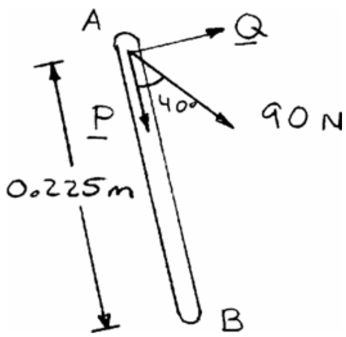
Chapter 3, Solution 1.



Resolve 90 N force into vector components **P** and **Q**

where $Q = (90\text{ N})\sin 40°$

$= 57.851\text{ N}$

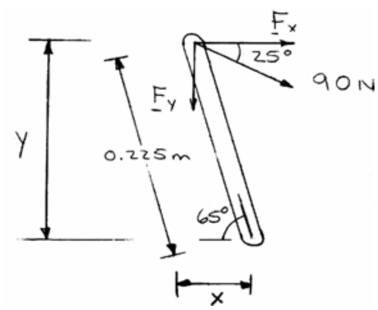
Then $M_B = -r_{A/B}Q$

$= -(0.225\text{ m})(57.851\text{ N})$

$= -13.0165\text{ N}\cdot\text{m}$

$\mathbf{M}_B = 13.02\text{ N}\cdot\text{m}$ ↻ ◀

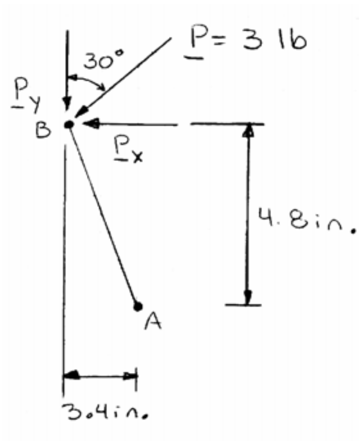
Chapter 3, Solution 2.



$$F_x = (90\text{ N})\cos 25°$$
$$= 81.568\text{ N}$$
$$F_y = (90\text{ N})\sin 25°$$
$$= 38.036\text{ N}$$
$$x = (0.225\text{ m})\cos 65°$$
$$= 0.095089\text{ m}$$
$$y = (0.225\text{ m})\sin 65°$$
$$= 0.20392\text{ m}$$
$$M_B = xF_y - yF_x$$
$$= (0.095089\text{ m})(38.036\text{ N}) - (0.20392\text{ m})(81.568\text{ N})$$
$$= -13.0165\text{ N}\cdot\text{m}$$

$$\mathbf{M}_B = 13.02\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

Chapter 3, Solution 3.



$$P_x = (3\text{ lb})\sin 30°$$

$$= 1.5\text{ lb}$$

$$P_y = (3\text{ lb})\cos 30°$$

$$= 2.5981\text{ lb}$$

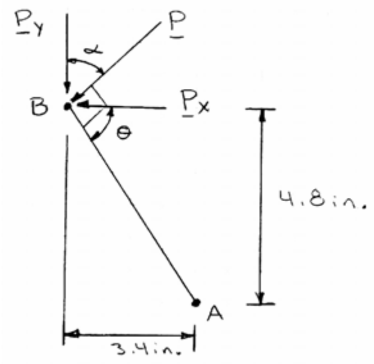
$$M_A = x_{B/A}P_y + y_{B/A}P_x$$

$$= (3.4\text{ in.})(2.5981\text{ lb}) + (4.8\text{ in.})(1.5\text{ lb})$$

$$= 16.0335\text{ lb}\cdot\text{in.}$$

$$\mathbf{M}_A = 16.03\text{ lb}\cdot\text{in.} ↻ ◀$$

Chapter 3, Solution 4.



For **P** to be a minimum, it must be perpendicular to the line joining points *A* and *B*

with $r_{AB} = \sqrt{(3.4\text{ in.})^2 + (4.8\text{ in.})^2}$

$= 5.8822\text{ in.}$

$$\alpha = \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\left(\frac{4.8\text{ in.}}{3.4\text{ in.}}\right)$$

$$= 54.689°$$

Then $M_A = r_{AB}P_{\min}$

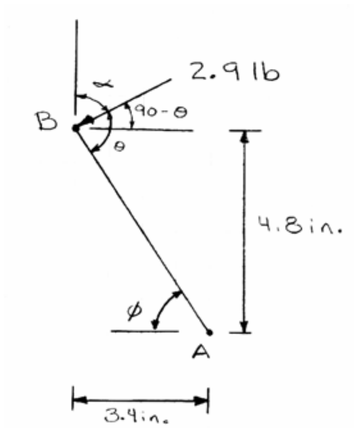
or $P_{\min} = \dfrac{M_A}{r_{AB}} = \dfrac{19.5\text{ lb}\cdot\text{in.}}{5.8822\text{ in.}}$

$= 3.3151\text{ lb}$

$\therefore P_{\min} = 3.32\text{ lb}$ ⦣ $54.7°$

or $P_{\min} = 3.32\text{ lb}$ ⦣ $35.3°$ ◀

Chapter 3, Solution 5.



By definition $M_A = r_{B/A} P \sin\theta$

where $\theta = \phi + (90^\circ - \alpha)$

and $\phi = \tan^{-1}\left(\dfrac{4.8 \text{ in.}}{3.4 \text{ in.}}\right)$

$= 54.689^\circ$

Also $r_{B/A} = \sqrt{(3.4 \text{ in.})^2 + (4.8 \text{ in.})^2}$

$= 5.8822 \text{ in.}$

Then $(17 \text{ lb}\cdot\text{in.}) = (5.8822 \text{ in.})(2.9 \text{ lb})\sin(54.689^\circ + 90^\circ - \alpha)$

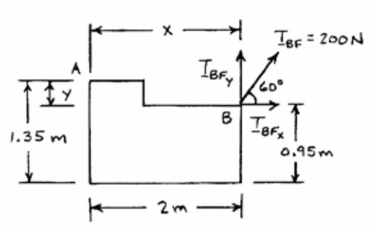
or $\sin(144.689^\circ - \alpha) = 0.99658$

or $144.689^\circ - \alpha = 85.260^\circ; 94.740^\circ$

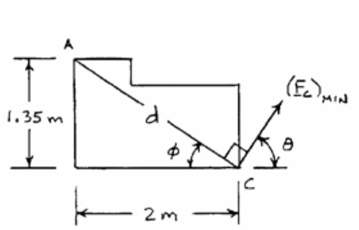
$\therefore \alpha = 49.9^\circ, 59.4^\circ$ ◀

Chapter 3, Solution 6.

(*a*)



(*b*)



(*a*) $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BF}$

$$M_A = xT_{BF_y} + yT_{BF_x}$$

$$= (2 \text{ m})(200 \text{ N})\sin 60° + (0.4 \text{ m})(200 \text{ N})\cos 60°$$

$$= 386.41 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 386 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) For $\mathbf{F}_C$ to be a minimum, it must be perpendicular to the line joining *A* and *C*.

$$\therefore M_A = d\left(F_C\right)_{\min}$$

with $d = \sqrt{(2 \text{ m})^2 + (1.35 \text{ m})^2}$

$$= 2.4130 \text{ m}$$

Then $386.41 \text{ N}\cdot\text{m} = (2.4130 \text{ m})\left(F_C\right)_{\min}$

$$\left(F_C\right)_{\min} = 160.137 \text{ N}$$

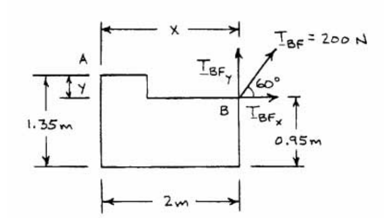
and $\phi = \tan^{-1}\left(\dfrac{1.35 \text{ m}}{2 \text{ m}}\right) = 34.019°$

$$\theta = 90 - \phi = 90° - 34.019° = 55.981°$$

$\therefore \left(\mathbf{F}_C\right)_{\min} = 160.1 \text{ N} \measuredangle 56.0°$ ◀

Chapter 3, Solution 7.

(*a*)



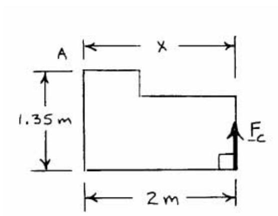
$$M_A = xT_{BF_y} + yT_{BF_x}$$

$$= (2\text{ m})(200\text{ N})\sin 60° + (0.4\text{ m})(200\text{ N})\cos 60°$$

$$= 386.41\text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 386\text{ N}\cdot\text{m}$ ↻ ◀

(*b*)



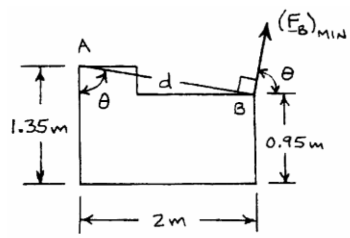
Have $M_A = xF_C$

or $F_C = \dfrac{M_A}{x} = \dfrac{386.41\text{ N}\cdot\text{m}}{2\text{ m}}$

$= 193.205\text{ N}$

$\therefore \mathbf{F}_C = 193.2\text{ N}$ ↑ ◀

(*c*)



For $\mathbf{F}_B$ to be minimum, it must be perpendicular to the line joining *A* and *B*

$$\therefore M_A = d(F_B)_{\min}$$

with $d = \sqrt{(2\text{ m})^2 + (0.40\text{ m})^2} = 2.0396\text{ m}$

Then $386.41\text{ N}\cdot\text{m} = (2.0396\text{ m})(F_C)_{\min}$

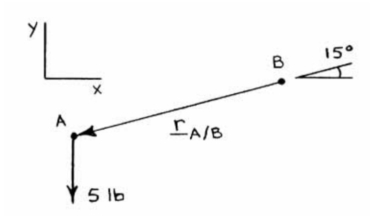
$$(F_C)_{\min} = 189.454\text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{2\text{ m}}{0.4\text{ m}}\right) = 78.690°$

or $(\mathbf{F}_C)_{\min} = 189.5\text{ N} \measuredangle 78.7°$ ◀

Chapter 3, Solution 8.

(*a*)



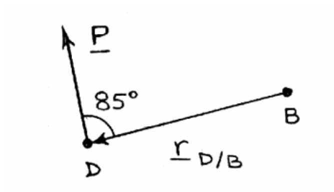
$$M_B = \left(r_{A/B}\cos 15°\right)W$$

$$= (14 \text{ in.})(\cos 15°)(5 \text{ lb})$$

$$= 67.615 \text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_B = 67.6 \text{ lb}\cdot\text{in.}$ ↻ ◀

(*b*)

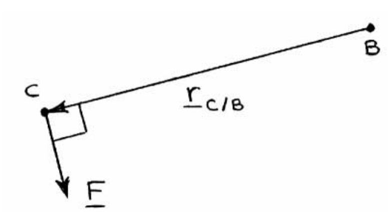


$$M_B = r_{D/B} P \sin 85°$$

$$67.615 \text{ lb}\cdot\text{in.} = (3.2 \text{ in.}) P \sin 85°$$

or $P = 21.2 \text{ lb}$ ◀

(*c*)



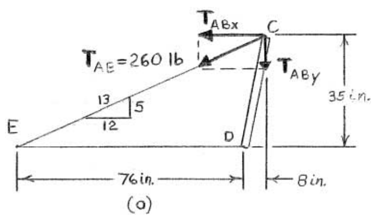
For $(\mathbf{F})_{\text{min}}$, $\mathbf{F}$ must be perpendicular to *BC*.

Then, $M_B = r_{C/B} F$

$$67.615 \text{ lb}\cdot\text{in.} = (18 \text{ in.}) F$$

or $\mathbf{F} = 3.76 \text{ lb}$ ⦪ $75.0°$ ◀

Chapter 3, Solution 9.



(*a*) Slope of line $EC = \dfrac{35\text{ in.}}{76\text{ in.} + 8\text{ in.}} = \dfrac{5}{12}$

Then $T_{ABx} = \dfrac{12}{13}(T_{AB})$

$= \dfrac{12}{13}(260\text{ lb}) = 240\text{ lb}$

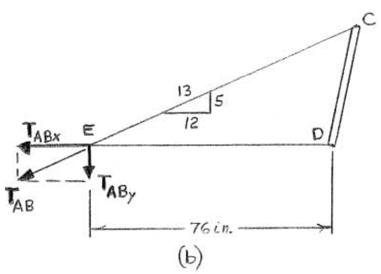
and $T_{ABy} = \dfrac{5}{13}(260\text{ lb}) = 100\text{ lb}$

Then $M_D = T_{ABx}(35\text{ in.}) - T_{ABy}(8\text{ in.})$

$= (240\text{ lb})(35\text{ in.}) - (100\text{ lb})(8\text{ in.})$

$= 7600\text{ lb}\cdot\text{in.}$

or $\mathbf{M}_D = 7600\text{ lb}\cdot\text{in.}$ ↻ ◀



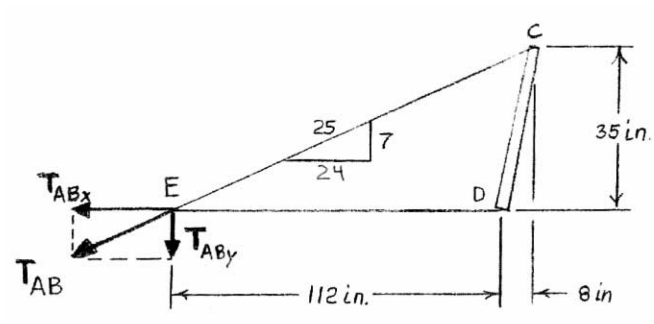
(*b*) Have $M_D = T_{ABx}(y) + T_{ABy}(x)$

$= (240\text{ lb})(0) + (100\text{ lb})(76\text{ in.})$

$= 7600\text{ lb}\cdot\text{in.}$

or $\mathbf{M}_D = 7600\text{ lb}\cdot\text{in.}$ ↻ ◀

Chapter 3, Solution 10.



$$\text{Slope of line } EC = \frac{35 \text{ in.}}{112 \text{ in.} + 8 \text{ in.}} = \frac{7}{24}$$

Then
$$T_{ABx} = \frac{24}{25} T_{AB}$$

and
$$T_{ABy} = \frac{7}{25} T_{AB}$$

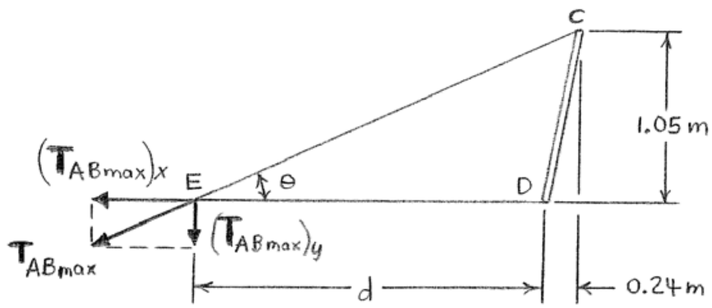
Have
$$M_D = T_{ABx}(y) + T_{ABy}(x)$$

$$\therefore \ 7840 \text{ lb} \cdot \text{in.} = \frac{24}{25} T_{AB}(0) + \frac{7}{25} T_{AB}(112 \text{ in.})$$

$$T_{AB} = 250 \text{ lb}$$

or $T_{AB} = 250$ lb ◀

Chapter 3, Solution 11.



The minimum value of d can be found based on the equation relating the moment of the force $\mathbf{T}_{AB}$ about D:

$$M_D = \left(T_{AB\,\max}\right)_y (d)$$

where

$$M_D = 1152 \text{ N}\cdot\text{m}$$

$$\left(T_{AB\,\max}\right)_y = T_{AB\,\max}\sin\theta = (2880 \text{ N})\sin\theta$$

Now

$$\sin\theta = \frac{1.05 \text{ m}}{\sqrt{(d + 0.24)^2 + (1.05)^2} \text{ m}}$$

$$\therefore\ 1152 \text{ N}\cdot\text{m} = 2880 \text{ N}\left[\frac{1.05}{\sqrt{(d + 0.24)^2 + (1.05)^2}}\right](d)$$

or

$$\sqrt{(d + 0.24)^2 + (1.05)^2} = 2.625d$$

or

$$(d + 0.24)^2 + (1.05)^2 = 6.8906d^2$$

or

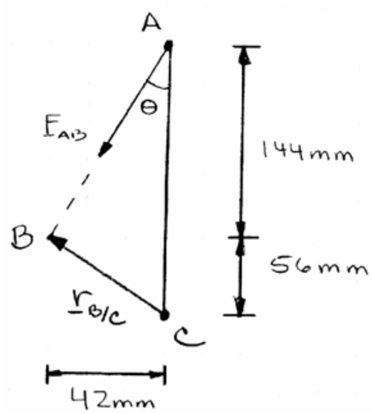
$$5.8906d^2 - 0.48d - 1.1601 = 0$$

Using the quadratic equation, the minimum values of d are 0.48639 m and –0.40490 m.

Since only the positive value applies here, $d = 0.48639$ m

or $d = 486$ mm ◀

Chapter 3, Solution 12.



with $d_{AB} = \sqrt{(42\text{ mm})^2 + (144\text{ mm})^2}$

$= 150\text{ mm}$

$\sin\theta = \dfrac{42\text{ mm}}{150\text{ mm}}$

$\cos\theta = \dfrac{144\text{ mm}}{150\text{ mm}}$

and $\mathbf{F}_{AB} = -F_{AB}\sin\theta\,\mathbf{i} - F_{AB}\cos\theta\,\mathbf{j}$

$= \dfrac{2.5\text{ kN}}{150\text{ mm}}\left[(-42\text{ mm})\mathbf{i} - (144\text{ mm})\mathbf{j}\right]$

$= -(700\text{ N})\mathbf{i} - (2400\text{ N})\mathbf{j}$

Also $\mathbf{r}_{B/C} = -(0.042\text{ m})\mathbf{i} + (0.056\text{ m})\mathbf{j}$

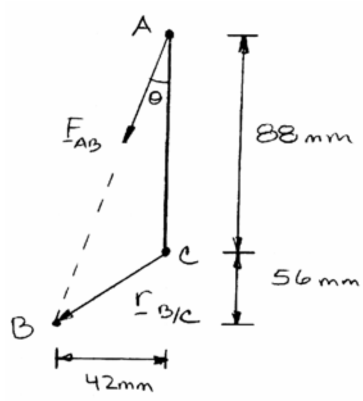
Now $\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_{AB}$

$= (-0.042\mathbf{i} + 0.056\mathbf{j}) \times (-700\mathbf{i} - 2400\mathbf{j})\text{ N}\cdot\text{m}$

$= (140.0\text{ N}\cdot\text{m})\mathbf{k}$

or $\mathbf{M}_C = 140.0\text{ N}\cdot\text{m}$ ↺ ◀

Chapter 3, Solution 13.



with $d_{AB} = \sqrt{(42\text{ mm})^2 + (144\text{ mm})^2}$

$= 150\text{ mm}$

$$\sin\theta = \frac{42\text{ mm}}{150\text{ mm}}$$

$$\cos\theta = \frac{144\text{ mm}}{150\text{ mm}}$$

$$\mathbf{F}_{AB} = -F_{AB}\sin\theta\mathbf{i} - F_{AB}\cos\theta\mathbf{j}$$

$$= \frac{2.5\text{ kN}}{150\text{ mm}}\left[(-42\text{ mm})\mathbf{i} - (144\text{ mm})\mathbf{j}\right]$$

$$= -(700\text{ N})\mathbf{i} - (2400\text{ N})\mathbf{j}$$

Also $\mathbf{r}_{B/C} = -(0.042\text{ m})\mathbf{i} - (0.056\text{ m})\mathbf{j}$

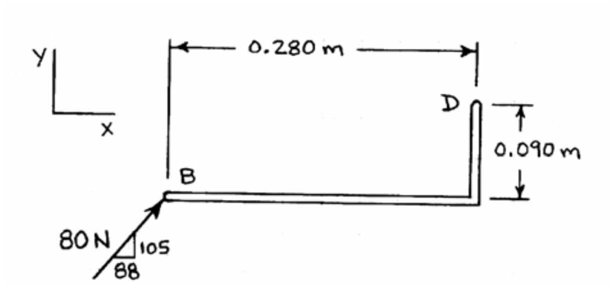
Now $\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_{AB}$

$$= (-0.042\mathbf{i} - 0.056\mathbf{j}) \times (-700\mathbf{i} - 2400\mathbf{j})\text{N}\cdot\text{m}$$

$$= (61.6\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_C = 61.6\text{ N}\cdot\text{m}$ ↺ ◀

Chapter 3, Solution 14.



$$+\circlearrowleft \Sigma M_D: \quad M_D = (0.090\text{ m})\left(\frac{88}{137} \times 80\text{ N}\right) - (0.280\text{ m})\left(\frac{105}{137} \times 80\text{ N}\right)$$

$$= -12.5431\text{ N}\cdot\text{m}$$

or $\mathbf{M}_D = 12.54\text{ N}\cdot\text{m}$ ↻ ◀

Chapter 3, Solution 15.

Note: $\mathbf{B} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})$

$\mathbf{B}' = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})$

$\mathbf{C} = C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$

By definition: $|\mathbf{B}\times\mathbf{C}| = BC\sin(\alpha - \beta)$ (1)

$|\mathbf{B}'\times C| = BC\sin(\alpha + \beta)$ (2)

Now ... $\mathbf{B}\times\mathbf{C} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$

$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k}$ (3)

and $\mathbf{B}'\times\mathbf{C} = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$

$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k}$ (4)

Equating the magnitudes of $\mathbf{B}\times\mathbf{C}$ from equations (1) and (3) yields:

$BC\sin(\alpha - \beta) = BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)$ (5)

Similarly, equating the magnitudes of $\mathbf{B}'\times\mathbf{C}$ from equations (2) and (4) yields:

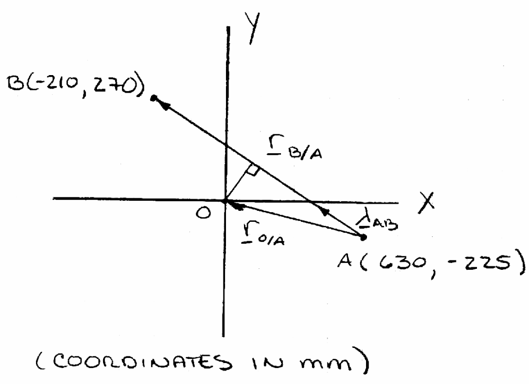
$BC\sin(\alpha + \beta) = BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)$ (6)

Adding equations (5) and (6) gives:

$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\cos\beta\sin\alpha$

or $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha + \beta) + \frac{1}{2}\sin(\alpha - \beta)$ ◀

Chapter 3, Solution 16.



Have $d = \left|\boldsymbol{\lambda}_{AB} \times \mathbf{r}_{O/A}\right|$

where $\boldsymbol{\lambda}_{AB} = \dfrac{\mathbf{r}_{B/A}}{r_{B/A}}$

and $\mathbf{r}_{B/A} = (-210\text{ mm} - 630\text{ mm})\mathbf{i}$

$+ (270\text{ mm} - (-225\text{ mm}))\mathbf{j}$

$= -(840\text{ mm})\mathbf{i} + (495\text{ mm})\mathbf{j}$

$$r_{B/A} = \sqrt{(-840\text{ mm})^2 + (495\text{ mm})^2}$$

$= 975\text{ mm}$

$$\text{Then } \lambda_{AB} = \frac{-(840\text{ mm})\mathbf{i} + (495\text{ mm})\mathbf{j}}{975\text{ mm}}$$

$$= \frac{1}{65}(-56\mathbf{i} + 33\mathbf{j})$$

Also $\mathbf{r}_{O/A} = (0 - 630)\mathbf{i} + (0 - (-225))\mathbf{j}$

$= -(630\text{ mm})\mathbf{i} + (225\text{ mm})\mathbf{j}$

$$\therefore d = \left|\frac{1}{65}(-56\mathbf{i} + 33\mathbf{j}) \times \left[-(630\text{ mm})\mathbf{i} + (225\text{ mm})\mathbf{j}\right]\right|$$

$= 126.0\text{ mm}$

$d = 126.0\text{ mm}$ ◀

Chapter 3, Solution 17.

(*a*)
$$\boldsymbol{\lambda} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

where
$$\mathbf{A} = 12\mathbf{i} - 6\mathbf{j} + 9\mathbf{k}$$
$$\mathbf{B} = -3\mathbf{i} + 9\mathbf{j} - 7.5\mathbf{k}$$

Then
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -6 & 9 \\ -3 & 9 & -7.5 \end{vmatrix}$$
$$= (45 - 81)\mathbf{i} + (-27 + 90)\mathbf{j} + (108 - 18)\mathbf{k}$$
$$= 9(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})$$

And $|\mathbf{A} \times \mathbf{B}| = 9\sqrt{(-4)^2 + (7)^2 + (10)^2} = 9\sqrt{165}$

$$\therefore \boldsymbol{\lambda} = \frac{9(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})}{9\sqrt{165}}$$

or $\boldsymbol{\lambda} = \dfrac{1}{\sqrt{165}}(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})$ ◀

(*b*)
$$\boldsymbol{\lambda} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

where
$$\mathbf{A} = -14\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}$$
$$\mathbf{B} = 3\mathbf{i} + 1.5\mathbf{j} - \mathbf{k}$$

Then
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -14 & -2 & 8 \\ 3 & 1.5 & -1 \end{vmatrix}$$
$$= (2 - 12)\mathbf{i} + (24 - 14)\mathbf{j} + (-21 + 6)\mathbf{k}$$
$$= 5(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

and
$$|\mathbf{A} \times \mathbf{B}| = 5\sqrt{(-2)^2 + (2)^2 + (-3)^2} = 5\sqrt{17}$$
$$\therefore \boldsymbol{\lambda} = \frac{5(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})}{5\sqrt{17}}$$

or $\boldsymbol{\lambda} = \dfrac{1}{\sqrt{17}}(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$ ◀

Chapter 3, Solution 18.

(*a*) Have $A = |\mathbf{P} \times \mathbf{Q}|$

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 7 & -2 \\ -5 & 1 & 3 \end{vmatrix} \text{ in.}^2$$

$$= \left[(21 + 2)\mathbf{i} + (10 - 9)\mathbf{j} + (3 + 35)\mathbf{k}\right]\text{in.}^2$$

$$= \left(23 \text{ in.}^2\right)\mathbf{i} + \left(1 \text{ in.}^2\right)\mathbf{j} + \left(38 \text{ in.}^2\right)\mathbf{k}$$

$$\therefore A = \sqrt{(23)^2 + (1)^2 + (38)^2} = 44.430 \text{ in.}^2$$

or $A = 44.4 \text{ in.}^2$ ◀

(*b*) $A = |\mathbf{P} \times \mathbf{Q}|$

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 3 \\ 6 & -1 & 5 \end{vmatrix} \text{ in.}^2$$

$$= \left[(-20 - 3)\mathbf{i} + (-18 - 10)\mathbf{j} + (-2 + 24)\mathbf{k}\right] \text{in.}^2$$

$$= -\left(23 \text{ in.}^2\right)\mathbf{i} - \left(28 \text{ in.}^2\right)\mathbf{j} + \left(22 \text{ in.}^2\right)\mathbf{k}$$

$$\therefore A = \sqrt{(-23)^2 + (-28)^2 + (22)^2} = 42.391 \text{ in.}^2$$

or $A = 42.4 \text{ in.}^2$ ◀

Chapter 3, Solution 19.

(*a*) Have $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 3 & 1.5 \\ 7.5 & 3 & -4.5 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= \left[(-13.5 - 4.5)\mathbf{i} + (11.25 - 27)\mathbf{j} + (-18 - 22.5)\mathbf{k}\right] \text{N}\cdot\text{m}$$

$$= \left(-18.00\mathbf{i} - 15.75\mathbf{j} - 40.5\mathbf{k}\right) \text{N}\cdot\text{m}$$

or $\mathbf{M}_O = -(18.00\ \text{N}\cdot\text{m})\mathbf{i} - (15.75\ \text{N}\cdot\text{m})\mathbf{j} - (40.5\,\text{N}\cdot\text{m})\mathbf{k}$ ◀

(*b*) Have $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -0.75 & -1 \\ 7.5 & 3 & -4.5 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= \left[(3.375 + 3)\mathbf{i} + (-7.5 + 9)\mathbf{j} + (6 + 5.625)\mathbf{k}\right] \text{N}\cdot\text{m}$$

$$= \left(6.375\mathbf{i} + 1.500\mathbf{j} + 11.625\mathbf{k}\right) \text{N}\cdot\text{m}$$

or $\mathbf{M}_O = (6.38\ \text{N}\cdot\text{m})\mathbf{i} + (1.500\ \text{N}\cdot\text{m})\mathbf{j} + (11.63\ \text{N}\cdot\text{m})\mathbf{k}$ ◀

(*c*) Have $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2.5 & -1 & 1.5 \\ 7.5 & 3 & 4.5 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= \left[(4.5 - 4.5)\mathbf{i} + (11.25 - 11.25)\mathbf{j} + (-7.5 + 7.5)\mathbf{k}\right] \text{N}\cdot\text{m}$$

or $\mathbf{M}_O = 0$ ◀

This answer is expected since **r** and **F** are proportional $(\mathbf{F} = -3\mathbf{r})$. Therefore, vector **F** has a line of action passing through the origin at *O*.

Chapter 3, Solution 20.

(*a*) Have $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7.5 & 3 & -6 \\ 3 & -6 & 4 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= \left[(12 - 36)\mathbf{i} + (-18 + 30)\mathbf{j} + (45 - 9)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

or $\mathbf{M}_O = -(24.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (12.00 \text{ lb}\cdot\text{ft})\mathbf{j} + (36.0 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

(*b*) Have $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7.5 & 1.5 & -1 \\ 3 & -6 & 4 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= \left[(6 - 6)\mathbf{i} + (-3 + 3)\mathbf{j} + (4.5 - 4.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

or $\mathbf{M}_O = 0$ ◀

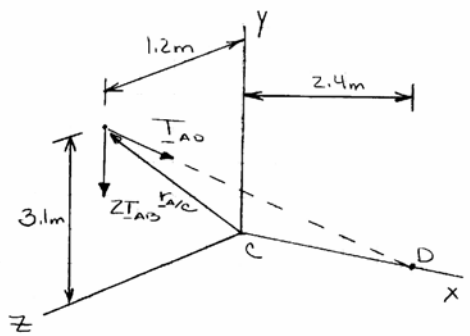
(*c*) Have $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 2 & -14 \\ 3 & -6 & 4 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= \left[(8 - 84)\mathbf{i} + (-42 + 32)\mathbf{j} + (48 - 6)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

or $\mathbf{M}_O = -(76.0 \text{ lb}\cdot\text{ft})\mathbf{i} - (10.00 \text{ lb}\cdot\text{ft})\mathbf{j} + (42.0 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

Chapter 3, Solution 21.



With $\quad \mathbf{T}_{AB} = -(369\text{ N})\mathbf{j}$

$$\mathbf{T}_{AB} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (369\text{ N})\frac{(2.4\text{ m})\mathbf{i} - (3.1\text{ m})\mathbf{j} - (1.2\text{ m})\mathbf{k}}{\sqrt{(2.4\text{ m})^2 + (-3.1\text{ m})^2 + (-1.2\text{ m})^2}}$$

$$\mathbf{T}_{AD} = (216\text{ N})\mathbf{i} - (279\text{ N})\mathbf{j} - (108\text{ N})\mathbf{k}$$

Then $\quad \mathbf{R}_A = 2\,\mathbf{T}_{AB} + \mathbf{T}_{AD}$

$$= (216\text{ N})\mathbf{i} - (1017\text{ N})\mathbf{j} - (108\text{ N})\mathbf{k}$$

Also $\quad \mathbf{r}_{A/C} = (3.1\text{ m})\mathbf{i} + (1.2\text{ m})\mathbf{k}$

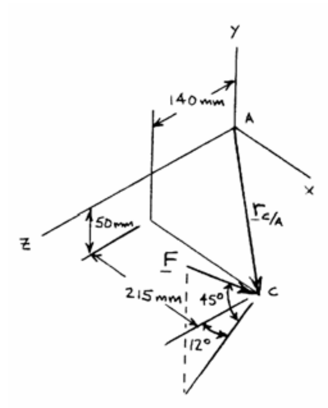
Have $\quad \mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{R}_A$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3.1 & 1.2 \\ 216 & -1017 & -108 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (885.6\text{ N}\cdot\text{m})\mathbf{i} + (259.2\text{ N}\cdot\text{m})\mathbf{j} - (669.6\text{ N}\cdot\text{m})\mathbf{k}$$

$$\mathbf{M}_C = (886\text{ N}\cdot\text{m})\mathbf{i} + (259\text{ N}\cdot\text{m})\mathbf{j} - (670\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 22.



Have $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}$

where $\mathbf{r}_{C/A} = (215 \text{ mm})\mathbf{i} - (50 \text{ mm})\mathbf{j} + (140 \text{ mm})\mathbf{k}$

$$F_x = -(36 \text{ N})\cos 45° \sin 12°$$

$$F_y = -(36 \text{ N})\sin 45°$$

$$F_z = -(36 \text{ N})\cos 45° \cos 12°$$

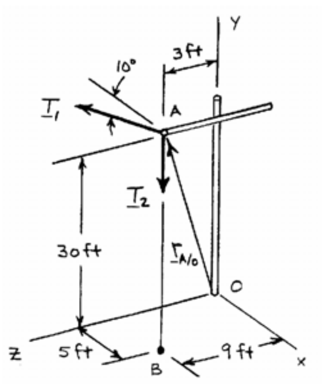
$$\therefore \mathbf{F} = -(5.2926 \text{ N})\mathbf{i} - (25.456 \text{ N})\mathbf{j} - (24.900 \text{ N})\mathbf{k}$$

and $$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.215 & -0.050 & 0.140 \\ -5.2926 & -25.456 & -24.900 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (4.8088 \text{ N}\cdot\text{m})\mathbf{i} + (4.6125 \text{ N}\cdot\text{m})\mathbf{j} - (5.7377 \text{ N}\cdot\text{m})\mathbf{k}$$

$$\mathbf{M}_A = (4.81 \text{ N}\cdot\text{m})\mathbf{i} + (4.61 \text{ N}\cdot\text{m})\mathbf{j} - (5.74 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 23.



Have $\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{R}$

where $\mathbf{r}_{A/D} = (30\text{ ft})\mathbf{j} + (3\text{ ft})\mathbf{k}$

$$\mathbf{T}_1 = -\left[(62\text{ lb})\cos 10°\right]\mathbf{i} - \left[(62\text{ lb})\sin 10°\right]\mathbf{j}$$

$$= -(61.058\text{ lb})\mathbf{i} - (10.766\text{ lb})\mathbf{j}$$

$$\mathbf{T}_2 = T_2 \frac{\overline{AB}}{AB}$$

$$= (62\text{ lb})\frac{(5\text{ ft})\mathbf{i} - (30\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k}}{\sqrt{(5\text{ ft})^2 + (-30\text{ ft})^2 + (6\text{ ft})^2}}$$

$$= (10\text{ lb})\mathbf{i} - (60\text{ lb})\mathbf{j} + (12\text{ lb})\mathbf{k}$$

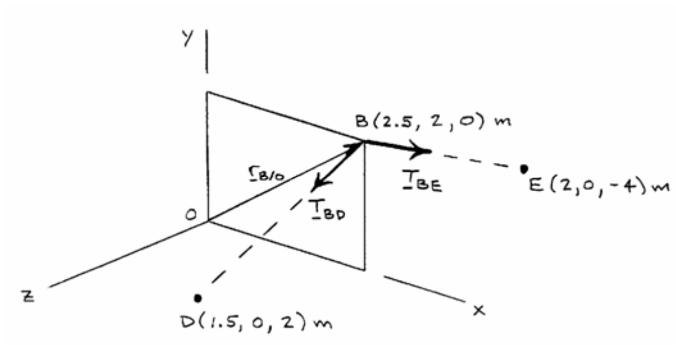
$$\therefore \mathbf{R} = -(51.058\text{ lb})\mathbf{i} - (70.766\text{ lb})\mathbf{j} + (12\text{ lb})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 30 & 3 \\ -51.058 & -70.766 & 12 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= (572.30\text{ lb}\cdot\text{ft})\mathbf{i} - (153.17\text{ lb}\cdot\text{ft})\mathbf{j} + (1531.74\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\mathbf{M}_O = (572\text{ lb}\cdot\text{ft})\mathbf{i} - (153.2\text{ lb}\cdot\text{ft})\mathbf{j} + (1532\text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 24.



(*a*) Have $\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BD}$

where $\mathbf{r}_{B/O} = (2.5\text{ m})\mathbf{i} + (2\text{ m})\mathbf{j}$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD}$$

$$= (900\text{ N})\frac{\left[-(1\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} + (2\text{ m})\mathbf{k}\right]}{\sqrt{(-1\text{ m})^2 + (-2\text{ m})^2 + (2\text{ m})^2}}$$

$$= -(300\text{ N})\mathbf{i} - (600\text{ N})\mathbf{j} + (600\text{ N})\mathbf{k}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 2 & 0 \\ -300 & -600 & 600 \end{vmatrix}\text{ N}\cdot\text{m}$$

$$\mathbf{M}_O = (1200\text{ N}\cdot\text{m})\mathbf{i} - (1500\text{ N}\cdot\text{m})\mathbf{j} - (900\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

continued

(*b*) Have $\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BE}$

where $\mathbf{r}_{B/O} = (2.5 \text{ m})\mathbf{i} + (2 \text{ m})\mathbf{j}$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE}$$

$$= (675 \text{ N})\frac{\left[-(0.5 \text{ m})\mathbf{i} - (2 \text{ m})\mathbf{j} - (4 \text{ m})\mathbf{k}\right]}{\sqrt{(0.5 \text{ m})^2 + (-2 \text{ m})^2 + (-4 \text{ m})^2}}$$

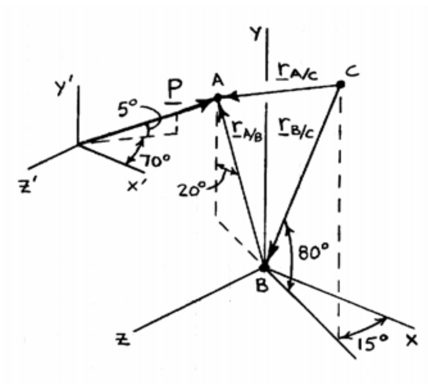
$$= -(75 \text{ N})\mathbf{i} - (300 \text{ N})\mathbf{j} - (600 \text{ N})\mathbf{k}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 2 & 0 \\ -75 & -300 & -600 \end{vmatrix} \text{N}\cdot\text{m}$$

$$\mathbf{M}_O = -(1200 \text{ N}\cdot\text{m})\mathbf{i} + (1500 \text{ N}\cdot\text{m})\mathbf{j} - (600 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 25.



Have $\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{P}$

where

$$\mathbf{r}_{A/C} = \mathbf{r}_{B/C} + \mathbf{r}_{A/B}$$

$$= (16 \text{ in.})(-\cos 80° \cos 15° \mathbf{i} - \sin 80° \mathbf{j} - \cos 80° \sin 15° \mathbf{k})$$

$$+(15.2 \text{ in.})(-\sin 20° \cos 15° \mathbf{i} + \cos 20° \mathbf{j} - \sin 20° \sin 15° \mathbf{k})$$

$$= -(7.7053 \text{ in.})\mathbf{i} - (1.47360 \text{ in.})\mathbf{j} - (2.0646 \text{ in.})\mathbf{k}$$

and $$\mathbf{P} = (150 \text{ lb})(\cos 5° \cos 70° \mathbf{i} + \sin 5° \mathbf{j} - \cos 5° \sin 70° \mathbf{k})$$

$$= (51.108 \text{ lb})\mathbf{i} + (13.0734 \text{ lb})\mathbf{j} - (140.418 \text{ lb})\mathbf{k}$$

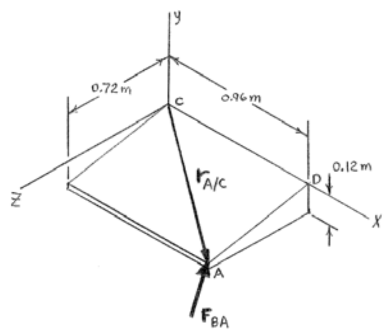
Then

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7.7053 & -1.47360 & -2.0646 \\ 51.108 & 13.0734 & -140.418 \end{vmatrix} \text{ lb}\cdot\text{in.}$$

$$= (233.91 \text{ lb}\cdot\text{in.})\mathbf{i} - (1187.48 \text{ lb}\cdot\text{in.})\mathbf{j} - (25.422 \text{ lb}\cdot\text{in.})\mathbf{k}$$

or $\mathbf{M}_C = (19.49 \text{ lb}\cdot\text{ft})\mathbf{i} - (99.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (2.12 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

Chapter 3, Solution 26.



Have $\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$

where $\mathbf{r}_{A/C} = (0.96\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$

and $\mathbf{F}_{BA} = \lambda_{BA} F_{BA}$

$$= \left[\frac{-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k}}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}\right](228\text{ N})$$

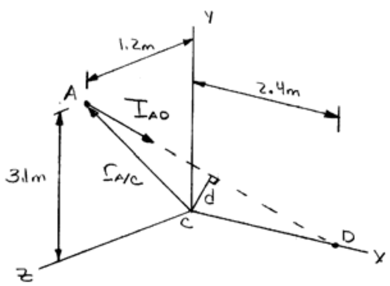
$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} + (60.480\text{ N}\cdot\text{m})\mathbf{j} + (205.92\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_C = -(146.9\text{ N}\cdot\text{m})\mathbf{i} + (60.5\text{ N}\cdot\text{m})\mathbf{j} + (206\text{ N}\cdot\text{m})\mathbf{k}$ ◀

Chapter 3, Solution 27.



