CHAPTER 6

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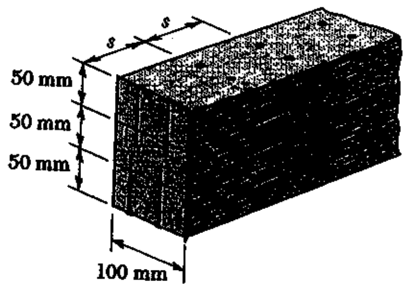
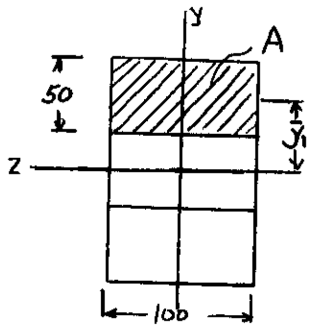
PROBLEM 6.1



6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$
$$= 28.125 \times 10^{-6} \text{ m}^4$$



$$A = (100)(50) = 5000 \text{ mm}^2$$

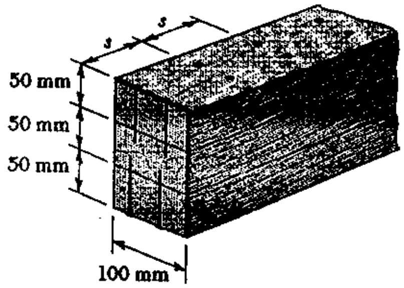
$$\bar{y}_1 = 50 \text{ mm}$$

$$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3$$
$$= 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(1500)(250 \times 10^{-6})}{28.125 \times 10^{-6}} = 13.333 \times 10^3 \text{ N/m}$$

$$q s = 2 F_{nail} \qquad s = \frac{2F_{nail}}{q} = \frac{(2)(400)}{13.333 \times 10^3} = 60 \times 10^{-3} \text{ m}$$
$$= 60 \text{ mm}$$

PROBLEM 6.2

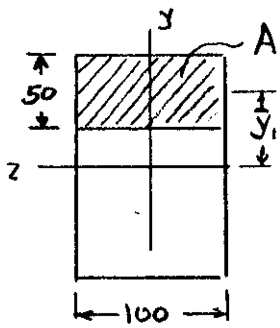


6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing *s* that can be used between each pair of nails.

6.2 For the built-up beam of Prob. 6.1, determine the allowable shear if the spacing between each pair of nails is *s* = 45 mm.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$
$$= 28.125 \times 10^{-6} \text{ m}^4$$



$A = (100)(50) = 5000 \text{ mm}^2$

$\bar{y}_1 = 50 \text{ mm}$

$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$

$q = \frac{VQ}{I}$ $\qquad qs = 2F_{nail}$

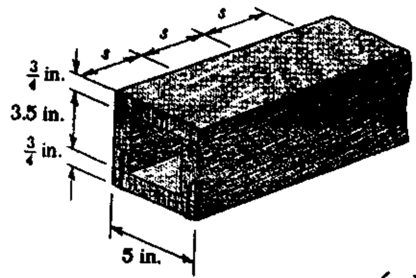
Eliminating q $\qquad \frac{VQ}{I} = \frac{2F_{nail}}{s}$

Solving for V $\qquad V = \frac{2IF_{nail}}{Qs} = \frac{(2)(28.125 \times 10^{-6})(400)}{(250 \times 10^{-6})(45 \times 10^{-3})}$

$= 2 \times 10^3 \text{ N} = 2 \text{ kN}$ ◄

PROBLEM 6.3

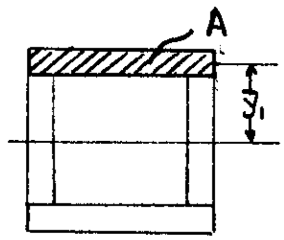
6.3 A square box beam is made of two $\frac{3}{4} \times 3.5$-in. planks and two $\frac{3}{4} \times 5$-in. planks nailed together as shown. Knowing that the spacing between nails is $s = 1.25$ in. and that the vertical shear in the beam is $V = 250$ lb, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.



SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(5)(5)^3 - \frac{1}{12}(3.5)(3.5)^3 = 39.578 \text{ in}^4$$



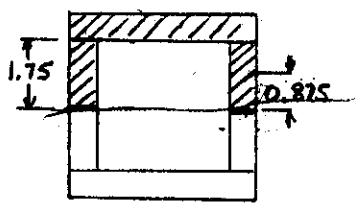
(a) $A = (5)(\frac{3}{4}) = 3.75 \text{ in}^2$

$$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in}$$

$$Q_1 = A\bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$$

$$q = \frac{VQ_1}{I} = \frac{(250)(7.969)}{39.578} = 50.34 \text{ lb/in}$$

$qs = 2F_{nail}$ $\qquad$ $F_{nail} = \frac{qs}{2} = \frac{(50.34)(1.25)}{2} = 31.5 \text{ lb.}$ ◄



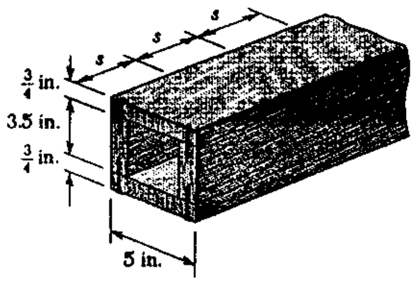
(b) $Q_2 = Q_1 + (2)(1.75)(\frac{3}{4})(0.875)$

$$= 7.969 + 2.297 = 10.266 \text{ in}^3$$

$$t = (2)(\tfrac{3}{4}) = 1.5 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(250)(10.266)}{(39.578)(1.5)} = 43.2 \text{ psi}$$ ◄

PROBLEM 6.4

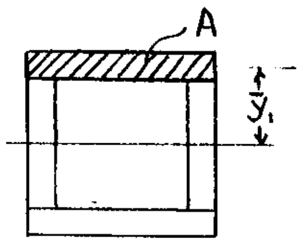


6.4 A square box beam is made of two $\frac{3}{4} \times 3.5$-in. planks and two $\frac{3}{4} \times 5$-in. planks nailed together as shown. Knowing that the spacing between nails is $s = 2$ in. and that the allowable shearing force in each nail is 75 lb, determine (*a*) the largest allowable vertical shear in the beam, (*b*) the corresponding maximum shearing stress in the beam.

SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(5)(5)^3 - \frac{1}{12}(3.5)(3.5)^3 = 39.578 \text{ in}^4$$



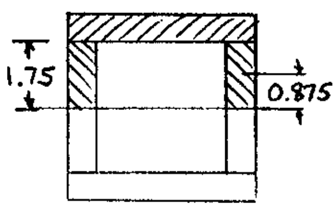
(a) $A = (5)(\frac{3}{4}) = 3.75 \text{ in}^2$

$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in.}$

$Q_1 = A\bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$

$q_{all} = \frac{2F_{nail}}{s} = \frac{(2)(75)}{2} = 75 \text{ lb/in}$

$q = \frac{VQ}{I}$ $\qquad V_{all} = \frac{I q_{all}}{Q_1} = \frac{(39.578)(75)}{7.969} = 372 \text{ lb}$ ◄



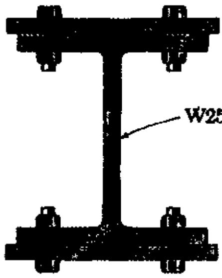
(b) $Q = Q_1 + (2)(1.75)(\frac{3}{4})(0.875)$

$= 7.969 + 2.297 = 10.266 \text{ in}^3$

$t = (2)(\frac{3}{4}) = 1.5 \text{ in}$

$\tau_{max} = \frac{VQ}{It} = \frac{(372)(10.266)}{(39.578)(1.5)} = 64.4 \text{ psi}$ ◄

PROBLEM 6.5 *



6.5 The beam shown has been reinforced by attaching to it two 12 × 175-mm plates, using bolts of 18-mm diameter spaced longitudinally every 125 mm. Knowing that the average allowable shearing stress in the bolts is 85 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia

Part	A (mm²)	d (mm)	Ad² (10⁶ mm⁴)	Ī (10⁶ mm⁴)
Top plate	2100	* 139	40.574	0.025
W250 × 44.8	5720	0	0	71.1
Bot. plate	2100	* 139	40.574	0.025
Σ			81.148	71.15

$* \quad d = \frac{266}{2} + \frac{12}{2} = 139 \text{ mm}$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 152.30 \times 10^6 \text{ mm}^4 = 152.30 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate} d_{plate} = (2100)(139) = 291.9 \times 10^3 \text{ mm}^3 = 291.9 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

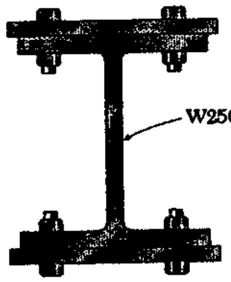
$$F_{bolt} = \tau_{all} A_{bolt} = (85 \times 10^6)(254.47 \times 10^{-6}) = 21.63 \times 10^3 \text{ N}$$

$$q = \frac{2 F_{bolt}}{s} = \frac{(2)(21.63 \times 10^3)}{125 \times 10^{-3}} = 346.1 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(152.30 \times 10^{-6})(346.1 \times 10^3)}{291.9 \times 10^{-6}} = 180.6 \times 10^3 \text{ N}$$

$$= 180.6 \text{ kN}$$

PROBLEM 6.6



6.5 The beam shown has been reinforced by attaching to it two 12 × 175-mm plates, using bolts of 18-mm diameter spaced longitudinally every 125 mm. Knowing that the average allowable shearing stress in the bolts is 85 MPa, determine the largest permissible vertical shearing force.

6.6 Solve Prob. 6.5, assuming that the reinforcing plates are only 9 mm thick.

SOLUTION

Calculate moment of inertia

Part	A (mm^2)	d (mm)	Ad^2 (10^6 mm^4)	$\bar{I}$ (10^6 mm^4)
Top plate	1575	* 137.5	29.777	0.011
W250×44.8	5720	0	0	71.1
Bot. plate	1575	137.5	29.777	0.011
Σ			59.555	71.121

$$* \quad d = \frac{266}{2} + \frac{9}{2} = 137.5 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 130.68 \times 10^6 \text{ mm}^4 = 130.68 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate} d_{plate} = (1575)(137.5) = 216.56 \times 10^3 \text{ mm}^3 = 216.56 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

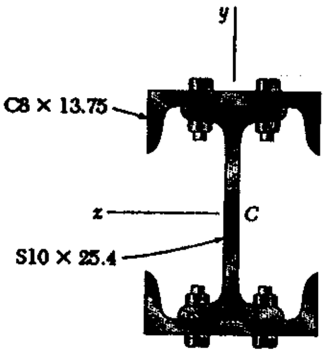
$$F_{bolt} = \tau_{all} A_{bolt} = (85 \times 10^6)(254.47 \times 10^{-6}) = 21.63 \times 10^3 \text{ N}$$

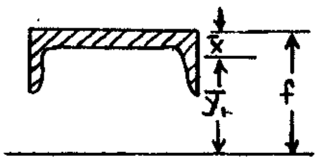
$$q = \frac{2 F_{bolt}}{s} = \frac{(2)(21.63 \times 10^3)}{125 \times 10^{-3}} = 346.1 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(130.68 \times 10^{-6})(346.1 \times 10^3)}{216.56 \times 10^{-6}} = 209 \times 10^3 \text{ N}$$

$$= 209 \text{ kN}$$

PROBLEM 6.7





6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$ -in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.

SOLUTION

Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{10}{2} + 0.303 = 5.303 \text{ in}$$

$$\bar{x} = 0.533 \text{ in}$$

$$\bar{y}_1 = f - \bar{x} = 5.303 - 0.533 = 4.770 \text{ in.}$$

Determine moment of inertia.

Part	A (in^2)	d (in)	Ad^2 (in^4)	$\bar{I}$ (in^4)
C8 × 13.75	4.04	4.770	91.92	1.53
S10 × 25.4	7.46	0	0	124
C8 × 13.75	4.04	4.770	91.92	1.53
Σ			183.84	127.06

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 183.84 + 127.06 = 310.9 \text{ in}^4$$

$$Q = A\bar{y}_1 = (4.04)(4.770) = 19.271 \text{ in}^3$$

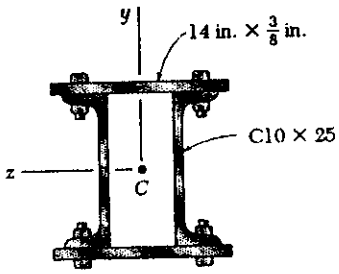
$$q = \frac{VQ}{I} = \frac{(30)(19.271)}{310.9} = 1.8595 \text{ kip/in}$$

$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.8595)(5) = 4.649 \text{ kip}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.649}{0.4418} = 10.52 \text{ ksi}$$

PROBLEM 6.8



6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$ -in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis

SOLUTION

Calculate moment of inertia

Part	A (in²)	d (in)	Ad² (in⁴)	$\bar{I}$ (in⁴)
Top plate	5.25	*5.1875	141.28	0.06
C10×25	7.35	0		91.2
C10×25	7.35	0		91.2
Bot. plate	5.25	*5.1875	141.28	0.06
Σ			282.56	182.52

$* \; d = \frac{10}{2} + \frac{1}{2}\left(\frac{3}{8}\right)$

$= 5.1875 \text{ in}$

$= \bar{y}_1$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 282.56 + 182.52 = 465.08 \text{ in}^4$$

$$Q = A_{plate}\bar{y}_1 = (5.25)(5.1875) = 27.234 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(27.234)}{465.08} = 1.7567 \text{ kips/in}$$

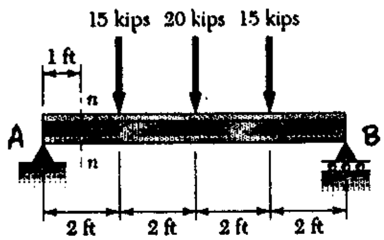
$$F_{bolt} = \frac{1}{2} q s = (\tfrac{1}{2})(1.7567)(5) = 4.392 \text{ kips}$$

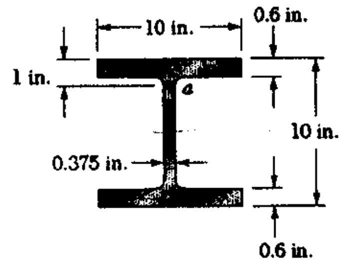
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.392}{0.4418} = 9.94 \text{ ksi}$$

PROBLEM 6.9

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.





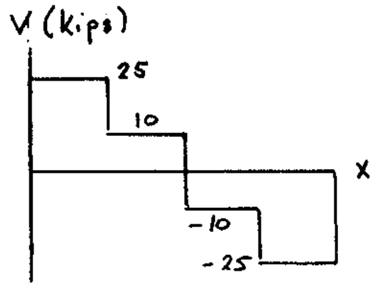
SOLUTION

By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0$

$R_A + R_B - 15 - 20 - 15 = 0$

$R_A = R_B = 25$ kips



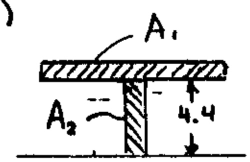
From shear diagram $V = 30$ kips at n-n.

Determine moment of inertia.

Part	A (in²)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
Top Flng	6	4.7	132.54	0.18
Web	3.30	0	0	21.30
Bot. Flng	6	4.7	132.54	0.18
Σ			265.08	21.66

$I = \Sigma Ad^2 + \Sigma \bar{I} = 286.74$ in⁴

(a)



Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)
1	6	4.7	28.2
2	1.65	2.2	3.63
Σ			31.83

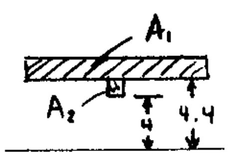
$Q = \Sigma A\bar{y}$

$= 31.83$ in³

$t = 0.375$ in

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{(25)(31.83)}{(286.74)(0.375)} = 7.40 \text{ ksi}$$ ◄

(b)



Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)
1	6	4.7	28.2
2	0.15	4.2	0.63
Σ			28.83

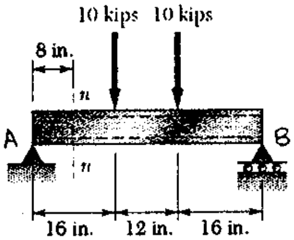
$Q = \Sigma A\bar{y} = 28.83$ in³

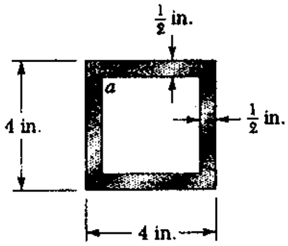
$t = 0.375$ in

$$\tau = \frac{VQ}{It} = \frac{(25)(28.83)}{(286.74)(0.375)} = 6.70 \text{ ksi}$$ ◄

PROBLEM 6.10

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



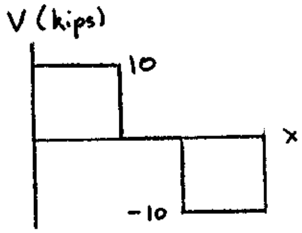


SOLUTION

By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - 10 - 10 = 0$

$R_A = R_B = 10$ kips

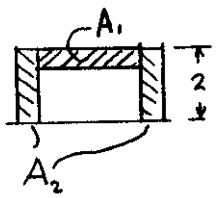


From the shear diagram $V = 10$ kips at n-n.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4$$

(a)



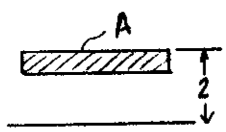
$$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)(\tfrac{1}{2})(1.75) + (2)(\tfrac{1}{2})(2)(1)$$

$$= 4.625 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17 \text{ ksi}$$ ◄

(b)



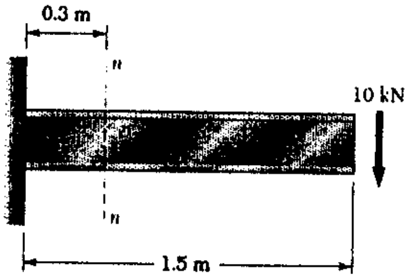
$$Q = A\bar{y} = (4)(\tfrac{1}{2})(1.75) = 3.5 \text{ in}^3$$

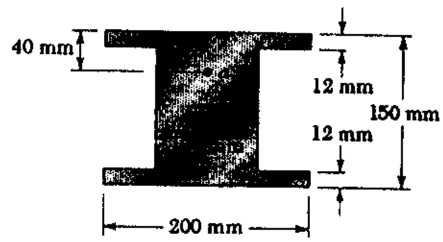
$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40 \text{ ksi}$$ ◄

PROBLEM 6.11

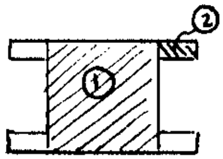
6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.





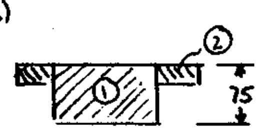
SOLUTION

At section n-n $V = 10$ kN



$$I = I_1 + 4I_2$$
$$= \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right)$$
$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$
$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$
$$= 39.58\times10^6 \text{ mm}^4 = 39.58\times10^{-6} \text{ m}^4$$

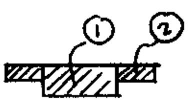
(a)



$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$
$$= (100)(75)(37.5) + (2)(50)(12)(69)$$
$$= 364.05\times10^3 \text{ mm}^3 = 364.05\times10^{-6} \text{ m}^3$$
$$t = 100 \text{ mm} = 0.100 \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3 \text{ Pa} = 920 \text{ kPa}$$ ◄

(b)

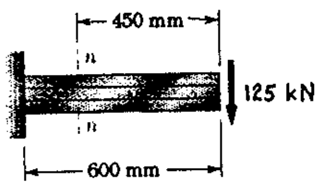


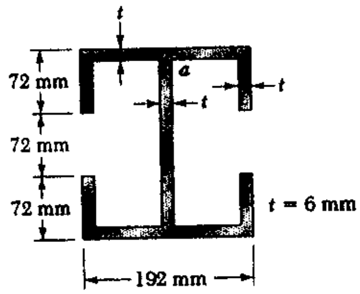
$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$
$$= (100)(40)(55) + (2)(50)(12)(69)$$
$$= 302.8\times10^3 \text{ mm}^3 = 302.8\times10^{-6} \text{ m}^3$$
$$t = 100 \text{ mm} = 0.100 \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3 \text{ Pa} = 765 \text{ kPa}$$ ◄

PROBLEM 6.12

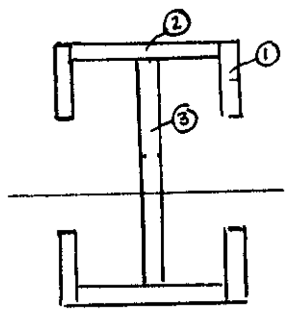
6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.





SOLUTION

At section n-n $\quad V = 125$ kN



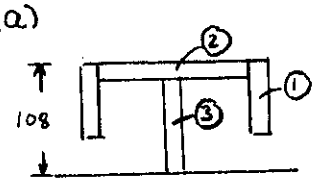
$$I_1 = \frac{1}{12}(6)(72)^3 + (6)(72)(72)^2 = 2.4261 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(180)(6)^3 + (180)(6)(105)^2 = 11.910 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(6)(204)^3 = 4.2448 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + I_3 = 37.77 \times 10^6 \text{ mm}^4 = 37.77 \times 10^{-6} \text{ m}^4$$

(a)



$$Q = 2A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3$$

$$= (2)(6)(72)(72) + (180)(6)(105) + (6)(102)(51)$$

$$= 206.82 \times 10^3 \text{ mm}^3 = 206.82 \times 10^{-6} \text{ m}^4$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(125 \times 10^3)(206.82 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 114.1 \times 10^6 \text{ Pa} = 114.1 \text{ MPa}$$

(b) $\quad Q = 2A_1\bar{y}_1 + A_2\bar{y}_2$

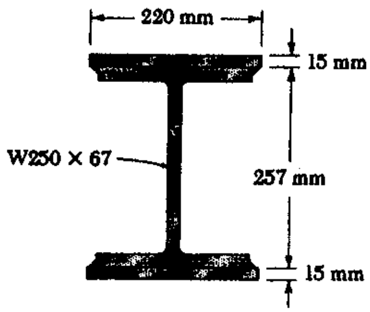
$$= (2)(6)(72)(72) + (180)(6)(105) = 175.61 \times 10^3 \text{ mm}^3 = 175.61 \times 10^{-6} \text{ m}^3$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(125 \times 10^3)(175.61 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 96.9 \times 10^6 \text{ Pa} = 96.9 \text{ MPa}$$

PROBLEM 6.13

6.13 Two steel plates of 15 × 220-mm rectangular cross section are welded to the W250 × 67 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 100 MPa.



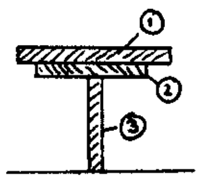
SOLUTION

Calculate moment of inertia

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	3300	* 136	61.036	0.062
W 250 × 67		0	0	104
Bot. plate	3300	136	61.036	0.062
Σ			122.072	104.124

$$* \ d = \frac{257}{2} + \frac{15}{2} = 136 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 226.2 \times 10^6 \text{ mm}^4 = 226.2 \times 10^{-6} \text{ m}^4$$



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)
① Top plate	3300	136	448.8
② Top flange	3203	120.65	386.4
③ Half web	1004	56.40	56.6
Σ			891.8

$$Q = \Sigma A\bar{y} = 891.8 \times 10^3 \text{ mm}^3 = 891.8 \times 10^{-6} \text{ m}^3$$

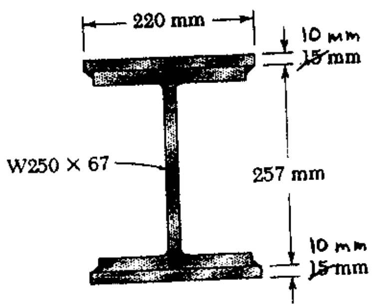
$$t = t_w = 8.9 \text{ mm} = 8.9 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It}$$

$$V = \frac{It\,\tau_{max}}{Q} = \frac{(226.2 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{891.8 \times 10^{-6}} = 226 \times 10^3 \text{ N}$$

$$= 226 \text{ kN}$$

PROBLEM 6.14



6.13 Two steel plates of 15 × 220-mm rectangular cross section are welded to the W250 × 67 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 100 MPa.

6.14 Solve Prob. 6.13, assuming that the two steel plates are (*a*) replaced by steel plates of 10 × 220-mm rectangular cross section, (*b*) removed.

SOLUTION

Calculate moment of inertia for part (a)

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	2200	*133.5	39.209	0.018
W 250 × 67		0	0	104
Bot. plate	2200	*133.5	39.209	0.018
Σ			78.42	104.04

$* \; d = \frac{257}{2} + \frac{10}{2} = 133.5 \text{ mm}$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 182.46 \times 10^6 \text{ mm}^4 = 182.46 \times 10^{-6} \text{ m}^4$$

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)
① Top plate	2200	133.5	293.7
② Top flange	3203	120.65	386.4
③ Half web	1004	56.40	56.6
Σ			736.7

$$Q = \Sigma A\bar{y} = 736.7 \times 10^3 \text{ mm}^3 = 736.7 \times 10^{-6} \text{ m}^3$$

$$t = t_w = 8.9 \text{ mm} = 8.9 \times 10^{-3} \text{ m} \qquad \tau_{max} = \frac{VQ}{It}$$

$$V = \frac{I t \tau_{max}}{Q} = \frac{(182.46\times10^{-6})(8.9\times10^{-3})(100\times10^6)}{736.7\times10^{-6}} = 220\times10^3 \text{ N} = 220 \text{ kN}$$

(b) $I = 104 \times 10^6 \text{ mm}^4 = 104 \times 10^{-6} \text{ m}^4$

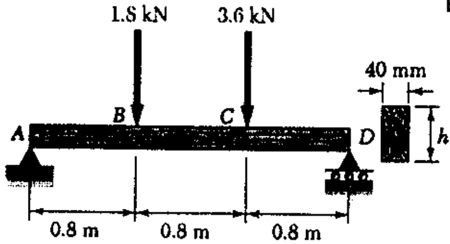
Consider Q for top flange and half web

$$Q = A_2\bar{y}_2 + A_3\bar{y}_3 = 386.4\times10^3 + 56.6\times10^3 = 443\times10^3 \text{ mm}^3 = 443\times10^{-6} \text{ m}^3$$

$$V = \frac{I t \tau_{max}}{Q} = \frac{(104\times10^{-6})(8.9\times10^{-3})(100\times10^6)}{443\times10^{-6}} = 209\times10^3 \text{ N} = 209 \text{ kN}$$

PROBLEM 6.15

6.15 Knowing that the allowable shearing stress for the timber used is 825 kPa, check whether the design obtained for the beam indicated is acceptable and, if not, redesign the cross section of the beam. Consider the beam of (*a*) Prob. 5.75, (*b*) Prob. 5.76.



(a) **SOLUTION**

From solution to PROBLEM 5.75

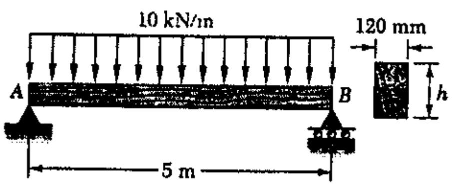
$|V|_{max} = 2.4$ kN $\qquad h = 173.2$ mm

$A = bh = (40)(173.2) = 6928\ \text{mm}^2$
$= 6928 \times 10^{-6}\ \text{m}^2$

For a rectangular cross section $\quad \tau_{max} = \frac{3}{2}\frac{|V|_{max}}{A}$

$\tau_{max} = \frac{3}{2}\frac{2.4 \times 10^3}{6928 \times 10^{-6}} = 520 \times 10^3$ Pa $= 520$ kPa < 825 kPa

Design is acceptable.



(b) **SOLUTION**

From solution to PROBLEM 5.76

$|V|_{max} = 25$ kN $\qquad h = 361$ mm

$A = bh = (120)(361) = 43.32 \times 10^3\ \text{mm}^2$
$= 43.32 \times 10^{-3}\ \text{m}^2$

For a rectangular cross section $\quad \tau_{max} = \frac{3}{2}\frac{|V|_{max}}{A}$

$\tau_{max} = \frac{3}{2}\frac{25 \times 10^3}{43.32 \times 10^{-3}} = 865 \times 10^3$ Pa $= 865$ kPa > 825 kPa

Design is not acceptable.

