CHAPTER 2

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PROBLEM 2.1

2.1 A steel rod is 2.2 m long and must not stretch more than 1.2 mm when a 8.5 kN load is applied to it. Knowing that E = 200 GPa, determine (a) the smallest diameter rod which should be used, (b) the corresponding normal stress caused by the load.

SOLUTION

(a) $\delta = \dfrac{PL}{AE} \quad \therefore \quad A = \dfrac{PL}{E\delta} = \dfrac{(8.5\times10^{3})(2.2)}{(200\times10^{9})(1.2\times10^{-3})} = 77.92\times10^{-6}\ \text{m}^2$

$A = \frac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(77.92\times10^{-6})}{\pi}} = 9.96\times10^{-3}\ \text{m}$

$= 9.96\ \text{mm}$ ◄

(b) $\sigma = \dfrac{P}{A} = \dfrac{8.5\times10^{3}}{77.92\times10^{-6}} = 109.1\times10^{6}\ \text{Pa} = 109.1\ \text{MPa}$ ◄

PROBLEM 2.2

2.2 A 4.8-ft-long steel wire of $\frac{1}{4}$-in. diameter steel wire is subjected to a 750-lb tensile load. Knowing that $E = 29\times10^{6}$ psi, determine (a) the elongation of the wire, (b) the corresponding normal stress.

SOLUTION

(a) $L = 4.8\ \text{ft} = 57.6\ \text{in.} \qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{1}{4}\right)^2 = 49.087\times10^{-3}\ \text{in}^2$

$\delta = \dfrac{PL}{AE} = \dfrac{(750)(57.6)}{(49.087\times10^{-3})(29\times10^{6})} = 30.3\times10^{-3}\ \text{in} = 0.0303\ \text{in}$ ◄

(b) $\sigma = \dfrac{P}{A} = \dfrac{750}{49.087\times10^{-3}} = 15.28\times10^{3}\ \text{psi} = 15.28\ \text{ksi}$ ◄

PROBLEM 2.3

2.3 Two gage marks are placed exactly 10 inches apart on a $\frac{1}{2}$-in.-diameter aluminum rod with $E = 10.1\times10^{6}$ psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (a) the stress in the rod, (b) the factor of safety.

SOLUTION

(a) $\delta = 10.009 - 10.000 = 0.009\ \text{in.}$

$\dfrac{\delta}{L} = \dfrac{\sigma}{E} \quad \therefore \quad \sigma = \dfrac{E\delta}{L} = \dfrac{(10.1\times10^{6})(0.009)}{10} = 9.09\times10^{3}\ \text{psi}$

$= 9.09\ \text{ksi}$ ◄

(b) $\text{F.S.} = \dfrac{\sigma_U}{\sigma} = \dfrac{16}{9.09} = 1.760$ ◄

PROBLEM 2.4

2.4 A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that E = 105 GPa and that the maximum allowable normal stress is 180 MPa, determine (a) the smallest diameter that can be selected for the rod, (b) the corresponding maximum length of the rod.

SOLUTION

(a) $\sigma = \dfrac{P}{A} \quad \therefore \quad A = \dfrac{P}{\sigma} = \dfrac{4\times10^{3}}{180\times10^{6}} = 22.222\times10^{-6}\ \text{m}^2$

$A = \frac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(22.222\times10^{-6})}{\pi}} = 5.32\times10^{-3}\ \text{m}$

$= 5.32\ \text{mm}$ ◄

(b) $\delta = \dfrac{PL}{AE} \quad \therefore \quad L = \dfrac{AE\delta}{P} = \dfrac{(22.222\times10^{-6})(105\times10^{9})(3\times10^{-3})}{4\times10^{3}}$

$= 1.750\ \text{m}$ ◄

PROBLEM 2.5

2.5 A 9-m length of 6-mm-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 18 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(a) $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.006)^2 = 28.274 \times 10^{-6}\ \text{m}^2$

$\delta = \frac{PL}{AE} \therefore P = \frac{AE\delta}{L} = \frac{(28.274\times10^{-6})(200\times10^{9})(18\times10^{-3})}{9}$

$= 11.31\times10^{3}\ \text{N} = 11.31\ \text{kN}$ ◄

(b) $\sigma = \frac{P}{A} = \frac{11.31\times10^{3}}{28.274\times10^{-3}} = 400\times10^{6}\ \text{Pa} = 400\ \text{MPa}$ ◄

PROBLEM 2.6

2.6 A 4.5-ft. aluminum pipe should not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the allowable tensile strength is 14 ksi., determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(a) $\delta = \frac{PL}{AE} \therefore L = \frac{EA\delta}{P} = \frac{E\delta}{\sigma} = \frac{(10.1\times10^{6})(0.05)}{14\times10^{3}} = 36.1\ \text{in}$ ◄

(b) $\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{127.5\times10^{3}}{14\times10^{3}} = 9.11\ \text{in}^2$ ◄

PROBLEM 2.7

2.7 A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3$ GPa and that the length of the thread increases by 1.1 %, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(a) $\frac{\delta}{L} = \frac{1.1}{100} \therefore \frac{L}{\delta} = 90.909$

$\delta = \frac{PL}{AE} \therefore A = \frac{PL}{E\delta} = \frac{(8.5)(90.909)}{3.3\times10^{9}} = 234.16\times10^{-9}\ \text{m}^2$

$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = 0.546\times10^{-3}\ \text{m} = 0.546\ \text{mm}$ ◄

(b) $\sigma = \frac{P}{A} = \frac{8.5}{234.16\times10^{-9}} = 36.3\times10^{6}\ \text{Pa} = 36.3\ \text{MPa}$ ◄

PROBLEM 2.8

2.8 A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025 percent, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(a) $\frac{\delta}{L} = \frac{0.025}{100} = 0.00025$

$\sigma = \frac{E\delta}{L} = (10\times10^{6})(0.00025) = 2.5\times10^{3}\ \text{psi} = 2.5\ \text{ksi}$ ◄

(b) $\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{1600}{2.5\times10^{3}} = 0.640\ \text{in}^2$

$A = \frac{\pi}{4}(d_o^2 - d_i^2)$

$d_i^2 = d_o^2 - \frac{4A}{\pi} = 2.0^2 - \frac{(4)(0.64)}{\pi} = 3.1851\ \text{in}^2 \therefore d_i = 1.7847\ \text{in.}$

$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 - 1.7847) = 0.1077\ \text{in.}$ ◄

PROBLEM 2.9

SOLUTION

2.9 A block of 10-in. length and 1.8×1.6 in. cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 14 \times 10^6$ psi. Determine the largest load which can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12 percent of its original length.

Considering allowable stress $\sigma = 18$ ksi $= 18 \times 10^3$ psi

$A = (1.8)(1.6) = 2.88 \text{ in}^2 \qquad \sigma = \frac{P}{A}$

$P = \sigma A = (18 \times 10^3)(2.88) = 51.8 \times 10^3$ lb

Considering allowable deformation $\frac{\delta}{L} = \frac{0.12}{100} = 0.0012$

$\delta = \frac{PL}{AE} \therefore \quad P = AE\frac{\delta}{L} = (2.88)(14 \times 10^6)(0.0012) = 48.4 \times 10^3$ lb

Smaller value governs $\quad P = 48.4 \times 10^3$ lb $= 48.4$ kips

PROBLEM 2.10

SOLUTION

2.10 A 9-kN tensile load will be applied to a 50-m length of steel wire with $E = 200$ GPa. Determine the smallest diameter wire which can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in the length of the wire should be at most 25 mm.

Considering allowable stress $\sigma = 150 \times 10^6$ Pa

$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{9 \times 10^3}{150 \times 10^6} = 60 \times 10^{-6} \text{ m}^2$

Considering allowable elongation $\delta = 25 \times 10^{-3}$ m

$\delta = \frac{PL}{AE} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(9 \times 10^3)(50)}{(200 \times 10^9)(25 \times 10^{-3})} = 90 \times 10^{-6} \text{ m}^2$

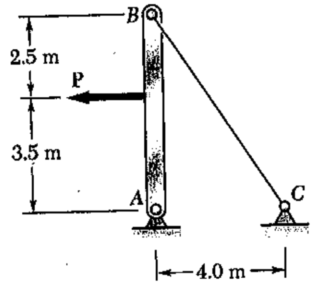
Larger area governs $\quad A = 90 \times 10^{-6} \text{ m}^2$

$A = \frac{\pi}{4} d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(90 \times 10^{-6})}{\pi}} = 10.70 \times 10^{-3}$ m

$= 10.70$ mm

PROBLEM 2.11

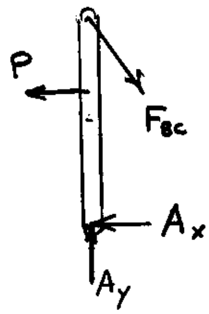
2.11 The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



SOLUTION

$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111$ m

Use bar AB as a free body



$\circlearrowleft \Sigma M_A = 0 \qquad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$

$P = 0.9509\ F_{BC}$

Considering allowable stress $\sigma = 190 \times 10^6$ Pa

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6}\ m^2$

$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3$ N

Considering allowable elongation $\delta = 6 \times 10^{-3}$ m

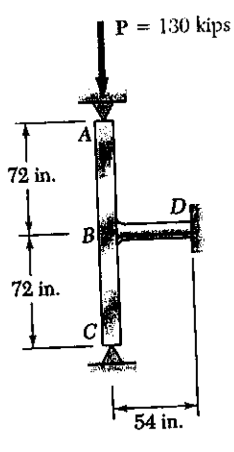
$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3$ N

Smaller value governs $\qquad F_{BC} = 2.091 \times 10^3$ N

$P = 0.9509\ F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3$ N $= 1.988$ kN ◄

PROBLEM 2.12

2.12 Rod *BD* is made of steel ($E = 29 \times 10^6$ psi) and is used to brace the axially compressed member *ABC*. The maximum force that can be developed in member *BD* is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of *BD* must not exceed 0.001 times the length of *ABC*, determine the smallest diameter rod that can be used for member *BD*.



SOLUTION

$F_{BD} = 0.02P = (0.02)(130) = 2.6$ kips $= 2.6 \times 10^3$ lb.

Considering stress $\sigma = 18$ ksi $= 18 \times 10^3$ psi

$\sigma = \frac{F_{BD}}{A} \therefore A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444\ in^2$

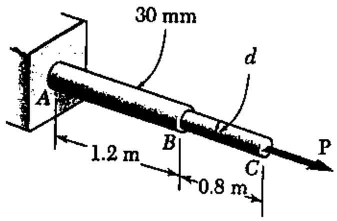
Considering deformation $\delta = (0.001)(144) = 0.144$ in.

$\delta = \frac{F_{BD} L_{BD}}{AE} \therefore A = \frac{F_{BD} L_{BD}}{E\delta} = \frac{(2.6 \times 10^3)(54)}{(29 \times 10^6)(0.144)} = 0.03362\ in^2$

Larger area governs $\qquad A = 0.14444\ in^2$

$A = \frac{\pi}{4} d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}} = 0.429$ in. ◄

PROBLEM 2.13



2.13 A single axial load of magnitude $P = 58$ kN is applied at end C of the brass rod ABC. Knowing that $E = 105$ GPa, determine the diameter d of portion BC for which the deflection of point C will be 3 mm.

SOLUTION

$$\delta_C = \sum \frac{P_i L_i}{A_i E} = \frac{P}{E}\left\{\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right\}$$

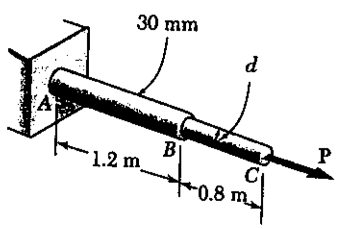
$$\frac{L_{BC}}{A_{BC}} = \frac{E\delta_C}{P} - \frac{L_{AB}}{A_{AB}} = \frac{(105\times10^9)(3\times10^{-3})}{58\times10^3} - \frac{1.2}{\frac{\pi}{4}(0.030)^2} = 3.7334\times10^3\ m^{-1}$$

$$A_{BC} = \frac{L_{BC}}{3.7334\times10^3} = \frac{0.8}{3.7334\times10^3} = 214.28\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 \quad \therefore \quad d_{BC} = \sqrt{\frac{4A_{BC}}{\pi}} = \sqrt{\frac{(4)(214.28\times10^{-6})}{\pi}} = 16.52\times10^{-3}\ m$$

$$= 16.52\ mm$$

PROBLEM 2.14



2.14 Both portions of the rod ABC are made of an aluminum for which $E = 73$ GPa. Knowing that the diameter of portion BC is $d = 20$ mm, determine the largest force **P** that can be applied if $\sigma_{all} = 160$ MPa and the corresponding deflection at point C is not to exceed 4 mm.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(0.030)^2 = 706.86\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\ m^2$$

Considering allowable stress $\sigma = 160\times10^6$ Pa

$$\sigma = \frac{P}{A} \quad \therefore \quad P = A\sigma$$

Portion AB $\quad P = (706.86\times10^{-6})(160\times10^6) = 113.1\times10^3\ N$

Portion BC $\quad P = (314.16\times10^{-6})(160\times10^6) = 50.3\times10^3\ N$

Considering allowable deflection $\delta_C = 4\times10^{-3}$ m

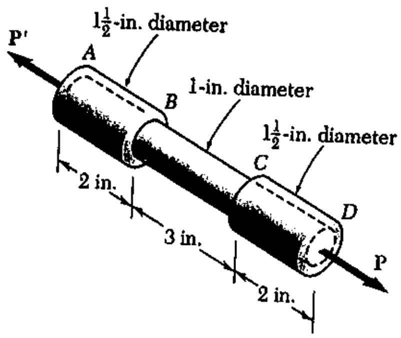
$$\delta_C = \sum \frac{PL_i}{AE} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$P = E\delta_C\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)^{-1} = (73\times10^9)(4\times10^{-3})\left(\frac{1.2}{706.86\times10^{-6}} + \frac{0.8}{314.16\times10^{-6}}\right)^{-1}$$

$$= 68.8\times10^3\ N$$

Smallest value for P governs $\quad P = 50.3\times10^3\ N = 50.3\ kN$

PROBLEM 2.15



2.15 The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(a) $\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$

$P = E\delta \left(\sum \frac{L_i}{A_i}\right)^{-1}$ $\qquad A_i = \frac{\pi}{4} d_i^2$

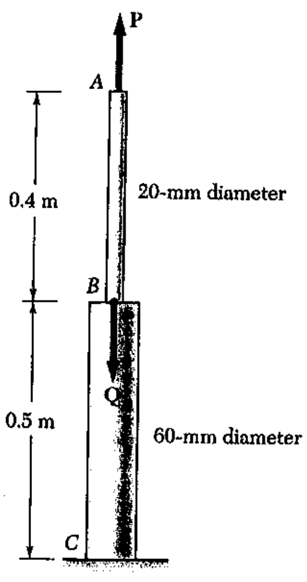
	L, in.	d, in	A_i, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$P = (29\times10^6)(0.002)(6.083)^{-1} = 9.535\times10^3$ lb. $= 9.53$ kips

(b) $\delta_{BC} = \frac{P L_{BC}}{A_{BC} E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{9.535\times10^3}{29\times10^6}(3.8197) = 1.254\times10^{-3}$ in

PROBLEM 2.16

2.16 Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.



SOLUTION

(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}$ m^2

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}$ m^2

Force in member AB is P tension

Elongation $\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})}$

$= 72.756\times10^{-6}$ m

Force in member BC is Q − P compression

Shortening $\delta_{BC} = \frac{(Q-P)L_{BC}}{E A_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})}$

$= 2.5263\times10^{-9}(Q-P)$

For zero deflection at A $\qquad \delta_{BC} = \delta_{AB}$

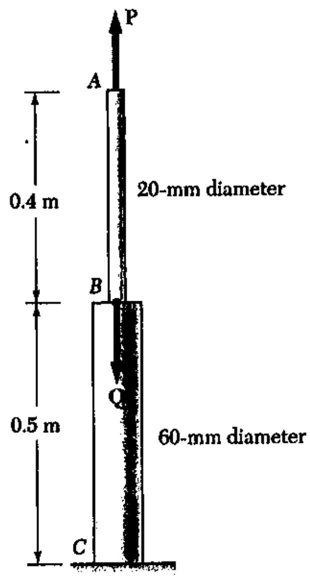
$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3$ N

$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3$ N $= 32.8$ kN

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}$ m $= 0.0728$ mm

PROBLEM 2.17

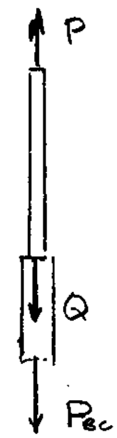
2.17 The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.



SOLUTION

(a) $$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\ m^2$$



$$P_{AB} = P = 6\times10^3\ N$$

$$P_{BC} = P - Q = 6\times10^3 - 42\times10^3 = -36\times10^3\ N$$

$$L_{AB} = 0.4\ m \qquad L_{BC} = 0.5\ m$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6\times10^3)(0.4)}{(314.16\times10^{-6})(70\times10^9)}$$

$$= 109.135\times10^{-6}\ m$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36\times10^3)(0.5)}{(2.8274\times10^{-3})(70\times10^9)}$$

$$= -90.947\times10^{-6}\ m$$

$$\delta_A = \delta_{AB} + \delta_{BC} = 109.135\times10^{-6} - 90.947\times10^{-6}\ m = 18.19\times10^{-6}\ m$$
$$= 0.01819\ mm$$

(b) $$\delta_B = \delta_{BC} = -90.9\times10^{-6}\ m = -0.0909\ mm$$

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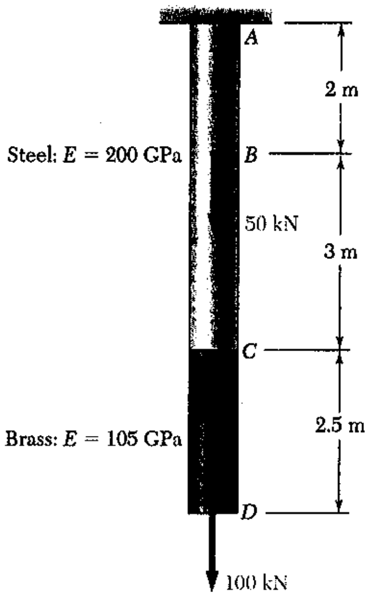
PROBLEM 2.18



2.18 The 36-mm-diameter steel rod *ABC* and a brass rod *CD* of the same diameter are joined at point *C* to form the 7.5-m rod *ABCD*. For the loading shown, and neglecting the weight of the rod, determine the deflection of (*a*) point *C*, (*b*) point *D*.

SOLUTION

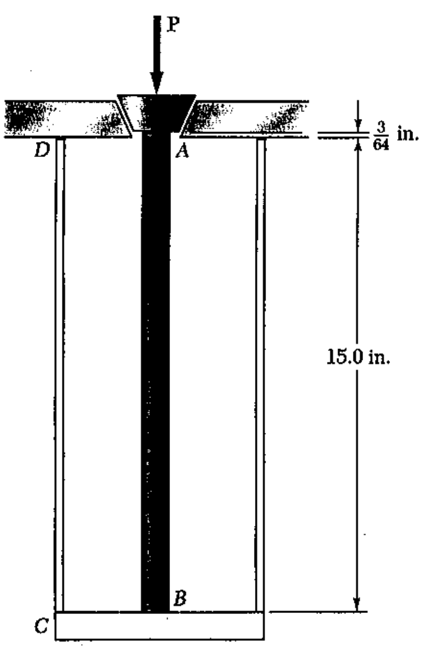
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.036)^2 = 1.01787 \times 10^{-3}\ \text{m}^2$$

Portion	P_i	L_i	E_i	$P_i L_i / A E_i$
AB	150 kN	2 m	200 GPa	1.474×10^{-3} m
BC	100 kN	3 m	200 GPa	1.474×10^{-3} m
CD	100 kN	2.5 m	105 GPa	2.339×10^{-3} m

(a) $\delta_C = \delta_{AB} + \delta_{BC} = 1.474 \times 10^{-3} + 1.474 \times 10^{-3}$
$= 2.948 \times 10^{-3}\ \text{m} = 2.95\ \text{mm}$

(b) $\delta_D = \delta_C + \delta_{CD} = 2.948 \times 10^{-3} + 2.339 \times 10^{-3}$
$= 5.287 \times 10^{-3}\ \text{m} = 5.29\ \text{mm}$

PROBLEM 2.19



2.19 The brass tube *AB* ($E = 15 \times 10^6$ psi) has a cross-sectional area of 0.22 in^2 and is fitted with a plug at *A*. The tube is attached at *B* to a rigid plate which is itself attached at *C* to the bottom of an aluminum cylinder ($E = 10.4 \times 10^6$ psi) with a cross-sectional area of 0.40 in^2. The cylinder is then hung from a support at *D*. In order to close the cylinder, the plug must move down through $\frac{3}{64}$ in. Determine the force **P** that must be applied to the cylinder.