Have $\left|\mathbf{M}_C\right| = T_{AD}\, d$

where $d =$ Perpendicular distance from C to line $\overrightarrow{AD}$

with $\mathbf{M}_C = \mathbf{r}_{A/C}\ \mathbf{T}_{AD}$

and $\mathbf{r}_{A/C} = (3.1\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}$

$$\mathbf{T}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$

$$\mathbf{T}_{AD} = (369\text{ N})\frac{\left[(2.4\text{ m})\mathbf{i} - (3.1\text{ m})\mathbf{j} - (1.2\text{ m})\mathbf{k}\right]}{\sqrt{(2.4\text{ m})^2 + (-3.1\text{ m})^2 + (-1.2\text{ m})^2}}$$

$$= (216\text{ N})\mathbf{i} - (279\text{ N})\mathbf{j} - (108\text{ N})\mathbf{k}$$

Then

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3.1 & 1.2 \\ 216 & -279 & -108 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (259.2\text{ N}\cdot\text{m})\mathbf{j} - (669.6\text{ N}\cdot\text{m})\mathbf{k}$$

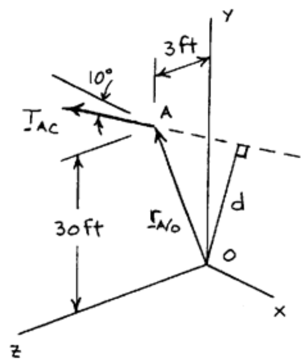
and $\left|\mathbf{M}_C\right| = \sqrt{(259.2\text{ N}\cdot\text{m})^2 + (-669.6\text{ N}\cdot\text{m})^2}$

$$= 718.02\text{ N}\cdot\text{m}$$

$$\therefore 718.02\text{ N}\cdot\text{m} = (369\text{ N})d$$

or $d = 1.946$ m ◀

Chapter 3, Solution 28.



Have $|\mathbf{M}_O| = \mathbf{T}_{AC}\, d$

where d = Perpendicular distance from O to rope AC

with $\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AC}$

and $\mathbf{r}_{A/O} = (30\text{ ft})\mathbf{j} + (3\text{ ft})\mathbf{k}$

$$\mathbf{T}_{AC} = -\big[(62\text{ lb})\cos 10°\big]\mathbf{i} - \big[(62\text{ lb})\sin 10°\big]\mathbf{j}$$

$$= -(61.058\text{ lb})\mathbf{i} - (10.766\text{ lb})\mathbf{j}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 30 & 3 \\ -61.058 & -10.766 & 0 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= (32.298\text{ lb}\cdot\text{ft})\mathbf{i} - (183.174\text{ lb}\cdot\text{ft})\mathbf{j} + (1831.74\text{ lb}\cdot\text{ft})\mathbf{k}$$

and

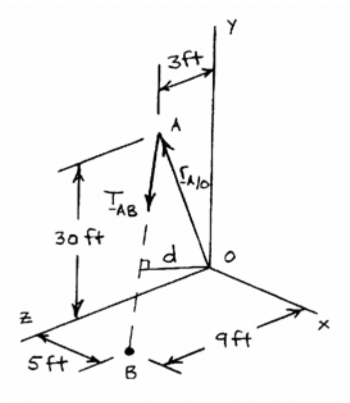
$$|\mathbf{M}_O| = \sqrt{(32.298\text{ lb}\cdot\text{ft})^2 + (-183.174\text{ lb}\cdot\text{ft})^2 + (1831.74\text{ lb}\cdot\text{ft})^2}$$

$$= 1841.16\text{ lb}\cdot\text{ft}$$

$$\therefore 1841.16\text{ lb}\cdot\text{ft} = (62\text{ lb})d$$

or $d = 29.7$ ft ◀

Chapter 3, Solution 29.



Have $|\mathbf{M}_O| = T_{AB}\, d$

where d = Perpendicular distance from O to rope AB

with $\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$

and $\mathbf{r}_{A/O} = (30 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$

$$\mathbf{T}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= (62 \text{ lb})\frac{\left[(5 \text{ ft})\mathbf{i} - (30 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}\right]}{\sqrt{(5 \text{ ft})^2 + (-30 \text{ ft})^2 + (6 \text{ ft})^2}}$$

$$= (10 \text{ lb})\mathbf{i} - (60 \text{ lb})\mathbf{j} + (12 \text{ lb})\mathbf{k}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 30 & 3 \\ 10 & -60 & 12 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= (540 \text{ lb}\cdot\text{ft})\mathbf{i} + (30 \text{ lb}\cdot\text{ft})\mathbf{j} - (300 \text{ lb}\cdot\text{ft})\mathbf{k}$$

and

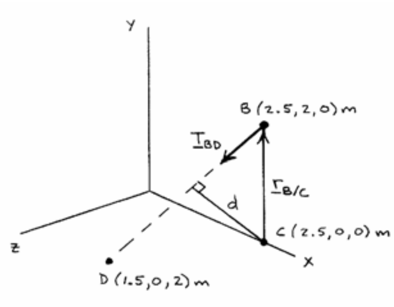
$$|\mathbf{M}_O| = \sqrt{(540 \text{ lb}\cdot\text{ft})^2 + (30 \text{ lb}\cdot\text{ft})^2 + (-300 \text{ lb}\cdot\text{ft})^2}$$

$$= 618.47 \text{ lb}\cdot\text{ft}$$

$$\therefore 618.47 \text{ lb}\cdot\text{ft} = (62 \text{ lb})d$$

or $d = 9.98$ ft ◀

Chapter 3, Solution 30.



Have $\mathbf{M}_C = T_{BD}\, d$

where $d =$ Perpendicular distance from C to cable BD

with $\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{T}_{B/D}$

and $\mathbf{r}_{B/C} = (2\text{ m})\mathbf{j}$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD}$$

$$= (900\text{ N})\frac{\left[-(1\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} + (2\text{ m})\mathbf{k}\right]}{\sqrt{(-1\text{ m})^2 + (-2\text{ m})^2 + (2\text{ m})^2}}$$

$$= -(300\text{ N})\mathbf{i} - (600\text{ N})\mathbf{j} + (600\text{ N})\mathbf{k}$$

Then

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ -300 & -600 & 600 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (1200\text{ N}\cdot\text{m})\mathbf{i} + (600\text{ N}\cdot\text{m})\mathbf{k}$$

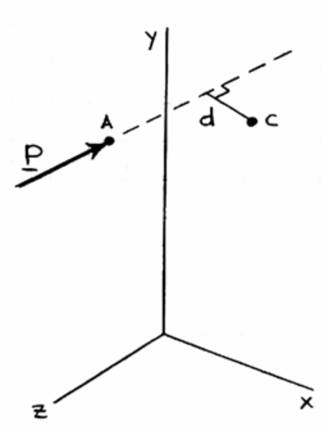
and $$|\mathbf{M}_C| = \sqrt{(1200\text{ N}\cdot\text{m})^2 + (600\text{ N}\cdot\text{m})^2}$$

$$= 1341.64\text{ N}\cdot\text{m}$$

$$\therefore 1341.64 = (900\text{ N})d$$

or $d = 1.491$ m ◀

Chapter 3, Solution 31.



Have $M_C = Pd$

From the solution of problem 3.25

$$\mathbf{M}_C = (233.91\text{ lb}\cdot\text{in.})\mathbf{i} - (1187.48\text{ lb}\cdot\text{in.})\mathbf{j} - (25.422\text{ lb}\cdot\text{in.})\mathbf{k}$$

Then

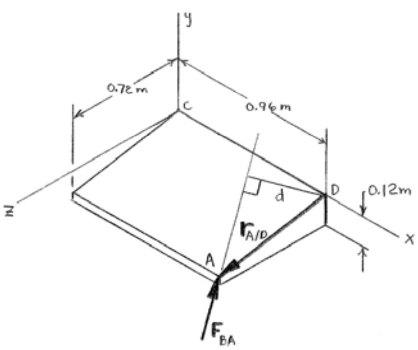
$$M_C = \sqrt{(233.91)^2 + (-1187.48)^2 + (-25.422)^2}$$

$$= 1210.57\text{ lb}\cdot\text{in.}$$

and $$d = \frac{M_C}{P} = \frac{1210.57\text{ lb.in.}}{150\text{ lb}}$$

or $d = 8.07$ in. ◀

Chapter 3, Solution 32.



Have $\quad |\mathbf{M}_D| = F_{BA}d$

where $\quad d$ = perpendicular distance from D to line AB.

$$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/D} = -(0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

$$\mathbf{F}_{BA} = \lambda_{BA}F_{BA} = \frac{(-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k})}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}(228\text{ N})$$

$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} - (8.64\text{ N}\cdot\text{m})\mathbf{j} - (1.44\text{ N}\cdot\text{m})\mathbf{k}$$

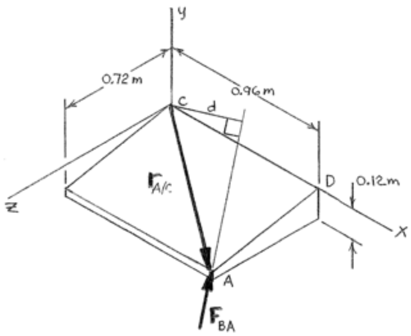
and $\quad |\mathbf{M}_D| = \sqrt{(146.88)^2 + (8.64)^2 + (1.44)^2} = 147.141\text{ N}\cdot\text{m}$

$$\therefore\ 147.141\text{ N}\cdot\text{m} = (228\text{ N})d$$

$$d = 0.64536\text{ m}$$

or $\quad d = 0.645\text{ m}$ ◀

Chapter 3, Solution 33.



Have $|\mathbf{M}_C| = F_{BA}d$

where d = perpendicular distance from C to line AB.

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/C} = (0.96\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

$$\mathbf{F}_{BA} = \lambda_{BA}F_{BA} = \frac{\left(-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6)\mathbf{k}\right)}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}(228\text{ N})$$

$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix}\text{ N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} - (60.48\text{ N}\cdot\text{m})\mathbf{j} + (205.92\text{ N}\cdot\text{m})\mathbf{k}$$

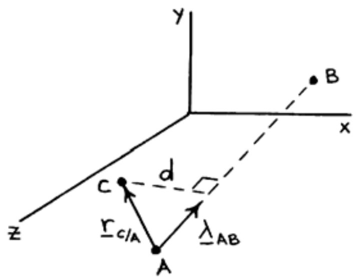
and $|\mathbf{M}_C| = \sqrt{(146.88)^2 + (60.48)^2 + (205.92)^2} = 260.07\text{ N}\cdot\text{m}$

$$\therefore\ 260.07\text{ N}\cdot\text{m} = (228\text{ N})d$$

$$d = 1.14064\text{ m}$$

or $d = 1.141\text{ m}$ ◀

Chapter 3, Solution 34.



(*a*) Have $d = r_{C/A}\sin\theta = \left|\lambda_{AB} \times \mathbf{r}_{C/A}\right|$

where d = Perpendicular distance from C to pipe AB

$$\text{with } \lambda_{AB} = \frac{\mathbf{AB}}{AB} = \frac{7\mathbf{i} + 4\mathbf{j} - 32\mathbf{k}}{\sqrt{(7)^2 + (4)^2 + (-32)^2}}$$

$$= \frac{1}{33}(7\mathbf{i} + 4\mathbf{j} - 32\mathbf{k})$$

$$\text{and } \mathbf{r}_{C/A} = -(14 \text{ ft})\mathbf{i} + (5 \text{ ft})\mathbf{j} + \left[(L - 22)\text{ft}\right]\mathbf{k}$$

Then
$$\lambda_{AB} \times \mathbf{r}_{C/A} = \frac{1}{33}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 4 & -32 \\ -14 & 5 & L - 22 \end{vmatrix}\text{ft}$$

$$= \frac{1}{33}\left\{\left[4(L - 22) + 32(5)\right]\mathbf{i} + \left[32(14) - 7(L - 22)\right]\mathbf{j} + \left[7(5) + 4(14)\right]\mathbf{k}\right\}\text{ft}$$

$$= \frac{1}{33}\left[(4L + 72)\mathbf{i} + (-7L + 602)\mathbf{j} + 91\mathbf{k}\right]\text{ft}$$

and
$$d = \frac{1}{33}\sqrt{(4L + 72)^2 + (-7L + 602)^2 + (91)^2}$$

For
$$(d)_{\text{min}},\quad \frac{dd^2}{dL} = \frac{1}{33^2}\left[2(4)(4L + 72) + 2(-7)(-7L + 602)\right] = 0$$

or $\quad 65L - 3926 = 0$

or $\quad L = 60.400$ ft

But $\quad L > L_{\text{greenhouse}}$ $\quad$ so $\quad L = 30.0$ ft ◀

(*b*) with $\quad L = 30$ ft,
$$d = \frac{1}{33}\sqrt{(4 \times 30 + 72)^2 + (-7 \times 30 + 602)^2 + (91)^2}$$

or $d = 13.51$ ft ◀

Note: $\quad$ with $\quad L = 60.4$ ft,

$$d = \frac{1}{33}\sqrt{(4 \times 60.4 + 72)^2 + (-7 \times 60.4 + 602)^2 + (91)^2} = 11.29 \text{ ft}$$

Chapter 3, Solution 35.

$$\mathbf{P}\cdot\mathbf{Q} = (-4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k})\cdot(9\mathbf{i} - \mathbf{j} - 7\mathbf{k})$$

$$= (-4)(9) + (8)(-1) + (-3)(-7)$$

$$= -23$$

or $\mathbf{P}\cdot\mathbf{Q} = -23$ ◀

$$\mathbf{P}\cdot\mathbf{S} = (-4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k})\cdot(5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$= (-4)(5) + (8)(-6) + (-3)(2)$$

$$= -74$$

or $\mathbf{P}\cdot\mathbf{S} = -74$ ◀

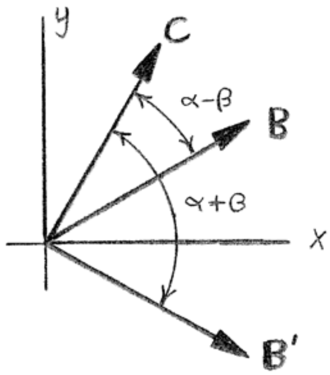
$$\mathbf{Q}\cdot\mathbf{S} = (9\mathbf{i} - \mathbf{j} - 7\mathbf{k})\cdot(5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$= (9)(5) + (-1)(-6) + (-7)(2)$$

$$= 37$$

or $\mathbf{Q}\cdot\mathbf{S} = 37$ ◀

Chapter 3, Solution 36.



By definition

$$\mathbf{B}\cdot\mathbf{C} = BC\cos(\alpha - \beta)$$

where $\mathbf{B} = B\left[(\cos\beta)\mathbf{i} + (\sin\beta)\mathbf{j}\right]$

$$\mathbf{C} = C\left[(\cos\alpha)\mathbf{i} + (\sin\alpha)\mathbf{j}\right]$$

$$\therefore\ (B\cos\beta)(C\cos\alpha) + (B\sin\beta)(C\sin\alpha) = BC\cos(\alpha - \beta)$$

or $\cos\beta\cos\alpha + \sin\beta\sin\alpha = \cos(\alpha - \beta)$ (1)

By definition

$$\mathbf{B}'\cdot\mathbf{C} = BC\cos(\alpha + \beta)$$

where $\mathbf{B}' = \left[(\cos\beta)\mathbf{i} - (\sin\beta)\mathbf{j}\right]$

$$\therefore\ (B\cos\beta)(C\cos\alpha) + (-B\sin\beta)(C\sin\alpha) = BC\cos(\alpha + \beta)$$

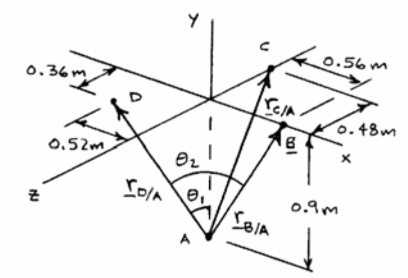
or $\cos\beta\cos\alpha - \sin\beta\sin\alpha = \cos(\alpha + \beta)$ (2)

Adding Equations (1) and (2),

$$2\cos\beta\cos\alpha = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

or $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha + \beta) + \frac{1}{2}\cos(\alpha - \beta)$ ◀

Chapter 3, Solution 37.



First note:

$$\mathbf{r}_{B/A} = (0.56 \text{ m})\mathbf{i} + (0.9 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (0.9 \text{ m})\mathbf{j} - (0.48 \text{ m})\mathbf{k}$$

$$\mathbf{r}_{D/A} = -(0.52 \text{ m})\mathbf{i} + (0.9 \text{ m})\mathbf{j} + (0.36 \text{ m})\mathbf{k}$$

$$|\mathbf{r}_{B/A}| = \sqrt{(0.56 \text{ m})^2 + (0.9 \text{ m})^2} = 1.06 \text{ m}$$

$$|\mathbf{r}_{C/A}| = \sqrt{(0.9 \text{ m})^2 + (-0.48 \text{ m})^2} = 1.02 \text{ m}$$

$$|\mathbf{r}_{D/A}| = \sqrt{(-0.52 \text{ m})^2 + (0.9 \text{ m})^2 + (0.36 \text{ m})^2} = 1.10 \text{ m}$$

By definition $\quad \mathbf{r}_{B/A} \cdot \mathbf{r}_{D/A} = |\mathbf{r}_{B/A}||\mathbf{r}_{D/A}|\cos\theta$

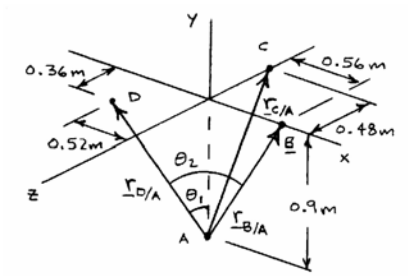
or $(0.56\mathbf{i} + 0.9\mathbf{j}) \cdot (-0.52\mathbf{i} + 0.9\mathbf{j} + 0.36\mathbf{k}) = (1.06)(1.10)\cos\theta$

$$(0.56)(-0.52) + (0.9)(0.9) + (0)(0.36) = 1.166\cos\theta$$

$$\cos\theta = 0.44494$$

$$\theta = 63.6° \blacktriangleleft$$

Chapter 3, Solution 38.



From the solution to problem 3.37

$$|\mathbf{r}_{C/A}| = 1.02 \text{ m} \quad \text{with} \quad \mathbf{r}_{C/A} = (0.9 \text{ m})\mathbf{i} - (0.48 \text{ m})\mathbf{j}$$

$$|\mathbf{r}_{D/A}| = 1.10 \text{ m} \quad \text{with} \quad \mathbf{r}_{D/A} = -(0.52 \text{ m})\mathbf{i} + (0.9 \text{ m})\mathbf{j} + (0.36 \text{ m})\mathbf{k}$$

Now by definition

$$\mathbf{r}_{C/A} \cdot \mathbf{r}_{D/A} = |\mathbf{r}_{C/A}||\mathbf{r}_{D/A}|\cos\theta$$

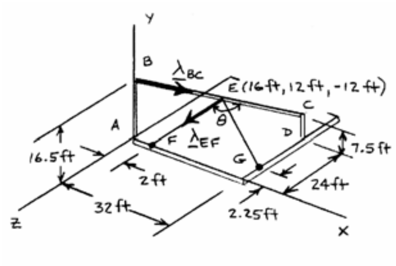
$$\text{or } (0.9\mathbf{j} - 0.48\mathbf{k}) \cdot (-0.52\mathbf{i} + 0.9\mathbf{j} + 0.36\mathbf{k}) = (1.02)(1.10)\cos\theta$$

$$0(-0.52) + (0.9)(0.9) + (-0.48)(0.36) = 1.122\cos\theta$$

$$\cos\theta = 0.56791$$

or $\theta = 55.4°$ ◀

Chapter 3, Solution 39.



(*a*) By definition

$$\boldsymbol{\lambda}_{BC} \cdot \boldsymbol{\lambda}_{EF} = (1)(1)\cos\theta$$

where

$$\boldsymbol{\lambda}_{BC} = \frac{(32\text{ ft})\mathbf{i} - (9\text{ ft})\mathbf{j} - (24\text{ ft})\mathbf{k}}{\sqrt{(32)^2 + (-9)^2 + (-24)^2}\text{ ft}}$$

$$= \frac{1}{41}(32\mathbf{i} - 9\mathbf{j} - 24\mathbf{k})$$

$$\boldsymbol{\lambda}_{EF} = \frac{-(14\text{ ft})\mathbf{i} - (12\text{ ft})\mathbf{j} + (12\text{ ft})\mathbf{k}}{\sqrt{(-14)^2 + (-12)^2 + (12)^2}\text{ ft}}$$

$$= \frac{1}{11}(-7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})$$

Therefore

$$\frac{(32\mathbf{i} - 9\mathbf{j} - 24\mathbf{k})}{41} \cdot \frac{(-7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})}{11} = \cos\theta$$

$$(32)(-7) + (-9)(-6) + (-24)(6) = (41)(11)\cos\theta$$

$$\cos\theta = -0.69623$$

or $\theta = 134.1° \blacktriangleleft$

(*b*) By definition

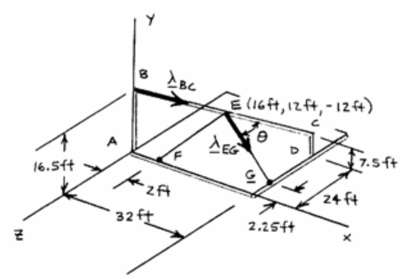
$$(T_{EG})_{BC} = (T_{EF})\cos\theta$$

$$= (110\text{ lb})(-0.69623)$$

$$= -76.585\text{ lb}$$

or $(T_{EF})_{BC} = -76.6\text{ lb} \blacktriangleleft$

Chapter 3, Solution 40.



(*a*) By definition

$$\lambda_{BC} \cdot \lambda_{EG} = (1)(1)\cos\theta$$

where

$$\lambda_{BC} = \frac{(32 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} - (24 \text{ ft})\mathbf{k}}{\sqrt{(32)^2 + (-9)^2 + (-24)^2} \text{ ft}}$$

$$= \frac{1}{41}(32\mathbf{i} - 9\mathbf{j} - 24\mathbf{k})$$

$$\lambda_{EG} = \frac{(16 \text{ ft})\mathbf{i} - (12 \text{ ft})\mathbf{j} + (9.75)\mathbf{k}}{\sqrt{(16)^2 + (-12)^2 + (9.75)^2} \text{ ft}}$$

$$= \frac{1}{22.25}(16\mathbf{i} - 12\mathbf{j} + 9.75\mathbf{k})$$

Therefore
$$\frac{(32\mathbf{i} - 9\mathbf{j} - 24\mathbf{k})}{41} \cdot \frac{(16\mathbf{i} - 12\mathbf{j} + 9.75\mathbf{k})}{22.25} = \cos\theta$$

$$(32)(16) + (-9)(-12) + (-24)(9.75) = (41)(22.25)\cos\theta$$

$$\cos\theta = 0.42313$$

or $\theta = 65.0° \blacktriangleleft$

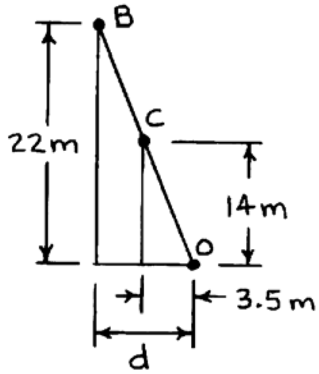
(*b*) By definition
$$(T_{EG})_{BC} = (T_{EG})\cos\theta$$

$$= (178 \text{ lb})(0.42313)$$

$$= 75.317 \text{ lb}$$

or $(T_{EG})_{BC} = 75.3 \text{ lb} \blacktriangleleft$

Chapter 3, Solution 41.



First locate point B:

$$\frac{d}{22} = \frac{3.5}{14}$$

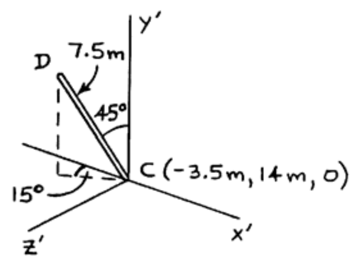
or $\quad d = 5.5 \text{ m}$

(*a*) $\quad d_{BA} = \sqrt{(5.5 + 0.5)^2 + (-22)^2 + (-3)^2} = 23 \text{ m}$

Locate point D:

$$\left[(-3.5 - 7.5\sin 45°\cos 15°), (14 + 7.5\cos 45°), (0 + 7.5\sin 45°\sin 15°)\right]\text{m}$$

or $(-8.6226 \text{ m}, 19.3033 \text{ m}, 1.37260 \text{ m})$



Then

$$d_{BD} = \sqrt{(-8.6226 + 5.5)^2 + (19.3033 - 22)^2 + (1.37260 - 0)^2} \text{ m}$$

$$= 4.3482 \text{ m}$$

and $\quad \cos\theta_{ABD} = \dfrac{\mathbf{d}_{BA} \cdot \mathbf{d}_{BD}}{d_{BA}\, d_{BD}} = \dfrac{(6\mathbf{i} - 22\mathbf{j} - 3\mathbf{k})\cdot(-3.1226\mathbf{i} - 2.6967\mathbf{j} + 1.37260\mathbf{k})}{(23)(4.3482)}$

$$= 0.36471$$

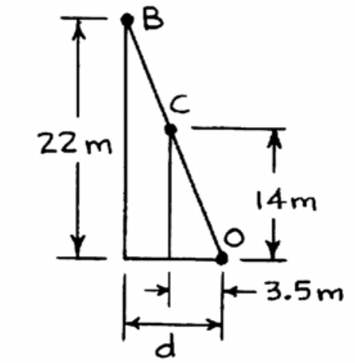
or $\quad \theta_{ABD} = 68.6°$ ◀

(*b*) $\quad (T_{BA})_{BD} = T_{BA}\cos\theta_{ABD}$

$$= (230 \text{ N})(0.36471)$$

or $(T_{BA})_{BD} = 83.9 \text{ N}$ ◀

Chapter 3, Solution 42.



First locate point B:

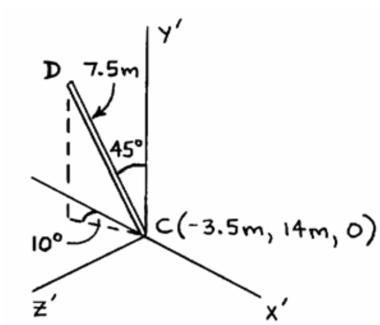
$$\frac{d}{22} = \frac{3.5}{14}$$

or $\quad d = 5.5 \text{ m}$

(*a*) Locate point D:

$$\left[\left(-3.5 - 7.5\sin 45°\cos 10°\right),\ \left(14 + 7.5\cos 45°\right),\ \left(0 + 7.5\sin 45°\sin 10°\right)\right]\text{m}$$

or $\left(-8.7227 \text{ m}, 19.3033 \text{ m}, 0.92091 \text{ m}\right)$



Then

$$\mathbf{d}_{DC} = \left(5.2227 \text{ m}\right)\mathbf{i} - \left(5.3033 \text{ m}\right)\mathbf{j} - \left(0.92091 \text{ m}\right)\mathbf{k}$$

and

$$d_{DB} = \sqrt{\left(-5.5 + 8.7227\right)^2 + \left(22 - 19.3033\right)^2 + \left(0 - 0.92091\right)^2} \text{ m}$$

$$= 4.3019 \text{ m}$$

and $\quad \cos\theta_{BDC} = \dfrac{d_{DB} \cdot d_{DC}}{d_{DB}\ d_{DC}} = \dfrac{\left(3.2227\mathbf{i} + 2.6967\mathbf{j} - 0.92091\mathbf{k}\right)\cdot\left(5.2227\mathbf{i} - 5.3033\mathbf{j} - 0.92091\mathbf{k}\right)}{\left(4.3019\right)\left(7.5\right)}$

$$= 0.104694$$

or $\quad \theta_{BDC} = 84.0°$ ◀

(*b*) $\quad \left(T_{BD}\right)_{DC} = T_{BD}\cos\theta_{BDC} = \left(250 \text{ N}\right)\left(0.104694\right)$

or $\quad \left(T_{BD}\right)_{DC} = 26.2 \text{ N}$ ◀

Chapter 3, Solution 43.

Volume of parallelopiped is found using the mixed triple product

(*a*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$

$$= \begin{vmatrix} 3 & -4 & 1 \\ -7 & 6 & -8 \\ 9 & -2 & -3 \end{vmatrix} \text{in.}^3$$

$$= (-54 + 288 + 14 - 48 + 84 - 54)\text{in.}^3$$

$$= 230 \text{ in.}^3$$

or Volume = 230 in.^{3} ◀

(*b*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$

$$= \begin{vmatrix} -5 & -7 & 4 \\ 6 & -2 & 5 \\ -4 & 8 & -9 \end{vmatrix} \text{in.}^3$$

$$= (-90 + 140 + 192 + 200 - 378 - 32)\text{ in.}^3$$

$$= 32 \text{ in.}^3$$

or Volume = 32 in.^{3} ◀

Chapter 3, Solution 44.

For the vectors to all be in the same plane, the mixed triple product is zero.

$$\mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S}) = 0$$

$$\therefore \quad 0 = \begin{vmatrix} -3 & -7 & 5 \\ -2 & 1 & -4 \\ 8 & S_y & -6 \end{vmatrix}$$

$$0 = 18 + 224 - 10S_y - 12S_y + 84 - 40$$

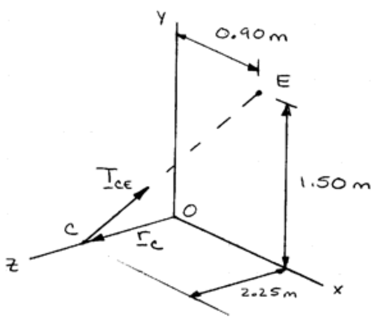
So that

$$22\ S_y = 286$$

$$S_y = 13$$

or $S_y = 13.00$ ◀

Chapter 3, Solution 45.



Have $\mathbf{r}_C = (2.25\text{ m})\mathbf{k}$

$$\mathbf{T}_{CE} = T_{CE}\frac{\overrightarrow{CE}}{CE}$$

$$\mathbf{T}_{CE} = (1349\text{ N})\frac{[(0.90\text{ m})\mathbf{i} + (1.50\text{ m})\mathbf{j} - (2.25\text{ m})\mathbf{k}]}{\sqrt{(0.90)^2 + (1.50)^2 + (-2.25)^2}\text{ m}}$$

$$= (426\text{ N})\mathbf{i} + (710\text{ N})\mathbf{j} - (1065\text{ N})\mathbf{k}$$

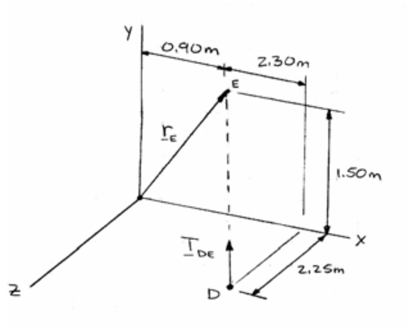
Now $\mathbf{M}_O = \mathbf{r}_C \times \mathbf{T}_{CE}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.25 \\ 426 & 710 & -1065 \end{vmatrix}\text{N}\cdot\text{m}$$

$$= -(1597.5\text{ N}\cdot\text{m})\mathbf{i} + (958.5\text{ N}\cdot\text{m})\mathbf{j}$$

$$\therefore\ M_x = -1598\text{ N}\cdot\text{m},\ M_y = 959\text{ N}\cdot\text{m},\ M_z = 0 \blacktriangleleft$$

Chapter 3, Solution 46.



Have $\quad \mathbf{r}_E = (0.90\text{ m})\mathbf{i} + (1.50\text{ m})\mathbf{j}$

$$\mathbf{T}_{DE} = T_{DE}\frac{\overrightarrow{DE}}{DE}$$

$$= (1349\text{ N})\frac{\left[-(2.30\text{ m})\mathbf{i} + (1.50\text{ m})\mathbf{j} - (2.25\text{ m})\mathbf{k}\right]}{\sqrt{(-2.30)^2 + (1.50)^2 + (-2.25)^2}\text{ m}}$$

$$= -(874\text{ N})\mathbf{i} + (570\text{ N})\mathbf{j} - (855\text{ N})\mathbf{k}$$

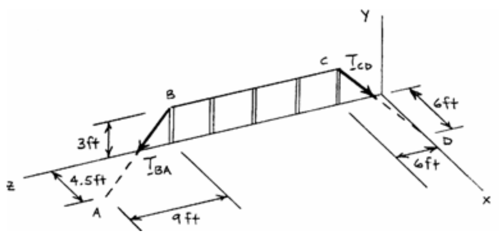
Now $\quad \mathbf{M}_O = \mathbf{r}_E \times \mathbf{T}_{DE}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.90 & 1.50 & 0 \\ -874 & 570 & -855 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(1282.5\text{ N}\cdot\text{m})\mathbf{i} + (769.5\text{ N}\cdot\text{m})\mathbf{j} + (1824\text{ N}\cdot\text{m})\mathbf{k}$$

$$\therefore M_x = -1283\text{ N}\cdot\text{m},\ M_y = 770\text{ N}\cdot\text{m},\ M_z = 1824\text{ N}\cdot\text{m} \blacktriangleleft$$

Chapter 3, Solution 47.



Have $$\left|\mathbf{M}_z\right| = \mathbf{k}\cdot\left[\left(\mathbf{r}_B\right)_y \times \mathbf{T}_{BA}\right] + \mathbf{k}\left[\left(\mathbf{r}_C\right)_y \times \mathbf{T}_{CD}\right]$$

where $$\mathbf{M}_z = -(48\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\left(\mathbf{r}_B\right)_y = \left(\mathbf{r}_C\right)_y = (3\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BA} = T_{BA}\frac{\overrightarrow{BA}}{BA} = (14\text{ lb})\frac{\left[(4.5\text{ ft})\mathbf{i} - (3\text{ ft})\mathbf{j} + (9\text{ ft})\mathbf{k}\right]}{\sqrt{(4.5)^2 + (-3)^2 + (9)^2}\text{ ft}}$$

$$= (6\text{ lb})\mathbf{i} - (4\text{ lb})\mathbf{j} + (12\text{ lb})\mathbf{k}$$

$$\mathbf{T}_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}\frac{\left[(6\text{ ft})\mathbf{i} - (3\text{ ft})\mathbf{j} - (6\text{ ft})\mathbf{k}\right]}{\sqrt{(6)^2 + (-3)^2 + (-6)^2}\text{ ft}}$$

$$= \frac{T_{CD}}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

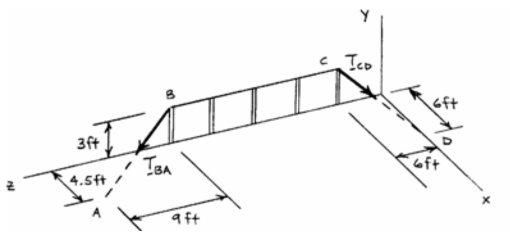
Then $$-(48\text{ lb}\cdot\text{ft}) = \mathbf{k}\cdot\left\{(3\text{ ft})\mathbf{j}\times\left[(6\text{ lb})\mathbf{i} - (4\text{ lb})\mathbf{j} + (12\text{ lb})\mathbf{k}\right]\right\}$$

$$+\mathbf{k}\cdot\left\{(3\text{ ft})\mathbf{j}\times\left[\frac{T_{CD}}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})\right]\right\}$$

or $$-48 = -18 - 2T_{CD}$$

$$T_{CD} = 15.00\text{ lb} \blacktriangleleft$$

Chapter 3, Solution 48.



Have $$\left|\mathbf{M}_y\right| = \mathbf{j}\cdot\left[\left(\mathbf{r}_B\right)_z \times \mathbf{T}_{BA}\right] \times \mathbf{j}\cdot\left[\left(\mathbf{r}_C\right)_z \times \mathbf{T}_{CD}\right]$$

where $$\mathbf{M}_y = 156 \text{ lb}\cdot\text{ft}$$

$$\left(\mathbf{r}_B\right)_z = \left(24 \text{ ft}\right)\mathbf{k}; \qquad \left(\mathbf{r}_C\right)_z = \left(6 \text{ ft}\right)\mathbf{k}$$

$$\mathbf{T}_{BA} = T_{BA}\frac{\overrightarrow{BA}}{BA} = T_{BA}\frac{\left[\left(4.5 \text{ ft}\right)\mathbf{i} - \left(3 \text{ ft}\right)\mathbf{j} + \left(9 \text{ ft}\right)\mathbf{k}\right]}{\sqrt{\left(4.5\right)^2 + \left(-3\right)^2 + \left(9\right)^2} \text{ ft}}$$

$$= \frac{T_{BA}}{10.5}\left(4.5\mathbf{i} - 3\mathbf{j} + 9\mathbf{k}\right)$$

$$\mathbf{T}_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = \left(7.5 \text{ lb}\right)\frac{\left[\left(6 \text{ ft}\right)\mathbf{i} - \left(3 \text{ ft}\right)\mathbf{j} + \left(9 \text{ ft}\right)\mathbf{k}\right]}{\sqrt{\left(6\right)^2 + \left(-3\right)^2 + \left(9\right)^2} \text{ ft}}$$

$$= \left(5 \text{ lb}\right)\mathbf{i} - \left(2.5 \text{ lb}\right)\mathbf{j} - \left(5 \text{ lb}\right)\mathbf{k}$$

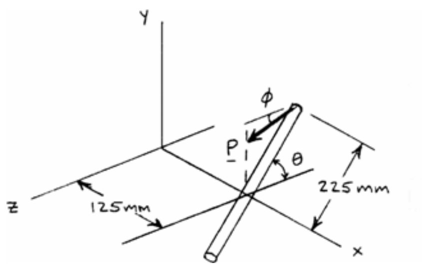
Then $$\left(156 \text{ lb}\cdot\text{ft}\right) = \mathbf{j}\cdot\left\{\left(24 \text{ ft}\right)\mathbf{k} \times \frac{T_{BA}}{10.5}\left(4.5\mathbf{i} - 3\mathbf{j} + 9\mathbf{k}\right)\right\}$$

$$+\mathbf{j}\cdot\left\{\left(6 \text{ ft}\right)\mathbf{k} \times \left[\left(5 \text{ lb}\right)\mathbf{i} - \left(2.5 \text{ lb}\right)\mathbf{j} - \left(5 \text{ lb}\right)\mathbf{k}\right]\right\}$$

or $$156 = \frac{108}{10.5}T_{BA} + 30$$

$T_{BA} = 12.25 \text{ lb}$ ◀

Chapter 3, Solution 49.



Based on $M_x = (P\cos\phi)\left[(0.225 \text{ m})\sin\theta\right] - (P\sin\phi)\left[(0.225 \text{ m})\cos\theta\right]$ (1)

$M_y = -(P\cos\phi)(0.125 \text{ m})$ (2)

$M_z = -(P\sin\phi)(0.125 \text{ m})$ (3)

By $\dfrac{\text{Equation}(3)}{\text{Equation}(2)}$: $\dfrac{M_z}{M_y} = \dfrac{-(P\sin\phi)(0.125)}{-(P\cos\phi)(0.125)}$

or $\dfrac{-4}{-23} = \tan\phi \therefore \quad \phi = 9.8658°$

or $\phi = 9.87°$ ◀

From Equation (2)

$-23 \text{ N}\cdot\text{m} = -(P\cos 9.8658°)(0.125 \text{ m})$

$P = 186.762 \text{ N}$

or $P = 186.8 \text{ N}$ ◀

From Equation (1)

$26 \text{ N}\cdot\text{m} = \left[(186.726 \text{ N})\cos 9.8658°\right]\left[(0.225 \text{ m})\sin\theta\right]$

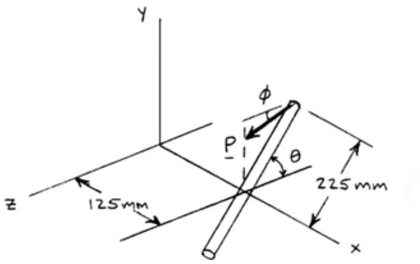
$-\left[(186.726 \text{ N})\sin 9.8658°\right]\left[(0.225 \text{ m})\cos\theta\right]$

or $0.98521\sin\theta - 0.171341\cos\theta = 0.61885$

Solving numerically,

$\theta = 48.1°$ ◀

Chapter 3, Solution 50.