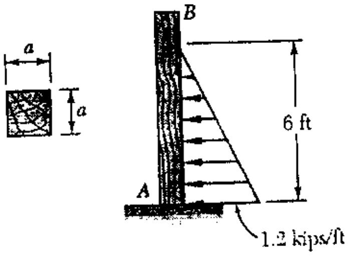
Resign $\quad A = \frac{3}{2}\frac{|V|_{max}}{\tau_{all}} = \frac{3}{2}\frac{25 \times 10^3}{825 \times 10^3} = 45.45 \times 10^{-3}\ \text{m}^2$

$= 45.45 \times 10^3\ \text{mm}^2$

$h = \frac{A}{b} = \frac{45.45 \times 10^3}{120} = 379$ mm $\qquad h = 379$ mm

PROBLEM 6.16

6.16 Knowing that the allowable shearing stress for the timber used is 130 psi, check whether the design obtained for the beam indicated is acceptable and, if not, redesign the cross section of the beam. Consider the beam of (*a*) Prob. 5.77, (*b*) Prob. 5.78.



(a) **SOLUTION**

$V_{max} = \frac{1}{2}(6)(1.2) = 3.6$ kips

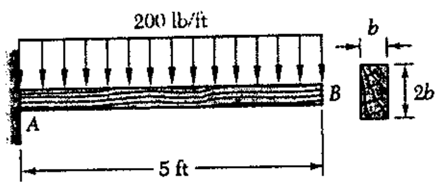
From solution to PROBLEM 5.77

$a = 6.67$ in. $\qquad A = a^2 = 44.45$ in^2

For a rectangular section $\qquad \tau_{max} = \frac{3}{2}\frac{V_{max}}{A}$

$\tau_{max} = \frac{3}{2}\frac{3.6}{44.45} = 0.1215$ ksi $= 121.5$ psi < 130 psi

Design is acceptable.



(b) **SOLUTION**

From solution to PROBLEM 5.78

$|V|_{max} = 1000$ lb $\qquad b = 2.95$ in.

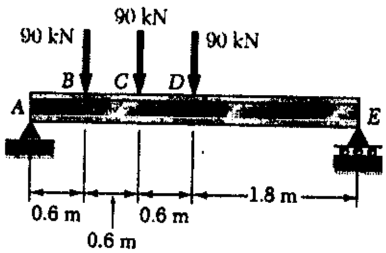
$A = (b)(2b) = 2b^2 = 17.40$ in^2

For a rectangular cross section $\qquad \tau_{max} = \frac{3}{2}\frac{|V|_{max}}{A}$

$\tau_{max} = \frac{3}{2}\frac{1000}{17.40} = 86.2$ psi < 130 psi

Design is acceptable.

PROBLEM 6.17



6.17 Determine the average shearing stress in the web of the beam indicated and check whether the design obtained earlier for that beam is acceptable, knowing that the allowable shearing stress for the steel used is 100 MPa. Consider the beam of (*a*) Prob. 5.81, (*b*) Prob. 5.82.

(a) **SOLUTION**

From the solution to PROBLEM 5.81

$|V|_{max} = 180$ kN

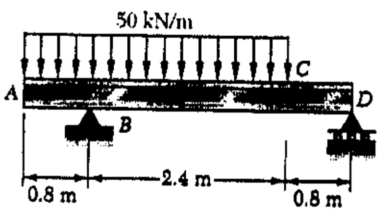
The selected section is W 410 × 60

For that section $t_w = 7.7$ mm $\quad d = 407$ mm

$A_{web} = t_w d = 3.13 \times 10^3\ mm^2 = 3.13 \times 10^{-3}\ m^2$

$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{180 \times 10^3}{3.13 \times 10^{-3}} = 57.4 \times 10^6\ Pa = 57.4\ MPa < 100\ MPa$$

Design is acceptable.



(b) **SOLUTION**

From the solution to PROBLEM 5.82

$|V|_{max} = 80$ kN

The selected section is W 250 × 28.4

For that section $t_w = 6.4$ mm $\quad d = 260$ mm

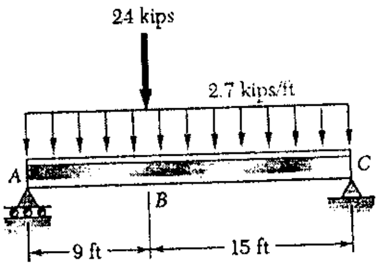
$A_{web} = t_w d = (6.4)(260) = 1664\ mm^2 = 1664 \times 10^{-6}\ m^2$

$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{80 \times 10^3}{1664 \times 10^{-6}} = 48.1 \times 10^6\ Pa = 48.1\ MPa < 100\ MPa$$

Design is acceptable.

PROBLEM 6.18

6.18 Determine the average shearing stress in the web of the beam indicated and check whether the design obtained earlier for that beam is acceptable, knowing that the allowable shearing stress for the steel used is 14.5 ksi. Consider the beam of (*a*) Prob. 5.83, (*b*) Prob. 5.84



(a) **SOLUTION**

From the solution to PROBLEM 5.83

$|V|_{max} = 48$ kips

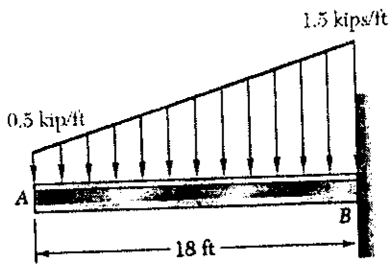
The selected section is W 27 × 84

For that section $t_w = 0.460$ in. $d = 26.71$ in.

$$A_{web} = t_w d = (0.460)(26.71) = 12.29 \text{ in}^2$$

$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{48}{12.29} = 3.91 \text{ ksi} < 14.5 \text{ ksi}$$

Design is acceptable.



(b) **SOLUTION**

From the solution to PROBLEM 5.84

$|V|_{max} = \frac{1}{2}(18)(0.5 + 1.5) = 18$ kips

The selected section is W 18 × 50

For that section $t_w = 0.355$ in. $d = 17.99$ in.

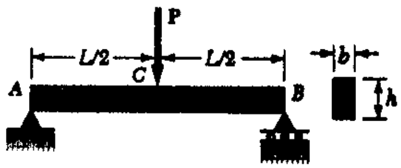
$$A_{web} = t_w d = (0.355)(17.99) = 6.39 \text{ in}^2$$

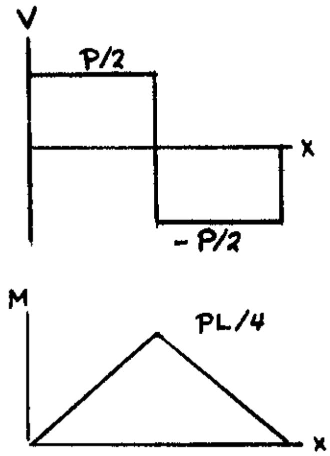
$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{18}{6.39} = 2.82 \text{ ksi} < 14.5 \text{ ksi}$$

Design is acceptable.

PROBLEM 6.19

6.19 A simply supported timber beam *AB* of rectangular cross section carries a single concentrated load **P** at its midpoint *C*. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $h/2L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.





SOLUTION

Reactions $\quad R_A = R_B = P/2$

(1) $V_{max} = R_A = \frac{P}{2}$

(2) $A = bh$ for rectangular section

(3) $\tau_m = \frac{3}{2}\frac{V_{max}}{A} = \frac{3P}{4bh}$ for rectangular section

(4) $M_{max} = \frac{PL}{4}$

(5) $S = \frac{1}{6}bh^2$ for rectangular section

(6) $\sigma_m = \frac{M_{max}}{S} = \frac{3PL}{2bh^2}$

(a) $\frac{\tau_m}{\sigma_m} = \frac{h}{2L}$

(b) Solving for h $\quad h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960\times10^3)}{12\times10^6} = 320\times10^{-3}$ m

$= 320$ mm

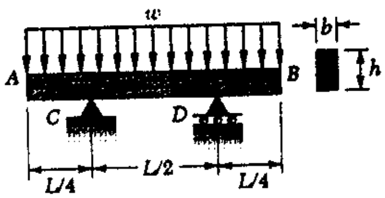
Solving equation (3) for b

$b = \frac{3P}{4h\tau_m} = \frac{(3)(40\times10^3)}{(4)(320\times10^{-3})(960\times10^3)} = 97.7\times10^{-3}$ m

$= 97.7$ mm

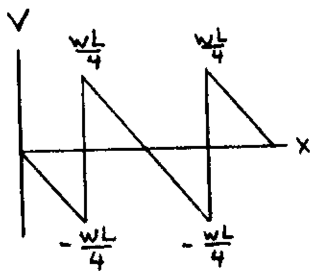
PROBLEM 6.20

6.20 A timber beam *AB* of length *L* and rectangular cross section carries a uniformly distributed load *w* and is supported as shown. (*a*) Show that the ratio τ_m / σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where *h* and *L* are, respectively, the depth and the length of the beam. (*b*) Determine the depth *h* and width *b* of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.



SOLUTION

$$R_A = R_B = \frac{wL}{2}$$



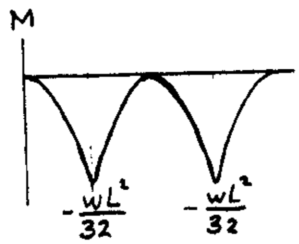
From shear diagram $\quad |V|_m = \frac{wL}{4} \quad (1)$

For rectangular section $\quad A = bh \quad (2)$

$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \quad (3)$$

From bending moment diagram

$$|M|_m = \frac{wL^2}{32} \quad (4)$$



For a rectangular cross section

$$S = \frac{1}{6}bh^2 \quad (5)$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \quad (6)$$

(a) Dividing eq. (3) by eq. (6) $\quad \frac{\tau_m}{\sigma_m} = \frac{2h}{L}$

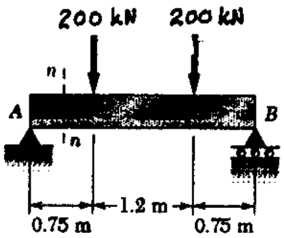
(b) Solving for h $\quad h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08\times10^6)}{(2)(12\times10^6)} = 225\times10^{-3}$ m $= 225$ mm

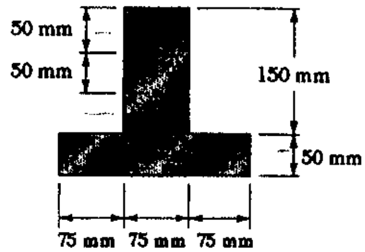
Solving eq. (3) for b $\quad b = \frac{3wL}{8h\tau_m} = \frac{(3)(8\times10^3)(5)}{(8)(225\times10^{-3})(1.08\times10^6)}$

$= 61.7\times10^{-3}$ m $= 61.7$ mm

PROBLEM 6.21

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



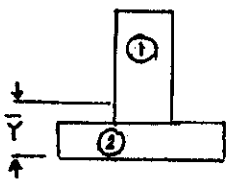


SOLUTION

$R_A = R_B = 200\ \text{kN}$

At section n-n $\quad V = 200\ \text{kN}$

Locate centroid and calculate moment of inertia.

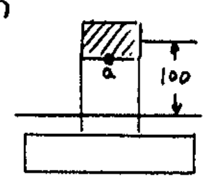


Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
①	11250	125	1406.25	50	28.125	21.094
②	11250	25	281.25	50	28.125	2.344
Σ	22500		1687.5		56.25	23.438

$$\bar{Y} = \frac{1687.5 \times 10^3}{22500} = 75\,\text{mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 79.688 \times 10^6\ \text{mm}^4 = 79.688 \times 10^{-6}\ \text{m}^4$$

(a)



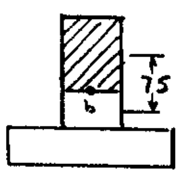
$$Q_a = A\bar{y} = (75)(50)(100) = 375 \times 10^3\ \text{mm}^4 = 375 \times 10^{-6}\ \text{m}^4$$

$$t = 75\ \text{mm} = 75 \times 10^{-3}\ \text{m}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(375 \times 10^{-6})}{(79.768\ 10^{-6})(75 \times 10^{-3}} = 12.55 \times 10^6\ \text{Pa}$$

$$= 12.55\ \text{MPa}$$ ◄

(b)



$$Q_b = A\bar{y} = (75)(100)(75) = 562.5 \times 10^3\ \text{mm}^3 = 562.5 \times 10^{-6}\ \text{m}^3$$

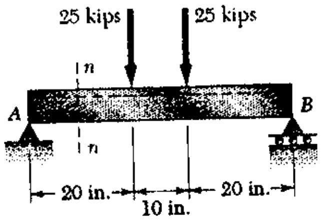
$$t = 75\ \text{mm} = 75 \times 10^{-3}\ \text{m}$$

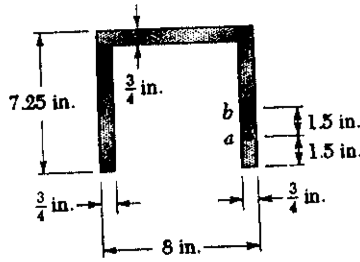
$$\tau_b = \frac{VQ_b}{It} = \frac{(200 \times 10^3)(562.5 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 18.82 \times 10^6\ \text{Pa}$$

$$= 18.82\ \text{MPa}$$ ◄

PROBLEM 6.22

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



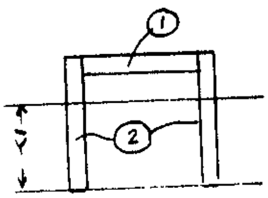


SOLUTION

$R_A = R_B = 25$ kips

At section n-n $V = 25$ kips.

Locate centroid and calculate moment of inertia.

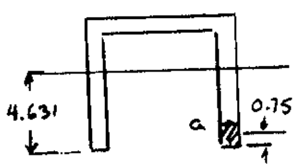


Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

(a)

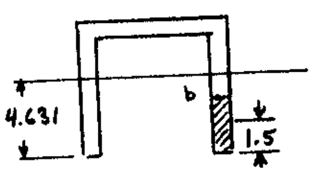


$$Q_a = A\bar{y} = (\tfrac{3}{4})(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \tfrac{3}{4} = 0.75 \text{ in}$$

$$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} = 1.745 \text{ ksi}$$

(b)



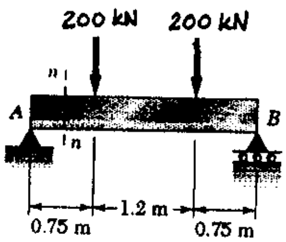
$$Q_b = A\bar{y} = (\tfrac{3}{4})(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

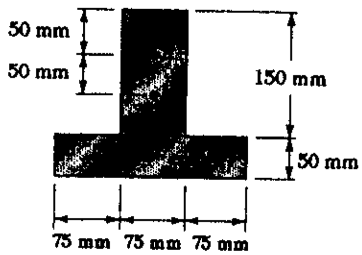
$$t = 0.75 \text{ in.}$$

$$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} = 2.82 \text{ ksi}$$

PROBLEM 6.23

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.



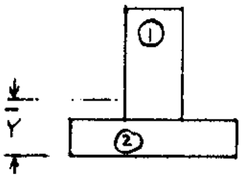


SOLUTION

$R_A = R_B = 200$ kN

At section n-n $V = 200$ kN

Locate centroid and calculate moment of inertia.

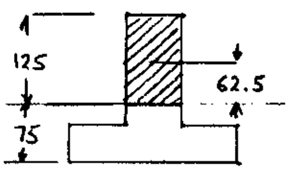


Part	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (10^3\ mm^3)$	$d\ (mm)$	$Ad^2\ (10^6\ mm^4)$	$\bar{I}\ (10^6\ mm^4)$
1	11250	125	1406.25	50	28.125	21.094
2	11250	25	281.25	50	28.125	2.344
Σ	22500		1687.5		56.25	23.438

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1687.5 \times 10^3}{22500} = 75 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 79.688 \times 10^6 \text{ mm}^4 = 79.688 \times 10^{-6} \text{ m}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



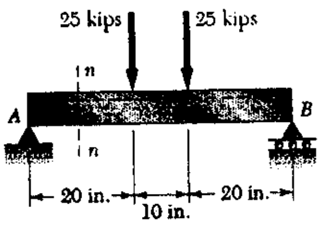
$$Q = A\bar{y} = (75)(125)(62.5) = 585.94 \times 10^3 \text{ mm}^3$$
$$= 585.94 \times 10^{-6} \text{ m}^3$$

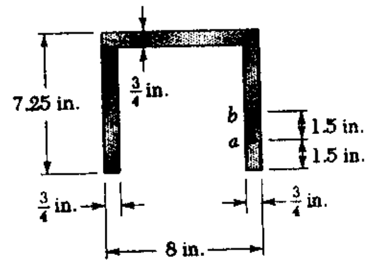
$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(200 \times 10^3)(585.94 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 19.61 \times 10^6 \text{ Pa}$$
$$= 19.61 \text{ MPa}$$

PROBLEM 6.24

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section n-n.



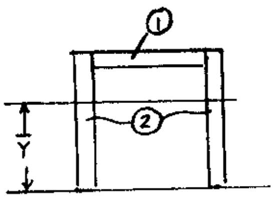


SOLUTION

$R_A = R_B = 25$ kips

At section n-n $\quad V = 25$ kips

Locate centroid and calculate moment of inertia

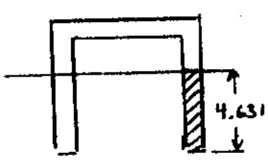


Part	A (in^2)	$\bar{y}$ (in)	$A\bar{y}$ (in^3)	d (in)	Ad^2 (in^4)	$\bar{I}$ (in^4)
1	4.875	6.875	33.52	2.244	24.55	0.23
2	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.

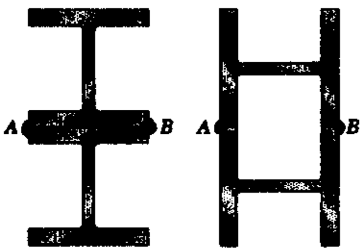


$$Q = A\bar{y} = \left(\tfrac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \tfrac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$

PROBLEM 6.25



(*a*) (*b*)

6.25 Two W200 × 46.1 rolled steel sections are to be welded at *A* and *B* in either of the two ways shown to form a composite beam. Knowing that for each weld the allowable horizontal shearing force is 500 kN per meter of weld, determine the maximum allowable shear in the composite beam for each of the two arrangements shown.

SOLUTION

For rolled steel section W 200 × 46.1

$A = 5860\ \text{mm}^2 \quad d = 203\ \text{mm} \quad b_f = 203\ \text{mm}$

$I_x = 45.5 \times 10^6\ \text{mm}^4 \quad I_y = 15.3 \times 10^6\ \text{mm}^4$

(a) $I = 2\left[I_x + A\left(\frac{d}{2}\right)^2\right] = 2\left[45.5 \times 10^6 + (5860)\left(\frac{203}{2}\right)^2\right] = 211.7 \times 10^6\ \text{mm}^4$
$= 211.7 \times 10^{-6}\ \text{m}^4$

$Q = A\,\frac{d}{2} = (5860)\left(\frac{203}{2}\right) = 594.8 \times 10^3\ \text{mm}^3 = 594.8 \times 10^{-6}\ \text{m}^3$

$q_1 = 500\ \text{kN/m}$ for one weld. For 2 welds $q_{all} = 1000\ \text{kN/m}$

$q_{all} = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q} = \frac{(211.7 \times 10^{-6})(1000 \times 10^3)}{594.8 \times 10^{-6}} = 356 \times 10^3\ \text{N}$
$= 356\ \text{kN}$ ◄

(b) $I = 2\left[I_y + A\left(\frac{b_f}{2}\right)^2\right] = 2\left[15.3 \times 10^6 + 5860\left(\frac{203}{2}\right)^2\right] = 151.34 \times 10^6\ \text{mm}^4$
$= 151.34 \times 10^{-6}\ \text{m}^4$

$Q = A\,\frac{b_f}{2} = (5860)\left(\frac{203}{2}\right) = 594.8 \times 10^3\ \text{mm}^3 = 594.8 \times 10^{-6}\ \text{m}^3$

$V_{all} = \frac{I q_{all}}{Q} = \frac{(151.34 \times 10^{-6})(1000 \times 10^3)}{594.8 \times 10^{-6}} = 254 \times 10^3\ \text{N} = 254\ \text{kN}$ ◄

PROBLEM 6.26

6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear V. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$I = \frac{\pi}{4}c^4$ and $A = \pi c^2$

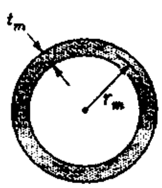
For semicircle $A_s = \frac{\pi}{2}c^2 \quad \bar{y} = \frac{4c}{3\pi}$

$Q = A\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$

τ_{max} occurs at center where $t = 2c$

$$\tau_{max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3}\frac{V}{A} \qquad k = \frac{4}{3} = 1.333 \blacktriangleleft$$

PROBLEM 6.27



6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear V. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

For a thin walled circular section $\quad A = 2\pi r_m t_m$

$J = A r_m^2 = 2\pi r_m^3 t_m \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$

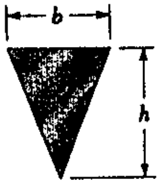
For a semicircular arc $\quad \bar{y} = \frac{2r_m}{\pi}$

$A_s = \pi r_m t_m \qquad Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$

$t = 2t_m$

$$\tau_{max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A} \qquad k = 2.00 \blacktriangleleft$$

PROBLEM 6.28



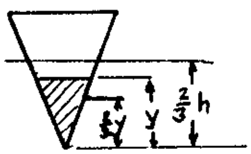
6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$A = \tfrac{1}{2}bh \qquad I = \tfrac{1}{36}bh^3$$



For a cut at location y

$$A(y) = \tfrac{1}{2}\left(\tfrac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \tfrac{2}{3}h - \tfrac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$t(y) = \frac{by}{h}$$

$$\tau(y) = \frac{VQ}{It} = \frac{V\,\frac{by^2}{3}(h-y)}{\left(\frac{1}{36}bh^3\right)\frac{by}{h}} = \frac{12\,V\,y\,(h-y)}{bh^3} = \frac{12V}{bh^3}(hy-y^2)$$

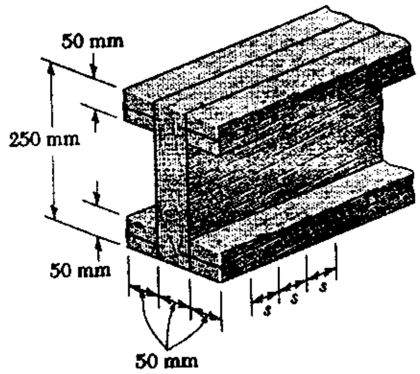
To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$

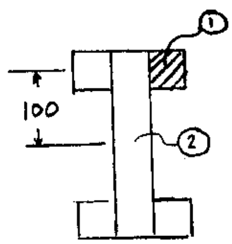
$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h-2y_m) = 0 \qquad y_m = \tfrac{1}{2}h$$

$$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\tfrac{1}{2}h^2 - \left(\tfrac{1}{2}h\right)^2\right] = \frac{3V}{bh^2} = \frac{3}{2}\frac{V}{A}$$

$$k = \tfrac{3}{2} = 1.500$$

PROBLEM 6.29





6.29 The built-up wooden beam shown is subjected to a vertical shear of 5 kN. Knowing that the longitudinal spacing of the nails is $s = 45$ mm and that each nail is 90 mm long, determine the shearing force in each nail.

SOLUTION

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(50)(50)^3 + (50)(50)(100)^2 = 25.52 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(50)(250)^3 = 65.10 \times 10^6 \text{ mm}^4$$

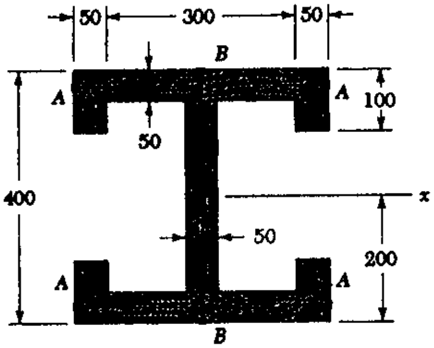
$$I = 4I_1 + I_2 = 167.18 \times 10^6 \text{ mm}^4 = 167.18 \times 10^{-6} \text{ m}^4$$

$$Q = Q_1 = A_1 \bar{y}_1 = (50)(50)(100) = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

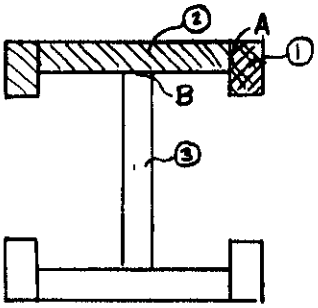
$$q = \frac{VQ}{I} = \frac{(5 \times 10^3)(250 \times 10^{-6})}{167.18 \times 10^{-6}} = 7.477 \times 10^3 \text{ N/m}$$

$$F_{nail} = qs = (7.477 \times 10^3)(45 \times 10^{-3}) = 336 \text{ N}$$

PROBLEM 6.30



Dimensions in mm



6.30 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (*a*) at A, (*b*) at B. (*Given:* $I_x = 1.504 \times 10^9\ \text{mm}^4$.)

SOLUTION

$$I_x = 1.504 \times 10^9\ \text{mm}^4 = 1504 \times 10^{-6}\ \text{m}^4$$

$$s_A = 60\ \text{mm} = 0.060\ \text{m}$$

$$s_B = 25\ \text{mm} = 0.025\ \text{m}$$

(a) $Q_A = Q_1 = A_1\bar{y}_1 = (50)(100)(150) = 750 \times 10^3\ \text{mm}^3$
$= 750 \times 10^{-6}\ \text{m}^3$

$$F_A = q_A s_A$$

$$= \frac{VQ_1 s_A}{I} = \frac{(8\times10^3)(750\times10^{-6})(0.060)}{1504\times10^{-6}}$$

$$= 239\ \text{N}$$

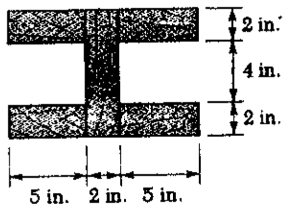
(b) $Q_2 = A_2\bar{y}_2 = (300)(50)(175) = 2625 \times 10^3\ \text{mm}^3$

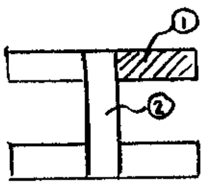
$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3\ \text{mm}^3$
$= 4125 \times 10^{-6}\ \text{m}^3$

$$F_B = q_B s_B = \frac{VQ_B s_B}{I} = \frac{(8\times10^3)(4125\times10^{-6})(0.025)}{1504\times10^{-6}} = 549\ \text{N}$$

PROBLEM 6.31

6.31 The built-up beam shown is made up by gluing together five planks. Knowing that the allowable average shearing stress in the glued joints is 60 psi, determine the largest permissible vertical shear in the beam.





$$\tau = \frac{VQ}{It}$$

SOLUTION

$$I_1 = \tfrac{1}{12}(5)(2)^3 + (5)(2)(3)^2 = 93.33 \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(2)(8)^3 = 85.33 \text{ in}^4$$

$$I = 4I_1 + I_2 = 458.66 \text{ in}^4$$

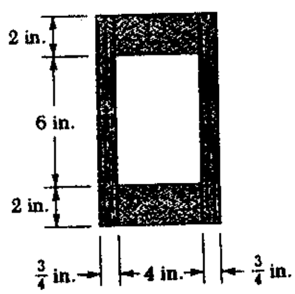
$$Q = A_1\bar{y}_1 = (5)(2)(3) = 30 \text{ in}^3$$

For each glued joint $t = 2$ in.

$$V = \frac{It\tau}{Q} = \frac{(458.66)(2)(60)}{30} = 1835 \text{ lb.}$$

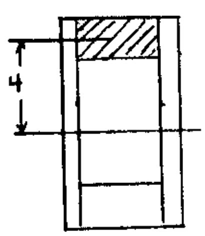
PROBLEM 6.32

6.32 The built-up beam shown is made up by gluing together two $\frac{3}{4} \times 10$-in. plywood strips and two 2×4-in. planks. Knowing that the allowable average shearing stress in the glued joints is 50 psi, determine the largest permissible vertical shear in the beam.