Shortening of brass tube AB

$L_{AB} = 15 + \frac{3}{64} = 15.047$ in $\qquad A_{AB} = 0.22\ \text{in}^2$

$E_{AB} = 15 \times 10^6$ psi

$$\delta_{AB} = \frac{P L_{AB}}{E_{AB} A_{AB}} = \frac{P(15.047)}{(15 \times 10^6)(0.22)} = 4.5597 \times 10^{-6}\ P$$

Lengthening of aluminum cylinder CD

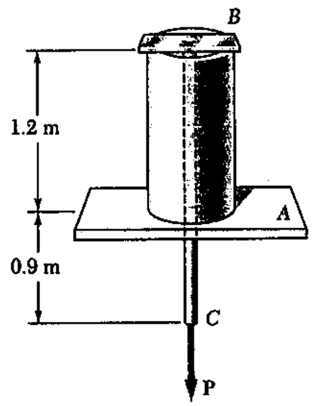
$L_{CD} = 15$ in, $A_{CD} = 0.40\ \text{in}^2$, $E_{CD} = 10.4 \times 10^6$ psi

$$\delta_{CD} = \frac{P L_{CD}}{E_{CD} A_{CD}} = \frac{P(15)}{(10.4 \times 10^6)(0.40)} = 3.6058 \times 10^{-6}\ P$$

Total deflection $\quad \delta_A = \delta_{AB} + \delta_{CD}$

$\frac{3}{64} = (4.5597 \times 10^{-6} + 3.6058 \times 10^{-6})P \quad \therefore \quad P = 5.74 \times 10^3$ lb.
$= 5.74$ kips

PROBLEM 2.20



2.20 A 1.2-m section of aluminum pipe of cross-sectional area 1100 mm^2 rests on a fixed support at A. The 15-mm-diameter steel rod BC hangs from a rigid bar that rests on the top of the pipe at B. Knowing that the modulus of elasticity is 200 GPa for steel and 72 GPa for aluminum, determine the deflection of point C when a 60 kN force is applied at C.

SOLUTION

Rod BC $\quad L_{BC} = 2.1\ m, \quad E_{BC} = 200\times10^9\ Pa$

$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.015)^2 = 176.715\times10^{-6}\ m^2$$

$$\delta_{C/B} = \frac{PL_{BC}}{E_{BC}A_{BC}} = \frac{(60\times10^3)(2.1)}{(200\times10^9)(176.715\times10^{-6})}$$

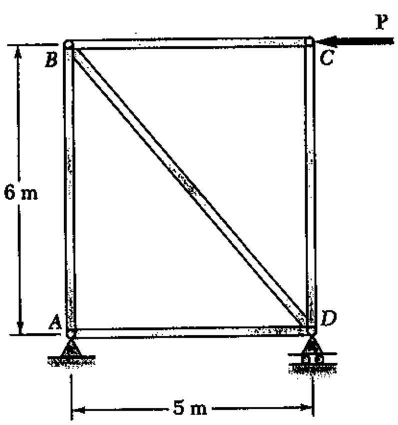
$$= 3.565\times10^{-3}\ m$$

Pipe AB: $\quad L_{AB} = 1.2\ m, \quad E_{AB} = 72\times10^9\ Pa, \quad A_{AB} = 1100\ mm^2 = 1100\times10^{-6}\ m^2$

$$\delta_{B/A} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{(60\times10^3)(1.2)}{(72\times10^9)(1100\times10^{-6})} = 909.1\times10^{-6}\ m^2$$

$$\delta_C = \delta_{B/A} + \delta_{C/B} = 909.1\times10^{-6} + 3.565\times10^{-3} = 4.47\times10^{-3}\ m = 4.47\ mm$$

PROBLEM 2.21



2.21 The steel frame ($E = 200$ GPa) shown has a diagonal brace BD with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed 1.6 mm.

SOLUTION

$\delta_{BC} = 1.6\times10^{-3}\ m, \quad A_{BD} = 1920\ mm^2 = 1920\times10^{-6}\ m^2$

$L_{BC} = \sqrt{5^2 + 6^2} = 7.810\ m, \quad E_{BC} = 200\times10^9\ Pa$

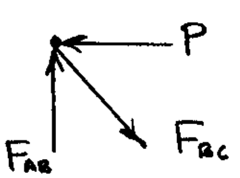
$$\delta_{BC} = \frac{F_{BC}L_{BC}}{E_{BC}A_{BC}}$$

$$F_{BC} = \frac{E_{BC}A_{BC}\delta_{BC}}{L_{BC}} = \frac{(200\times10^9)(1920\times10^{-6})(1.6\times10^{-3})}{7.81}$$

$$= 78.67\times10^3\ N$$

Use joint B as a free body: $\xrightarrow{+}\Sigma F_x = 0$

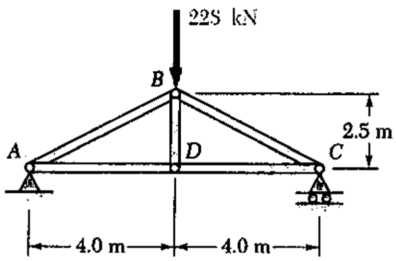
$$\frac{5}{7.810}F_{BC} - P = 0$$



$$P = \frac{5}{7.810}F_{BC} = \frac{(5)(78.67\times10^3)}{7.810}$$

$$= 50.4\times10^3\ N = 50.4\ kN$$

PROBLEM 2.22



2.22 For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of members AB and AD, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

SOLUTION

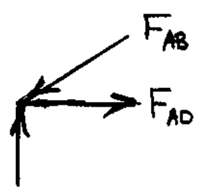
Statics: Reactions are 114 kN upward at A & C.

Member BD is a zero force member

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint A as a free body: $+\uparrow \Sigma F_y = 0 \qquad 114 - \frac{2.5}{4.717} F_{AB} = 0$

$$F_{AB} = 215.10 \text{ kN}$$



$$\xrightarrow{+} \Sigma F_x = 0 \qquad F_{AD} - \frac{4}{4.717} F_{AB} = 0$$

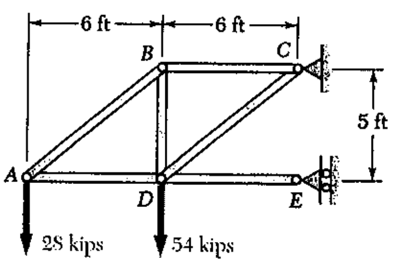
$$F_{AD} = \frac{(4)(215.10)}{4.717} = 182.4 \text{ kN}$$

Member AB: $\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(215.10\times10^3)(4.717)}{(200\times10^9)(2400\times10^{-6})}$

$$= 2.11\times10^{-3} \text{ m} = 2.11 \text{ mm}$$

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4\times10^3)(4.0)}{(200\times10^9)(1800\times10^{-6})} = 2.03\times10^{-3} \text{ m} = 2.03 \text{ mm}$$

PROBLEM 2.23

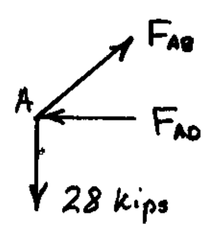


2.23 Members AB and BC are made of steel ($E = 29 \times 10^6$ psi) with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member AB, (*b*) member BC.

SOLUTION

(a) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft.} = 93.72 \text{ in}$

Use joint A as a free body

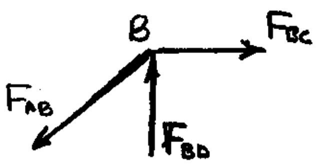


$$+\uparrow \Sigma F_y = 0 \qquad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74\times10^3 \text{ lb.}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74\times10^3)(93.72)}{(29\times10^6)(0.80)} = 0.1767 \text{ in.}$$

(b) Use joint B as a free body

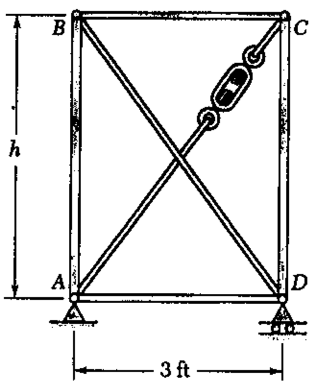


$$\xrightarrow{+} \Sigma F_x = 0 \qquad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60\times10^3 \text{ lb.}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60\times10^3)(72)}{(29\times10^6)(0.64)} = 0.1304 \text{ in.}$$

PROBLEM 2.24



2.24 Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

SOLUTION

$\delta_{AB} = \delta_{CD} = 0.04 \text{ in} \qquad h = 4 \text{ ft.} = 48 \text{ in.} = L_{CD}$

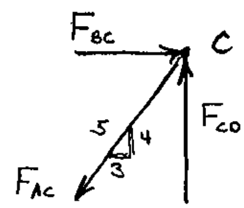
$A_{CD} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$

$\delta_{CD} = \dfrac{F_{CD} L_{CD}}{E A_{CD}}$

$F_{CD} = \dfrac{E A_{CD} \delta_{CD}}{L_{CD}} = \dfrac{(29\times10^6)(0.99402)(0.04)}{48}$

$= 24.022 \times 10^3 \text{ lb.}$

Use joint C as a free body



$+\uparrow \Sigma F_y = 0: \quad F_{CD} - \frac{4}{5} F_{AC} = 0 \qquad \therefore F_{AC} = \frac{5}{4} F_{CD}$

$F_{AC} = \frac{5}{4}(24.022 \times 10^3) = 30.0 \times 10^3 \text{ lb.}$

30.0 kips ◄

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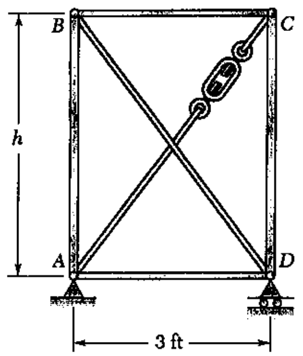
PROBLEM 2.25

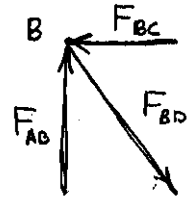


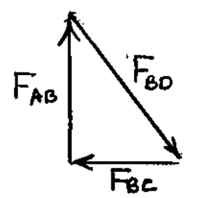
2.24 Members *AB* and *CD* are $1\frac{1}{8}$-in.-diameter steel rods, and members *BC* and *AD* are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in *AC* so that the deformations in members *AB* and *CD* do not exceed 0.04 in.

2.25 For the structure in Prob. of 2.24, determine (*a*) the distance *h* so that the deformations in members *AB, BC, CD* and *AD* are all equal to 0.04 in., (*b*) the corresponding tension in member *AC*.

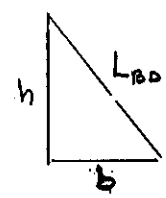
SOLUTION

(a) Statics: Use joint B as a free body





Force Triangle



Geometry

From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BC}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$

For equal deformations

$$\delta_{AB} = \delta_{BC} \therefore \quad \frac{F_{AB}\, h}{E A_{AB}} = \frac{F_{BC}\, b}{E A_{BC}} \qquad F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \qquad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{9/8}{7/8} = \frac{9}{7} \qquad b = 3\text{ ft} = 36\text{ in.}$$

$$h = \frac{9}{7} b = \frac{9}{7}(3) = 3.86\text{ ft} = 46.3\text{ in.}$$

(b) Setting $\delta_{AB} = \delta_{BC} = 0.04$ in.

$$\delta_{BC} = \frac{F_{BC}\, b}{E A_{BC}} \therefore \qquad F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(29 \times 10^6) \frac{\pi}{4} \left(\frac{7}{8}\right)^2 (0.04)}{36}$$

$$= 19.376 \times 10^3 \text{ lb.}$$

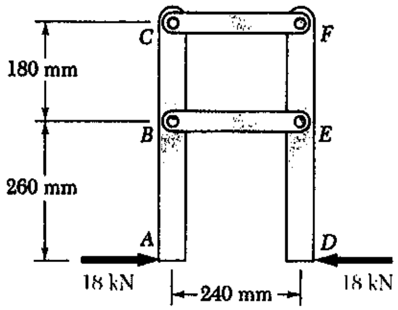
$$F_{AB} = \frac{h}{b} F_{BC} = \frac{9}{7}(19.376 \times 10^3) = 24.912 \times 10^3 \text{ lb}$$

From the force triangle

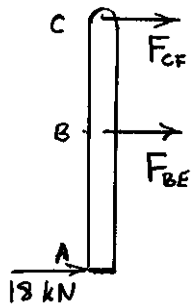
$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 31.6 \times 10^3 \text{ lb.}$$

PROBLEM 2.26

2.26 Members *ABC* and *DEF* are joined with steel links ($E = 200$ GPa). Each of the links is made of a pair of 25×35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.



SOLUTION



Use member ABC as a free body

$$\circlearrowleft \Sigma M_B = 0$$

$$(0.260)(18\times10^3) - (0.180)F_{CF} = 0$$

$$F_{CF} = \frac{(0.260)(18\times10^3)}{0.180} = 26\times10^3 \text{ N}$$

$$\circlearrowleft \Sigma M_C = 0 \qquad (0.440)(18\times10^3) + (0.180)F_{BE} = 0$$

$$F_{BE} = -\frac{(0.440)(18\times10^3)}{0.180} = -44\times10^3 \text{ N}$$

Area for link made of two plates

$$A = (2)(0.025)(0.035) = 1.75\times10^{-3} \text{ m}^2$$

(a) $$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA} = \frac{(-44\times10^3)(0.240)}{(200\times10^9)(1.75\times10^{-3})} = -30.2\times10^{-6} \text{ m} = -0.0302 \text{ mm}$$ ◄

(b) $$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA} = \frac{(26\times10^3)(0.240)}{(200\times10^9)(1.75\times10^{-3})} = 17.83\times10^{-6} \text{ m} = 0.01783 \text{ mm}$$ ◄

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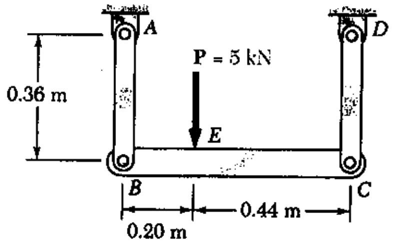
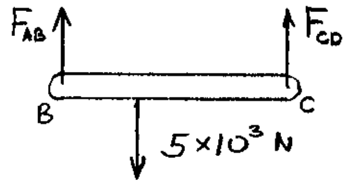
PROBLEM 2.27



2.27 Each of the links AB and CD is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of 125 mm². Knowing that they support the rigid member BC, determine the deflection of point E.

SOLUTION



Use member BC as a free body

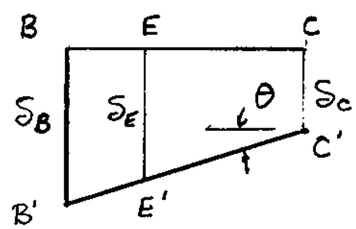
$\circlearrowleft + \ \Sigma M_C = 0 \qquad -(0.64)F_{AB} + (0.44)(5\times10^3) = 0 \qquad F_{AB} = 3.4375\times10^3 \text{ N}$

$\circlearrowleft + \ \Sigma M_B = 0 \qquad (0.64)F_{CD} - (0.20)(5\times10^3) = 0 \qquad F_{CD} = 1.5625\times10^3 \text{ N}$

For links AB and CD $\qquad A = 125 \text{ mm}^2 = 125\times10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(3.4375\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 132.00\times10^{-6} \text{ m} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(1.5625\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 60.00\times10^{-6} \text{ m} = \delta_C$$



Deformation diagram

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00\times10^{-6}}{0.64}$$

$$= 112.5\times10^{-6} \text{ rad}$$

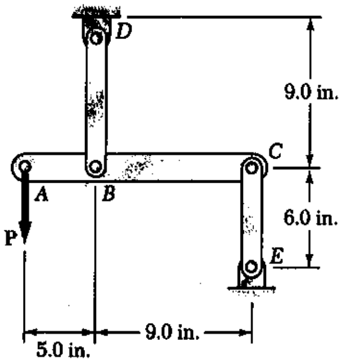
$$\delta_E = \delta_C + l_{EC}\,\theta$$

$$= 60.00\times10^{-6} + (0.44)(112.5\times10^{-6})$$

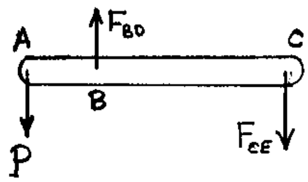
$$= 109.5\times10^{-6} \text{ m} = 0.1095 \text{ mm}$$

PROBLEM 2.28

2.28 Link *BD* is made of brass ($E = 15 \times 10^6$ psi) and has a cross-sectional area of 0.40 in^2. Link *CE* is made of aluminum ($E = 10.4 \times 10^6$ psi) and has a cross-sectional area of 0.50 in^2. Determine the maximum force **P** that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.014 in.



SOLUTION



Use member ABC as a free body.

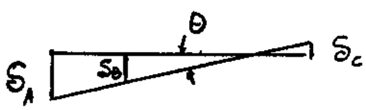
$\circlearrowleft \Sigma M_C = 0, \quad 14P - 9F_{BD} = 0, \qquad F_{BD} = 1.5556\ P$

$\circlearrowleft \Sigma M_B = 0 \quad 5P - 9F_{CE} = 0, \qquad F_{CE} = 0.5556\ P$

$$\delta_B = \delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}} = \frac{(1.5556\ P)(9.0)}{(15 \times 10^6)(0.40)} = 2.3333 \times 10^{-6}\ P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE} L_{CE}}{E_{CE} A_{CE}} = \frac{(0.5556\ P)(6.0)}{(10.4 \times 10^6)(0.50)} = 0.6410 \times 10^{-6}\ P \uparrow$$

From the deformation diagram



Deformation Diagram

$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{l_{BC}} = \frac{2.9743 \times 10^{-6}\ P}{9}$$

$$= 0.3305 \times 10^{-6}\ P$$

$$\delta_A = \delta_B + l_{AB}\,\theta$$

$$= 2.3333 \times 10^{-6} P + (5)(0.3305 \times 10^{-6}) P$$

$$= 3.9858 \times 10^{-6}\ P$$

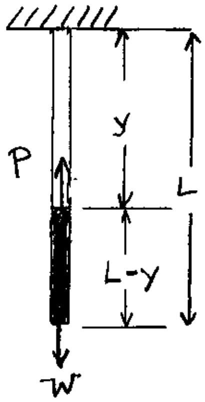
Apply displacement limit $\quad \delta_A = 0.014 \text{ in} = 3.9858 \times 10^{-6}\ P$

$$P = \frac{0.014}{3.9858 \times 10^{-6}} = 3.51 \times 10^3 \text{ lb} = 3.51 \text{ kips}$$

PROBLEM 2.29

2.29 A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Assuming now the cable to be horizontal, determine the force that should be applied to each end of the cable to obtain the same elongation as in part *a*.

SOLUTION



(a) For element at point identified by coordinate y

P = weight of portion below the point

$= \rho g A(L-y)$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g (L-y)}{E}\,dy$$

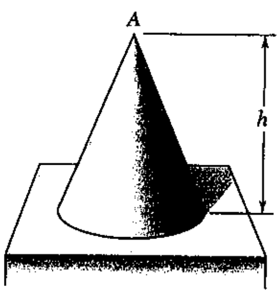
$$\delta = \int_0^L \frac{\rho g (L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \tfrac{1}{2}y^2\right)\Big|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) = \frac{1}{2}\,\frac{\rho g L^2}{E}$$

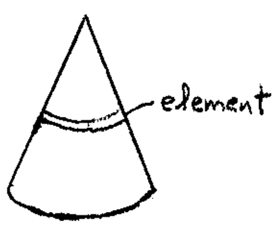
(b) For $\delta = \dfrac{PL}{EA}$ $\quad P = \dfrac{EA\delta}{L} = \dfrac{EA}{L}\,\dfrac{\rho g L^2}{2E} = \tfrac{1}{2}\rho g L = \tfrac{1}{2}W$

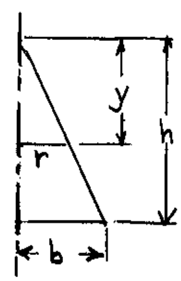
PROBLEM 2.30

2.30 Determine the deflection of the apex A of a homogeneous circular cone of height h, density ρ, and modulus of elasticity E, due to its own weight.



SOLUTION





Let b = radius of the base and r = radius at section with coordinate y.

$$r = \frac{b}{h}y$$

Volume of portion above element $\quad V = \frac{1}{3}\pi r^2 y = \frac{1}{3}\pi \frac{b^2}{h^2} y^3$

$$P = \rho g V = \frac{\pi \rho g b^2 y^3}{3h^2} \qquad A = \pi r^2 = \frac{\pi b^2}{h^2} y^2$$

$$\delta = \Sigma \frac{P\,\Delta y}{EA} = \int_0^h \frac{P\,dy}{EA} = \int_0^h \frac{\pi \rho g b^2 y^3}{3h^2}\cdot\frac{h^2}{E\pi b^2 y^2}\,dy = \int_0^h \frac{\rho g y}{3E}\,dy$$

$$= \frac{\rho g}{3E}\,\frac{y^2}{2}\Big|_0^h = \frac{\rho g h^2}{6E}$$

PROBLEM 2.31

2.31 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\epsilon_t = 2\ln(d_1/d)$.