Based on $M_x = (P\cos\phi)\left[(0.225\text{ m})\sin\theta\right] - (P\sin\phi)\left[(0.225\text{ m})\cos\theta\right]$ (1)

$$M_y = -(P\cos\phi)(0.125\text{ m}) \quad (2)$$

$$M_z = -(P\sin\phi)(0.125\text{ m})$$

By $\dfrac{\text{Equation}(3)}{\text{Equation}(2)}$: $\dfrac{M_z}{M_y} = \dfrac{-(P\sin\phi)(0.125)}{-(P\cos\phi)(0.125)}$

or $\dfrac{-3.5}{-20} = \tan\phi; \quad \phi = 9.9262°$

From Equation (3):

$$-3.5\text{ N}\cdot\text{m} = -(P\sin 9.9262°)(0.125\text{ m})$$

$$P = 162.432\text{ N}$$

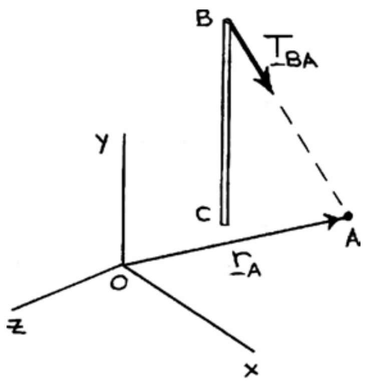
From Equation (1):

$$M_x = (162.432\text{ N})(0.225\text{ m})(\cos 9.9262°\sin 60° - \sin 9.9262°\cos 60°)$$

$$= 28.027\text{ N}\cdot\text{m}$$

or $M_x = 28.0\text{ N}\cdot\text{m}$ ◀

Chapter 3, Solution 51.



First note:

$$\mathbf{T}_{BA} = T_{BA}\frac{\overline{BA}}{BA}$$

$$= (70\text{ lb})\frac{(4)\mathbf{i} + [1.5 - (L_{BC} + 1)]\mathbf{j} + (-6)\mathbf{k}}{\sqrt{(4)^2 + [1.5 - (L_{BC} + 1)]^2 + (-6)^2}}$$

$$= (70\text{ lb})\frac{4\mathbf{i} + (0.5 - L_{BC})\mathbf{j} - 6\mathbf{k}}{\sqrt{52 + (0.5 - L_{BC})^2}}$$

$$\mathbf{r}_A = (4\text{ ft})\mathbf{i} + (1.5\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}$$

Have $\mathbf{M}_O = \mathbf{r}_A \times \mathbf{T}_{BA}$

$$= \frac{70\text{ lb}}{\sqrt{52 + (0.5 - L_{BC})^2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\text{ ft} & 1.5\text{ ft} & -12\text{ ft} \\ 4 & (0.5 - L_{BC}) & -6 \end{vmatrix}$$

For the **i** components: $-763\text{ lb}\cdot\text{ft} = \dfrac{70}{\sqrt{52 + (0.5 - L_{BC})^2}}\left[1.5(-6) + 12(0.5 - L_{BC})\right]\text{lb}\cdot\text{ft}$

or $\quad 10.9\sqrt{52 + (0.5 - L_{BC})^2} = 3 + 12L_{BC}$

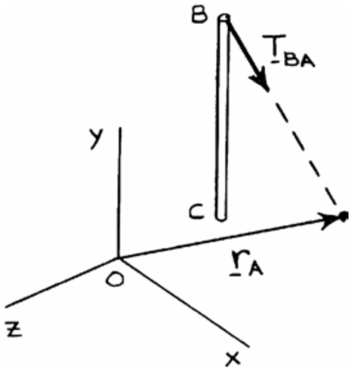
or $\quad (10.9)^2\left[52 + (0.5 - L_{BC})^2\right] = 9 + 72L_{BC} + 144L_{BC}^2$

or $\quad 25.19L_{BC}^2 + 190.81L_{BC} - 6198.8225 = 0$

Then $\quad L_{BC} = \dfrac{-190.81 \pm \sqrt{(190.81)^2 - 4(25.19)(-6198.8225)}}{2(25.19)}$

Taking the positive root $\quad L_{BC} = 12.35\text{ ft}$ ◀

Chapter 3, Solution 52.



First note:

$$\mathbf{T}_{BA} = T_{BA}\frac{\overline{BA}}{BA}$$

$$= (70\text{ lb})\frac{(4)\mathbf{i} + \left[1.5 - (L_{BC} + 1)\right]\mathbf{j} + (-6)\mathbf{k}}{\sqrt{(4)^2 + \left[1.5 - (L_{BC}+1)\right]^2 + (-6)^2}}$$

$$= (70\text{ lb})\frac{4\mathbf{i} + (0.5 - L_{BC})\mathbf{j} - 6\mathbf{k}}{\sqrt{52 + (0.5 - L_{BC})^2}}$$

$$\mathbf{r}_A = (4\text{ ft})\mathbf{i} + (1.5\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}$$

Have $\mathbf{M}_O = \mathbf{r}_A \times \mathbf{T}_{BA}$

$$= \frac{T_{BA}}{\sqrt{52 + (0.5 - L_{BC})^2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\text{ ft} & 1.5\text{ ft} & -12\text{ ft} \\ 4 & (0.5 - L_{BC}) & -6 \end{vmatrix}$$

For the **i** components: $-900\text{ lb}\cdot\text{ft} = \dfrac{T_{BA}}{\sqrt{52 + (0.5 - L_{BC})^2}}\left[1.5(-6) + 12(0.5 - L_{BC})\right]\text{lb}\cdot\text{ft}$

or $$300 = \frac{T_{BA}}{\sqrt{52 + (0.5 - L_{BC})^2}}(1 + 4L_{BC}) \qquad (1)$$

For the **k** components: $-315\text{ lb}\cdot\text{ft} = \dfrac{T_{BA}}{\sqrt{52 + (0.5 - L_{BC})^2}}\left[4(0.5 - L_{BC}) - 1.5(4)\right]\text{lb}\cdot\text{ft}$

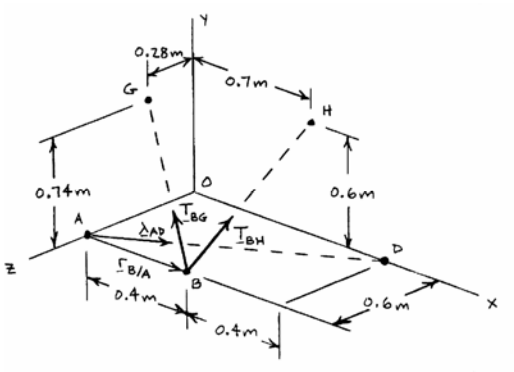
or $$315 = \frac{4T_{BA}}{\sqrt{52 + (0.5 - L_{BC})^2}}(1 + L_{BC}) \qquad (2)$$

Then, $\dfrac{(1)}{(2)} \Rightarrow \quad \dfrac{300}{315} = \dfrac{1 + 4L_{BC}}{4(1 + L_{BC})}$

or $L_{BC} = \dfrac{59}{4}\text{ ft}$

$L_{BC} = 14.75\text{ ft}$ ◀

Chapter 3, Solution 53.



Have $\quad M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BH} \right)$

where $\quad \boldsymbol{\lambda}_{AD} = \dfrac{(0.8 \text{ m})\mathbf{i} - (0.6 \text{ m})\mathbf{k}}{\sqrt{(0.8 \text{ m})^2 + (-0.6 \text{ m})^2}} = 0.8\mathbf{i} - 0.6\mathbf{k}$

$$\mathbf{r}_{B/A} = (0.4 \text{ m})\mathbf{i}$$

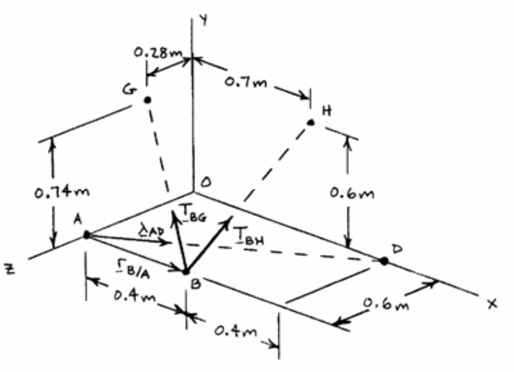
$$\mathbf{T}_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH} = (1125 \text{ N})\frac{\left[(0.3 \text{ m})\mathbf{i} + (0.6 \text{ m})\mathbf{j} - (0.6 \text{ m})\mathbf{k}\right]}{\sqrt{(0.3)^2 + (0.6)^2 + (-0.6)^2} \text{ m}}$$

Then

$$M_{AD} = \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.4 & 0 & 0 \\ 375 & 750 & -750 \end{vmatrix} = -180 \text{ N}\cdot\text{m}$$

or $M_{AD} = -180.0 \text{ N}\cdot\text{m}$ ◀

Chapter 3, Solution 54.



Have $\qquad M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BG}\right)$

where $\qquad \boldsymbol{\lambda}_{AD} = (0.8\text{ m})\mathbf{i} - (0.6\text{ m})\mathbf{k}$

$$\mathbf{r}_{B/A} = (0.4\text{ m})\mathbf{i}$$

$$\mathbf{T}_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG} = (1125\text{ N})\frac{\left[-(0.4\text{ m})\mathbf{i} + (0.74)\mathbf{j} - (0.32\text{ m})\mathbf{k}\right]}{\sqrt{(-0.4\text{ m})^2 + (0.74\text{ m})^2 + (-0.32\text{ m})^2}}$$

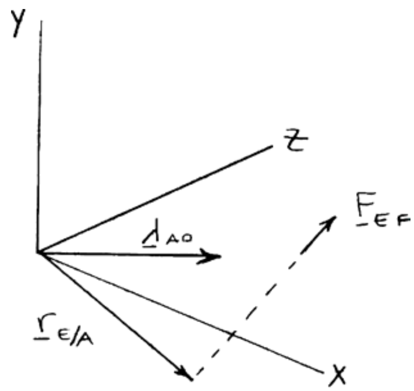
$$= -(500\text{ N})\mathbf{i} + (925\text{ N})\mathbf{j} - (400\text{ N})\mathbf{k}$$

Then

$$M_{AD} = \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.4 & 0 & 0 \\ -500 & 925 & -400 \end{vmatrix}$$

or $\quad M_{AD} = -222\text{ N}\cdot\text{m}$ ◀

Chapter 3, Solution 55.



Have

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{E/A} \times \mathbf{F}_{EF}\right)$$

where
$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD}$$

$$\boldsymbol{\lambda}_{AD} = \frac{(7.2 \text{ m})\mathbf{i} + (0.9 \text{ m})\mathbf{j}}{\sqrt{(7.2 \text{ m})^2 + (0.9 \text{ m})^2}}$$

$$= 0.99228\mathbf{i} + 0.124035\mathbf{j}$$

$$\mathbf{r}_{E/A} = (2.1 \text{ m})\mathbf{i} - (0.9 \text{ m})\mathbf{j}$$

$$\mathbf{F}_{EF} = F_{EF}\frac{\overrightarrow{EF}}{EF} = (24.3 \text{ kN})\frac{\left[(0.3 \text{ m})\mathbf{i} + (1.2 \text{ m})\mathbf{j} + (2.4 \text{ m})\mathbf{k}\right]}{\sqrt{(0.3 \text{ m})^2 + (1.2 \text{ m})^2 + (2.4 \text{ m})^2}}$$

$$= (2.7 \text{ kN})\mathbf{i} + (10.8 \text{ kN})\mathbf{j} + (21.6 \text{ kN})\mathbf{k}$$

Then

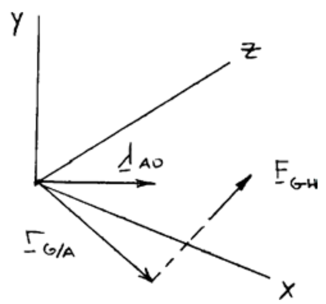
$$M_{AD} = \begin{vmatrix} 0.99228 & 0.124035 & 0 \\ 2.1 & -0.9 & 0 \\ 2.7 & 10.8 & 21.6 \end{vmatrix} \text{kN} \cdot \text{m}$$

$$= -19.2899 - 5.6262$$

$$= -24.916 \text{ kN} \cdot \text{m}$$

or $M_{AD} = -24.9 \text{ kN} \cdot \text{m}$ ◀

Chapter 3, Solution 56.



Have $M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{E}_{EF}\right)$

Where $$\boldsymbol{\lambda}_{AD} = \frac{(7.2\text{ m})\mathbf{i} + (0.9\text{ m})\mathbf{j}}{\sqrt{(7.2\text{ m})^2 + (0.9\text{ m})^2}}$$

$$= 0.99228\,\mathbf{i} + 0.124035\,\mathbf{j}$$

$$\mathbf{r}_{G/A} = (6\text{ m})\mathbf{i} - (1.8\text{ m})\mathbf{j}$$

$$\mathbf{F}_{GH} = F_{GH}\frac{\overrightarrow{GH}}{GH} = (21.3\text{ kN})\frac{\left[(-1.2\text{ m})\mathbf{i} + (2.4\text{ m})\mathbf{j} + (2.4\text{ m})\mathbf{k}\right]}{\sqrt{(-1.2\text{ m})^2 + (2.4\text{ m})^2 + (2.4\text{ m})^2}}$$

$$= -(7.1\text{ kN})\mathbf{i} + (14.2\text{ kN})\mathbf{j} + (14.2\text{ kN})\mathbf{k}$$

Then

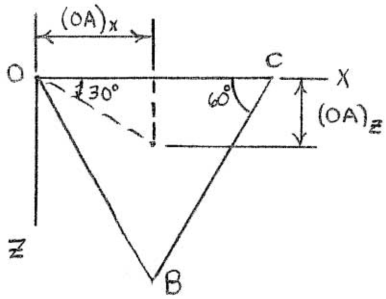
$$M_{AD} = \begin{vmatrix} 0.99228 & 0.124035 & 0 \\ 6 & -1.8 & 0 \\ -7.1 & 14.2 & 14.2 \end{vmatrix}\text{ kN}\cdot\text{m}$$

$$= -25.363 - 10.5678$$

$$= -35.931\text{ kN}\cdot\text{m}$$

or $M_{AD} = -35.9\text{ kN}\cdot\text{m}$ ◀

Chapter 3, Solution 57.



Have $M_{OA} = \boldsymbol{\lambda}_{OA} \cdot \left(\mathbf{r}_{C/O} \times \mathbf{P}\right)$

where

From triangle *OBC*

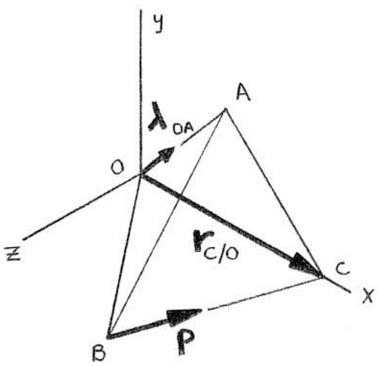
$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30^\circ = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

Since $(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA_z)^2$

or $a^2 = \left(\frac{a}{2}\right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$

$$\therefore \quad (OA)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$



Then $\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$

and $\boldsymbol{\lambda}_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$

$$\mathbf{P} = \boldsymbol{\lambda}_{BC} P$$

$$= \frac{(a\sin 30^\circ)\mathbf{i} - (a\cos 30^\circ)\mathbf{k}}{a}(P)$$

$$= \frac{P}{2}\left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)$$

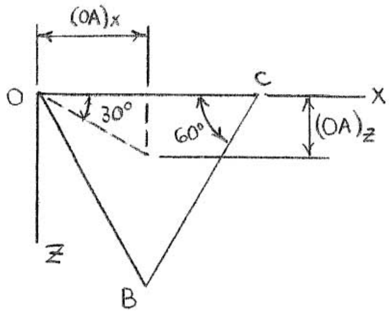
$$\mathbf{r}_{C/O} = a\mathbf{i}$$

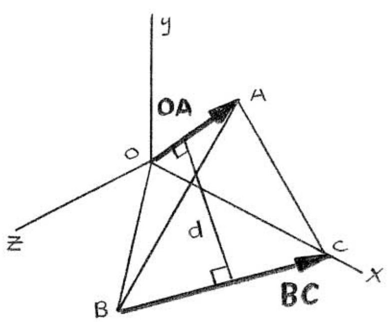
$$\therefore \quad M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix}(a)\left(\frac{P}{2}\right)$$

$$= \frac{aP}{2}\left(-\sqrt{\frac{2}{3}}\right)(1)\left(-\sqrt{3}\right)$$

$$= \frac{aP}{\sqrt{2}} \qquad M_{OA} = \frac{aP}{\sqrt{2}}$$

Chapter 3, Solution 58.





(*a*) For edge *OA* to be perpendicular to edge *BC*,

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

where

From triangle *OBC*

$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

$$\therefore \quad \overrightarrow{OA} = \left(\frac{a}{2}\right)\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}$$

and $\overrightarrow{BC} = (a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}$

$$= \frac{a}{2}\mathbf{i} - \frac{a\sqrt{3}}{2}\mathbf{k}$$

$$= \frac{a}{2}\left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)$$

Then $\left[\frac{a}{2}\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}\right] \cdot \left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)\frac{a}{2} = 0$

or $\frac{a^2}{4} + (OA)_y(0) - \frac{a^2}{4} = 0$

$$\therefore \quad \overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

so that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$. ◀

(*b*) Have $M_{OA} = Pd$, with *P* acting along *BC* and *d* the perpendicular distance from $\overrightarrow{OA}$ to $\overrightarrow{BC}$.

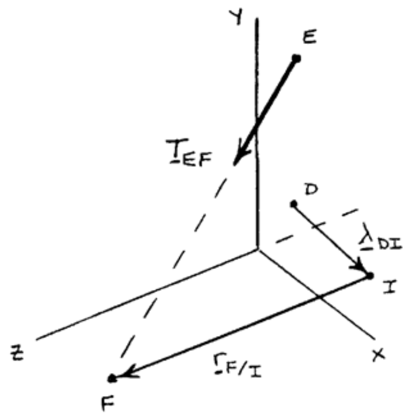
From the results of Problem 3.57,

$$M_{OA} = \frac{Pa}{\sqrt{2}}$$

$$\therefore \quad \frac{Pa}{\sqrt{2}} = Pd$$

or $d = \frac{a}{\sqrt{2}}$ ◀

Chapter 3, Solution 59.



Have $$M_{DI} = \boldsymbol{\lambda}_{DI} \cdot \left(\mathbf{r}_{F/I} \times \mathbf{T}_{EF}\right)$$

where $$\boldsymbol{\lambda}_{DI} = \frac{\overrightarrow{DI}}{DI} = \frac{(4.8\text{ ft})\mathbf{i} - (1.2\text{ ft})\mathbf{j}}{\sqrt{(4.8\text{ ft})^2 + (-1.2\text{ ft})^2}}$$

$$= 0.97014\mathbf{i} - 0.24254\mathbf{j}$$

$$\mathbf{r}_{F/I} = (16.2\text{ ft})\mathbf{k}$$

$$\mathbf{T}_{EF} = T_{EF}\frac{\overrightarrow{EF}}{EF} = (29.7\text{ lb})\frac{\left[(3.6\text{ ft})\mathbf{i} - (10.8\text{ ft})\mathbf{j} + (16.2\text{ ft})\mathbf{k}\right]}{\sqrt{(3.6\text{ ft})^2 + (-10.8\text{ ft})^2 + (16.2\text{ ft})^2}}$$

$$= (5.4\text{ lb})\mathbf{i} - (16.2\text{ lb})\mathbf{j} + (24.3\text{ lb})\mathbf{k}$$

Then

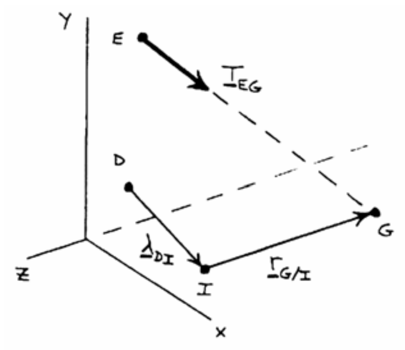
$$M_{DI} = \begin{vmatrix} 0.97014 & -0.24254 & 0 \\ 0 & 0 & 16.2 \\ 5.4 & -16.2 & 24.3 \end{vmatrix} \text{lb}\cdot\text{ft}$$

$$= -21.217 + 254.60$$

$$= 233.39\text{ lb}\cdot\text{ft}$$

or $M_{DI} = 233\text{ lb}\cdot\text{ft}$ ◀

Chapter 3, Solution 60.



Have $M_{DI} = \boldsymbol{\lambda}_{DI} \cdot \left(\mathbf{r}_{G/I} \times \mathbf{T}_{EG}\right)$

where $$\boldsymbol{\lambda}_{DI} = \frac{\overrightarrow{DI}}{DI} = \frac{(4.8 \text{ ft})\mathbf{i} - (1.2 \text{ ft})\mathbf{j}}{\sqrt{(4.8 \text{ ft})^2 + (-1.2 \text{ ft})^2}}$$

$$= 0.97014\mathbf{i} - 0.24254\mathbf{j}$$

$$\mathbf{r}_{G/I} = -(35.1 \text{ ft})\mathbf{k}$$

$$\mathbf{T}_{EG} = T_{EG}\frac{\overrightarrow{EG}}{EG} = (24.6 \text{ lb})\frac{\left[(3.6 \text{ ft})\mathbf{i} - (10.8 \text{ ft})\mathbf{j} - (35.1 \text{ ft})\mathbf{k}\right]}{\sqrt{(3.6 \text{ ft})^2 + (-10.8 \text{ ft})^2 + (-35.1 \text{ ft})^2}}$$

$$= (2.4 \text{ lb})\mathbf{i} - (7.2 \text{ lb})\mathbf{j} - (23.4 \text{ lb})\mathbf{k}$$

Then

$$M_{DI} = \begin{vmatrix} 0.97014 & -0.24254 & 0 \\ 0 & 0 & -35.1 \\ 2.4 & -7.2 & -23.4 \end{vmatrix} \text{lb}\cdot\text{ft}$$

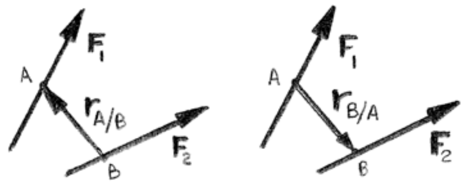
$$= 20.432 - 245.17$$

$$= -224.74 \text{ lb}\cdot\text{ft}$$

or $M_{DI} = -225 \text{ lb}\cdot\text{ft}$ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 3, Solution 61.



First note that $\mathbf{F}_1 = F_1\boldsymbol{\lambda}_1$ and $\mathbf{F}_2 = F_2\boldsymbol{\lambda}_2$

Let $M_1 =$ moment of $\mathbf{F}_2$ about the line of action of $\mathbf{M}_1$

and $M_2 =$ moment of $\mathbf{F}_1$ about the line of action of $\mathbf{M}_2$

Now, by definition

$$M_1 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \mathbf{F}_2\right) = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F_2$$

$$M_2 = \boldsymbol{\lambda}_2 \cdot \left(\mathbf{r}_{A/B} \times \mathbf{F}_1\right) = \boldsymbol{\lambda}_2 \cdot \left(\mathbf{r}_{A/B} \times \boldsymbol{\lambda}_1\right) F_1$$

Since $F_1 = F_2 = F$ and $\mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$

$$M_1 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F$$

$$M_2 = \boldsymbol{\lambda}_2 \cdot \left(-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1\right) F$$

Using Equation (3.39)

$$\boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) = \boldsymbol{\lambda}_2 \cdot \left(-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1\right)$$

so that $M_2 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F$

$$\therefore \quad M_{12} = M_{21} \blacktriangleleft$$

Chapter 3, Solution 62.

From the solution of Problem 3.53:

$$\boldsymbol{\lambda}_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BH} = (375 \text{ N})\mathbf{i} + (750 \text{ N})\mathbf{j} - (750 \text{ N})\mathbf{k}; \quad T_{BH} = 1125 \text{ N}$$

$$M_{AD} = -180 \text{ N}\cdot\text{m}$$

Only the perpendicular component of $\mathbf{T}_{BH}$ contributes to the moment of $\mathbf{T}_{BH}$ about line *AD*. The parallel component of $\mathbf{T}_{BH}$ will be used to find the perpendicular component.

Have $\quad (\mathbf{T}_{BH})_{\text{Parallel}} = \boldsymbol{\lambda}_{AD} \cdot \mathbf{T}_{BH}$

$$= [0.8\mathbf{i} - 0.6\mathbf{k}] \cdot [(375 \text{ N})\mathbf{i} + (750 \text{ N})\mathbf{j} - (750 \text{ N})\mathbf{k}]$$

$$= (300 + 450)\text{N}$$

$$= 750 \text{ N}$$

Since $\quad \mathbf{T}_{BH} = (\mathbf{T}_{BH})_{\text{Perpendicular}} + (\mathbf{T}_{BH})_{\text{Parallel}}$

Then $\quad (T_{BH})_{\text{Perpendicular}} = \sqrt{(\mathbf{T}_{BH})^2 - (\mathbf{T}_{BH})^2_{\text{Parallel}}}$

$$= \sqrt{(1125 \text{ N})^2 - (750 \text{ N})^2}$$

$$= 838.53 \text{ N}$$

and $\quad |\mathbf{M}_{AD}| = (\mathbf{T}_{BH})_{\text{Perpendicular}}\, d$

$$180 \text{ N}\cdot\text{m} = (838.53 \text{ N})d$$

$$d = 0.21466 \text{ m}$$

or $\quad d = 215 \text{ mm}$ ◀

Chapter 3, Solution 63.

From the solution of Problem 3.54:

$$\boldsymbol{\lambda}_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BG} = -(500\text{ N})\mathbf{i} + (925\text{ N})\mathbf{j} - (400\text{ N})\mathbf{k}$$

$$T_{BG} = 1125\text{ N}$$

$$M_{AD} = -222\text{ N}\cdot\text{m}$$

Only the perpendicular component of $\mathbf{T}_{BG}$ contributes to the moment of $\mathbf{T}_{BG}$ about line *AD*. The parallel component of $\mathbf{T}_{BG}$ will be used to find the perpendicular component.

Have $\quad (\mathbf{T}_{BG})_{\text{Parallel}} = \boldsymbol{\lambda}_{AD}\cdot\mathbf{T}_{BG}$

$$= [0.8\mathbf{i} - 0.6\mathbf{k}]\cdot[-(500\text{ N})\mathbf{i} + (925\text{ N})\mathbf{j} - (400\text{ N})\mathbf{k}]$$

$$= (-400 + 240)\text{N}$$

$$= -160\text{ N}$$

Since $\quad \mathbf{T}_{BG} = (\mathbf{T}_{BG})_{\text{Perpendicular}} + (\mathbf{T}_{BG})_{\text{Parallel}}$

Then $\quad (T_{BG})_{\text{Perpendicular}} = \sqrt{(T_{BG})^2 - (T_{BG})^2_{\text{Parallel}}}$

$$= \sqrt{(1125\text{ N})^2 - (-160\text{ N})^2}$$

$$= 1113.56\text{ N}$$

and $\quad |M_{AD}| = (T_{BG})_{\text{Perpendicular}}\, d$

$$222\text{ N}\cdot\text{m} = (1113.56\text{ N})d$$

$$d = 0.199361\text{ m}$$

or $\quad d = 199.4\text{ mm}$ ◀

Chapter 3, Solution 64.

From the solution of Problem 3.59:

$$\lambda_{DI} = 0.97014\mathbf{i} - 0.24254\mathbf{j}$$

$$\mathbf{T}_{EF} = (5.4 \text{ lb})\mathbf{i} - (16.2 \text{ lb})\mathbf{j} + (24.3 \text{ lb})\mathbf{k}$$

$$T_{EF} = 29.7 \text{ lb}$$

$$M_{DI} = 233.39 \text{ lb}\cdot\text{ft}$$

Only the perpendicular component of $\mathbf{T}_{EF}$ contributes to the moment of $\mathbf{T}_{EF}$ about line *DI*. The parallel component of $\mathbf{T}_{EF}$ will be used to find the perpendicular component.

Have $\quad (\mathbf{T}_{EF})_{\text{Parallel}} = \lambda_{DI}\cdot\mathbf{T}_{EF}$

$$= [0.97014\mathbf{i} - 0.24254\mathbf{j}]\cdot[(5.4 \text{ lb})\mathbf{i} - (16.2 \text{ lb})\mathbf{j} + (24.3 \text{ lb})\mathbf{k}]$$

$$= (5.2388 + 3.9291)$$

$$= 9.1679 \text{ lb}$$

Since $\quad \mathbf{T}_{EF} = (\mathbf{T}_{EF})_{\text{Perpendicular}} + (\mathbf{T}_{EF})_{\text{Parallel}}$

Then $\quad (T_{EF})_{\text{Perpendicular}} = \sqrt{(\mathbf{T}_{EF})^2 - (\mathbf{T}_{EF})^2_{\text{Parallel}}}$

$$= \sqrt{(29.7)^2 - (9.1679)^2}$$

$$= 28.250 \text{ lb}$$

and $\quad |\mathbf{M}_{DI}| = (\mathbf{T}_{EF})_{\text{Perpendicular}}\, d$

$$233.39 \text{ lb}\cdot\text{ft} = (28.250 \text{ lb})d$$

$$d = 8.2616 \text{ ft}$$

or $\quad d = 8.26 \text{ ft}$ ◀

Chapter 3, Solution 65.

From the solution of Problem 3.60:

$$\boldsymbol{\lambda}_{DI} = 0.97014\,\mathbf{i} - 0.24254\,\mathbf{j}$$

$$\mathbf{T}_{EG} = (2.4\text{ lb})\mathbf{i} - (7.2\text{ lb})\mathbf{j} - (23.4\text{ lb})\mathbf{k}$$

$$T_{EG} = 24.6\text{ lb}$$

$$M_{DI} = -\,224.74\text{ lb}\cdot\text{ft}$$

Only the perpendicular component of $\mathbf{T}_{EG}$ contributes to the moment of $\mathbf{T}_{EG}$ about line *DI*. The parallel component of $\mathbf{T}_{EG}$ will be used to find the perpendicular component.

Have $\quad (\mathbf{T}_{EG})_{\text{Parallel}} = \boldsymbol{\lambda}_{DI}\cdot\mathbf{T}_{EG}$

$$= [0.97014\,\mathbf{i} - 0.24254\,\mathbf{j}]\cdot[(2.4\text{ lb})\mathbf{i} - (7.2\text{ lb})\mathbf{j} - (23.4\text{ lb})\mathbf{k}]$$

$$= (2.3283 + 1.74629)$$

$$= 4.0746\text{ lb}$$

Since $\quad \mathbf{T}_{EG} = (\mathbf{T}_{EG})_{\text{Perpendicular}} + (T_{EG})_{\text{Parallel}}$

Then $\quad (T_{EG})_{\text{Perpendicular}} = \sqrt{(T_{EG})^2 - (T_{EG})^2_{\text{Parallel}}}$

$$= \sqrt{(24.6)^2 - (4.0746)^2}$$

$$= 24.260\text{ lb}$$

and $\quad |M_{DI}| = (T_{EG})_{\text{Perpendicular}}\, d$

$$224.74\text{ lb}\cdot\text{ft} = (24.260\text{ lb})d$$

$$d = 9.2638\text{ ft}$$

or $d = 9.26$ ft ◀

Chapter 3, Solution 66.

From the solution of Prob. 3.55:

$$\lambda_{AD} = 0.99228\mathbf{i} + 0.124035\mathbf{j}$$

$$\mathbf{F}_{EF} = (2.7\text{ kN})\mathbf{i} + (10.8\text{ kN})\mathbf{j} + (21.6\text{ kN})\mathbf{k}$$

$$F_{EF} = 24.3\text{ kN}$$

$$M_{AD} = -24.916\text{ kN}\cdot\text{m}$$

Only the perpendicular component of $\mathbf{F}_{EF}$ contributes to the moment of $\mathbf{F}_{EF}$ about edge AD. The parallel component of $\mathbf{F}_{EF}$ will be used to find the perpendicular component.

Have $(\mathbf{F}_{EF})_{\text{Parallel}} = \lambda_{AD}\cdot\mathbf{F}_{EF}$

$$= [0.99228\mathbf{i} + 0.124035\mathbf{j}]\cdot[(2.7\text{ kN})\mathbf{i} + (10.8\text{ kN})\mathbf{j} + (21.6\text{ kN})\mathbf{k}]$$

$$= 4.0187\text{ kN}$$

Since $\mathbf{F}_{EF} = (\mathbf{F}_{EF})_{\text{Perpendicular}} + (\mathbf{F}_{EF})_{\text{Parallel}}$

Then $(F_{EF})_{\text{Perpendicular}} = \sqrt{(F_{EF})^2 - (F_{EF})^2_{\text{Parallel}}}$

$$= \sqrt{(24.3)^2 - (4.0187)^2}$$

$$= 23.965\text{ kN}$$

and $|M_{AD}| = (F_{EF})_{\text{Perpendicular}}\, d$

$$24.916\text{ kN}\cdot\text{m} = (23.965\text{ kN})d$$

$$d = 1.039683\text{m}$$

or $d = 1.040$ m ◀

Chapter 3, Solution 67.

From the solution of Prob. 3.56:

$$\boldsymbol{\lambda}_{AD} = 0.99228\mathbf{i} + 0.124035\mathbf{j}$$

$$\mathbf{F}_{GH} = -(7.1 \text{ kN})\mathbf{i} + (14.2 \text{ kN})\mathbf{j} + (14.2 \text{ kN})\mathbf{k}$$

$$F_{GH} = 21.3 \text{ kN}$$

$$M_{AD} = -35.931 \text{ kN}\cdot\text{m}$$

Only the perpendicular component of $\mathbf{F}_{GH}$ contributes to the moment of $\mathbf{F}_{GH}$ about edge *AD*. The parallel component of $\mathbf{F}_{GH}$ will be used to find the perpendicular component.

Have $\left(F_{GH}\right)_{\text{Parallel}} = \boldsymbol{\lambda}_{AD} \cdot \mathbf{F}_{GH}$

$$= (0.99228\mathbf{i} + 0.124035\mathbf{j}) \cdot \left[-(7.1 \text{ kN})\mathbf{i} + (14.2 \text{ kN})\mathbf{j} + (14.2 \text{ kN})\mathbf{k}\right]$$

$$= -5.2839 \text{ kN}$$

Since $\mathbf{F}_{GH} = \left(\mathbf{F}_{GH}\right)_{\text{Perpendicular}} + \left(\mathbf{F}_{GH}\right)_{\text{Parallel}}$

Then $\left(F_{GH}\right)_{\text{Perpendicular}} = \sqrt{\left(F_{GH}\right)^2 - \left(F_{GH}\right)^2_{\text{Parallel}}}$

$$= \sqrt{(21.3)^2 - (5.2839)^2}$$

$$= 20.634 \text{ kN}$$

and $\left|M_{AD}\right| = \left(F_{GH}\right)_{\text{Perpendicular}} d$

$$35.931 \text{ kN}\cdot\text{m} = (20.634 \text{ kN})d$$

$$d = 1.741349\text{m}$$

or $d = 1.741 \text{ m}$ ◀

Chapter 3, Solution 68.

(*a*) Have $M_1 = d_1 F_1$

Where $d_1 = 0.6 \text{ m}$ and $F_1 = 40 \text{ N}$

$\therefore M_1 = (0.6 \text{ m})(40 \text{ N})$

or $M_1 = 24.0 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) Have $\mathbf{M}_{\text{Total}} = \mathbf{M}_1 + \mathbf{M}_2$

$8 \text{ N}\cdot\text{m} = 24.0 \text{ N}\cdot\text{m} - (0.820 \text{ m})(\cos\alpha)(24 \text{ N})$

$\therefore \cos\alpha = 0.81301$

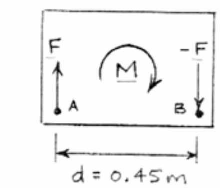
or $\alpha = 35.6°$ ◀

(*c*) Have $\mathbf{M}_1 + \mathbf{M}_2 = 0$

$24 \text{ N}\cdot\text{m} - d_2(24 \text{ N}) = 0$

or $d_2 = 1.000 \text{ m}$ ◀

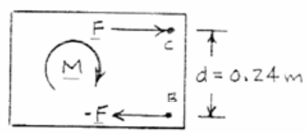
Chapter 3, Solution 69.



(*a*) $M = Fd$

$$12\ \text{N}\cdot\text{m} = F(0.45\ \text{m})$$

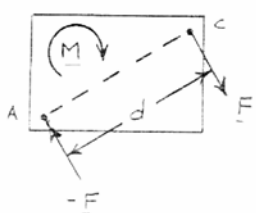
or $F = 26.7$ N ◀



(*b*) $M = Fd$

$$12\ \text{N}\cdot\text{m} = F(0.24\ \text{m})$$

or $F = 50.0$ N ◀



(*c*) $M = Fd$

Where $d = \sqrt{(0.45\ \text{m})^2 + (0.24\ \text{m})^2}$

$= 0.51$ m

$$12\ \text{N}\cdot\text{m} = F(0.51\ \text{m})$$

or $F = 23.5$ N ◀

Chapter 3, Solution 70.

(*a*) Note when $a = 8$ in., $\mathbf{r}_{C/F}$ is perpendicular to the inclined 10 lb forces.

Have $M = \Sigma F_d \,(+\circlearrowleft)$

$$= -(10\text{ lb})\left[a + 8\text{ in.} + 2(1\text{ in.})\right] - (10\text{ lb})\left[2a\sqrt{2} + 2(1\text{ in.})\right]$$

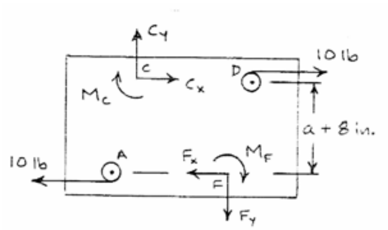
For $a = 8$ in.,

$$M = -(10\text{ lb})(18\text{ in.} + 24.627\text{ in.})$$

$$= -426.27\text{ lb}\cdot\text{in.}$$

or $M = 426\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

(*b*)



Have $\mathbf{M} = 480\text{ lb}\cdot\text{in.} \circlearrowright$

Also $M = \Sigma(M + F_d)\,(+\circlearrowleft)$

= Moment of couple due to horizontal forces at *A* and *D* + Moment of force-couple systems at *C* and *F* about *C*.

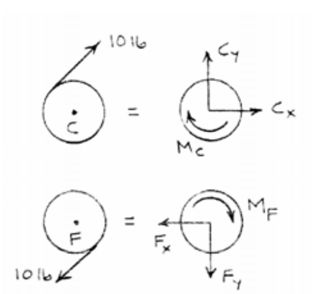
Then $-480\text{ lb}\cdot\text{in.} = -10\text{ lb}\left[a + 8\text{ in.} + 2(1\text{ in.})\right] + \left[M_C + M_F + F_X(a + 8\text{ in.}) + F_y(2a)\right]$

Where $M_C = -(10\text{ lb})(1\text{ in.}) = -10\text{ lb}\cdot\text{in.}$

$M_F = M_C = -10\text{ lb}\cdot\text{in.}$

$$F_x = \frac{-10}{\sqrt{2}}\text{lb}$$

$$F_y = \frac{-10}{\sqrt{2}}\text{lb}$$



$$\therefore -480\text{ lb}\cdot\text{in.} = -10\text{ lb}(a + 10\text{ in.}) - 10\text{ lb}\cdot\text{in.} - 10\text{ lb}\cdot\text{in.}$$

$$-\frac{10\text{ lb}}{\sqrt{2}}(a + 8\text{ in.}) - \frac{10\text{ lb}}{\sqrt{2}}(2a)$$

$$303.43 = 31.213\,a$$

or $a = 9.72\text{in.}$ ◀

Chapter 3, Solution 71.

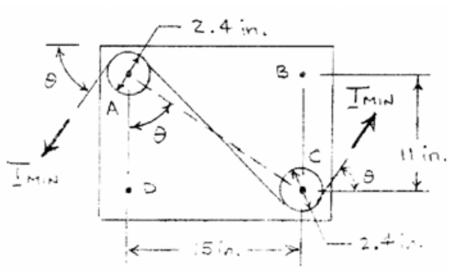
(a) Have $M = \Sigma F_d \, (+\circlearrowright)$

$= (9 \text{ lb})(13.8 \text{ in.}) - (2.5 \text{ lb})(15.2 \text{ in.})$

$= (86.2 \text{ lb} \cdot \text{in.})$

$\mathbf{M} = 86.2 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀

(b)



Have $M = Td = 86.2 \text{ lb} \cdot \text{in.}$

For T to be a minimum, d must be maximum.

$\therefore T_{\min}$ must be perpendicular to line AC.

$$\tan\theta = \frac{15.2 \text{ in.}}{11.4 \text{ in.}}$$

$\theta = 53.130°$

or $\theta = 53.1°$ ◀

(c) Have $M = T_{\min} d_{\max}$ Where $M = 86.2 \text{ lb} \cdot \text{in.}$

$$d_{\max} = \sqrt{(15.2 \text{ in.})^2 + (11.4 \text{ in.})^2} + 2(1.2 \text{ in.})$$

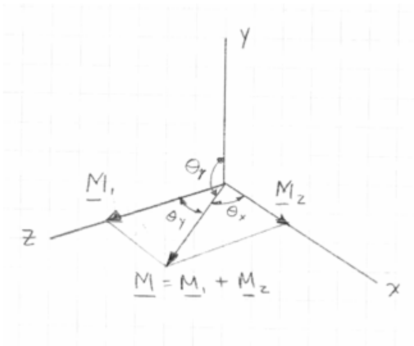
$= 21.4 \text{ in.}$

$\therefore 86.2 \text{ lb} \cdot \text{in.} = T_{\min}(21.4 \text{ in.})$

$T_{\min} = 4.0280 \text{ lb}$

or $T_{\min} = 4.03 \text{ lb}$ ◀

Chapter 3, Solution 72.