SOLUTION

$$I = \tfrac{1}{12}(5.5)(10)^3 - \tfrac{1}{12}(4)(6)^3 = 386.33 \text{ in}^4$$



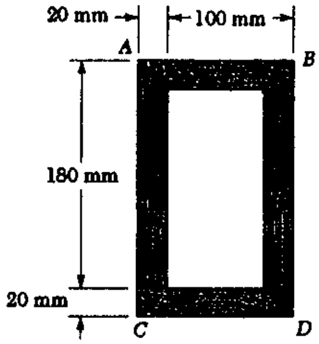
$$Q = A\bar{y} = (4)(2)(4) = 32 \text{ in}^4$$

$$t = 2 + 2 = 4 \text{ in}$$

$$\tau = \frac{VQ}{It}$$

$$V = \frac{It\tau}{Q} = \frac{(386.33)(4)(50)}{32} = 2410 \text{ lb.}$$

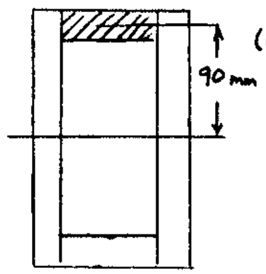
PROBLEM 6.33



6.33 Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

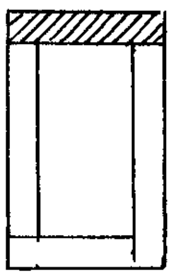
$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4$$
$$= 52.693 \times 10^{-6} \text{ m}^4$$



(a) $Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3$
$$= 144 \times 10^{-6} \text{ m}^3$$

$$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$$

$$\tau_A = \frac{VQ_A}{It_A} = \frac{(3.5\times10^3)(144\times10^{-6})}{(52.693\times10^{-6})(0.040)}$$
$$= 239 \times 10^3 \text{ Pa} = 239 \text{ kPa}$$



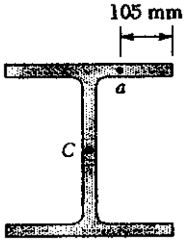
(b) $Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$

$$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{(3.5\times10^3)(216\times10^{-6})}{(52.693\times10^{-6})(0.040)} = 359 \times 10^3 \text{ Pa}$$
$$= 359 \text{ kPa}$$

PROBLEM 6.34

6.34 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (*a*) at point *A*, (*b*) at the centroid *C* of the section.

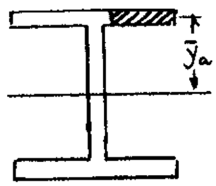


SOLUTION

For W360×122, $d = 363$ mm, $b_f = 257$ mm, $t_f = 21.70$ mm, $t_w = 13.0$ mm

$I = 365 \times 10^6 \text{ mm}^4 = 365 \times 10^{-6} \text{ m}^4$

(a)



$A_a = (105)(21.70) = 2278.5 \text{ mm}^2$

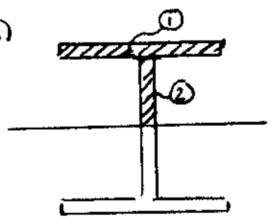
$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$

$Q_a = A_a \bar{y}_a = 388.8 \times 10^3 \text{ mm}^3 = 388.8 \times 10^{-6} \text{ m}^{-6}$

$t_a = t_f = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6 \text{ Pa} = 12.27 \text{ MPa}$$

(b)



$A_1 = b_f t_f = (257)(21.70) = 5577 \text{ mm}^2$

$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$

$A_2 = t_w\left(\frac{d}{2} - t_f\right) = (13.0)(159.8) = 2077 \text{ mm}^2$

$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_f\right) = 79.9 \text{ mm}$

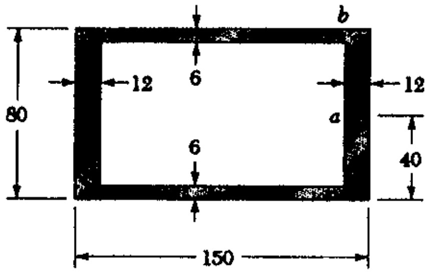
$Q_c = \Sigma A\bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3$
$= 1117.7 \times 10^{-6} \text{ m}^3$

$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$

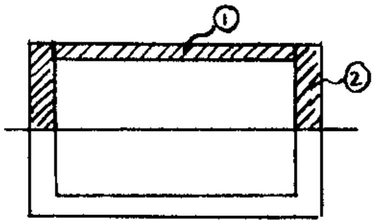
$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa}$$

PROBLEM 6.35

6.35 and 6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



Dimesions in mm



SOLUTION

$$I = \frac{1}{12}(150)(80)^3 - \frac{1}{12}(126)(68)^3$$

$$= 3.098 \times 10^6 \text{ mm}^4 = 3.098 \times 10^{-6} \text{ m}^4$$

(a) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (126)(6)(37) + (2)(12)(40)(20)$$

$$= 47.172 \times 10^3 \text{ mm}^3 = 47.172 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(47.172 \times 10^{-6})}{(3.098 \times 10^{-6})(0.024)}$$

$$= 95.2 \times 10^6 \text{ Pa} = 95.2 \text{ MPa}$$

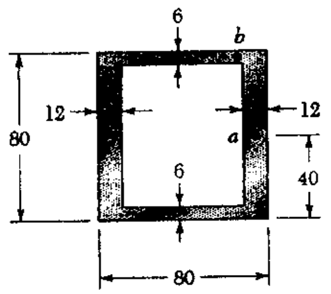
(b) $Q_b = A_1\bar{y}_1 = (126)(6)(37) = 27.97 \times 10^3 \text{ mm}^3 = 27.97 \times 10^{-6} \text{ m}^3$

$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

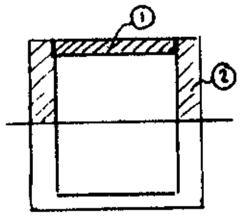
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(27.97 \times 10^{-6})}{(3.098 \times 10^{-6})(0.012)} = 112.9 \times 10^6 \text{ Pa} = 112.9 \text{ MPa}$$

PROBLEM 6.36

6.35 and 6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



Dimesions in mm



SOLUTION

$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460\times10^6\ \text{mm}^4$$
$$= 1.946\times10^{-6}\ \text{m}^4$$

(a) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632\times10^3\ \text{mm}^3$$
$$= 31.632\times10^{-6}\ \text{m}^4$$

$$t_a = (2)(12) = 24\ \text{mm} = 0.024\ \text{m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150\times10^3)(31.632\times10^{-6})}{(1.946\times10^{-6})(0.024)} = 101.6\times10^6\ \text{Pa}$$
$$= 101.6\ \text{MPa}$$

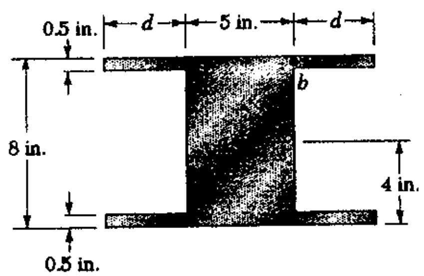
(b) $Q_b = A_1\bar{y}_1 = (56)(6)(37) = 12.432\times10^3\ \text{mm}^3$
$$= 12.432\times10^{-6}\ \text{m}^3$$

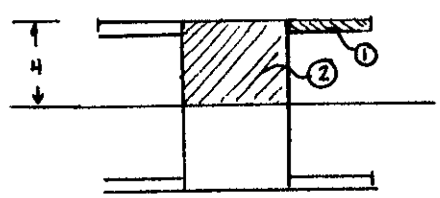
$$t_b = (2)(6) = 12\ \text{mm} = 0.012\ \text{m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150\times10^3)(12.432\times10^{-6})}{(1.946\times10^{-6})(0.012)} = 79.9\times10^6\ \text{Pa} = 79.9\ \text{MPa}$$

PROBLEM 6.37

6.37 The vertical shear is 1200 lb in a beam having the cross section shown. Knowing that $d = 4$ in., determine the shearing stress (*a*) at point *a*, (*b*) at point *b*.





SOLUTION

$$I_1 = \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(3.75)^2 = 28.167\ \text{in}^4$$

$$I_2 = \frac{1}{3}(5)(4)^3 = 106.67\ \text{in}^4$$

$$I = 4I_1 + 2I_2 = 326\ \text{in}^4$$

(a) $Q_a = 2A_1\bar{y}_1 + A_2\bar{y}_2$
$$= (2)(4)(0.5)(3.75) + (5)(4)(2) = 55\ \text{in}^3$$

$$t_a = 5\ \text{in.}$$

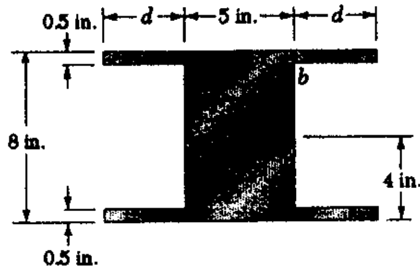
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(55)}{(326)(5)} = 40.5\ \text{psi}$$

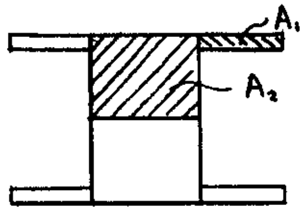
(b) $Q_b = A_1\bar{y}_1 = (4)(0.5)(3.75) = 7.5\ \text{in}^3$ $\qquad t_b = 0.5\ \text{in.}$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(7.5)}{(326)(0.5)} = 55.2\ \text{psi}$$

PROBLEM 6.38

6.38 The vertical shear is 1200 lb in a beam having the cross section shown. Determine (*a*) the distance *d* for which $\tau_a = \tau_b$, (*b*) the corresponding shearing stress at points *a* and *b*.





SOLUTION

$A_1 = 0.5\,d \text{ in}^2, \quad \bar{y}_1 = 3.75 \text{ in} \qquad t_b = 0.5 \text{ in}$

$A_2 = (5)(4) = 20 \text{ in}^2, \quad \bar{y}_2 = 2 \text{ in} \qquad t_a = 5 \text{ in}$

$Q_b = A_1 y_1 = 1.875\,d \text{ in}^3$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{V}{I}\frac{1.875\,d}{0.5} = 3.75\frac{Vd}{I}$$

$Q_a = A_2\bar{y}_2 + 2Q_b = (20)(2) + (2)(1.875\,d)$
$= 40 + 3.75\,d$

$t_a = 5$ in.

(a) $$\tau_a = \frac{VQ_a}{It_a} = \frac{V(40 + 3.75d)}{I\ (5)} = 8\frac{V}{I} + 0.75\frac{Vd}{I} = \tau_b = 3.75\frac{Vd}{I}$$

$8 + 0.75d = 3.75\,d \qquad d = \frac{8}{3} = 2.667$ in. ◄

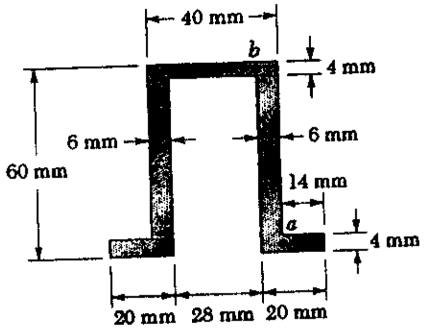
(b) $I_1 = \frac{1}{12}(2.667)(0.5)^3 + (2.667)(0.5)(3.75)^2 = 18.78 \text{ in}^4$

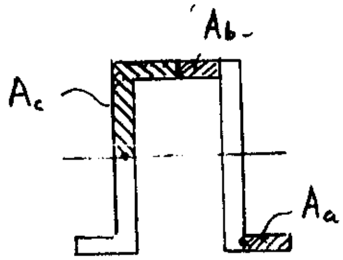
$I_2 = \frac{1}{3}(5)(4)^3 = 106.67 \text{ in}^4$

$I = 4I_1 + 2I_2 = 288.45 \text{ in}^4$

$$\tau_a = \tau_b = 3.75\frac{Vd}{I} = \frac{(3.75)(1200)(2.667)}{288.45} = 41.6 \text{ psi}$$ ◄

PROBLEM 6.39





6.39 Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*.

SOLUTION

Neutral axis lies 30 mm above bottom

$$\tau_c = \frac{VQ_c}{It} \qquad \tau_a = \frac{VQ_a}{It_a} \qquad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a} \qquad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$

$t_c = 6 \text{ mm}$

$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$

$t_a = 4 \text{ mm}$

$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$

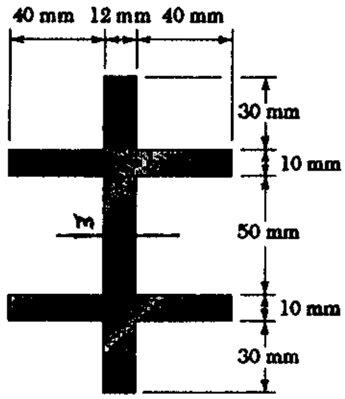
$t_b = 4 \text{ mm}$

$\tau_c = 75 \text{ MPa}$

$$\tau_a = \frac{Q_a}{Q_c} \cdot \frac{t_c}{t_a} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

$$\tau_b = \frac{Q_b}{Q_c} \cdot \frac{t_c}{t_b} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

PROBLEM 6.40



6.40 Knowing that a given vertical shear **V** causes a maximum shearing stress of 50 MPa in a thin-walled member having the cross section shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*, (*c*) at point *c*.

SOLUTION

$$Q_a = (12)(30)(25+10+15) = 18 \times 10^3 \text{ mm}^3$$

$$Q_b = (40)(10)(25+5) = 12 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_a + 2Q_b + (12)(10)(25+5) = 45.6 \times 10^3 \text{ mm}^3$$

$$Q_m = Q_c + (12)(25)\left(\tfrac{25}{2}\right) = 49.35 \times 10^3 \text{ mm}^3$$

$$t_a = t_c = t_m = 12 \text{ mm}$$

$$t_b = 10 \text{ mm}$$

$$\tau_m = 50 \text{ MPa}$$

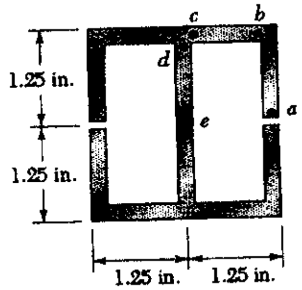
(a) $\dfrac{\tau_a}{\tau_m} = \dfrac{Q_a}{Q_m} \cdot \dfrac{t_m}{t_a} = \dfrac{18}{49.35} \cdot \dfrac{12}{12} = 0.3647$ $\qquad \tau_a = 18.23 \text{ MPa}$ ◄

(b) $\dfrac{\tau_b}{\tau_m} = \dfrac{Q_b}{Q_m} \cdot \dfrac{t_m}{t_b} = \dfrac{12}{49.35} \cdot \dfrac{12}{10} = 0.2918$ $\qquad \tau_b = 14.59 \text{ MPa}$ ◄

(c) $\dfrac{\tau_c}{\tau_m} = \dfrac{Q_c}{Q_m} \cdot \dfrac{t_m}{t_c} = \dfrac{45.6}{49.35} \cdot \dfrac{12}{12} = 0.9240$ $\qquad \tau_c = 46.2 \text{ MPa}$ ◄

PROBLEM 6.41

6.41* and *6.42 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ - in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

$t = 0.125$ in at all sections.

$V = 2$ kips

$$Q_a = 0 \qquad \tau_a = \frac{VQ_a}{It} = 0$$

$$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.09766 \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} = 1.26 \text{ ksi}$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in}^3$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} = 3.30 \text{ ksi}$$

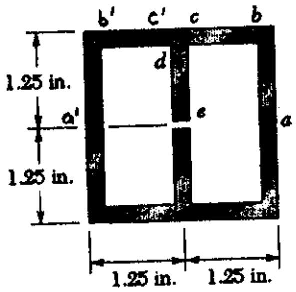
$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} = 6.84 \text{ ksi}$$

$$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$$

$$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} = 7.86 \text{ ksi}$$

PROBLEM 6.42



6.41 and 6.42 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ - in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

Add symmetric points c', b', and a'.

$$Q_e = 0$$

$$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.07910 \text{ in}^3 \qquad t_d = 0.125 \text{ in}$$

$$Q_c = Q_e = (0.125)^2(1.1875) = 0.09765 \text{ in}^4 \qquad t_c = 0.25 \text{ in.}$$

$$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3 \qquad t_b = 0.25 \text{ in.}$$

$$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60839 \text{ in}^3 \qquad t_a = 0.25 \text{ in}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60839)}{(1.2382)(0.25)} = 3.93 \text{ ksi}$$

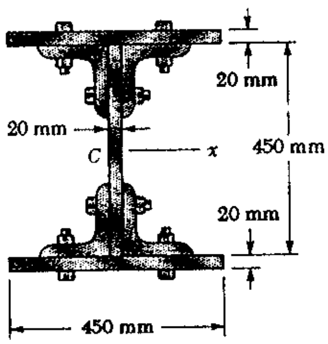
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi}$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi}$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2882)(0.125)} = 1.02 \text{ ksi}$$

$$\tau_e = \frac{VQ_e}{It_e} = 0$$

PROBLEM 6.43



6.43 Three 20 × 450-mm steel plates are bolted to four L152 × 152 × 19.0 angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (*Given:* $I_x = 1896 \times 10^6$ mm^4.)

SOLUTION

Flange: $I_f = \frac{1}{12}(450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6 \text{ mm}^4$

Web: $I_w = \frac{1}{12}(20)(450)^3 = 151.9 \times 10^6 \text{ mm}^4$

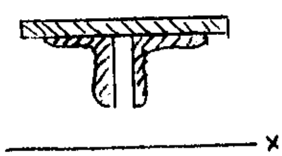
Angle: $\bar{I} = 11.6 \times 10^6 \text{ m}^4, \quad A = 5420 \text{ mm}^2$

$y = 44.9 \text{ mm} \qquad d = 225 - 44.9 = 180.1 \text{ mm}$

$I_a = \bar{I} + Ad^2$

$= 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6 \text{ mm}^4$

$I = 2I_f + I_w + 4I_a = 1896 \times 10^6 \text{ mm}^4 = 1896 \times 10^{-6} \text{ m}^4$



$Q_f = (450)(20)(235) = 2115 \times 10^3 \text{ mm}^3$

$Q_a = (5420)(180.1) = 976 \times 10^3 \text{ mm}^3$

$Q = Q_f + 2Q_a = 4067 \times 10^3 \text{ mm}^3 = 4067 \times 10^{-6} \text{ m}^3$

$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(22)^2 = 380.1 \text{ mm}^2 = 380.1 \times 10^{-6} \text{ m}^2$

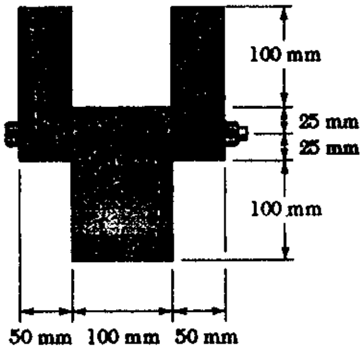
$F_{bolt} = 2\tau_{bolt} A_b = (2)(90 \times 10^6)(380.1 \times 10^{-6}) = 68.42 \times 10^3 \text{ N}$

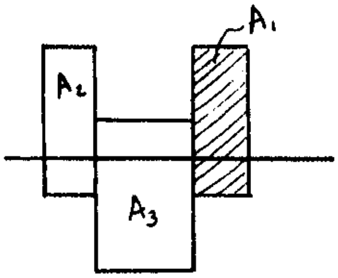
$q_{all} = \frac{F_{bolt}}{s} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3 \text{ N/m}$

$q = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3 \text{ N}$

$= 255 \text{ kN}$

PROBLEM 6.44





6.44 A beam consists of three planks connected by steel bolts with a longitudinal spacing of 225 mm. Knowing that the shear in the beam is vertical and equal to 6 kN and that the allowable average shearing stress in each bolt is 60 MPa, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}^2\,(10^6\,mm^4)$	$\bar{I}\,(10^6\,mm^4)$
①	7500	50	18.75	14.06
②	7500	50	18.75	14.06
③	15000	-50	37.50	28.12
Σ			75.00	56.25

$$I = \Sigma A\bar{y}^2 + \Sigma \bar{I} = 131.25 \times 10^6\ mm^4 = 131.25 \times 10^{-6}\ m^4$$

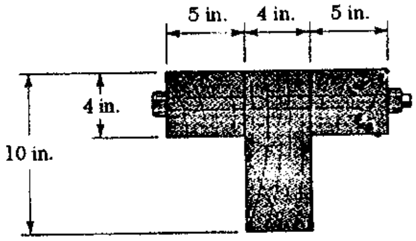
$$Q = A_1\bar{y}_1 = (7500)(50) = 375 \times 10^3\ mm^3 = 375 \times 10^{-6}\ m^3$$

$$F_{bolt} = \tau_{bolt} A_{bolt} = qs = \frac{VQs}{I}$$

$$A_{bolt} = \frac{VQs}{\tau_{bolt} I} = \frac{(6\times10^3)(375\times10^{-6})(0.225)}{(60\times10^6)(131.25\times10^{-6})} = 64.286\times10^{-6}\ m^2 = 64.286\ mm^2$$

$$d_{bolt} = \sqrt{\frac{4A_{bolt}}{\pi}} = \sqrt{\frac{(4)(64.286)}{\pi}} = 9.05\ mm$$

PROBLEM 6.45



6.45 and 6.46 Three planks are connected as shown by bolts of $\frac{3}{8}$-in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

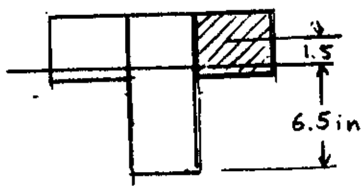
SOLUTION

Locate neutral axis.

$$\Sigma A = (2)(5)(4) + (4)(10) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(5)(4)(8) + (4)(10)(5) = 520 \text{ in}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 6.5 \text{ in}$$



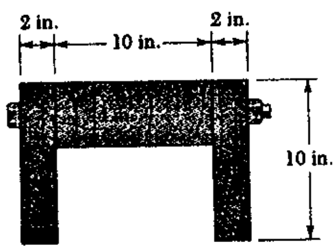
$$Q = (5)(4)(1.5) = 30 \text{ in}^3$$

$$I = 2\left[\tfrac{1}{12}(5)(4)^3 + (5)(4)(1.5)^2\right] + \tfrac{1}{12}(4)(10)^3 + (4)(10)(1.5)^2 = 566.7 \text{ in}^4$$

$$F = qs = \frac{VQs}{I} = \frac{(2.5)(30)(6)}{566.7} = 0.7941 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 \quad 0.1104 \text{ in}^2 \qquad \tau_{bolt} = \frac{F}{A_{bolt}} = \frac{0.7941}{0.1104} = 7.19 \text{ ksi}$$

PROBLEM 6.46



6.45 and 6.46 Three planks are connected as shown by bolts of $\frac{3}{8}$-in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

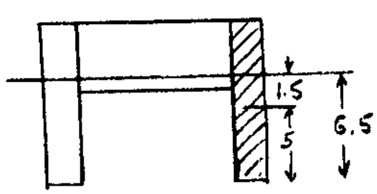
SOLUTION

Locate neutral axis

$$\Sigma A = (2)(2)(10) + (10)(4) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(2)(10)(5) + (10)(4)(8) = 520 \text{ in}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{520}{80} = 6.5 \text{ in}$$



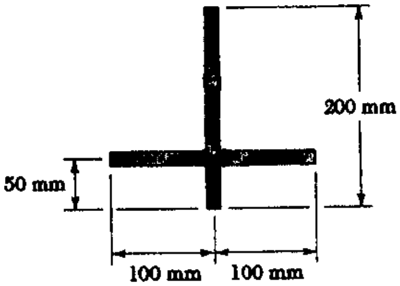
$$I = 2\left[\tfrac{1}{12}(2)(10)^3 + (2)(10)(1.5)^2\right] + \tfrac{1}{12}(10)(4)^3 + (10)(4)(1.5)^2 = 566.7 \text{ in}^4$$

$$Q = (2)(10)(1.5) = 30 \text{ in}^3$$

$$F = qs = \frac{VQs}{I} = \frac{(2.5)(30)(6)}{566.7} = 0.7941 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2 \qquad \tau_{bolt} = \frac{F}{A_{bolt}} = \frac{0.7941}{0.1104} = 7.19 \text{ ksi}$$

PROBLEM 6.47



6.47 Three plates, each 12-mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.

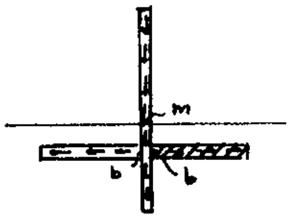
SOLUTION

Locate neutral axis

$$\Sigma A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^2$$

$$\Sigma A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 75.77 \text{ mm}$$



$$I = \tfrac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2\left[\tfrac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2\right]$$

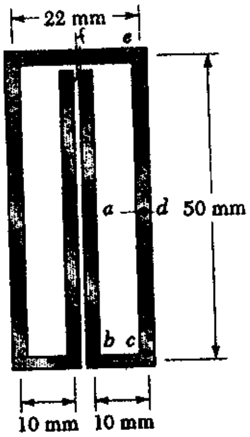
$$= 10.934 \times 10^6 \text{ mm}^3 = 10.934 \times 10^{-6} \text{ m}^3$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

PROBLEM 6.48

6.48 A plate of 2-mm thickness is bent as shown and then used as a beam. For a vertical shear of 5 kN, determine the shearing stress at the five points indicated and sketch the shear flow in the cross section.



SOLUTION

$$I = 2\left[\tfrac{1}{12}(2)(48)^3 + \tfrac{1}{12}(2)(52)^3 + \tfrac{1}{12}(20)(2)^3 + (20)(2)(25)^2\right]$$

$$= 133.76 \times 10^3\ \text{mm}^4 = 133.75 \times 10^{-9}\ \text{m}^3$$

$$Q_a = (2)(24)(12) = 576\ \text{mm}^3 = 576 \times 10^{-9}\ \text{m}^3$$

$$Q_b = 0$$

$$Q_c = Q_b - (12)(2)(25) = -600\ \text{mm}^3 = -600 \times 10^{-9}\ \text{m}^3$$

$$Q_d = Q_c - (2)(24)(12) = -1.176 \times 10^3\ \text{mm}^3 = -1.176 \times 10^{-6}\ \text{m}^3$$

$$Q_e = Q_d + (2)(26)(13) = -600\ \text{mm}^3 = -500 \times 10^{-9}\ \text{m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(5\times10^3)(576\times10^{-9})}{(133.75\times10^{-9})(2\times10^{-3})} = 10.77\times10^6\ \text{Pa} = 10.76\ \text{MPa}$$

$$\tau_b = \frac{VQ_b}{It} = 0$$

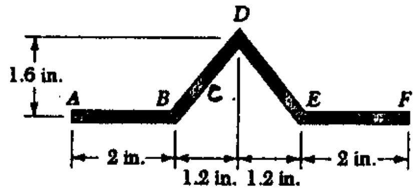
$$\tau_c = \frac{VQ_c}{It} = \frac{(5\times10^3)(600\times10^{-9})}{(133.75\times10^{-9})(2\times10^{-3})} = 11.21\times10^6\ \text{Pa} = 11.21\ \text{MPa}$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(5\times10^3)(1.176\times10^{-6})}{(133.75\times10^{-9})(2\times10^{-3})} = 22.0\times10^6\ \text{Pa} = 22.0\ \text{MPa}$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(5\times10^3)(500\times10^{-9})}{(133.75\times10^{-9})(2\times10^{-3})} = 9.35\times10^6\ \text{Pa} = 9.35\ \text{MPa}$$

PROBLEM 6.49

6.49 A plate of $\frac{1}{4}$-in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (*a*) the maximum shearing stress in the section, (*b*) the shearing stress at point *B*. Also sketch the shear flow in the cross section.



