$

$T_2 = 1029\ K$

Example 4.4

The figure below shows a schematic diagram, at a … head (Surface 1), piston crown (Surface 2) and c…

a) Using the dimensions indicated on the diagram, … relevant view factors.

b) The cylinder head can be represented as a black body at 1700 K and the … emissivity of $\varepsilon_2 = 0.75$ … piston crown … body radiation … crown (Surface 2) and show that the radiosity …

$$J_2 = 42.5\times10^{-9}\,T_2^4 + 71035 + 0.1\,J_3$$

The following equation may be used without proof…

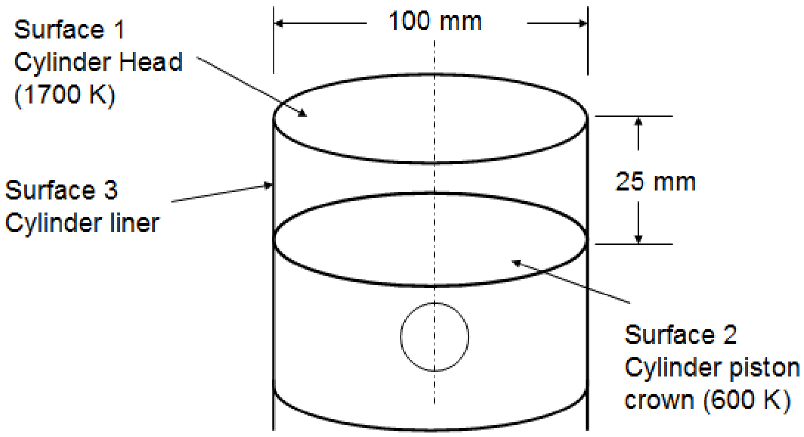
$$\frac{E_{b,i} - J_i}{(1-\varepsilon_i)/\varepsilon_i} = \sum_{j=1}^{N} \left(J_i - J_j\right) F_{ij}$$

c) Similar analysis applied to the cylinder liner …

$$J_3 = 107210 + 0.222\,J_2$$

If the surface temperature of the piston crown, $T_2 = 600$ K, calculate the … the piston crown.

d) Briefly explain how this analysis could be ext…



Solution

a) $A_1 = A_2 = \pi r^2 = \pi \times 50^2$ mm^2

$$A_3 = \pi D L = \pi \times 50 \times 100 = 2\pi \times 50^2\ mm^2$$

$F_{11} = 0$ (Flat surface)

$F_{12} = 0.6$ (Given)

$$F_{13} = 1 - F_{12} = 1.0 - 0.6 = 0.4$$

By Symmetry:

$$F_{12} = F_{21} = 0.6$$

$$F_{32} = F_{23} = 0.4$$

$F_{22} = 0$

$F_{31} = \dfrac{A_1}{A_3} F_{13} = 0.4$ Since $A_1 = A_3$

$F_{32} = 0.4$ (By symmetry)

$$F_{33} = 1.0 - F_{31} - F_{32} = 1.0 - 0.4 - 0.4 = 0.2$$

b) For surface 2, i = 2

$$\frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2}} = (J_2 - J_1)F_{21} + (J_2 - J_3)F_{23}$$

$J_1 = \sigma T_1^4$ (Black body)

$$\varepsilon_2 = 0.75, \quad \frac{1-0.75}{0.75} = \frac{1}{3}$$

$E_{b,2} = \sigma T_2^4$

$$\frac{\sigma T_2^4 - J_2}{1/3} = (J_2 - \sigma T_1^4)F_{21} + (J_2 - J_3)F_{23}$$

$$J_2 = \frac{\sigma T_2^4 + \frac{1}{3}\left(\sigma T_1^4 F_{21} + J_3 F_{23}\right)}{1 + \frac{1}{3}\left(F_{21} + F_{23}\right)}$$