SOLUTION

If the volume is constant $\quad \frac{\pi}{4} d^2 L = \frac{\pi}{4} d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2 = 2\ln\frac{d_1}{d}$$

PROBLEM 2.32

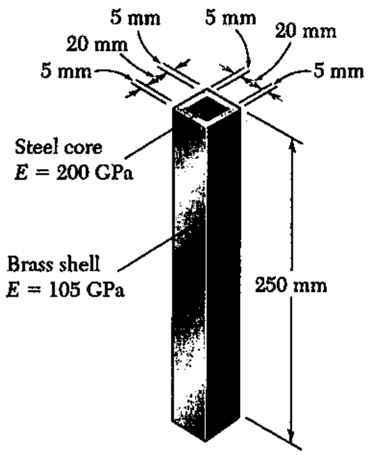
2.32 Denoting by ϵ the "engineering strain" in a tensile specimen, show that the true strain is $\epsilon_t = \ln(1 + \epsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0 + \delta}{L_0} = \ln\left(1 + \frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus $\quad \varepsilon_t = \ln(1+\varepsilon)$

PROBLEM 2.33



2.33 An axial force of 60 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the brass shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_b = portion of axial force carried by brass shell

P_s = portion of axial force carried by steel core

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_b A_b + E_s A_s}$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\frac{\delta}{L} = \varepsilon = \frac{60 \times 10^3}{(105 \times 10^9)(500 \times 10^{-6}) + (200 \times 10^9)(400 \times 10^{-6})} = 452.83 \times 10^{-6}$$

(a) $\sigma_b = E_b \varepsilon = (105 \times 10^9)(452.83 \times 10^{-6}) = 47.5 \times 10^6 \text{ Pa}$
$= 47.5 \text{ MPa}$ ◄

(b) $\delta = L\varepsilon = (250 \times 10^{-3})(452.83 \times 10^{-6}) = 113.2 \times 10^{-6} \text{ m}$
$= 0.1132 \times 10^{-3} \text{ m}$
$= 0.1132 \text{ mm}$ ◄

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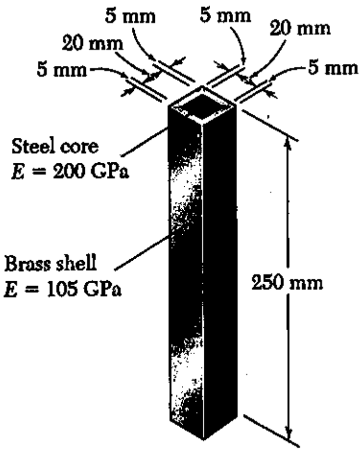
PROBLEM 2.34



2.34 The length of the assembly decreases by 0.15 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (b) the corresponding stress in the steel core.

SOLUTION

Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6}\ m^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6}\ m^2$$

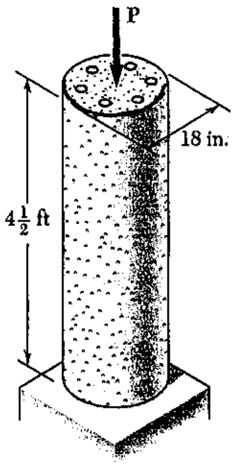
(a) $$P = [(105\times10^9)(500\times10^{-6}) + (200\times10^9)(400\times10^{-6})]\frac{0.15\times10^{-3}}{250\times10^{-3}}$$

$$= 79.5\times10^3\ N \qquad = 75.9\ kN$$

(b) $$\sigma_s = E_s\varepsilon = \frac{E_s\delta}{L} = \frac{(200\times10^9)(0.15\times10^{-3})}{250\times10^{-3}} = 120\times10^6\ Pa$$

$$= 120\ MPa$$

PROBLEM 2.35



2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete

P_s = portion carried by the six steel rods

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(1.125)^2 = 5.964 \text{ in}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(18)^2 - 5.964 = 248.5 \text{ in}^2$$

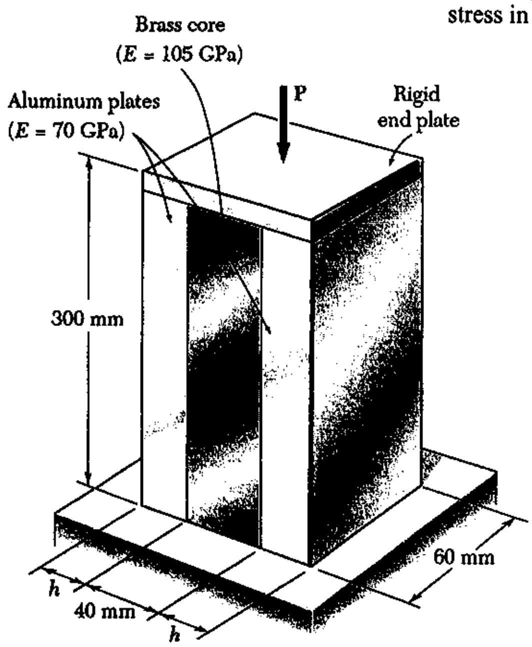
$$L = 4.5 \text{ ft} = 54 \text{ in}$$

$$\varepsilon = -\frac{350 \times 10^3}{(4.2\times10^6)(248.5) + (29\times10^6)(5.964)} = -287.67\times10^{-6}$$

$$\sigma_s = E_s\varepsilon = (29\times10^6)(-287.67\times10^{-6}) = -8.34\times10^3 \text{ psi} = -8.34 \text{ ksi}$$

$$\sigma_c = E_c\varepsilon = (4.2\times10^6)(-287.67\times10^{-6}) = -1.208\times10^3 \text{ psi} = -1.208 \text{ ksi}$$

PROBLEM 2.36



2.36 An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

SOLUTION

Let P_b = portion of axial force carried by brass core

P_a = portion carried by two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

$$P = P_b + P_a = (E_b A_b + E_a A_a)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$A_a = (2)(60)(10) = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

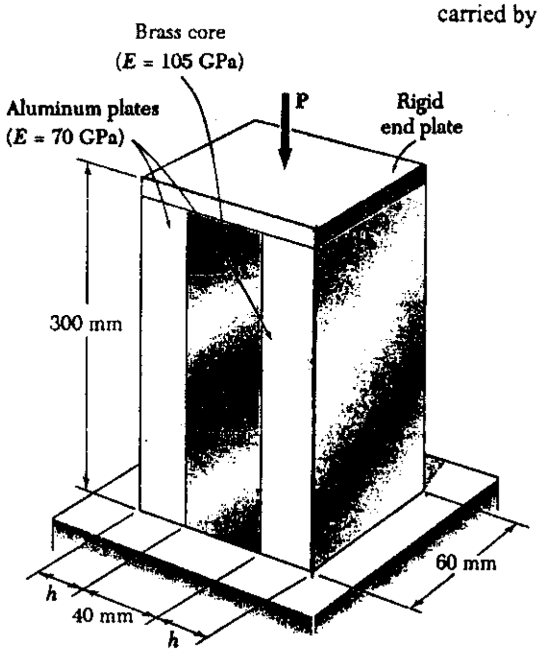
$$\varepsilon = \frac{450 \times 10^3}{(105 \times 10^9)(2400 \times 10^{-6}) + (70 \times 10^9)(1200 \times 10^{-6})} = 1.3393 \times 10^{-3}$$

(a) $\sigma_b = E_b \varepsilon = (105 \times 10^9)(1.3393 \times 10^{-3}) = 140.6 \times 10^6 \text{ Pa} = 140.6 \text{ MPa}$ ◀

(b) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(1.3393 \times 10^{-3}) = 93.75 \times 10^6 \text{ Pa} = 93.75 \text{ MPa}$ ◀

PROBLEM 2.37

2.37 For the composite block shown in Prob. 2.36, determine (*a*) the value of *h* if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (*b*) the total load if the stress in the brass is 80 MPa.



SOLUTION

Let P_b = portion of axial force carried by brass core

P_a = portion carried by the two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

(a) Given $P_a = \frac{1}{2} P_b$

$$\frac{E_a A_a \delta}{L} = \frac{1}{2} \frac{E_b A_b \delta}{L}$$

$$A_a = \frac{1}{2} \frac{E_b}{E_a} A_b$$

$$A_b = (40)(60) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{1}{2} \frac{105 \times 10^9}{70 \times 10^9} 2400 = 1800 \text{ mm}^2 = (2)(60)\, h$$

$$h = \frac{1800}{(2)(60)} = 15 \text{ mm}$$

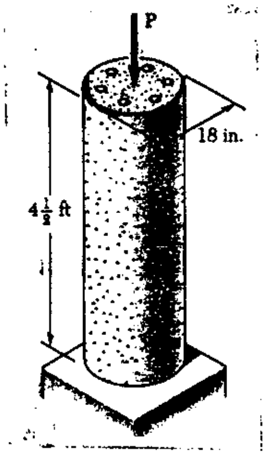
(b) $$\sigma_b = \frac{P_b}{A_b}$$

$$P_b = A_b \sigma_b = (2400 \times 10^{-6})(80 \times 10^6) = 192 \times 10^3 \text{ N}$$

$$P_a = \tfrac{1}{2} P_b = 96 \times 10^3 \text{ N}$$

$$P = P_b + P_a = 288 \times 10^3 \text{ N} = 288 \text{ kN}$$

PROBLEM 2.38



2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

2.38 For the post of Prob. 2.35, determine the maximum centric force which may be applied if the allowable normal stress is 20 ksi in the steel and 2.4 ksi in the concrete.

SOLUTION

Determine allowable strain in each material

Steel: $\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{20\times10^3}{29\times10^6} = 689.97\times10^{-6}$

Concrete: $\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{2.4\times10^3}{4.2\times10^6} = 571.43\times10^{-6}$

Smaller value governs $\quad \varepsilon = \frac{\delta}{L} = 571.43\times10^{-6}$

Let P_c = portion of load carried by concrete

P_s = portion carried by six steel rods

$\delta = \frac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \frac{\delta}{L} = E_c A_c \varepsilon$

$\delta = \frac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \frac{\delta}{L} = E_s A_s \varepsilon$

$P = P_c + P_s = (E_c A_c + E_s A_s)\varepsilon$

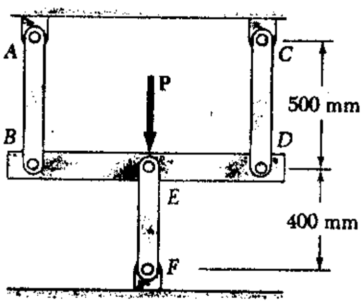
$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(1.125)^2 = 5.964 \text{ in}^2$

$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(18)^2 - 5.964 = 248.5 \text{ in}^2$

$P = [(4.2\times10^6)(248.5) + (29\times10^6)(5.964)](571.43\times10^{-6})$

$= 695\times10^3 \text{ lb} = 695 \text{ kips}$ ◄

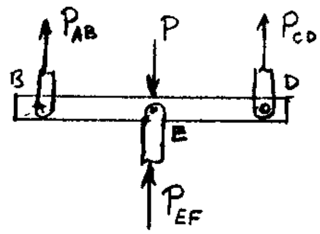
PROBLEM 2.39



2.39 Three steel rods ($E = 200$ GPa) support a 36-kN load **P**. Each of the rods AB and CD has a 200-mm^2 cross-sectional area and rod EF has a 625- mm^2 cross-sectional area. Determine the (a) the change in length of rod EF, (b) the stress in each rod.

SOLUTION

Use member BED as a free body



By symmetry, or by $\Sigma M_E = 0$

$$P_{CD} = P_{AB}$$

$$\Sigma F_y = 0$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{EA_{AB}}, \quad \delta_{CD} = \frac{P_{CD}L_{CD}}{EA_{CD}}, \quad \delta_{EF} = \frac{P_{EF}L_{EF}}{EA_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{P_{EF}L_{EF}}{EA_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256\, P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512\, P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

(a) $$\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

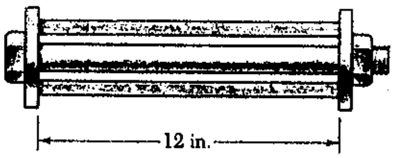
$$= 0.0762 \text{ mm} \blacktriangleleft$$

or $$\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

(b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.40



2.40 A brass bolt ($E_b = 15 \times 10^6$ psi) with a $\frac{3}{8}$-in. diameter is fitted inside a steel tube ($E_s = 29 \times 10^6$ psi) with a $\frac{7}{8}$-in. outer diameter and $\frac{1}{8}$-in wall thickness. After the nut has been fit snugly, it is tighened one quarter of a full turn. Knowing that the bolt is single-threaded with a 0.1-in. pitch, determine the normal stress (*a*) in the bolt, (*b*) in the tube.

SOLUTION

The movement of the nut along the bolt after a quarter turn is equal to $\frac{1}{4} \times$ pitch.

$$\delta = (\tfrac{1}{4})(0.1) = 0.025 \text{ in}$$

Also $\delta = \delta_{bolt} + \delta_{tube}$ where δ_{bolt} = elongation of the bolt and δ_{tube} = shortening of the tube

Let P_{bolt} = axial tensile force in the bolt
P_{tube} = axial compressive force in the tube

For equilibrium of each end plate $P_{bolt} = P_{tube} = P$

$$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}\left(\left(\frac{7}{8}\right)^2 - \left(\frac{5}{8}\right)^2\right) = 0.29452 \text{ in}^2$$

$$\delta_{bolt} = \frac{P_{bolt} L}{E A_{bolt}} = \frac{(P)(12)}{(15\times10^6)(0.11045)} = 7.2431\times10^{-6} P$$

$$\delta_{tube} = \frac{P_{tube} L}{E A_{tube}} = \frac{(P)(12)}{(29\times10^6)(0.29452)} = 1.4050\times10^{-6} P$$

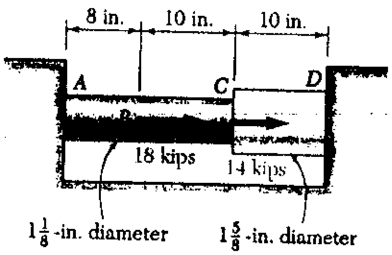
$$0.025 = 7.2431\times10^{-6} P + 1.4050\times10^{-6} P = 8.6481\times10^{6} P$$

$$P = 2.8908\times10^3 \text{ lb.}$$

(a) $$\sigma_{bolt} = \frac{P}{A_{bolt}} = \frac{2.8908\times10^3}{0.11045} = 26.2\times10^3 \text{ psi} = 26.2 \text{ ksi}$$

(b) $$\sigma_{tube} = -\frac{P}{A_{tube}} = -\frac{2.8908\times10^3}{0.29452} = -9.82\times10^3 \text{ psi} = -9.82 \text{ ksi}$$

PROBLEM 2.41

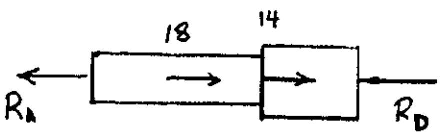


2.41 Two cylindrical rods, *CD* made of steel ($E = 29 \times 10^6$ psi) and *AC* made of aluminum ($E = 10.4 \times 10^6$ psi), are joined at *B* and restrained by rigid supports at *A* and *D*. Determine (*a*) the reactions at *A* and *D*, (*b*) the deflection of point *C*.

SOLUTION

AB: $P = R_A, \quad L_{AB} = 8 \text{ in}$

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$



$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(8)}{(10.4\times10^6)(0.99402)} = 0.77386\times10^{-6}\, R_A$$

BC: $P = R_A - 18\times10^3, \quad L = 10 \text{ in}, \quad A = 0.99402 \text{ in}^2$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 18\times10^3)(10)}{(10.4\times10^6)(0.99402)} = 0.96732\times10^{-6} R_A - 17.412\times10^{-3}$$

CD: $P = R_A - 18\times10^3 - 14\times10^3 = R_A - 32\times10^3$

$L = 10 \text{ in} \qquad A = \frac{\pi}{4} d_{CD}^2 = \frac{\pi}{4}(1.625)^2 = 2.0739 \text{ in}^2$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 32\times10^3)(10)}{(29\times10^6)(2.0739)} = 0.16627\times10^{-6} R_A - 5.321\times10^{-3}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 1.9075\times10^{-6} R_A - 22.733\times10^{-3}$$

Since point D cannot move relative to A $\quad \delta_{AD} = 0$

(a) $1.9075\times10^{-6} R_A - 22.733\times10^{-3} = 0 \qquad R_A = 11.92\times10^3 \text{ lb.} \leftarrow$

$R_D = 32\times10^3 - R_A = 20.08\times10^3 \text{ lb} \leftarrow$

(b) $\delta_C = \delta_{AB} + \delta_{CD}$

$= 1.7412\times10^{-6} R_A - 17.412\times10^{-3}$

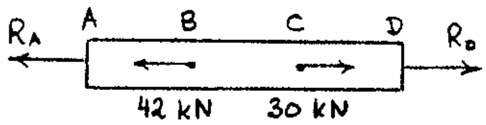
$= (1.7412\times10^{-6})(11.92\times10^3) - 17.412\times10^{-3} = 3.34\times10^{-3} \text{ in}$

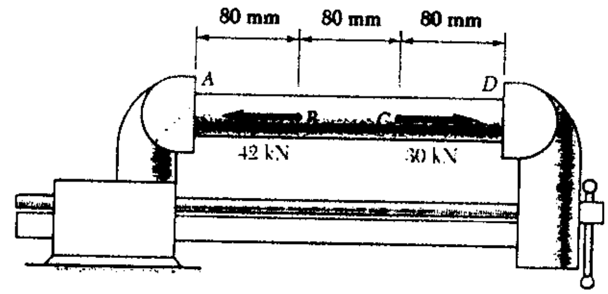
or $\delta_C = \dfrac{R_D L_{CD}}{E_{CD} A_{CD}} = \dfrac{(20.08\times10^3)(10)}{(29\times10^6)(2.0739)} = 3.34\times10^{-3} \text{ in.}$

PROBLEM 2.42

2.42 A steel tube ($E = 200$ GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.

SOLUTION





For the tube $d_i = d_o - 2t$
$= 32 - (2)(4) = 24$ mm

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(32^2 - 24^2)$$
$$= 351.86 \text{ mm}^2 = 351.86 \times 10^{-6} \text{ m}^2$$

AB: $P = R_A, \quad L = 0.080$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A$$

BC: $P = R_A + 42\times10^3, \quad L = 0.080$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 42\times10^3)(10)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A + 47.746\times10^{-6}$$

CD: $P = R_A + 12\times10^3, \quad L = 0.080$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 12\times10^3)(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A + 13.642\times10^{-6}$$

Total: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 3.4104\times10^{-9} R_A + 61.388\times10^{-6}$

Given jaw movement $\quad \delta_{AD} = -0.2 \text{ mm} = 0.2\times10^{-3}$ m

(a) $-0.2\times10^{-3} = 3.4104\times10^{-9} R_A + 61.388\times10^{-6} \therefore \quad R_A = -76.6\times10^3$ N
$= -76.6$ kN

$R_D = R_A + 12\times10^3 \qquad R_D = -64.6\times10^3$ N
$= -64.6$ kN

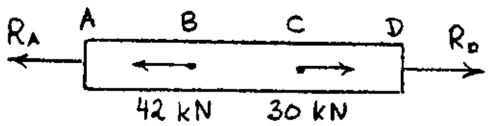
(b) $\delta_{BC} = (1.1368\times10^{-9})(-76.6\times10^3) + 47.746\times10^{-6} = -39.4\times10^{-6}$ m
$= -0.0394$ mm

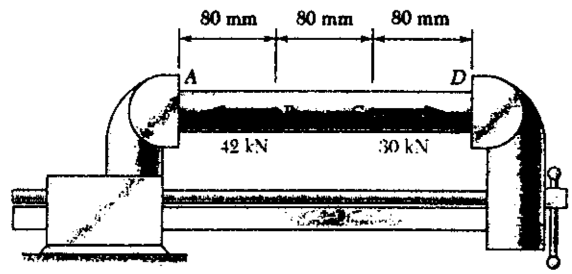
PROBLEM 2.43

SOLUTION

2.42 A steel tube ($E = 200$ GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.

2.43 Solve Prob. 2.42, assuming that after the forces have been applied, the vise is adjusted to decrease the distance between its jaws by 0.1 mm.





For the tube $d_i = d_o - 2t$

$= 32 - (2)(4) = 24$ mm

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(32^2 - 24^2)$$

$$= 351.86 \text{ mm}^2 = 351.86 \times 10^{-6} \text{ m}^2$$

AB: $P = R_A$, $L = 0.080$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A$$

BC: $P = R_A + 42\times10^3$, $L = 0.080$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 42\times10^3)(10)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A + 47.746\times10^{-6}$$

CD: $P = R_A + 12\times10^3$, $L = 0.080$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 12\times10^3)(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A + 13.642\times10^{-6}$$

Total: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 3.4104\times10^{-9} R_A + 61.388\times10^{-6}$

Due to the movement of the jaws $\delta_{AD} = -0.1$ mm $= -0.1\times10^{-3}$ m

(a) $-0.1\times10^{-3} = 3.4104\times10^{-9} R_A + 61.388\times10^{-6}$ $\quad R_A = -47.322\times10^3$ N

$= -47.3$ kN

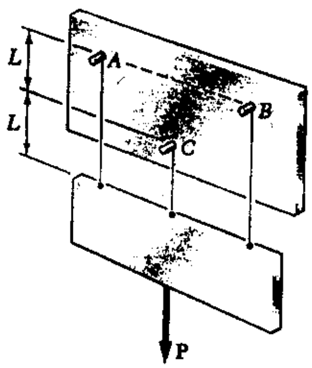
$R_D = R_A + 12\times10^3$ $\quad = -35.3\times10^3$ N

$= -35.3$ kN

(b) $\delta_{BC} = (1.1368\times10^{-9})(-47.322\times10^3) + 47.746\times10^{-6} = -6.05\times10^{-6}$ m

$= -0.00605$ mm

PROBLEM 2.44



2.44 Three wires are used to suspend the plate shown. Aluminum wires are used at *A* and *B* with a diameter of $\frac{1}{8}$ in. and a steel wire is used at *C* with a diameter of $\frac{1}{12}$ in. Knowing that the allowable stress for aluminum ($E = 10.4 \times 10^6$ psi) is 14 ksi and that the allowable stress for steel ($E = 29 \times 10^6$ psi) is 18 ksi, determine the maximum load **P** that may be applied.