Based on $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_1 = (18\ \text{N}\cdot\text{m})\mathbf{k}$$

$$\mathbf{M}_2 = (7.5\ \text{N}\cdot\text{m})\mathbf{i}$$

$$\therefore \mathbf{M} = (7.5\ \text{N}\cdot\text{m})i + (18\ \text{N}\cdot\text{m})\mathbf{k}$$

and $$M = \sqrt{(7.5\ \text{N}\cdot\text{m})^2 + (18\ \text{N}\cdot\text{m})^2}$$

$$= 19.5\ \text{N}\cdot\text{m}$$

or $M = 19.50\ \text{N}\cdot\text{m}$ ◀

With $$\lambda = \frac{\mathbf{M}}{M} = \frac{(7.5\ \text{N}\cdot\text{m})\mathbf{i} + (18\ \text{N}\cdot\text{m})\mathbf{k}}{19.5\ \text{N}\cdot\text{m}}$$

$$= \frac{5}{13}\mathbf{i} + \frac{12}{13}\mathbf{k}$$

or cos¤ x =(7.5/19.5)
cos¤ z =(18/19.5)
and ¤ y =90¤

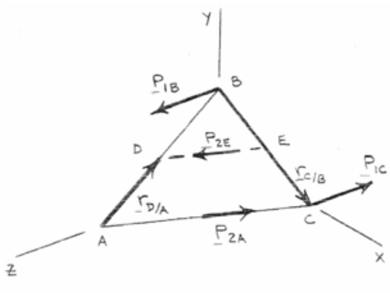
Then $$\cos\theta_x = \frac{5}{13} \qquad \therefore \theta_x = 67.380°$$

$$\cos\theta_y = 0 \qquad \therefore \theta_y = 90°$$

$$\cos\theta_z = \frac{12}{13} \qquad \therefore \theta_z = 22.620°$$

or $\theta_x = 67.4°$, $\theta_y = 90.0°$, $\theta_z = 22.6°$ ◀

Chapter 3, Solution 73.



Have $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

Where $\mathbf{M}_1 = \mathbf{r}_{C/B} \times \mathbf{P}_{IC}$

$$\mathbf{r}_{C/B} = (38.4 \text{ in.})\mathbf{i} - (16 \text{ in.})\mathbf{j}$$

$$\mathbf{P}_{IC} = -(25 \text{ lb})\mathbf{k}$$

$$\therefore \mathbf{M}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 38.4 & -16 & 0 \\ 0 & 0 & -25 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= (400 \text{ lb}\cdot\text{in.})\mathbf{i} + (960 \text{ lb}\cdot\text{in.})\mathbf{j}$$

and $\mathbf{M}_2 = \mathbf{r}_{D/A} \times \mathbf{P}_{ZE}$

$$\mathbf{r}_{D/A} = (8 \text{ in.})\mathbf{j} - (22 \text{ in.})\mathbf{k}$$

$$\mathbf{P}_{ZE} = P_{ZE}\frac{\overrightarrow{ED}}{ED} = (36.5 \text{ lb})\frac{\left[-(19.2 \text{ in.})\mathbf{i} + (22 \text{ in.})\mathbf{k}\right]}{\sqrt{(-19.2 \text{ in.})^2 + (22 \text{ in.})^2}}$$

$$= -(24 \text{ lb})\mathbf{i} + (27.5 \text{ lb})\mathbf{k}$$

$$\therefore \mathbf{M}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -22 \\ -24 & 0 & 27.5 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$\mathbf{M}_2 = (220 \text{ lb}\cdot\text{in.})\mathbf{i} + (528 \text{ lb}\cdot\text{in.})\mathbf{j} + (192 \text{ lb}\cdot\text{in.})\mathbf{k}$$

and $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

$$= \left[(400 \text{ lb}\cdot\text{in.})\mathbf{i} + (960 \text{ lb}\cdot\text{in.})\mathbf{j}\right] + \left[(220 \text{ lb}\cdot\text{in.})\mathbf{i} + (528 \text{ lb}\cdot\text{in.})\mathbf{j} + (192 \text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$= (620 \text{ lb}\cdot\text{in.})\mathbf{i} + (1488 \text{ lb}\cdot\text{in.})\mathbf{j} + (192 \text{ lb}\cdot\text{in.})\mathbf{k}$$

continued

$$M = \left[\sqrt{(620)^2 + (1488)^2 + (192)^2}\right] \text{lb}\cdot\text{in.}$$

$$= 1623.39 \text{ lb}\cdot\text{in.}$$

or $M = 1.623$ kip·in. ◀

$$\lambda = \frac{\mathbf{M}}{M} = \frac{(620 \text{ lb}\cdot\text{in.})\mathbf{i} + (1488 \text{ lb}\cdot\text{in.})\mathbf{j} + (192 \text{ lb}\cdot\text{in.})\mathbf{k}}{1623.39 \text{ lb}\cdot\text{in.}}$$

$$= 0.38192\mathbf{i} + 0.91660\mathbf{j} + 0.118271\mathbf{k}$$

$\cos\theta_x = 0.38192$ or $\theta_x = 67.5°$ ◀

$\cos\theta_y = 0.91660$ or $\theta_y = 23.6°$ ◀

$\cos\theta_z = 0.118271$ or $\theta_z = 83.2°$ ◀

Chapter 3, Solution 74.

Have $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

Where $\mathbf{M}_1 = \mathbf{r}_{E/D} \times \mathbf{F}_D$

$$= -(0.7 \text{ m})\mathbf{k} \times (80 \text{ N})\mathbf{j}$$

$$= (56.0 \text{ N}\cdot\text{m})\mathbf{i}$$

And $\mathbf{M}_2 = \mathbf{r}_{G/F} \times \mathbf{F}_B$

Now $d_{BF} = \sqrt{(-0.300 \text{ m})^2 + (0.540 \text{ m})^2 + (0.350 \text{ m})^2}$

$$= 0.710 \text{ m}$$

Then $\mathbf{F}_B = \lambda_{BF} F_B$

$$= \frac{(-0.300 \text{ m})\mathbf{i} + (0.540 \text{ m})\mathbf{j} + (0.350 \text{ m})\mathbf{k}}{0.710 \text{ m}}(71 \text{ N})$$

$$= -(30 \text{ N})\mathbf{i} + (54 \text{ N})\mathbf{j} + (35 \text{ N})\mathbf{k}$$

$\therefore$ $\mathbf{M}_2 = (0.54 \text{ m})\mathbf{j} \times \left[-(30 \text{ N})\mathbf{i} + (54 \text{ N})\mathbf{j} + (35 \text{ N})\mathbf{k}\right]$

$$= (18.90 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{k}$$

Finally $\mathbf{M} = (56.0 \text{ N}\cdot\text{m})\mathbf{i} + \left[(18.90 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{k}\right]$

$$= (74.9 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{k}$$

and $M = \sqrt{(74.9 \text{ N}\cdot\text{m})^2 + (16.20 \text{ N}\cdot\text{m})^2}$

$$= 76.632 \text{ N}\cdot\text{m}$$

or $\mathbf{M} = 76.6 \text{ N}\cdot\text{m}$ ◀

$$\cos\theta_x = \frac{74.9}{76.632} \qquad \cos\theta_y = \frac{0}{76.632} \qquad \cos\theta_z = \frac{16.20}{76.632}$$

or $\theta_x = 12.20°$ $\quad \theta_y = 90.0°$ $\quad \theta_z = 77.8°$ ◀

Chapter 3, Solution 75.

Have $\mathbf{M} = (\mathbf{M}_1 + \mathbf{M}_2) + \mathbf{M}_P$

From the solution to Problem 3.74

$$(\mathbf{M}_1 + \mathbf{M}_2) = (74.9\ \text{N}\cdot\text{m})\mathbf{i} + (16.20\ \text{N}\cdot\text{m})\mathbf{k}$$

Now $\mathbf{M}_P = \mathbf{r}_{D/E} \times \mathbf{P}_E$

$$= \left[(0.54\ \text{m})\mathbf{j} + (0.70\ \text{m})\mathbf{k}\right] \times (90\ \mathbf{N})\mathbf{i}$$

$$= (63.0\ \text{N}\cdot\text{m})\mathbf{j} - (48.6\ \text{N}\cdot\text{m})\mathbf{k}$$

$\therefore$ $\mathbf{M} = (74.9\mathbf{i} + 16.20\mathbf{k}) + (63.0\ \mathbf{j} - 48.6\ \mathbf{k})$

$$= (74.9\ \text{N}\cdot\text{m})\mathbf{i} + (63.0\ \text{N}\cdot\text{m})\mathbf{j} - (32.4\ \text{N}\cdot\text{m})\mathbf{k}$$

and $M = \sqrt{(74.9\ \text{N}\cdot\text{m})^2 + (63.0\ \text{N}\cdot\text{m})^2 + (-32.4\ \text{N}\cdot\text{m})^2}$

$$= 103.096\ \text{N}\cdot\text{m}$$

or $M = 103.1\ \text{N}\cdot\text{m}$ ◀

and $\cos\theta_x = \dfrac{74.9}{103.096}$ $\cos\theta_y = \dfrac{63.0}{103.096}$ $\cos\theta_z = \dfrac{-32.4}{103.096}$

or $\theta_x = 43.4°$ $\theta_y = 52.3°$ $\theta_z = 108.3°$ ◀

Chapter 3, Solution 76.

Have $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_P$

From Problem 3.73 solution:

$$\mathbf{M}_1 = (400 \text{ lb}\cdot\text{in.})\mathbf{i} + (960 \text{ lb}\cdot\text{in.})\mathbf{j}$$

$$\mathbf{M}_2 = (220 \text{ lb}\cdot\text{in.})\mathbf{i} + (528 \text{ lb}\cdot\text{in.})\mathbf{j} + (192 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Now $\mathbf{M}_P = \mathbf{r}_{E/A} \times \mathbf{P}_E$

$$\mathbf{r}_{E/A} = (19.2 \text{ in.})\mathbf{i} + (8 \text{ in.})\mathbf{j} - (44 \text{ in.})\mathbf{k}$$

$$\mathbf{P}_E = (52.5 \text{ lb})\mathbf{j}$$

Therefore

$$\mathbf{M}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19.2 & 8 & -44 \\ 0 & 52.5 & 0 \end{vmatrix}$$

$$= (2310 \text{ lb. in.})\mathbf{i} + (1008 \text{ lb. in.})\mathbf{k}$$

and $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_P$

$$= \left[(400 + 220 + 2310)\mathbf{i} + (960 + 528)\mathbf{j} + (192 + 1008)\mathbf{k}\right] \text{lb}\cdot\text{in.}$$

$$= (2930 \text{ lb}\cdot\text{in.})\mathbf{i} + (1488 \text{ lb}\cdot\text{in.})\mathbf{j} + (1200 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{(2930)^2 + (1488)^2 + (1200)^2}$$

$$= 3498.4 \text{ lb}\cdot\text{in.}$$

or $\mathbf{M} = 3.50 \text{ kip}\cdot\text{in.}$ ◀

continued

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{2930\mathbf{i} + 1488\mathbf{j} + 1200\mathbf{k}}{3498.4}$$

$$= 0.83753\mathbf{i} + 0.42534\mathbf{j} + 0.34301\mathbf{k}$$

$$\cos\theta_x = 0.83753$$

or $\theta_x = 33.1°$ ◀

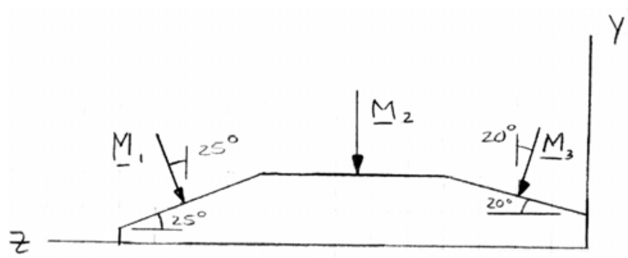
$$\cos\theta_y = 0.42534$$

or $\theta_y = 64.8°$ ◀

$$\cos\theta_z = 0.34301$$

or $\theta_z = 69.9°$ ◀

Chapter 3, Solution 77.



Have $|\mathbf{M}| = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

Where $\mathbf{M}_1 = -(1.2\text{ lb}\cdot\text{ft})\cos 25°\mathbf{j} - (1.2\text{ lb}\cdot\text{ft})\sin 25°\mathbf{k}$

$\mathbf{M}_2 = -(1.3\text{ lb}\cdot\text{ft})\mathbf{j}$

$\mathbf{M}_3 = -(1.4\text{ lb}\cdot\text{ft})\cos 20°\mathbf{j} + (1.4\text{ lb}\cdot\text{ft})\sin 20°\mathbf{k}$

$\therefore \quad \mathbf{M} = (-1.08757 - 1.3 - 1.31557)\mathbf{j} + (-0.507142 + 0.478828)\mathbf{k}$

$= -(3.7031\text{ lb}\cdot\text{ft})\mathbf{j} - (0.028314\text{ lb}\cdot\text{ft})\mathbf{k}$

and $|\mathbf{M}| = \sqrt{(-3.7031)^2 + (-0.028314)^2} = 3.7032\text{ lb}\cdot\text{ft}$

or $\mathbf{M} = 3.70\text{ lb}\cdot\text{ft}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{-3.7031\mathbf{j} - 0.028314\mathbf{k}}{3.7032}$$

$$= -0.99997\mathbf{j} - 0.0076458\mathbf{k}$$

$\cos\theta_x = 0$

or $\theta_x = 90°$ ◀

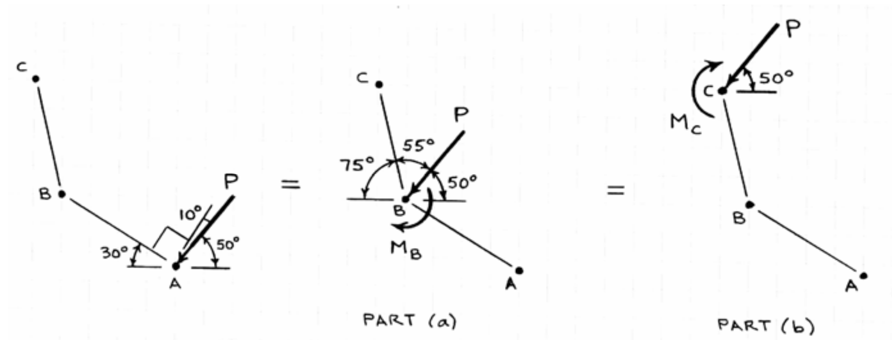
$\cos\theta_y = -0.99997$

or $\theta_y = 179.6°$ ◀

$\cos\theta_z = -0.0076458$

or $\theta_z = 90.4°$ ◀

Chapter 3, Solution 78.



(*a*) $\mathbf{F}_B = \mathbf{P}$: $\quad \therefore F_B = 160.0 \text{ N} \measuredangle 50.0°$ ◀

$$M_B = -r_{BA}P\cos 10°$$

$$= -(0.355 \text{ m})(160 \text{ N})\cos 10°$$

$$= -55.937 \text{ N}\cdot\text{m}$$

or $M_B = 55.9 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) $\mathbf{F}_C = \mathbf{P}$: $\quad \therefore F_C = 160.0 \text{ N} \measuredangle 50.0°$ ◀

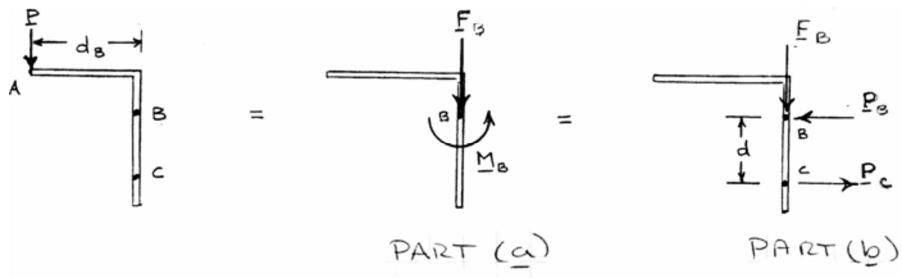
$$M_C = M_B - r_{CB}(F_B)_\perp$$

$$= M_B - r_{CB}F_B \sin 55°$$

$$= -55.937 \text{ N}\cdot\text{m} - (0.305 \text{ m})(160 \text{ N})\sin 55°$$

or $\mathbf{M}_C = 95.9 \text{ N}\cdot\text{m}$ ↻ ◀

Chapter 3, Solution 79.



(*a*) $\Sigma\mathbf{F}$: $\mathbf{F}_B = 135 \text{ N}$

or $\mathbf{F}_B = 135 \text{ N} \downarrow$ ◀

$\Sigma\mathbf{M}$: $M_B = P\, d_B$

$= (135 \text{ N})(0.125 \text{ m})$

$= 16.875 \text{ N}\cdot\text{m}$

or $\mathbf{M}_B = 16.88 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) $\Sigma\mathbf{M}_B$: $M_B = F_C\, d$

$16.875 \text{ N}\cdot\text{m} = F_C (0.075 \text{ m})$

$F_C = 225 \text{ N}$

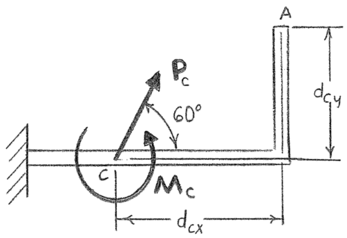
or $\mathbf{F}_C = 225 \text{ N} \longrightarrow$ ◀

$\Sigma\mathbf{F}$: $0 = -F_B + F_C$

$F_B = F_C = 225 \text{ N}$

or $\mathbf{F}_B = 225 \text{ N} \longleftarrow$ ◀

Chapter 3, Solution 80.



(*a*) Based on ΣF: $P_C = P = 700$ N

or $\mathbf{P}_C = 700$ N $\measuredangle$ 60° ◀

ΣM_C: $M_C = -P_x d_{Cy} + P_y d_{Cx}$

where $P_x = (700 \text{ N})\cos 60° = 350$ N

$P_y = (700 \text{ N})\sin 60° = 606.22$ N

$d_{Cx} = 1.6$ m

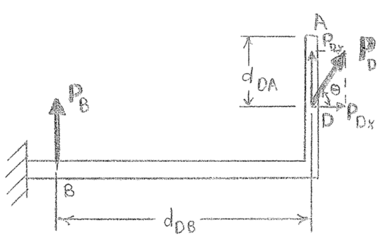
$d_{Cy} = 1.1$ m

$$\therefore\ M_C = -(350 \text{ N})(1.1 \text{ m}) + (606.22 \text{ N})(1.6 \text{ m})$$

$$= -385 \text{ N}\cdot\text{m} + 969.95 \text{ N}\cdot\text{m}$$

$$= 584.95 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 585$ N·m ↺ ◀



(*b*) Based on ΣF_x: $P_{Dx} = P\cos 60°$

$$= (700 \text{ N})\cos 60°$$

$$= 350 \text{ N}$$

ΣM_D: $(P\cos 60°)(d_{DA}) = P_B(d_{DB})$

$$\left[(700 \text{ N})\cos 60°\right](0.6 \text{ m}) = P_B(2.4 \text{ m})$$

$$P_B = 87.5 \text{ N}$$

or $\mathbf{P}_B = 87.5$ N ↑ ◀

$$\Sigma F_y: \quad P\sin 60° = P_B + P_{Dy}$$

$$(700\text{ N})\sin 60° = 87.5\text{ N} + P_{Dy}$$

$$P_{Dy} = 518.72\text{ N}$$

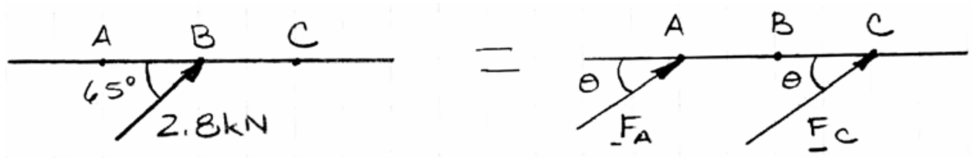
$$P_D = \sqrt{(P_{Dx})^2 + (P_{Dy})^2}$$

$$= \sqrt{(350)^2 + (518.72)^2} = 625.76\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{P_{Dy}}{P_{Dx}}\right) = \tan^{-1}\left(\frac{518.72}{350}\right) = 55.991°$$

or $P_D = 626\text{ N} \measuredangle\ 56.0°$ ◀

Chapter 3, Solution 81.



ΣF_x: $\quad 2.8\cos 65° = F_A \cos\theta + F_C \cos\theta$

$$= \left(F_A + F_C\right)\cos\theta \qquad (1)$$

ΣF_y: $\quad 2.8\sin 65° = F_A \sin\theta + F_C \sin\theta$

$$= \left(F_A + F_C\right)\sin\theta \qquad (2)$$

Then $\quad \dfrac{(2)}{(1)} \Rightarrow \tan 65° = \tan\theta$

or $\theta = 65.0°$

ΣM_A: $\quad (27 \text{ m})(2.8 \text{ kN})\sin 65° = (72 \text{ m})(F_C)\sin 65°$

or $F_C = 1.050$ kN

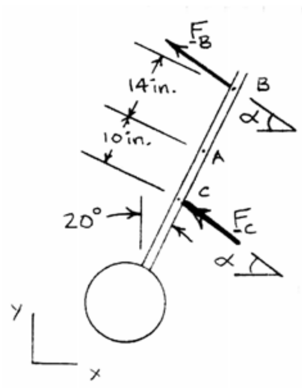
From Equation (1): $\;2.8 \text{ kN} = F_A + 1.050 \text{ kN}$

or $F_A = 1.750$ kN

$\therefore\; \mathbf{F}_A = 1.750 \text{ kN} \measuredangle\, 65.0°$ ◀

$\mathbf{F}_C = 1.050 \text{ kN} \measuredangle\, 65.0°$ ◀

Chapter 3, Solution 82.



Based on

$$\Sigma F_x: \; -(54 \text{ lb})\cos 30° = -F_B \cos\alpha - F_C \cos\alpha$$

$$(F_B + F_C)\cos\alpha = (54 \text{ lb})\cos 30° \qquad (1)$$

$$\Sigma F_y: \quad (54 \text{ lb})\sin 30° = F_B \sin\alpha + F_C \sin\alpha$$

$$\text{or } (F_B + F_C)\sin\alpha = (54 \text{ lb})\sin 30° \qquad (2)$$

From $\dfrac{Eq(2)}{Eq(1)}: \tan\alpha = \tan 30°$

$$\therefore \alpha = 30°$$

Based on ΣM_C : $\quad \left[(54 \text{ lb})\cos(30° - 20°)\right](10 \text{ in.}) = (F_B \cos 10°)(24 \text{ in.})$

$$\therefore F_B = 22.5 \text{ lb}$$

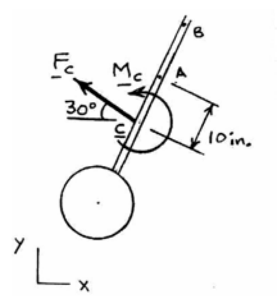
or $\mathbf{F}_B = 22.5$ lb ⦣ 30° ◀

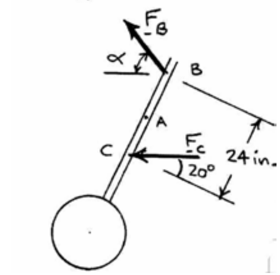
From Eq. (1), $\quad (22.5 + F_C)\cos 30° = (54)\cos 30°$

$$F_C = 31.5 \text{ lb}$$

or $\mathbf{F}_C = 31.5$ lb ⦣ 30° ◀

Chapter 3, Solution 83.





(*a*) Based on

ΣF_x: $\quad -(54\text{ lb})\cos 30° = -F_C\cos 30°$

$\therefore \quad F_C = 54\text{ lb}$

or $\mathbf{F}_C = 54.0\text{ lb} \searrow 30°$ ◀

ΣM_C: $\quad \left[(54\text{ lb})\cos 10°\right](10\text{ in.}) = M_C$

$\therefore \quad M_C = 531.80\text{ lb}\cdot\text{in.}$

or $\mathbf{M}_C = 532\text{ lb}\cdot\text{in.}$ ↻ ◀

(*b*) Based on

ΣF_y: $\quad (54\text{ lb})\sin 30° = F_B\sin\alpha$

or $\quad F_B\sin\alpha = 27 \qquad (1)$

ΣM_B: $\quad 531.80\text{ lb}\cdot\text{in.} - \left[(54\text{ lb})\cos 10°\right](24\text{ in.})$

$= -F_C\left[(24\text{ in.})\cos 20°\right]$

$F_C = 33.012\text{ lb}$

or $\mathbf{F}_C = 33.0\text{ lb} \longleftarrow$ ◀

And $\quad \Sigma F_x$: $\quad -(54\text{ lb})\cos 30° = -33.012\text{ lb} - F_B\cos\alpha$

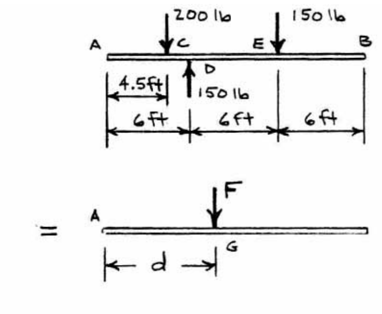
$F_B\cos\alpha = 13.7534 \qquad (2)$

From $\quad \dfrac{Eq(1)}{Eq(2)}$: $\quad \tan\alpha = \dfrac{27}{13.7534} \quad \therefore \alpha = 63.006°$

From Eq. (1), $\quad F_B = \dfrac{27}{\sin(63.006°)} = 30.301\text{ lb}$

or $\mathbf{F}_B = 30.3\text{ lb} \searrow 63.0°$ ◀

Chapter 3, Solution 84.



(*a*) Have $\uparrow \Sigma F_y$: $\quad F_C + F_D + F_E = F$

$$F = -200\text{ lb} + 150\text{ lb} - 150\text{ lb}$$

$$F = -200\text{ lb}$$

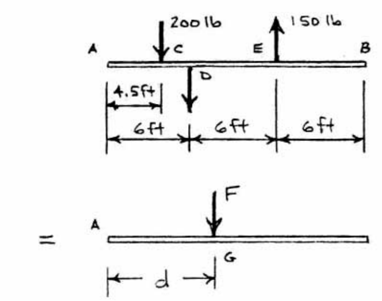
or $\mathbf{F} = 200\text{ lb}\downarrow$ ◀

Have $\quad +\circlearrowleft \Sigma M_G$: $\quad F_C\left(d - 4.5\text{ ft}\right) - F_D\left(6\text{ ft}\right) = 0$

$$\left(200\text{ lb}\right)\left(d - 4.5\text{ ft}\right) - \left(150\text{ lb}\right)\left(6\text{ ft}\right) = 0$$

$$d = 9\text{ ft}$$

or $d = 9.00\text{ ft}$ ◀



(*b*) Changing directions of the two 150-lb forces only changes the sign of the couple.

$$\therefore\ F = -200\text{ lb}$$

or $\mathbf{F} = 200\text{ lb}\downarrow$ ◀

And $\quad +\circlearrowleft \Sigma M_G$: $\quad F_C\left(d - 4.5\text{ ft}\right) + F_D\left(6\text{ ft}\right) = 0$

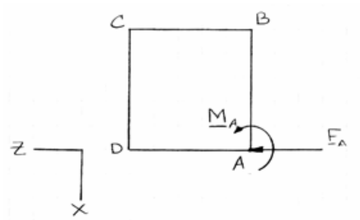
$$\left(200\text{ lb}\right)\left(d - 4.5\text{ ft}\right) + \left(150\text{ lb}\right)\left(6\text{ ft}\right) = 0$$

$$d = 0$$

or $d = 0$ ◀

Chapter 3, Solution 85.

(*a*)



Based on ΣF_z:

$$-200 \text{ N} + 200 \text{ N} + 240 \text{ N} = F_A$$

$$F_A = 240 \text{ N}$$

or $\mathbf{F}_A = (240 \text{ N})\mathbf{k}$ ◀

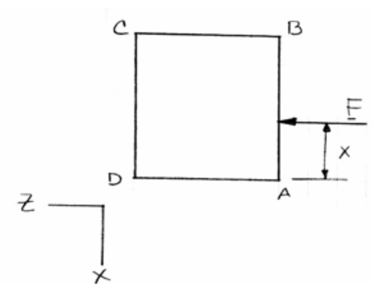
Based on ΣM_A:

$$(200 \text{ N})(0.7 \text{ m}) - (200 \text{ N})(0.2 \text{ m}) = M_A$$

$$M_A = 100 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = (100.0 \text{ N}\cdot\text{m})\mathbf{j}$ ◀

(*b*)



Based on ΣF_z:

$$-200 \text{ N} + 200 \text{ N} + 240 \text{ N} = F$$

$$F = 240 \text{ N}$$

or $\mathbf{F} = (240 \text{ N})\mathbf{k}$ ◀

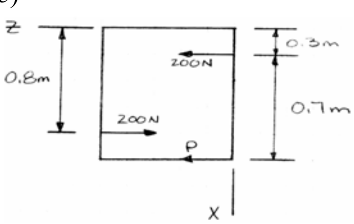
Based on ΣM_A:

$$100 \text{ N}\cdot\text{m} = (240 \text{ N})(x)$$

$$x = 0.41667 \text{ m}$$

or $x = 0.417$ m From A along AB ◀

(*c*)



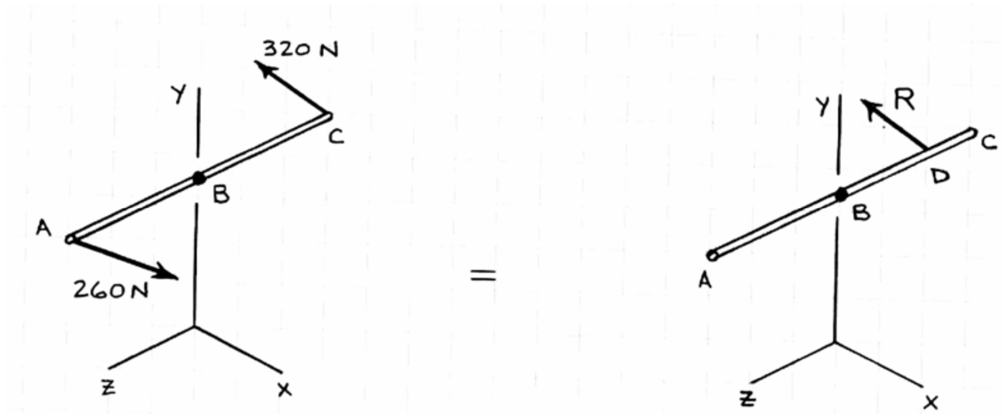
Based on ΣM_B:

$$-(200 \text{ N})(0.3 \text{ m}) + (200 \text{ N})(0.8 \text{ m}) - P(1 \text{ m}) = R(0)$$

$$P = 100 \text{ N}$$

or $P = 100.0$ N ◀

Chapter 3, Solution 86.



Let **R** be the single equivalent force...

$\Sigma \mathbf{F}$: $\quad \mathbf{R} = \mathbf{F}_A + \mathbf{F}_C$

$$= (260\text{ N})(\cos 10°\,\mathbf{i} - \sin 10°\,\mathbf{k}) + (320\text{ N})(-\cos 8°\,\mathbf{i} - \sin 8°\,\mathbf{k})$$

$$= -(60.836\text{ N})\mathbf{i} - (89.684\text{ N})\mathbf{k}$$

or $\mathbf{R} = -(60.8\text{ N})\mathbf{i} - (89.7\text{ N})\mathbf{k}$ ◀

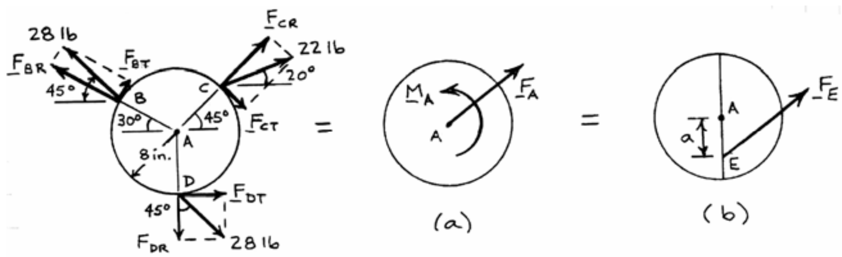
ΣM_A: $\quad r_{AD}R_x = r_{AC}F_C\cos 8°$

$$r_{AD}(60.836\text{ N}) = (0.690\text{ m})(320\text{ N})\cos 8°$$

$$r_{AD} = 3.5941\text{ m}$$

∴ **R** Would have to be applied 3.59 m to the right of *A* ◀
on an extension of handle *ABC*.

Chapter 3, Solution 87.



(*a*) Have $\Sigma\mathbf{F}$: $\mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{F}_A$

Since $\mathbf{F}_B = -\mathbf{F}_D$

$\therefore$ $\mathbf{F}_A = \mathbf{F}_C = 22 \text{ lb} \measuredangle 20°$

or $\mathbf{F}_A = 22.0 \text{ lb} \measuredangle 20°$ ◀

Have $+\circlearrowright \Sigma M_A$: $-F_{BT}(r) - F_{CT}(r) + F_{DT}(r) = M_A$

$$-[(28 \text{ lb})\sin 15°](8 \text{ in.}) - [(22 \text{ lb})\sin 25°](8 \text{ in.})$$

$$+[(28 \text{ lb})\sin 45°](8 \text{ in.}) = M_A$$

$$M_A = 26.036 \text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_A = 26.0 \text{ lb}\cdot\text{in.} \circlearrowright$ ◀

(*b*) Have $\Sigma\mathbf{F}$: $\mathbf{F}_A = \mathbf{F}_E$

or $\mathbf{F}_E = 22.0 \text{ lb} \measuredangle 20°$ ◀

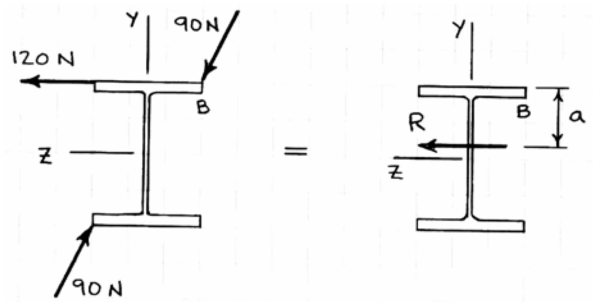
$+\circlearrowright \Sigma\mathbf{M}$: $M_A = [F_E \cos 20°](a)$

$\therefore$ $26.036 \text{ lb}\cdot\text{in.} = [(22 \text{ lb})\cos 20°](a)$

$$a = 1.25941 \text{ in.}$$

or $a = 1.259$ in. Below A ◀

Chapter 3, Solution 88.



(*a*) Let **R** be the single equivalent force. Then

$$\mathbf{R} = (120 \text{ N})\mathbf{k}$$

$R = 120$ N ◀

$+\circlearrowleft \Sigma M_B$: $-a(120 \text{ N}) = -(0.165 \text{ m})(90 \text{ N})\cos 15° + (0.201 \text{ m})(90 \text{ N})\sin 15°$

$$a = 0.080516 \text{ m}$$

$$\therefore \text{The line of action is } y = \frac{201}{2}\text{mm} - 80.516 \text{ mm} = 19.984 \text{ mm}$$

or $y = 19.98$ mm ◀

(*b*) $+\circlearrowleft \Sigma M_B$: $-[(0.201 - 0.040)\text{m}](120 \text{ N}) = -(0.165 \text{ m})(90 \text{ N})\cos\theta + (0.201 \text{ m})(90 \text{ N})\sin\theta$

or $\cos\theta - 1.21818\sin\theta = 1.30101$

or $\cos^2\theta = (1.30101 + 1.21818\sin\theta)^2$

or $1 - \sin^2\theta = 1.69263 + 3.1697\sin\theta + 1.48396\sin^2\theta$

or $2.48396\sin^2\theta + 3.1697\sin\theta + 0.69263 = 0$

Then $$\sin\theta = \frac{-3.1697 \pm \sqrt{(3.1697)^2 - 4(2.48396)(0.69263)}}{2(2.48396)}$$

or $\theta = -16.26°$ and $\theta = -85.0°$ ◀

Chapter 3, Solution 89.

(*a*) First note that **F** = **P** and that **F** must be equivalent to (**P**, $\mathbf{M}_D$) at point *D*,

Where $\mathbf{M}_D = 57.6 \text{ N}\cdot\text{m}$ ↻

For $F = (F)_{\min}$ **F** must act as far from *D* as possible

zira gashtavar barabar ba niru dar fasele ast, Fmin =>dmax

$\therefore$ Point of application is at point *B* ◀

(*b*) For $(F)_{\min}$ **F** must be perpendicular to *BD*

Now $d_{DB} = \sqrt{(630 \text{ mm})^2 + (-160 \text{ mm})^2}$

$= 650 \text{ mm}$

$\tan\alpha = \dfrac{63}{16}$

$\alpha = 75.7°$

faseleye beyne D va B => bar har do amud ast.

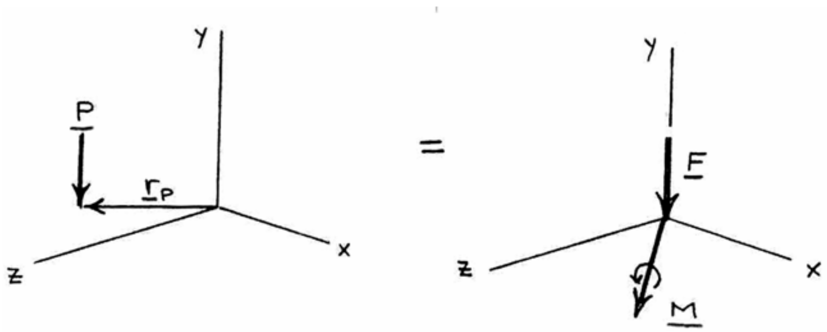
Then $M_D = d_{DB}F$

$57.6 \text{ N}\cdot\text{m} = (0.650 \text{ m})F$

$F = 88.6 \text{ N}$

or $\mathbf{F} = 88.6 \text{ N} \measuredangle 75.7°$ ◀

Chapter 3, Solution 90.



Have $\Sigma\mathbf{F}$: $-(250\text{ kN})\mathbf{j} = \mathbf{F}$

or $\mathbf{F} = -(250\text{ kN})\mathbf{j}$ ◀

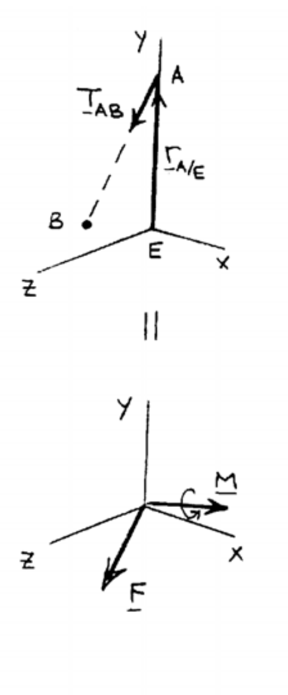
Also have $\Sigma\mathbf{M}_G$: $\mathbf{r}_P \times \mathbf{P} = \mathbf{M}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.030 & 0 & 0.060 \\ 0 & -250 & 0 \end{vmatrix} \text{kN}\cdot\text{m} = \mathbf{M}$$

$$\therefore\ \mathbf{M} = (15\text{ kN}\cdot\text{m})\mathbf{i} + (7.5\text{ kN}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = (15.00\text{ kN}\cdot\text{m})\mathbf{i} + (7.50\text{ kN}\cdot\text{m})\mathbf{k}$ ◀

Chapter 3, Solution 91.



Have $\Sigma\mathbf{F}$: $\quad \mathbf{T}_{AB} = \mathbf{F}$

where $\mathbf{T}_{AB} = T_{AB}\dfrac{\overrightarrow{AB}}{AB}$

$$= (54 \text{ lb})\frac{2.25\mathbf{i} - 18\mathbf{j} + 9\mathbf{k}}{\sqrt{(2.25)^2 + (-18)^2 + (9)^2}}$$

$$= (6 \text{ lb})\mathbf{i} - (48 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}$$

So that $\quad \mathbf{F} = (6.00 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j} + (24.0 \text{ lb})\mathbf{k}$ ◀

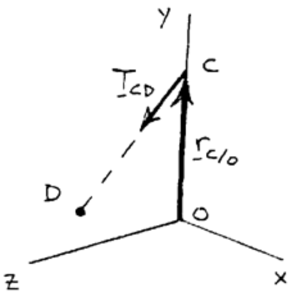
Have $\Sigma\mathbf{M}_E$: $\quad \mathbf{r}_{A/E} \times \mathbf{T}_{AB} = \mathbf{M}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 22.5 & 0 \\ 6 & -48 & 24 \end{vmatrix} \text{lb}\cdot\text{ft} = \mathbf{M}$$

$$\therefore \mathbf{M} = (540 \text{ lb}\cdot\text{ft})\mathbf{i} - (135 \text{ lb}\cdot\text{ft})\mathbf{k}$$

or $\quad \mathbf{M} = (540 \text{ lb}\cdot\text{ft})\mathbf{i} - (135.0 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

Chapter 3, Solution 92.