SOLUTION

$$L_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in}$$

$$A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

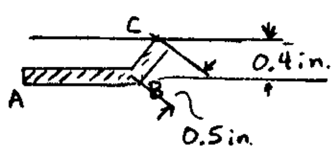
Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
AB	0.5	0	0	0.4	0.080	neglect
BD	0.5	0.8	0.4	0.4	0.080	*0.1067
DE	0.5	0.8	0.4	0.4	0.080	*0.1067
EF	0.5	0	0	0.4	0.080	neglect
Σ	2.0		0.8		0.320	0.2133

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{0.8}{2.0} = 0.4 \text{ in}$$

$$* \ \frac{1}{12} A_{BD} h^2 = \frac{1}{12}(0.5)(1.6)^2 = 0.1067 \text{ in}^4$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 0.5333 \text{ in}^4$$

(a)



$$Q_m = Q_{AB} + Q_{BC}$$

$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$

$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$

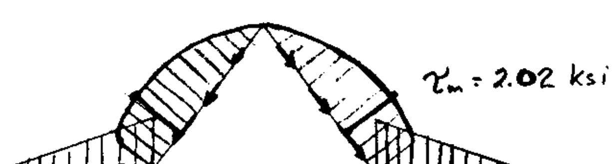
$$Q_m = 0.225 \text{ in}^3$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5333)(0.25)} = 2.025 \text{ ksi}$$

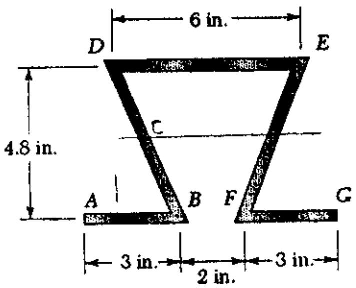
(b) $Q_B = Q_{AB} = 0.2 \text{ in}^3$

$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5333)(0.25)} = 1.80 \text{ ksi}$$

$$\tau_D = 0$$



PROBLEM 6.50



6.50 A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (a) the thickness t for which the maximum shearing stress is 300 psi, (b) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.

SOLUTION

$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2$ in.

Neutral axis lies at 2.4 in. above AB

Calculate I

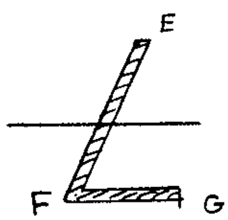
$$I_{AB} = (3t)(2.4)^2 = 17.28\,t$$
$$I_{BD} = \tfrac{1}{12}(5.2\,t)(4.8)^2 = 9.984\,t$$
$$I_{DE} = (6\,t)(2.4)^2 = 34.56\,t$$
$$I_{EF} = I_{DB} = 9.984\,t$$
$$I_{FG} = I_{AB} = 17.28\,t$$

$$I = \Sigma I = 89.09\,t$$

(a) At point C $\quad Q_c = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6\,t)(1.2) = 10.32\,t$

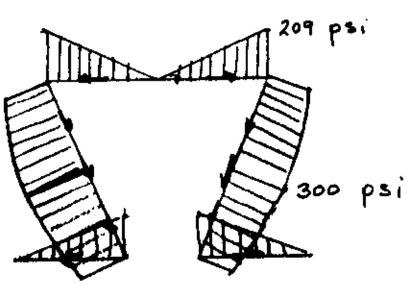
$$\tau = \frac{VQ_c}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(600)(10.32\,t)}{(300)(89.09\,t)} = 0.23168 \text{ in}$$

(b) $I = (89.09)(0.23168) = 20.64 \text{ in}^3$



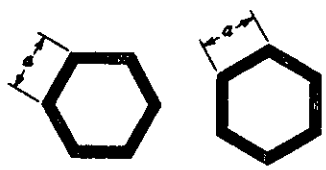
$$Q_E = Q_{EF} + Q_{FG}$$
$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^3$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)} = 209 \text{ psi}$$



PROBLEM 6.51

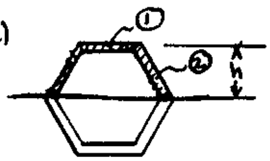
6.51 and 6.52 An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a) (b)

SOLUTION

(a)



$h = \frac{\sqrt{3}}{2} a$

$A_1 = A_2 = at$

$I_1 = A_1 h^2 = a t h^2 = \frac{3}{4} a^3 t$

$I_2 = \frac{1}{3} A_2 h^2 = \frac{1}{3} a t \frac{3}{4} a^2 = \frac{1}{4} a^3 t$

$I = 2I_1 + 4I_2 = \frac{5}{2} a^3 t$

$Q_1 = A_1 h = \frac{\sqrt{3}}{2} a^2 t$

$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4} a^2 t$

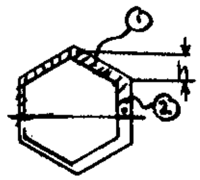
$Q_m = Q_1 + 2Q_2 = \sqrt{3} a^2 t$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \sqrt{3} a^2 t}{(\frac{5}{2} a^3 t) 2t} = \frac{\sqrt{3}}{5} \frac{V}{at}$$

$$= \frac{6\sqrt{3}}{5} \frac{V}{6at} = \frac{6\sqrt{3}}{5} \frac{V}{A} = k \frac{V}{A}$$

$k = \frac{6\sqrt{3}}{5} = 2.08$ ◀

(b)



$h = \frac{a}{2}$

$A_1 = at \quad A_2 = \frac{1}{2} at$

$$I_1 = \bar{I}_1 + A_1 d^2 = \frac{1}{12} a t h^2 + a t \left(\frac{a}{2} + \frac{h}{2}\right)^2 = \frac{1}{48} a^3 t + \frac{9}{16} a^3 t = \frac{7}{12} a^3 t$$

$I_2 = \frac{1}{3} t \left(\frac{a}{2}\right)^3 = \frac{1}{24} a^3 t$

$I = 4I_1 + 4I_2 = \frac{5}{2} a^3 t$

$Q_1 = at \left(\frac{a}{2} + \frac{h}{2}\right) = \frac{3}{4} a^2 t$

$Q_2 = \left(\frac{1}{2} at\right)\left(\frac{a}{4}\right) = \frac{1}{8} a^2 t$

$Q = 2Q_1 + 2Q_2 = \frac{7}{4} a^3 t$

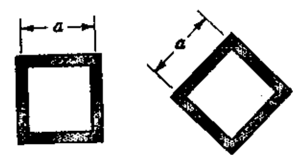
$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4} a^3 t}{(\frac{5}{2} a^3 t)(2t)}$$

$$= \frac{7}{20} \frac{V}{at} = \frac{42}{20} \frac{V}{6at} = \frac{21}{10} \frac{V}{A}$$

$= k \frac{V}{A} \qquad k = \frac{21}{10} = 2.10$ ◀

PROBLEM 6.52

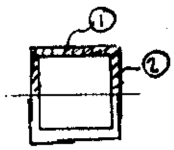
6.51 and 6.52 An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a) (b)

SOLUTION

(a)



$$I_1 = (at)\left(\tfrac{a}{2}\right)^2 = \tfrac{1}{4}a^3t$$

$$I_2 = \tfrac{1}{3}t\left(\tfrac{a}{2}\right)^3 = \tfrac{1}{24}a^3t$$

$$I = 2I_1 + 4I_2 = \tfrac{2}{3}a^3t$$

$$Q_1 = (at)\left(\tfrac{a}{2}\right) = \tfrac{1}{2}a^2t$$

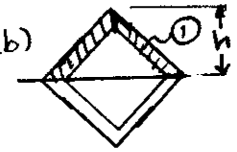
$$Q_2 = \left(\tfrac{1}{2}at\right)\left(\tfrac{a}{4}\right) = \tfrac{1}{8}a^2t$$

$$Q = Q_1 + 2Q_2 = \tfrac{3}{4}a^2t$$

$$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V(\tfrac{3}{4}a^2t)}{(\tfrac{2}{3}a^3t)(2t)} = \frac{9}{16}\frac{V}{at} = \frac{9}{4}\frac{V}{4at} = \frac{9}{4}\frac{V}{A}$$

$$= k\frac{V}{A} \quad \therefore \quad k = \tfrac{9}{4} = 2.25$$

(b)



$$h = \tfrac{1}{2}\sqrt{2}\,a$$

$$I_1 = \tfrac{1}{3}A_1h^2 = \left(\tfrac{1}{3}at\right)\left(\tfrac{\sqrt{2}}{2}a\right)^2 = \tfrac{1}{6}a^3t$$

$$I = 4I_1 = \tfrac{2}{3}a^3t$$

$$Q_1 = at\left(\tfrac{h}{2}\right) = \tfrac{1}{4}\sqrt{2}\,a^2t$$

$$Q = 2Q_1 = \tfrac{1}{2}\sqrt{2}\,a^2t$$

$$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V(\tfrac{1}{2}\sqrt{2}\,a^2t)}{(\tfrac{2}{3}a^3t)(2t)}$$

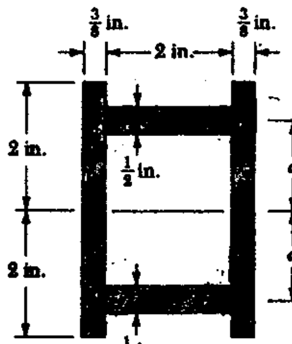
$$= \frac{3\sqrt{2}}{8}\frac{V}{at} = \frac{3\sqrt{2}}{2}\frac{V}{4at}$$

$$= \frac{3\sqrt{2}}{2}\frac{V}{A} = k\frac{V}{A}$$

$$k = \frac{3\sqrt{2}}{2} = 2.12$$

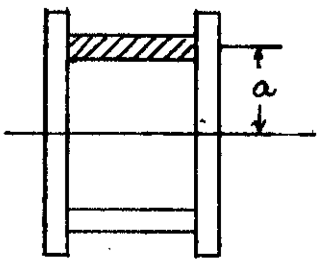
PROBLEM 6.53

6.53 The design of a beam calls for connecting two vertical rectangular $\frac{3}{8} \times 4$-in. plates by welding them to two horizontal $\frac{1}{2} \times 2$-in. plates as shown. For a vertical shear V, determine the dimension a for which the shear flow through the welded surfaces is maximum.



SOLUTION

$$I = (2)(\tfrac{1}{12})(\tfrac{3}{8})(4)^3 + (2)(\tfrac{1}{12})(2)(\tfrac{1}{2})^3 + (2)(2)(\tfrac{1}{2})\,a^2$$
$$= 4.041667 + 2a^2 \quad in^4$$



$$Q = (2)(\tfrac{1}{2})\,a = a \quad in^3$$

$$q = \frac{VQ}{I} = \frac{Va}{4.041667 + 2a^2} \qquad \text{Set } \frac{dq}{da} = 0$$

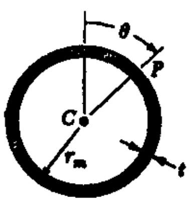
$$\frac{dq}{da} = \left[\frac{(4.041667 + 2a^2) - (a)(4a)}{(4.041667 + 2a^2)^2}\right] V = 0$$

$$2a^2 = 4.041667$$

$$a = 1.422 \text{ in.}$$

PROBLEM 6.54

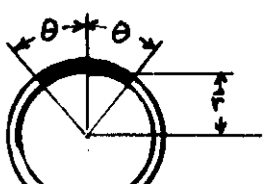
6.54 (*a*) Determine the shearing stress at point *P* of a thin-walled pipe of the cross section shown caused by a vertical shear V. (*b*) Show that the maximum shearing stress occurs for $\theta = 90°$ and is equal to $2V/A$, where *A* is the cross-sectional area of the pipe.



SOLUTION

$$A = 2\pi r_m t \qquad J = A r_m^2 = 2\pi r_m^3 t \qquad I = \tfrac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin\theta}{\theta} \quad \text{for a circular arc}$$



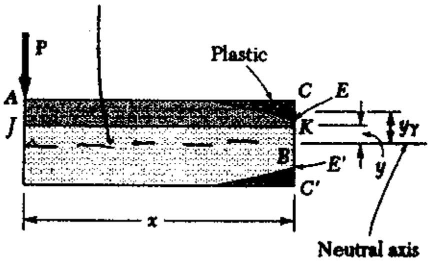
$$A_p = 2r\theta t$$

$$Q_p = A_p\bar{r} = 2rt\sin\theta$$

(a) $$\tau_p = \frac{VQ_p}{I(2t)} = \frac{(V)(2rt\sin\theta)}{(\pi r_m^3 t)(2t)} = \frac{V\sin\theta}{\pi r_m t}$$

(b) $$\tau_m = \frac{2V\sin\frac{\pi}{2}}{2\pi r_m t} = \frac{2V}{A}$$

PROBLEM 6.55



6.55 Consider the cantilever beam AB discussed in Sec. 6.8 and the portion $ACKJ$ of the beam that is located to the left of the transverse section CC' and above the horizontal plane JK, where K is a point at a distance $y < y_Y$ above the neutral axis (Fig. P6.55). (*a*) Recalling that $\sigma_x = \sigma_Y$ between C and E and $\sigma_x = (\sigma_Y / y_Y)y$ between E and K, show that the magnitude of the horizontal shearing force **H** exerted on the lower face of the portion of beam $ACKJ$ is

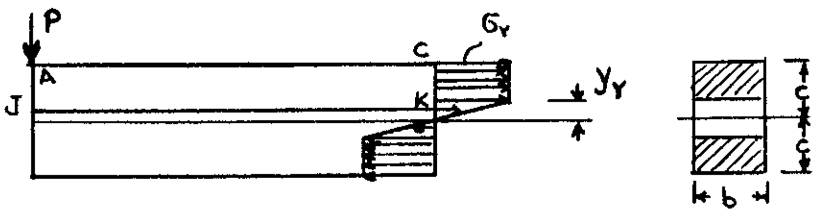
$$H = \tfrac{1}{2} b\sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y}\right)$$

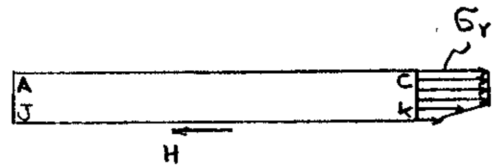
(*b*) Observing that the shearing stress at K is

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \to 0} \frac{1}{b}\frac{\Delta H}{\Delta x} = \frac{1}{b}\frac{\partial H}{\partial x}$$

and recalling that y_Y is a function of x defined by Eq. (6.14), derive Eq. (6.15).

SOLUTION





Point K is located a distance y above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_Y \frac{y}{y_Y} \quad \text{for } 0 \le y < y_Y \quad \text{and} \quad \sigma = \sigma_Y \quad \text{for } y_Y \le y \le c.$$

For equilibrium of horizontal forces acting on ACKJ

$$H = \int \sigma\, dA = \int_y^{y_Y} \frac{\sigma_Y y b}{y_Y}\, dy + \int_{y_Y}^{c} \sigma_Y b\, dy = \frac{\sigma_Y b}{y_Y}\left(\frac{y_Y^2 - y^2}{2}\right) + \sigma_Y b (c - y_Y)$$

$$= \tfrac{1}{2} b \sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y}\right) \quad \blacktriangleleft \text{ (a)}$$

Note that y_Y is a function of x

$$\tau_{xy} = \frac{1}{b}\frac{\partial H}{\partial x} = \tfrac{1}{2}\sigma_Y\left(-\frac{\partial y_Y}{\partial x} + \frac{y^2}{y_Y^2}\frac{dy_Y}{dx}\right) = -\tfrac{1}{2}\sigma_Y\left(1 - \frac{y^2}{y_Y^2}\right)\frac{dy_Y}{dx}$$

But $\quad M = Px = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right)$

Differentiating $\quad \dfrac{dM}{dx} = P = \frac{3}{2} M_Y \left(-\frac{2}{3}\frac{y_Y}{c^2}\frac{dy_Y}{dx}\right)$

$$\frac{dy_Y}{dx} = -\frac{Pc^2}{y_Y M_Y} = -\frac{Pc^2}{y_Y \frac{2}{3}\sigma_Y b c^2} = -\frac{3}{2}\frac{P}{\sigma_Y b y_Y}$$

Then $\quad \tau_{xy} = \tfrac{1}{2}\sigma_Y\left(1 - \frac{y^2}{y_Y^2}\right)\frac{3}{2}\frac{P}{\sigma_Y b \sigma_Y} = \dfrac{3P}{4 b y_Y}\left(1 - \dfrac{y^2}{y_Y^2}\right) \quad \blacktriangleleft \text{ (b)}$

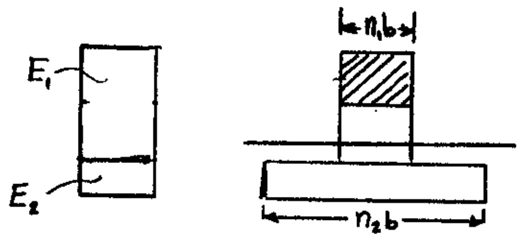
PROBLEM 6.56

6.56 For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

$$\tau = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed using the transformed section of the beam (see Sec. 4.6) and provided further that t is the actual width of the beam at the point where τ is computed.

SOLUTION



Actual Section   Transformed Section

Let E_{ref} be a reference modulus of elasticity

$$n_1 = \frac{E_1}{E_{ref}}, \quad n_2 = \frac{E_2}{E_{ref}}, \quad \text{etc.}$$

Widths b of actual section are multiplied by n's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section

The horizontal shearing force over length Δx is

$$\Delta H = -\int (\Delta\sigma_x)\, dA = \int \frac{n(\Delta M)y}{I}\, dA = \frac{(\Delta M)}{I}\int ny\, dA = \frac{Q(\Delta M)}{I}$$

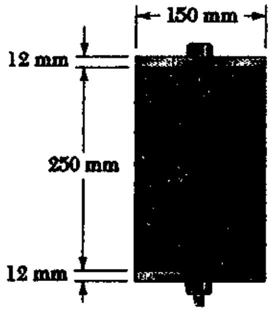
$Q = \int ny\, dA$ = first moment of transformed section.

Shear flow $\quad q = \dfrac{\Delta H}{\Delta x} = \dfrac{\Delta M}{\Delta x}\dfrac{Q}{I} = \dfrac{VQ}{I}$

q is distributed over actual width t, thus $\tau = \dfrac{q}{t}$

$$\tau = \frac{VQ}{It}$$

PROBLEM 6.57



6.57* and *6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. Determine the average shearing stress in the bolts caused by a vertical shearing force of 4 kN. (*Hint*. Use the method indicated in Prob. 6.56.)

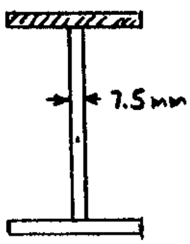
SOLUTION

Let $E_{ref} = E_s = 200$ GPa

$n_s = 1 \qquad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$

Widths of transformed section

$b_s = 150$ mm $\qquad b_w = (\frac{1}{20})(150) = 7.5$ mm



$$I = 2\left[\tfrac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right]$$
$$\tfrac{1}{12}(7.5)(250)^3$$
$$= 2\left[0.0216\times10^6 + 30.890\times10^6\right] + 9.766\times10^6$$
$$71.589 \times 10^6 \text{ mm}^4 = 71.589\times10^{-6} \text{ m}^4$$

$$Q = (150)(12)(125+6) = 235.8\times10^3 \text{ mm}^3$$
$$= 235.8\times10^{-6} \text{ mm}^3$$

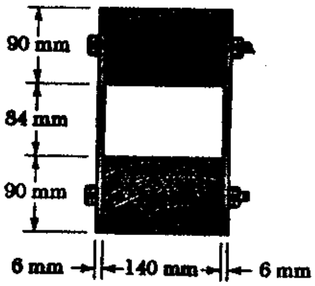
$$q = \frac{VQ}{I} = \frac{(4\times10^3)(235.8\times10^{-6})}{71.589\times10^{-6}} = 13.175\times10^3 \text{ N/m}$$

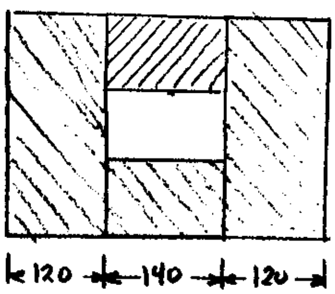
$$F_{bolt} = qs = (23.187\times10^3)(200\times10^{-3}) = 2.635\times10^3 \text{ N}$$

$$A_{bolt} = \frac{\pi}{4}d_{bolt}^2 = (\tfrac{\pi}{4})(12)^2 = 113.1 \text{ mm}^2 = 113.1\times10^{-6} \text{ m}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.635\times10^3}{113.1\times10^{-6}} = 23.3\times10^6 \text{ Pa} = 23.3 \text{ MPa}$$

PROBLEM 6.58





6.57* and *6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. Determine the average shearing stress in the bolts caused by a vertical shearing force of 4 kN. (*Hint.* Use the method indicated in Prob. 6.56.)

SOLUTION

Let wood be the reference material

$$n_w = 1.0 \qquad n_s = \frac{E_s}{E_w} = \frac{200\text{ GPa}}{10\text{ GPa}} = 20$$

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(380)(264)^3 - \frac{1}{12}(140)(84)^3 = 575.7\times10^6\ \text{mm}^4$$

$$= 575.7\times10^{-6}\ \text{m}^4$$

$$Q = (140)(90)(42+45) = 1.0962\times10^6\ \text{mm}^3$$

$$= 1.096\times10^{-3}\ \text{m}^3$$

$$q = \frac{VQ}{I} = \frac{(4\times10^3)(1.096\times10^{-3})}{575.7\times10^{-6}} = 7.615\times10^3\ \text{N/m}$$

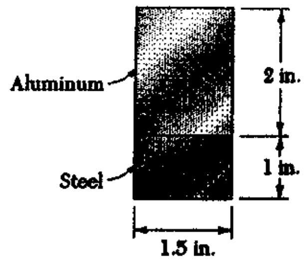
$$F_{bolt} = qs = (7.615\times10^3)(200\times10^{-3}) = 1.523\times10^3\ \text{N}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(12)^2 = 113.1\ \text{mm}^2 = 113.1\times10^{-6}\ \text{m}^2$$

Double shear $\quad \tau_{bolt} = \dfrac{F_{bolt}}{2A_{bolt}} = \dfrac{1.523\times10^3}{(2)(113.1\times10^{-6})} = 6.73\times10^6\ \text{Pa}$

$$= 6.73\ \text{MPa}$$

PROBLEM 6.59

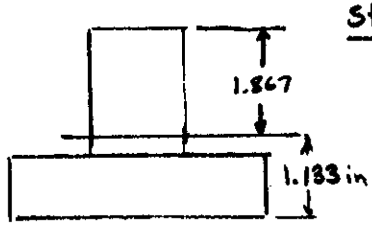


6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum stress in the beam. (*Hint.* Use the method indicated in Prob. 6.56.)

SOLUTION

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in²)	$\bar{y}$ (in)	$nA\bar{y}$ (in³)	d (in)	nAd^2 (in⁴)	$n\bar{I}$ (in⁴)
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$$

(a) At the bonded surface $\qquad Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

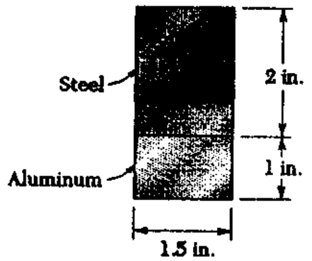
$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} = 1.323 \text{ ksi}$$

(b) At the neutral axis $\qquad Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^2$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2414)(1.5)} = 1.329 \text{ ksi}$$

PROBLEM 6.60

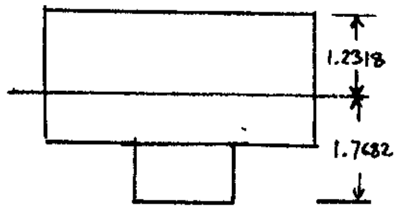
6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum stress in the beam. (*Hint.* Use the method indicated in Prob. 6.56.)



SOLUTION

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in²)	$\bar{y}$ (in)	$nA\bar{y}$ (in³)	d (in)	nAd^2 (in⁴)	$n\bar{I}$ (in⁴)
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$

(a) At the bonded surface $\qquad Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

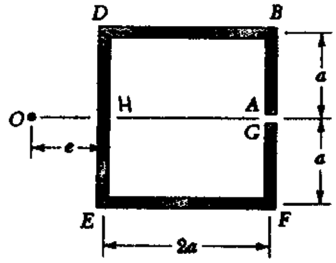
$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888 \text{ ksi} \blacktriangleleft$$

(b) At the neutral axis $\qquad Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453 \text{ ksi} \blacktriangleleft$$

PROBLEM 6.61

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



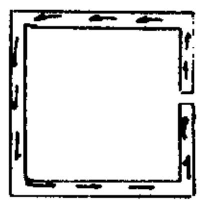
SOLUTION

$I_{AB} = I_{FG} = \frac{1}{3}ta^3 \qquad I_{DB} = I_{EF} = 2ata^3 + \frac{1}{12}2at\,t^3 \approx 2ta^3$

$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \qquad I = \Sigma I = \frac{16}{3}ta^3$

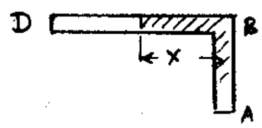
Part AB $\qquad A = ty \qquad \bar{y} = \frac{y}{2} \qquad Q = \frac{1}{2}ty^2$

$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{1}{2}ty^2}{\frac{16}{3}ta^3t} = \frac{3Vy^2}{32a^3t}$$



$$F_1 = \int \tau\, dA = \int_0^a \tau t\, dy = \frac{3V}{32a^3}\int_0^a y^2\, dt = \frac{1}{32}V$$

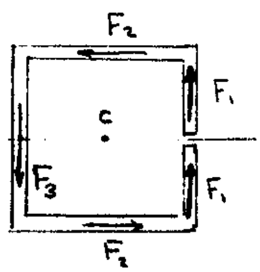
Part BD



$$Q = Q_B + txa = \frac{1}{2}ta^2 + tax$$

$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3}a^3t}\left(\frac{1}{2}a^2 + ax\right)$$

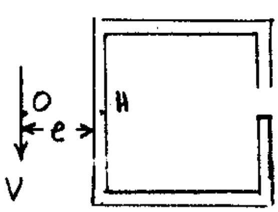
$$= \frac{3V}{32a^2}(a + 2x)$$



$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{3V}{32a^2}(a + 2x)\, dx$$

$$= \frac{3V}{32a^2}(ax + x^2)\Big|_0^{2a} = \frac{3V}{32a^2}(2a^2 + 4a^2) = \frac{9}{16}V$$

$$+\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H$$

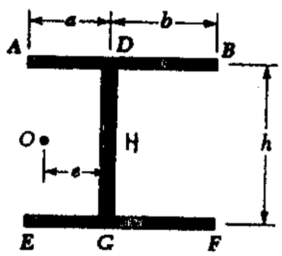


$$Ve = (2a)(2F_1) + (2a)(F_2)$$

$$= \frac{1}{8}Va + \frac{9}{8}Va = \frac{5}{4}Va \qquad e = \frac{5}{4}a$$

PROBLEM 6.62

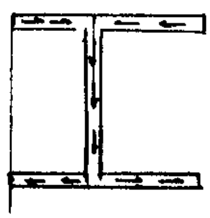
6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



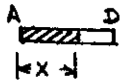
SOLUTION

$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$



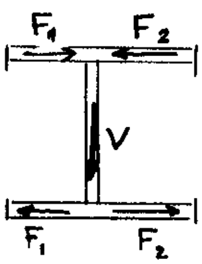
Part AD $\qquad Q = tx\frac{h}{2} = \frac{1}{2}thx$



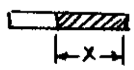
$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^a x\,dx$$

$$= \frac{Vht}{2I}\left.\frac{x^2}{2}\right|_0^a = \frac{Vhta^2}{4I}$$



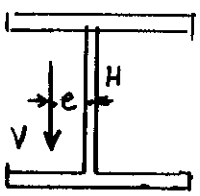
Part BD $\qquad Q = tx\frac{h}{2} = \frac{1}{2}thx$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^b x^2\,dx$$

$$= \frac{Vht}{2I}\left.\frac{x^2}{2}\right|_0^b = \frac{Vhtb^2}{4I}$$



$$\circlearrowright \Sigma M_H = \circlearrowright \Sigma M_H$$

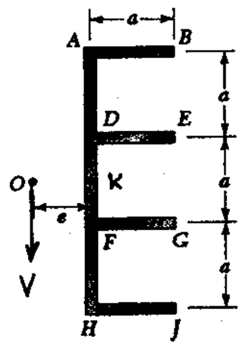
$$Ve = F_2 h - F_1 h = \frac{Vh^2t(b^2-a^2)}{4I}$$

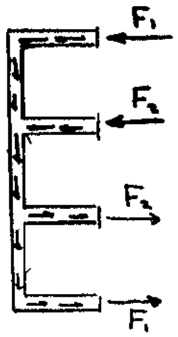
$$= \frac{Vh^2t(b^2-a^2)}{4\cdot\frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2-a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2-a^2)}{6(a+b)+h}$$

PROBLEM 6.63

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.