$$J_2 = \frac{56.7\times10^{-9}T_2^4 + \frac{1}{3}\left(0.6\times56.7\times10^{-9}\times1000^4 + 0.4\times J_3\right)}{1 + \frac{1}{3}\left(0.6 + 0.4\right)}$$

$$J_2 = 42.5\times10^{-9}T_2^4 + 0.1J_3 + 1.7010\times35$$

We are also given that

$$J_2 + 0.2J_3 = 27010$$

$$0.2J_2 + J_3 = 11270$$

Hence

$_2$ $-$ ^{49} 0 2 2 $_2$2 . 0 1 0 7 2 1 7 1 0 3 5 6 0 0 1 0 5 . 4

$_2$ $+8=1U$ 7 5 6 5 5 0 8 9 7 7 7 8 . 0

$_2$ $/ 8^2$ 9 2 4 7 $m W J =$

Also

$$q_2 = \frac{E_b - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{-8\times 9 \times 2\ 4\ 7\ 6\ 0\ 0\ 1\ 0\ 7\ ^{49}\ 5\ 6}{3/1} \times - = /m W\ 0\ 2\ 4\ 6$$

Negative sign $E_{b,2} > J_2$ indicates net flux is into the pist

c) To make the analysis more realistic, it needs piston crown, and cylinder liner. Radiation fr We then carry out analysis over a complete eng

Example 4.5

The figure below shows the geometric view factor $F_{i,j}$ ... the case of two rectangular surfaces at right an radiative heat transfer between a turbocharger h system. The horizontal rectangle, W = 0.12 m and denoted Surface 1. The vertical rectangle, h = 0 denoted by Surface 2. The surrounds, which may b of 60°C.

a) Using the graph and also view factor algebra, find $F_{1,2}$, $F_{2,1}$, $F_{1,3}$ and $F_{2,3}$

b) By applying a grey-body radiation analysis, show that ... $\varepsilon_1 = 0.5$ ... J_1 is:

J_1 = 28.3 (10^{-9} T_1^4 + 0.1 J_2 + 325.4) (W/m^2)

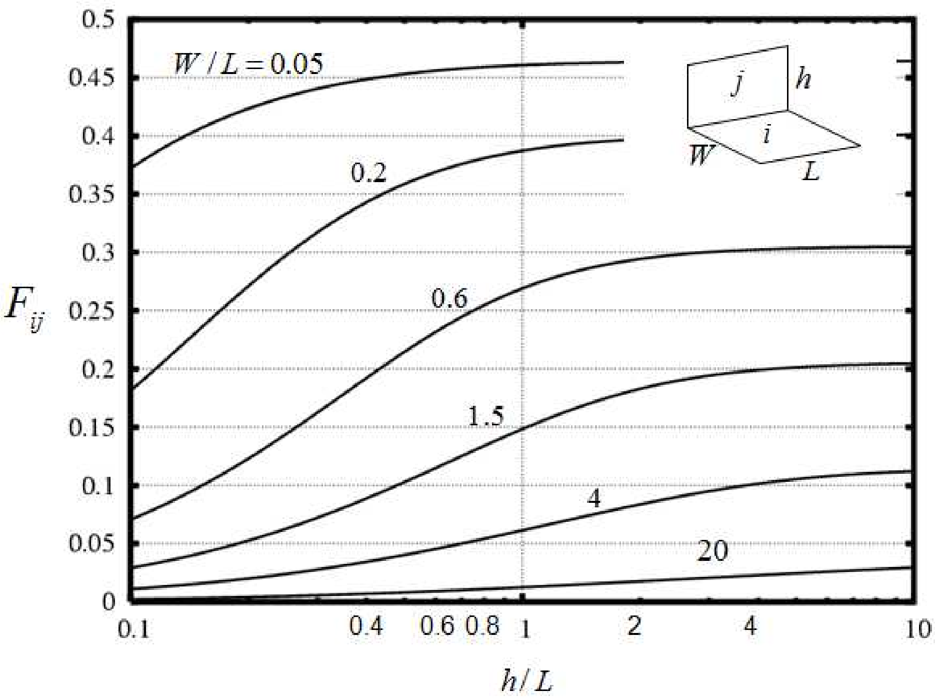
The following equation may be used without proof

$$\frac{E_{b,i} - J_i}{(1-\varepsilon_i)/\varepsilon_i} = \sum_{j=1}^{N} F_{i,j}\left(J_i - J_j\right)$$

c) A similar analysis applied to surface 2 with $\varepsilon_2 = 0.4$ results:

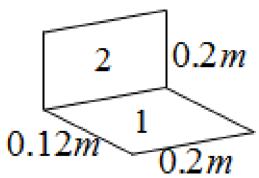
$J_2 = 22.7\times10^{-9}\,T_2^4 + 0.09735\,J_1$ (W/m²)

Use this to estimate the surface temperature of
housing has a surface temperature of $T_2 = 700$ K.



Solution

$$\frac{h}{L} = \frac{0.2}{0.2} = 1, \qquad \frac{W}{L} = \frac{0.12}{0.2} = 0.6$$



From the figure: $F_{12} = 0.27$

$$F_{21}A_2 = F_{12}A_1$$

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{w}{h}F_{12} = \frac{0.12}{0.2}\times 0.27 = 0.162$$

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{11} = 0$$

$$F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

$$F_{21} + F_{22} + F_{23} = 1$$

$$F_{22} = 0$$

$$F_{23} = 1 - F_{21} = 1 - 0.162 = 0.838$$

For a grey body radiative heat transfer in an e

$$\frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{n} F_{ij}\left(J_i - J_j\right)$$

Applying for surface 1, i = 1 (the casing)

$$\frac{E_{b,1}-J_1}{\frac{1-\varepsilon_1}{\varepsilon_1}} = (J_1-J_2)F_{1-2} + (J_1-J_3)F_{1-3}$$

$$E_{b,1} = \sigma T_1^4$$

$$J_3 = \sigma T_3^4$$

$$\frac{1-\varepsilon_1}{\varepsilon_1} = \frac{1-0.5}{0.5} = 1$$

So

$$J_1 = \frac{\sigma T_1^4 + J_2 F_{12} + \sigma T_3^4 F_{13}}{1 + F_{12} + F_{13}}$$

$$J_1 = \frac{5.67\ 3.027.0107.56\ T_1^{4}\ 9\ J_2\ \times 3\times 3\times 3+1+0\times 7}{1 + 0.72 + 0.37}$$

$$J_1 = 2\ 5\ 4\ 1\ 3\ 5.0\ 1\ 0\ 3\ 5.2\ 8\ \times 10^{-9}\ T_1^4 + J_2 + \ldots\ W/m^2$$

c)

Given: $J_2 = 3\ 5\ 0\ 0\ 9\ 7\ 2.0\ 1\ 0\ 6\ 8.2\ 2\ \times 10^{-9}\ T_2^4 + J_1 \ldots\ W/m^2$

$$J_2 = 3\ 5\ 0\ 0\ 9\ 7\ 2.0\ 7\ 0\ 0\ 1\ 0\ 6\ 8.2\ 2\ \ldots 10^{-9}\ \ldots^4\ J_1\ W/m^2$$

$$J_2 = 0\ 9\ 7\ 2.0\ 5\ 7\ 9\ 6\ J_1 + \ldots \qquad (2)$$

Substituting from equation 2 into equation 1:

$$J_1 = 2\ 5\ 4\ 0\ 9\ 7\ 2.0\ 5\ 7\ 9\ 6\ 1\ 3\ 5.0\ 1\ldots \times 10^{-9}\ T_1^4 + (\ \ldots J_1\ )\ W/m^2$$