SOLUTION

By symmetry $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \qquad \varepsilon_C = \frac{\delta}{L} = 2\,\varepsilon_A$$

Determine allowable strain

A & B $\qquad \varepsilon_A = \frac{\sigma_A}{E_A} = \frac{14 \times 10^3}{10.4 \times 10^6} = 1.3462 \times 10^{-3}$

$$\varepsilon_C = 2\,\varepsilon_A = 2.6924 \times 10^{-4}$$

C $\qquad \varepsilon_C = \frac{\sigma_C}{E_C} = \frac{18 \times 10^3}{29 \times 10^6} = 0.6207 \times 10^{-3}$

$$\varepsilon_A = \varepsilon_B = \tfrac{1}{2}\varepsilon_C = 0.3103 \times 10^{-6}$$

Allowable strain for wire C governs $\therefore$ $\sigma_C = 18 \times 10^3$ psi

$$\sigma_A = E_A \varepsilon_A \qquad P_A = A_A E_A \varepsilon_A = \frac{\pi}{4}\left(\frac{1}{8}\right)^2 (10.4 \times 10^6)(0.3103 \times 10^{-6})$$

$$= 139.61 \text{ lb}$$

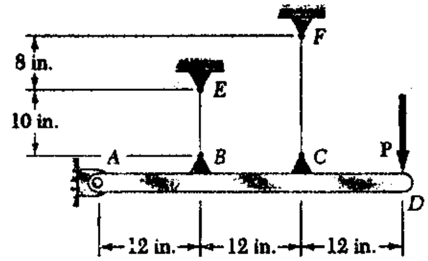
$$P_B = 139.61 \text{ lb.}$$

$$\sigma_C = E_C \varepsilon_C \qquad P_C = A_C \sigma_C = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 (18 \times 10^3) = 98.17 \text{ lb.}$$

For equilibrium of the plate

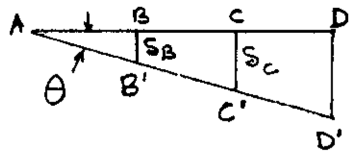
$$P = P_A + P_B + P_C = 177.4 \text{ lb.}$$

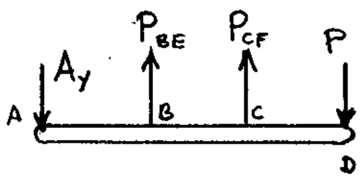
PROBLEM 2.45



2.45 The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at D. Knowing that the wires were initially taught, determine (*a*) the additional tension in each wire when a 220-lb load **P** is applied at D, (*b*) the corresponding deflection of point D.

SOLUTION





Let θ be the rotation of bar ABCD

Then $\quad \delta_B = 12\,\theta$

$$\delta_C = 24\,\theta$$

$$\delta_B = \frac{P_{BE} L_{BE}}{AE}$$

$$P_{BE} = \frac{EA\,\delta_{BE}}{L_{BE}} = \frac{(29\times10^6)\,\frac{\pi}{4}\left(\frac{1}{16}\right)^2 (12\,\theta)}{10}$$

$$= 106.77\times10^3\ \theta$$

$$\delta_c = \frac{P_{CF} L_{CF}}{EA}$$

$$P_{CF} = \frac{EA\,\delta_{CF}}{L_{CF}} = \frac{(29\times10^6)\,\frac{\pi}{4}\left(\frac{1}{16}\right)^2 (24\,\theta)}{18}$$

$$= 118.63\times10^3\,\theta$$

Using free body ABCD

$$\circlearrowleft \Sigma M_A = 0 \qquad 12\,P_{BE} + 24\,P_{CF} - 36\,P = 0$$

$$(12)(106.77\times10^3\,\theta) + (24)(118.63\times10^6\,\theta) - (36)(220) = 0$$

$$4.1283\times10^6\ \theta = (36)(220)$$

$$\theta = 1.9185\times10^{-3}\ \text{rad}$$

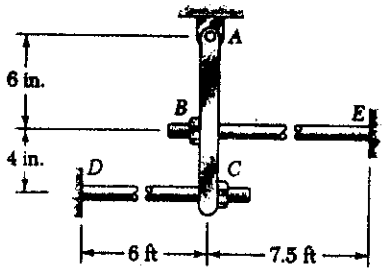
(a) $\quad P_{BE} = (106.77\times10^3)(1.9185\times10^{-3}) = 204.8\ \text{lb.}$

$\quad P_c = (118.63\times10^3)(1.9185\times10^{-3}) = 227.6\ \text{lb}$

(b) $\quad \delta_D = 36\ \theta = (36)(1.9185\times10^{-3}) = 69.1\times10^{-3}\ \text{in}$

$$= 0.0691\ \text{in.}$$

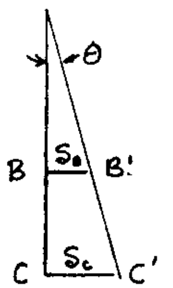
PROBLEM 2.46



2.46 The steel rods *BE* and *CD* each have a diameter of $\frac{5}{8}$ in. ($E = 29 \times 10^6$ psi). The ends are threaded with a pitch of 0.1 in. Knowing that after being snugly fit, the nut at *B* is tightened one full turn, determine (*a*) the tension in rod *CD*, (*b*) the deflection of point *C* of the rigid member *ABC*.

SOLUTION

Let θ be the rotation of bar ABC as shown



Then, $\delta_B = 6\theta$ and $\delta_C = 10\,\theta$

But $\delta_B = \delta_{turn} - \dfrac{P_{BE} L_{BE}}{E_{BE} A_{BE}}$

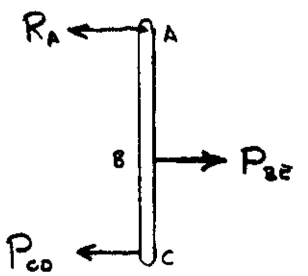
$$P_{BE} = (E_{BE} A_{BE})(\delta_{turn} - \delta_B)/L_{BE}$$

$L_{BE} = 7.5$ ft $= 90$ in., $\delta_{turn} = 0.1$ in

$$A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$P_{BE} = \frac{(29\times10^6)(0.3068)(0.1 - 6\,\theta)}{90}$$

$$= 9.886 \times 10^3 - 593.15 \times 10^3\,\theta$$



$$\delta_C = \frac{P_{CD} L_{CD}}{E A_{CD}} \quad \therefore \quad P_{CD} = \frac{E A \delta_C}{L_{CD}}$$

$L_{CD} = 6$ ft $= 72$ in, $A_{CD} = 0.3068$ in^2

$$P_{CD} = \frac{(29\times10^6)(0.3068)(10\,\theta)}{72}$$

$$= 1.23572 \times 10^6\,\theta$$

$+\circlearrowleft\ \Sigma M_A = 0 \qquad 6 P_{BE} - 10 P_{CD} = 0$

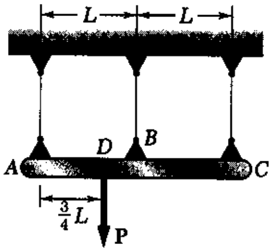
$$(6)(9.886\times10^3 - 593.16\times10^3\,\theta) - (10)(1.23572\times10^6)\,\theta = 0$$

$$59.316\times10^3 - 15.916\times10^6\,\theta = 0 \qquad \theta = 3.7268\times10^{-3} \text{ rad}$$

(a) $P_{CD} = (1.23572\times10^6)(3.7268\times10^{-3}) = 4.61\times10^3$
$= 4.61$ kips ◄

(b) $\delta_C = 10\,\theta = (10)(3.7268\times10^{-3}) = 37.3\times10^{-3}$ in
$= 0.0373$ in ◄

PROBLEM 2.47



2.47 The rigid rod *ABCD* is suspended from three wires of the same material. The cross-sectional area of the wire at *B* is equal to half of the cross-sectional area of the wires *A* and *C*. Determine the tension in each wire caused by the load **P**.

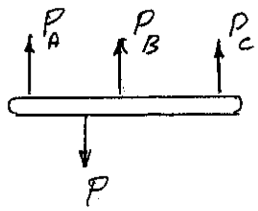
SOLUTION

$$+\circlearrowleft \Sigma M_A = 0 \qquad 2LP_C + LP_B - \tfrac{3}{4}LP = 0$$

$$P_C = \tfrac{3}{8}P - \tfrac{1}{2}P_B$$

$$+\circlearrowleft M_C = 0 \qquad -2LP_A - LP_B + \tfrac{5}{4}LP = 0$$

$$P_A = \tfrac{5}{8}P - \tfrac{1}{2}P_B$$

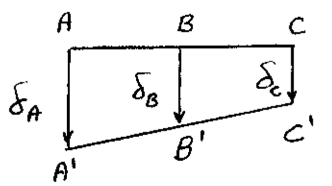


LET ℓ BE THE LENGTH OF THE WIRES

$$\delta_A = \frac{P_A \ell}{EA} = \frac{\ell}{EA}\left(\tfrac{5}{8}P - \tfrac{1}{2}P_B\right)$$

$$\delta_B = \frac{P_B \ell}{E(A/2)} = \frac{2\ell}{EA}P_B$$

$$\delta_C = \frac{P_C \ell}{EA} = \frac{\ell}{EA}\left(\tfrac{3}{8}P - \tfrac{1}{2}P_B\right)$$



FROM THE DEFORMATION DIAGRAM.

$$\delta_A - \delta_B = \delta_B - \delta_C$$

OR

$$\delta_B = \tfrac{1}{2}(\delta_A + \delta_C)$$

$$\frac{\ell}{E(A/2)}P_B = \frac{1}{2}\frac{\ell}{EA}\left(\tfrac{5}{8}P - \tfrac{1}{2}P_B + \tfrac{3}{8}P - \tfrac{1}{2}P_B\right)$$

$$\tfrac{5}{2}P_B = \tfrac{1}{2}P \;;\; P_B = \tfrac{1}{5}P \qquad P_B = 0.200P$$

$$P_A = \tfrac{5}{8}P - \tfrac{1}{2}\left(\tfrac{P}{5}\right) = \tfrac{21}{40}P \qquad P_A = 0.525P$$

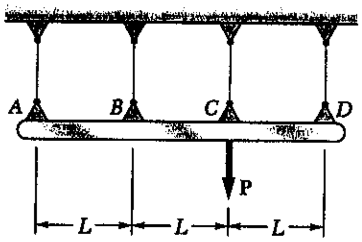
$$P_C = \tfrac{3}{8}P - \tfrac{1}{2}\left(\tfrac{P}{5}\right) = \tfrac{11}{40}P \qquad P_C = 0.275P$$

CHECK:

$P_A + P_B + P_C = 1.000P$. OK

PROBLEM 2.48

2.48 The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P**.



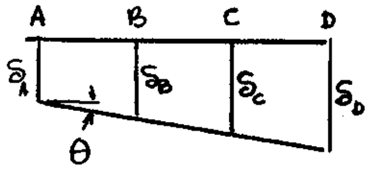
SOLUTION

Let θ be the slope of bar ABCD after deformation

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta$$

$$\delta_D = \delta_A + 3L\theta$$

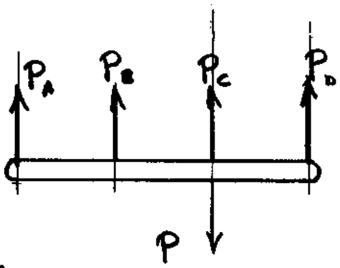


$$P_A = \frac{EA}{\ell}\delta_A$$

$$P_B = \frac{EA}{\ell}\delta_B = \frac{EA}{\ell}\delta_A + \frac{EAL}{\ell}\theta$$

$$P_C = \frac{EA}{\ell}\delta_C = \frac{EA}{\ell}\delta_A + \frac{2EAL}{\ell}\theta$$

$$P_D = \frac{EA}{\ell}\delta_D = \frac{EA}{\ell}\delta_A + \frac{3EAL}{\ell}\theta$$



$+\uparrow \Sigma F_y = 0$

$$P_A + P_B + P_C + P_D - P = 0$$

$$\frac{4EA}{\ell}\delta_A + \frac{6EA}{\ell}L\theta = P$$

$$4\delta_A + 6L\theta = \frac{P\ell}{EA} \qquad (1)$$

$\circlearrowleft \Sigma M_A = 0$

$$LP_B + 2LP_C + 3LP_D - 2LP = 0$$

$$\frac{6EAL}{\ell}\delta_A + \frac{14EAL}{\ell}L\theta = 2LP$$

$$6\delta_A + 14L\theta = \frac{2P\ell}{EA} \qquad (2)$$

Solving (1) and (2) simultaneously

$$L\theta = \frac{1}{10}\frac{P\ell}{EA}$$

$$\delta_A = -\frac{1}{10}\frac{P\ell}{EA}$$

$$P_A = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \frac{1}{10}P$$

$$P_B = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \frac{1}{5}P$$

$$P_C = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + 2\frac{EA}{\ell}\frac{1}{10}\frac{P\ell}{EA} = \frac{3}{10}P$$

$$P_D = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + 3\frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \frac{2}{5}P$$

PROBLEM 2.49

2.49 A steel railroad track (E = 200 GPa, $\alpha = 11.7 \times 10^{-6}/°\text{C}$) was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48 °C, assuming that the rails (a) are welded to form a continuous track, (b) are 10 m long with 3-mm gaps between them.

SOLUTION

(a) $\delta_T = \alpha(\Delta T)L = (11.7\times10^{-6})(48-6)(10) = 4.914\times10^{-3}\ \text{m}$

$\delta_P = \dfrac{PL}{AE} = \dfrac{L\sigma}{E} = \dfrac{(10)\sigma}{200\times10^{9}} = 50\times10^{-12}\,\sigma$

$\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\,\sigma = 0 \quad \therefore\ \sigma = -98.3\times10^{6}\ \text{Pa}$

$= -98.3\ \text{MPa}$ ◄

(b) $\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\,\sigma = 3\times10^{-3}$

$\sigma = \dfrac{3\times10^{-3} - 4.914\times10^{-3}}{50\times10^{-12}} = -38.3\times10^{6}\ \text{Pa} = -38.3\ \text{MPa}$ ◄

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PROBLEM 2.50

2.50 The aluminum shell is fully bonded to the brass core, and the assembly is unstressed at a temperature of 78°F. Considering only axial deformations, determine the stress when the temperature reaches 180 °F (*a*) in the brass core, (*b*) in the aluminum shell.

Brass core
$E = 15 \times 10^6$ psi
$\alpha = 11.6 \times 10^{-6}/°F$

Aluminum shell
$E = 10.6 \times 10^6$ psi
$\alpha = 12.9 \times 10^{-6}/°F$

SOLUTION

$\Delta T = 180 - 78 = 102$ F

Let P_b be the tensile force developed in the brass core

For equilibrium with zero total force, the compressive force in the aluminum shell is P_b

Strains $\varepsilon_b = \frac{P_b}{E_b A_b} + \alpha_b(\Delta T)$, $\varepsilon_a = -\frac{P_b}{E_a A_a} + \alpha_a(\Delta T)$

Matching $\varepsilon_b = \varepsilon_a$ $\frac{P_b}{E_b A_b} + \alpha_b(\Delta T) = -\frac{P_b}{E_a A_a} + \alpha_a(\Delta T)$

$$\left(\frac{1}{E_b A_b} + \frac{1}{E_a A_a}\right) P_b = (\alpha_a - \alpha_b)(\Delta T)$$

$A_b = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1)^2 = 0.7854$ in^2

$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0^2) = 4.1233$ in^2

$\alpha_a - \alpha_b = 1.3 \times 10^{-6}$ /°F

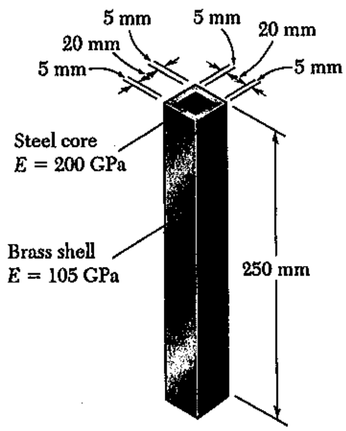
$$\left[\frac{1}{(15 \times 10^6)(0.7854)} + \frac{1}{(10.6 \times 10^6)(4.1233)}\right] P_b = (1.3 \times 10^{-6})(102)$$

$P_b = 1.2305 \times 10^3$ lb

$\sigma_b = \frac{P_b}{A_b} = \frac{1.2305 \times 10^3}{0.7854} = 1.567 \times 10^3$ psi $= 1.567$ ksi

$\sigma_a = -\frac{P_b}{A_a} = -\frac{1.2305 \times 10^3}{4.1233} = -0.298 \times 10^3$ psi $= -0.298$ ksi

PROBLEM 2.51



2.51 The brass shell ($\alpha_b = 20.9 \times 10^{-6}/°C$) is fully bonded to the steel core ($\alpha_s = 11.7 \times 10^{-6}/°C$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 55 MPa.

SOLUTION

Let P_s = axial force developed in the steel core

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains $\varepsilon_s = \dfrac{P_s}{E_s A_s} + \alpha_s(\Delta T)$

$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching $\varepsilon_s = \varepsilon_b$

$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T)$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\alpha_b - \alpha_s = 9.2 \times 10^{-6}/°C$$

$$P_s = \sigma_s A_s = (55 \times 10^6)(400 \times 10^{-6}) = 22 \times 10^3 \text{ N}$$

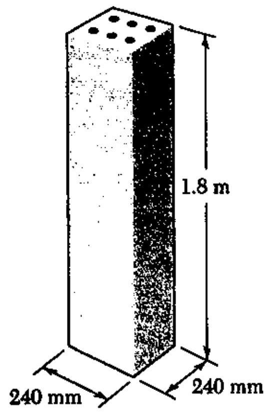
$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \times 10^9)(400 \times 10^{-6})} + \frac{1}{(105 \times 10^9)(500 \times 10^{-6})} = 31.55 \times 10^{-9} \text{ N}^{-1}$$

$$(31.55 \times 10^{-9})(22 \times 10^3) = (9.2 \times 10^{-6})(\Delta T)$$

$$\Delta T = 75.4 \text{ °C} \blacktriangleleft$$

PROBLEM 2.52

2.52 The concrete post (E_c = 25 GPa and $\alpha_c = 9.9 \times 10^{-6}/°C$) is reinforced with six steel bars, each of 22-mm diameter (E_s = 200 GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 35 °C.



SOLUTION

$$A_s = 6\,\frac{\pi}{4} d^2 = 6 \cdot \frac{\pi}{4}(22)^2 = 2.2808 \times 10^3\ \text{mm}^2 = 2.2808 \times 10^{-3}\ \text{m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808 \times 10^3 = 55.32 \times 10^3\ \text{mm}^2 = 55.32 \times 10^{-3}\ \text{m}^2$$

Let P_c = tensile force developed in the concrete

For equilibrium with zero total force, the compressive force in the six steel rods is P_c

Strains: $\varepsilon_s = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$, $\varepsilon_c = \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s$ $\quad \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right) P_c = (\alpha_s - \alpha_c)(\Delta T)$$

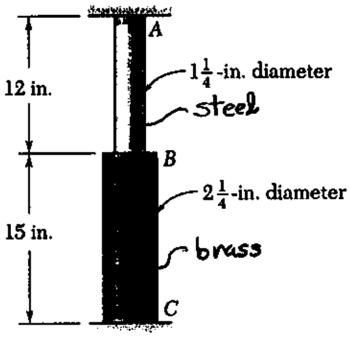
$$\left[\frac{1}{(25 \times 10^9)(55.32 \times 10^{-3})} + \frac{1}{(200 \times 10^9)(2.2808 \times 10^{-3})}\right] P_c = (1.8 \times 10^{-6})(35)$$

$$P_c = 21.61 \times 10^3\ \text{N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61 \times 10^3}{55.32 \times 10^{-3}} = 0.391 \times 10^6\ \text{Pa} = 0.391\ \text{MPa}$$ ◄

$$\sigma_s = -\frac{P_c}{A_c} = \frac{21.61 \times 10^3}{2.2808 \times 10^{-3}} = -9.47 \times 10^6\ \text{Pa} = -9.47\ \text{MPa}$$ ◄

PROBLEM 2.53



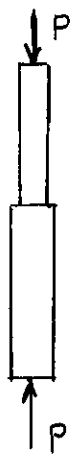
2.53 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion BC is made of brass ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 65°F, (*b*) the corresponding deflection of point B.

SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2$$

Free thermal expansion



$$\delta_T = L'_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$

$$= (12)(6.5\times10^{-6})(65) + (15)(10.4\times10^{-6})(65)$$

$$= 15.21\times10^{-3} \text{ in}$$

Shortening due to induced compressive force P

$$\delta_P = \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}}$$

$$= \frac{12\,P}{(29\times10^6)(1.2272)} + \frac{15\,P}{(15\times10^6)(3.9761)} = 588.69\times10^{-9}\,P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$(588.69\times10^{-9})P = 15.21\times10^{-3}$$

$$P = 25.84\times10^3 \text{ lb}$$

(a) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{25.84\times10^3}{1.2272} = -21.1\times10^3 \text{ psi} = -21.1 \text{ ksi}$ ◄

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{25.84\times10^3}{3.9761} = -6.50\times10^3 \text{ psi} = -6.50 \text{ ksi}$ ◄

(b) $\delta_B = +\dfrac{PL_{AB}}{E_s A_{AB}} - L_{AB}\alpha_s(\Delta T)$

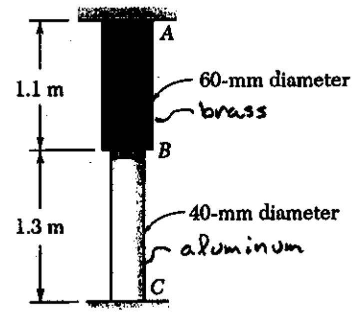
$$= +\frac{(25.84\times10^3)(12)}{(29\times10^6)(1.2272)} + (12)(6.5\times10^{-6})(65) = +3.64\times10^{-3} \text{ in} \uparrow$$

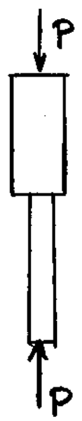
i.e. 3.64×10^{-3} in ↑

$= 0.00364$ in ↑ ◄

PROBLEM 2.54

2.54 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) and portion BC is made of aluminum ($E_a = 72$ GPa, $\alpha_a = 23.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 42°C, (*b*) the corresponding deflection of point B.





SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_b(\Delta T) + L_{BC}\alpha_a(\Delta T)$$
$$= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42)$$
$$= 2.2705 \times 10^{-3} \text{ m}$$

Shortening due to induced compressive force

$$\delta_P = \frac{P L_{AB}}{E_b A_{AB}} + \frac{P L_{BC}}{E_a A_{BC}}$$
$$= \frac{1.1\ P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3\ P}{(72 \times 10^9)(1.2566 \times 10^{-3})}$$
$$= 18.074 \times 10^{-9}\ P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$18.074 \times 10^{-9}\ P = 2.2705 \times 10^{-3}$$
$$P = 125.62 \times 10^3 \text{ N}$$

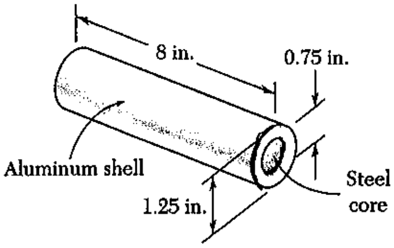
(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$ ◄

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$ ◄

(b) $$\delta_B = +\frac{P L_{AB}}{E_b A_{AB}} - L_{AB}\alpha_b(\Delta T)$$
$$= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42)$$
$$= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm}$$

i.e. 0.500 mm ↓ ◄

PROBLEM 2.55



2.55 The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, $\alpha_a = 12.9 \times 10^{-6}/°\text{F}$) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.

SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise

Let $\sigma_a = -6$ ksi $= -6 \times 10^3$ psi

$$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$ where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

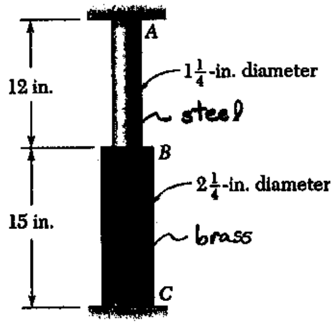
(a) $$\Delta T = 145.91\ °\text{F}$$

(b) $$\varepsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

or $$\varepsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

$$\delta = L\varepsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053 \text{ in.}$$

PROBLEM 2.56



2.53 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°F$) and portion *BC* is made of brass ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°F$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 65°F, (*b*) the corresponding deflection of point *B*.

2.56 For the rod of Prob. 2.53, determine the maximum allowable temperature change if the stress in the steel portion *AB* is not to exceed 18 ksi and if the stress in the brass portion *CB* is not to exceed 7 ksi.

SOLUTION

Allowable force in each portion

AB: $\sigma_{AB} = -18$ ksi $= -18 \times 10^3$ psi, $\quad A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272$ in^2

$$P = \sigma_{AB} A_{AB} = (-18 \times 10^3)(1.2272) = -22.090 \times 10^3 \text{ lb.}$$

BC: $\sigma_{BC} = -7$ ksi $= -7 \times 10^3$ psi, $\quad A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761$ in^2

$$P = \sigma_{BC} A_{BC} = (-7 \times 10^3)(3.9761) = -27.833 \times 10^3 \text{ lb.}$$

Smaller absolute value governs $\therefore$ $P = -22.090 \times 10^3$ lb.

Deformation due to P

$$\delta_P = \frac{P L_{AB}}{E_{AB} A_{AB}} + \frac{P L_{BC}}{E_{BC} A_{BC}} = -\frac{(22.090 \times 10^3)(12)}{(29 \times 10^6)(1.2272)} - \frac{(22.090 \times 10^3)(15)}{(15 \times 10^6)(3.9761)}$$

$$= -13.004 \times 10^{-3} \text{ in}$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T) = (12)(6.5 \times 10^{-6})(\Delta T) + (15)(10.4 \times 10^{-6})(\Delta T)$$

$$= (234 \times 10^{-6})(\Delta T)$$

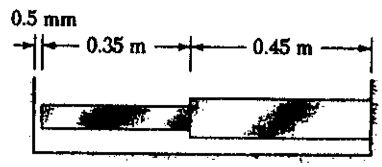
Total deformation is zero

$$\delta_T + \delta_P = (234 \times 10^{6})(\Delta T) - 13.004 \times 10^{-3} = 0$$

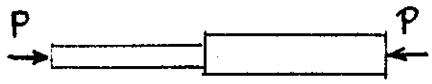
$$\Delta T = 55.6\,°F$$

PROBLEM 2.57

2.57 Determine (*a*) the compressive force in the bars shown after a temperature rise of 96°C, (*b*) the corresponding change in length of the bronze bar.



Bronze	Aluminum
$A = 1500\ \text{mm}^2$	$A = 1800\ \text{mm}^2$
$E = 105$ GPa	$E = 73$ GPa
$\alpha = 21.6 \times 10^{-6}/°\text{C}$	$\alpha = 23.2 \times 10^{-6}/°\text{C}$



SOLUTION

Calculate free thermal expansion

$$\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a \Delta T$$

$$= (0.35)(21.6\times10^{-6})(96) + (0.45)(23.2\times10^{-6})(96)$$

$$= 1.728 \times 10^{-3}\ \text{m}$$

Constrained expansion

$$\delta = 0.5\ \text{mm} = 0.500\times10^{-3}\ \text{m}$$

Shortening due to induced compressive force P

$$\delta_P = 1.728\times10^{-3} - 0.500\times10^{-3} = 1.228\times10^{-3}\ \text{m}$$

But, in terms of P

$$\delta_P = \frac{PL_b}{A_bE_b} + \frac{PL_a}{A_aE_a} = \left(\frac{L_b}{A_bE_b} + \frac{L_a}{A_aE_a}\right)P$$

$$= \left(\frac{0.35}{(1500\times10^{-6})(105\times10^{9})} + \frac{0.45}{(1800\times10^{-6})(73\times10^{9})}\right)P$$

$$= 5.6496\times10^{-9}\ P$$

(a) Equating $\quad 5.6496\times10^{-9}P = 1.228\times10^{-3} \quad \therefore\ P = 217.46\times10^{3}\ \text{N}$

$$= 217\ \text{kN}$$

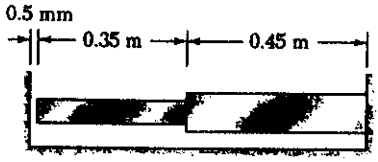
(b) $$\delta_b = L_b\alpha_b(\Delta T) - \frac{PL_b}{A_bE_b}$$

$$= (0.35)(21.6\times10^{-6})(96) - \frac{(217.46\times10^{3})(0.35)}{(1500\times10^{-6})(105\times10^{9})}$$

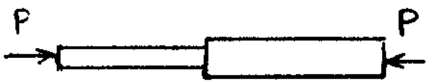
$$= 725.76\times10^{-6} - 483.24\times10^{-6} = 242.5\times10^{-6}\ \text{m}$$

$$= 0.2425\ \text{mm}$$

PROBLEM 2.58



Bronze	Aluminum
$A = 1500\ \text{mm}^2$	$A = 1800\ \text{mm}^2$
$E = 105\ \text{GPa}$	$E = 73\ \text{GPa}$
$\alpha = 21.6 \times 10^{-6}/°\text{C}$	$\alpha = 23.2 \times 10^{-6}/°\text{C}$



2.58 Knowing that a 0.5-mm gap exists when the temperature is 20 °C, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to -90 MPa, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$\sigma_a = -90 \times 10^6\ \text{Pa} \qquad A_a = 1800 \times 10^{-6}\ \text{m}^2$

$P = -\sigma_a A_a = (90 \times 10^6)(1800 \times 10^{-6}) = 162 \times 10^3\ \text{N}$

Shortening due to P

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(162 \times 10^3)(0.35)}{(105 \times 10^9)(1500 \times 10^{-6})} + \frac{(162 \times 10^3)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})}$$

$$= 914.79 \times 10^{-6}\ \text{m} = 0.91479\ \text{mm}$$

Available length for thermal expansion

$$\delta_T = 0.5\ \text{mm} + 0.91479\ \text{mm} = 1.41479\ \text{mm} = 1.41479 \times 10^{-3}\ \text{m}$$

But $\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$

$$= (0.35)(21.6 \times 10^{-6})\Delta T + (0.45)(23.2 \times 10^{-6})\Delta T$$

$$= 18.00 \times 10^{-6} (\Delta T)$$

Equating $\quad 18.00 \times 10^{-6}(\Delta T) = 1.41479 \times 10^{-3} \quad \therefore \quad \Delta T = 78.6\ °\text{C}$

(a)

$$T_{hot} = T_{cold} + \Delta T$$

$$= 20 + 78.6 = 98.6\ °\text{C}$$

(b) $\delta_a = L_a \alpha_a (\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (0.45)(23.2 \times 10^{-6})(78.6) - \frac{(162 \times 10^3)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})}$$

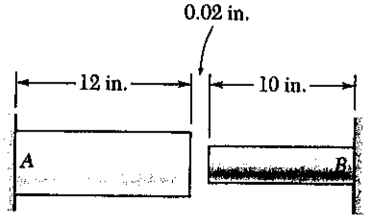
$$= 820.58 \times 10^{-6} - 554.79 \times 10^{-6} = 265.78 \times 10^{-6}\ \text{m}$$

$L_{exact} = L_a + \delta_a = 0.45\text{m} + 265.78 \times 10^{-6}\ \text{m}$

$$= 0.450266\ \text{m} = 450.0266\ \text{mm}$$

PROBLEM 2.59

2.59 At room temperature (70°F) a 0.02-in. gap exists between the ends of the rods shown. At a later time when the temperature has reached 320°F, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.



Aluminum	Stainless steel
$A = 2.8$ in^2	$A = 1.2$ in^2
$E = 10.4 \times 10^6$ psi	$E = 28.0 \times 10^6$ psi
$\alpha = 13.3 \times 10^{-6}/°\mathrm{F}$	$\alpha = 9.6 \times 10^{-6}/°\mathrm{C}$

SOLUTION

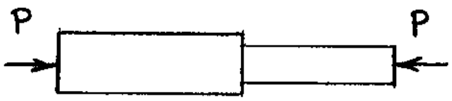
$$\Delta T = 320 - 70 = 250\ °\mathrm{F}$$

Free thermal expansion

$$\delta_T = L_a \alpha_a (\Delta T) + L_s \alpha_s (\Delta T)$$

$$= (12)(13.3 \times 10^{-6})(250) + (10)(9.6 \times 10^{-6})(250)$$

$$= 63.9 \times 10^{-3} \text{ in} = 0.0639 \text{ in.}$$



Shortening due to P to meet constraint

$$\delta_P = 0.0639 - 0.02 = 0.0439 \text{ in.}$$

$$\delta_P = \frac{PL_a}{A_a E_a} + \frac{PL_s}{A_s E_s} = \left(\frac{L_a}{A_a E_a} + \frac{L_s}{A_s E_s} \right) P$$

$$= \left(\frac{12}{(2.8)(10.4 \times 10^6)} + \frac{10}{(1.2)(28.0 \times 10^6)} \right) P = 709.71 \times 10^{-9} P$$

Equating $\quad 709.71 \times 10^{-9} P = 0.0439 \qquad P = 61.857 \times 10^3$ lb

(a) $$\sigma_a = -\frac{P}{A_a} = -\frac{61.857 \times 10^3}{2.8} = -22.09 \times 10^3 \text{ psi}$$

$$= -22.09 \text{ ksi}$$

(b) $$\delta_a = L_a \alpha_a (\Delta T) - \frac{PL_a}{A_a E_a}$$

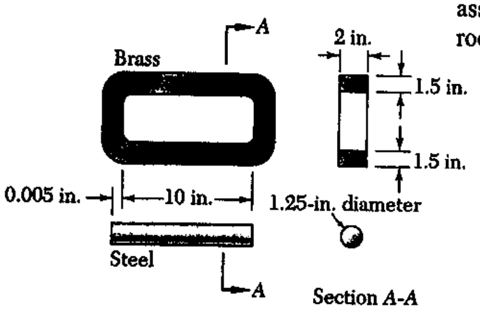
$$= (12)(13.3 \times 10^{-6})(250) - \frac{(61.857 \times 10^3)(12)}{(2.8)(10.4 \times 10^6)}$$

$$= 39.90 \times 10^{-3} - 25.49 \times 10^{-3} = 14.41 \times 10^{-3} \text{ in}$$

$$= 0.01441 \text{ in}$$

PROBLEM 2.60

2.60 A brass link ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°F$) and a steel rod ($E_s = 29 \times 10^6$ ksi, $\alpha_s = 6.5 \times 10^{-6}/°F$) have the dimensions shown at a temperature of 65°F. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 100°F. Determine (*a*) the final normal stress in the steel rod, (*b*) the final length of the steel rod.



SOLUTION

ΔT associated with difference between final and initial dimensions

$$\Delta T = 100 - 65 = 35\ °F$$

Free thermal expansion of each part

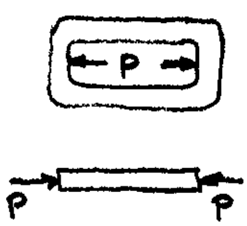
Brass link $(\delta_T)_b = \alpha_b (\Delta T)(L) = (10.4\times10^{-6})(35)(10) = 3.64\times10^{-3}$ in

Steel rod $(\delta_T)_s = \alpha_s (\Delta T)(L) = (6.5\times10^{-6})(35)(10) = 2.275\times10^{-3}$ in

At the final temperature the free length of the steel rod

$0.005 + 2.275\times10^{-3} - 3.64\times10^{-3} = 3.635\times10^{-3}$ in

longer than the brass link



Add equal but opposite forces P to elongate the brass link and contract the steel rod.

Brass link $(\delta_P)_b = \dfrac{PL}{AE} = \dfrac{P(10)}{(2)(1.5)(2)(15\times10^6)}$

$= 111.11\times10^{-9}\,P$

Steel rod $(\delta_P)_s = \dfrac{PL}{AE} = \dfrac{P(10)}{\frac{\pi}{4}(1.25)^2(29\times10^6)}$

$= 280.99\times10^{-9}\,P$

$(\delta_P)_b + (\delta_P)_s = 3.635\times10^{-3}$

$(392.10\times10^{-9})P = 3.635\times10^{-3}$ $\qquad P = 9.2705\times10^3$ lb

(a) Final stress in steel rod $\quad \sigma_s = -\dfrac{P}{A_s} = -\dfrac{9.2705\times10^3}{\frac{\pi}{4}(1.25)^2}$

$= -7.55\times10^3$ psi $= -7.55$ ksi ◄

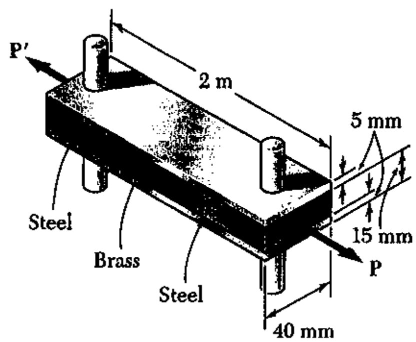
(b) Final length of steel rod

$L_f = 10.000 + 0.005 + (\delta_T)_s - (\delta_P)_s$

$= 10.005 + 2.275\times10^{-3} - (280.99\times10^{-9})(9.2705\times10^3)$

$= 10.00467$ in. ◄

PROBLEM 2.61



2.61 Two steel bars (E_s = 200 GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar (E_b = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load P = 25 kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

(a) Required temperature change for fabrication

$$\delta_T = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

(a) Temperature change required to expand steel bar by this amount

$$\delta_T = L\alpha_s \Delta T \qquad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$$

$$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$

$$\Delta T = 21.368 \text{ °C} \qquad 21.4 \text{ °C}$$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_s = \frac{FL}{A_s E_s} = \frac{P(2.00)}{(400 \times 10^{-6})(200 \times 10^9)} = 25 \times 10^{-9} P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_P)_b = \frac{P^* L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^9)} = 31.746 \times 10^{-9} P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit

$$(\delta_P)_s + (\delta_P)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.811 \times 10^3 \text{ N}$$

Stresses due to fabrication

Steel: $$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.811 \times 10^3}{400 \times 10^{-6}} = 22.03 \times 10^6 \text{ Pa} = 22.03 \text{ MPa}$$

Brass: $$\sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811 \times 10^3}{600 \times 10^{-6}} = -14.68 \times 10^6 \text{ Pa} = -14.68 \text{ MPa}$$

To these stresses must be added the stresses due to the 25 kN load.

continued

Problem 2.61 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^{9})}{2.00}\delta' = 40\times10^{6}\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^{9})}{2.00}\delta' = 31.5\times10^{6}\,\delta'$$

Total $\quad P = P_s + P_b = 25\times10^{3}$ N

$$40\times10^{6}\,\delta' + 31.5\times10^{6}\,\delta' = 25\times10^{3} \qquad \delta' = 349.65\times10^{-6}\ \text{m}$$

$$P_s = (40\times10^{6})(349.65\times10^{-6}) = 13.986\times10^{3}\ \text{N}$$

$$P_b = (31.5\times10^{6})(349.65\times10^{-6}) = 11.140\times10^{3}\ \text{N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^{3}}{400\times10^{-6}} = 34.97\times10^{6}\ \text{Pa}$$

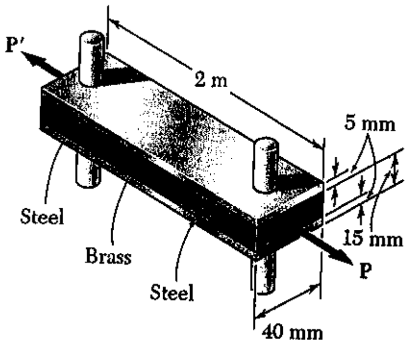
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^{3}}{600\times10^{-6}} = 18.36\times10^{6}\ \text{Pa}$$

Add stress due to fabrication

$$\sigma_s = 34.97\times10^{6} + 22.03\times10^{6} = 57.0\times10^{6}\ \text{Pa} = 57.0\ \text{MPa}$$

$$\sigma_b = 18.36\times10^{6} - 14.68\times10^{6} = 3.68\times10^{6}\ \text{Pa} = 3.68\ \text{MPa}$$

PROBLEM 2.62



2.61 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

2.62 Determine the maximum load P that may be applied to the brass bar of Prob. 2.61 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

SOLUTION

See solution to PROBLEM 3.61 to obtain the fabrication stresses

$\sigma_s^* = 22.03$ MPa $\qquad \sigma_b^* = -14.68$ MPa

Allowable stresses: $\sigma_{s,all} = 30$ MPa , $\sigma_{b,all} = 25$ MPa

Available stress increase from load

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$
$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$
$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.85 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$
$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

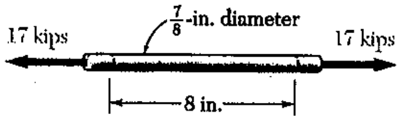
$$P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$$

$$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^3 \text{ N}$$

Total allowable additional force

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N}$$
$$= 5.70 \text{ kN}$$

PROBLEM 2.63



2.63 In a standard tensile test a steel rod of $\frac{7}{8}$-in. diameter is subjected to a tension force of 17 kips. Knowing that $\nu = 0.3$ and $E = 29 \times 10^6$ psi, determine (*a*) the elongation of the rod in an 8-in. gage length, (*b*) the change in diameter of the rod.

SOLUTION

$$P = 17 \text{ kip} = 17 \times 10^3 \text{ lb.} \qquad A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{17 \times 10^3}{0.60132} = 28.27 \times 10^3 \text{ psi} \qquad \varepsilon_x = \frac{\sigma}{E} = \frac{28.27 \times 10^3}{29 \times 10^6} = 974.9 \times 10^{-6}$$

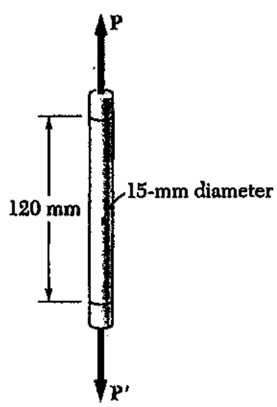
$$\delta_x = L\varepsilon_x = (8.0)(974.9 \times 10^{-6}) = 7.80 \times 10^{-3} \text{ in} = 0.00780 \text{ in.}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.3)(974.9 \times 10^{-6}) = -292.5 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = \left(\frac{7}{8}\right)(-292.5 \times 10^{-6}) = -256 \times 10^{-6} \text{ in} = -0.000256 \text{ in.}$$

PROBLEM 2.64

2.64 A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a 15-mm-diameter rod and it is subjected to a 3.5 kN tensile force. Knowing that an elongation of 11 mm and a decrease in diameter of 0.62 mm are observed in a 120-mm gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio of the material.



SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(15)^2 = 176.715 \text{ mm}^2 = 176.715 \times 10^{-6} \text{ m}^2$$

$$P = 3.5 \times 10^3 \text{ N}$$

$$\sigma = \frac{P}{A} = \frac{3.5 \times 10^3}{176.715 \times 10^{-6}} = 19.806 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\delta_x}{L} = \frac{11}{120} = 91.667 \times 10^{-3}$$

$$E = \frac{\sigma}{\varepsilon_x} = \frac{19.806 \times 10^6}{91.667 \times 10^{-3}} = 216 \times 10^6 \text{ Pa} = 216 \text{ MPa}$$

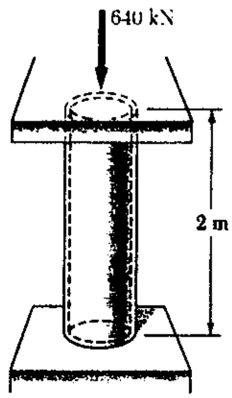
$$\delta_y = -0.62 \text{ mm}$$

$$\varepsilon_y = \frac{\delta_y}{d} = -\frac{0.62}{15} = -41.333 \times 10^{-3}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{41.333 \times 10^{-3}}{91.667 \times 10^{-3}} = 0.4509$$

$$G = \frac{E}{2(1+\nu)} = \frac{216 \times 10^6}{2(1+0.4509)} = 74.5 \times 10^6 \text{ Pa} = 74.5 \text{ MPa}$$

PROBLEM 2.65



2.65 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$d_o = 240$ mm $\quad t = 10$ mm $\quad d_i = d_o - 2t = 220$ mm

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3 \text{ mm}^2 = 7.2257 \times 10^{-3} \text{ m}^2$$

$$P = 640 \times 10^3 \text{ N}$$

$$\text{(a)} \quad \delta = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3} \text{ m} = -2.43 \text{ mm}$$

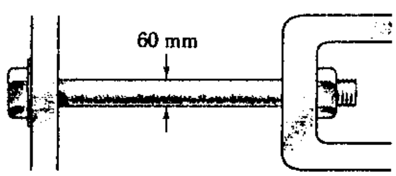
$$\varepsilon = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

$$\varepsilon_{lat} = -\nu\varepsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

$$\text{(b)} \quad \Delta d_o = d_o \varepsilon_{lat} = (240)(400.4 \times 10^{-6}) = 0.0961 \text{ mm}$$

$$\text{(c)} \quad \Delta t = t \varepsilon_{lat} = (10)(400.4 \times 10^{-6}) = 0.00400 \text{ mm}$$

PROBLEM 2.66



2.66 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by 13 μm.

SOLUTION

$\delta_y = -13 \times 10^{-6}$ m $\qquad d = 60 \times 10^{-3}$ m

$$\varepsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \quad \therefore \quad \varepsilon_x = -\frac{\varepsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$$

$$\sigma_x = E\varepsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2$$

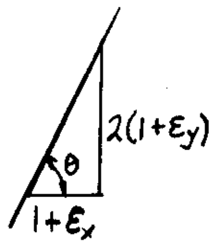
$$F = \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N} = 422 \text{ kN}$$

PROBLEM 2.67

2.67 An aluminum plate (E = 74 GPa, ν = 0.33) plate is subjected to a centric axial load which causes a normal stress σ. Knowing that before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when σ = 125 MPa.

SOLUTION

The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$



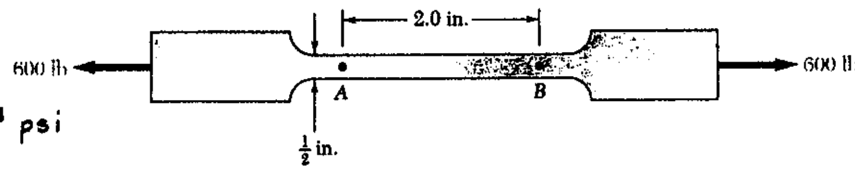
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125\times10^6}{74\times10^9} = 1.6892\times10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(1.6892\times10^{-3}) = 0.5574\times10^{-3}$$

$$\tan\theta = \frac{2(1-0.0005574)}{1+0.0016892} = 1.99551$$

PROBLEM 2.68

2.68 A 600 lb tensile load is applied to a test coupon made from $\frac{1}{16}$ in. flat steel plate ($E = 29\times10^6$ psi, $\nu = 0.30$). Determine the resulting change (a) in the 2.00-in. gage length, (b) in the width of portion AB of the test coupon, (c) in the thickness of portion AB, (d) in the cross-sectional area of portion AB.



SOLUTION

$$A = (\tfrac{1}{2})(\tfrac{1}{16}) = 0.03125\ \text{in}^2$$

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2\times10^3\ \text{psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{19.2\times10^3}{29\times10^6} = 662.07\times10^{-6}$$

(a) $\delta_x = L_0\varepsilon_x = (2.0)(662.07\times10^{-6}) = 1.324\times10^{-3}$ in.

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07\times10^{-6}) = -198.62\times10^{-6}$$

(b) $\delta_{width} = w_0\varepsilon_y = (\tfrac{1}{2})(-198.62\times10^{-6}) = -99.3\times10^{-6}$ in.

(c) $\delta_{thickness} = t_0\varepsilon_z = (\tfrac{1}{16})(-198.62\times10^{-6} = -12.41\times10^{-6}$ in.

(d) $A = wt = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z)$

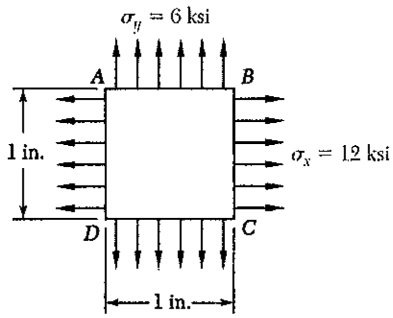
$$= w_0 t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$\Delta A = A - A_0 = w_0 t_0(\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$= (\tfrac{1}{2})(\tfrac{1}{16})(-198.62\times10^{-6} - 198.62\times10^{-6} + \text{negligible term})$$

$$= -12.41\times10^{-6}\ \text{in}^2$$

PROBLEM 2.69



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{29\times10^6}\left[12\times10^3 - (0.30)(6\times10^3)\right]$$

$$= 351.72 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{29\times10^6}\left[6\times10^3 - (0.30)(12\times10^3)\right]$$

$$= 82.76 \times 10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})_0\varepsilon_x = (1.00)(351.72\times10^{-6}) = 351.7 \times 10^{-6}$ in.

(b) $\delta_{BC} = (\overline{BC})_0\varepsilon_y = (1.00)(82.76\times10^{-6}) = 82.8\times10^{-6}$ in.

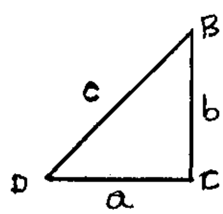
(c) $(\overline{AC}) = \sqrt{(\overline{AB})^2 + (\overline{BC})^2} = \sqrt{(\overline{AB}_0 + \delta_x)^2 + (\overline{BC}_0 + \delta_y)^2}$

$$= \sqrt{(1 + 352\times10^{-6})^2 + (1 + 82.8\times10^{-6})^2}$$

$$= 1.41452$$

$(\overline{AC})_0 = \sqrt{2}$ $\qquad \overline{AC} - (\overline{AC})_0 = 307\times10^{-6}$ in.

or use calculus as follows:



Lable sides using a, b, and c as shown

$$c^2 = a^2 + b^2$$

Obtain differentials $\quad 2c\,dc = 2a\,da + 2b\,dc$

from which $\quad dc = \frac{a}{c}\,da + \frac{b}{c}\,dc$

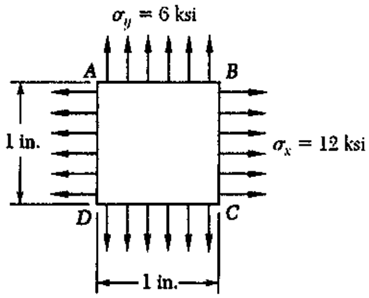
But $\quad a = 1.00$ in, $b = 1.00$ in, $c = \sqrt{2}$ in.

$da = \delta_{AB} = 351.72\times10^{-6}$ in, $\quad db = \delta_{BC} = 82.8\times10^{-6}$ in

$$\delta_{AC} = dc = \frac{1.00}{\sqrt{2}}(351.7\times10^{-6}) + \frac{1.00}{\sqrt{2}}(82.8\times10^{6})$$

$$= 307\times10^{-6} \text{ in.}$$

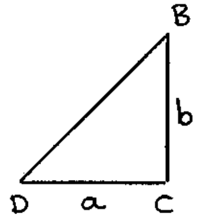
PROBLEM 2.70



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

2.70 For the square of Prob. 2.69, determine the percent change in the slope of diagonal *DB* due to the pressurization of the vessel.

SOLUTION



Label sides a and b as shown.

The slope is $s = \frac{b}{a}$

The change in slope is calculated from differential calculus

$$ds = \frac{a\,db - b\,da}{a^2} \quad \text{or} \quad \frac{ds}{s} = \frac{a}{b}\,\frac{a\,db - b\,da}{a^2} = \frac{db}{b} - \frac{da}{a}$$

$$\%\text{ change in slope} = \frac{ds}{s} \times 100\,\%$$

$$\frac{da}{a} = \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{29 \times 10^6}\left[12 \times 10^3 - (0.29)(6 \times 10^3)\right]$$

$$= 351.72 \times 10^{-6}$$

$$\frac{db}{b} = \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{29 \times 10^6}\left[6 \times 10^3 - (0.29)(12 \times 10^3)\right]$$

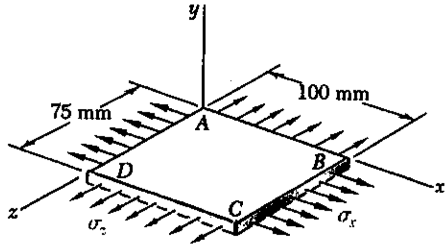
$$= 82.76 \times 10^{-6}$$

$$\frac{ds}{s} = 351.72 \times 10^{-6} - 82.76 \times 10^{-6} = 268.96 \times 10^{-6}$$

$$\%\text{ change in slope} = 268.96 \times 10^{-4}\ \%$$

$$= 0.0269\ \%$$

PROBLEM 2.71



2.71 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\sigma_x = 120\times10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160\times10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{87\times10^9}\left[120\times10^6 - (0.34)(160\times10^6)\right]$$

$$= 754.02\times10^{-6}$$

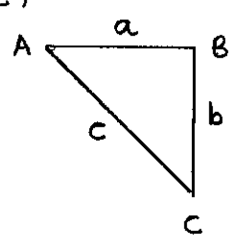
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87\times10^9}\left[-(0.34)(120\times10^6) + 160\times10^6\right]$$

$$= 1.3701\times10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02\times10^{-6}) = 0.0754 \text{ mm}$

(b) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75 \text{ mm})(1.3701\times10^{-6}) = 0.1028 \text{ mm}$

(c)



Label sides of right triangle ABC as a, b, and c

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus

$$2c\,dc = 2a\,da + 2b\,db$$

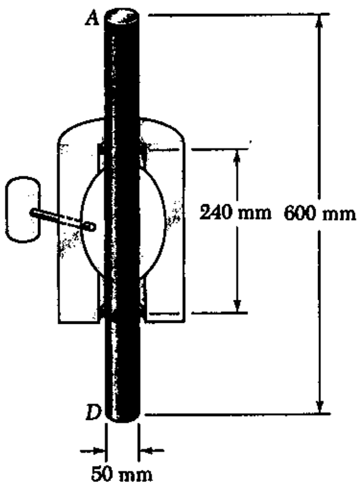
$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 100$ mm, $b = 100$ mm $\quad c = \sqrt{100^2 + 75^2} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm $\qquad db = \delta_{BC} = 0.1370$ mm

$$\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028) = 0.1220 \text{ mm}$$

PROBLEM 2.72



2.72 The brass rod *AD* is fitted with a jacket that is used to apply an hydrostatic pressure of 48 MPa to the 250-mm portion *BC* of the rod. Knowing that $E = 105$ GPa, and $\nu = 0.33$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter of portion *BC* of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -p = -48\times10^6 \text{ Pa}, \quad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{105\times10^9}\left[-48\times10^6 - (0.33)(0) - (0.33)(-48\times10^6)\right]$$

$$= -306.29\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{105\times10^9}\left[-(0.33)(-48\times10^6) + 0 - (0.33)(-48\times10^6)\right]$$

$$= -301.71\times10^{-6}$$

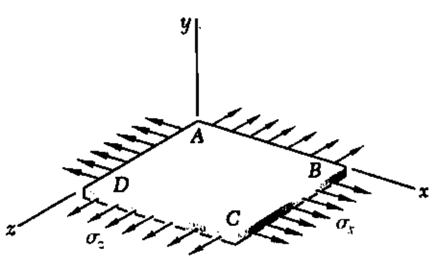
(a) Change in length: Only portion $\overline{BC}$ is strained. $L = 240$ mm

$$\delta_y = L\varepsilon_y = (240)(-301.71\times10^{-6}) = -0.0724 \text{ mm}$$

(b) Change in diameter: $d = 50$ mm

$$\delta_x = \delta_z = d\,\varepsilon_x = (50)(-306.29\times10^{-6}) = -0.01531 \text{ mm}$$

PROBLEM 2.73



2.73 The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the *x* direction must be zero, that is, $\varepsilon_x = 0$. Denoting by *E* the modulus of elasticity and by ν Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0/ε_z.

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0)$$

(a) $\sigma_x = \nu\sigma_0$

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$$

PROBLEM 2.74

2.74 For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypothenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (*b*) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.

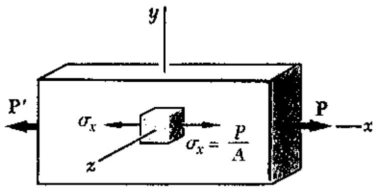


Fig 1.40 (*a*)

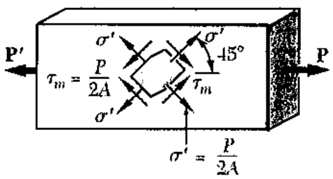


Fig 1.40 (*b*)

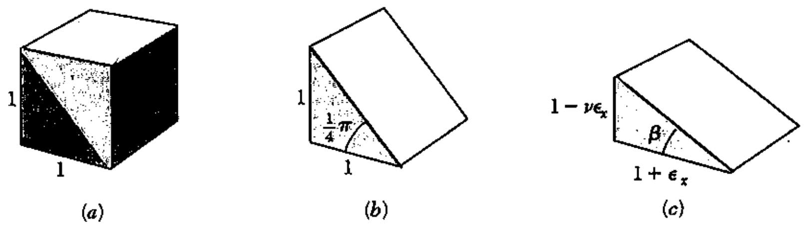
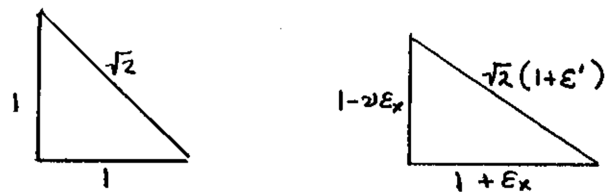


Fig 2.54

SOLUTION



Before deformation After deformation

(a)

$$\left[\sqrt{2}(1+\varepsilon')\right]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

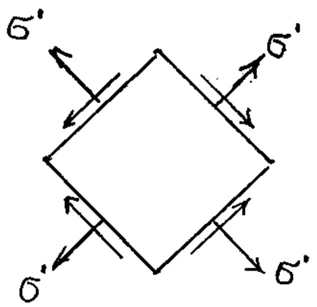
$$2(1 + 2\varepsilon' + \varepsilon'^2) = 1 + 2\varepsilon_x + \varepsilon_x^2 + 1 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

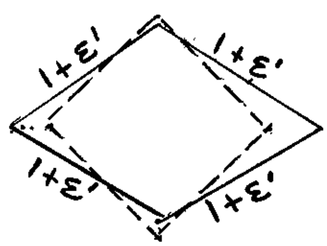
$$4\varepsilon' + 2\varepsilon'^2 = 2\varepsilon_x + \varepsilon_x^2 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

Neglect squares as small $\quad 4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x$$

(b)





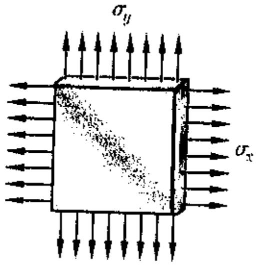
$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$

$$= \frac{1-\nu}{E}\cdot\frac{P}{2A}$$

$$= \frac{1-\nu}{2E}\sigma_x$$

$$= \frac{1-\nu}{2}\varepsilon_x$$

PROBLEM 2.75



2.75 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad (1) \qquad \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \qquad \text{or} \qquad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

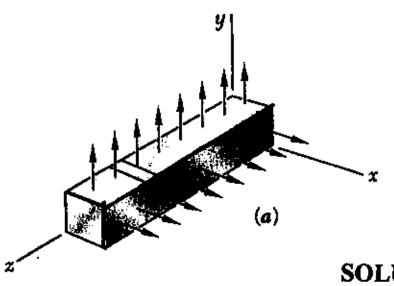
Multiplying (1) by ν and adding to (2)

$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \qquad \text{or} \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{\cancel{E}}\cdot\frac{\cancel{E}}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

PROBLEM 2.76

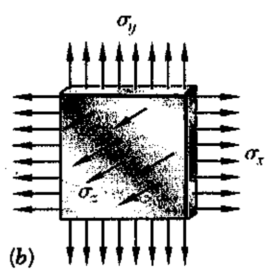


2.76 In many situations physical constraints prevent strain from occurring in a given direction, for example $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for the this situation, which is known as *plane strain*, we can express σ_z, ε_x and ε_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

SOLUTION



$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \qquad \text{or} \qquad \sigma_z = \nu(\sigma_x + \sigma_y)$$

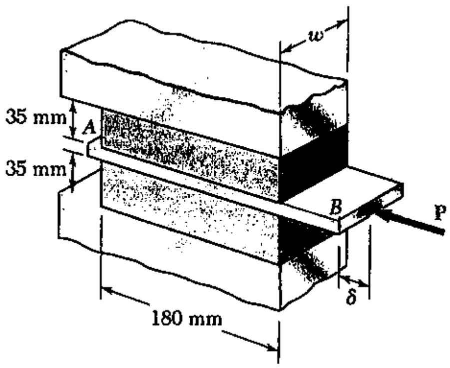
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y]$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

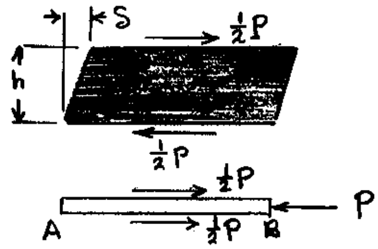
PROBLEM 2.77



2.77 Two blocks of rubber, each of width $w = 60$ mm, are bonded to rigid supports and to the movable plate AB. Knowing that a force of magnitude $P = 19$ kN causes a deflection $\delta = 3$ mm, determine the modulus of rigidity of the rubber used.

SOLUTION

Consider upper block of rubber. The force carried is $\frac{1}{2}P$.



The shearing stress is

$$\tau = \frac{\frac{1}{2}P}{A}$$

where $A = (180\text{ mm})(60\text{ mm}) = 10.8 \times 10^3\text{ mm}^2 = 10.8 \times 10^{-3}\text{ m}^2$

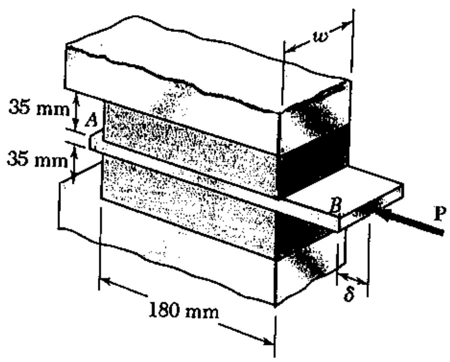
$P = 19 \times 10^3$ N

$$\tau = \frac{(\frac{1}{2})(19 \times 10^3)}{10.8 \times 10^{-3}} = 0.87963 \times 10^6\text{ Pa}$$

$$\gamma = \frac{\delta}{h} = \frac{3\text{ mm}}{35\text{ mm}} = 0.085714$$

$$G = \frac{\tau}{\gamma} = \frac{0.87963 \times 10^6}{0.085714} = 10.26 \times 10^6\text{ Pa} = 10.26\text{ MPa}$$ ◄

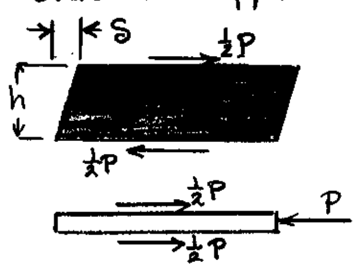
PROBLEM 2.78



2.78 Two blocks of rubber, for which $G = 7.5$ MPa, are bonded to rigid supports and to the movable plate AB. Knowing that the width of each block is $w = 80$ mm, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION

Consider the upper block of rubber. The force carried is $\frac{1}{2}P$.



The shearing stress is

$$\tau = \frac{\frac{1}{2}P}{A}$$

from which

$$P = 2A\tau$$

The shearing strain is $\gamma = \frac{\delta}{h}$ from which $\delta = h\gamma$

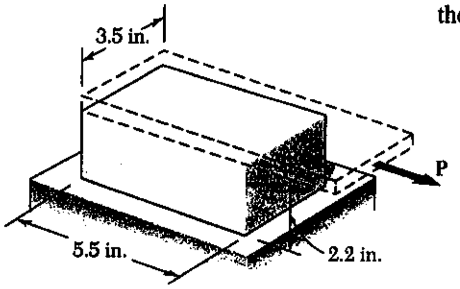
Effective spring constant $k = \frac{P}{\delta} = \frac{2A\tau}{h\gamma}$

Noting that $\tau = G\gamma$,

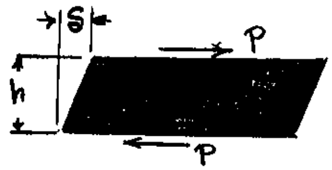
$$k = \frac{2AG}{h} = \frac{(2)(0.180)(0.080)(7.5 \times 10^6)}{0.035} = 6.17 \times 10^6\text{ N/m}$$

6.17×10^3 kN/m ◄

PROBLEM 2.79



2.79 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.



Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb.