SOLUTION

$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^3$$

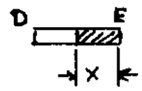
$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3 \qquad I = \Sigma I = \frac{29}{4}ta^3$$

Part AB $\qquad A = tx \qquad \bar{y} = \frac{3a}{2} \qquad Q = \frac{3}{2}atx$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2}atx}{\frac{29}{4}ta^3 t} = \frac{6Vx}{29a^2t}$$

$$F_1 = \int \tau dA = \int_0^a \frac{6Vx}{29a^2t}t\,dx = \frac{6V}{29a^2}\int_0^a x\,dx = \frac{3}{29}V$$

Part DE $\qquad A = tx \qquad \bar{y} = \frac{a}{2} \qquad Q = \frac{1}{2}atx$



$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}atx}{\frac{29}{4}ta^3 t} = \frac{2Vx}{29a^2t}$$

$$F_2 = \int \tau dA = \int_0^a \frac{2Vx}{29a^2t}t\,dx = \frac{2V}{29a^2}\int_0^a x\,dx = \frac{1}{29}V$$

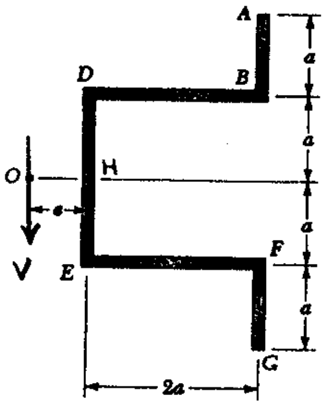
$$\circlearrowright \Sigma M_K = \circlearrowright \Sigma M_K$$

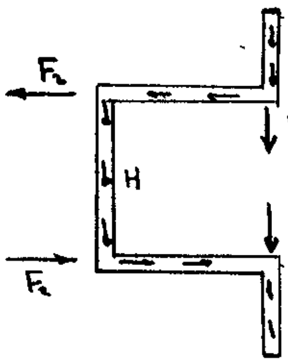
$$Ve = F_1(3a) + F_2(a) = \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a$$

PROBLEM 6.64

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.





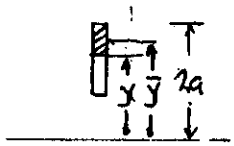
SOLUTION

$$I_{AB} = I_{FG} = \tfrac{1}{12} t a^3 + (ta)\left(\tfrac{3a}{2}\right)^2 = \tfrac{7}{3} t a^3$$

$$I_{DB} = I_{EF} = (2at)a^2 + \tfrac{1}{12}(2a)t^3 \approx 2a^3 t$$

$$I_{DE} = \tfrac{1}{12} t (2a)^3 = \tfrac{2}{3} t a^3 \qquad I = \Sigma I = \tfrac{28}{3} t a^3$$

Part AB $\qquad A = t(2a-y), \quad \bar{y} = \frac{2a+y}{2}$



$$Q = A\bar{y} = \tfrac{1}{2} t (2a-y)(2a+y) = \tfrac{1}{2} t (4a^2 - y^2)$$

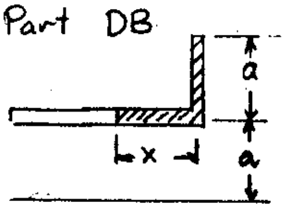
$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$

$$F_1 = \int \tau \, dA = \int_a^{2a} \frac{V}{2I}(4a^2 - y^2)\, t\, dy$$

$$= \frac{Vt}{2I}\left(4a^2 y - \frac{y^3}{3}\right)\Big|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\frac{Vta^3}{I} = \frac{5}{56} V$$

Part DB



$$Q = (ta)\frac{3a}{2} + txa = ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$

$$F_2 = \int \tau \, dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right) t\, dx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right) dx$$

$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Big|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

$$= 5\frac{Vta^3}{I} = \frac{15}{28} V$$

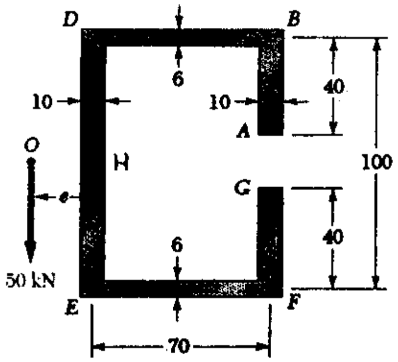
$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H$$

$$Ve = F_2(2a) - 2F_1(2a) = \frac{30}{28} Va - \frac{20}{56} Va = \frac{5}{7} Va$$

$$e = \frac{5}{7} a$$

PROBLEM 6.65

6.65 and 6.66 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a 50-kN vertical shearing force applied at *O*.



Dimensions in mm

SOLUTION

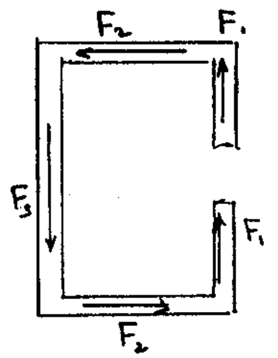
All quantities in mm, mm², etc.

$$I_{AB} = \tfrac{1}{12}(10)(40)^3 + (10)(40)(30)^2 = 0.41333 \times 10^6 \text{ mm}^4$$

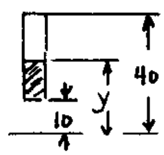
$$I_{DB} \approx (70)(6)(50)^2 + \tfrac{1}{12}(70)(6)^3 = 1.05126 \times 10^6 \text{ mm}^4$$

$$I_{DE} = \tfrac{1}{12}(10)(100)^3 = 0.83333 \times 10^6 \text{ mm}^4$$

$$I = \Sigma I = 3.7625 \times 10^6 \text{ mm}^4$$



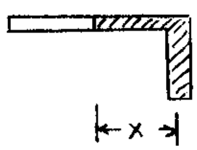
Part AB:



$$A = 10(y-10)$$
$$\bar{y} = \tfrac{1}{2}(y+10)$$
$$Q = A\bar{y} = 5(y-10)(y+10) = 5(y^2 - 100)$$
$$\tau = \frac{VQ}{It} \qquad Q_B = 5(50^2 - 100) = 12 \times 10^3 \text{ mm}^3$$
$$F_1 = \int \tau\, dA = \int_{10}^{50} \frac{V\, 5(y^2-100)}{I\, t}\, t\, dy$$

$$\frac{F_1}{V} = \frac{5}{I}\cdot\int_{10}^{50}(100-y^2)\,dy = \frac{5}{I}\cdot\left(\frac{y^3}{3} - 100y\right)\Bigg|_{10}^{50} = \frac{5}{I}\left[\frac{50^3}{3} - (100)(50) - \frac{10^3}{3} + (100)(10)\right]$$
$$= \frac{(5)(36.667\times10^3)}{3.7625\times10^6} = 0.048726$$

Part DB



$$Q = Q_B + (6x)(50) = 12\times10^3 + 300x$$
$$\tau = \frac{VQ}{It} \qquad Q_D = 12\times10^3 + (300)(70) = 33\times10^3 \text{ mm}^3$$
$$F_2 = \int \tau\, dA = \int_0^{70} \frac{V(12\times10^3 + 300x)}{I\,t}\, t\, dx = \frac{V}{I}\int_0^{70}(12\times10^3 + 300x)\,dx$$
$$\frac{F_2}{V} = \frac{1}{I}\left[(12\times10^3)x + 300\frac{x^2}{2}\right]_0^{70} = \frac{(12\times10^3)(70) + (300)(70^2)/2}{3.7625\times10^6}$$
$$= 0.41860$$

$$\circlearrowright \Sigma M_H = \circlearrowright M_H \qquad Ve = 2F_1(70) + F_2(100) = (2)(0.048726V)(70) + (0.41860V)(100)$$
$$= 48.7\,V \qquad e = 48.7 \text{ mm} \blacktriangleleft$$

At point H $\quad Q_H = Q_D + (10)(50)(25) = 33\times10^3 + 12.5\times10^3 = 45.5\times10^3$ mm

continued

PROBLEM 6.65 (continued)

$V = 50 \times 10^3$ N $\qquad I = 3.7625 \times 10^6\ mm^4 = 3.7625 \times 10^{-6}\ m^4$

Part AB, Point A $\qquad Q = 0 \qquad \tau = 0$

Part AB, Point B $\qquad Q = Q_B = 12 \times 10^3\ mm^3 = 12 \times 10^{-6}\ m^3, \qquad t = 10 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(12 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 15.95 \times 10^6\ Pa = 15.95\ MPa$$

Part BD, Point B $\qquad Q = 12 \times 10^{-6}\ m^3 \qquad t = 6 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(12 \times 10^{-6})}{(3.7625 \times 10^{-6})(6 \times 10^{-3})} = 26.6 \times 10^6\ Pa = 26.6\ MPa$$

Part BD, Point D $\qquad Q = 33 \times 10^{-6}\ m^3 \qquad t = 6 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(33 \times 10^{-6})}{(3.7625 \times 10^{-6})(6 \times 10^{-3})} = 73.1 \times 10^6\ Pa = 73.1\ MPa$$

Part DE, Point D $\qquad Q = 33 \times 10^{-6}\ m^3 \qquad t = 10 \times 10^{-3}$ m

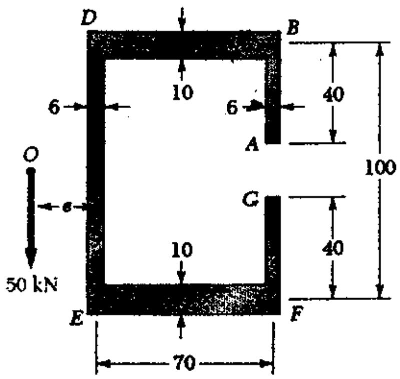
$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(33 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 43.9 \times 10^6\ Pa = 43.9\ MPa$$

Point H $\qquad Q = 45.5 \times 10^{-6}\ m^3 \qquad t = 10 \times 10^{-3}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(45.5 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 60.5 \times 10^6\ Pa = 60.5\ MPa$$

PROBLEM 6.66

6.65 and 6.66 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a 50-kN vertical shearing force applied at *O*.



Dimensions in mm

SOLUTION

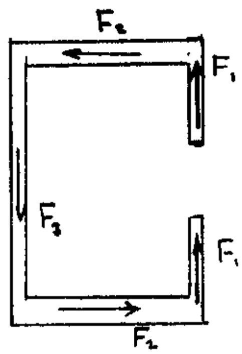
All quantities in mm, mm², etc

$$I_{AB} = \tfrac{1}{12}(6)(40)^3 + (6)(40)(30)^2 = 0.248\times10^6 \text{ mm}^4$$

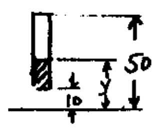
$$I_{BD} = (70)(10)(50)^2 + \tfrac{1}{12}(70)(10)^3 = 1.75583\times10^6 \text{ mm}^4$$

$$I_{DE} = \tfrac{1}{12}(6)(100)^3 = 0.500\times10^6 \text{ mm}^4$$

$$I = \Sigma I = 4.5076\times10^6 \text{ mm}^4$$



Part AB: $A = 6(y-10) \quad \bar{y} = \tfrac{1}{2}(y+10)$



$$Q = A\bar{y} = 3(y-10)(y+10) = 3(y^2-100)$$

$$Q_B = 3(50^2-100) = 7.2\times10^3 \text{ mm}^3$$

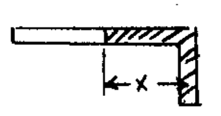
$$\tau = \frac{VQ}{It}$$

$$F_1 = \int \tau\, dA = \int_{10}^{50} \frac{V\,3(y^2-100)}{I\cancel{t}}\cancel{t}\,dy = \frac{3V}{I}\int_{10}^{50}(y^2-100)\,dy$$

$$\frac{F_1}{V} = \frac{3}{I}\int_{10}^{50}(y^2-100)\,dy = \frac{3}{I}\left(\frac{y^3}{3}-100y\right)\Bigg|_{10}^{50} = \frac{3}{I}\left[\frac{50^3}{3}-(100)(50)-\frac{10^3}{3}+(100)(10)\right]$$

$$= \frac{(3)(36.667\times10^3)}{4.5076\times10^6} = 0.02440$$

Part DB: $Q = Q_B + (10x)(50) = 7.2\times10^3 + 500x$



$$\tau = \frac{VQ}{It} \qquad Q_D = 7.2\times10^3 + (500)(70) = 42.2\times10^3 \text{ mm}^3$$

$$F_2 = \int \tau\, dA = \int_0^{70} \frac{V(7.2\times10^3+500x)}{I\cancel{t}}\cancel{t}\,dx$$

$$\frac{F_2}{V} = \frac{1}{I}\int_0^{70}[(7.2\times10^3)+500x]\,dx \quad \frac{1}{I}\left[(7.2\times10^3)x + 500\frac{x^2}{2}\right] = \frac{1}{I}\left[(7.2\times10^3)(70)+(500)\frac{(70)^2}{2}\right]$$

$$= \frac{1.729\times10^4}{4.5076\times10^6} = 0.38357$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft M_H \qquad Ve = (2F_1)(70)+(F_2)(100) = (2)(0.02440V)(70)+(0.38357V)(100)$$

$$= 41.8\ V \qquad e = 41.8 \text{ mm}$$

At point H $\quad Q_H = Q_D + (6)(50)(25) = 42.2\times10^3 + 7.5\times10^3 = 49.7\times10^3 \text{ mm}^3$

continued

PROBLEM 6.66 (continued

$V = 50\times10^3$ N $\quad I = 4.5076\times10^6\ \text{mm}^4 = 4.5076\times10^{-6}\ \text{m}^4$

Point A $\quad Q = 0 \quad \tau = 0$

Part AB, Point B $\quad Q_B = 7.2\times10^{-6}\ \text{m}^3 \quad t = 6\times10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50\times10^3)(7.2\times10^{-6})}{(4.5076\times10^{-6})(6\times10^{-3})} = 13.31\times10^6\ \text{Pa} = 13.31\ \text{MPa}$$

Part BD, Point B $\quad Q = 7.2\times10^{-6}\ \text{m}^3 \quad t = 10\times10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50\times10^3)(7.2\times10^{-6})}{(4.5076\times10^{-6})(10\times10^{-3})} = 7.99\times10^6\ \text{Pa} = 7.99\ \text{MPa}$$

Part BD, Point D $\quad Q = 42.2\times10^{-6}\ \text{m}^3 \quad t = 10\times10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50\times10^3)(42.2\times10^{-6})}{(4.5076\times10^{-6})(10\times10^{-3})} = 46.8\times10^6\ \text{Pa} = 46.8\ \text{MPa}$$

Part DE, Point D $\quad Q = 42.2\times10^{-6}\ \text{m}^3 \quad t = 6\times10^{-3}$

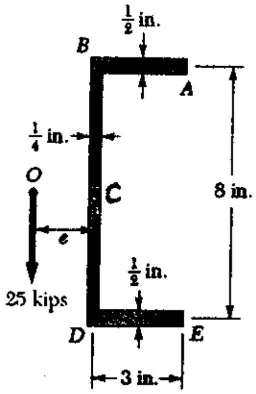
$$\tau = \frac{VQ}{It} = \frac{(50\times10^3)(42.2\times10^{-6})}{(4.5076\times10^{-6})(6\times10^{-3})} = 78.0\times10^6\ \text{Pa} = 78.0\ \text{MPa}$$

Point H $\quad Q = 49.7\times10^{-6}\ \text{m}^3 \quad t = 6\times10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50\times10^3)(49.7\times10^{-6})}{(4.5076\times10^{-6})(6\times10^{-3})} = 91.9\times10^6\ \text{Pa} = 91.9\ \text{MPa}$$

PROBLEM 6.67

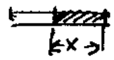
6.67* and *6.68 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at *O*.



SOLUTION

$$I = 2\left[\tfrac{1}{12}(3)(\tfrac{1}{2})^3 + (3)(\tfrac{1}{2})(4)^2\right] + \tfrac{1}{12}(\tfrac{1}{4})(8)^3 = 58.729 \text{ in}^4$$

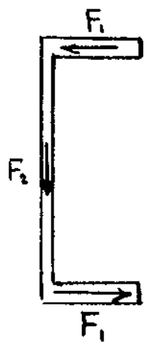
Part AB $\quad A = \tfrac{1}{2}x, \quad \bar{y} = 4, \quad Q = A\bar{y} = 2x$



$$\tau = \frac{VQ}{It} = \frac{(25)(2x)}{(58.729)(\frac{1}{2})} = 1.7027x$$

Point A $\quad x = 0 \qquad \tau = 0$

Point B $\quad x = 3 \qquad \tau = 5.11$ ksi



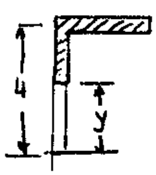
$$F_1 = \int \tau \, dA = \int_0^3 1.7027x \cdot \tfrac{1}{2}\,dx = \frac{1.7027}{4}x^2\Big|_0^3$$

$$= \frac{(1.7027)(3)^2}{4} = 3.8311 \text{ kips}$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H \qquad 25e = (F_1)(8)$$

$$e = \frac{(3.8311)(8)}{25} = 1.226 \text{ in}$$

Part BD $\quad Q = (2)(3) + (\tfrac{1}{4})(4-y)(\tfrac{4+y}{2})$



$$= 6 + \tfrac{1}{8}(16 - y^2) = 8 - \tfrac{1}{8}y^2$$

$$\tau = \frac{VQ}{It} = \frac{25(8 - \frac{1}{8}y^2)}{(58.729)(\frac{1}{4})}$$

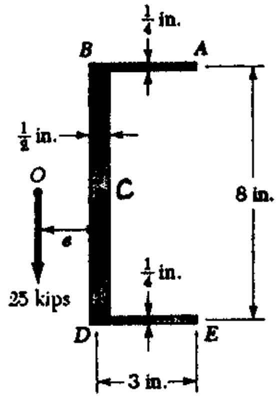
$$= 13.622 - 0.2128y^2$$

Point B $\quad y = 4$ in $\qquad \tau = 10.22$ ksi

Point C $\quad y = 0 \qquad \tau = 13.62$ ksi

PROBLEM 6.68

6.67 and 6.68 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at *O*.



SOLUTION

$$I = 2\left[\tfrac{1}{12}(3)(\tfrac{1}{4})^3 + (3)(\tfrac{1}{4})(4)^2\right] + \tfrac{1}{12}(\tfrac{1}{2})(8)^3 = 45.341 \text{ in}^4$$

Part AB $\quad A = \frac{1}{4}x, \quad \bar{y} = 4 \quad Q = A\bar{y} = x$

$$\tau = \frac{VQ}{It} = \frac{(25)(x)}{(45.341)(\frac{1}{4})} = 2.2055\,x$$

Point A $\quad x = 0 \quad \tau = 0$

Point B $\quad x = 3 \quad \tau = 6.62$ ksi

$$F_1 = \int \tau\, dA = \int_0^3 (2.2055\,x)\tfrac{1}{4}\,dx = \frac{2.2055}{4}\,\frac{x^2}{2}\Big|_0^3$$

$$= \frac{(2.2055)(3)^2}{(4)(2)} = 2.4812 \text{ kips}$$

$$+\circlearrowright M_H = +\circlearrowright M_H \qquad 25e = F_1(8)$$

$$e = 0.794 \text{ in.}$$

Part BD $\quad Q = 3 + \frac{1}{2}(4-y)\frac{4+y}{2} =$

$$= 3 + \tfrac{1}{4}(16-y^2) = 7 - \tfrac{1}{4}y^2$$

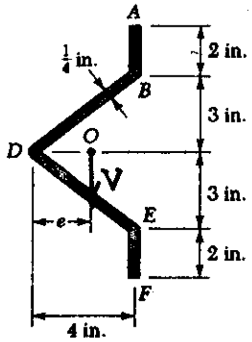
$$\tau = \frac{VQ}{It} = \frac{(25)(7-\frac{1}{4}y^2)}{(45.341)(\frac{1}{2})} = 7.7193 - 0.2757\,y^2$$

Point B $\quad y = 4 \quad \tau = 3.31$ ksi

Point C $\quad y = 0 \quad \tau = 7.72$ ksi

PROBLEM 6.69

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



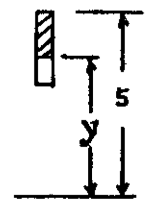
SOLUTION

$$L_{DB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{DB} = L_{DB} t = (5)(\tfrac{1}{4}) = 1.25 \text{ in}^2$$

$$I_{DB} = \frac{1}{3} A_{AB} h^2 = (\tfrac{1}{3})(1.25)(3)^2 = 3.75 \text{ in}^4$$

$$I_{AB} = \tfrac{1}{12}(\tfrac{1}{4})(2)^3 + (\tfrac{1}{4})(2)(4)^2 = 8.1667 \text{ in}^4$$

$$I = (2)(3.75) + (2)(8.1667) = 23.833 \text{ in}^4$$



Part AB : $A = \frac{1}{4}(5-y) \text{ in}^2, \quad \bar{y} = \frac{1}{2}(5+y) \text{ in}$

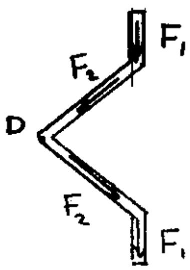
$$Q = A\bar{y} = \tfrac{1}{8}(5-y)(5+y) = \tfrac{1}{8}(25-y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(25-y^2)}{(8)(23.833)(0.25)} = \frac{V(25-y^2)}{47.667}$$

$$F_1 = \int \tau \, dA = \int_3^5 \frac{V(25-y^2)}{47.667} \cdot \frac{1}{4} \, dy$$

$$= \frac{V}{190.667}\left[25y - \tfrac{1}{3}y^3\right]\Big|_3^5 =$$

$$= \frac{V}{190.667}\left[(25)(5) - \tfrac{1}{3}(5)^3 - (25)(3) + \tfrac{1}{3}(3)^3\right] = 0.09091 V$$

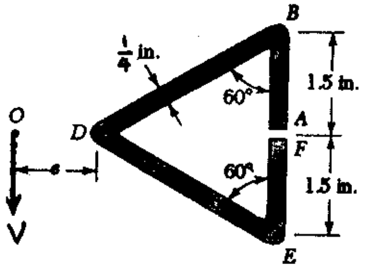


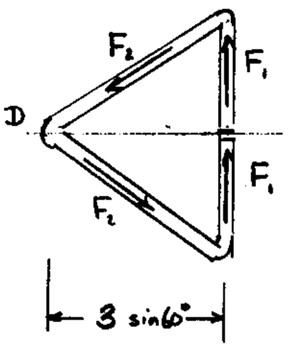
$$\circlearrowright M_D = \circlearrowright M_D \qquad -Ve = -2F_1(4) = -0.7273 V$$

$$e = 0.727 \text{ in.}$$

PROBLEM 6.70

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.





SOLUTION

$$I_{AB} = \frac{1}{3}\left(\frac{1}{4}\right)(1.5)^3 = 0.28125 \text{ in}^4$$

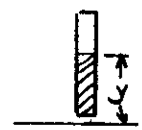
$$L_{BD} = 3 \text{ in} \qquad A_{BD} = (3)\left(\frac{1}{4}\right) = 0.75 \text{ in}^2$$

$$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$

Part AB: $\quad A = \frac{1}{4}y \quad \bar{y} = \frac{1}{2}y \quad Q = A\bar{y} = \frac{1}{8}y^2$

$$\tau = \frac{VQ}{It} = \frac{V\,y^2}{(8)(1.6875)(0.25)} = \frac{Vy^2}{3.375}$$



$$F_1 = \int \tau\, dA = \int_0^{1.5} \frac{Vy^2}{3.375}\cdot(0.25\,dy)$$

$$= \frac{(0.25)V}{3.375}\,\frac{y^3}{3}\Big|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.08333\ V$$

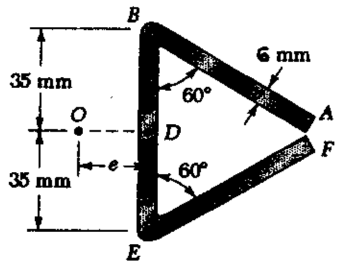
$$\circlearrowleft M_D = \circlearrowleft M_D \qquad Ve = 2\,F_1\,(3\sin 60°)$$

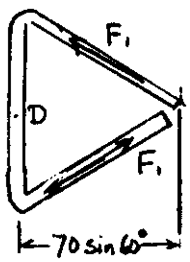
$$Ve = (2)(0.08333)V(3\sin 60°)$$

$$e = (2)(0.08333)(3\sin 60°) = 0.433 \text{ in.}$$

PROBLEM 6.71

6.69 through 6.74 Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.