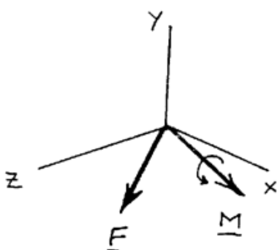
Have $\Sigma \mathbf{F}$: $\quad \mathbf{T}_{CD} = \mathbf{F}$

where $\mathbf{T}_{CD} = T_{CD}\dfrac{\overrightarrow{CD}}{CD}$

$$= (61\text{ lb})\frac{-0.9\mathbf{i} - 16.8\mathbf{j} + 7.2\mathbf{k}}{\sqrt{(-0.9)^2 + (-16.8)^2 + (7.2)^2}}$$

$$= -(3\text{ lb})\mathbf{i} - (56\text{ lb})\mathbf{j} + (24\text{ lb})\mathbf{k}$$

So that $\quad \mathbf{F} = -(3.00\text{ lb})\mathbf{i} - (56.0\text{ lb})\mathbf{j} + (24.0\text{ lb})\mathbf{k}$ ◀



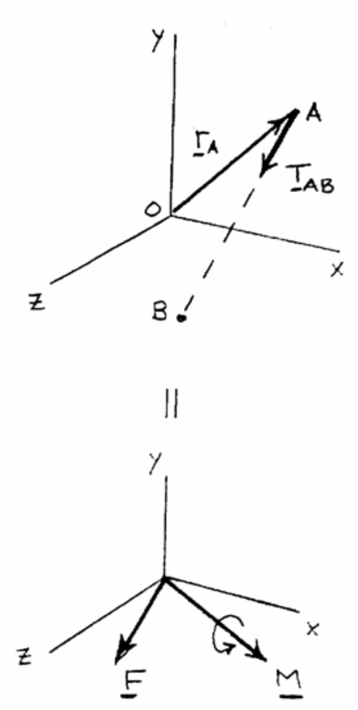
Have $\Sigma \mathbf{M}_O = \mathbf{r}_{C/D} \times \mathbf{T}_{CD} = \mathbf{M}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 22.5 & 0 \\ -3 & -56 & 24 \end{vmatrix} \text{lb}\cdot\text{ft} = \mathbf{M}$$

$$\therefore \mathbf{M} = (540\text{ lb}\cdot\text{ft})\mathbf{i} + (67.5\text{ lb}\cdot\text{ft})\mathbf{k}$$

$\mathbf{M} = (540\text{ lb}\cdot\text{ft})\mathbf{i} + (67.5\text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

Chapter 3, Solution 93.



Have $\Sigma \mathbf{F}$: $\quad \mathbf{T}_{AB} = \mathbf{F}$

where $\mathbf{T}_{AB} = T_{AB}\dfrac{\overline{AB}}{AB}$

$$= (10.5\text{ kN})\frac{-\mathbf{i} - 4.75\mathbf{j} + 2\mathbf{k}}{\sqrt{(-1)^2 + (-4.75)^2 + (2)^2}}$$

$$= -(2\text{ kN})\mathbf{i} - (9.5\text{ kN})\mathbf{j} + (4\text{ kN})\mathbf{k}$$

So that $\quad \mathbf{F} = -(2.00\text{ kN})\mathbf{i} - (9.50\text{ kN})\mathbf{j} + (4.00\text{ kN})\mathbf{k}$ ◀

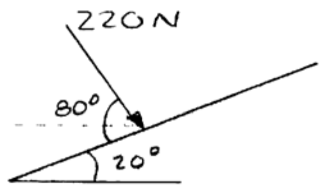
Have $\Sigma \mathbf{M}_O$: $\quad \mathbf{r}_A \times \mathbf{T}_{AB} = \mathbf{M}$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 4.75 & 0 \\ -2 & -9.5 & 4 \end{vmatrix} \text{kN}\cdot\text{m} = \mathbf{M}$$

$$\therefore \mathbf{M} = (19\text{ kN}\cdot\text{m})\mathbf{i} - (12\text{ kN}\cdot\text{m})\mathbf{j} - (19\text{ kN}\cdot\text{m})\mathbf{k}$$

$\mathbf{M} = (19.00\text{ kN}\cdot\text{m})\mathbf{i} - (12.00\text{ kN}\cdot\text{m})\mathbf{j} - (19.00\text{ kN}\cdot\text{m})\mathbf{k}$ ◀

Chapter 3, Solution 94.



Let $(\mathbf{R}, \mathbf{M}_O)$ be the equivalent force-couple system

Then $\mathbf{R} = (220 \text{ N})(-\sin 60° \mathbf{j} - \cos 60° \mathbf{k})$

$$= (110 \text{ N})(-\sqrt{3}\mathbf{j} - \mathbf{k})$$

or $\mathbf{R} = -(190.5 \text{ N})\mathbf{j} - (110 \text{ N})\mathbf{k}$ ◀

Now $\Sigma \mathbf{M}_O$: $\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{R}$

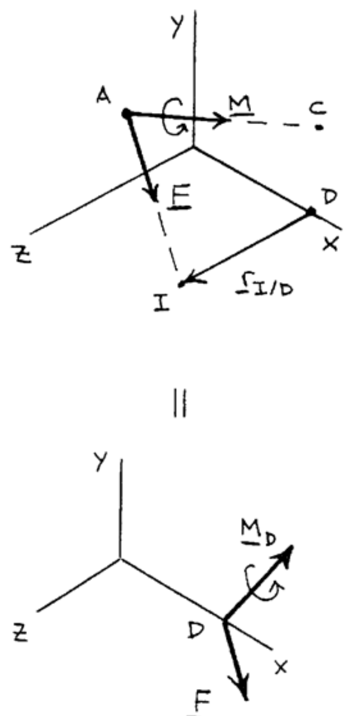
Where $\mathbf{r}_{OC} = (0.2 \text{ m})\mathbf{i} + \left[(0.1 - 0.4\sin 20°)\text{m}\right]\mathbf{j} + (0.4\cos 20° \text{m})\mathbf{k}$

Then $$\mathbf{M}_O = -(0.1)(110 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & (1 - 4\sin 20°) & 4\cos 20° \\ 0 & \sqrt{3} & 1 \end{vmatrix}(\text{m})$$

$$= -(11 \text{ N}\cdot\text{m})\left\{\left[(1 - 4\sin 20°)(1) - (4\cos 20°)(\sqrt{3})\right]\mathbf{i} - 2\mathbf{j} + 2\sqrt{3}\mathbf{k}\right\}$$

or $\mathbf{M}_O = (75.7 \text{ N}\cdot\text{m})\mathbf{i} + (22.0 \text{ N}\cdot\text{m})\mathbf{j} - (38.1 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

Chapter 3, Solution 95.



Have $\Sigma\mathbf{F}$: $\quad \mathbf{F} = \mathbf{F}_D$

where $\mathbf{F} = F\dfrac{\overrightarrow{AI}}{AI}$

$$= (63\text{ lb})\frac{14.4\mathbf{i} - 4.8\mathbf{j} + 7.2\mathbf{k}}{\sqrt{(14.4)^2 + (-4.8)^2 + (7.2)^2}}$$

So that $\quad \mathbf{F} = (54.0\text{ lb})\mathbf{i} - (18.00\text{ lb})\mathbf{j} + (27.0\text{ lb})\mathbf{k}$ ◀

Have $\Sigma\mathbf{M}_D$: $\quad \mathbf{M} + \mathbf{r}_{I/O} \times \overline{\mathbf{F}} = \mathbf{M}_D$

where $\mathbf{M} = M\dfrac{\overrightarrow{AC}}{AC}$

$$= (560\text{ lb}\cdot\text{in.})\frac{9.6\mathbf{i} - 7.2\mathbf{k}}{\sqrt{(9.6)^2 + (-7.2)^2}}$$

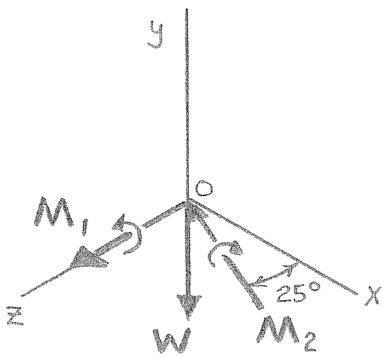
$$= (448\text{ lb}\cdot\text{in.})\mathbf{i} - (336\text{ lb}\cdot\text{in.})\mathbf{k}$$

Then $\mathbf{M}_D = (448\text{ lb}\cdot\text{in.})\mathbf{i} - (336\text{ lb}\cdot\text{in.})\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 14.4 \\ 54 & -18 & 27 \end{vmatrix}\text{ lb}\cdot\text{in.}$

$$= (448\text{ lb}\cdot\text{in.})\mathbf{i} - (336\text{ lb}\cdot\text{in.})\mathbf{k} + \left[(259.2\text{ lb}\cdot\text{in.})\mathbf{i} + (777.6\text{ lb}\cdot\text{in.})\mathbf{j}\right]$$

or $\quad \mathbf{M}_D = (707\text{ lb}\cdot\text{in.})\mathbf{i} + (778\text{ lb}\cdot\text{in.})\mathbf{j} - (336\text{ lb}\cdot\text{in.})\mathbf{k}$ ◀

Chapter 3, Solution 96.



First assume that the given force **W** and couples $\mathbf{M}_1$ and $\mathbf{M}_2$ act at the origin.

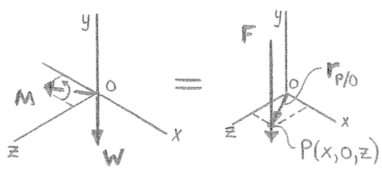
Now $$\mathbf{W} = -W_{\mathbf{j}}$$

and $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = -\left(M_2 \cos 25°\right)\mathbf{i} + \left(M_1 - M_2 \sin 25°\right)\mathbf{k}$$

Note that since **W** and **M** are perpendicular, it follows that they can be replaced with a single equivalent force.

(*a*) Have $F = \mathbf{W}$ or $\mathbf{F} = -W_{\mathbf{j}} = -(2.4 \text{ N})\mathbf{j}$

or $\mathbf{F} = -(2.40 \text{ N})\mathbf{j}$ ◀



(*b*) Assume that the line of action of **F** passes through point *P (x,* 0*, z)*.

Then for equivalence

$$\mathbf{M} = \mathbf{r}_{P/O} \times \mathbf{F}$$

where $$\mathbf{r}_{P/O} = x\mathbf{i} + z\mathbf{k}$$

$$\therefore \quad -\left(M_2 \cos 25°\right)\mathbf{i} + \left(M_1 - M_2 \sin 25°\right)\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & -W & 0 \end{vmatrix} = (Wz)\mathbf{i} - (Wx)\mathbf{k}$$

Equating the **i** and **k** coefficients,

$$z = \frac{-M_z \cos 25°}{W} \quad \text{and} \quad x = -\left(\frac{M_1 - M_2 \sin 25°}{W}\right)$$

(*b*) For $W = 2.4 \text{ N}, M_1 = 0.068 \text{ N}\cdot\text{m}, M_2 = 0.065 \text{ N}\cdot\text{m}$

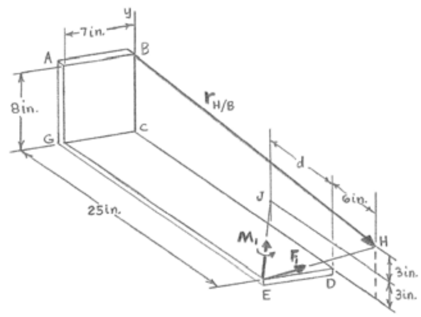
$$x = \frac{0.068 - 0.065 \sin 25°}{-2.4} = -0.0168874 \text{ m}$$

or $x = -16.89$ mm ◀

$$z = \frac{-0.065 \cos 25°}{2.4} = -0.024546 \text{ m}$$

or $z = -24.5$ mm ◀

Chapter 3, Solution 97.



(*a*) Have $\Sigma M_{Bz}: \quad M_{2z} = 0$

$$\mathbf{k}\cdot\left(\mathbf{r}_{H/B}\times\mathbf{F}_1\right) + M_{1z} = 0 \qquad (1)$$

where $\mathbf{r}_{H/B} = (31\text{ in.})\mathbf{i} - (2\text{ in.})\mathbf{j}$

$$\mathbf{F}_1 = \lambda_{EH}F_1$$

$$= \frac{(6\text{ in.})\mathbf{i} + (6\text{ in.})\mathbf{j} - (7\text{ in.})\mathbf{k}}{11.0\text{ in.}}(20\text{ lb})$$

$$= \frac{20\text{ lb}}{11.0}(6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$$

$$M_{1z} = \mathbf{k}\cdot\mathbf{M}_1$$

$$\mathbf{M}_1 = \lambda_{EJ}M_1$$

$$= \frac{-d\mathbf{i} + (3\text{ in.})\mathbf{j} - (7\text{ in.})\mathbf{k}}{\sqrt{d^2 + 58}\text{ in.}}(480\text{ lb}\cdot\text{in.})$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 31 & -2 & 0 \\ 6 & 6 & -7 \end{vmatrix}\frac{20\text{ lb}\cdot\text{in.}}{11.0} + \frac{(-7)(480\text{ lb}\cdot\text{in.})}{\sqrt{d^2 + 58}} = 0$$

continued

Solving for d, Equation (1) reduces to

$$\frac{20\ \text{lb}\cdot\text{in.}}{11.0}\left(186+12\right)-\frac{3360\ \text{lb}\cdot\text{in.}}{\sqrt{d^2+58}}=0$$

From which $$d = 5.3955\ \text{in.}$$

$$\text{or } d = 5.40\ \text{in.} \blacktriangleleft$$

(b) $$\mathbf{F}_2 = \mathbf{F}_1 = \frac{20\ \text{lb}}{11.0}\left(6\mathbf{i}+6\mathbf{j}-7\mathbf{k}\right)$$

$$= \left(10.9091\mathbf{i}+10.9091\mathbf{j}-12.7273\mathbf{k}\right)\text{lb}$$

$$\text{or } \mathbf{F}_2 = \left(10.91\ \text{lb}\right)\mathbf{i}+\left(10.91\ \text{lb}\right)\mathbf{j}-\left(12.73\ \text{lb}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}_2 = \mathbf{r}_{H/B}\times\mathbf{F}_1+\mathbf{M}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 31 & -2 & 0 \\ 6 & 6 & -7 \end{vmatrix}\frac{20\ \text{lb}\cdot\text{in.}}{11.0}+\frac{\left(-5.3955\right)\mathbf{i}+3\mathbf{j}-7\mathbf{k}}{9.3333}\left(480\ \text{lb}\cdot\text{in.}\right)$$

$$= \left(25.455\mathbf{i}+394.55\mathbf{j}+360\mathbf{k}\right)\text{lb}\cdot\text{in.}$$

$$+\left(-277.48\mathbf{i}+154.285\mathbf{j}-360\mathbf{k}\right)\text{lb}\cdot\text{in.}$$

$$\mathbf{M}_2 = -\left(252.03\ \text{lb}\cdot\text{in.}\right)\mathbf{i}+\left(548.84\ \text{lb}\cdot\text{in.}\right)\mathbf{j}$$

$$\text{or } \mathbf{M}_2 = -\left(21.0\ \text{lb}\cdot\text{ft}\right)\mathbf{i}+\left(45.7\ \text{lb}\cdot\text{ft}\right)\mathbf{j} \blacktriangleleft$$

Chapter 3, Solution 98.

(a) a: ΣF_y: $\quad R_a = -400 \text{ N} - 600 \text{ N}$

or $\mathbf{R}_a = 1000\ N \downarrow$ ◀

ΣM_B: $\quad M_a = (2 \text{ kN}\cdot\text{m}) + (2 \text{ kN}\cdot\text{m}) + (5 \text{ m})(400 \text{ N})$

or $\mathbf{M}_a = 6.00 \text{ kN}\cdot\text{m}$ ↺ ◀

b: ΣF_y: $\quad R_b = -1200 \text{ N} + 200 \text{ N}$

or $\mathbf{R}_b = 1000 \text{ N} \downarrow$ ◀

ΣM_B: $\quad M_b = (0.6 \text{ kN}\cdot\text{m}) + (5 \text{ m})(1200 \text{ N})$

or $\mathbf{M}_b = 6.60 \text{ kN}\cdot\text{m}$ ↺ ◀

c: ΣF_y: $\quad R_c = 200 \text{ N} - 1200 \text{ N}$

or $\mathbf{R}_c = 1000 \text{ N} \downarrow$ ◀

ΣM_B: $\quad M_c = -(4 \text{ kN}\cdot\text{m}) - (1.6 \text{ kN}\cdot\text{m}) - (5 \text{ m})(200 \text{ N})$

or $\mathbf{M}_c = 6.60 \text{ kN}\cdot\text{m}$ ↻ ◀

d: ΣF_y: $\quad R_d = -800 \text{ N} - 200 \text{ N}$

or $\mathbf{R}_d = 1000 \text{ N} \downarrow$ ◀

ΣM_B: $\quad M_d = -(1.6 \text{ kN}\cdot\text{m}) + (4.2 \text{ kN}\cdot\text{m}) + (5 \text{ m})(800 \text{ N})$

or $\mathbf{M}_d = 6.60 \text{ kN}\cdot\text{m}$ ↺ ◀

continued

e: ΣF_y: $R_e = -500 \text{ N} - 400 \text{ N}$

or $\mathbf{R}_e = 900 \text{ N} \downarrow$ ◀

ΣM_B: $M_e = (3.8 \text{ kN}\cdot\text{m}) + (0.3 \text{ kN}\cdot\text{m}) + (5 \text{ m})(500 \text{ N})$

or $\mathbf{M}_e = 6.60 \text{ kN}\cdot\text{m}$ ↻ ◀

f: ΣF_y: $R_f = 400 \text{ N} - 1400 \text{ N}$

or $\mathbf{R}_f = 1000 \text{ N} \downarrow$ ◀

ΣM_B: $M_f = (8.6 \text{ kN}\cdot\text{m}) - (0.8 \text{ kN}\cdot\text{m}) - (5 \text{ m})(400 \text{ N})$

or $\mathbf{M}_f = 5.80 \text{ kN}\cdot\text{m}$ ↻ ◀

g: ΣF_y: $R_g = -1200 \text{ N} + 300 \text{ N}$

or $\mathbf{R}_g = 900 \text{ N} \downarrow$ ◀

ΣM_B: $M_g = (0.3 \text{ kN}\cdot\text{m}) + (0.3 \text{ kN}\cdot\text{m}) + (5 \text{ m})(1200 \text{ N})$

or $\mathbf{M}_g = 6.60 \text{ kN}\cdot\text{m}$ ↻ ◀

h: ΣF_y: $R_h = -250 \text{ N} - 750 \text{ N}$

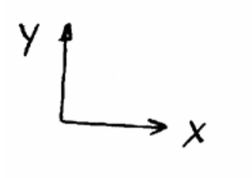
or $\mathbf{R}_h = 1000 \text{ N} \downarrow$ ◀

ΣM_B: $M_h = -(0.65 \text{ kN}\cdot\text{m}) + (6 \text{ kN}\cdot\text{m}) + (5 \text{ m})(250 \text{ N})$

or $\mathbf{M}_h = 6.60 \text{ kN}\cdot\text{m}$ ↻ ◀

(*b*) The equivalent loadings are (*b*), (*d*), (*h*) ◀

Chapter 3, Solution 99.



The equivalent force-couple system at B is...

ΣF_y: $R = -650 \text{ N} - 350 \text{ N}$

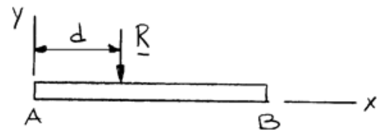
or $\mathbf{R} = 1000 \text{ N} \downarrow$

ΣM_B: $M = (1.6 \text{ m})(800 \text{ N}) + (1.27 \text{ kN}\cdot\text{m}) + (5 \text{ m})(650 \text{ N})$

or $\mathbf{M} = 5.80 \text{ kN}\cdot\text{m}$ ↻

∴ The equivalent loading of Problem 3.98 is (*f*) ◀

Chapter 3, Solution 100.



Equivalent force system...

(*a*) ΣF_y: $R = -400\text{ N} - 200\text{ N}$

or $\mathbf{R} = 600\text{ N}\downarrow$ ◀

ΣM_A: $-d(600\text{ N}) = -(200\text{ N}\cdot\text{m}) + (100\text{ N}\cdot\text{m}) - (4\text{ m})(200\text{ N})$

or $d = 1.500\text{ m}$ ◀

(*b*) ΣF_y: $R = -400\text{ N} + 100\text{ N}$

or $\mathbf{R} = 300\text{ N}\downarrow$ ◀

ΣM_A: $-d(300\text{ N}) = -(200\text{ N}\cdot\text{m}) - (600\text{ N}\cdot\text{m}) + (4\text{ m})(100\text{ N})$

or $d = 1.333\text{ m}$ ◀

(*c*) ΣF_y: $R = -400\text{ N} - 100\text{ N}$

or $\mathbf{R} = 500\text{ N}\downarrow$ ◀

ΣM_A: $-d(500\text{ N}) = -(200\text{ N}\cdot\text{m}) - (200\text{ N}\cdot\text{m}) - (4\text{ m})(100\text{ N})$

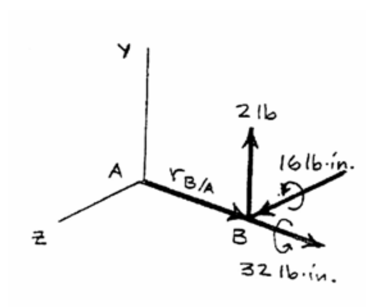
or $d = 1.600\text{ m}$ ◀

Chapter 3, Solution 101.

The equivalent force-couple system at *A* for each of the five force-couple systems will be determined and compared to

$$\mathbf{F} = (2\text{ lb})\mathbf{j} \qquad \mathbf{M} = (48\text{ lb}\cdot\text{in.})\mathbf{i} + (32\text{ lb}\cdot\text{in.})\mathbf{k}$$

To determine if they are equivalent



Force-couple system at *B*:

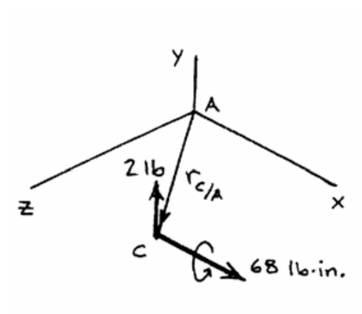
Have $\Sigma\mathbf{F}$: $\quad \mathbf{F} = (2\text{ lb})\mathbf{j}$

and $\Sigma\mathbf{M}_A$: $\quad \mathbf{M} = \Sigma\mathbf{M}_B + (\mathbf{r}_{B/A} \times \mathbf{F}_B)$

$$\mathbf{M} = (32\text{ lb}\cdot\text{in.})\mathbf{i} + (16\text{ lb}\cdot\text{in.})\mathbf{k} + [(8\text{ in.})\mathbf{i} \times (2\text{ lb})\mathbf{j}]$$

$$= (32\text{ lb}\cdot\text{in.})\mathbf{i} + (32\text{ lb}\cdot\text{in.})\mathbf{k}$$

$\therefore$ is not equivalent ◀



Force-couple system at *C*:

Have $\Sigma\mathbf{F}$: $\quad \mathbf{F} = (2\text{ lb})\mathbf{j}$

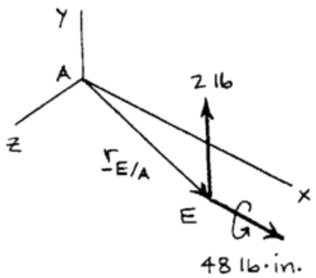
And $\Sigma\mathbf{M}_A$: $\quad \mathbf{M} = \mathbf{M}_C + (\mathbf{r}_{C/A} \times \mathbf{F}_C)$

$$\mathbf{M} = (68\text{ lb}\cdot\text{in.})\mathbf{i} + [(8\text{ in.})\mathbf{i} + (10\text{ in.})\mathbf{k}] \times (2\text{ lb})\mathbf{j}$$

$$= (48\text{ lb}\cdot\text{in.})\mathbf{i} + (16\text{ lb}\cdot\text{in.})\mathbf{k}$$

$\therefore$ is not equivalent ◀

continued



Force-couple system at *E*:

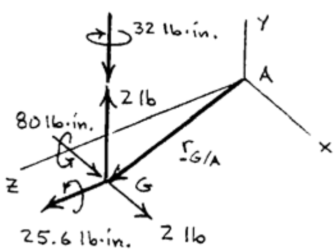
Have $\Sigma\mathbf{F}$: $\quad \mathbf{F} = (2 \text{ lb})\mathbf{j}$

and $\Sigma\mathbf{M}_A$: $\quad \mathbf{M} = \mathbf{M}_E + (\mathbf{r}_{E/A} \times \mathbf{F}_E)$

$$\mathbf{M} = (48 \text{ lb}\cdot\text{in.})\mathbf{i} + \left[(16 \text{ in.})\mathbf{i} - (3.2 \text{ in.})\mathbf{j}\right] \times (2 \text{ lb})\mathbf{j}$$

$$= (48 \text{ lb}\cdot\text{in.})\mathbf{i} + (32 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$\therefore$ is equivalent ◀



Force-couple system at *G*:

Have $\Sigma\mathbf{F}$: $\quad \mathbf{F} = (2 \text{ lb})\mathbf{i} + (2 \text{ lb})\mathbf{j}$

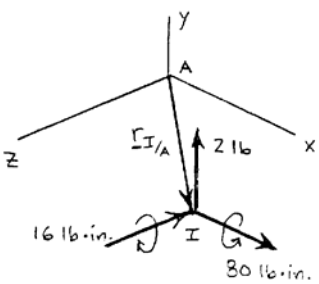
F has two force components

$\therefore$ is not equivalent ◀

Force-couple system at *I*:

Have $\Sigma\mathbf{F}$: $\quad \mathbf{F} = (2 \text{ lb})\mathbf{j}$

and $\Sigma\mathbf{M}_A$: $\quad \Sigma\mathbf{M}_I + (\mathbf{r}_{I/A} \times \mathbf{F}_I)$



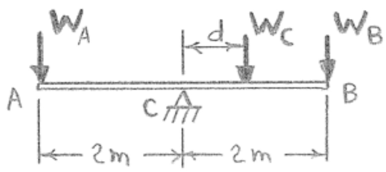
$$\mathbf{M} = (80 \text{ lb}\cdot\text{in.})\mathbf{i} - (16 \text{ in.})\mathbf{k}$$

$$+\left[(16 \text{ in.})\mathbf{i} - (8 \text{ in.})\mathbf{j} + (16 \text{ in.})\mathbf{k}\right] \times (2 \text{ lb})\mathbf{j}$$

$$\mathbf{M} = (48 \text{ lb}\cdot\text{in.})\mathbf{i} + (16 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$\therefore$ is not equivalent ◀

Chapter 3, Solution 102.



First $\quad W_A = m_A g = (38\text{ kg})g$

$$W_B = m_B g = (29\text{ kg})g$$

(*a*) $\quad W_C = m_C g = (27\text{ kg})g$

For resultant weight to act at *C*, important → $\quad \Sigma M_C = 0$

Then $\quad [(38\text{ kg})g](2\text{ m}) - [(27\text{ kg})g](d) - [(29\text{ kg})g](2\text{ m}) = 0$

$$\therefore \quad d = \frac{76 - 58}{27} = 0.66667\text{ m}$$

or $d = 0.667$ m ◀

(*b*) $\quad W_C = m_C g = (24\text{ kg})g$

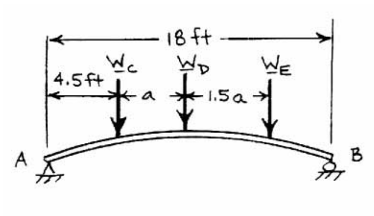
For resultant weight to act at *C*, $\quad \Sigma M_C = 0$

Then $\quad [(38\text{ kg})g](2\text{ m}) - [(24\text{ kg})g](d) - [(29\text{ kg})g](2\text{ m}) = 0$

$$\therefore \quad d = \frac{76 - 58}{24} = 0.75\text{ m}$$

or $d = 0.750$ m ◀

Chapter 3, Solution 103.



(*a*) Have ΣF: $\quad -W_C - W_D - W_E = R$

$$\therefore R = -200 \text{ lb} - 175 \text{ lb} - 135 \text{ lb}$$

$$= -510 \text{ lb}$$

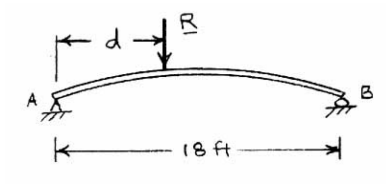
or $\mathbf{R} = 510 \text{ lb} \downarrow$ ◀

Have ΣM_A:

$$-(200 \text{ lb})(4.5 \text{ ft}) - (175 \text{ lb})(7.8 \text{ ft}) - (135 \text{ lb})(12.75 \text{ ft}) = -R(d)$$

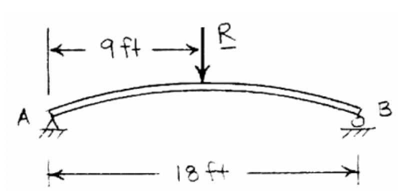
$$\therefore -3986.3 \text{ lb}\cdot\text{ft} = (-510 \text{ lb})d$$

or $d = 7.82$ ft ◀



(*b*) For equal reactions at *A* and *B*,

The resultant **R** must act at midspan.



From $\quad \Sigma M_A = -R\left(\frac{L}{2}\right)$

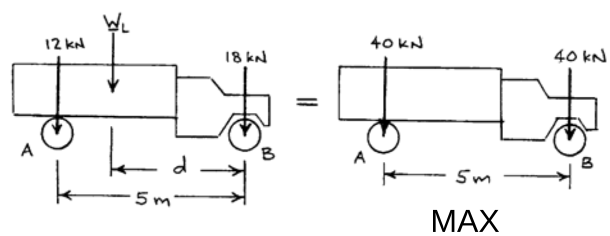
$$\therefore -(200 \text{ lb})(4.5 \text{ ft}) - (175 \text{ lb})(4.5 \text{ ft} + a) - (135 \text{ lb})(4.5 \text{ ft} + 2.5\, a)$$

$$= -(510 \text{ lb})(9 \text{ ft})$$

or $2295 + 512.5\, a = 4590$

and $a = 4.48$ ft ◀

Chapter 3, Solution 104.



Note:
baraye inke nirooye vared shode dar do tarafe barabar bashad , bayad barayand (R) daghighan be vasat vared beshavad.

Have $\Sigma\mathbf{F}$: $\quad -12\text{ kN} - W_L - 18\text{ kN} = -40\text{ kN} - 40\text{ kN}$

$$W_L = 50\text{ kN}$$

or $W_L = 50.0\text{ kN}$ ◀

$\Sigma\mathbf{M}_B$: $\quad (12\text{ kN})(5\text{ m}) + (50\text{ kN})d = (40\text{ kN})(5\text{ m})$

$$d = 2.8\text{ m}$$

or heaviest load (50 kN) is located ◀
2.80 m from front axle

Chapter 3, Solution 105.

(*a*) $\Sigma\mathbf{F}$: $\mathbf{R} = (80\text{ N})\mathbf{i} - (40\text{ N})\mathbf{j} - (60\text{ N})\mathbf{j} + (90\text{ N})(-\sin 50°\mathbf{i} - \cos 50°\mathbf{j})$

$$= (11.0560\text{ N})\mathbf{i} - (157.851\text{ N})\mathbf{j}$$

$$R = \sqrt{(11.0560\text{ N})^2 + (-157.851\text{ N})^2}$$

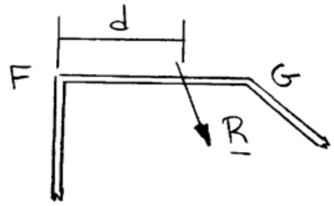
$$= 158.2\text{ N}$$

$$\tan\theta = \frac{-157.851}{11.0560}$$

$$\theta = 86.0°$$

or $\mathbf{R} = 158.2\text{ N}$ ⦩ $86.0°$ ◀

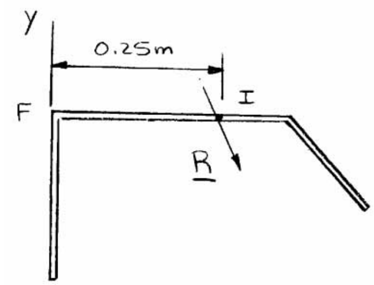
(*b*)



$$\Sigma M_F:\ d-(157.851\text{ N}) = (0.32\text{ m})(80\text{ N}) - (0.15\text{ m})(40\text{ N}) - (0.35\text{ m})(60\text{ N})$$
$$-(0.61\text{ m})(90\text{ N})\cos 50° - (0.16\text{ m})(90\text{ N})\sin 50°$$

or $d = 302\text{ mm}$ to the right of F ◀

Chapter 3, Solution 106.



(*a*) ΣM_I: $0 = (0.32\text{ m})(80\text{ N}) + (0.1\text{ m})(40\text{ N}) - (0.1\text{ m})(60\text{ N}) - (0.36\text{ m})(90\text{ N})\cos\alpha$

$-(0.16\text{ m})(90\text{ N})\sin\alpha$

or $4\sin\alpha + 9\cos\alpha = 6.5556$

$(9\cos\alpha)^2 = (6.5556 - 4\sin\alpha)^2$

$81(1 - \sin^2\alpha) = 42.976 - 52.445\sin\alpha + 16\sin^2\alpha$

$97\sin^2\alpha - 52.445\sin\alpha - 38.024 = 0$

Solving by the quadratic formula gives for the positive root

$\sin\alpha = 0.95230$

$\alpha = 72.233°$

or $\alpha = 72.2°$ ◀

Note: The second root $(\alpha = -24.3°)$ is rejected since $0 < \alpha < 90°$.

(*b*) $\Sigma\mathbf{F}$: $\mathbf{R} = (80\text{ N})\mathbf{i} - (40\text{ N})\mathbf{j} - (60\text{ N})\mathbf{j}$

$+(90\text{ N})(-\sin 72.233°\mathbf{i} - \cos 72.233°\mathbf{j})$

$= -(5.7075\text{ N})\mathbf{i} - (127.463\text{ N})\mathbf{j}$

$R = \sqrt{(-5.7075\text{ N})^2 + (-127.463\text{ N})^2}$

$= 127.6\text{ N}$

$\tan\theta = \dfrac{-127.463}{-5.7075}$

$\theta = 87.4°$

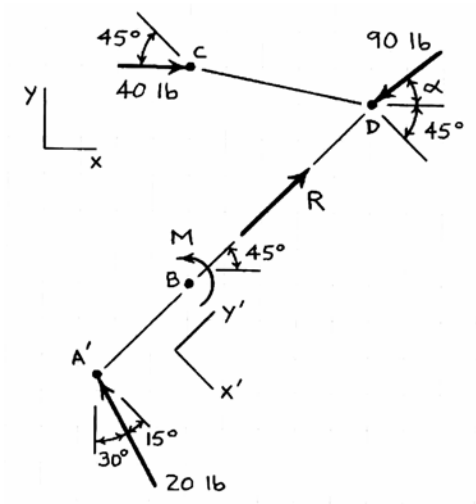
or $\mathbf{R} = 127.6\text{ N}$ ↙ $87.4°$ ◀

Chapter 3, Solution 107.

(*a*) Have $+\circlearrowright\ \Sigma M_D$: $\quad 0 = M - (0.8\text{ in.})(40\text{ lb}) - (2.9\text{ in.})(20\text{ lb})\cos 30° - (3.3\text{ in.})(20\text{ lb})\sin 30°$

or $M = 115.229\text{ lb}\cdot\text{in.}$

or $\mathbf{M} = 115.2\text{ lb}\cdot\text{in.}\ \circlearrowright$ ◀



Now, **R** is oriented at $45°$ as shown (since its line of action passes through *B* and *D*).

Have $\Sigma F_{x'}$: $\quad 0 = (40\text{ lb})\cos 45° - (20\text{ lb})\cos 15°$

$$-(90\text{ lb})\cos(\alpha + 45°)$$

or $\alpha = 39.283°$

or $\alpha = 39.3°$ ◀

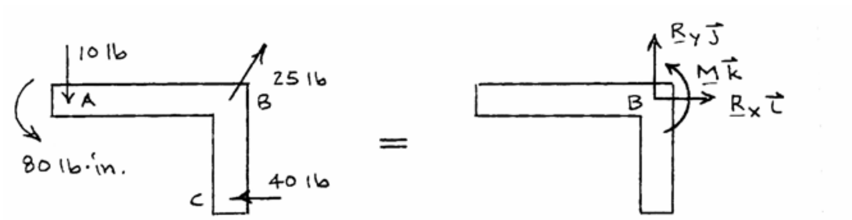
(*b*) ΣF_x: $\quad R_x = 40 - 20\sin 30° - 90\cos 39.283°$

$= -39.663\text{ lb}$

Now $\quad R = \sqrt{2}R_x$ $\quad$ or $\quad \mathbf{R} = 56.1\text{ lb} \measuredangle\ 45.0°$ ◀

Chapter 3, Solution 108.

(*a*) Reduce system to a force and couple at *B*:



Have $\mathbf{R} = \Sigma\mathbf{F} = -(10 \text{ lb})\mathbf{j} + (25 \text{ lb})\cos 60°\mathbf{i} + (25 \text{ lb})\sin 60°\mathbf{j} - (40 \text{ lb})\mathbf{i}$

$$= -(27.5 \text{ lb})\mathbf{i} + (11.6506 \text{ lb})\mathbf{j}$$

or $R = \sqrt{(-27.5 \text{ lb})^2 + (11.6506 \text{ lb})^2} = 29.866 \text{ lb}$

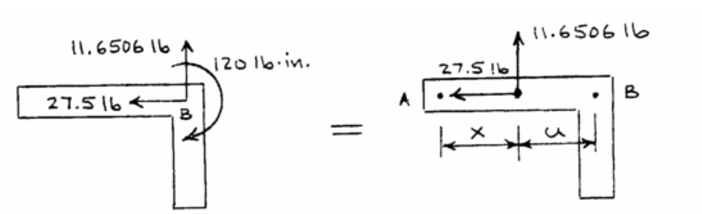
$$\theta = \tan^{-1}\left(\frac{11.6506}{27.5}\right) = 22.960°$$

or $\mathbf{R} = 29.9 \text{ lb}$ ↗ $23.0°$ ◀

Also $\mathbf{M}_B = \Sigma\mathbf{M}_B = (80 \text{ lb}\cdot\text{in.})\mathbf{k} - (12 \text{ in.})\mathbf{i} \times (-10 \text{ lb})\mathbf{j} - (8 \text{ in.})\mathbf{j} \times (-40 \text{ lb})\mathbf{i}$

$$= -(120 \text{ lb}\cdot\text{in.})\mathbf{k}$$

(*b*)

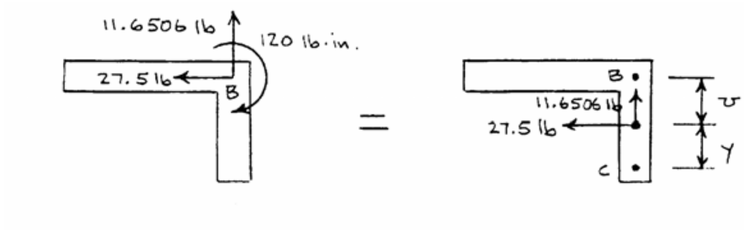


Have $\mathbf{M}_B = -(120 \text{ lb}\cdot\text{in.})\mathbf{k} = -(u)\mathbf{i} \times (11.6506 \text{ lb})\mathbf{j}$

$$-(120 \text{ lb}\cdot\text{in.})\mathbf{k} = -(11.6506 \text{ lb})(u)\mathbf{k}$$

$u = 10.2999 \text{ in.}$ and $x = 12 \text{ in.} - 10.2999 \text{ in.}$

$$= 1.7001 \text{ in.}$$



Have $\mathbf{M}_B = -(120\text{ lb}\cdot\text{in.})\mathbf{k} = -(v)\mathbf{j} \times (-27.5\text{ lb})\mathbf{i}$

$$-(120\text{ lb}\cdot\text{in.})\mathbf{k} = -(27.5\text{ lb})(v)\mathbf{k}$$

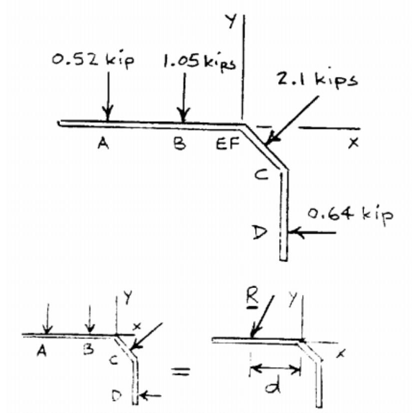
$$v = 4.3636\text{ in.}$$

and $y = 8\text{ in.} - 4.3636\text{ in.} = 3.6364\text{ in.}$

or $\quad 1.700\text{ in.}$ to the right of A and 3.64 in. above C ◀

Chapter 3, Solution 109.

(*a*)



Position origin along centerline of sheet metal at the intersection with line *EF*.