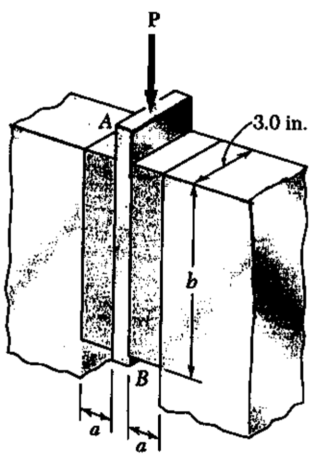
The area is $A = (3.5)(5.5) = 19.25 \text{ in}^2$

Shearing stress $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52$ psi

Shearing strain $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h}$ $\therefore$ $\delta = h\gamma = (2.2)(0.0085006) = 0.0187$ in. ◀

PROBLEM 2.80



2.80 A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 220$ psi and $G = 1800$ psi. Knowing that a centric vertical force of magnitude $P = 3.2$ kips must cause a 0.1 in. vertical deflection of the plate AB, determine the smallest allowable dimensions a and b of the block.

SOLUTION

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$.

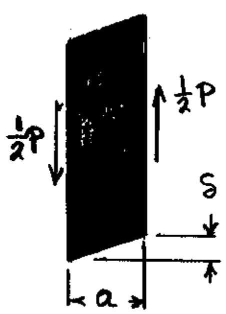
The shearing stress is $\tau = \dfrac{\frac{1}{2}P}{A}$

or required $A = \dfrac{P}{2\tau} = \dfrac{3.2 \times 10^3}{(2)(220)} = 7.2727 \text{ in}^2$

But $A = (3.0)\, b$

Hence $b = \dfrac{A}{3.0} = 2.42$ in. ◀

Use $b = 2.42$ in and $\tau = 220$ psi

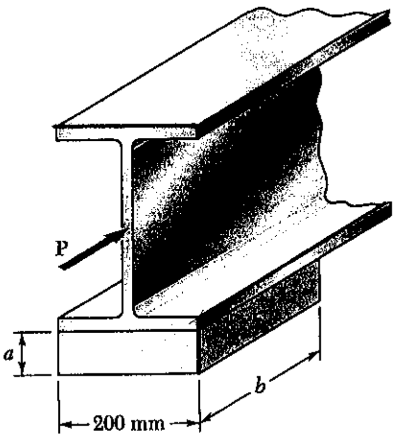


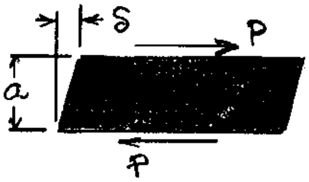
Shearing strain $\gamma = \dfrac{\tau}{G} = \dfrac{220}{1800} = 0.12222$

But $\gamma = \dfrac{\delta}{a}$

Hence $a = \dfrac{\delta}{\gamma} = \dfrac{0.1}{0.12222} = 0.818$ in. ◀

PROBLEM 2.81





2.81 An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN lateral load is applied as shown. Determine *(a)* the smallest allowable dimension b, *(b)* the smallest required thickness a if the maximum allowable shearing stress is 420 kPa.

SOLUTION

Shearing force $P = 22 \times 10^3$ N

Shearing stress $\tau = 420 \times 10^3$ Pa

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3}\ m^2$$

$$= 52.381 \times 10^3\ mm^2$$

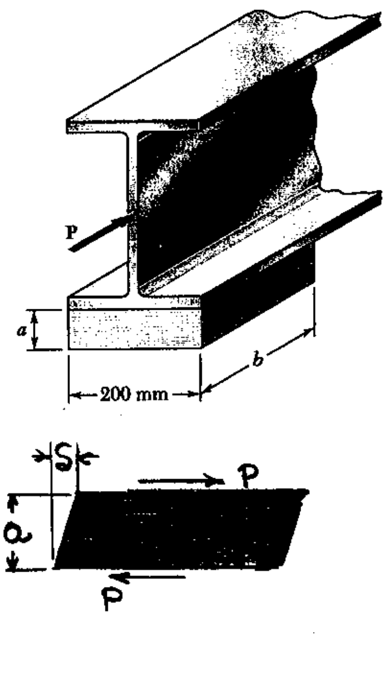
$$A = (200\ mm)(b)$$

$$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262\ mm$$

$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

But $\gamma = \frac{\delta}{a} \quad \therefore \quad a = \frac{\delta}{\gamma} = \frac{10\ mm}{466.67 \times 10^{-3}} = 21.4\ mm$

PROBLEM 2.82



2.82 For the elastomeric bearing in Prob. 2.81 with $b = 220$ mm and $a = 30$ mm, determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 19$ kN and a maximum displacement $\delta = 12$ mm.

SOLUTION

Shearing force $P = 19 \times 10^3$ N

Area $A = (200\ mm)(220\ mm) = 44 \times 10^3\ mm^2$
$= 44 \times 10^{-3}\ m^2$

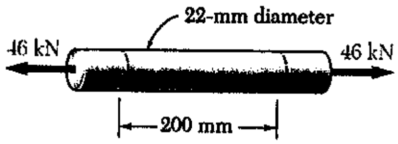
$$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^3\ Pa$$
$$= 431\ kPa$$

Shearing strain $\gamma = \frac{\delta}{a} = \frac{12\ mm}{30\ mm} = 0.400$

Shearing modulus

$$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6\ Pa$$
$$= 1.080\ MPa$$

PROBLEM 2.83



*2.83 Determine the dilatation e and the change in volume of the 200-mm length of the rod shown if (*a*) the rod is made of steel with $E = 200$ GPa and $\nu = 0.30$, (*b*) the rod is made of aluminum with $E = 70$ GPa and $\nu = 0.35$.

SOLUTION

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(22)^2 = 380.13\ \text{mm}^2 = 380.13 \times 10^{-6}\ \text{m}^2$$

$$P = 46 \times 10^3\ \text{N} \qquad \sigma_x = \frac{P}{A} = 121.01 \times 10^6\ \text{Pa} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} \qquad \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\frac{\sigma_x}{E}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x - \nu\sigma_x - \nu\sigma_x) = \frac{(1-2\nu)\sigma_x}{E}$$

Volume $\mathcal{V} = AL = (380.13\ \text{mm}^2)(200\ \text{mm}) = 76.026 \times 10^3\ \text{mm}^3$

$$\Delta\mathcal{V} = \mathcal{V}e$$

(a) steel: $e = \dfrac{(0.4)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6}$ ◄

$$\Delta\mathcal{V} = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40\ \text{mm}^3$$ ◄

(b) aluminum: $e = \dfrac{(0.3)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6}$ ◄

$$\Delta\mathcal{V} = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4\ \text{mm}^3$$ ◄

PROBLEM 2.84

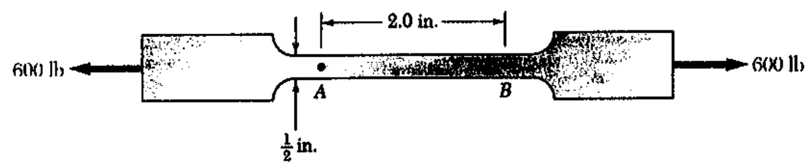
*2.84 Determine the change in volume of the 2-in. gage length segment AB in Prob. 2.68 (a) by computing the dilatation of the material, (b) by subtracting the original volume of portion AB from its final volume.

From PROBLEM 2.68

thickness $= \frac{1}{16}$ in

$E = 29 \times 10^6$ psi

$\nu = 0.30$



SOLUTION

(a) $A = (\frac{1}{2})(\frac{1}{16}) = 0.03125 \text{ in}^2$

Volume: $V_0 = A L_0 = (0.03125)(2.00) = 0.0625 \text{ in}^3$

$$\sigma_x = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 264.83 \times 10^{-6}$$

$$\Delta V = V_0 e = (0.0625)(264.83 \times 10^{-6}) = 16.55 \times 10^{-6} \text{ in}^3$$ ◄

(b) From the solution to PROBLEM 2.68

$$\delta_x = 1.324 \times 10^{-3} \text{ in}, \quad \delta_y = -99.3 \times 10^{-6} \text{ in} \quad \delta_z = -12.41 \times 10^{-6} \text{ in}$$

The dimensions when under a 600 lb tensile load are:

length $\quad L = L_0 + \delta_x = 2 + 1.324 \times 10^{-3} = 2.001324$ in.

width $\quad w = w_0 + \delta_y = \frac{1}{2} - 99.3 \times 10^{-6} = 0.4999007$ in.

thickness $\quad t = t_0 + \delta_z = \frac{1}{16} - 12.41 \times 10^{-6} = 0.06248759$ in

volume $\quad V = Lwt = 0.062516539 \text{ in}^3$

$$\Delta V = V - V_0 = 0.062516539 - 0.0625 = 16.54 \times 10^{-6} \text{ in}^3$$ ◄

PROBLEM 2.85

***2.85** A 6-in. diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere $\quad \mathcal{V}_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6}(6.00)^3 = 113.097 \text{ in}^3$

$$\sigma_x = \sigma_y = \sigma_z = -p = -7.1 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6} = -97.93\times10^{-6}$$

Likewise $\quad \varepsilon_y = \varepsilon_z = -97.93\times10^{-6}$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79\times10^{-6}$$

(a) $-\Delta d = -d_0\varepsilon_x = -(6.00)(97.93\times10^{-6}) = 588\times10^{-6}$ in. ◄

(b) $-\Delta\mathcal{V} = -\mathcal{V}_0 e = -(113.097)(-293.79\times10^{-6}) = 33.2\times10^{-3}$ in^3 ◄

(c) Let m = mass of sphere $\qquad m$ = constant.

$$m = \rho_0\mathcal{V}_0 = \rho\mathcal{V} = \rho\mathcal{V}_0(1+e)$$

$$\frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{\mathcal{V}_0(1+e)}\cdot\frac{\mathcal{V}_0}{m} - 1 = \frac{1}{1+e} - 1$$

$$= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots$$

$$\approx -e = 293.79\times10^{-6}$$

$$\frac{\rho-\rho_0}{\rho_0}\times100\% = (293.79\times10^{-6})(100\%) = 0.0294\%$$ ◄

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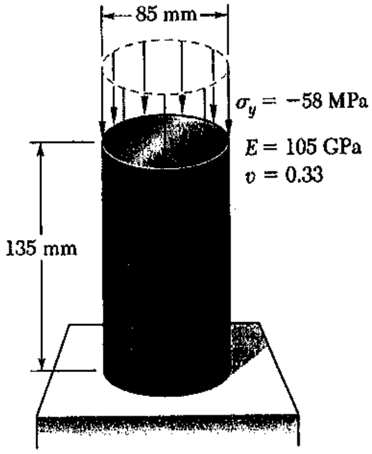
PROBLEM 2.86



*2.86 (*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a* assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_o = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(a) $\sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E}$$

$$= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9}$$

$$= -187.81 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3$$

(b) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y$$

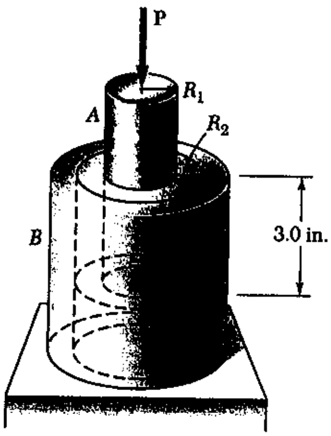
$$= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

$$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3$$

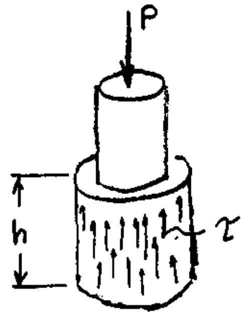
PROBLEM 2.87



*2.87 A vibration isolation support consists of a rod A of radius $R_1 = \frac{3}{8}$-in. and a tube B of inner radius $R_2 = 1$ in. bonded to a . 3-in.-long hollow rubber cylinder with a modulus of rigidity $G = 1.8$ ksi. Determine the largest allowable force **P** which may be applied to rod A if its deflection is not to exceed 0.1 in.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

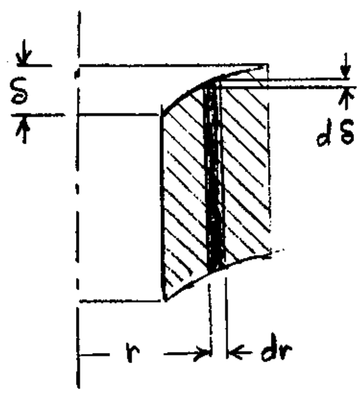


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

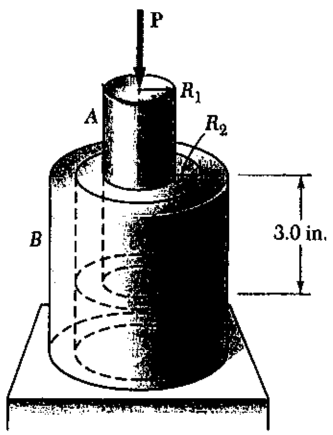
$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h \delta}{\ln(R_2/R_1)}$$

Data: $R_1 = \frac{3}{8}\text{ in} = 0.375\text{ in.}, \quad R_2 = 1.0\text{ in.}, \quad h = 3.0\text{ in}$

$G = 1.8\times10^3\text{ psi}, \quad \delta = 0.1\text{ in}$

$$P = \frac{(2\pi)(1.8\times10^3)(3.0)(0.1)}{\ln(1.0/0.375)} = 3.46\times10^3\text{ lb} = 3.46\text{ kips}$$

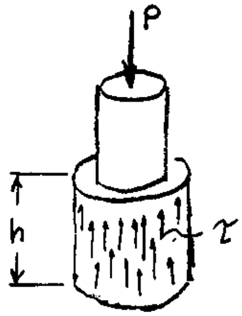
PROBLEM 2.88



***2.88** A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 3-in.-long hollow rubber cylinder with a modulus of rigidity $G = 1.6$ ksi. Determine the required value of the ratio R_2/R_1 if if a 2-kip force **P** is to cause a 0.12-in. deflection of rod A.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

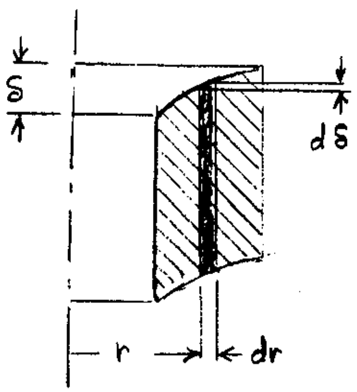


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

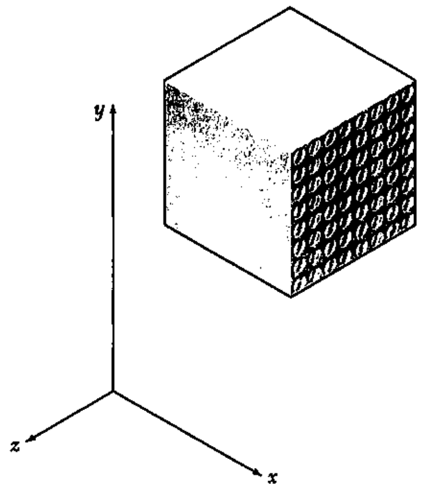
$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi G h \delta}{P} = \frac{(2\pi)(1.6\times10^3)(3.0)(0.12)}{2\times10^3} = 1.8096$$

$$\frac{R_2}{R_1} = \exp(1.8096) = 6.11$$

PROBLEM 2.89



***2.89** A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x, σ_y, and σ_z.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \quad (7)$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{\nu_{xz}}{E_x}\sigma_x \quad (8)$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

Solving simultaneously $\quad \sigma_y = \sigma_z = 0.134993\,\sigma_x$

Using (4) and (5) in (1) $\quad \varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$\varepsilon_x = \frac{1}{E_x}\left[1 - (0.254)(0.134993) - (0.254)(0.134993)\right]\sigma_x$$

$$= \frac{0.93142\,\sigma_x}{E_x}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600\times10^{-6}\ \text{m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6\ \text{Pa}$$

continued

Problem 2.89 continued

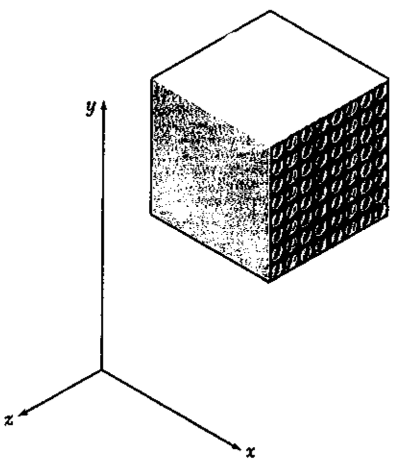
$$\varepsilon_x = \frac{(0.93142)(40.625\times10^{3})}{50\times10^{9}} = 756.78\times10^{-6}$$

(a) $\delta_x = L_x\varepsilon_x = (40\text{ mm})(756.78\times10^{-6}) = 0.0303\text{ mm}$

(b) $\sigma_x = 40.625\times10^{6}\text{ Pa} = 40.6\text{ MPa}$

$$\sigma_y = \sigma_z = (0.134993)(40.625\times10^{6}) = 5.48\times10^{6}\text{ Pa} = 5.48\text{ MPa}$$

PROBLEM 2.90



2.90 The composite cube of Prob. 2.89 is constrained against deformation in the *z* direction and elongated in the *x* direction by 0.035 mm due to a tensile load in the *x* direction.. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the *y* direction.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

Constraint condition $\quad \varepsilon_z = 0$

Load condition $\quad \sigma_y = 0$

From equation (3) $\quad 0 = -\dfrac{\nu_{xz}}{E_x}\sigma_x + \dfrac{1}{E_z}\sigma_z$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

continued

Problem 2.90 continued

From equation (1) with $\sigma_y = 0$

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}\left[\sigma_x - 0.254\,\sigma_z\right] = \frac{1}{E_x}\left[1 - (0.254)(0.077216)\right]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x \varepsilon_x}{0.98039}$$

But $\varepsilon_x = \frac{\delta_x}{L_x} = \frac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(a) $\sigma_x = \frac{(50 \times 10^9)(875 \times 10^{-6})}{0.98039} = 44.625 \times 10^3 \text{ Pa} = 44.6 \text{ MPa}$

$\sigma_y = 0$

$\sigma_z = (0.077216)(44.625 \times 10^6) = 3.446 \times 10^6 \text{ Pa} = 3.45 \text{ MPa}$

From (2) $\varepsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z$

$$= -\frac{(0.254)(44.625 \times 10^6)}{50 \times 10^9} + 0 - \frac{(0.428)(3.446 \times 10^6)}{15.2 \times 10^9}$$

$$= -323.73 \times 10^{-6}$$

$\delta_y = L_y \varepsilon_y = (40 \text{ mm})(-323.73 \times 10^{-6}) = -0.0129 \text{ mm}$

PROBLEM 2.91

***2.91** Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$ [*Hint*: Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

$$G = \frac{E}{2(1+\nu)} \qquad \text{or} \qquad \frac{E}{G} = 2(1+\nu)$$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus

Applying the bounds $\quad 2 \leq \frac{E}{G} < 2(1+\frac{1}{2}) = 3$

Taking the reciprocals $\quad \frac{1}{2} \geq \frac{G}{E} > \frac{1}{3}$

$$\text{or} \quad \frac{1}{3} \leq \frac{G}{E} \leq \frac{1}{2}$$

PROBLEM 2.92

***2.92** The material constants E, G, k, and ν are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (*a*) $k = GE/(9G - 3E)$ and (*b*) $\nu = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2\nu)} \qquad \text{and} \qquad G = \frac{E}{2(1+\nu)}$$

(a) $\quad 1 + \nu = \frac{E}{2G} \qquad \text{or} \qquad \nu = \frac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 6E}$$

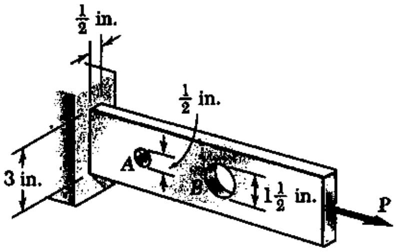
(b) $\quad \frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$

$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

PROBLEM 2.93



2.93 Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 6.5$ kips, determine the maximum value of the stress (*a*) at *A*, (*b*) at *B*.

SOLUTION

(a) At hole A $\quad r = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ in

$d = 3 - \frac{1}{2} = 2.50$ in

$A_{net} = dt = (2.50)(\frac{1}{2}) = 1.25 \text{ in}^2$

$\sigma_{nom} = \frac{P}{A_{net}} = \frac{6.5}{1.25} = 5.2$ ksi

$\frac{r}{d} = \frac{1/4}{2.50} = 0.10 \quad$ From Fig 2.64a $\quad K = 2.70$

$\sigma_{max} = K\sigma_{nom} = (2.70)(5.2) = 14.04$ ksi

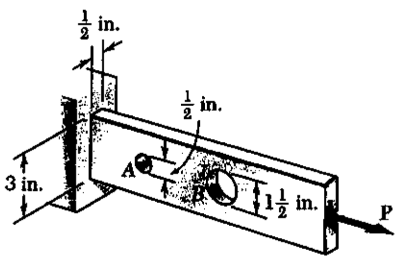
(b) At hole B $\quad r = \frac{1}{2}(1.5) = 0.75, \quad d = 3 - 1.5 = 1.5$ in

$A_{net} = dt = (1.5)(\frac{1}{2}) = 0.75 \text{ in}^2, \quad \sigma_{nom} = \frac{P}{A_{net}} = \frac{6.5}{0.75} = 8.667$ ksi

$\frac{r}{d} = \frac{0.75}{1.5} = 0.5 \quad$ From Fig 2.64a $\quad K = 2.10$

$\sigma_{max} = K\sigma_{nom} = (2.10)(8.667) = 18.2$ ksi

PROBLEM 2.94



2.94 Knowing that $\sigma_{all} = 16$ ksi, determine the maximum allowable value of the centric axial load **P**.

SOLUTION

At hole A $\quad r = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ in

$d = 3 - \frac{1}{2} = 2.50$ in.