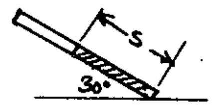
SOLUTION

$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$

$L_{AB} = 70 \text{ mm} \quad A_{AB} = (70)(6) = 420 \text{ mm}^2$

$I_{AB} = \frac{1}{3} A_{AB} h^2 = (\frac{1}{3})(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$

$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$



Part AB $\quad A = ts = 6s$

$\bar{y} = \frac{1}{2} s \sin 30° = \frac{1}{4} s$

$Q = A\bar{y} = \frac{3}{2} s^2$

$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$

$F_1 = \int \tau \, dA = \int_0^{70} \frac{3Vs^2}{2I\cancel{t}} \cancel{t} \, ds = \frac{3V}{I} \int_0^{70} s^2 \, ds$

$= \frac{(3)(70)^3}{(2)(3)\,I} V = \frac{1}{3} V$

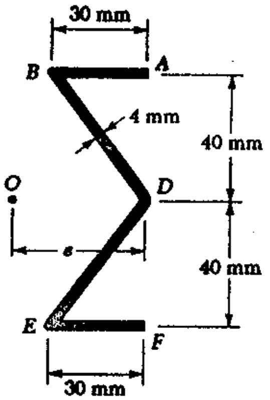
$+\circlearrowleft \Sigma M_D = +\circlearrowleft \Sigma M_D \quad Ve = 2[(F_1 \cos 60°)(70 \sin 60°)]$

$= 20.2 \; V$

$e = 20.2 \text{ mm}$

PROBLEM 6.72

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

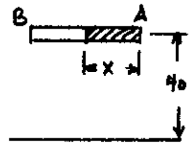
$$I_{AB} = (30)(4)(40)^2 = 192 \times 10^3 \text{ mm}^4$$

$$L_{BD} = \sqrt{30^2 + 40^2} = 50 \text{ mm} \qquad A_{BD} = (50)(4) = 200 \text{ mm}^2$$

$$I_{BD} = \tfrac{1}{3} A_{BD} h^2 = \tfrac{1}{3}(200)(40)^2 = 106.67 \times 10^3 \text{ mm}^4$$

$$I = 2 I_{BD} + 2 I_{AB} = 597.33 \times 10^3 \text{ mm}^4$$

Part AB $\qquad A = 4x \qquad \bar{y} = 40 \qquad Q = A\bar{y} = 160x$



$$\tau = \frac{VQ}{It} = \frac{V(160x)}{I(4)} = \frac{40V}{I}x$$

$$F_1 = \int \tau \, dA = \int_0^{30} \frac{40V}{I} 4\,dx = \frac{160V}{I}\int_0^{30} x\,dx$$

$$= \frac{160 V x^2}{2I}\Big|_0^{30} = \frac{(160)(30)^2}{(2)(597.33 \times 10^3)} V$$

$$= 0.12054 \, V$$

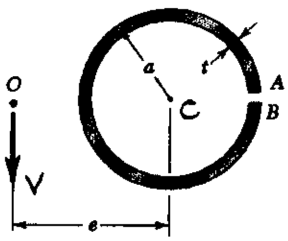
$$+\circlearrowright M_D = +\circlearrowright M_D \qquad Ve = (2)F_1(40) = 9.64 \, V$$

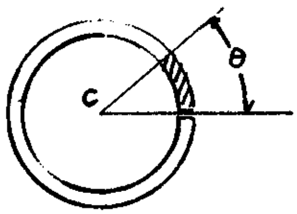
$$e = 9.64 \text{ mm}$$

PROBLEM 6.73

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION





For whole cross section $\quad A = 2\pi a t$

$$J = Aa^2 = 2\pi a^3 t \qquad I = \tfrac{1}{2}J = \pi a^3 t$$

Use polar coordinate θ for partial cross section.

$$A = st = a\theta t \qquad s = \text{arc length}$$

$$\bar{r} = a\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \tfrac{1}{2}\theta$$

$$\bar{y} = \bar{r}\sin\alpha = a\frac{\sin^2\alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t\, a\frac{\sin^2\alpha}{\alpha} = a^2 t\; 2\sin^2\alpha$$

$$= a^2 t\; 2\sin^2\frac{\theta}{2} = a^2 t(1-\cos\theta)$$

$$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1-\cos\theta)$$

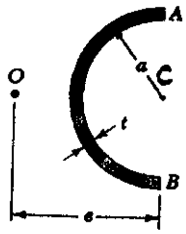
$$M_c = \int a\tau\, dA = \int_0^{2\pi}\frac{Va^3}{I}(1-\cos\theta)\,t\,a\,d\theta = \frac{Va^4 t}{I}(\theta - \sin\theta)\Big|_0^{2\pi}$$

$$= \frac{2\pi V a^4 t}{\pi a^3 t} = 2aV$$

But $M_c = Ve$, hence $e = 2a$ ◄

PROBLEM 6.74

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

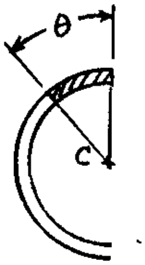


SOLUTION

For a thin-walled hollow circular cross section $A = 2\pi a t$

$J = a^2 A = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$

For the half-pipe section $I = \frac{\pi}{2} a^3 t$



Use polar coordinate θ for partial cross section

$A = st = a\theta t \qquad s = \text{arc length}$

$\bar{r} = a \frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{\theta}{2}$

$\bar{y} = \bar{r}\cos\alpha = a \frac{\sin\alpha \cos\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t \, a \frac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\,(2\sin\alpha\cos\alpha)$

$= a^2 t \sin 2\alpha = a^2 t \sin\theta$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}\sin\theta$

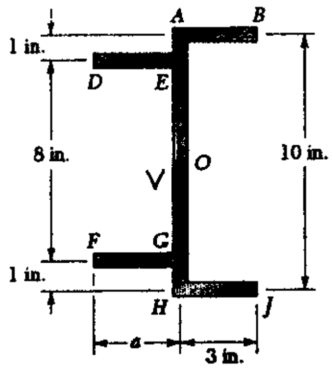
$M_c = \int a\tau\, dA = \int_0^\pi a \frac{Va^2}{I}\sin\theta \, t a\, d\theta = \frac{Va^4 t}{I} - \cos\theta \Big|_0^\pi$

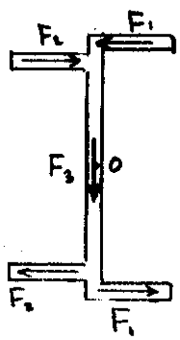
$= 2\frac{Va^4 t}{I} = \frac{4}{\pi} Va$

But $M_c = Ve$, hence $e = \frac{4}{\pi} a = 1.273\, a$

PROBLEM 6.75

6.75 and 6.76 A thin-walled beam of uniform thickness has the cross section shown Determine the dimension a for which the shear center O of the cross section is located at the point indicated.





SOLUTION

Part AB $\quad A = tx \quad \bar{y} = 5 \text{ in} \quad Q = A\bar{y} = 5tx$

$$\tau = \frac{VQ}{It} = \frac{V \cdot 5\cancel{t}x}{I\cancel{t}} = \frac{5Vx}{I}$$

$$F_1 = \int \tau \, dA = \int_0^3 \frac{5Vx}{I} t \, dx = \frac{5Vt}{I}\int_0^3 x \, dx$$

$$= \frac{(5)(3)^2}{2}\frac{Vt}{I} = 22.5 \frac{Vt}{I}$$

Part DE $\quad A = tx \quad \bar{y} = 4 \text{ in.} \quad Q = A\bar{y} = 4tx$

$$\tau = \frac{VQ}{It} = \frac{V\,4tx}{It} = \frac{4Vx}{I}$$

$$F_2 = \int \tau \, dA = \int_0^a \frac{4Vx}{I} \cdot t \, dx = \frac{4Vt}{I}\int_0^a x \, dx$$

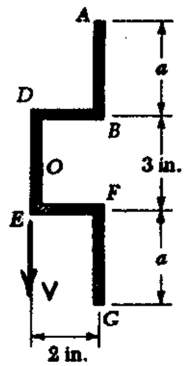
$$= \frac{2Vta^2}{I}$$

$$\circlearrowleft \Sigma M_o = \circlearrowleft \Sigma M_o \qquad 0 = (10)(22.5 \frac{Vt}{I}) - (8)\frac{2Vta^2}{I}$$

$$a^2 = \frac{(10)(22.5)}{(8)(2)} = 14.0625 \text{ in}^2 \qquad a = 3.75 \text{ in.}$$

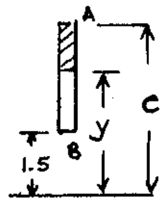
PROBLEM 6.76

6.75 and 6.76 A thin-walled beam of uniform thickness has the cross section shown Determine the dimension a for which the shear center O of the cross section is located at the point indicated.



SOLUTION

Part AB   Let $c = 1.5 + a$ as shown.



$$A = t(c-y) \qquad \bar{y} = \tfrac{1}{2}(c+y)$$

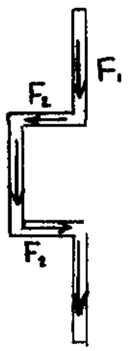
$$Q = A\bar{y} = \tfrac{1}{2}t(c-y)(c+y) = \tfrac{1}{2}t(c^2-y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(c^2-y^2)}{2I}$$

$$F_1 = \int \tau\, dA = \int_{1.5}^{c} \frac{V(c^2-y^2)}{2I}\, t\, dy = \frac{Vt}{2I}\int_{1.5}^{c}(c^2-y^2)\,dy$$

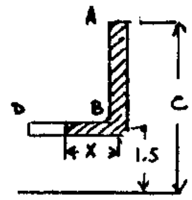
$$= \frac{Vt}{2I}\left(c^2y - \frac{y^3}{3}\right)\Big|_{1.5}^{c} = \frac{Vt}{2I}\left[c^3 - \frac{c^3}{3} - 1.5c^2 + \frac{(1.5)^3}{3}\right]$$

$$= \frac{Vt}{2I}\left[\tfrac{2}{3}c^3 - 1.5c^2 + 1.125\right]$$



Part BD $Q = Q_{AB} + t x \bar{y}_{BD}$

$$= \tfrac{1}{2}t(c^2 - 1.5^2) + t x (1.5)$$



$$\tau = \frac{VQ}{It} = \frac{V}{2I}(c^2 - 1.5^2 + 3x)$$

$$F_2 = \int \tau\, dA = \int_0^2 \frac{V}{2I}(c^2 - 1.5^2 + 3x)\, t\, dx$$

$$= \frac{Vt}{2I}\left[(c^2 - 1.5^2)x + 1.5x^2\right]\Big|_0^2 = \frac{Vt}{2I}\left[2c^2 - (2)(1.5)^2 + (1.5)(2)^2\right]$$

$$= \frac{Vt}{2I}\left[2c^2 + 1.5\right]$$

$$+\circlearrowright \Sigma M_O = +\circlearrowright \Sigma M_O \qquad 0 = 3F_2 - (2)(2)F_1$$

$$\frac{Vt}{2I}\left\{3(2c^2 + 1.5) - 4\left(\tfrac{2}{3}c^3 - 1.5c^2 + 1.125\right)\right] = 0$$

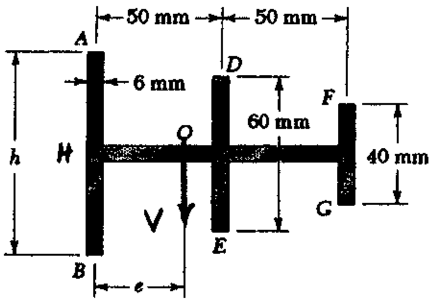
$$-\tfrac{8}{3}c^3 + 12c^2 = 0$$

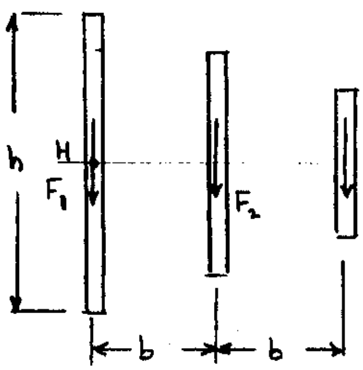
$$c = \frac{(12)(3)}{8} = 4.5 \text{ in}$$

$$a = 4.5 - 1.5 = 3.00 \text{ in.}$$

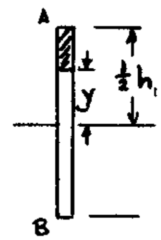
PROBLEM 6.77

6.77 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center O of the cross section, knowing that $h = 80$ mm.





SOLUTION



Let $h_1 = \overline{AB} = h$, $h_2 = \overline{DE}$, $h_3 = \overline{FG}$

$$I = \frac{1}{12} t\,(h_1^3 + h_2^3 + h_3^3)$$

Part AB: $A = (\frac{1}{2}h_1 - y)\,t$

$$\bar{y} = \frac{1}{2}(\frac{1}{2}h_1 + y)$$

$$Q = A\bar{y} = \frac{1}{2}t\,(\frac{1}{2}h_1 - y)(\frac{1}{2}h_1 + y) = \frac{1}{2}t\,(\frac{1}{4}h_1^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)$$

$$F_1 = \int \tau\, dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)\,t\,dy$$

$$= \frac{Vt}{2I}\left(\frac{1}{4}h_1^2 y - \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left(\frac{1}{4}h_1^2\,\frac{1}{2}h_1 - \frac{1}{3}(\frac{h_1}{2})^3\right) = \frac{Vt\,h_1^3}{12\,I}$$

$$= \frac{h_1^3\,V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE $\quad F_2 = \dfrac{h_2^3\,V}{h_1^3 + h_2^3 + h_3^3}$

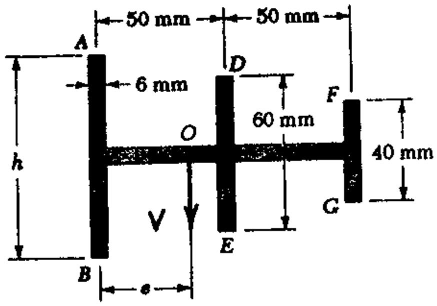
and for Part FG $\quad F_3 = \dfrac{h_3^3\,V}{h_1^3 + h_2^3 + h_3^3}$

$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H \qquad Ve = bF_2 + 2bF_3 = \frac{bh_2^3 + 2bh_3^3}{h_1^3 + h_2^3 + h_3^3}V$$

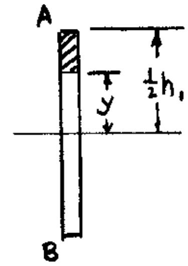
$$e = \frac{h_2^3 + 2h_3^3}{h_1^3 + h_2^3 + h_3^3}\,b = \frac{(60)^3 + (2)(40)^3}{(80)^3 + (60)^3 + (40)^3}(50) = 21.7 \text{ mm}$$

PROBLEM 6.78

6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension h for which the shear center O of the cross section is located at a distance e = 25 mm from the center of the flange AB.



SOLUTION



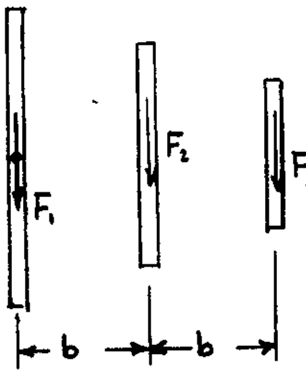
Let $h_1 = \overline{AB} = h$, $h_2 = \overline{DE}$, $h_3 = \overline{FG}$

$$I = \frac{1}{12}t\,(h_1^3 + h_2^3 + h_3^3)$$

Part AB $\quad A = (\frac{1}{2}h_1 - y)t$

$$\bar{y} = \frac{1}{2}(\frac{1}{2}h_1 + y)$$

$$Q = A\bar{y} = \frac{1}{2}t(\frac{1}{2}h_1 - y)(\frac{1}{2}h_1 + y) = \frac{1}{2}t(\frac{1}{4}h_1^2 - y^2)$$



$$\tau = \frac{VQ}{It} = \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)$$

$$F_1 = \int \tau\, dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)\,t\,dy$$

$$= \frac{Vt}{2I}(\frac{1}{4}h_1^2 y - \frac{1}{3}y^3)\Big|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left[(\frac{1}{4}h_1^2)(\frac{1}{2}h_1) - \frac{1}{3}(\frac{1}{2}h_1)^3\right] = \frac{Vt\,h_1^3}{12\,I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE $\quad F_2 = \dfrac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$

and for Part FG $\quad F_3 = \dfrac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$

$$+\circlearrowleft \Sigma M_O = \circlearrowleft \Sigma M_O \qquad 0 = eF_1 - (b-e)F_2 - (2b-e)F_3$$

$$\frac{eh_1^3 - (b-e)h_2^3 - (2b-e)h_3^3}{h_1^3 + h_2^3 + h_3^3}V = 0$$

$$h_1^3 = \frac{b-e}{e}h_2^3 + \frac{(2b-e)}{e}h_3^3 = \frac{(25)(60)^3}{25} + \frac{(75)(40)^3}{25} = 408 \times 10^3 \text{ mm}^3$$

$$h_1 = 74.2 \text{ mm}$$

PROBLEM 6.79

6.79 For the angle shape and loading of Sample Prob. 6.5, check that $\int q\,dz = 0$ along the horizontal leg of the angle and $\int q\,dy = P$ along its vertical leg.

SOLUTION

Referring to Sample Prob. 6.5

Along horizontal leg $\quad \tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

$$\int q\,dz = \int_0^a \tau_f t\,dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2)\,dz = \frac{3P}{4a^3}\left(a^2 z - 4a\frac{z^2}{2} + 3\frac{z^3}{3}\right)\Big|_0^a$$

$$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0$$

Along vertical leg $\quad \tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\int q\,dy = \int_0^a \tau_e t\,dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2)\,dy = \frac{3P}{4a^3}\left(a^2 y + 4a\frac{y^2}{2} - 5\frac{y^3}{3}\right)\Big|_0^a$$

$$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3\right) = \frac{3P}{4a^3}\cdot\frac{4}{3}a^3 = P$$

PROBLEM 6.80

6.80 For the angle shape and loading of Sample Prob. 6.5, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

SOLUTION

Referring to Sample Prob. 6.5

(a) Along vertical leg $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \qquad y = \tfrac{2}{5}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 + (4a)(\tfrac{2}{5}a) - (5)(\tfrac{2}{5}a)^2\right] = \frac{3P}{4ta^3}(\tfrac{9}{5}a^2) = \frac{27}{20}\frac{P}{ta}$$

Along horizontal leg $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

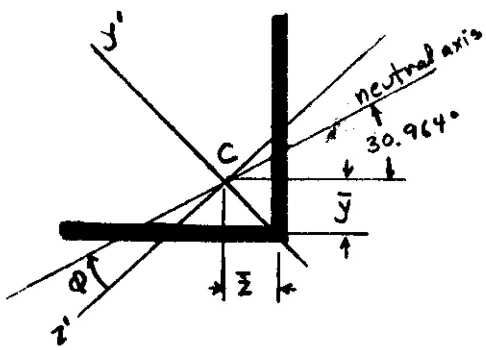
$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \qquad z = \tfrac{2}{3}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 - (4a)(\tfrac{2}{3}a) + (3)(\tfrac{2}{3}a)^2\right] = \frac{3P}{4ta^3}(-\tfrac{5}{3}a^2) = -\frac{1}{4}\frac{P}{ta}$$

At the corner $y = 0,\ z = 0 \qquad \tau = \dfrac{3}{4}\dfrac{P}{ta}$

(b) $I_{y'} = \tfrac{1}{3}ta^3 \qquad I_{z'} = \tfrac{1}{12}ta^3 \qquad \theta = 45^\circ$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \qquad \varphi = 14.036^\circ$$



$$\theta - \varphi = 45 - 14.036 = 30.964^\circ$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at(a/2)}{2at} = \tfrac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at(a/2)}{2at} = \tfrac{1}{4}a$$

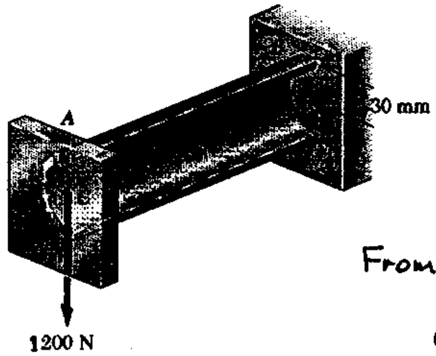
Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z}\tan 30.964^\circ$$
$$= (\tfrac{1}{4} + \tfrac{1}{4}\tan 30.964^\circ)a = 0.400\,a$$
$$= \tfrac{2}{5}a$$

Neutral axis intersects horizontal leg at

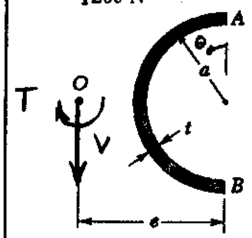
$$z = \bar{z} + \bar{y}\tan(45^\circ + \varphi)$$
$$= (\tfrac{1}{4} + \tfrac{1}{4}\tan 59.036^\circ)a = 0.6667\,a$$
$$= \tfrac{2}{3}a$$

PROBLEM 6.81



*6.81 A cantilever beam *AB*, consisting of half of a thin-walled pipe of 30-mm mean radius and 6-mm wall thickness, is subjected to a 1200-N vertical load. Knowing that the line of action of the load passes through the centroid *C* of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum stress in the beam. (*Hint:* The location of the shear center of the cross section was determined in Prob. 6.74.)

SOLUTION



From the solution to PROBLEM 6.74

$e = \frac{4}{\pi} a \qquad I = \frac{\pi}{2} a^3 t$

$Q = a^2 t \sin\theta \qquad \tau = \frac{VQ}{It}$

$Q_{max} = a^2 t$ at $\theta = 90°$

Due to shearing force $\quad \tau_{90°} = \frac{V Q_{max}}{It}$

$V = 1200$ N $\qquad t = 6 \times 10^{-3}$ m

$I = \frac{\pi}{2}(30)^3(6) = 254.47 \times 10^3 \text{ mm}^4 = 254.47 \times 10^{-9} \text{ m}^4$

$Q_{max} = (30)^2(6) = 5.4 \times 10^3 \text{ mm}^3 = 5.4 \times 10^{-6} \text{ m}^4$

$\tau_{90°} = \frac{(1200)(5.4 \times 10^{-6})}{(254.47 \times 10^{-9})(6 \times 10^{-3})} = 4.24 \times 10^6 \text{ Pa} = 4.24 \text{ MPa}$

$e = \frac{4}{\pi} a, \qquad \bar{x} = \frac{2}{\pi} a \qquad e - \bar{x} = \frac{2}{\pi} a$

Torque $T = (e - \bar{x})V = \frac{2}{\pi}(30 \times 10^{-3})(1200) = 22.92$ N·m

$l = \pi a = \pi(30) = 94.248 \text{ mm} = 94.248 \times 10^{-3} \text{ m}$

For torsion of a rectangular bar $\quad c_1 = c_2 = \frac{1}{3}\left(1 - 0.630 \frac{t}{l}\right)$

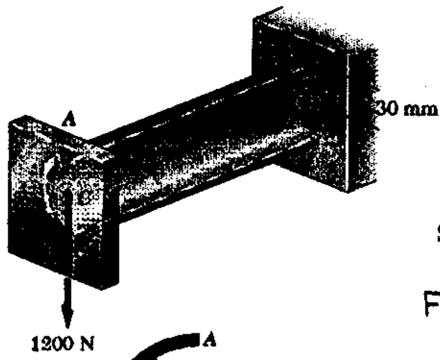
$= \frac{1}{3}\left(1 - \frac{(0.630)(6)}{94.248}\right) = 0.31996$

$\tau_{torsion} = \frac{T}{c_1 l t^2} = \frac{22.92}{(0.31996)(94.248 \times 10^{-3})(6 \times 10^{-3})^2} = 21.11 \times 10^6 \text{ Pa}$

$= 21.11$ MPa

By superposition $\quad \tau_{max} = 4.24 + 21.11 = 25.35$ MPa

PROBLEM 6.82

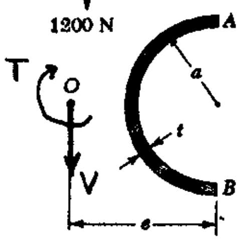


*6.81 A cantilever beam AB, consisting of half of a thin-walled pipe of 30-mm mean radius and 6-mm wall thickness, is subjected to a 1200-N vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum stress in the beam. (*Hint:* The location of the shear center of the cross section was determined in Prob. 6.74.)

*6.82 Solve Prob. 6.81, assuming that the thickness of the beam is reduced to 5 mm.

SOLUTION

From the solution to PROBLEM 6.74

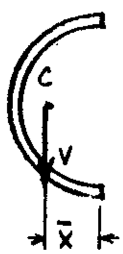


$e = \frac{4}{\pi} a \qquad I = \frac{\pi}{2} a^3 t$

$Q = a^2 t \sin\theta \qquad \tau = \frac{VQ}{It} = \frac{Va^2}{I}$

$Q_{max} = a^2 t \quad \text{at } \theta = 90°$

Due to shearing force $\quad \tau_{90°} = \frac{VQ_{max}}{It}$



$V = 1200\ N \qquad t = 5 \times 10^{-3}\ m$

$I = \frac{\pi}{2}(30)^3(5) = 212.06 \times 10^3\ mm^4 = 212.06 \times 10^{-9}\ m^4$

$Q_{max} = (30)^2(5) = 4.5 \times 10^3\ mm^3 = 4.5 \times 10^{-6}\ m^3$

$\tau_{90°} = \frac{(1200)(4.5 \times 10^{-6})}{(212.06 \times 10^{-9})(5 \times 10^{-3})} = 5.09 \times 10^6\ Pa = 5.09\ MPa$

$e = \frac{4}{\pi} a \qquad \bar{x} = \frac{2}{\pi} a \qquad e - x = \frac{2}{\pi} a$

Torque $\quad T = (e - \bar{x})V = \frac{2}{\pi}(30 \times 10^{-3})(1200) = 22.92\ N \cdot m$ ◄

For torsion of a rectangular bar $\quad c_1 = c_2 = \frac{1}{3}\left[1 - 0.630 \frac{t}{\ell}\right]$

$= \frac{1}{3}\left[1 - \frac{(0.630)(5)}{94.248}\right] = 0.32219$

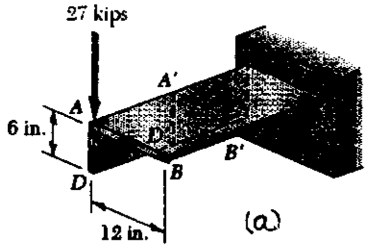
$\tau_{torsion} = \frac{T}{c_1 \ell t^2} = \frac{22.92}{(0.32219)(94.248 \times 10^{-3})(5 \times 10^{-3})^2} = 30.19 \times 10^6\ Pa$

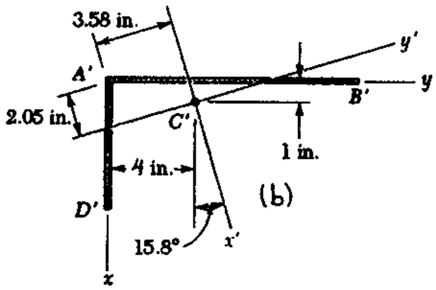
$= 30.19\ MPa$

By superposition $\quad \tau_{max} = 5.09 + 30.19 = 35.3\ MPa$ ◄

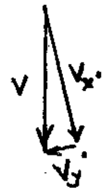
PROBLEM 6.83

*6.83 The cantilever beam shown consists of an angle shape of $\frac{3}{8}$ - in. thickness. For the given loading, determine the location and magnitude of the largest shearing stress along line $A'B'$ in the horizontal leg of the angle shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 115.7\ \text{in}^4$ and $I_{y'} = 12.61\ \text{in}^4$





SOLUTION



$V = 27$ kips $\qquad \beta = 15.8°$

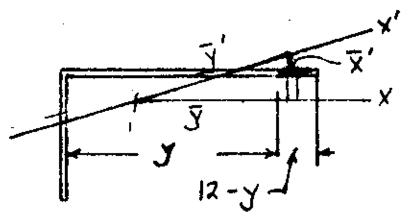
$V_{x'} = V\cos\beta \qquad V_{y'} = V\sin\beta$

In the horizontal leg use coordinate y as shown.

$A = \frac{3}{8}(12 - y)\ \text{in}^2 \qquad t = \frac{3}{8}$ in

$\bar{y} = \frac{1}{2}(12 + y) - 4 = 2 + \frac{1}{2}y$ in.

$\bar{x} = 1$ in



$\bar{x}' = \bar{x}\cos\beta - \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta + \bar{x}\sin\beta$

Due to $V_{x'}$ $\qquad \tau_1 = \dfrac{V_{x'} A \bar{x}'}{I_{y'} t} = \dfrac{(V\cos\beta)(\frac{3}{8})(12-y)[(1)\cos\beta - (2+\frac{1}{2}y)\sin\beta]}{(12.61)(\frac{3}{8})}$

$= 2.0603\,(12-y)(0.41765 - 0.13614\,y)$ ksi

Due to $V_{y'}$ $\qquad \tau_2 = \dfrac{V_{y'} A \bar{y}'}{I_{x'} t} = \dfrac{(V\sin\beta)(\frac{3}{8})(12-y)[(2+\frac{1}{2}y)\cos\beta + (1)\sin\beta]}{(115.6)(\frac{3}{8})}$

$= 0.063595(12-y)(2.19672 + 0.48111\,y)$ ksi

Total $\quad \tau_1 + \tau_2 = (12-y)(1.000 - 0.250\,y)$ ksi

y (in)	0	2	4	6	8	10	12
τ (ksi)	12.00	5.00	0	-3.00	-4.00	-3.00	0

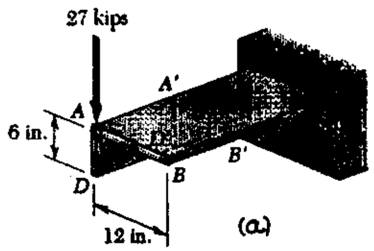
$\tau = 12$ ksi at corner

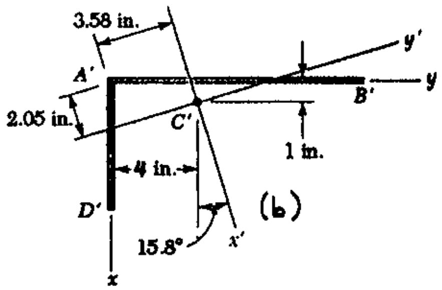
$\tau = -4$ ksi at $y = 8$ in

$\dfrac{d\tau}{dx} = -(0.25)(12 - y_m) - (1 - 0.25y_m)$

$= 0.5y_m - 4 = 0 \qquad y_m = 8$ in

PROBLEM 6.84



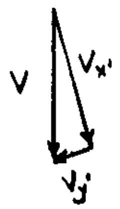


*6.83 The cantilever beam shown consists of an angle shape of $\frac{3}{8}$ - in. thickness. For the given loading, determine the location and magnitude of the largest shearing stress along line $A'B'$ in the horizontal leg of the angle shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 115.7\ \text{in}^4$ and $I_{y'} = 12.61\ \text{in}^4$.