Which gives:

$$J_1 = 1\ 0\ 5\ 0\ 1\ 0\ 7.2\ 8\ \times 10^{-9}\ T_1^4 + \ldots\ W/m^2$$

Applying a heat balance to surface 1

$$q_{out} = q_{in}$$

$$q_{in} = -\left[\frac{E_{b,1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1}}\right] \quad {}^{9} \quad {}_{1} \quad T_1^4 \; {}^{-9\,4\,4} T + 1 \times 0 - 5 \times 0 - 1 = 0\ 7\ .\ 2\ 8\ 1\ 0\ 9\ .\ 5\ 7$$

$$_{in} \quad {}^{-9} T_1^4 q + 1 \times 0 - 5 = 0\ 1\ 0\ .\ 2\ 8$$

$$_{out} \quad \sigma(\varepsilon_{1\,1} \quad {}_{\infty}) \quad {}^{-9}\left(T_1^4\ T\ T\ 3\ q^{4}\ 3\right) 1\ 0\ 7\ .\ 5\ 6\ 5\ .\ 0 - \times \times = - =$$

Combining and solving for T_1, q gives

$T_1 = 396\ K$

Note that $q_{in} = -q$ since q is out of the surface

5. Heat Exchangers

Example 5.1

A heat exchanger consists of numerous recta… an adjacent pair of channels, there are two … K, separated by a 18 mm wide and 0.5 mm thi… resistances for … $2 \times 10^{-4}\ m^2K/W$ and $5 \times 10^{-4}\ m^2K/W$ respectivel… number given by $Nu_{Dh} = 5.95$ where the subscript 'Dh' refers to the hydrau…

a) Calculate the overall heat transfer coeffi… separating wall and the two fouling resist…
b) Calculate the overall heat transfer coeffi…
c) Which is the controlling heat transfer co…

Solution:

Hydraulic Diameter = 4 x Area / Wetted peri…

$$D_h = \frac{4 \times 2.5 \times 10^{-3} \times 18 \times 10^{-3}}{2(2.5 \times 10^{-3} + 18 \times 10^{-3})} = 4 \times 10^{-3}$$

$$h = \frac{Nu_D\, k}{D_h}$$

$$h_{water} = \frac{5.95 \times 0.625}{4 \times 10^{-3}} = 930\ W/m^2K$$

$$h_{air} = \frac{5.95 \times 0.0371}{4 \times 10^{-3}} = 18.65\ W/m^2K$$

$$U = \left[\frac{1}{930} + \frac{1}{18.65}\right]^{-1} = 18.55\ W/m^2K$$

b) $$U = \left[\frac{1}{930} + \frac{0.5 \times 10^{-3}}{16} + 2 \times 10^{-4} + 5 \times 10^{-4} + \frac{1}{18.65}\right]^{-1} = 18.05\ W/m^2K$$

c) The controlling heat transfer coefficien…

Example 5.2

A heat exchanger tube of D = 20 mm diameter conv
K, ρ = 1000 kg/m³, μ = 0.651×10⁻³ kg/m s) on the inside which is us
outside where the external h_o = 100 W/m²K
thermal resistance of the tube walls, evaluate t
internal heat transfer coefficient is given by t
pipe flow:

$$Nu = 0.023\,Re_D^{0.8}\,Pr_D^{0.4}$$

Solution:

$$\dot{m} = \rho V A$$

$$V = \frac{\dot{m}}{\rho A}$$

$$Re_D = \frac{\rho V D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.0983}{\pi \times 0.02 \times 0.651 \times 10^{-3}} = 9613$$

$$Nu_D = 0.023 \times 9613^{0.8} \times 4.3^{0.4} = 63.3$$

$$Nu_D = \frac{hD}{k}$$

$$h = \frac{k\,Nu_D}{D} = \frac{0.63 \times 63.3}{0.02} = 2000\ W/m^2K$$

$$U = \left[\frac{1}{2000} + \frac{1}{100}\right]^{-1} = 95\ W/m^2K$$

Example 5.3

a) Show that the overall heat transfer coefficien
relation:

$$U_o = \left[\frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{r_o}{r_i h_i} + \frac{1}{h_o}\right]^{-1}$$