(*a*) Have $\Sigma \mathbf{F} = \mathbf{R}$

$$\mathbf{R} = \left[-0.52\,\mathbf{j} - 1.05\,\mathbf{j} - 2.1(\sin 45°\,\mathbf{i} + \cos 45°\,\mathbf{j}) - 0.64\,\mathbf{i}\right]\text{kips}$$

$$\mathbf{R} = -(2.1249\text{ kips})\,\mathbf{i} - (3.0549\text{ kips})\,\mathbf{j}$$

$$R = \sqrt{(-2.1249)^2 + (-3.0549)^2}$$

$$= 3.7212\text{ kips}$$

$$\theta = \tan^{-1}\left(\frac{-3.0549}{-2.1249}\right) = 55.179°$$

or $\mathbf{R} = 3.72\text{ kips} \measuredangle\ 55.2°$ ◀

Have $M_{EF} = \Sigma M_{EF}$

Where $M_{EF} = (0.52\text{ kip})(3.6\text{ in.}) + (1.05\text{ kips})(1.6\text{ in.})$

$$-(2.1\text{ kips})(0.8\text{ in.}) - (0.64\text{ kip})\left[(1.6\text{ in.})\sin 45° + 1.6\text{ in.}\right]$$

$$= 0.123923\text{ kip}\cdot\text{in.}$$

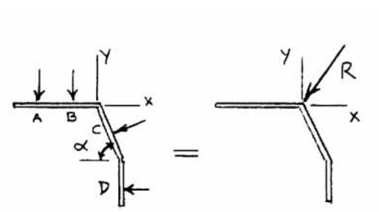
To obtain distance *d* left of *EF*,

Have $M_{EF} = dR_y = d(-3.0549\text{ kips})$

$$d = \frac{0.123923\text{ kip}\cdot\text{in.}}{-3.0549\text{ kips}} = -0.040565\text{ in.}$$

or $d = 0.0406\text{ in. left of } EF$ ◀

(*b*)



Have $M_{EF} = \Sigma M_{EF} = 0$

$$M_{EF} = (0.52\text{ kip})(3.6\text{ in.}) + (1.05\text{ kips})(1.6\text{ in.})$$

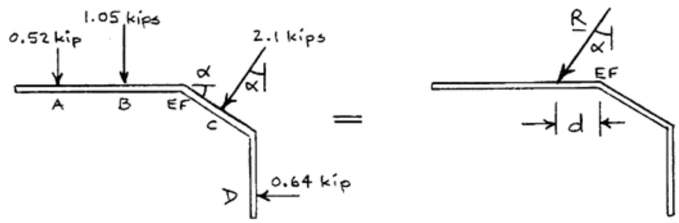
$$-(2.1\text{ kips})(0.8\text{ in.})$$

$$-(0.64\text{ kip})\left[(1.6\text{ in.})\sin\alpha + 1.6\text{ in.}\right]$$

$$\therefore\quad (1.024\text{ kip}\cdot\text{in.})\sin\alpha = 0.848\text{ kip}\cdot\text{in.}$$

or $\alpha = 55.9°$ ◀

Chapter 3, Solution 110.



(*a*) Have $\Sigma\mathbf{F} = \mathbf{R}$

$$\mathbf{R} = \left[-0.52\mathbf{j} - 1.05\mathbf{j} - 2.1(\sin\alpha\mathbf{i} + \cos\alpha\mathbf{j}) - 0.64\mathbf{i}\right]\text{kips}$$

$$= -\left[0.64\text{ kip} + (2.1\text{ kips})(\sin\alpha)\mathbf{i}\right] - \left[1.57\text{ kips} + (2.1\text{ kips})\cos\alpha\right]\mathbf{j}$$

Then $\tan\alpha = \dfrac{R_x}{R_y} = \dfrac{0.64 + 2.1\sin\alpha}{1.57 + 2.1\cos\alpha}$

$$1.57\tan\alpha + 2.1\sin\alpha = 0.64 + 2.1\sin\alpha$$

$$\tan\alpha = \frac{0.64}{1.57}$$

$$\alpha = 22.178°$$

or $\alpha = 22.2°$ ◀

(*b*) From $\alpha = 22.178°$

$$R_x = -0.64\text{ kip} - (2.1\text{ kips})\sin 22.178°$$

$$= -1.43272\text{ kips}$$

$$R_y = -1.57\text{ kips} - (2.1\text{ kips})\cos 22.178°$$

$$= -3.5146\text{ kips}$$

$$R = \sqrt{(-1.43272)^2 + (-3.5146)^2}$$

$$= 3.7954 \text{ kips}$$

or $\mathbf{R} = 3.80 \text{ kips} \measuredangle 67.8°$ ◀

Then $M_{EF} = \Sigma M_{EF}$

Where $M_{EF} = (0.52 \text{ kip})(3.6 \text{ in.}) + (1.05 \text{ kips})(1.6 \text{ in.}) - (2.1 \text{ kips})(0.8 \text{ in.})$

$$-(0.64 \text{ kip})\left[(1.6 \text{ in.})\sin 22.178° + 1.6 \text{ in.}\right]$$

$$= 0.46146 \text{ kip}\cdot\text{in.}$$

To obtain distance d left of EF,

Have $M_{EF} = dR_y$

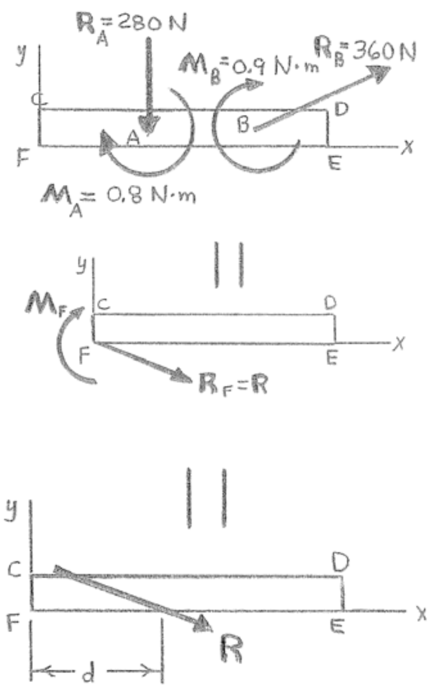
$$= d(-3.5146 \text{ kips})$$

$$d = \frac{0.46146 \text{ kip}\cdot\text{in.}}{-3.5146 \text{ kips}}$$

$$= -0.131298 \text{ in.}$$

or $d = 0.1313$ in. left of EF ◀

Chapter 3, Solution 111.



Equivalent force-couple at A due to belts on pulley A

Have $\Sigma\mathbf{F}$: $-120\text{ N} - 160\text{ N} = R_A$

$\therefore\ \mathbf{R}_A = 280\text{ N}\downarrow$

Have $\Sigma\mathbf{M}_A$: $-40\text{ N}(0.02\text{ m}) = M_A$

$\therefore\ \mathbf{M}_A = 0.8\text{ N}\cdot\text{m}$ ↻

Equivalent force-couple at B due to belts on pulley B

Have $\Sigma\mathbf{F}$: $(210\text{ N} + 150\text{ N}) \measuredangle\ 25° = \mathbf{R}_B$

$\therefore\ \mathbf{R}_B = 360\text{ N} \measuredangle\ 25°$

Have $\Sigma\mathbf{M}_B$: $-60\text{ N}(0.015\text{ m}) = M_B$

$\therefore\ \mathbf{M}_B = 0.9\text{ N}\cdot\text{m}$ ↻

Equivalent force-couple at F

Have $\Sigma\mathbf{F}$: $\mathbf{R}_F = (-280\text{ N})\mathbf{j} + (360\text{ N})(\cos 25°\mathbf{i} + \sin 25°\mathbf{j})$

$= (326.27\text{ N})\mathbf{i} - (127.857\text{ N})\mathbf{j}$

$$R = R_F = \sqrt{R_{Fx}^2 + R_{Fy}^2} = \sqrt{(326.27)^2 + (127.857)^2} = 350.43\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_{Fy}}{R_{Fx}}\right) = \tan^{-1}\left(\frac{-127.857}{326.27}\right) = -21.399°$$

or $\mathbf{R}_F = \mathbf{R} = 350\text{ N}$ ⦪ $21.4°$ ◀

Have

$$\Sigma \mathbf{M}_F: \quad M_F = -(280\text{ N})(0.06\text{ m}) - 0.80\text{ N}\cdot\text{m}$$

$$-\left[(360\text{ N})\cos 25^\circ\right](0.010\text{ m})$$

$$+\left[(360\text{ N})\sin 25^\circ\right](0.120\text{ m}) - 0.90\text{ N}\cdot\text{m}$$

$$\mathbf{M}_F = -(3.5056\text{ N}\cdot\text{m})\mathbf{k}$$

To determine where a single resultant force will intersect line *FE*,

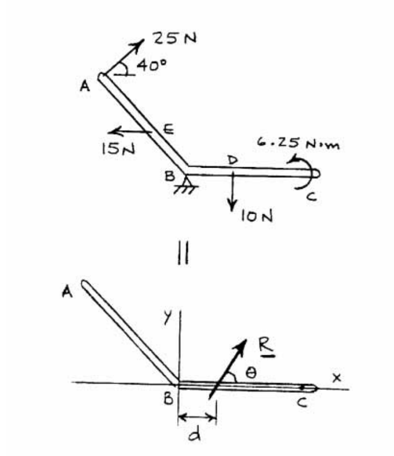
$$M_F = dR_y$$

$$\therefore \quad d = \frac{M_F}{R_y} = \frac{-3.5056\text{ N}\cdot\text{m}}{-127.857\text{ N}} = 0.027418\text{ m} = 27.418\text{ mm}$$

or $d = 27.4\text{ mm}$ ◀

Chapter 3, Solution 112.

(*a*)



Have $\mathbf{R} = \Sigma\mathbf{F}$

$$\mathbf{R} = (25\text{ N})(\cos 40°\mathbf{i} + \sin 40°\mathbf{j}) - (15\text{ N})\mathbf{i} - (10\text{ N})\mathbf{j}$$

$$= (4.1511\text{ N})\mathbf{i} + (6.0696\text{ N})\mathbf{j}$$

$$\mathbf{R} = \sqrt{(4.1511)^2 + (6.0696)^2}$$

$$= 7.3533\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{6.0696}{4.1511}\right)$$

$$= 55.631°$$

or $\mathbf{R} = 7.35\text{ N} \measuredangle 55.6°$ ◀

(*b*) From $M_B = \Sigma M_B = dR_y$

where
$$M_B = -[(25\text{ N})\cos 40°][(0.375\text{ m})\sin 50°]$$
$$-[(25\text{ N})\sin 40°][(0.375\text{ m})\cos 50°]$$
$$+(15\text{ N})[(0.150\text{ m})\sin 50°] - (10\text{ N})(0.150\text{ m})$$
$$+ 6.25\text{ N}\cdot\text{m}$$

$$\therefore\ M_B = -2.9014\text{ N}\cdot\text{m}$$

and
$$d = \frac{M_B}{R_y}$$

$$= \frac{-2.9014\text{ N}\cdot\text{m}}{6.0696\text{ N}}$$

$$= 0.47802\text{ m}$$

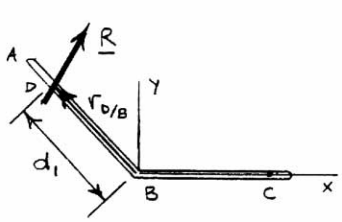
or $d = 478\text{ mm}$ to the left of B ◀

(*c*) From $\mathbf{M}_B = \mathbf{r}_{D/B} \times \mathbf{R}$

$$-(2.9014\text{ N}\cdot\text{m})\mathbf{k} = (-d_1\cos 50°\mathbf{i} + d_1\sin 50°\mathbf{j})$$
$$\times[(4.1511\text{ N})\mathbf{i} + (6.096\text{ N})\mathbf{j}]$$

$$-(2.9014\text{ N}\cdot\text{m})\mathbf{k} = -(7.0814 d_1)\mathbf{k}$$

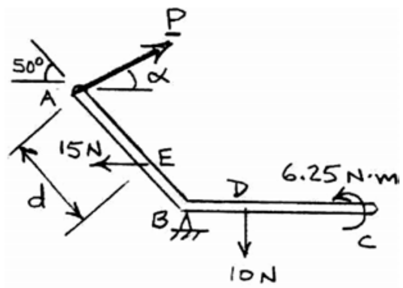
$$\therefore\ d_1 = 0.40972\text{ m}$$



or $d_1 = 410\text{ mm}$ from B along line AB

or 34.7 mm above and to left of A ◀

Chapter 3, Solution 113.



Based on $\Sigma F_x = 0$

$$P\cos\alpha - 15\text{ N} = 0$$

$$\therefore \quad P\cos\alpha = 15\text{ N} \qquad (1)$$

and $\Sigma F_y = 0$

$$P\sin\alpha - 10\text{ N} = 0$$

$$\therefore \quad P\sin\alpha = 10\text{ N} \qquad (2)$$

Dividing Equation (2) by Equation (1),

$$\tan\alpha = \frac{10}{15}$$

$$\therefore \quad \alpha = 33.690°$$

Substituting into Equation (1),

$$P = \frac{15\text{ N}}{\cos 33.690°} = 18.0278\text{ N}$$

or $\mathbf{P} = 18.03\text{ N} \measuredangle\ 33.7°$

(*a*) Based on $\Sigma M_B = 0$

$$-\left[(18.0278\text{ N})\cos 33.690°\right]\left[(d + 0.150\text{ m})\sin 50°\right]$$

$$-\left[(18.0278\text{ N})\sin 33.690°\right]\left[(d + 0.150\text{ m})\cos 50°\right]$$

$$+(15\text{ N})\left[(0.150\text{ m})\sin 50°\right] - (10\text{ N})(0.150\text{ m}) + 6.25\text{ N}\cdot\text{m} = 0$$

$$-17.9186d = -3.7858$$

$$\therefore \; d = 0.21128\text{ m}$$

or $d = 211\text{ mm}$ ◀

(*b*) Based on $\Sigma M_D = 0$

$$-\left[(18.0278\text{ N})\cos 33.690°\right]\left[(d + 0.150\text{ m})\sin 50°\right]$$

$$-\left[(18.0278\text{ N})\sin 33.690°\right]\left[(d + 0.150\text{ m})\cos 50° + 0.150\text{ m}\right]$$

$$+(15\text{ N})\left[(0.150\text{ m})\sin 50°\right] + 6.25\text{ N}\cdot\text{m} = 0$$

$$-17.9186d = -3.7858$$

$$\therefore\ d = 0.21128\text{ m}$$

or $d = 211\text{ mm}$ ◀

This result is expected, since $\mathbf{R} = 0$ and $\mathbf{M}_B^R = 0$ for $d = 211\text{ mm}$ implies that $\mathbf{R} = 0$ and $\mathbf{M} = 0$ at any other point for the value of *d* found in part *a*.

Chapter 3, Solution 114.

(*a*) Let $(\mathbf{R}, \mathbf{M}_D)$ be the equivalent force-couple system at *D*.

First note... At $x = b;\ y = h$

For $y = kx^2$

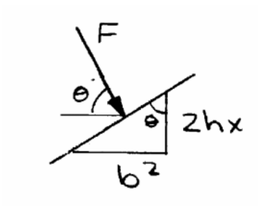
We have $h = kb^2$

or $k = \dfrac{h}{b^2}$

$$\therefore\ y = \left(\frac{h}{b^2}\right)x^2$$

For any contact point *c* alone the surface $y = \left(\dfrac{h}{b^2}\right)x^2$

$$\frac{dy}{dx} = 2\frac{h}{b^2}x$$



$$\mathbf{R} = F \ \measuredangle\ \tan^{-1}\left(\frac{b^2}{2hx}\right) \blacktriangleleft$$

and ΣM_D: $M_D = -(x)F\sin\theta + (h - y)F\cos\theta$

$$= -xF\left(\frac{b^2}{\sqrt{b^4 + 4h^2x^2}}\right) + \left(h - \frac{h}{b^2}x^2\right)F\frac{2hx}{\sqrt{b^4 + 4h^2x^2}}$$

or

$$\mathbf{M}_D = F\frac{\left[-xb^2 + \left(h - \dfrac{h}{b^2}x^2\right)(2hx)\right]}{\sqrt{b^4 + 4h^2x^2}}$$

$$= F\frac{\left[-xb^2 + 2h^2x - \dfrac{2h^2x^3}{b^2}\right]}{\sqrt{b^4 + 4h^2x^2}}$$

$$\text{or } \boldsymbol{M}_D = F\left[\frac{(2h^2 - b^2)x - \dfrac{2h^2x^3}{b^2}}{\sqrt{b^4 + 4h^2x^2}}\right] \circlearrowleft \blacktriangleleft$$

(*b*) With $b = 1 \text{ ft}, \quad h = 2 \text{ ft}$

$$M_D = \left[\frac{7x - 8x^3}{1 + 16x^2}\right]F$$

For M_D to be a maximum

Then $$\frac{dM_D}{dx} = 0 = F\left[\frac{\left(7 - 24x^2\right)\sqrt{1 + 16x^2} - \left(7x - 8x^3\right)\left(\frac{1}{2}\right)(32x)\left(1 + 16x^2\right)^{-\frac{1}{2}}}{\left(1 + 16x^2\right)}\right]$$

For the non-trivial solution:

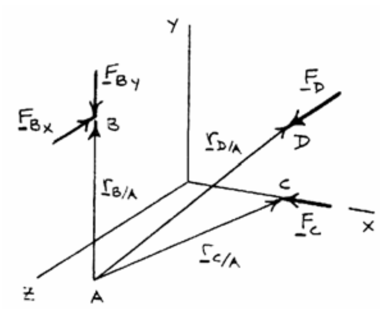
$$0 = \left(7 - 24x^2\right)\left(1 + 16x^2\right) - 16x\left(7x - 8x^3\right)$$

$$0 = 256x^4 + 24x^2 - 7$$

Solving by the quadratic formula gives for the positive root.

$x = 0.354 \text{ ft}$ ◀

Chapter 3, Solution 115.



For equivalence

$$\Sigma\mathbf{F}:\quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}_A$$

$$\mathbf{R}_A = -(240\text{ N})\mathbf{j} - (125\text{ N})\mathbf{k} - (300\text{ N})\mathbf{i} + (150\text{ N})\mathbf{k}$$

$$\therefore\quad \mathbf{R}_A = -(300\text{ N})\mathbf{i} - (240\text{ N})\mathbf{j} + (25\text{ N})\mathbf{k} \blacktriangleleft$$

Also for equivalence

$$\Sigma\mathbf{M}_A:\quad \mathbf{r}_{B/A}\times\mathbf{F}_B + \mathbf{r}_{C/A}\times\mathbf{F}_C + \mathbf{r}_{D/A}\times\mathbf{F}_D = \mathbf{M}_A$$

$$\text{or } M_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.12\text{ m} & 0 \\ 0 & -240\text{ N} & -125\text{ N} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06\text{ m} & 0.03\text{ m} & -0.075\text{ m} \\ -300\text{ N} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06\text{ m} & 0.08\text{ m} & -0.75\text{ m} \\ 0 & 0 & 150\text{ N} \end{vmatrix}$$

$$= \left[-(15\text{ N}\cdot\text{m})\mathbf{i}\right] + \left[(22.5\text{ N}\cdot\text{m})\mathbf{j} + (9\text{ N}\cdot\text{m})\mathbf{k}\right] + \left[(12\text{ N}\cdot\text{m})\mathbf{i} - (9\text{ N}\cdot\text{m})\mathbf{j}\right]$$

$$\text{or } \mathbf{M}_A = -(3\text{ N}\cdot\text{m})\mathbf{i} + (13.5\text{ N}\cdot\text{m})\mathbf{j} + (9\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 116.

Let $(\mathbf{R}, \mathbf{M}_D)$ be the equivalent force-couple system at O.

Now $\Sigma\mathbf{F}$: $\mathbf{R} = \Sigma\mathbf{F}$

$$= (1.8 \text{ lb})(-\sin 40°\mathbf{i} - \cos 40°\mathbf{k}) + (11 \text{ lb})(-\sin 12°\mathbf{j} - \cos 12°\mathbf{k}) + (18 \text{ lb})(-\sin 15°\mathbf{j} - \cos 15°\mathbf{k})$$

$$\text{or } \boldsymbol{R} = -(1.157 \text{ lb})\mathbf{i} - (6.95 \text{ lb})\mathbf{j} - (29.5 \text{ lb})\mathbf{k} \blacktriangleleft$$

Note that each belt force may be replaced by a force-couple that is equivalent to the same force plus the moment of the force about the shaft (x axis) of the sander. Then ...

$$\Sigma\mathbf{M}_O: \quad \mathbf{M}_O = \Sigma\mathbf{M}_O$$

$$= (1.8 \text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.75 \text{ in.} & 2.2 \text{ in.} \\ -\sin 40° & 0 & -\cos 40° \end{vmatrix}$$

$$-\left[(2.5 \text{ in.})(11 \text{ lb})\right]\mathbf{i} - (9 \text{ in.})\mathbf{i} \times (11 \text{ lb})(-\sin 12°\mathbf{j} - \cos 12°\mathbf{k})$$

$$+\left[(2.5 \text{ in.})(18 \text{ lb})\right]\mathbf{i} - (9 \text{ in.})\mathbf{i} \times (18 \text{ lb})(-\sin 15°\mathbf{j} - \cos 15°\mathbf{k})$$

$$= \left[(1.8)(-0.75\cos 40°\mathbf{i} - 2.2\sin 40°\mathbf{j} + 0.75\sin 40°\mathbf{k}) - 27.5\mathbf{i}\right.$$

$$\left. +(99)(\sin 12°\mathbf{k} - \cos 12°\mathbf{j}) + 45\mathbf{i} + (162)(\sin 15°\mathbf{k} - \cos 15°\mathbf{j})\right](\text{lb}\cdot\text{in.})$$

$$= \left[(-1.03416 - 27.5 + 45)\mathbf{i} + (-2.5454 - 96.837 - 156.480)\mathbf{j}\right.$$

$$\left. +(0.86776 + 20.583 + 41.929)\mathbf{k}\right](\text{lb}\cdot\text{in.})$$

$$\text{or } \mathbf{M}_O = (16.47 \text{ lb}\cdot\text{in.})\mathbf{i} - (256 \text{ lb}\cdot\text{in.})\mathbf{j} + (63.4 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 117.

Have ΣF_x: $-10\text{ N} = A_x + B_x \quad \Rightarrow B_x = -10 - A_x$

ΣF_y: $0 = A_y + B_y \quad \Rightarrow B_y = -A_y$

ΣF_z: $6\text{ N} = A_z + B_z \quad \Rightarrow B_z = 6 - A_z$

and $\Sigma \mathbf{M}_O$: $\mathbf{M}_O = \mathbf{r}_{O/A} \times \mathbf{A} + \mathbf{r}_{O/B} \times \mathbf{B}$

Now d_{BA}: $372\text{ mm} = \sqrt{(60\text{ mm})^2 + (-72\text{ mm})^2 + (d_{BA})_z^2}$

or $(d_{BA})_z = 360\text{ mm}$

Then $\mathbf{r}_{O/A} = (135\text{ mm})\mathbf{i} - (72\text{ mm})\mathbf{j} + (310\text{ mm})\mathbf{k}$

$\mathbf{r}_{O/B} = (75\text{ mm})\mathbf{i} - (50\text{ mm})\mathbf{k}$

$$(60\text{ N}\cdot\text{m})\mathbf{i} + (0.05\text{ N}\cdot\text{m})\mathbf{j} - (10\text{ N}\cdot\text{m})\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.135 & -0.72 & 0.310 \\ A_x & A_y & A_z \end{vmatrix}(\text{N}\cdot\text{m}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.075 & 0 & -0.050 \\ B_x & B_y & B_z \end{vmatrix}(\text{N}\cdot\text{m})$$

i: $60 = (-0.072\,A_z - 0.310\,A_y) + (0.050)B_y$

or $60 = -0.072\,A_z - 0.360\,A_y \qquad (1)$

j: $0.05 = (0.310A_x - 0.135A_z) + (-0.050\,B_x - 0.075\,B_z)$

$= 0.310A_x - 0.050(-10 - A_x) - 0.135A_z - 0.075(6 - A_z)$

or $0 = 0.360A_x - 0.060\,A_z$

$A_z = 6A_x$

Now $A_x = 2$ N $\quad \therefore A_z = 12.00$ **N**

From equation (1) $\quad 60 = -0.072(12.00) - 0.360A_y$

or $A_y = -169.1$ N

Then $\quad B_x = -12.00$ N

$B_y = 169.1$ N

$B_z = -6.00$ N

$\therefore \mathbf{A} = (2.00\text{ N})\mathbf{i} - (169.1\text{ N})\mathbf{j} + (12.00\text{ N})\mathbf{k}$ ◀

$\mathbf{B} = -(12.00\text{ N})\mathbf{i} + (169.1\text{ N})\mathbf{j} - (6.00\text{ N})\mathbf{k}$ ◀

Chapter 3, Solution 118.

Have $\Sigma \mathbf{F}: \quad \mathbf{B} + \mathbf{C} = \mathbf{R}$

$\Sigma F_x: \quad B_x + C_x = 3.9 \text{ lb} \quad \text{or} \quad B_x = 3.9 \text{ lb} - C_x \qquad (1)$

$\Sigma F_y: \quad C_y = R_y \qquad (2)$

$\Sigma F_z: \quad C_z = -1.1 \text{ lb} \qquad (3)$

Have $\Sigma \mathbf{M}_A: \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{C/A} \times \mathbf{C} + \mathbf{M}_B = \mathbf{M}_A^R$

$$\therefore \frac{1}{12}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & 4.5 \\ B_x & 0 & 0 \end{vmatrix} + \frac{1}{12}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 2.0 \\ C_x & C_y & -1.1 \end{vmatrix} + (2 \text{ lb}\cdot\text{ft})\mathbf{i} = M_x\mathbf{i} + (1.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (1.1 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$(2 - 0.166667C_y)\mathbf{i} + (0.375B_x + 0.166667C_x + 0.36667)\mathbf{j} + (0.33333C_y)\mathbf{k}$$

$$= M_x\mathbf{i} + (1.5)\mathbf{j} - (1.1)\mathbf{k}$$

From **i** - coefficient $\quad 2 - 0.166667C_y = M_x \qquad (4)$

j - coefficient $\quad 0.375B_x + 0.166667C_x + 0.36667 = 1.5 \qquad (5)$

k - coefficient $\quad 0.33333C_y = -1.1 \quad \text{or} \quad C_y = -3.3 \text{ lb} \qquad (6)$

(*a*) From Equations (1) and (5):

$$0.375(3.9 - C_x) + 0.166667C_x = 1.13333$$

$$C_x = \frac{0.32917}{0.20833} = 1.58000 \text{ lb}$$

From Equation (1): $\quad B_x = 3.9 - 1.58000 = 2.32 \text{ lb}$

$\therefore \quad \mathbf{B} = (2.32 \text{ lb})\mathbf{i}$ ◀

$\mathbf{C} = (1.580 \text{ lb})\mathbf{i} - (3.30 \text{ lb})\mathbf{j} - (1.110 \text{ lb})\mathbf{k}$ ◀

(*b*) From Equation (2): $\quad R_y = C_y = -3.30 \text{ lb}$

or $\mathbf{R}_y = -(3.30 \text{ lb})$ ◀

From Equation (4): $\quad M_x = -0.166667(-3.30) + 2.0 = 2.5500 \text{ lb}\cdot\text{ft}$

or $\mathbf{M}_x = (2.55 \text{ lb}\cdot\text{ft})$ ◀

Chapter 3, Solution 119.

(*a*) Duct *AB* will not have a tendency to rotate about the vertical or *y*-axis if:

$$M_{By}^R = \mathbf{j}\cdot\Sigma\mathbf{M}_B^R = \mathbf{j}\cdot\left(\mathbf{r}_{F/B}\times\mathbf{F}_F + \mathbf{r}_{E/B}\times\mathbf{F}_E\right) = 0$$

where

$$\mathbf{r}_{F/B} = (1.125\text{ m})\mathbf{i} - (0.575\text{ m})\mathbf{j} + (0.7\text{ m})\mathbf{k}$$

$$\mathbf{r}_{E/B} = (1.35\text{ m})\mathbf{i} - (0.85\text{ m})\mathbf{j} + (0.7\text{ m})\mathbf{k}$$

$$\mathbf{F}_F = 50\text{ N}\left[(\sin\alpha)\mathbf{j} + (\cos\alpha)\mathbf{k}\right]$$

$$\mathbf{F}_E = -(25\text{ N})\mathbf{k}$$

$$\therefore\quad \Sigma\mathbf{M}_B^R = (50\text{ N})\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1.125\text{ m} & -0.575\text{ m} & 0.7\text{ m}\\ 0 & \sin\alpha & \cos\alpha\end{vmatrix} + (25\text{ N})\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1.35\text{ m} & -0.85\text{ m} & 0.70\\ 0 & 0 & -1\end{vmatrix}$$

$$= \left[(-28.75\cos\alpha - 35\sin\alpha + 21.25)\mathbf{i} - (56.25\cos\alpha - 33.75)\mathbf{j} + (56.25\sin\alpha)\mathbf{k}\right]\text{N}\cdot\text{m}$$

Thus,

$$M_{By}^R = -56.25\cos\alpha + 33.75 = 0$$

$$\cos\alpha = 0.60$$

$$\alpha = 53.130°$$

or $\alpha = 53.1°$ ◀

(*b*) $\mathbf{R} = \mathbf{F}_E + \mathbf{F}_F$

where

$$\mathbf{F}_E = -(25\text{ N})\mathbf{k}$$

$$\mathbf{F}_F = (50\text{ N})(\sin 53.130°\mathbf{j} + \cos 53.130°\mathbf{k}) = (40\text{ N})\mathbf{j} + (30\text{ N})\mathbf{k}$$

$$\therefore\quad \mathbf{R} = (40\text{ N})\mathbf{j} + (5\text{ N})\mathbf{k}$$ ◀

and

$$\mathbf{M} = \Sigma\mathbf{M}_B^R = -\left[28.75(0.6) + 35(0.8) - 21.25\right]\mathbf{i} - \left[56.25(0.6) - 33.75\right]\mathbf{j} + \left[56.25(0.8)\right]\mathbf{k}$$

$$= -(24\text{ N}\cdot\text{m})\mathbf{i} - (0)\mathbf{j} + (45\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = -(24.0\text{ N}\cdot\text{m})\mathbf{i} + (45.0\text{ N}\cdot\text{m})\mathbf{k}$ ◀

Chapter 3, Solution 120.

(*a*) Have $\mathbf{R} = \Sigma\mathbf{F} = \mathbf{F}_F + \mathbf{F}_E$

where $\mathbf{F}_F = 50\text{ N}\left[(\sin 60°)\mathbf{j} + (\cos 60°)\mathbf{k}\right] = (43.301\text{ N})\mathbf{j} + (25\text{ N})\mathbf{k}$

$\mathbf{F}_E = -(25\text{ N})\mathbf{k}$

$\therefore\ \mathbf{R} = (43.301\text{ N})\mathbf{j}$ or $\mathbf{R} = (43.3\text{ N})\mathbf{j}$ ◀

Have $\mathbf{M}_C^R = \Sigma(\mathbf{r}\times\mathbf{F}) = \mathbf{r}_{F/C}\times\mathbf{F}_F + \mathbf{r}_{E/C}\times\mathbf{F}_E$

where $\mathbf{r}_{F/C} = (0.225\text{ m})\mathbf{i} - (0.050\text{ m})\mathbf{j}$

$\mathbf{r}_{E/C} = (0.450\text{ m})\mathbf{i} - (0.325\text{ m})\mathbf{j}$

$$\therefore\ \mathbf{M}_C^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.225 & -0.050 & 0 \\ 0 & 43.301 & 25 \end{vmatrix}\text{N}\cdot\text{m} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.450 & -0.325 & 0 \\ 0 & 0 & -25 \end{vmatrix}\text{N}\cdot\text{m}$$

$$= (6.875\text{ N}\cdot\text{m})\mathbf{i} + (5.625\text{ N}\cdot\text{m})\mathbf{j} + (9.7427\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_C^R = (6.88\text{ N}\cdot\text{m})\mathbf{i} + (5.63\text{ N}\cdot\text{m})\mathbf{j} + (9.74\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(*b*) To determine which direction duct section *CD* has a tendency to turn, have

$$M_{CD}^R = \boldsymbol{\lambda}_{DC}\cdot\mathbf{M}_C^R$$

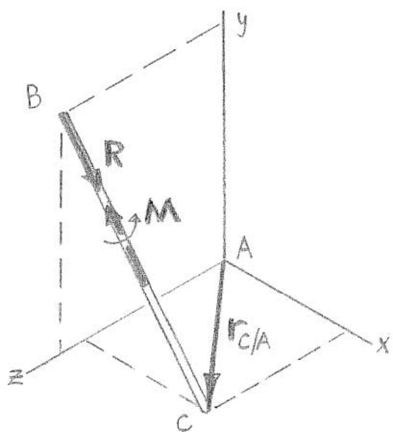
where

$$\boldsymbol{\lambda}_{DC} = \frac{-(0.45\text{ m})\mathbf{i} + (0.1\text{ m})\mathbf{j}}{\sqrt{(-0.45)^2 + (0.1)^2}} = -0.97619\mathbf{i} + 0.21693\mathbf{j}$$

Then
$$M_{CD}^R = (-0.97619\mathbf{i} + 0.21693\mathbf{j})\cdot(6.875\mathbf{i} + 5.625\mathbf{j} + 9.7427\mathbf{k})\text{N}\cdot\text{m}$$
$$= (-6.7113 + 1.22023)\text{N}\cdot\text{m}$$
$$= -5.4911\text{ N}\cdot\text{m}$$

Since $\boldsymbol{\lambda}_{DC}\cdot\mathbf{M}_C^R < 0$, duct *DC* tends to rotate *counterclockwise* relative to elbow *C* as viewed from *D* to *C*. ◀

Chapter 3, Solution 121.



Have $\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{R}_A = R\boldsymbol{\lambda}_{BC}$

where $\boldsymbol{\lambda}_{BC} = \dfrac{(42 \text{ in.})\mathbf{i} - (96 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{106 \text{ in.}}$

$$\therefore \quad \mathbf{R}_A = \frac{21.2 \text{ lb}}{106}(42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

or $\mathbf{R}_A = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$ ◀

Have $\Sigma\mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$

where $\mathbf{r}_{C/A} = (42 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{k} = \dfrac{1}{12}(42\mathbf{i} + 48\mathbf{k}) \text{ ft}$

$$= (3.5 \text{ ft})\mathbf{i} + (4.0 \text{ ft})\mathbf{k}$$

$$\mathbf{R} = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$$

$$\mathbf{M} = -\boldsymbol{\lambda}_{BC} M$$

$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106}(13.25 \text{ lb}\cdot\text{ft})$$

$$= -(5.25 \text{ lb}\cdot\text{ft})\mathbf{i} + (12 \text{ lb}\cdot\text{ft})\mathbf{j} + (2 \text{ lb}\cdot\text{ft})\mathbf{k}$$

Then $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{lb}\cdot\text{ft} + (-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k})\text{lb}\cdot\text{ft} = \mathbf{M}_A$

$$\therefore \quad \mathbf{M}_A = (71.55 \text{ lb}\cdot\text{ft})\mathbf{i} + (56.80 \text{ lb}\cdot\text{ft})\mathbf{j} - (65.20 \text{ lb}\cdot\text{ft})\mathbf{k}$$

or $\mathbf{M}_A = (71.6 \text{ lb}\cdot\text{ft})\mathbf{i} + (56.8 \text{ lb}\cdot\text{ft})\mathbf{j} - (65.2 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

Chapter 3, Solution 122.

From $$\mathbf{R}_C = \mathbf{R} = (60 \text{ lb})\,\lambda_{AB} = 60 \text{ lb}\left[\frac{-(0.6 \text{ ft})\mathbf{i} + (4.2 \text{ ft})\mathbf{j} - (1.5 \text{ ft})\mathbf{k}}{\sqrt{(-0.6 \text{ ft})^2 + (4.2 \text{ ft})^2 + (-1.5 \text{ ft})^2}}\right]$$

$$\mathbf{R}_C = -(8.00 \text{ lb})\mathbf{i} + (56.0 \text{ lb})\mathbf{j} - (20.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

From $$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{R} + \mathbf{M}$$

where

$$\mathbf{r}_{A/C} = (7.8 \text{ ft})\mathbf{i} + (1.5 \text{ ft})\mathbf{k}$$

$$\mathbf{M} = (22.5 \text{ lb}\cdot\text{ft})\,\lambda_{BA} = (22.5 \text{ lb}\cdot\text{ft})\left[\frac{(0.6 \text{ ft})\mathbf{i} - (4.2 \text{ ft})\mathbf{j} + (1.5 \text{ ft})\mathbf{k}}{\sqrt{(0.6 \text{ ft})^2 + (-4.2 \text{ ft})^2 + (1.5 \text{ ft})^2}}\right]$$

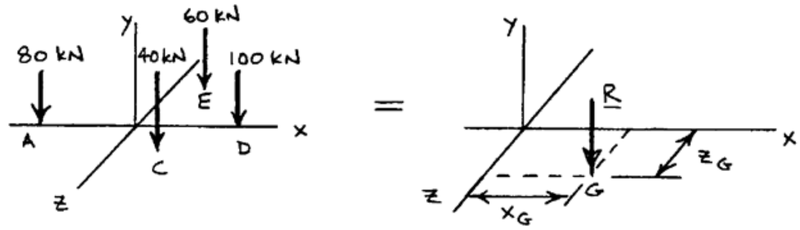
$$= (3 \text{ lb}\cdot\text{ft})\mathbf{i} - (21 \text{ lb}\cdot\text{ft})\mathbf{j} + (7.5 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\therefore\ \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7.8 & 0 & 1.5 \\ -8 & 56 & -20 \end{vmatrix} \text{lb}\cdot\text{ft} + (3\mathbf{i} - 21\mathbf{j} + 7.5\mathbf{k})\text{lb}\cdot\text{ft}$$

$$= \left[(-84 + 3)\text{lb}\cdot\text{ft}\right]\mathbf{i} + \left[(144 - 21)\text{lb}\cdot\text{ft}\right]\mathbf{j} + \left[(436.8 + 7.5)\text{lb}\cdot\text{ft}\right]\mathbf{k}$$

or $$\mathbf{M}_C = -(81.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (123.0 \text{ lb}\cdot\text{ft})\mathbf{j} + (444 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 123.



Have: $\Sigma\mathbf{F}$: $\mathbf{F}_A + \mathbf{F}_C + \mathbf{F}_D + \mathbf{F}_E = \mathbf{R}$

$$\mathbf{R} = -(80 \text{ kN})\mathbf{j} - (40 \text{ kN})\mathbf{j} - (100 \text{ kN})\mathbf{j} - (60 \text{ kN})\mathbf{j}$$

$$= -(280 \text{ kN})\mathbf{j}$$

or $R = 280$ kN ◀

Have: ΣM_x: $F_A(z_A) + F_C(z_C) + F_D(z_D) + F_E(z_E) = R(z_G)$

$$(80 \text{ kN})(0) + (40 \text{ kN})\left[(3 \text{ m})\sin 60°\right] + 60 \text{ kN}(0)$$

$$+(60 \text{ kN})\left[-(3 \text{ m})\sin 60°\right] = (280 \text{ kN})Z_G$$

$\therefore$ $Z_G = -0.185577$ m

or $Z_G = -0.1856$ m ◀

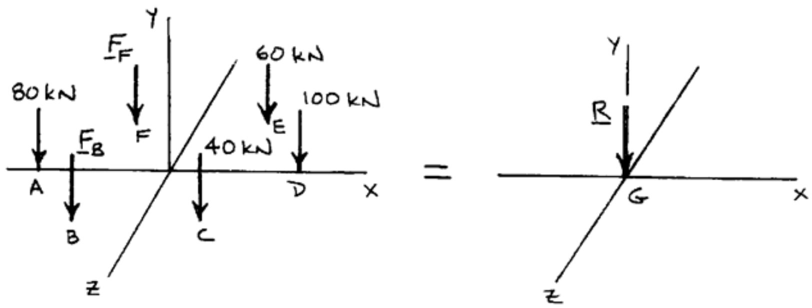
ΣM_z: $F_A(x_A) + F_C(x_C) + F_D(x_D) + F_E(x_E) = R(x_G)$

$$(80 \text{ kN})\left[-(3 \text{ m})\cos 60° - 1.5 \text{ m}\right] + (40 \text{ kN})(1.5 \text{ m}) + 60 \text{ kN}(1.5 \text{ m})$$

$$+ (100 \text{ kN})\left[(3 \text{ m})\cos 60° + 1.5 \text{ m}\right] = (280 \text{ kN})x_G$$

or $x_G = 0.750$ m ◀

Chapter 3, Solution 124.



Have: ΣM_x: $F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) + F_E(z_E) + F_F(z_F) = R(z_G)$

$$(80\text{ kN})(0) + F_B[(3\text{ m})\sin 60°] + (40\text{ kN})[(3\text{ m})\sin 60°] + (100\text{ kN})(0)$$

$$+(60\text{ kN})[-(3\text{ m})\sin 60°] + F_F[-(3\text{ m})\sin 60°] = R(0)$$

$$F_B - F_F = 20\text{ kN} \qquad (1)$$

Also ΣM_z: $F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) + F_E(x_E) + F_F(x_F) = R(x_G)$

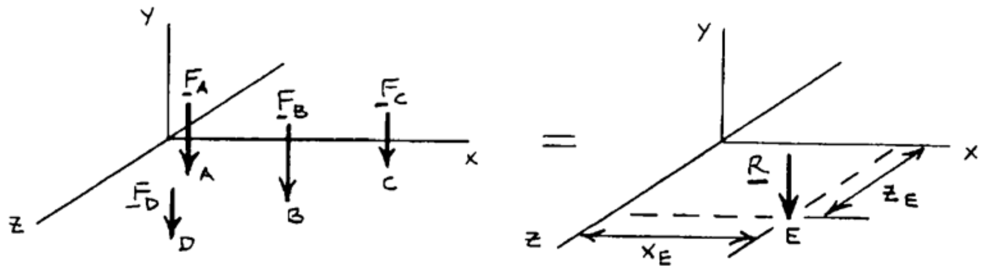
$$(80\text{ kN})[-(3\text{ m})\cos 60° - 1.5\text{ m}] + F_B(-1.5\text{ m}) + (40\text{ kN})(1.5\text{ m})$$
$$+ (100\text{ kN})[(3\text{ m})\cos 60° + 1.5\text{ m}] + (60\text{ kN})(1.5\text{ m}) + F_F(-1.5\text{ m}) = R(0)$$

$$F_B + F_F = 140\text{ kN} \qquad (2)$$

Solving equations (1) and (2): $F_B = 80.0\text{ kN}$ ◀

$F_F = 60.0\text{ kN}$ ◀

Chapter 3, Solution 125.