*6.84 For the cantilever beam and loading of Prob. 6.83, determine the location and magnitude of the largest shearing stress along line $A'D'$ in the vertical leg of the angle shape..

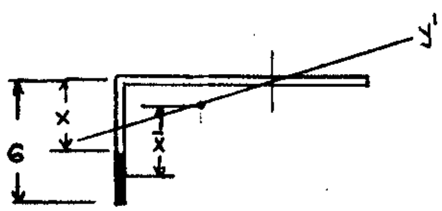
SOLUTION

$V = 27$ kips $\qquad \beta = 15.8°$



$V_{x'} = V\cos\beta \qquad V_{y'} = V\sin\beta$

In vertical leg use coordinate x as shown.



$A = \frac{3}{8}(6-x)\ \text{in}^2 \qquad t = \frac{3}{8}\ \text{in.}$

$\bar{y} = 4\ \text{in}$

$\bar{x} = \frac{1}{2}(6+x) - 1 = 2 + \frac{1}{2}x$

$\bar{x}' = \bar{x}'\cos\beta - \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta + \bar{x}\sin\beta$

Due to $V_{x'}$ $\quad \tau_1 = \dfrac{V_{x'}A\bar{x}'}{I_{y'}t} = \dfrac{(V\cos\beta)(\frac{3}{8})(6-x)\left[(2+\frac{1}{2}x)\cos\beta - 4\sin\beta\right]}{(12.61)(\frac{3}{8})}$

$= 2.0603\,(6-x)(0.83531 + 0.48111\,x)$

Due to $V_{y'}$ $\quad \tau_2 = \dfrac{V_{y'}A\bar{y}'}{I_{x'}t} = \dfrac{(V\sin\beta)(\frac{3}{8})(6-x)\left[4\cos\beta + (2+\frac{1}{2}x)\sin\beta\right]}{(115.6)(\frac{3}{8})}$

$= 0.06359\,(6-x)(4.3934 + 0.13614\,x)$

Total: $\tau_1 + \tau_2 = (6-x)(2.000 + 1.000\,x)$

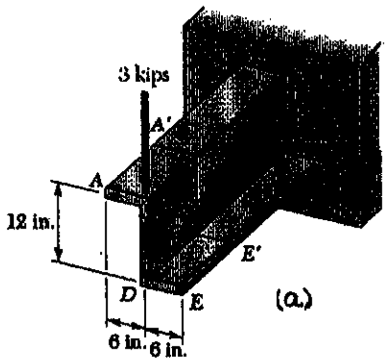
x (in)	0	1	2	3	4	5	6
τ (ksi)	12.00	15.00	16.00	15.00	12.00	7.00	0

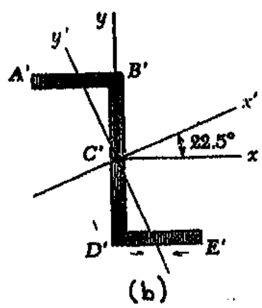
$\tau_{max} = 16$ ksi at $x = 2$ in. ◄

$\dfrac{d\tau}{dx} = (6-x_m)(1) + (2+x_m)(-1)$

$= 4 - 2x_m = 0 \qquad x_m = 2\ \text{in}$

PROBLEM 6.87



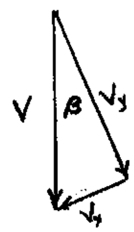


*6.87 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$ - in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3\ \text{in}^4$ and $I_{y'} = 13.61\ \text{in}^4$.

SOLUTION

$V = 3$ kips $\qquad \beta = 22.5°$

$V_{x'} = V \sin\beta \qquad V_{y'} = V\cos\beta$



In upper horizontal leg use coordinate x $\quad (-6\text{ in} \le x \le 0)$

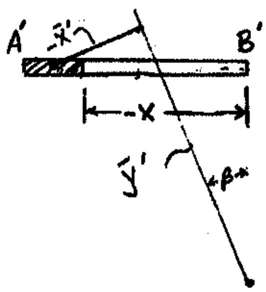
$$A = \tfrac{1}{4}(6 + x)\ \text{in.}$$

$$\bar{x} = \tfrac{1}{2}(-6+x)\ \text{in}$$

$$\bar{y} = 6\ \text{in.}$$

$$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$$

$$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$$



Due to $V_{x'}$ $\qquad \tau_1 = \dfrac{V_{x'} A \bar{x}'}{I_{y'} t}$

$$\tau_1 = \frac{(V\sin\beta)(\frac{1}{4})(6+x)\left[\frac{1}{2}(-6+x)\cos\beta + 6\sin\beta\right]}{(13.61)(\frac{1}{4})}$$

$$= 0.084353(6+x)(-0.47554 + 0.46194x)$$

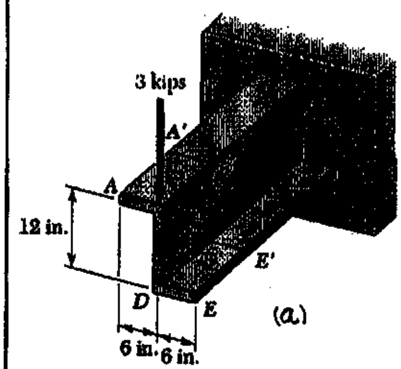
Due to $V_{y'}$ $\qquad \tau_2 = \dfrac{V_{y'} A \bar{y}'}{I_{x'} t} = \dfrac{(V\cos\beta)(\frac{1}{4})(6+x)\left[6\cos\beta - \frac{1}{2}(-6+x)\sin\beta\right]}{(166.3)(\frac{1}{4})}$

$$= 0.0166665(6+x)\left[6.69132 - 0.19134x\right]$$

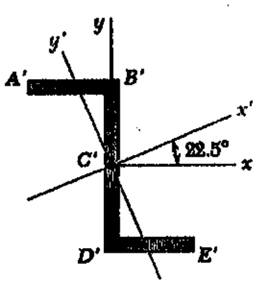
Total $\tau_1 + \tau_2 = (6+x)\left[-0.07141 + 0.085396x\right]$

x (in)	-6	-5	-4	-3	-2	-1	0
τ (ksi)	0	-0.105	-0.140	-0.104	0.003	0.180	0.428

PROBLEM 6.88



(a)



(b)

*6.87 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$ - in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3\ \text{in}^4$ and $I_{y'} = 13.61\ \text{in}^4$.

*6.88 For the cantilever beam and loading of Prob. 6.87, determine the distribution of the shearing stresses along line $B'D'$ in the vertical web of the Z shape..

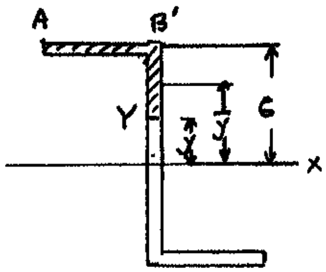
SOLUTION

$V = 3\ \text{kips} \qquad \beta = 22.5°$

$V_{y'} = V \sin\beta \qquad V_{y'} = V\cos\beta$

For part AB' $\quad A = (\frac{1}{4})(6) = 1.5\ \text{in}^2$

$\bar{x} = -3\ \text{in},\ \bar{y} = 6\ \text{in}.$



For part $B'Y$

$A = \frac{1}{4}(6-y)$

$\bar{x} = 0 \qquad \bar{y} = \frac{1}{2}(6+y)$

$x' = x\cos\beta + y\sin\beta$
$y' = y\cos\beta - x\sin\beta$

Due to $V_{x'}$ $\qquad \tau_1 = \dfrac{V_{x'}(A_{AB}\bar{x}'_{AB} + A_{BY}\bar{x}'_{BY})}{I_{y'}\,t}$

$$\tau_1 = \frac{(V\sin\beta)[(1.5)(-3\cos\beta + 6\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\sin\beta]}{(13.61)(\frac{1}{4})}$$

$$= \frac{(V\sin\beta)[-0.7133 + 1.7221 - 0.047835y^2]}{3.4025} = 0.3404 - 0.01614\,y^2$$

Due to $V_{y'}$ $\qquad \tau_2 = \dfrac{V_{y'}(A_{AB}\bar{y}'_{AB} + A_{BY}\bar{y}')}{I_{x'}\,t}$

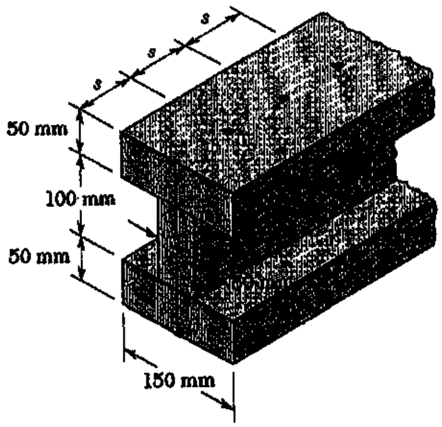
$$\tau_2 = \frac{(V\cos\beta)[(1.5)(6\cos\beta + 3\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\cos\beta]}{(166.3)(\frac{1}{4})}$$

$$= \frac{(V\cos\beta)[10.037 + 4.1575 - 0.11548y^2]}{(166.3)(\frac{1}{4})} = 0.9463 - 0.00770\,y^2$$

Total $\quad \tau_1 + \tau_2 = 1.2867 - 0.02384\,y^2$

y (in)	0	± 2	± 4	± 6
τ (ksi)	1.287	1.191	0.905	0.428

PROBLEM 6.89



6.89 Three boards, each 50 mm thick, are nailed together to form a beam that is subjected to a 1200-N vertical shear. Knowing that the allowable shearing force in each nail is 600 N, determine the largest permissible spacing s between the nails.

SOLUTION

Calculate moment of inertia

Part	A (mm²)	d (mm)	Ad² (10⁶ mm⁴)	$\bar{I}$ (10⁶ mm⁴)
Top	7500	75	42.19	1.56
Middle	5000	0	0	4.17
Bottom	7500	75	42.19	1.56
Σ			84.38	7.29

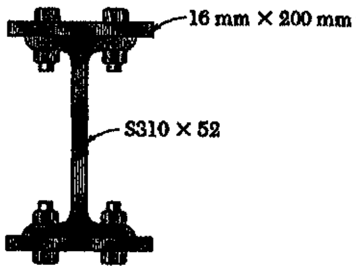
$$I = \Sigma A d^2 + \Sigma \bar{I} = 91.67 \times 10^6 \text{ mm}^4 = 91.67 \times 10^{-6} \text{ m}^4$$

$$Q = A_{top} d_{top} = (7500)(75) = 562.5 \times 10^3 \text{ mm}^3 = 562.5 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(562.5 \times 10^{-6})}{91.67 \times 10^{-6}} = 7.363 \times 10^3 \text{ N/m}$$

$$F_{nail} = q s \qquad s = \frac{F_{nail}}{q} = \frac{600}{7.363 \times 10^3} = 81.5 \times 10^{-3} \text{ m} = 81.5 \text{ mm}$$

PROBLEM 6.90



6.90 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using bolts of 18-mm diameter spaced longitudinally every 120 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible shearing force.

SOLUTION

Calculate moment of inertia

Part	A (mm^2)	d (mm)	Ad^2 ($10^6 mm^4$)	$\bar{I}$ ($10^6 mm^4$)
Top plate	3200	*160.5	82.43	0.07
S310×52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$* \ d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate} d_{plate} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

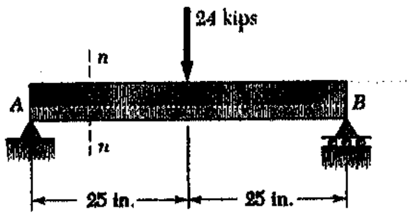
$$qs = 2F_{bolt} \qquad q = \frac{2F_{bolt}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

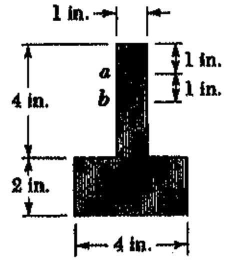
$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$= 193.5 \text{ kN}$$

PROBLEM 6.91

6.91 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



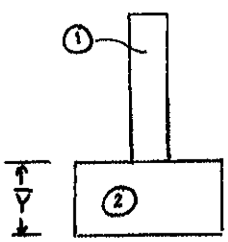


SOLUTION

$R_A = R_B = 12$ kips

At section n-n $V = 12$ kips.

Locate centroid and calculate moment of inertia.

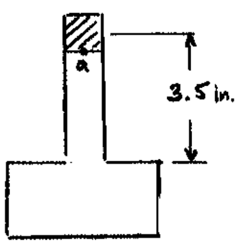


Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	4	4	16	2	16	5.33
②	8	1	8	1	8	2.67
Σ	12		24		24	8

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 24 + 8 = 32 \text{ in}^4$$

(a)

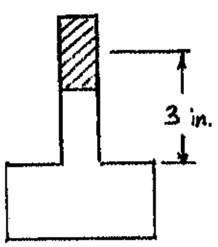


$$Q_a = A_a \bar{y}_a = (1)(1)(3.5) = 3.5 \text{ in}^3$$

$$t = 1 \text{ in}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} = 1.313 \text{ ksi}$$

(b)



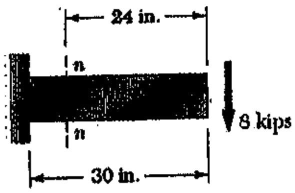
$$Q_b = A_b \bar{y}_b = (1)(2)(3) = 6 \text{ in}^3$$

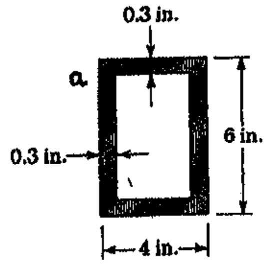
$$t = 1 \text{ in.}$$

$$\tau_b = \frac{VQ}{It} = \frac{(12)(6)}{(32)(1)} = 2.25 \text{ ksi}$$

PROBLEM 6.92

6.92 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.





SOLUTION

At section n-n $\quad V = 8$ kips

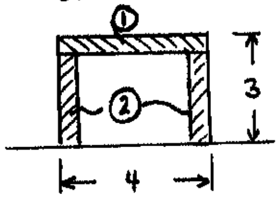
Moment of inertia

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(4)(6)^3 - \frac{1}{12}(3.4)(5.4)^3$$

$$= 27.3852 \text{ in}^4$$

(a) The largest shearing stress occurs on a section through the centroid of the entire cross section.

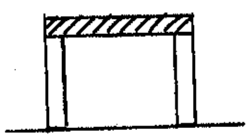


Part	A (in^2)	$\bar{y}$ (in)	$Q = A\bar{y}$ (in^3)
1	1.2	2.85	3.42
2	1.62	1.35	2.187
Σ	2.82		5.607

$Q_m = 4.5135$ in^3 $\qquad t = (2)(0.3) = 0.6$ in.

$$\tau_m = \frac{VQ_m}{It} = \frac{(8)(5.607)}{(27.3852)(0.6)} = 2.73 \text{ ksi}$$

(b)



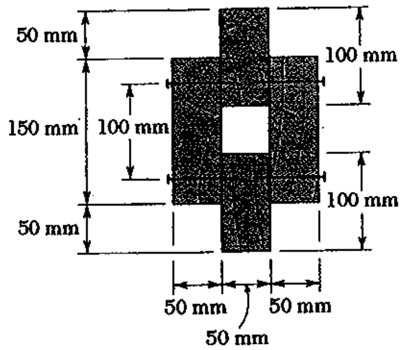
$$Q_a = A_a \bar{y}_a = (1.2)(2.85) = 3.42 \text{ in}^3$$

$$t = (2)(0.3) = 0.6 \text{ in.}$$

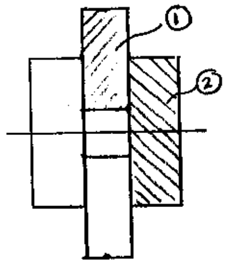
$$\tau_a = \frac{VQ_a}{It} = \frac{(8)(3.42)}{(27.3852)(0.6)} = 1.665 \text{ ksi}$$

PROBLEM 6.93

6.93 The built-up timber beam shown is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60$ mm and that each nail is 90 mm long, determine the shearing force in each nail.



SOLUTION



$$I_1 = \frac{1}{12} b h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(50)(100)^3 + (50)(100)(75)^2$$

$$= 32.292 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12}(50)(150)^3$$

$$= 14.0625 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \text{ mm}^4 = 92.71 \times 10^{-6} \text{ m}^4$$

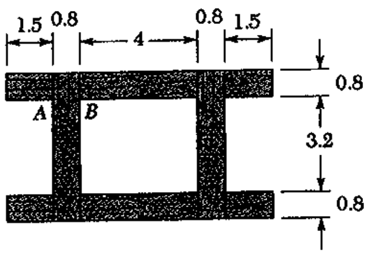
$$Q = Q_1 = A_1 \bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \text{ N/m} \qquad s = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$2F_{nail} = qs \qquad F_{nail} = \frac{1}{2} qs = \frac{1}{2}(24.27 \times 10^3)(60 \times 10^{-3}) = 728 \text{ N}$$

PROBLEM 6.94

6.94 The built-up beam shown was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb vertical shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.



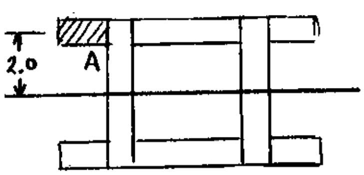
Dimensions in inches

SOLUTION

$$I = 2\left[\frac{1}{12}(0.8)(4.8)^3 + \frac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2\right]$$

$$= 60.143 \text{ in}^4$$

(a) $A_a = (1.5)(0.8) = 1.2 \text{ in}^2 \qquad \bar{y}_a = 2.0 \text{ in.}$

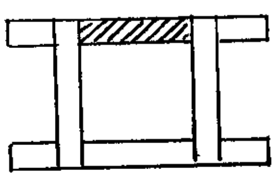


$Q_a = A_a \bar{y}_a = 2.4 \text{ in}^3$

$t_a = 0.8 \text{ in.}$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(1200)(2.4)}{(60.143)(0.8)} = 59.9 \text{ psi}$$

(b) $A_b = (4)(0.8) = 3.2 \text{ in}^2 \qquad \bar{y}_b = 2.0 \text{ in.}$

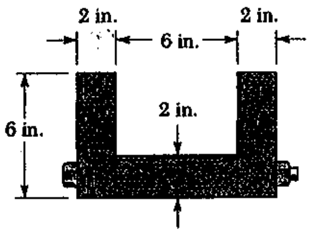


$Q_b = A_b \bar{y}_b = (3.2)(2.0) = 6.4 \text{ in}^3$

$t_b = (2)(0.8) = 1.6 \text{ in.}$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(1200)(6.4)}{(60.143)(1.6)} = 79.8 \text{ psi}$$

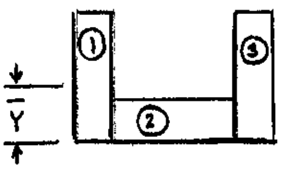
PROBLEM 6.95



6.95 A beam consists of three planks connected as shown by $\frac{3}{8}$ - in.-diameter bolts spaced every 12 in. along the longitudinal axis of the beam. Knowing that the beam is subjected to a 2500-lb vertical shear, determine the maximum shearing stress in the bolts.

SOLUTION

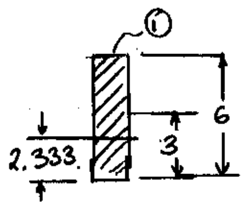
Locate neutral axis and compute moment of inertia.



Part	$A\ (in^2)$	$\bar{y}\ (in)$	$A\bar{y}\ in^3$	$d\ (in)$	$Ad^2\ (in^4)$	$\bar{I}\ (in^4)$
1	12	3	36	0.667	5.333	36
2	12	1	12	1.333	21.333	4
3	12	3	36	0.667	5.333	36
Σ	36		84		132	76

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{84}{36} = 2.333 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 108 \text{ in}^4$$



$$Q = A_1\bar{y}_1 = (2)(6)(3-2.333) = 8 \text{ in}^3$$

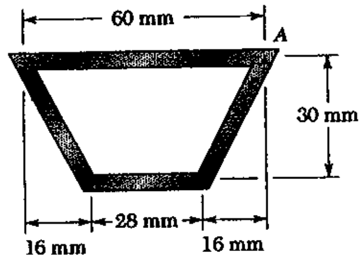
$$q = \frac{VQ}{I} = \frac{(2500)(8)}{108} = 185.2 \text{ lb/in}$$

$$F_{bolt} = qs = (185.2)(12) = 2.222 \times 10^3 \text{ lb.}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.222 \times 10^3}{0.1104} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

PROBLEM 6.96



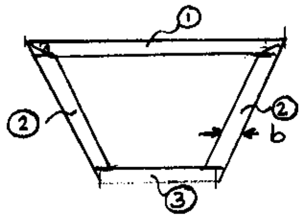
6.96 An extruded beam with the cross section shown and a 3-mm wall thickness is subjected to a 10-kN vertical shear. Determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

SOLUTION

For part ② height $h = 30$ mm

$$b = \frac{\sqrt{16^2+30^2}}{30}\, t = (1.13333)(3) = 3.4 \text{ mm}$$

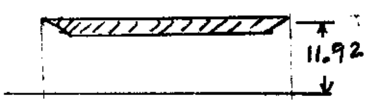
$$\bar{I}_x = \frac{1}{12}(2b)h^3 = \quad \times 10^3 \text{ mm}^4$$



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ mm³	d (mm)	Ad^2 (10^3 mm⁴)	$\bar{I}$ (10^3 mm⁴)
①	180	30	5400	11.92	25.58	0.135
②	204	15	3060	3.08	1.94	15.3
③	84	0	0	18.08	27.46	0.063
	468		8460		54.98	15.50

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8460}{46.8} = 18.08 \text{ mm}$$

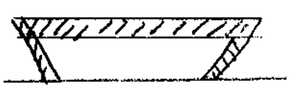
$$I = \Sigma A d^2 + \Sigma \bar{I} = 70.48 \times 10^3 \text{ mm}^4 = 70.48 \times 10^{-9} \text{ m}^4$$



$$Q_A = (60)(3)(11.92) = 2.146 \times 10^3 \text{ mm}^3 = 2.146 \times 10^{-6} \text{ m}^3$$

$$\tau_A = \frac{VQ_A}{It} = \frac{(10\times10^3)(2.146\times10^{-6})}{(70.48\times10^{-9})(6\times10^{-3})} = 50.7\times10^6 \text{ Pa}$$

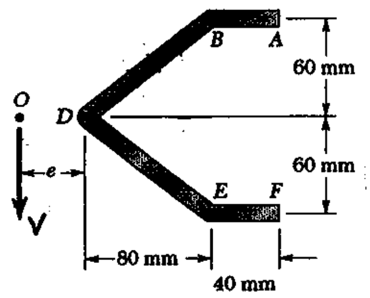
$$= 50.7 \text{ MPa}$$



$$Q_m = Q_A + (2)(3.4)(11.92)\frac{11.92}{2} = 2.629\times10^3 \text{ mm}^3 = 2.629\times10^{-6} \text{ m}^3$$

$$\tau_m = \frac{VQ}{It} = \frac{(10\times10^3)(2.629\times10^{-6})}{(70.48\times10^{-9})(6\times10^{-3})} = 62.6\times10^6 \text{ Pa} = 62.6 \text{ MPa}$$

PROBLEM 6.97



6.97 and 6.98 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center O of the cross section.

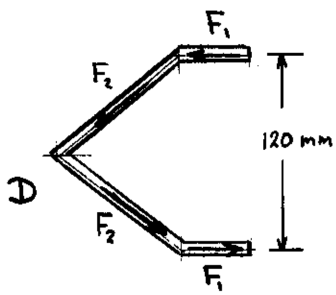
SOLUTION

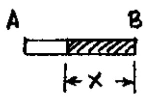
$I_{AB} = (40t)(60)^2 = 144 \times 10^3\ t$

$L_{DB} = \sqrt{80^2 + 60^2} = 100\ \text{mm} \qquad A_{DB} = 100\ t$

$I_{DB} = \frac{1}{3} A_{DB} h^2 = \frac{1}{3}(100t)(60)^2 = 120 \times 10^3\ t$

$I = 2 I_{AB} + 2 I_{DB} = 528 \times 10^3\ t$





Part AB: $A = tx \quad \bar{y} = 60\ \text{mm}$

$Q = A\bar{y} = 60\ tx\ \ \text{mm}^3$

$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$

$F_1 = \int \tau\, dA = \int_0^{40} \frac{60 V x}{I}\, t\, dx = \frac{60Vt}{I}\int_0^{40} x\, dx$

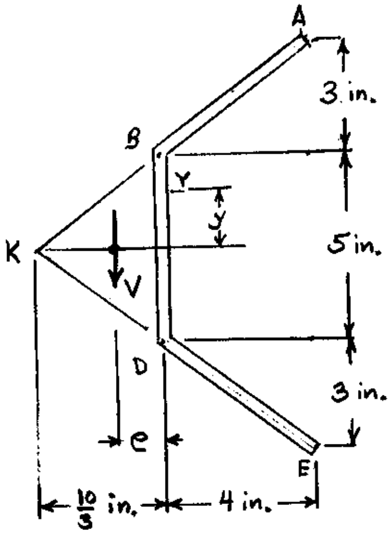
$= \frac{60Vt}{I}\left.\frac{x^2}{2}\right|_0^{30} = \frac{(60)(30)^2 V\cancel{t}}{(2)(528 \times 10^3)\cancel{t}} = 0.051136\ V$

$\circlearrowright \Sigma M_D = \circlearrowright \Sigma M_D \qquad Ve = (0.051136\ V)(120) \qquad e = (0.051136)(120) = 6.14\ \text{mm}$

PROBLEM 6.98

6.97 **and 6.98** A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center O of the cross section.

SOLUTION



$$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{AB} = 5t$$

$$I_{AB} = \tfrac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \tfrac{1}{12}(5t)(3)^2 + (5t)(4)^2$$

$$= 83.75\ t \ \text{in}^4$$

$$I_{BD} = \tfrac{1}{12}(t)(5)^3 = 10.417\ t \ \text{in}^4$$

$$I = 2I_{AB} + I_{BD} = 177.917\ t \ \text{in.}$$

In part BD $\quad Q = Q_{AB} + Q_{BY}$

$$Q = (5t)(4) + (2.5 - y)t(\tfrac{1}{2})(2.5 + y)$$

$$= 20t + 3.125t - \tfrac{1}{2} t y^2$$

$$= (23.125 - \tfrac{1}{2}y^2)\, t$$

$$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau\, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{I\, t} \cdot t\, dy$$

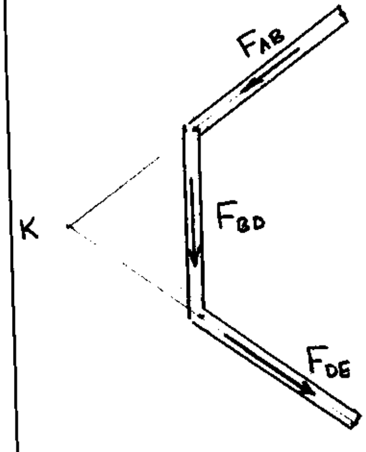
$$= \frac{Vt}{I} \int_{-2.5}^{2.5} (23.125 - \tfrac{1}{2}y^2)\, dy = \frac{Vt}{I}\left[23.125 y - \tfrac{1}{6}y^3\right]_{-2.5}^{2.5}$$

$$= \frac{Vt}{I} \cdot 2\left[(23.125)(2.5) - \frac{(2.5)^3}{6}\right] = \frac{Vt\,(110.417)}{177.917\ t}$$

$$= 0.62061\ V$$

$$\circlearrowright \Sigma M_K = \circlearrowright \Sigma M_K \qquad -V\left(\tfrac{10}{3} - e\right) = -\tfrac{10}{3}(0.62061\ V)$$

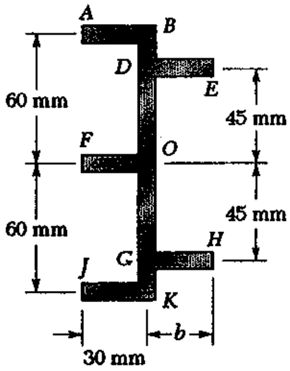
$$e = \tfrac{10}{3}\left[1 - 0.62061\right] = 1.265 \text{ in.}$$



Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.