With the terminology given by the figure below

b) A heat exchanger made of two concentric tubes
inner tube is made of 3mm wall thickness of st
inner tube radius is 25mm and has a water flow
90mm and has an oil flow rate of 0.12 kg/s. Gi

oil:

$_p$ μ $^{-2}$ $k = \times =$ K W/ m1 38 . 0

Water:

$_p$ μ $^{-6}$ $k = \times =$ K W/ m6 25 . 0

Using the relations:

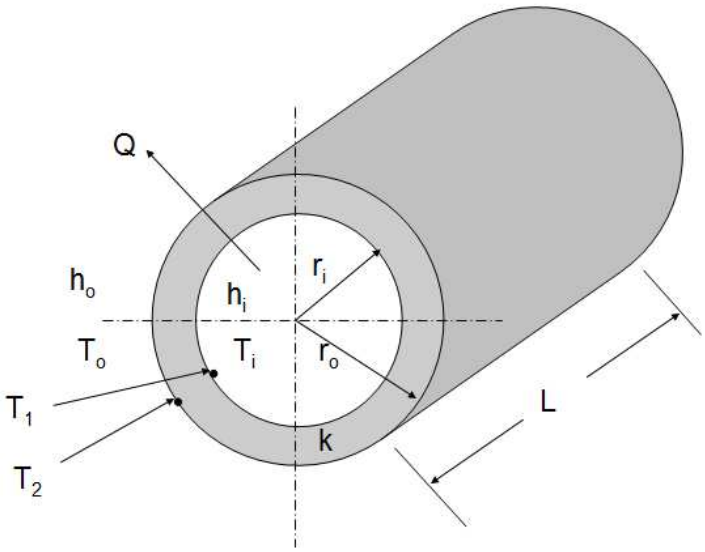
$Nu_D = 6.5 \quad Re_D < 2300$

$Nu_D = 0.023 Re_D^{0.8} Pr^{0.4} \quad Re_D > 2300$

Calculate the overall heat transfer coefficient.

Which is the controlling heat transfer coeffici

If the heat exchange °C ti °C, 5u5 s e id n g °C w a c t a e d r c u a l t a l e f t h e
the tube for a parallel flow heat exchanger



Solution:

a)

For the convection inside

$$Q = h_i A (T_i - T_1)$$

$$Q = 2\pi r_i L h_i (T_i - T_1) \qquad (1)$$

For the convection outside

$$Q = h_o A (T_2 - T_o)$$

$$Q = 2\pi r_o L h_o (T_2 - T_o) \qquad (2)$$

For conduction through the pipe material

$$Q = -2\pi k r L \frac{dT}{dr}$$

$$dT = -\left(\frac{Q}{2\pi L k}\right)\frac{dr}{r} \qquad (3)$$

Integrating between 1 and 2:

$$T_1 - T_2 = \left(\frac{Q}{2\pi L k}\right)\ln\left(\frac{r_o}{r_i}\right) \qquad (4)$$

From 1 and 2

$$T_i - T_1 = \left(\frac{Q}{2\pi L h_i r_i}\right) \qquad (5)$$

$$T_2 - T_o = \left(\frac{Q}{2\pi L h_o r_o}\right) \qquad (6)$$

Adding 4, 5 and 6

$$T_i - T_o = \frac{Q}{2\pi L}\left(\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h_o} + \frac{1}{r_i h_i}\right)$$

Rearranging

$$\frac{Q}{2\pi L r_o} = \frac{T_i - T_o}{\left(\frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} + \frac{r_o}{r_i h_i}\right)} = U_o\left(T_i - T_o\right)$$

Therefore, overall heat transfer coefficient is

$$U_o = \left(\frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o} + \frac{r_o}{r_i h_i}\right)^{-1}$$

b)

i) To calculate the overall heat transfer coeffi
coefficient both inside and outside.