Have $\quad \Sigma \mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}$

$$-(116 \text{ kips})\mathbf{j} - (470 \text{ kips})\mathbf{j} - (66 \text{ kips})\mathbf{j} - (28 \text{ kips})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(680 \text{ kips})\mathbf{j} \quad R = 680 \text{ kips} \blacktriangleleft$$

Have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_E)$

$$(116 \text{ kips})(24 \text{ ft}) + (470 \text{ kips})(48 \text{ ft}) + (66 \text{ kips})(18 \text{ ft}) + (28 \text{ kips})(100.5 \text{ ft}) = (680 \text{ kips})(z_E)$$

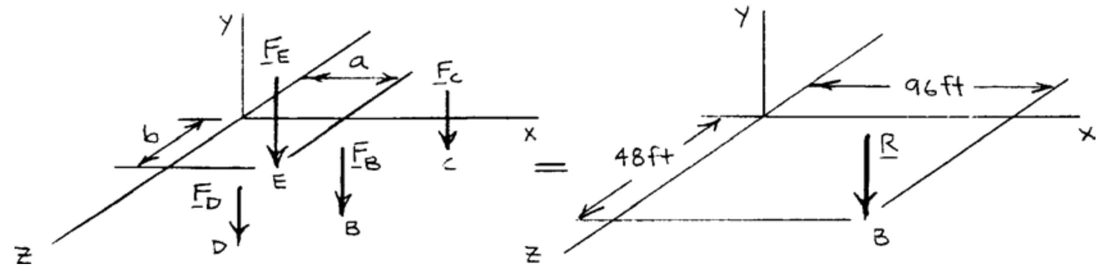
$$\therefore \quad z_E = 43.156 \text{ ft} \text{ or } z_E = 43.2 \text{ ft} \blacktriangleleft$$

Have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_E)$

$$(116 \text{ kips})(30 \text{ ft}) + (470 \text{ kips})(96 \text{ ft}) + (66 \text{ kips})(162 \text{ ft}) + (28 \text{ kips})(96 \text{ ft}) = (680 \text{ kips})(x_E)$$

$$\therefore \quad x_E = 91.147 \text{ or } x_E = 91.1 \text{ ft} \blacktriangleleft$$

Chapter 3, Solution 126.



Have $\qquad \Sigma\mathbf{F}: \quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{F}_E = \mathbf{R}$

$$-(470 \text{ kips})\mathbf{j} - (66 \text{ kips})\mathbf{j} - (28 \text{ kips})\mathbf{j} - (116 \text{ kips})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(680 \text{ kips})\mathbf{j}$$

Have $\qquad \Sigma M_x: \quad F_B(z_B) + F_C(z_C) + F_D(z_D) + F_E(z_E) = R(z_B)$

$$(470 \text{ kips})(48 \text{ ft}) + (66 \text{ kips})(18 \text{ ft}) + (28 \text{ kips})(100.5 \text{ ft}) + (116 \text{ kips})(b) = (680 \text{ kips})(48 \text{ ft})$$

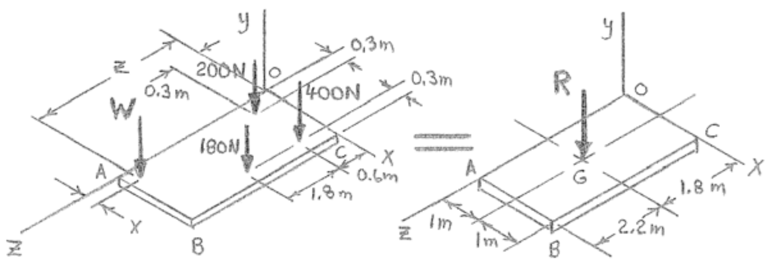
$$\therefore \quad b = 52.397 \text{ ft} \text{ or } b = 52.4 \text{ ft} \blacktriangleleft$$

Have $\qquad \Sigma M_z: \quad F_B(x_B) + F_C(x_C) + F_D(x_D) + F_E(x_E) = R(x_B)$

$$(470 \text{ kips})(96 \text{ ft}) + (66 \text{ kips})(162 \text{ ft}) + (28 \text{ kips})(96 \text{ ft}) + (116 \text{ kips})(a) = (680 \text{ kips})(96 \text{ ft})$$

$$\therefore \quad a = 58.448 \text{ ft} \text{ or } a = 58.4 \text{ ft} \blacktriangleleft$$

Chapter 3, Solution 127.



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.6 \times 0.6 \times 1.2$-m box should be placed adjacent to one of the edges of the trailer with the 0.6×0.6-m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

Have $\Sigma \mathbf{F}: \quad -(200\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} - (180\text{ N})\mathbf{j} = \mathbf{R}$

$$\therefore \quad \mathbf{R} = -(780\text{ N})\mathbf{j}$$

Have $\Sigma M_z: \quad (200\text{ N})(0.3\text{ m}) + (400\text{ N})(1.7\text{ m}) + (180\text{ N})(1.7\text{ m}) = (780\text{ N})(x)$

$$\therefore \quad x = 1.34103\text{ m}$$

Have $\Sigma M_x: \quad (200\text{ N})(0.3\text{ m}) + (400\text{ N})(0.6\text{ m}) + (180\text{ N})(2.4\text{ m}) = (780\text{ N})(z)$

$$\therefore \quad z = 0.93846\text{ m}$$

From the statement of the problem, it is known that the resultant of **R** from the original loading and the lightest load **W** passes through G, the point of intersection of the two center lines. Thus, $\Sigma \mathbf{M}_G = 0$.

Further, since the lightest load **W** is to be as small as possible, the fourth box should be placed as far from G as possible without the box overhanging the trailer. These two requirements imply

$$(0.3\text{ m} \le x \le 1\text{ m}) \quad (1.8\text{ m} \le z \le 3.7\text{ m})$$

continued

Let $x = 0.3$ m, $\Sigma M_{Gz}: \quad (200 \text{ N})(0.7 \text{ m}) - (400 \text{ N})(0.7 \text{ m}) - (180 \text{ N})(0.7 \text{ m}) + W(0.7 \text{ m}) = 0$

$$\therefore \quad W = 380 \text{ N}$$

$$\Sigma M_{Gx}: \quad -(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + (380 \text{ N})(z - 1.8 \text{ m}) = 0$$

$$\therefore \; z = 3.5684 \text{ m} < 3.7 \text{ m} \qquad \therefore \; \text{acceptable}$$

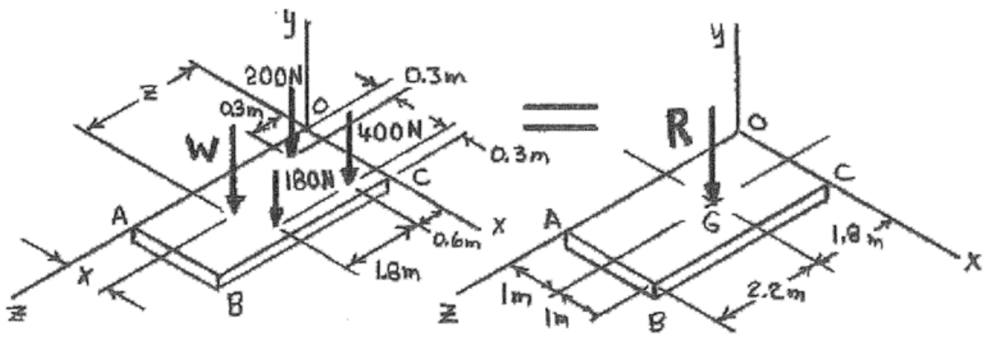
Let $z = 3.7$ m, $\Sigma M_{Gx}: \quad -(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + W(1.7 \text{ m}) = 0$

$$\therefore \quad W = 395.29 \text{ N} > 380 \text{ N}$$

Since the weight W found for $x = 0.3$ m is less than W found for $z = 3.7$ m, $x = 0.3$ m results in the smallest weight W.

or $W = 380$ N at $(0.3 \text{ m}, 0, 3.57 \text{ m})$ ◀

Chapter 3, Solution 128.



For the largest additional weight on the trailer with the box having at least one side coinsiding with the side of the trailer, the box must be as close as possible to point *G*. For $x = 0.6$ m, with a small side of the box touching the *z*-axis, satisfies this condition.

Let $x = 0.6$ m, ΣM_{Gz}: $(200 \text{ N})(0.7 \text{ m}) - (400 \text{ N})(0.7 \text{ m}) - (180 \text{ N})(0.7 \text{ m}) + W(0.4 \text{ m}) = 0$

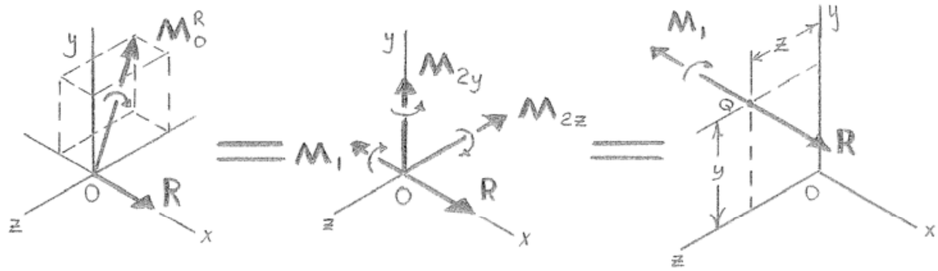
$$\therefore \; W = 665 \text{ N}$$

and ΣM_{GX}: $-(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + (665 \text{ N})(z - 1.8 \text{ m}) = 0$

$$\therefore \; z = 2.8105 \text{ m} \qquad (2 \text{ m} < z < 4 \text{ m}) \qquad \therefore \; \text{acceptable}$$

or $W = 665$ N at $(0.6 \text{ m}, 0, 2.81 \text{ m})$ ◀

Chapter 3, Solution 129.



First, reduce the given force system to a force-couple system at the origin.

Have $\Sigma\mathbf{F}$: $(2P)\mathbf{i} - (P)\mathbf{j} + (P)\mathbf{j} = \mathbf{R}$

$$\therefore \quad \mathbf{R} = (2P)\mathbf{i}$$

Have $\Sigma\mathbf{M}_O$: $\Sigma(\mathbf{r}_O \times \mathbf{F}) = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = Pa\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 2.5 \\ 2 & -1 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{vmatrix} = Pa(-1.5\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$$

(*a*) $\mathbf{R} = 2P\mathbf{i}$ or Magnitude of $\mathbf{R} = 2P$ ◀

Direction of $\mathbf{R}$: $\theta_x = 0°$, $\theta_y = -90°$, $\theta_z = 90°$ ◀

(*b*) Have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \dfrac{\mathbf{R}}{R}$

$$= \mathbf{i}\cdot(-1.5Pa\mathbf{i} + 5Pa\mathbf{j} - 6Pa\mathbf{k})$$

$$= -1.5Pa$$

and pitch $P = \dfrac{M_1}{R} = \dfrac{-1.5Pa}{2P} = -0.75a$ or $P = -0.75a$ ◀

(*c*) Have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (5Pa)\mathbf{j} - (6Pa)\mathbf{k}$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$(5Pa)\mathbf{j} - (6Pa)\mathbf{k} = (y\mathbf{j} + z\mathbf{k}) \times (2P\mathbf{i}) = -(2Py)\mathbf{k} + (2Pz)\mathbf{j}$$

From $\mathbf{i}: \quad 5Pa = 2Pz$

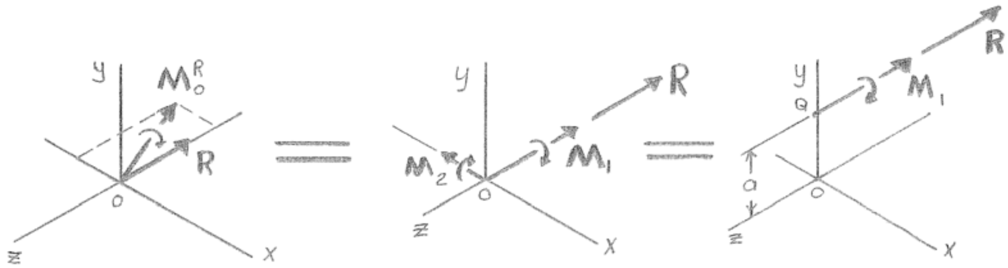
$$\therefore \quad z = 2.5a$$

From $\mathbf{k}: \quad -6Pa = -2Py$

$$\therefore \quad y = 3a$$

$\therefore$ The axis of the wrench is parallel to the *x*-axis and intersects the *yz*-plane at $y = 3a,\ z = 2.5a$ ◀

Chapter 3, Solution 130.



First, reduce the given force system to a force-couple at the origin.

Have $\Sigma\mathbf{F}: \quad P\mathbf{i} - P\mathbf{i} - P\mathbf{k} = \mathbf{R}$

$$\therefore \quad \mathbf{R} = -P\mathbf{k}$$

Have $\Sigma\mathbf{M}_O: \quad -P(3a)\mathbf{k} - P(3a)\mathbf{j} + P(-a\mathbf{i} + 3a\mathbf{j}) = \mathbf{M}_O^R$

$$\therefore \quad \mathbf{M}_O^R = Pa(-\mathbf{i} - 3\mathbf{k})$$

Then let vectors $(\mathbf{R}, \mathbf{M}_1)$ represent the components of the wrench, where their directions are the same.

(*a*) $\mathbf{R} = -P\mathbf{k}$ or Magnitude of $\mathbf{R} = P$ ◀

Direction of $\mathbf{R}$: $\theta_x = 90°, \ \theta_y = 90°, \ \theta_z = -180°$ ◀

(*b*) Have

$$M_1 = \lambda_R \cdot \mathbf{M}_O^R$$
$$= -\mathbf{k} \cdot \left[Pa(-\mathbf{i} - 3\mathbf{k})\right]$$
$$= 3Pa$$

and pitch $\quad P = \dfrac{M_1}{R} = \dfrac{3Pa}{P} = 3a \qquad$ or $P = 3a$ ◀

(*c*) Have $\quad \mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = Pa(-\mathbf{i} - 3\mathbf{k}) - (-3Pa\mathbf{k}) = -Pa\mathbf{i}$$

Require $\quad \mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

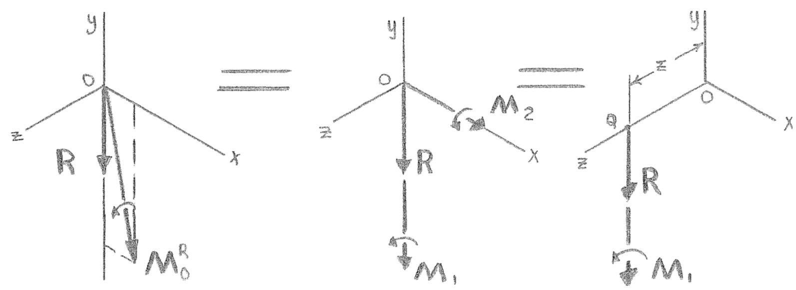
$$-Pa\mathbf{i} = (x\mathbf{i} + y\mathbf{j}) \times (-P)\mathbf{k} = Px\mathbf{j} - Py\mathbf{i}$$

From $\quad \mathbf{i}: \ -Pa = -Py \quad$ or $\quad y = a$

$\quad \mathbf{j}: \ x = 0$

$\therefore$ The axis of the wrench is parallel to the *z*-axis and intersects the *xy* plane at $x = 0, \ y = a$ ◀

Chapter 3, Solution 131.



First, reduce the given force system to a force-couple at the origin.

$$\text{Have } \Sigma\mathbf{F}: \quad -(10\text{ N})\mathbf{j} - (11\text{ N})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(21\text{ N})\mathbf{j}$$

$$\text{Have } \Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.5 \\ 0 & -10 & 0 \end{vmatrix} \text{N}\cdot\text{m} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.375 \\ 0 & -11 & 0 \end{vmatrix} \text{N}\cdot\text{m} - (12\text{ N}\cdot\text{m})\mathbf{j}$$

$$= (0.875\text{ N}\cdot\text{m})\mathbf{i} - (12\text{ N}\cdot\text{m})\mathbf{j}$$

(*a*) $\mathbf{R} = -(21\text{ N})\mathbf{j}$ or

$\mathbf{R} = -(21.0\text{ N})\mathbf{j}$ ◀

(*b*) $$\text{Have } M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \frac{\mathbf{R}}{R}$$

$$= (-\mathbf{j})\cdot\left[(0.875\text{ N}\cdot\text{m})\mathbf{i} - (12\text{ N}\cdot\text{m})\mathbf{j}\right]$$

$$= 12\text{ N}\cdot\text{m} \qquad \text{and} \qquad \mathbf{M}_1 = -(12\text{ N}\cdot\text{m})\mathbf{j}$$

$$\text{and pitch } P = \frac{M_1}{R} = \frac{12\text{ N}\cdot\text{m}}{21\text{ N}} = 0.57143\text{ m}$$ or $P = 0.571$ m ◀

(*c*) Have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (0.875\text{ N}\cdot\text{m})\mathbf{i}$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$\therefore \quad (0.875\text{ N}\cdot\text{m})\mathbf{i} = (x\mathbf{i} + z\mathbf{k})\times\left[-(21\text{ N})\mathbf{j}\right]$$

$$0.875\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

From **i**: $0.875 = 21z$

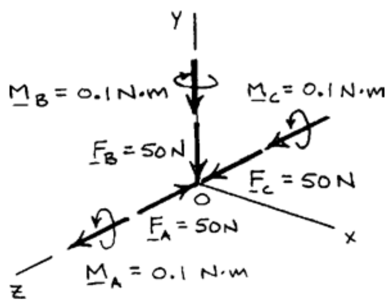
$\therefore \quad z = 0.041667 \text{ m}$

From **k**: $0 = -21x$

$\therefore \quad z = 0$

$\therefore$ The axis of the wrench is parallel to the y-axis and intersects the xz-plane at $x = 0$, $z = 41.7 \text{ mm}$ ◀

Chapter 3, Solution 132.



(*a*) First, reduce the given force system to a force-couple system.

Have $\Sigma\mathbf{F}$: $\quad -(50\text{ N})\mathbf{k} - (50\text{ N})\mathbf{j} + (50\text{ N})\mathbf{k} = \mathbf{R}$

$\mathbf{R} = -(50\text{ N})\mathbf{j}; \qquad R = 50\text{ N}$

$\boldsymbol{R} = -(50.0\text{ N})\mathbf{j}$ ◀

Have

$\Sigma\mathbf{M}_O$: $\quad (0.1\text{ N}\cdot\text{m})\mathbf{k} - (0.1\text{ N}\cdot\text{m})\mathbf{j} + (0.1\text{ N}\cdot\text{m})\mathbf{k} = \mathbf{M}_O^R$

$\mathbf{M}_O^R = -(0.1\text{ N}\cdot\text{m})\mathbf{j} + (0.2\text{ N}\cdot\text{m})\mathbf{k}$

(*b*) Have $\quad M_1 = \boldsymbol{\lambda}_R \times \mathbf{M}_O^R = \dfrac{\mathbf{R}}{R}\cdot\mathbf{M}_O^R$

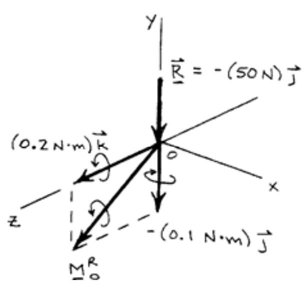
$= -\mathbf{j}\cdot\left[-(0.1\text{ N}\cdot\text{m})\mathbf{j} + (0.2\text{ N}\cdot\text{m})\mathbf{k}\right]$

$= 0.1\text{ N}\cdot\text{m}$

and pitch $\quad P = \dfrac{M_1}{R} = \dfrac{0.1\text{ N}\cdot\text{m}}{50\text{ N}} = 0.002\text{ m}$

or $P = 2.00\text{ mm}$ ◀

(*c*)



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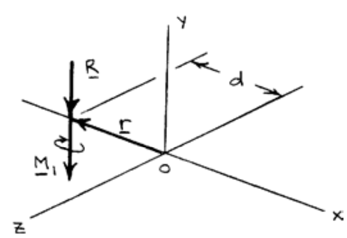


Have $\quad \mathbf{M}_1 = P\mathbf{R} = (0.002\text{ m})\left[-(50\text{ N})\mathbf{j}\right]$

$= -(0.1\text{ N}\cdot\text{m})\mathbf{j}$

Note that because $\mathbf{M}_z = (0.2\ \text{N}\cdot\text{m})\mathbf{k}$, the line of action of the wrench must pass through the x-axis to compensate for $\mathbf{M}_z$ as shown above:

With $\quad \mathbf{M}_1 + (\mathbf{r} \times \mathbf{R}) = \mathbf{M}_O^R$

Then $\quad -(0.1\ \text{N}\cdot\text{m})\mathbf{j} + \left[-(d)\mathbf{i} \times -(50\text{N})\mathbf{j}\right]$

$$= -(0.1\ \text{N}\cdot\text{m})\mathbf{j} + (0.2\ \text{N}\cdot\text{m})\mathbf{k}$$

or $\left[(50\ \text{N})(d)\right]\mathbf{k} = (0.2\ \text{N}\cdot\text{m})\mathbf{k}$

and $\quad d = 0.004\ \text{m}$

$$x = -d = -0.004\ \text{m}$$

or $x = -4.00\ \text{mm}, z = 0$ ◀

Chapter 3, Solution 133.

First replace the given couples with an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^B\right)$ at the origin.

$\Sigma\mathbf{F}$: $\quad \mathbf{R} = -(35\text{ lb})\mathbf{i} - (12\text{ lb})\mathbf{k}$

$\Sigma\mathbf{M}_O$: $\quad \mathbf{M}_O^R = -(200\text{ lb}\cdot\text{in.})\mathbf{i} + \left[(8\text{ in.})\mathbf{j} + (8\text{ in.})\mathbf{k}\right]\times\left[-(35\text{ lb})\mathbf{i}\right]$

$$-(140\text{ lb}\cdot\text{in.})\mathbf{k} + \left[(10\text{ in.})\mathbf{i} + (4\text{ in.})\mathbf{j}\right]\times\left[-(12\text{ lb})\mathbf{k}\right]$$

$$= (-200 - 48)\mathbf{i} + (-280 + 120)\mathbf{j} + (280 - 140)\mathbf{k}$$

$$= -(248\text{ lb}\cdot\text{in.})\mathbf{i} - (1600\text{ lb}\cdot\text{in.})\mathbf{j} + (140\text{ lb}\cdot\text{in.})\mathbf{k}$$

Now $\quad R = \sqrt{(-35\text{ lb})^2 + (-12\text{ lb})^2}$

$$= 37\text{ lb}$$

Then $\quad \boldsymbol{\lambda}_{\text{axis}} = \frac{1}{37}(-35\mathbf{i} - 12\mathbf{k})$

(*a*) $\quad \mathbf{R} = -(35.0\text{ lb})\mathbf{i} - (12.00)\mathbf{k}$ ◀

(*b*) $\quad M_1 = \boldsymbol{\lambda}_{\text{axis}}\cdot\mathbf{M}_O^R$

$$= \frac{1}{37}(-35\mathbf{i} - 12\mathbf{k})\cdot(-248\mathbf{i} - 160\mathbf{j} + 140\mathbf{k})(\text{lb}\cdot\text{in.})$$

$$= \frac{1}{37}(35\times 248 - 12\times 140)(\text{lb}\cdot\text{in.})$$

$$= \frac{7000}{37}\text{ lb}\cdot\text{in.}$$

Then $\quad P = \frac{M_1}{R} = \frac{\frac{7000}{37}\text{ lb}\cdot\text{in.}}{37\text{ lb}}$

or $P = 5.11$ in. ◀

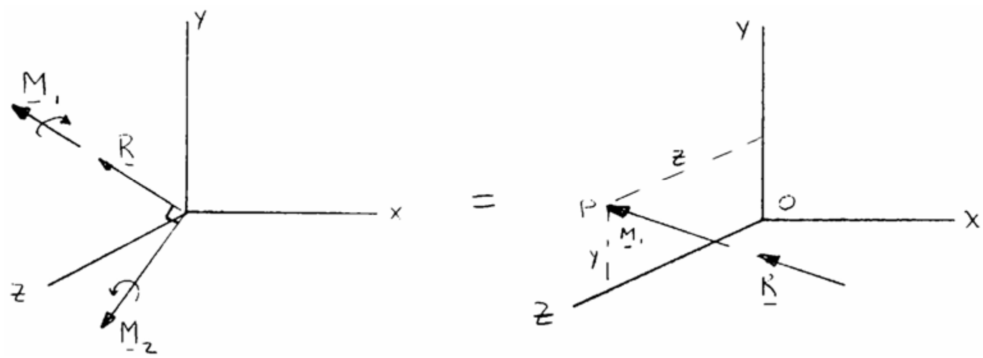
(*c*) Have $\mathbf{M}_1 = \boldsymbol{\lambda}_{\text{axis}} M_1$

$$= \frac{7000 \text{ lb}\cdot\text{in.}}{37^2}\left(-35\mathbf{i} - 12\mathbf{k}\right)$$

Then $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

or $$\mathbf{M}_2 = \left(-248\mathbf{i} - 160\mathbf{j} + 140\mathbf{k}\right) - \frac{7000}{37^2}\left(-35\mathbf{i} - 12\mathbf{k}\right)$$

$$= -\left(69.037 \text{ lb}\cdot\text{in.}\right)\mathbf{i} - \left(160 \text{ lb}\cdot\text{in.}\right)\mathbf{j} + \left(201.36 \text{ lb}\cdot\text{in.}\right)\mathbf{k}$$



Require $\mathbf{M}_z = \mathbf{r}_{O/P} \times \mathbf{R}$

or $$-69.037\mathbf{i} - 160\mathbf{j} + 201.36\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ -35 & 0 & -12 \end{vmatrix}$$

$\mathbf{j}$: $-160 = -35z$

or $z = 4.57$ in.

$\mathbf{k}$: $201.36 = 35y$

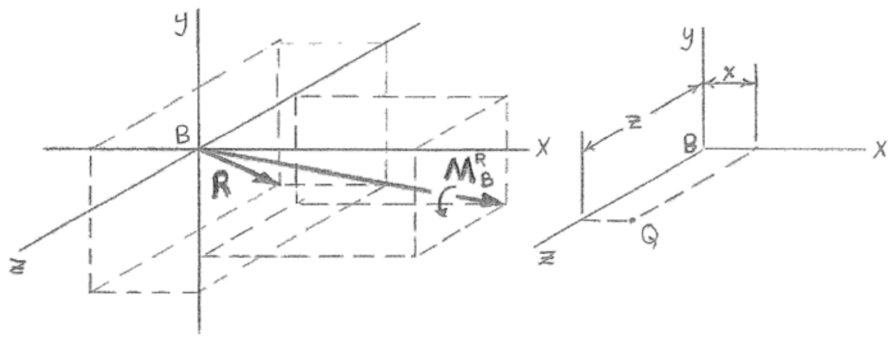
or $y = 5.75$ in.

$\therefore$ The point of intersection is defined by

$y = 5.75$ in. ◀

$z = 4.57$ in. ◀

Chapter 3, Solution 134.



First reduce the given force system to a force-couple at the origin at B.

(*a*) Have $\Sigma\mathbf{F}$: $-(79.2 \text{ lb})\mathbf{k} - (51 \text{ lb})\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = \mathbf{R}$

$$\therefore \quad \mathbf{R} = -(24.0 \text{ lb})\mathbf{i} - (45.0 \text{ lb})\mathbf{j} - (79.2 \text{ lb})\mathbf{k} \blacktriangleleft$$

and

$$R = 94.2 \text{ lb}$$

Have

$$\Sigma\mathbf{M}_B: \quad \mathbf{r}_{A/B} \times \mathbf{F}_A + \mathbf{M}_A + \mathbf{M}_B = \mathbf{M}_B^R$$

$$\mathbf{M}_B^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -20 & 0 \\ 0 & 0 & -79.2 \end{vmatrix} - 660\mathbf{k} - 714\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = 1584\mathbf{i} - 660\mathbf{k} - 42(8\mathbf{i} + 15\mathbf{j})$$

$$\therefore \quad \mathbf{M}_B^R = (1248 \text{ lb}\cdot\text{in.})\mathbf{i} - (630 \text{ lb}\cdot\text{in.})\mathbf{j} - (660 \text{ lb}\cdot\text{in.})\mathbf{k}$$

(*b*) Have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \dfrac{\mathbf{R}}{R}$

$$= \frac{-24.0\mathbf{i} - 45.0\mathbf{j} - 79.2\mathbf{k}}{94.2} \cdot \left[(1248 \text{ lb}\cdot\text{in.})\mathbf{i} - (630 \text{ lb}\cdot\text{in.})\mathbf{j} - (660 \text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$= 537.89 \text{ lb}\cdot\text{in.}$$

and $\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R$

$$= -(137.044\text{ lb}\cdot\text{in.})\mathbf{i} - (256.96\text{ lb}\cdot\text{in.})\mathbf{j} - (452.24\text{ lb}\cdot\text{in.})\mathbf{k}$$

Then pitch $p = \dfrac{M_1}{R} = \dfrac{537.89\text{ lb}\cdot\text{in.}}{94.2\text{ lb}} = 5.7101\text{ in.}$

or $p = 5.71\text{ in.}$ ◀

(*c*) Have $\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\therefore\ \mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (1248\mathbf{i} - 630\mathbf{j} - 660\mathbf{k}) - (-137.044\mathbf{i} - 256.96\mathbf{j} - 452.24\mathbf{k})$$

$$= (1385.04\text{ lb}\cdot\text{in.})\mathbf{i} - (373.04\text{ lb}\cdot\text{in.})\mathbf{j} - (207.76\text{ lb}\cdot\text{in.})\mathbf{k}$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/B} \times \mathbf{R}$

$$1385.04\mathbf{i} - 373.04\mathbf{j} - 207.76\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -24 & -45 & -79.2 \end{vmatrix}$$

$$= (45z)\mathbf{i} - (24z)\mathbf{j} + (79.2x)\mathbf{j} - (45x)\mathbf{k}$$

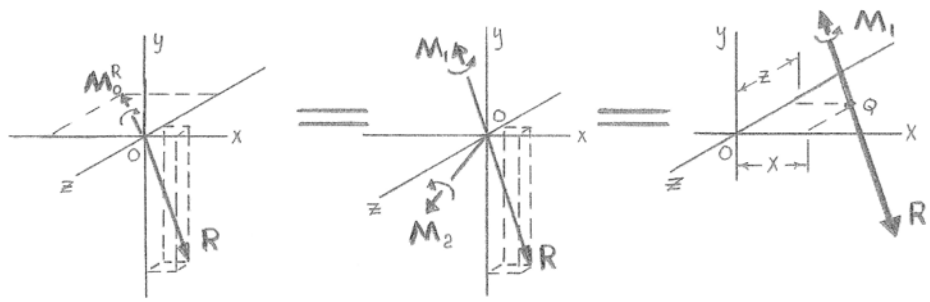
From **i**: $1385.04 = 45z \qquad \therefore\ z = 30.779\text{ in.}$

From **k**: $-207.76 = -45x \qquad \therefore\ x = 4.6169\text{ in.}$

$\therefore$ The axis of the wrench intersects the *xz*-plane at

$x = 4.62\text{ in.},\ z = 30.8\text{ in.}$ ◀

Chapter 3, Solution 135.



(*a*) First reduce the given force system to a force-couple at the origin.

Have $\Sigma\mathbf{F}$: $P\lambda_{BA} + P\lambda_{DC} + P\lambda_{DE} = \mathbf{R}$

$$\text{R} = P\left[\left(\frac{4}{5}\text{j} - \frac{3}{5}\text{k}\right) + \left(\frac{3}{5}\text{i} - \frac{4}{5}\text{j}\right) + \left(\frac{-9}{25}\text{i} - \frac{4}{5}\text{j} + \frac{12}{25}\text{k}\right)\right]$$

$$\therefore \quad \mathbf{R} = \frac{3P}{25}\left(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}\right) \blacktriangleleft$$

$$R = \frac{3P}{25}\sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25}P$$

Have

$$\Sigma\mathbf{M}: \quad \Sigma\left(\mathbf{r}_O \times P\right) = \mathbf{M}_O^R$$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5}\mathbf{j} - \frac{3P}{5}\mathbf{k}\right) + (20a)\mathbf{j} \times \left(\frac{3P}{5}\mathbf{i} - \frac{4P}{5}\mathbf{j}\right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25}\mathbf{i} - \frac{4P}{5}\mathbf{j} + \frac{12P}{25}\mathbf{k}\right) = \mathbf{M}_O^R$$

$$\therefore \quad \mathbf{M}_O^R = \frac{24Pa}{5}\left(-\mathbf{i} - \mathbf{k}\right)$$

(*b*) Have

$$M_1 = \lambda_R \cdot \mathbf{M}_O^R$$

$$\text{where } \lambda_R = \frac{\mathbf{R}}{R} = \frac{3P}{25}\left(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}\right)\frac{25}{27\sqrt{5}P} = \frac{1}{9\sqrt{5}}\left(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}\right)$$

Then $M_1 = \frac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \frac{-8Pa}{15\sqrt{5}}$

and pitch $p = \frac{M_1}{R} = \frac{-8Pa}{15\sqrt{5}}\left(\frac{25}{27\sqrt{5}P}\right) = \frac{-8a}{81}$ or $p = -0.0988a$ ◀

(*c*)

$$\mathbf{M}_1 = M_1\lambda_R = \frac{-8Pa}{15\sqrt{5}}\left(\frac{1}{9\sqrt{5}}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$$

Then

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \frac{8Pa}{675}(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$\left(\frac{8Pa}{675}\right)(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

$$= \left(\frac{3P}{25}\right)\left[20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}\right]$$

From **i**: $8(-403)\frac{Pa}{675} = 20z\left(\frac{3P}{25}\right)$ $\quad\therefore\ z = -1.99012a$

From **k**: $8(-406)\frac{Pa}{675} = -20x\left(\frac{3P}{25}\right)$ $\quad\therefore\ x = 2.0049a$

$\therefore$ The axis of the wrench intersects the *xz*-plane at

$x = 2.00a$, $z = -1.990a$ ◀

Chapter 3, Solution 136.

First reduce the given force-couple system to an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_B)$ at point B.

$$d_{BD} = \sqrt{(-480 \text{ mm})^2 + (560 \text{ mm})^2 + (-480 \text{ mm})^2}$$

$$= 880 \text{ mm}$$

$$\mathbf{F}_{BD} = F_{BD}\lambda_{BD} = \frac{132 \text{ N}}{880}(-480\mathbf{i} + 560\mathbf{j} - 480\mathbf{k})$$

$$= (12 \text{ N})(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$d_{EB} = \sqrt{(240 \text{ mm})^2 + (-220 \text{ mm})^2 + (480 \text{ mm})^2}$$

$$= 580 \text{ mm}$$

$$\mathbf{F}_{EB} = F_{EB}\lambda_{EB} = \frac{145 \text{ N}}{580}(240\mathbf{i} - 220\mathbf{j} + 480\mathbf{k})$$

$$= (5 \text{ N})(12\mathbf{i} - 11\mathbf{j} + 24\mathbf{k})$$

$\Sigma\mathbf{F}$: $\quad \mathbf{R} = \mathbf{F}_{BD} + \mathbf{F}_{EB}$

$$= (12 \text{ N})(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 5 \text{ N}(12\mathbf{i} - 11\mathbf{j} + 24\mathbf{k})$$

$$= -(12 \text{ N})\mathbf{i} + (29 \text{ N})\mathbf{j} + (48 \text{ N})\mathbf{k}$$

$$d_{BF} = \sqrt{(340 \text{ mm})^2 + (240 \text{ mm})^2 + (-60 \text{ mm})^2}$$

$$= 20\sqrt{442} \text{ mm}$$

Then

$$\mathbf{M}_B = \frac{20 \text{ N}\cdot\text{m}}{20\sqrt{442}}(340\mathbf{i} + 240\mathbf{j} - 60\mathbf{k})$$

$$= \frac{20 \text{ N}\cdot\text{m}}{\sqrt{442}}(17\mathbf{i} + 12\mathbf{j} - 3\mathbf{k})$$

Now determine whether **R** and $\mathbf{M}_B$ are perpendicular

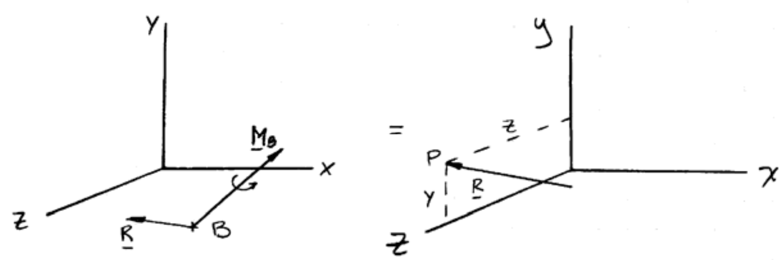
$$\mathbf{R}\cdot\mathbf{M}_B = \left(-12\mathbf{j} + 29\mathbf{j} + 48\mathbf{k}\right)\cdot\frac{20}{\sqrt{442}}\left(17\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\right)$$

$$= \frac{20}{\sqrt{442}}\left(-12\times 17 + 29\times 12 - 48\times 3\right)$$

$$= 0$$

$\therefore$ **R** and $\mathbf{M}_B$ are perpendicular so that $\left(\mathbf{R}, \mathbf{M}_B\right)$ can be reduced to the single equivalent force

$$R = -\left(12.00\text{ N}\right)\mathbf{i} + \left(29.0\text{ N}\right)\mathbf{j} + \left(48.0\text{ N}\right)\mathbf{k} \blacktriangleleft$$



Now require $\mathbf{M}_B$ $\;{}_{B/P}\times\mathbf{R}$

$$\text{or}\qquad \frac{20\text{ N}\cdot\text{m}}{\sqrt{442}}\left(17\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\right) = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -0.480 & y & z - 0.480\\ -12 & 29 & 48\end{vmatrix}\left(\text{N}\cdot\text{m}\right)$$

j: $\quad \dfrac{20\times 12}{\sqrt{442}} = -12\left(z - 0.480\right) + 0.480\left(48\right)$

or $\quad z = 1.449$ m

k: $\quad \dfrac{-20\times 3}{\sqrt{442}} = -0.480\left(29\right) + 12y$

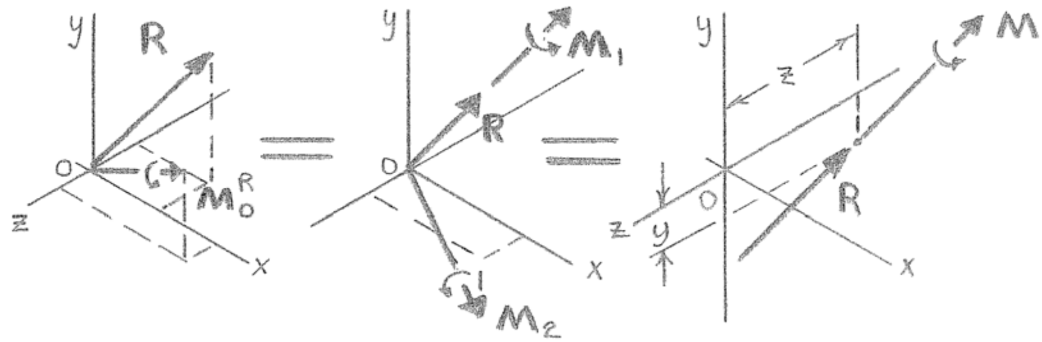
or $\quad y = 0.922$ m

$\therefore$ The point of intersection is defined by

$$y = 0.922\text{ m} \blacktriangleleft$$

$$z = 1.449\text{ m} \blacktriangleleft$$

Chapter 3, Solution 137.



First, reduce the given force system to a force-couple at the origin.