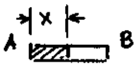
PROBLEM 6.99

6.99 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



SOLUTION

Part AB: $A = tx \qquad \bar{y} = 60 \text{ mm}$



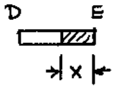
$$Q = A\bar{y} = 60\, t\, x \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{60\,Vx}{I}$$

$$F_1 = \int \tau\, dA = \int_0^{30} \frac{60\,Vx}{I}\, t\, dx = \frac{60\,Vt}{I}\int_0^{30} x\, dx$$

$$= \frac{60\,Vt}{I}\left.\frac{x^2}{2}\right|_0^{30} = \frac{(60)(30)^2}{2}\frac{Vt}{I} = 27\times10^3\ \frac{Vt}{I}$$

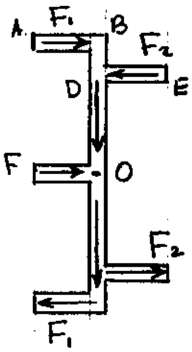
Part DE $\quad A = tx \qquad \bar{y} = 45 \text{ mm}$



$$Q = A\bar{y} = 45\, t\, x$$

$$\tau = \frac{VQ}{It} = \frac{45\,Vx}{I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{45\,Vx}{I}\, t\, dx = \frac{45\,Vt}{I}\int_0^b x\, dx = \frac{45\, b^2\, Vt}{2I}$$



$$+\circlearrowleft \Sigma M_o = +\circlearrowleft \Sigma M_o \qquad 0 = (2)(45)F_2 - (2)(60)F_1$$

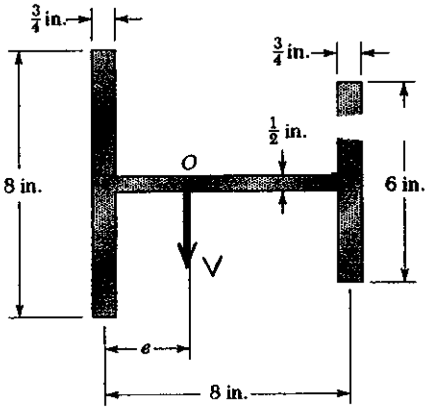
$$\left[(45)^2 b^2 - (2)(60)(27\times10^3)\right]\frac{Vt}{I} = 0$$

$$b^2 = \frac{(2)(60)(27\times10^3)}{45^2} = 1600 \text{ mm}^2 \qquad b = 40 \text{ mm}$$

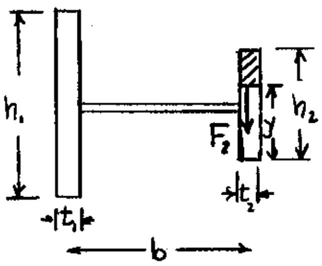
Note that the pair of F_1 forces form a couple. Likewise, the pair of F_2 forces. The lines of action of the forces in BDOGK pass through point O.

PROBLEM 6.100

6.100 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.



SOLUTION



$$I = \frac{1}{12} t_1 h_1^3 + \frac{1}{12} t_2 h_2^3$$

Right flange

$$A = \left(\tfrac{1}{2} h_2 - y\right) t_2$$

$$\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2} h_2 + y\right) t_2$$

$$Q = A\bar{y}$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2}h_2 - y\right)\left(\tfrac{1}{2}h_2 + y\right) t_2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{4}h_2^2 - y^2\right) t_2$$

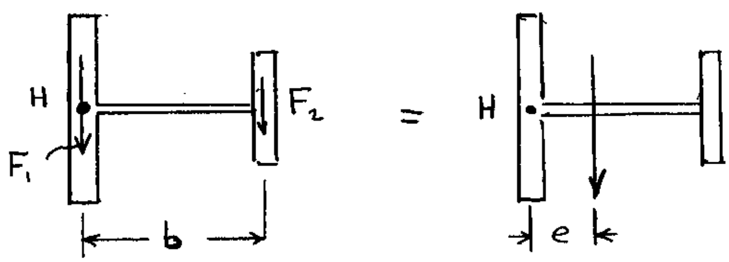
$$\tau = \frac{VQ}{I t_2} = \frac{V}{2 I t_2}\left(\tfrac{1}{4}h_2^2 - y^2\right) t_2$$

$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{V t_2}{2 I \cancel{t_2}}\left(\tfrac{1}{4}h_2^2 - y^2\right) \cancel{t_2}\, dy = \frac{V t_2}{2I}\left(\tfrac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Bigg|_{-h_2/2}^{h_2/2}$$

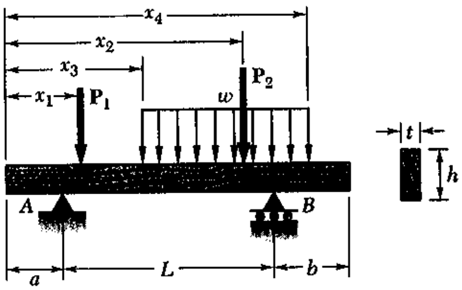
$$= \frac{V t_2}{2I}\left\{\tfrac{1}{4}h_2^2 \frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3 + \tfrac{1}{4}h_2^2 \frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{V t_2 h_2^3}{12 I} = \frac{V t_2 h_2^3}{t_1 h_1^3 + t_2 h_2^3}$$

$$+\circlearrowright M_H = +\circlearrowright M_H \qquad -Ve = -F_2 b = -V \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3}$$

$$e = \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3} = \frac{(0.75)(6)^3(8)}{(0.75)(8)^3 + (0.75)(6)^3} = 2.37 \text{ in.}$$



PROBLEM 6.C1



6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all}. Measuring x from end A and using SI units, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx, the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa, and using the increments indicated: (*a*) Prob. 5.75 ($\Delta x = 0.1$ m), (*b*) Prob. 5.76 ($\Delta x = 0.2$ m).

SOLUTION

See solution of P 5.C2 for the determination of R_A, R_B, $V(x)$, and $M(x)$

We recall that

$$V(x) = R_A\,STPA + R_B\,STPB - P_1\,STP1 - P_2\,STP2 - w(x-x_3)\,STP3 + w(x-x_4)\,STP4$$

$$M(x) = R_A(x-a)\,STPA + R_B(x-a-L)\,STPB - P_1(x-x_1)\,STP1 - P_2(x-x_2)\,STP2 - \tfrac{1}{2}w(x-x_3)^2\,STP3 + \tfrac{1}{2}w(x-x_4)^2\,STP4$$

where STPA, STPB, STP1, STP2, STP3, and STP4 are step functions defined in P 5.C2

(1) TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

If unknown dimension is h:

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $h_\sigma = h = \sqrt{6S/t}$

If unknown dimension is t:

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $t_\sigma = t = 6S/h^2$

(2) TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378: $\tau_{max} = \dfrac{3}{2}\dfrac{|V|}{A} = \dfrac{3}{2}\dfrac{|V|}{th}$

If unknown dimension is h: $h_\tau = h = \dfrac{3M}{2t\tau_{all}}$

If unknown dimension is t: $t_\tau = t = \dfrac{3M}{2h\tau_{all}}$

(CONTINUED)

PROBLEM 6.C1 CONTINUED

PROGRAM OUTPUTS

Prob. 5.75

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	-3.00	2.400	173.21	136.36
1.70	-3.00	2.100	162.02	136.36
1.80	-3.00	1.800	150.00	136.36
1.90	-3.00	1.500	136.93	136.36
2.00	-3.00	1.200	122.47	136.36
2.10	-3.00	0.900	106.07	136.36
2.20	-3.00	0.600	86.60	136.36
2.30	-3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.05	0.00

Prob. 5.76

RA = 25.00 kN RB = 25.00 kN

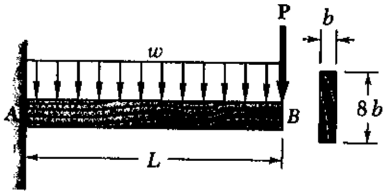
X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	25.00	0.000	0.00	378.79
0.20	23.00	4.800	141.42	348.48
0.40	21.00	9.200	195.79	318.18
0.60	19.00	13.200	234.52	287.88
0.80	17.00	16.800	264.58	257.58
1.00	15.00	20.000	288.68	227.27
1.20	13.00	22.800	308.22	196.97
1.40	11.00	25.200	324.04	166.67
1.60	9.00	27.200	336.65	136.36
1.80	7.00	28.800	346.41	106.06
2.00	5.00	30.000	353.55	75.76
2.20	3.00	30.800	358.24	45.45
2.40	1.00	31.200	360.56	15.15
2.60	-1.00	31.200	360.56	15.15
2.80	-3.00	30.800	358.24	45.45
3.00	-5.00	30.000	353.55	75.76
3.20	-7.00	28.800	346.41	106.06
3.40	-9.00	27.200	336.65	136.36
3.60	-11.00	25.200	324.04	166.67
3.80	-13.00	22.800	308.22	196.97
4.00	-15.00	20.000	288.68	227.27
4.20	-17.00	16.800	264.58	257.58
4.40	-19.00	13.200	234.52	287.88
4.60	-21.00	9.200	195.79	318.18
4.80	-23.00	4.800	141.42	348.48
5.00	0.00	0.000	0.00	0.00

The smallest allowable value of h is the largest of the values shown in the last two columns.

For Prob. 5.75, $h = h_\sigma = 173.2$ mm.

For Prob. 5.76, $h = h_\tau = 379$ mm

PROBLEM 6.C2



6.C2 A cantilever timber beam AB of length L and of the uniform rectangular section shown supports a concentrated load **P** at its free end and a uniformly distributed load w along its entire length. Write a computer program to determine the length L and the width b of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{all} = 1.8$ ksi and $\tau_{all} = 120$ psi, use this program to determine the dimensions L and b when (*a*) $P = 1000$ lb and $w = 0$, (*b*) $P = 0$ and $w = 12.5$ lb/in., (*c*) $P = 500$ lb and $w = 12.5$ lb/in.

SOLUTION

Both the maximum shear and the maximum bending moment occur at A. We have

$$V_A = P + wL \qquad M_A = PL + \frac{1}{2} wL^2$$

TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

$$\sigma_{all} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6} b(8b)^2} = \frac{3M_A}{32b^3} \qquad b_\sigma = b = \left[\frac{3}{32}\frac{M_A}{\sigma_{all}}\right]^{1/3}$$

TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:

$$\tau_{all} = \frac{3V}{2A} = \frac{3}{2}\frac{V_A}{b(8b)} = \frac{3V_A}{16b^2} \qquad b_\tau = b = \left[\frac{3}{16}\frac{V_A}{\tau_{all}}\right]^{1/2}$$

PROGRAM

For $L=0$, $V_A = P$ and $b_\tau > 0$, while $M_A = 0$ and $b_\sigma = 0$.
Starting with $L=0$ and using increments $\Delta L = 0.001$ in., we increase L until b_σ and b_τ become equal. We then print L and b.

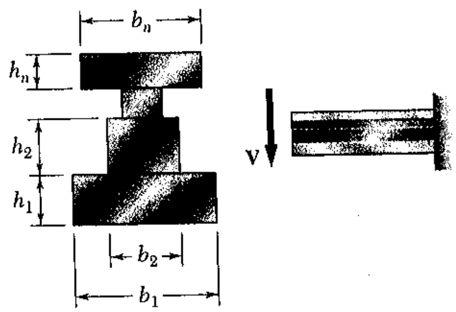
PROGRAM OUTPUTS

```
For P = 1000 lb, w =  0.0 lb/in.
Increment = 0.0010 in.
L = 37.5 in., b = 1.250 in.
```

```
For P =    0 lb, w = 12.5 lb/in.
Increment = 0.0010 in.
L = 70.3 in., b = 1.172 in.
```

```
For P =  500 lb, w = 12.5 lb/in.
Increment = 0.0010 in.
L = 59.8 in., b = 1.396 in.
```

PROBLEM 6.C3



6.C3 A beam having the cross section shown is subjected to a vertical shear **V**. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (*a*) Prob. 6.10, (*b*) Prob. 6.11, (*c*) Prob. 6.21, (*d*) Prob. 6.23.

SOLUTION

1. Enter V and the number n of rectangles.

2. For $i = 1$ to n, enter the dimensions b_i and h_i

3. Determine the area $A_i = b_i h_i$ of each rectangle.

4. Determine the elevation of the centroid of each rectangle

$$\bar{y}_i = \sum_{k=1}^{i} h_k - 0.5 h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section

$$\bar{y} = \left(\sum_i A_i \bar{y}_i\right) / \left(\sum_i A_i\right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2\right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface

$$Q_i = \sum_{k=1}^{i} = A_k (\bar{y}_k - \bar{y})$$

7. Select for t_i the smaller of b_i and b_{i+1}.

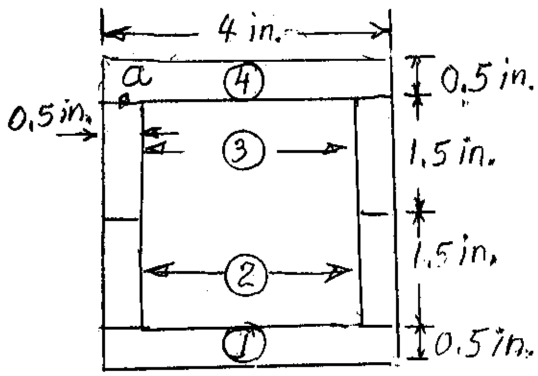
The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{V Q_i}{I t_i}$$

(CONTINUED)

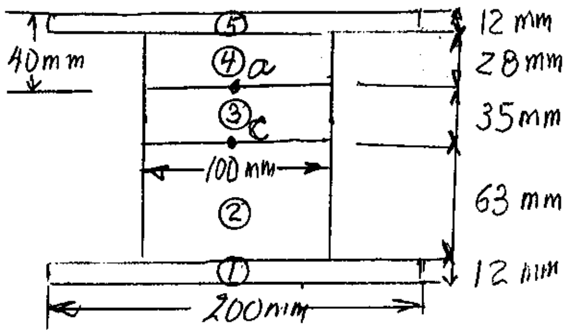
PROBLEM 6.C3 CONTINUED

PROGRAM OUTPUTS



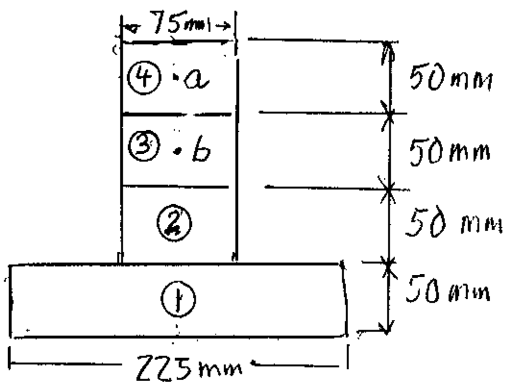
Problem 6.10

V=10.00 kips
YBAR of Section = 2.000 in.
I= 14.583 in^4
Between elements 1 and 2:
Tau = 2.400 ksi
Between elements 2 and 3: (a)
Tau = 3.171 ksi
Between elements 3 and 4: (b)
Tau = 2.400 ksi



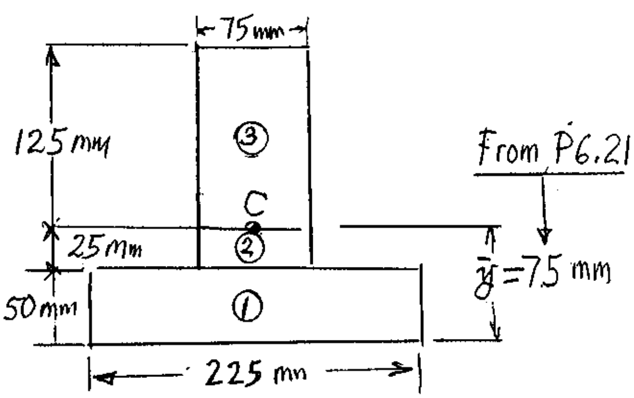
Problem 6.11

V= 10.00 kN
YBAR of Section = 75.00 mm
I= 39.580*10^-6 m^4
Between elements 1 and 2:
Tau = 418.39 kPa
Between elements 2 and 3: (a)
Tau = 919.78 kPa
Between elements 3 and 4: (b)
Tau = 765.03 kPa
Between elements 4 and 5:
Tau = 418.39 kPa



Problem 6.21

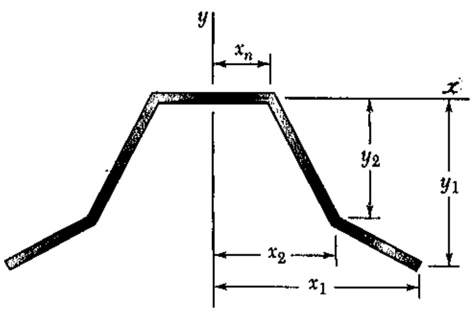
V=200.00 kN
YBAR of Section = 75.00 mm
I= 79.687*10^-6 m^4
Between elements 1 and 2:
Tau = 18.82 MPa
Between elements 2 and 3: (b)
Tau = 18.82 MPa
Between elements 3 and 4: (a)
Tau = 12.55 MPa



Problem 6.23

V=200.00 kN
YBAR of Section = 75.00 mm
I= 79.688*10^-6 m^4
Between elements 1 and 2:
Tau = 18.82 MPa
Between elements 2 and 3:
Tau = 19.61 MPa

PROBLEM 6.C4



6.C4 A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear **V**. Use this program (*a*) to solve Prob. 6.49, (*b*) to find the shearing stress at point E for the shape and load of Prob. 6.50, assuming a thickness $t = \frac{1}{4}$ in.

SOLUTION

For each element on the right-hand side, we compute (for $i = 1$ to n):

Length of element $= L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$

Area of element $= A_i = t L_i$ where $t = \frac{1}{4}$ in.

Distance from x axis to centroid of element $= \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$

Distance from x axis to centroid of section:

$$\bar{y} = (\Sigma A_i \bar{y}_i) / \Sigma A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2 \Sigma A_i \left[\frac{1}{12}(y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2\right]$$

Computation of Q at point P where stress is desired

$Q = \Sigma A_i (\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and point P.

Shearing stress at P:

$$\tau = \frac{VQ}{It}$$

NOTE: τ_{max} occurs on neutral axis, i.e., for $y_P = \bar{y}$.

PROGRAM OUTPUTS

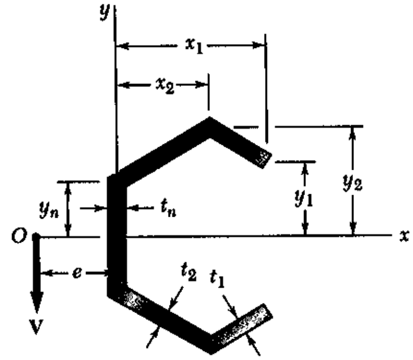
```
Part (a):

I = 0.5333 in^4
Taumax = 2.02 ksi
TauB = 1.800 ksi
```

```
Part (b):

I = 22.27 in^4
TauE = 194.0 psi
```

PROBLEM 6.C5



6.C5 The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (*a*) the location of the shear center O, (*b*) the distribution of shearing stresses caused by a vertical force applied at O. Use this program to solve Probs. 6.65, 6.68, 6.69, and 6.70.

SOLUTION

SINCE SECTION IS SYMMETRIC WITH x AXIS, COMPUTATIONS WILL BE DONE FOR TOP HALF.

FOR $i = 1$ TO $n+1$ (NOTE: $n+1$ IS THE ORIGIN)

ENTER t_i, x_i, y_i

COMPUTE LENGTH OF EACH SEGMENT

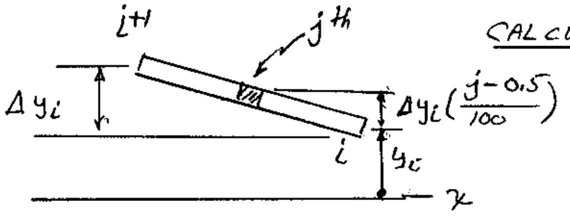
FOR $i = 1$ TO n

$\Delta x_i = x_{i+1} - x_i$

$\Delta y_i = y_{i+1} - y_i$

$L = (\Delta x_i^2 + \Delta y_i^2)^{1/2}$

CALCULATE MOMENT OF INERTIA I_x



CONSIDER EACH SEGMENT AS MADE OF 100 EQUAL PARTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$

FOR $j = 1$ TO 100

$y = y_i + \Delta y_i (j - 0.5)/100$

$\Delta I = (\Delta AREA) y^2$

$I_x = I_x + \Delta I$

SINCE ONLY TOP HALF WAS USED

$I_x = 2 I_x$

CALCULATE SHEARING STRESS AT ENDS OF SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$, $\tau_{new} = \tau_{next}$

FOR $j = 1$ TO 100

$y = y_i + \Delta y_i (j - 0.5)/100$

$\Delta Q = (\Delta AREA) y$

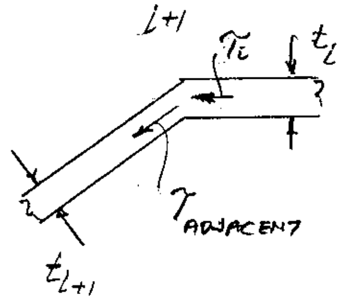
$\tau_{old} = \tau_{new}$, $Q = Q + \Delta Q$

$\tau_{new} = VQ / I_x t_i$

$\tau_{ave} = 0.5(\tau_{old} + \tau_{new})$

$\tau = \tau + \tau_{ave}$

CONTINUED



$FORCE_i = \tau(\Delta AREA)$

$\tau_i = VQ/I_x t_i$

$(\tau_{ADJACENT})_i = VQ/I_x t_{i+1}$

$Q_i = Q$

$\tau_{max} = (\tau_{ADJACENT})_i$

LOCATION OF SHEAR CENTER

CALCULATE MOMENT OF SHEAR FORCES ABOUT ORIGIN

FOR $i = 1$ TO n

$(F_x)_i = FORCE_i\,(\Delta x_i)/L_i$

$(F_y)_i = FORCE_i\,(\Delta y_i)/L_i$

$MOMENT_i = -(F_x)_i\, y_i + (F_y)_i\, x_i$

$MOMENT = MOMENT + MOMENT_i$

FOR WHOLE SECTION $MOMENT = 2(MOMENT)$

SHEAR CENTER IS AT

$e = MOMENT/V$

PROGRAM OUTPUT

Prob. 6.65

	T(i) mm	X(i) mm	Y(i) mm	L(i) mm
1	10.00	70.00	10.00	40.000
2	6.00	70.00	50.00	70.000
3	10.00	0.00	50.00	50.000
4	10.00	0.00	0.00	

Moment of inertia: Ix = 3759956 mm^4 Shear = 50000 N

Junction of segments	Q mm^3	Tau Before MPa	Tau After MPa	Force in segment kN
1 and 2	12000.000	15.96	26.60	2482.37
2 and 3	33000.000	73.14	43.88	20888.54
3 and 4	45500.000	60.51	60.51	27372.75

Moment of shear forces about origin: M = 2436.386 N·m
+ counterclockwise

Distance from origin to shear center: e = 48.728 mm

CONTINUED

PROBLEM 6.C5 - PROGRAM PRINTOUTS CONTINUED

Prob. 6.68

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	3.00	4.00	3.000
2	0.50	0.00	4.00	4.000
3	0.50	0.00	0.00	

Moment of inertia: Ix = 45.3328 in^4 Shear = 25.000 kips

Junction of segments	Q in^3	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	3.000	6.62	3.31	2.48
2 and 3	7.000	7.72	7.72	12.47

Moment of shear forces about origin: M = 19.853 kip·in.
+ counterclockwise

Distance from origin to shear center: e = 0.7941 in.

Prob. 6.69

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	4.00	5.00	2.000
2	0.25	4.00	3.00	5.000
3	0.25	0.00	0.00	

Moment of inertia: Ix = 23.8331 in^4 Shear = 10.000 kips

Junction of segments	Q in^3	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	2.000	3.36	3.36	0.91
2 and 3	3.875	6.50	6.50	6.80

Moment of shear forces about origin: M = -7.273 kip·in.
+ counterclockwise

Distance from origin to shear center: e = -0.7273 in.

Prob. 6.70

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	2.60	0.00	1.500
2	0.25	2.60	1.50	3.002
3	0.25	0.00	0.00	

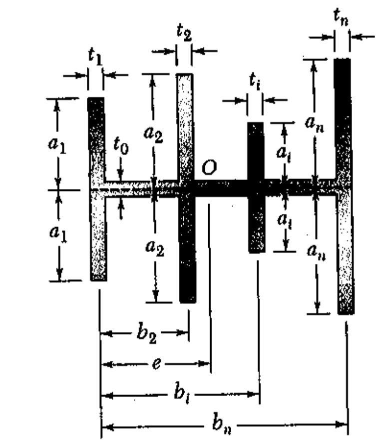
Moment of inertia: Ix = 1.6881 in^4 Shear = 10.000 kips

Junction of segments	Q in^3	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	0.281	6.66	6.66	0.83
2 and 3	0.844	20.00	20.00	11.65

Moment of shear forces about origin: M = 4.332 kip·in.
+ counterclockwise

Distance from origin to shear center: e = 0.4332 in.

PROBLEM 6.C6



6.C6 A thin-walled beam has the cross section shown. Write a computer program that, for dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center O of the cross section. Use this program to solve Prob. 6.100.

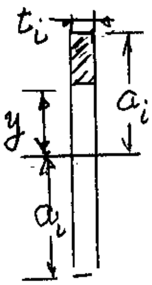
SOLUTION

Distribution of shearing stresses in element i

Let V = shear in cross section

$\bar{I}$ = Centroidal moment of inertia of section

We have for shaded area

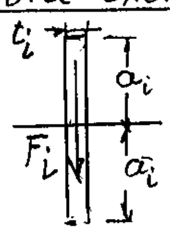


$$Q = A\bar{y} = t_i(a_i - y)\frac{a_i + y}{2}$$

$$= \frac{1}{2}t_i(a_i^2 - y^2)$$

$$\tau = \frac{QV}{It_i} = \frac{V}{2I}(a_i^2 - y^2)$$

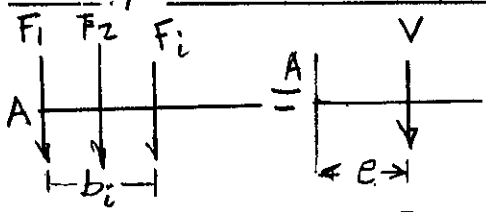
Force exerted on element i



$$F_i = \int_{-a_i}^{a_i} \tau(t_i\,dy) = \frac{Vt_i}{2I}\int_{-a_i}^{a_i}(a_i^2 - y^2)\,dy$$

$$= \frac{Vt_i}{I}\int_0^{a_i}(a_i^2 - y^2)\,dy = \frac{Vt_i}{I}\left(a_i^3 - \frac{1}{3}a_i^3\right) = \frac{2}{3}\frac{V}{I}t_i a_i^3$$

The system of the forces F_i must be equivalent to V at shear center.



$$\Sigma F = \Sigma F: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 = V \qquad (1)$$

$$\Sigma M_A = \Sigma M_A: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 b_i = eV \qquad (2)$$

Divide (2) by (1):
$$e = \frac{\Sigma t_i a_i^3 b_i}{\Sigma t_i a_i^3}$$

PROGRAM OUTPUT:

```
Prob. 6.100

For element 1:
t = 0.75 in., a = 4 in., b = 0

For element 2:
t = 0.75 in., a = 3 in., b= 8 in.

Answer: e = 2.37 in.
```