$$Re = \frac{\rho D_h V_m}{\mu}$$

For water:

$$V_m = \frac{\dot{m}}{\rho A}, \qquad A = \frac{\pi D^2}{4}$$

$$Re = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.25}{\pi \times 0.05 \times 725 \times 10^{-6}} = 8781$$

$$Pr = \frac{\mu C_p}{k} = \frac{725 \times 10^{-6} \times 4178}{0.625} = 4.85$$

Re > 2300 (turbulent flow)

Therefore: $Nu_D = 0.023\, Re_D^{0.8}\, Pr^{0.4} = 0.023 \times 8781^{0.8} \times 4.85^{0.4} = 58.26$

From which: $h_i = \frac{Nu_D k}{D} = \frac{625.062 \times}{05.0} = 77\ W/m^2 K$

For oil:

$$D_h = \frac{Area}{Perimeter} = \frac{4\pi (r_a^2 - r_b^2)}{2\pi (r_a + r_b)} = 2(r_a - r_b) = (0.045 - 0.025) = 0.034\ m \ldots$$

$$Re = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m}\, 2(r_a - r_b)}{\pi (r_a^2 - r_b^2)\mu} = \frac{2\dot{m}}{\pi (r_a + r_b)\mu} = \frac{2 \times 0.12}{\pi \times (0.045 + 0.028) \times 5 \ldots} = 33 \ldots$$

Re < 2300 (Laminar flow)

Therefore $Nu_D = 5.6$

$h_o = \frac{Nu_D k}{D_h} = \frac{5.6 \times 0.138}{0.034} = 22.7$ W/m2 K

$$U_o = \left(\frac{0.028}{16} \ln\left(\frac{28}{25}\right) + \frac{0.028}{0.025 \times 77} + \frac{1}{22.7} \right)^{-1} = 21.84\ W/m^2 K$$

ii) The controlling heat transfer coefficient is h_o because … h_o will cause similar changes in the overall heat … little changes. You can check that by doubling … check the effect on the overall heat transfer co…

iii) $T_{hi} = 90°C$, $T_{ci} = 10°C$, $T_{ho} = 55°C$

T_{co} is unknown. This can be computed from an ener…

For the oil side:

$$Q = \dot{m}_h C_{ph} (T_{hi} - T_{ho}) = 0.1 \times 2131 (90 - 55) = 7458\ W$$

$$Q = \dot{m}_c C_{pc} (T_{co} - T_{ci}) = 0.1 \times 4178 (T_{co} - 10) = 7458\ W$$

Therefore $T_{co} = 27.85°C$

Evaluate LMTD

$$\Delta T_1 = 90 - 10 = 80\ ^\circ C$$

$$\Delta T_2 = 44.36\ \ 56.18\ \ 55\ ^\circ C$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80 - 44.36}{\ln(80/44.36)} = 1.56\ ^\circ C$$

$$Q = UA\Delta T_{lm} = U\, 2\pi r_o L \times \Delta T_{lm}$$

$$L = \frac{Q}{2\pi r_o U \Delta T_{lm}} = \frac{8950}{\pi \times \ldots} = 41.5\,m$$

Example 5.4

Figure (a) below shows a cross-sectional view th heated by hot exhaust gases. Figure (b) shows a has a total of 50 channels for the hot exhaust g heat exchanger is 0.3m

Using the information tabulated below, together determine:

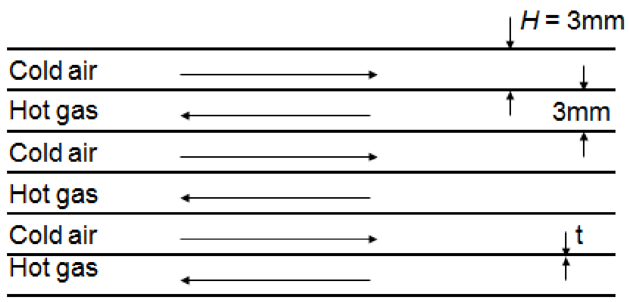
i. the hydraulic diameter for each passage;
ii. the appropriate Reynolds number;
iii. the overall heat transfer coefficient;
iv. the outlet temperature of the cold air;
v. and the length L