Have $$\Sigma \mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = (10 \text{ lb})\mathbf{k} + 14 \text{ lb}\left[\frac{(4 \text{ in.})\mathbf{i} + (6 \text{ in.})\mathbf{j} - (12 \text{ in.})\mathbf{k}}{14 \text{ in.}}\right] = (4 \text{ lb})\mathbf{i} + (6 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k} \blacktriangleleft$$

and $$R = \sqrt{56} \text{ lb}$$

Have $$\Sigma \mathbf{M}_O: \quad \sum(\mathbf{r}_O \times \mathbf{F}) + \sum \mathbf{M}_C = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \left[(12 \text{ in.})\mathbf{j} \times (10 \text{ lb})\mathbf{k}\right] + \left\{(16 \text{ in.})\mathbf{i} \times \left[(4 \text{ lb})\mathbf{i} + (6 \text{ lb})\mathbf{j} - (12 \text{ lb})\mathbf{k}\right]\right\}$$

$$+ (84 \text{ lb}\cdot\text{in.})\left[\frac{(16 \text{ in.})\mathbf{i} - (12 \text{ in.})\mathbf{j}}{20 \text{ in.}}\right] + (\ 120 \text{ lb}\cdot\text{in.})\left[\frac{(4 \text{ in.})\mathbf{i} - (12 \text{ in.})\mathbf{j} + (6 \text{ in.})\mathbf{k}}{14 \text{ in.}}\right]$$

$$\therefore \quad \mathbf{M}_0^R = (221.49 \text{ lb}\cdot\text{in.})\mathbf{i} + (38.743 \text{ lb}\cdot\text{in.})\mathbf{j} + (147.429 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$= (18.4572 \text{ lb}\cdot\text{ft})\mathbf{i} + (3.2286 \text{ lb}\cdot\text{ft})\mathbf{j} + (12.2858 \text{ lb}\cdot\text{ft})\mathbf{k}$$

The force-couple at O can be replaced by a single force if the direction of $\mathbf{R}$ is perpendicular to $\mathbf{M}_O^R$.

To be perpendicular $\mathbf{R}\cdot\mathbf{M}_O^R = 0$

Have $$\mathbf{R}\cdot\mathbf{M}_O^R = (4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})\cdot(18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k}) = 0?$$

$$= 73.829 + 19.3716 - 24.572$$

$$\neq 0$$

$\therefore$ System cannot be reduced to a single equivalent force.

To reduce to an equivalent wrench, the moment component along the line of action of $\mathbf{P}$ is found.

$$M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \frac{\mathbf{R}}{R}$$

$$= \left[\frac{(4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})}{\sqrt{56}}\right]\cdot(18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k})$$

$$= 9.1709 \text{ lb}\cdot\text{ft}$$

and $$\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R = (9.1709 \text{ lb}\cdot\text{ft})(0.53452\mathbf{i} + 0.80178\mathbf{j} - 0.26726\mathbf{k})$$

And pitch $$p = \frac{M_1}{R} = \frac{9.1709 \text{ lb}\cdot\text{ft}}{\sqrt{56} \text{ lb}} = 1.22551 \text{ ft}$$

or $p = 1.226$ ft ◀

Have

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k}) - (9.1709)(0.53452\mathbf{i} + 0.80178\mathbf{j} - 0.26726\mathbf{k})$$

$$= (13.5552 \text{ lb}\cdot\text{ft})\mathbf{i} - (4.1244 \text{ lb}\cdot\text{ft})\mathbf{j} + (14.7368 \text{ lb}\cdot\text{ft})\mathbf{k}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(13.5552\mathbf{i} - 4.1244\mathbf{j} + 14.7368\mathbf{k}) = (y\mathbf{j} + z\mathbf{k}) \times (4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$$

$$= -(2y + 6z)\mathbf{i} + (4z)\mathbf{j} - (4y)\mathbf{k}$$

From $\mathbf{j}$: $\quad -4.1244 = 4z \quad$ or $\quad z = -1.0311$ ft

From $\mathbf{k}$: $\quad 14.7368 = -4y \quad$ or $\quad y = -3.6842$ ft

$\therefore$ line of action of the wrench intersects the yz plane at

$y = -3.68$ ft, $z = 1.031$ ft ◀

Chapter 3, Solution 138.

Define $\mathbf{F}_A = (F_A)_x \mathbf{i} + (F_A)_y \mathbf{j}$

$\mathbf{F}_B = (F_B)_x \mathbf{i} + (F_B)_y \mathbf{j}$

Then ΣF_x: $(F_A)_x + (F_B)_x = 0$

$$(F_A)_x = -(F_B)_x$$

ΣF_y: $(F_A)_y + (F_B)_y = R$

$$(F_A)_y = R - (F_B)_y$$

and $\Sigma \mathbf{M}_A$: $b\mathbf{k} \times \left[(F_B)_x \mathbf{i} + (F_B)_y \mathbf{j}\right] = -a\mathbf{k} + R\mathbf{j} + M\mathbf{j}$

i: $-b(F_B)_y = aR$

or $(F_B)_y = -\dfrac{a}{b}R$

Then $(F_A)_y = R - \left(-\dfrac{a}{b}R\right)$

$$= R\left(1 + \frac{a}{b}\right)$$

j: $b(F_B)_x = M$

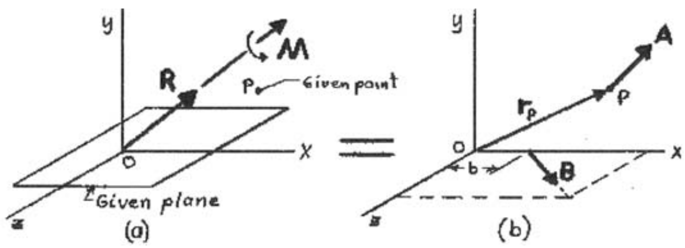
or $(F_B)_x = \dfrac{M}{b}$

Then $(F_A)_x = -\dfrac{M}{b}$

$$\therefore \quad \mathbf{F}_A = -\frac{M}{b}\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{j} \blacktriangleleft$$

$$\mathbf{F}_B = \frac{M}{b}\mathbf{i} - \frac{a}{b}R\mathbf{j} \blacktriangleleft$$

Chapter 3, Solution 139.



First, choose a coordinate system so that the *xy* plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (**R**, **M**) of the wrench are known, it follows that the scalar components of **R** and **M** are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let A be the force passing through the given point *P* and **B** be the force that lies in the given plane. Let *b* be the *x*-axis intercept of **B**.

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \quad \text{and} \quad M = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

while the unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Since the position vector of point *P* is given, it follows that the scalar components (*x*, *y*, *z*) of the position vector $\mathbf{r}_P$ are also known. p(x,y,z)

Then, for equivalence of the two systems

$$\Sigma F_x: \quad R_x = A_x + B_x \qquad (1)$$

$$\Sigma F_y: \quad R_y = A_y \qquad (2)$$

$$\Sigma F_z: \quad R_z = A_z + B_z \qquad (3)$$

$$\Sigma M_x: \quad M_x = yA_z - zA_y \qquad (4)$$

$$\Sigma M_y: \quad M_y = zA_x - xA_z - bB_z \qquad (5)$$

$$\Sigma M_z: \quad M_z = xA_y - yA_x \qquad (6)$$

continued

Based on the above six independent equations for the six unknowns $(A_x, A_y, A_z, B_x, B_z, b)$, there exists a unique solution for **A** and **B**.

From Equation (2) $$A_y = R_y \blacktriangleleft$$

Equation (6) $$A_x = \left(\frac{1}{y}\right)\left(xR_y - M_z\right) \blacktriangleleft$$

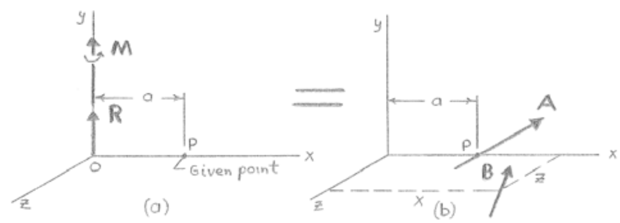
Equation (1) $$B_x = R_x - \left(\frac{1}{y}\right)\left(xR_y - M_z\right) \blacktriangleleft$$

Equation (4) $$A_z = \left(\frac{1}{y}\right)\left(M_x + zR_y\right) \blacktriangleleft$$

Equation (3) $$B_z = R_z - \left(\frac{1}{y}\right)\left(M_x + zR_y\right) \blacktriangleleft$$

Equation (5) $$b = \frac{\left(xM_x + yM_y + zM_z\right)}{\left(M_x - yR_z + zR_y\right)} \blacktriangleleft$$

Chapter 3, Solution 140.



First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures *a* and *b*.

Have $\mathbf{R} = R\mathbf{j}$ and $\mathbf{M} = M\mathbf{j}$ and are known.

The unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The distance *a* is known. It is assumed that force **B** intersects the *xz* plane at (*x*, 0, *z*). Then for equivalence

$$\Sigma F_x: \quad 0 = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA_z - xB_z + zB_x \tag{5}$$

$$\Sigma M_z: \quad 0 = aA_y + xB_y \tag{6}$$

Since **A** and **B** are made perpendicular,

$$\mathbf{A}\cdot\mathbf{B} = 0 \quad \text{or} \quad A_xB_x + A_yB_y + A_zB_z = 0 \tag{7}$$

There are eight unknowns: $A_x, A_y, A_z, B_x, B_y, B_z, x, z$

But only seven independent equations. Therefore, *there exists an infinite number of solutions.*

continued

Next consider Equation (4): $0 = -zB_y$

If $B_y = 0$, Equation (7) becomes $A_xB_x + A_zB_z = 0$

Using Equations (1) and (3) this equation becomes $A_x^2 + A_z^2 = 0$

Since the components of **A** must be real, a nontrivial solution is not possible. Thus, it is required that $B_y \neq 0$, so that from Equation (4), $z = 0$.

To obtain one possible solution, arbitrarily let $A_x = 0$.

(Note: Setting A_y, A_z, or B_z equal to zero results in unacceptable solutions.)

The defining equations then become.

$$0 = B_x \qquad (1)'$$

$$R = A_y + B_y \qquad (2)$$

$$0 = A_z + B_z \qquad (3)$$

$$M = -aA_z - xB_z \qquad (5)'$$

$$0 = aA_y + xB_y \qquad (6)$$

$$A_yB_y + A_zB_z = 0 \qquad (7)'$$

Then Equation (2) can be written $A_y = R - B_y$

Equation (3) can be written $B_z = -A_z$

Equation (6) can be written $x = -\dfrac{aA_y}{B_y}$

Substituting into Equation (5)′,

$$M = -aA_z - \left(-a\frac{R - B_y}{B_y}\right)\left(-A_z\right)$$

or

$$A_z = -\frac{M}{aR}B_y \qquad (8)$$

Substituting into Equation (7)′,

$$\left(R - B_y\right)B_y + \left(-\frac{M}{aR}B_y\right)\left(\frac{M}{aR}B_y\right) = 0$$

or
$$B_y = \frac{a^2R^3}{a^2R^2 + M^2}$$

Then from Equations (2), (8), and (3)

$$A_y = R - \frac{a^2R^3}{a^2R^2 + M^2} = \frac{RM^2}{a^2R^2 + M^2}$$

$$A_z = -\frac{M}{aR}\left(\frac{a^2R^3}{a^2R^2 + M^2}\right) = -\frac{aR^2M}{a^2R^2 + M^2}$$

$$B_z = \frac{aR^2M}{a^2R^2 + M^2}$$

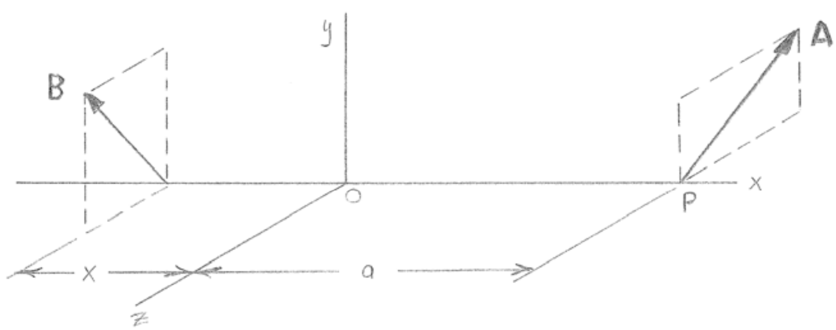
In summary

$$\mathbf{A} = \frac{RM}{a^2R^2 + M^2}\left(M\mathbf{j} - aR\mathbf{k}\right)$$

$$\mathbf{B} = \frac{aR^2}{a^2R^2 + M^2}\left(aR\mathbf{j} + M\mathbf{k}\right)$$

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

Lastly, if $R > 0$ and $M > 0$, it follows from the equations found for **A** and **B** that $A_y > 0$ and $B_y > 0$. From Equation (6), $x < 0$ (assuming $a > 0$). Then, as a consequence of letting $A_x = 0$, force **A** lies in a plane parallel to the *yz* plane and to the right of the origin, while force **B** lies in a plane parallel to the *yz* plane but to the left of the origin, as shown in the figure below.



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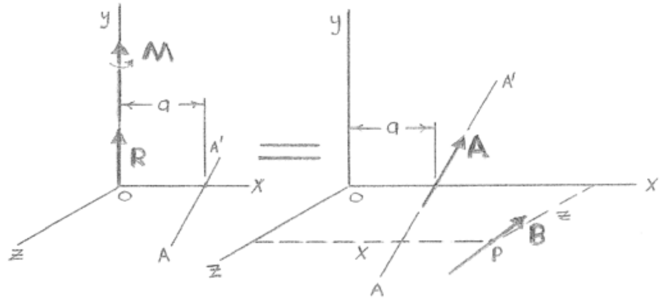


First, choose a rectangular coordinate system where one axis coincides with the axis of the wrench and another axis intersects the prescribed line of action (AA'). Note that it has been assumed that the line of action of force **B** intersects the *xz* plane at point $P(x,\ 0, z)$. Denoting the known direction of line AA' by

$$\boldsymbol{\lambda}_A = \lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}$$

it follows that force **A** can be expressed as

$$\mathbf{A} = A\boldsymbol{\lambda}_A = A\left(\lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}\right)$$

Force **B** can be expressed as

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Next, observe that since the axis of the wrench and the prescribed line of action AA' are known, it follows that the distance *a* can be determined. In the following solution, it is assumed that *a* is known.

Then, for equivalence

$$\Sigma F_x: \quad 0 = A\lambda_x + B_x \ (1)$$

$$\Sigma F_y: \quad R = A\lambda_y + B_y \ (2)$$

$$\Sigma F_z: \quad 0 = A\lambda_z + B_z \ (3)$$

$$\Sigma M_x: \quad 0 = -zB_y \qquad (4)$$

$$\Sigma M_y: \quad M = -aA\lambda_z + zB_x - xB_z \qquad (5)$$

$$\Sigma M_z: \quad 0 = aA\lambda_y + xB_y \qquad (6)$$

Since there are six unknowns $\left(A,\ B_x,\ B_y,\ B_z,\ x,\ z\right)$ and six independent equations, it will be possible to obtain a solution.

Case 1: Let $z = 0$ to satisfy Equation (4)

Now Equation (2) $$A\lambda_y = R - B_y$$

Equation (3) $$B_z = -A\lambda_z$$

Equation (6) $$x = -\frac{aA\lambda_y}{B_y} = -\left(\frac{a}{B_y}\right)\left(R - B_y\right)$$

Substitution into Equation (5)

$$M = -aA\lambda_z - \left[-\left(\frac{a}{B_y}\right)\left(R - B_y\right)\left(-A\lambda_z\right)\right]$$

$$\therefore \quad A = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y$$

Substitution into Equation (2)

$$R = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y\lambda_y + B_y$$

$$\therefore \quad B_y = \frac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}$$

Then $$A = -\frac{MR}{\lambda_z aR - \lambda_y M} = \frac{R}{\lambda_y - \frac{aR}{M}\lambda_z}$$

$$B_x = -A\lambda_x = \frac{\lambda_x MR}{\lambda_z aR - \lambda_y M}$$

$$B_z = -A\lambda_z = \frac{\lambda_z MR}{\lambda_z aR - \lambda_y M}$$

In summary

$$\mathbf{A} = \frac{P}{\lambda_y - \frac{aR}{M}\lambda_z}\boldsymbol{\lambda}_A \blacktriangleleft$$

$$\mathbf{B} = \frac{R}{\lambda_z aR - \lambda_y M}\left(\lambda_x M\mathbf{i} + \lambda_z aR\mathbf{j} + \lambda_z M\mathbf{k}\right) \blacktriangleleft$$

and $$x = a\left(1 - \frac{R}{B_y}\right) = a\left[1 - R\left(\frac{\lambda_z aR - \lambda_y M}{\lambda_z aR^2}\right)\right]$$

or $$x = \frac{\lambda_y}{\lambda_z}\frac{M}{R} \blacktriangleleft$$

Note that for this case, the lines of action of both **A** and **B** intersect the x axis.

continued

Case 2: Let $B_y = 0$ to satisfy Equation (4)

Now Equation (2) $$A = \frac{R}{\lambda_y}$$

Equation (1) $$B_x = -R\left(\frac{\lambda_x}{\lambda_y}\right)$$

Equation (3) $$B_z = -R\left(\frac{\lambda_z}{\lambda_y}\right)$$

Equation (6) $aA\lambda_y = 0$ which requires $a = 0$

Substitution into Equation (5)

$$M = z\left[-R\left(\frac{\lambda_x}{\lambda_y}\right)\right] - x\left[-R\left(\frac{\lambda_z}{\lambda_y}\right)\right] \quad \text{or} \quad \lambda_z x - \lambda_x z = \left(\frac{M}{R}\right)\lambda_y$$

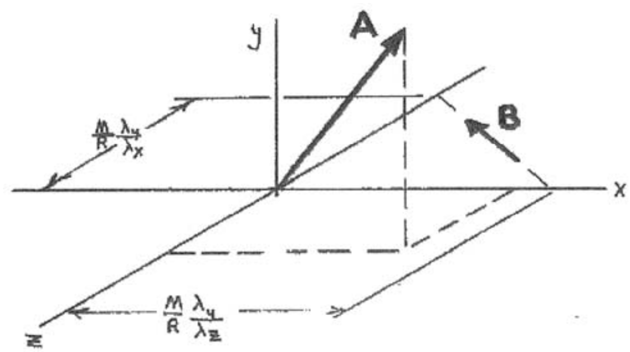
This last expression is the equation for the line of action of force **B**.

In summary

$$\mathbf{A} = \left(\frac{R}{\lambda_y}\right)\boldsymbol{\lambda}_A$$

$$\mathbf{B} = \left(\frac{R}{\lambda_y}\right)\left(-\lambda_x\mathbf{i} - \lambda_z\mathbf{k}\right)$$

Assuming that $\lambda_x,\ \lambda_y,\ \lambda_z > 0,$ the equivalent force system is as shown below.



Note that the component of **A** in the *xz* plane is parallel to **B**.

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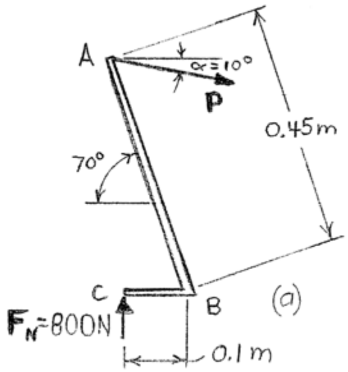


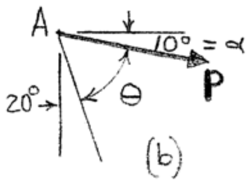
(*a*) Have

$$M_B = r_{C/B}F_N$$

$$= (0.1\text{ m})(800\text{ N})$$

$$= 80.0\text{ N}\cdot\text{m}$$

or $\mathbf{M}_B = 80.0\text{ N}\cdot\text{m}$ ↻ ◀



(*b*) By definition

$$M_B = r_{A/B}P\sin\theta$$

where

$$\theta = 90° - (90° - 70°) - \alpha$$

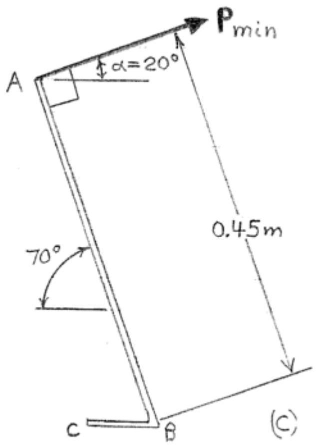
$$= 90° - 20° - 10°$$

$$= 60°$$

$$\therefore\quad 80.0\text{ N}\cdot\text{m} = (0.45\text{ m})P\sin 60°$$

$$P = 205.28\text{ N}$$

or $P = 205\text{ N}$ ◀



(*c*) For **P** to be minimum, it must be perpendicular to the line joining points *A* and *B*. Thus, **P** must be directed as shown.

Thus

$$M_B = dP_{\min} = r_{A/B}P_{\min}$$

or

$$80.0\text{ N}\cdot\text{m} = (0.45\text{ m})P_{\min}$$

$$\therefore\quad P_{\min} = 177.778\text{ N}$$

or $\mathbf{P}_{\min} = 177.8\text{ N} \measuredangle 20°$ ◀

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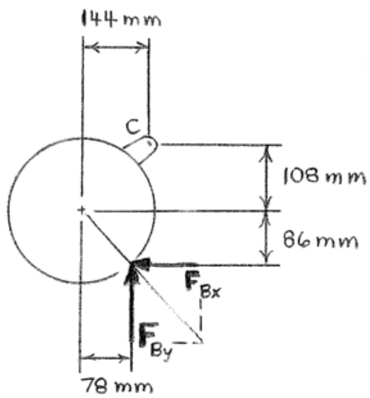

Have $$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$$

Noting the direction of the moment of each force component about C is clockwise,

$$M_C = xF_{By} + yF_{Bx}$$

where $$x = 144 \text{ mm} - 78 \text{ mm} = 66 \text{ mm}$$

$$y = 86 \text{ mm} + 108 \text{ mm} = 194 \text{ mm}$$

and $$F_{Bx} = \frac{78}{\sqrt{(78)^2 + (86)^2}}(580 \text{ N}) = 389.65 \text{ N}$$

$$F_{By} = \frac{86}{\sqrt{(78)^2 + (86)^2}}(580 \text{ N}) = 429.62 \text{ N}$$

$$\therefore \quad M_C = (66 \text{ mm})(429.62 \text{ N}) + (194 \text{ mm})(389.65 \text{ N})$$

$$= 103947 \text{ N}\cdot\text{mm}$$

$$= 103.947 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 103.9 \text{ N}\cdot\text{m}$ ↻ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

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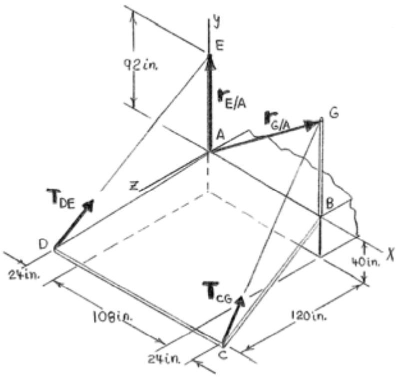

(*a*) Have $$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$$

where $$\mathbf{r}_{E/A} = (92 \text{ in.})\mathbf{j}$$

$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE}$$

$$= \frac{(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= (48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.} = -(22,080 \text{ lb}\cdot\text{in.})\mathbf{i} - (4416 \text{ lb}\cdot\text{in})\mathbf{k}$$

or $$\mathbf{M}_A = -(1840 \text{ lb}\cdot\text{ft})\mathbf{i} - (368 \text{ lb}\cdot\text{ft})\mathbf{k}$$ ◀

(*b*) Have $$\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$$

where $$\mathbf{r}_{G/A} = (108 \text{ in.})\mathbf{i} + (92 \text{ in.})\mathbf{j}$$

$$\mathbf{T}_{CG} = \lambda_{CG} T_{CG} = \frac{-(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= -(48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 108 & 92 & 0 \\ -48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= -(22,080 \text{ lb}\cdot\text{in.})\mathbf{i} + (25,920 \text{ lb}\cdot\text{in.})\mathbf{j} + (32,928 \text{ lb}\cdot\text{in.})\mathbf{k}$$

or $$\mathbf{M}_A = -(1840 \text{ lb}\cdot\text{ft})\mathbf{i} + (2160 \text{ lb}\cdot\text{ft})\mathbf{j} + (2740 \text{ lb}\cdot\text{ft})\mathbf{k}$$ ◀

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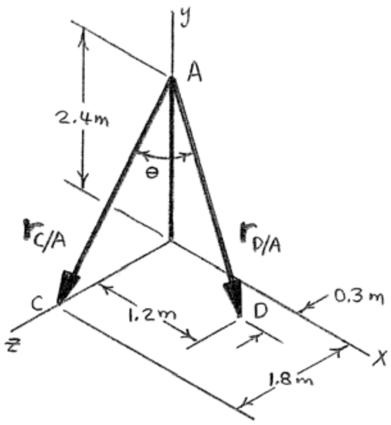

First note

$$AC = \left|\mathbf{r}_{C/A}\right| = \sqrt{(-2.4)^2 + (1.8)^2} \text{ m} = 3 \text{ m}$$

$$AD = \left|\mathbf{r}_{D/A}\right| = \sqrt{(1.2)^2 + (-2.4)^2 + (0.3)^2} \text{ m} = 2.7 \text{ m}$$

and $\mathbf{r}_{C/A} = -(2.4 \text{ m})\mathbf{j} + (1.8 \text{ m})\mathbf{k}$

$$\mathbf{r}_{D/A} = (1.2 \text{ m})\mathbf{i} - (2.4 \text{ m})\mathbf{j} + (0.3 \text{ m})\mathbf{k}$$

By definition

$$\mathbf{r}_{C/A} \cdot \mathbf{r}_{D/A} = \left|\mathbf{r}_{C/A}\right|\left|\mathbf{r}_{D/A}\right|\cos\theta$$

or $(-2.4\mathbf{j} + 1.8\mathbf{k}) \cdot (1.2\mathbf{i} - 2.4\mathbf{j} + 0.3\mathbf{k}) = (3)(2.7)\cos\theta$

$$(0)(1.2) + (-2.4)(-2.4) + (1.8)(0.3) = 8.1\cos\theta$$

and $\cos\theta = \dfrac{6.3}{8.1} = 0.77778$

$$\theta = 38.942°$$

or $\theta = 38.9°$ ◀

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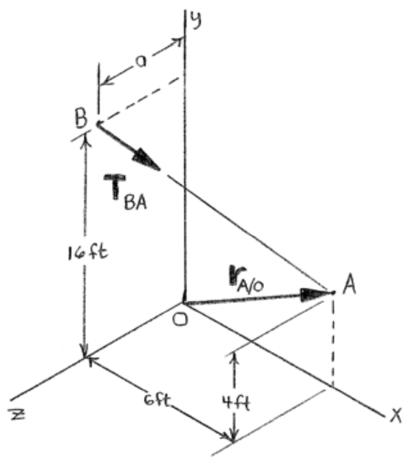

Based on $\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{BA}$

where

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= M_x\mathbf{i} + (100\text{ lb}\cdot\text{ft})\mathbf{j} - (400\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\mathbf{r}_{A/O} = (6\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BA} = \boldsymbol{\lambda}_{BA}T_{BA}$$

$$= \frac{(6\text{ ft})\mathbf{i} - (12\text{ ft})\mathbf{j} - (a)\mathbf{k}}{d_{BA}}T_{BA}$$

$$\therefore\ M_x\mathbf{i} + 100\mathbf{j} - 400\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 4 & 0 \\ 6 & -12 & -a \end{vmatrix} \frac{T_{BA}}{d_{BA}}$$

$$= \frac{T_{BA}}{d_{BA}}\left[-(4a)\mathbf{i} + (6a)\mathbf{j} - (96)\mathbf{k}\right]$$

From **j**-coefficient: $100d_{AB} = 6aT_{BA}$ or $T_{BA} = \dfrac{100}{6a}d_{BA}$ (1)

From **k**-coefficient: $-400d_{AB} = -96T_{BA}$ or $T_{BA} = \dfrac{400}{96}d_{BA}$ (2)

Equating Equations (1) and (2) yields $a = \dfrac{100(96)}{6(400)}$

or $a = 4.00$ ft ◀

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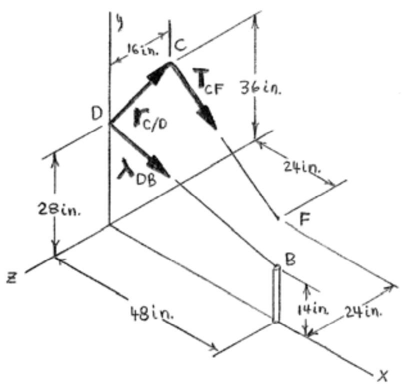

Have $$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CF}\right)$$

where $$\boldsymbol{\lambda}_{DB} = \frac{(48 \text{ in.})\mathbf{i} - (14 \text{ in.})\mathbf{j}}{50 \text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{C/D} = (8 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}$$

$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{(24 \text{ in.})\mathbf{i} - (36 \text{ in.})\mathbf{j} - (8 \text{ in.})\mathbf{k}}{44 \text{ in.}}(132 \text{ lb})$$

$$= (72 \text{ lb})\mathbf{i} - (108 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

$$\therefore\ M_{DB} = \begin{vmatrix} 0.96 & -0.28 & 0 \\ 0 & 8 & -16 \\ 72 & -108 & -24 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= 0.96\left[(8)(-24) - (-16)(-108)\right] + (-0.28)\left[(-16)(72) - 0\right]$$

$$= -1520.64 \text{ lb}\cdot\text{in.}$$

or $M_{DB} = -1521 \text{ lb}\cdot\text{in.}$ ◀

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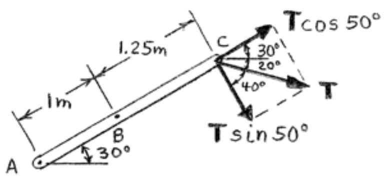


(*a*) Based on $\Sigma F: \quad F_A = T = 1000 \text{ N}$

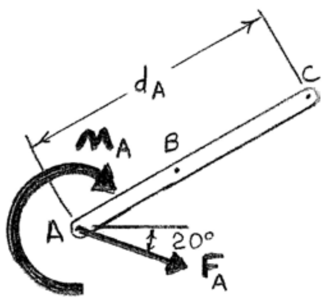
or $\mathbf{F}_A = 1000 \text{ N} \measuredangle 20°$ ◀

$$\Sigma M_A: \quad M_A = (T \sin 50°)(d_A)$$

$$= (1000 \text{ N})\sin 50°(2.25 \text{ m})$$

$$= 1723.60 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 1724 \text{ N}\cdot\text{m}$ ↻ ◀



(*b*) Based on $\Sigma F: \quad F_B = T = 1000 \text{ N}$

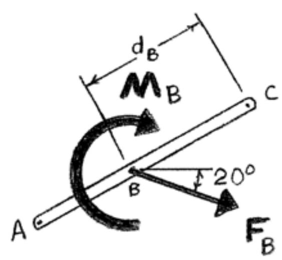
or $\mathbf{F}_B = 1000 \text{ N} \measuredangle 20°$ ◀

$$\Sigma M_B: \quad M_B = (T \sin 50°)(d_B)$$

$$= (1000 \text{ N})\sin 50°(1.25 \text{ m})$$

$$= 957.56 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_B = 958 \text{ N}\cdot\text{m}$ ↻ ◀



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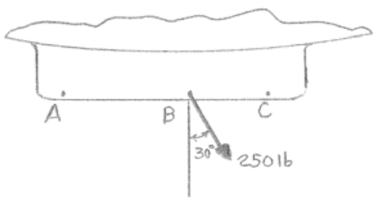
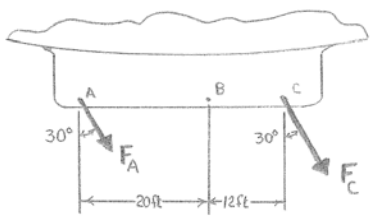




Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (250 \text{ lb})\sin 30^\circ = F_A \sin\alpha + F_B \sin\alpha \qquad (1)$$

$$\Sigma F_y: \quad -(250 \text{ lb})\cos 30^\circ = -F_A \cos\alpha - F_B \cos\alpha \qquad (2)$$

Dividing Equation (1) by Equation (2),

$$\frac{(250 \text{ lb})\sin 30^\circ}{-(250 \text{ lb})\cos 30^\circ} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30^\circ$

Based on

$$\Sigma M_C: \quad \left[(250 \text{ lb})\cos 30^\circ\right](12 \text{ ft}) = (F_A \cos 30^\circ)(32 \text{ ft})$$

$$\therefore \quad F_A = 93.75 \text{ lb}$$

or $\mathbf{F}_A = 93.8 \text{ lb} \searrow 60^\circ$ ◀

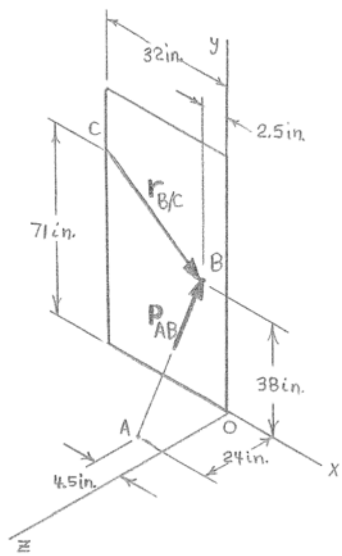
Based on

$$\Sigma M_A: \quad -\left[(250 \text{ lb})\cos 30^\circ\right](20 \text{ ft}) = (F_C \cos 30^\circ)(32 \text{ ft})$$

$$\therefore \quad F_C = 156.25 \text{ lb}$$

or $\mathbf{F}_C = 156.3 \text{ lb} \searrow 60^\circ$ ◀

Chapter 3, Solution 150.



Have

$$\Sigma \mathbf{F}: \quad \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\mathbf{P}_{AB} = \lambda_{AB} P_{AB}$$

$$= \frac{(2.0 \text{ in.})\mathbf{i} + (38 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}}{44.989 \text{ in.}}(45 \text{ lb})$$

$$\text{or } \mathbf{F}_C = (2.00 \text{ lb})\mathbf{i} + (38.0 \text{ lb})\mathbf{j} - (24.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Have

$$\Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$$

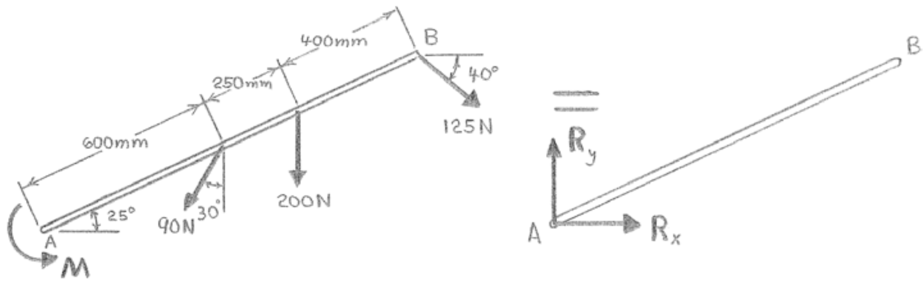
$$\mathbf{M}_C = 2\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 29.5 & -33 & 0 \\ 1 & 19 & -12 \end{vmatrix} \text{ lb}\cdot\text{in.}$$

$$= (2 \text{ lb}\cdot\text{in.})\{(-33)(-12)\mathbf{i} - (29.5)(-12)\mathbf{j}$$

$$+ [(29.5)(19) - (-33)(1)]\mathbf{k}\}$$

$$\text{or } \mathbf{M}_C = (792 \text{ lb}\cdot\text{in.})\mathbf{i} + (708 \text{ lb}\cdot\text{in.})\mathbf{j} + (1187 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 151.



For equivalence

$$\Sigma F_x: \quad -(90\text{ N})\sin 30° + (125\text{ N})\cos 40° = R_x$$

$$\text{or } R_x = 50.756\text{ N}$$

$$\Sigma F_y: \quad -(90\text{ N})\cos 30° - 200\text{ N} - (125\text{ N})\sin 40° = R_y$$

$$\text{or } R_y = -358.29\text{ N}$$

Then $$R = \sqrt{(50.756)^2 + (-358.29)^2} = 361.87\text{ N}$$

and $$\tan\theta = \frac{R_y}{R_x} = \frac{-358.29}{50.756} = -7.0591 \qquad \therefore \quad \theta = -81.937°$$

or $\mathbf{R} = 362\text{ N}$ ◣ $81.9°$ ◀

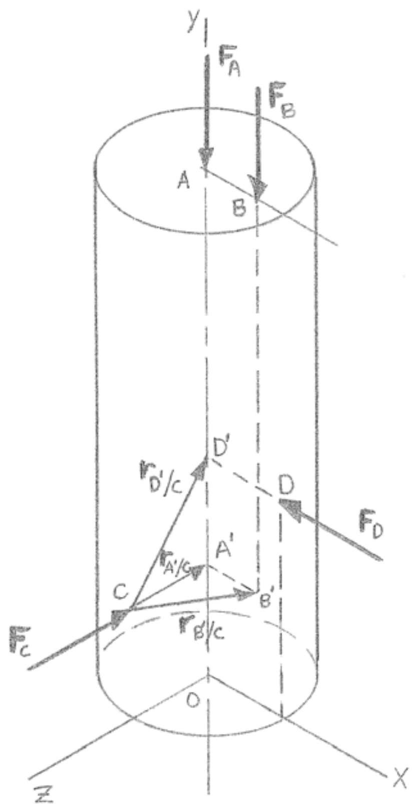
Also

$$\Sigma M_A: \quad M - [(90\text{ N})\sin 35°](0.6\text{ m}) - [(200\text{ N})\cos 25°](0.85\text{ m}) - [(125\text{ N})\sin 65°](1.25\text{ m}) = 0$$

$$\therefore \quad M = 326.66\text{ N}\cdot\text{m}$$

or $M = 327\text{ N}\cdot\text{m}$ ◀

Chapter 3, Solution 152.



For equivalence

$$\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}_C$$

$$\mathbf{R}_C = -(5\text{ lb})\mathbf{j} - (3\text{ lb})\mathbf{j} - (4\text{ lb})\mathbf{k} - (7\text{ lb})\mathbf{i}$$

$$\therefore \quad \mathbf{R}_C = (-7\text{ lb})\mathbf{i} - (8\text{ lb})\mathbf{j} - (4\text{ lb})\mathbf{k} \blacktriangleleft$$

Also for equivalence

$$\Sigma\mathbf{M}_C: \quad \mathbf{r}_{A'/C} \times \mathbf{F}_A + \mathbf{r}_{B'/C} \times \mathbf{F}_B + \mathbf{r}_{D'/C} \times \mathbf{F}_D = \mathbf{M}_C$$

or

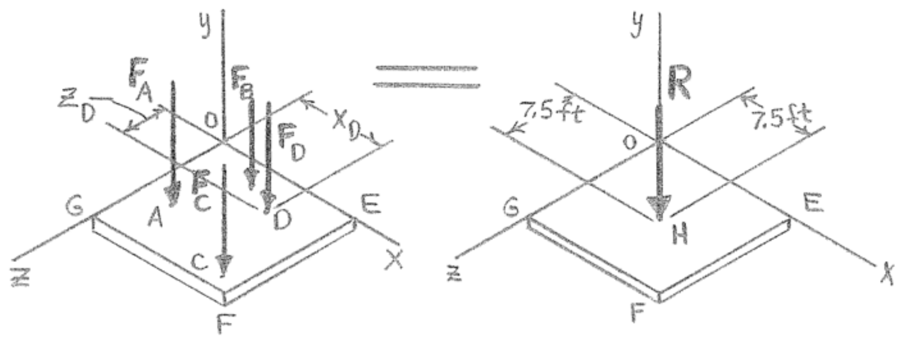
$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1.5\text{ in.} \\ 0 & 5\text{ lb} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1\text{ in.} & 0 & -1.5\text{ in.} \\ 0 & -3\text{ lb} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5\text{ in.} & 1.5\text{ in.} \\ -7\text{ lb} & 0 & 0 \end{vmatrix}$$

$$= \left[(-7.50\text{ lb}\cdot\text{in.} - 0)\mathbf{i}\right] + \left[(0 - 4.50\text{ lb}\cdot\text{in.})\mathbf{i} + (-3.0\text{ lb}\cdot\text{in.} - 0)\mathbf{k}\right]$$

$$+ \left[(10.5\text{ lb}\cdot\text{in.} - 0)\mathbf{j} + (0 + 10.5\text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$\text{or} \quad \mathbf{M}_C = -(12.0\text{ lb}\cdot\text{in.})\mathbf{i} + (10.5\text{ lb}\cdot\text{in.})\mathbf{j} + (7.5\text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

Chapter 3, Solution 153.



Have $$\Sigma\mathbf{F}:\quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}$$

$$-(85\text{ lb})\mathbf{j} - (60\text{ lb})\mathbf{j} - (90\text{ lb})\mathbf{j} - (95\text{ lb})\mathbf{j} = \mathbf{R}$$

$$\therefore\quad \mathbf{R} = -(330\text{ lb})\mathbf{j}$$

Have $$\Sigma M_x:\quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$$

$$(85\text{ lb})(9\text{ ft}) + (60\text{ lb})(1.5\text{ ft}) + (90\text{ lb})(14.25\text{ ft}) + (95\text{ lb})(z_D) = (330\text{ lb})(7.5\text{ ft})$$

$$\therefore\quad z_D = 3.5523\text{ ft}$$ or $z_D = 3.55$ ft ◀

Have $$\Sigma M_z:\quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$$

$$(85\text{ lb})(3\text{ ft}) + (60\text{ lb})(4.5\text{ ft}) + (90\text{ lb})(14.25\text{ ft}) + (95\text{ lb})(x_D) = (330\text{ lb})(7.5\text{ ft})$$

$$\therefore\quad x_D = 7.0263\text{ ft} \text{ or } x_D = 7.03\text{ ft}$$ ◀