Use the following relations:

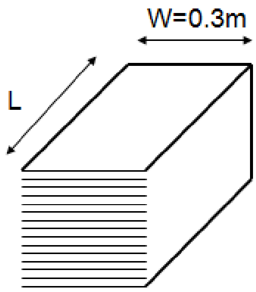
Using the relations:

$Nu_D = 4.6 \qquad Re_D < 2300$

$Nu_D = 0.023\, Re_D^{0.8}\, Pr^{1/3} \qquad Re_D > 2300$



(a) Cross sectional view through part of the Heat Exchanger



(b) Schematic diagram of the complete heat exchanger

Data for example 4.4

Hot exhaust inlet temperature	100 °C
Hot exhaust outlet temperature	70 °C
Cold air inlet temperature	30 °C
Hot exhaust total mass flow	0.1 kg/s
Cold air total mass flow	0.1 kg/s
Density for exhaust and cold air	1 kg/m³
Dynamic viscosity, exhaust and cold air	1.8x10-5 kg
Thermal conductivity, exhaust and cold air	0.02 W/m
Specific heat capacity, exhaust and cold air	1 kJ/kg
Heat exchanger wall thickness	0.5 mm
Heat Exchanger wall thermal conductivity	180 W/m K
Hot exhaust side fouling resistance	0.01 K m²/W
Cold air side fouling resistance	0.002 K m²/W

Solution:

$$Re = \frac{\rho V L}{\mu}$$

$$L = D_h \quad \text{(Hydraulic diameter)}$$

$$D_h = \frac{4 \times \text{area section}}{\text{periment}} = \frac{4 H w}{2(H + w)} = \frac{4 \times 0.003 \times 0.3}{2(0.003 + 0.3)} = 5.94\ mm$$

For a single passage:

$$V = \frac{(\dot{m})}{(\rho) \times H \times w} = \frac{(1.050/50)}{(0.003 \times 3 \times 1)} = 2.2\ m/s$$

$$Re = \frac{1.225 \times 2.2 \times 4.9 \times 10^{-3}}{1.8 \times 10^{-5}} = 733$$

$Re < 2300$ (laminar flow)

$$Nu_D = 4.6$$

$$h = \frac{Nu_D\, k}{D_h} = \frac{4.6 \times 0.020}{5.89 \times 10^{-3}} = 15\ W/m^2 K$$

Since the thermal properties are the same and th cold stream heat transfer coefficients are also

$$U = \left[\frac{1}{h_h} + R_{h,f} + \frac{t}{k} + \frac{1}{h_c} + R_{c,f}\right]^{-1} = \left[\frac{1}{15.5} + 0.01 + \frac{0.5\times10^{-3}}{180} + \frac{1}{15.5} + 0.002\right]^{-1}$$

$$= 7.1\ W/m^2K$$

Note that if the third term in the brackets that will not affect the overall heat transfer coeffi resistance.

$$Q = \dot{m}_h C_p (T_{h,i} - T_{h,o}) = \dot{m}_c C_p (T_{c,o} - T_{c,i})$$

$$(T_{h,i} - T_{h,o}) = (T_{c,o} - T_{c,i}) = (100 - 70) = 30\ °C$$

Also

$$Q = U A \Delta T_{lm}$$

T_{lm} is constant in a balanced flow heat exchanger

$$\Delta T_{lm} = 100 - 60 = 70 - 30 = 40\ °C$$

$$Q = \dot{m}_{h,i}\, C_p\, (T_{h,i} - T_{h,o}) / passage = \frac{0.1}{50} \times 1000 \times (100 - 70) = 60$$

Area of passage:

$$A = \frac{Q}{\Delta T_{lm}\, U} = \frac{60}{7.1 \times 40} = 0.211\ m^2$$

And since $A = w \times L$

$$L = \frac{0.211}{0.3} = 0.704\ m$$