$A_{net} = dt = (2.50)(\frac{1}{2}) = 1.25 \text{ in}^2$

$\frac{r}{d} = \frac{1/4}{2.50} = 0.10 \quad$ From Fig 2.64 a, $K = 2.70$

$\sigma_{max} = \frac{KP}{A_{net}} \quad \therefore \quad P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.25)(16)}{2.70} = 7.41$ kips

At hole B $\quad r = \frac{1}{2}(1.5) = 0.75$ in, $\quad d = 3 - 1.5 = 1.5$ in

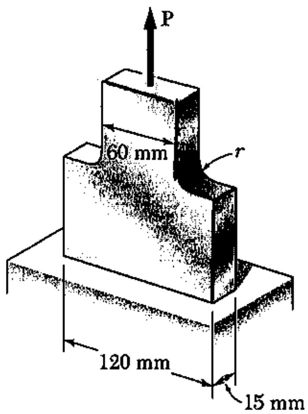
$A_{net} = dt = (1.5)(\frac{1}{2}) = 0.75 \text{ in}^2,$

$\frac{r}{d} = \frac{0.75}{1.5} = 0.5 \quad$ From Fig 2.64 a $\quad K = 2.10$

$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(0.75)(16)}{2.10} = 5.71$ kips

Smaller value for P controls $\quad P = 5.71$ kips

PROBLEM 2.95



2.95 Knowing that, for the plate shown, the allowable stress is 125 MPa, determine the maximum allowable value of P when (*a*) $r = 12$ mm, (*b*) $r = 18$ mm.

SOLUTION

$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$

$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$

(a) $r = 12 \text{ mm} \qquad \frac{r}{d} = \frac{12 \text{ mm}}{60 \text{ mm}} = 0.2$

From Fig. 2.64 b $\qquad K = 1.92 \qquad \sigma_{max} = K\frac{P}{A}$

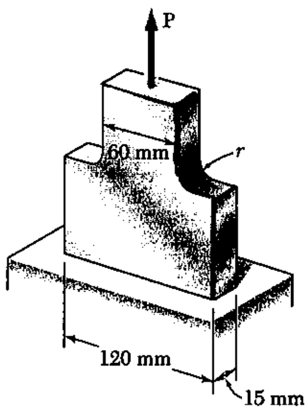
$P = \frac{A\sigma_{max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^6)}{1.92} = 58.6 \times 10^3 \text{ N}$

$= 58.3 \text{ kN}$

(b) $r = 18 \text{ mm}, \quad \frac{r}{d} = \frac{18 \text{ mm}}{60 \text{ mm}} = 0.30$, From Fig 2.64 b $\quad K = 1.75$

$P = \frac{A\sigma_{max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^6)}{1.75} = 64.3 \times 10^3 \text{ N} = 64.3 \text{ kN}$

PROBLEM 2.96



2.96 Knowing that $P = 38$ kN, determine the maximum stress when (*a*) $r = 10$ mm, (*b*) $r = 16$ mm, (*c*) $r = 18$ mm.

SOLUTION

$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$

$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$

(a) $r = 10 \text{ mm} \qquad \frac{r}{d} = \frac{10 \text{ mm}}{60 \text{ mm}} = 0.1667$

From Fig 2.64 b $\qquad K = 2.06 \qquad \sigma_{max} = \frac{KP}{A}$

$\sigma_{max} = \frac{(2.06)(38 \times 10^3)}{900 \times 10^{-6}} = 87.0 \times 10^6 \text{ Pa} = 87.0 \text{ MPa}$

(b) $r = 16 \text{ mm} \qquad \frac{r}{d} = \frac{16 \text{ mm}}{60 \text{ mm}} = 0.2667$

From Fig 2.64 b $\qquad K = 1.78$

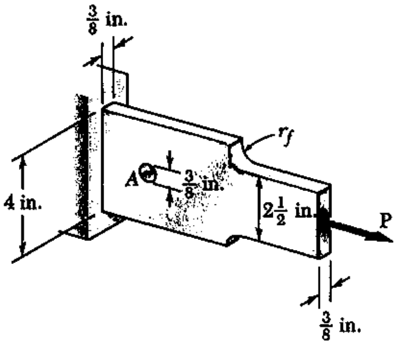
$\sigma_{max} = \frac{(1.78)(38 \times 10^3)}{900 \times 10^{-6}} = 75.2 \times 10^6 \text{ Pa} = 75.2 \text{ MPa}$

(c) $r = 18 \text{ mm} \qquad \frac{r}{d} = \frac{18 \text{ mm}}{60 \text{ mm}} = 0.30$

From Fig 2.64 b $\qquad K = 1.75$

$\sigma_{max} = \frac{(1.75)(38 \times 10^3)}{900 \times 10^{-6}} = 73.9 \times 10^6 \text{ Pa} = 73.9 \text{ MPa}$

PROBLEM 2.97



2.97 Knowing that the hole has a diameter of $\frac{3}{8}$-in., determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 15 ksi.

SOLUTION

For the circular hole $\quad r = (\frac{1}{2})(\frac{3}{8}) = 0.1875$ in

$d = 4 - \frac{3}{8} = 3.625$ in $\qquad \frac{r}{d} = \frac{0.1875}{3.625} = 0.0517$

$A_{net} = dt = (3.625)(\frac{3}{8}) = 1.3594$ in^2

From Fig 2.64 a $\qquad K_{hole} = 2.82$

$$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$$

(b) $\quad P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(1.3594)(15)}{2.82} = 7.23$ kips

(a) $\quad$ For fillet $\quad D = 4$ in, $\; d = 2.5$ in $\qquad \frac{D}{d} = \frac{4.0}{2.5} = 1.60$

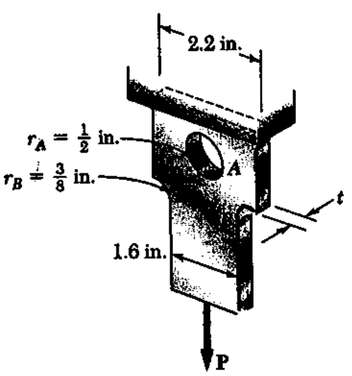
$A_{min} = dt = (2.5)(\frac{3}{8}) = 0.9375$ in^2

$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(0.9375)(15)}{7.23} = 1.945$

From Fig 2.64 b $\quad \frac{r_f}{d} \approx 0.17 \; \therefore \; r_f \approx 0.17d = (0.17)(2.5) = 0.43$ in

PROBLEM 2.98

2.98 For $P = 8.5$ kips, determine the minimum plate thickness *t* required if the allowable stress is 18 ksi.



SOLUTION

At the hole: $\quad r_A = \frac{1}{2}$ in $\qquad d_A = 2.2 - 1.0 = 1.2$ in.

$\frac{r_A}{d_A} = \frac{1/2}{1.2} = 0.417$

From Fig 2.64 a $\qquad K = 2.22$

$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{max}}$

$t = \frac{(2.22)(8.5)}{(1.2)(18)} = 0.87$ in.

At the fillet $\quad D = 2.2$ in, $\; d_B = 1.6$ in $\qquad \frac{D}{d_B} = \frac{2.2}{1.6} = 1.375$

$r_B = \frac{3}{8} = 0.375$ in $\qquad \frac{r_B}{d_B} = \frac{0.375}{1.6} = 0.2344$

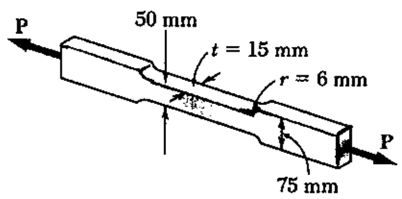
From Fig 2.64 b $\qquad K = 1.70 \qquad \sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(8.5)}{(1.6)(18)} = 0.50$ in

The larger value is the required minimum plate thickness

$t = 0.87$ in.

PROBLEM 2.99



2.99 (*a*) Knowing that the allowable stress is 140 MPa, determine the maximum allowable magnitude of the centric load **P**. (*b*) Determine the percent change in the maximum allowable magnitude of **P** if the raised portions are removed at the ends of the specimen.

SOLUTION

$$\frac{D}{d} = \frac{75\text{ mm}}{50\text{ mm}} = 1.50, \quad \frac{r}{d} = \frac{6\text{ mm}}{50\text{ mm}} = 0.12$$

From Fig 2.64 b $\quad K = 2.10$

$$A_{min} = t\,d = (15)(50) = 750\text{ mm}^2 = 750 \times 10^{-6}\text{ m}^2$$

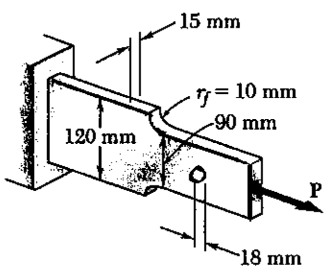
(a) $$\sigma_{max} = \frac{KP}{A_{min}} \quad \therefore \quad P = \frac{A_{min}\sigma_{max}}{K} = \frac{(750\times10^{-6})(140\times10^{6})}{2.10} = 50\times10^{3}\text{ N} = 50\text{ kN}$$

(b) Without raised section $\quad K = 1.00$

$$P = A_{min}\sigma_{max} = (750\times10^{-6})(140\times10^{6}) = 105\times10^{3} = 105\text{ kN}$$

$$\%\text{ change} = \left(\frac{105-50}{50}\right)\times 100\% = 110\%$$

PROBLEM 2.100



2.100 A centric axial force is applied to the steel bar shown. Knowing that σ_{all} is 135 MPa, determine the maximum allowable load **P**.

SOLUTION

At the hole: $\quad r = 9\text{ mm} \quad d = 90 - 18 = 72\text{ mm}$

$\frac{r}{d} = 0.125 \quad$ From Fig 2.64 a $\quad K = 2.65$

$$A_{net} = t\,d = (15)(72) = 1.08\times10^{3}\text{ mm}^2 = 1.08\times10^{-3}\text{ m}^2$$

$$\sigma_{max} = \frac{KP}{A_{net}}$$

$$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.08\times10^{-3})(135\times10^{6})}{2.65} = 55\times10^{3}\text{ N} = 55\text{ kN}$$

At the fillet $\quad D = 120\text{ mm}, \quad d = 90\text{ mm}, \quad \frac{D}{d} = \frac{120}{90} = 1.333$

$r = 10\text{ mm} \quad \frac{r}{d} = \frac{10}{90} = 0.1111 \quad$ From Fig 2.64 b $\quad K = 2.02$

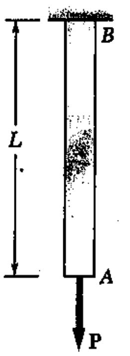
$$A_{min} = t\,d = (15)(90) = 1.35\times10^{3}\text{ mm}^2 = 1.35\times10^{-3}\text{ m}^2$$

$$\sigma_{max} = \frac{KP}{A_{min}}$$

$$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.35\times10^{-3})(135\times10^{6})}{2.02} = 90\times10^{3}\text{ N} = 90\text{ kN}$$

Smaller value for P controls $\quad P = 55\text{ kN}$

PROBLEM 2.101



2.101 The 30-mm square bar AB has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar until end A has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 4.5$ mm, (*b*) $\delta_m = 8$ mm.

SOLUTION

$A = (30)(30) = 900\ mm^2 = 900 \times 10^{-6}\ m^2$

$\delta_Y = L\varepsilon_Y = \dfrac{L\sigma_Y}{E} = \dfrac{(2.2)(345 \times 10^6)}{200 \times 10^9} = 3.795 \times 10^{-3} = 3.795\ mm$

If $\delta_m \geq \delta_Y \qquad P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3\ N$

Unloading $\delta' = \dfrac{P_m L}{AE} = \dfrac{\sigma_Y L}{E} = \delta_Y = 3.795\ mm$

$\delta_p = \delta_m - \delta'$

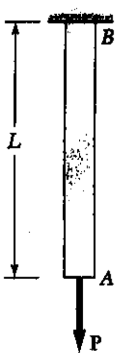
(a) $\delta_m = 4.5\ mm > \delta_Y \qquad P_m = 310.5 \times 10^3\ N = 310.5\ kN$ ◄

$\delta_{perm} = 4.5\ mm - 3.795\ mm = 0.705\ mm$ ◄

(b) $\delta_m = 8\ mm > \delta_Y \qquad P_m = 310.5 \times 10^3\ N = 310.5\ kN$ ◄

$\delta_p = 8.0\ mm - 3.795\ mm = 4.205\ mm$ ◄

PROBLEM 2.102



2.102 The 30-mm square bar AB has a length $L = 2.5$ m; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 3.5 mm, (*b*) 6.5 mm.

SOLUTION

$A = (30)(30) = 900\ mm^2 = 900 \times 10^{-6}\ m^2$

$\delta_Y = L\varepsilon_Y = \dfrac{L\sigma_Y}{E} = \dfrac{(2.5)(345 \times 10^6)}{200 \times 10^9} = 4.3125 \times 10^{3}\ m = 4.3125\ mm$

When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

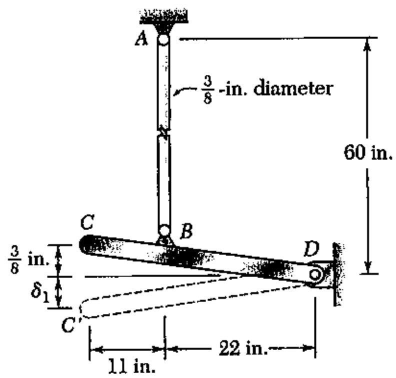
$P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3\ N = 310.5\ kN$ ◄

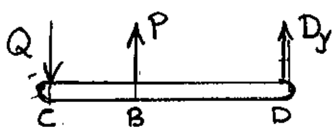
$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$

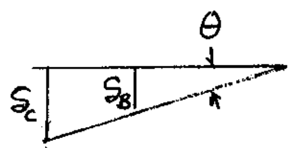
(a) $\delta_p = 3.5\ mm \qquad \delta_m = 3.5\ mm + 4.3125\ mm = 7.81\ mm$ ◄

(b) $\delta_p = 6.5\ mm \qquad \delta_m = 6.5\ mm + 4.3125\ mm = 10.81\ mm$ ◄

PROBLEM 2.103







2.103 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ ksi and $\sigma_Y = 36$ ksi. After the rod has been attached to a rigid lever *CD*, it is found that end *C* is $\frac{3}{8}$-in. too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C'*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

Since the rod AB is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2(36) = 3.976 \text{ kips}$$

Referring the free body diagram of lever CD

$$\Sigma M_D = 0 \qquad 33\,Q - 22\,P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(3.976)}{33} = 2.65 \text{ kips}$$

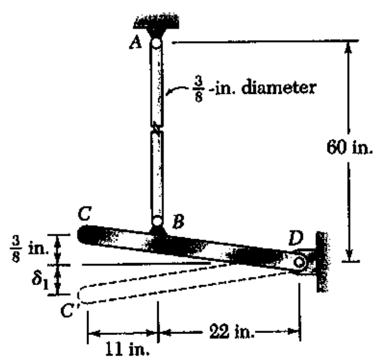
During unloading, the springback at B is

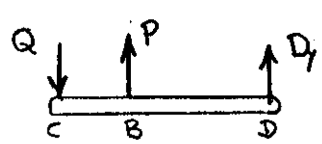
$$\delta_B = L_{AB}\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^6} = 0.0745 \text{ in}$$

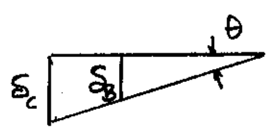
From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B}{22} = \frac{\delta_c}{33} \therefore \delta_c = \frac{33}{22}\delta_B = 0.1117 \text{ in.}$$

PROBLEM 2.104







2.103 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ ksi and $\sigma_Y = 36$ ksi. After the rod has been attached to a rigid lever *CD*, it is found that end *C* is $\frac{3}{8}$-in. too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C'*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

2.104 Solve Prob. 2.103, assuming that the yield point of the mild steel used is 50 ksi.

SOLUTION

Since the rod AB is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2(50) = 5.522 \text{ kips.}$$

Referring to the free body diagram of lever CD

$$\Sigma M_D = 0 \qquad 33Q - 22\,P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(5.522)}{33} = 3.68 \text{ kips}$$

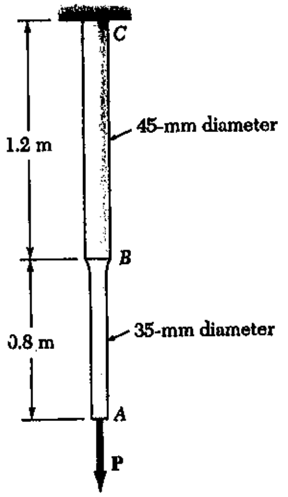
During unloading, the springback at B is

$$\delta_B = L_{AB}\,\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(50\times10^3)}{29\times10^6} = 0.1034 \text{ in.}$$

From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B}{22} = \frac{\delta_c}{33} \therefore \delta_c = \frac{33}{22}\delta_B = 0.1552 \text{ in.}$$

PROBLEM 2.105



2.105 Rods *AB* and *BC* are made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. The rods are stretched until end has moved down 9 mm. Neglecting stress concentrations, determine (*a*) the maximum value of the force **P**, (*b*) the permanent set measured at points *A* and *B* after the force has been removed.

SOLUTION

$A_{AB} = \frac{\pi}{4}(0.035)^2 = 962.1\times10^{-6}\ m^2;\quad A_{BC} = \frac{\pi}{4}(0.045)^2 = 1.5904\times10^{-3}\ m^2$

(a) $P_{max} = A_{min}\sigma_Y = (962.1\times10^{-6})(345\times10^6)$

$= 331.93\times10^3\ N = 332\ kN$

(b) Spring back $\delta' = \frac{PL_{AB}}{EA_{AB}} + \frac{PL_{BC}}{EA_{BC}} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$

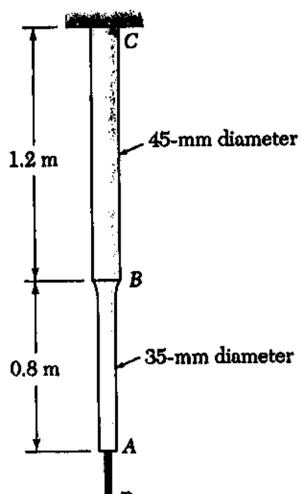
$\delta' = \frac{331.98\times10^3}{200\times10^9}\left(\frac{0.8}{962.1\times10^{-6}} + \frac{1.2}{1.5904\times10^{-3}}\right)$

$= 2.63\times10^{-3}\ m = 2.63\ m.$

At point A $\quad \delta_p = \delta_m - \delta' = 9\ mm - 2.63\ mm = 6.37\ mm$

At point B: No yielding in BC; hence $\delta_p = 0$

PROBLEM 2.106



2.105 Rods *AB* and *BC* are made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 345 MPa. The rods are stretched until end has moved down 9 mm. Neglecting stress concentrations, determine (*a*) the maximum value of the force **P**, (*b*) the permanent set measured at points *A* and *B* after the force has been removed.

2.106 Solve Prob. 2.105, assuming that the yield point of the mild steel used is 250 MPa.

SOLUTION

$A_{AB} = \frac{\pi}{4}(0.035)^2 = 962.1\times10^{-6}\ m^2,\quad A_{BC} = \frac{\pi}{4}(0.045)^2 = 1.5904\times10^{-3}\ m^2$

(a) $P_{max} = A_{min}\sigma_Y = (962.1\times10^{-6})(250\times10^6)$

$= 240.53\times10^3\ N = 241\ KN$

(b) Spring back $\delta' = \frac{PL_{AB}}{EA_{AB}} + \frac{PL_{BC}}{EA_{BC}} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$

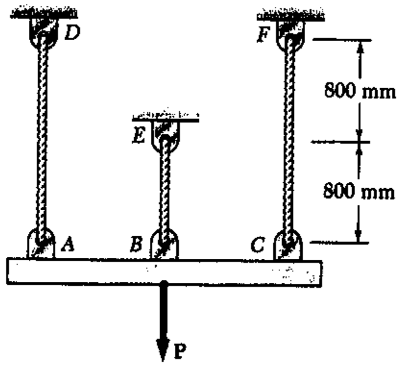
$\delta' = \frac{240.53\times10^3}{200\times10^9}\left(\frac{0.8}{962.1\times10^{-6}} + \frac{1.2}{1.5904\times10^{-3}}\right)$

$= 1.908\times10^{-3}\ m = 1.908\ mm.$

At point A $\quad \delta_p = \delta_m - \delta' = 9\ mm - 1.908\ mm = 7.09\ mm$

At point B, no yielding in BC; hence $\delta_p = 0$

PROBLEM 2.107



2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **P** is applied to the rigid bar ABC until the bar has moved downward a distance $\delta = 2$ mm. Knowing that the cables were initially taut, determine (*a*) the maximum value of **P**, (*b*) the maximum stress that occurs in cable AD, (*c*) the final displacement of the bar after the load is removed. (*Hint*: In part *c*, cable *BE* is not taut.)

SOLUTION

For each cable $A = \frac{\pi}{4}(0.006)^2 = 28.274\times10^{-6}\ \text{m}^2$

Strain at initial yielding

$$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{345\times10^6}{200\times10^9} = 1.725\times10^{-3}$$

Strain in cables AD and CF: $\varepsilon_{AD} = \varepsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2\text{ mm}}{1600\text{ mm}} = 1.25\times10^{-3}$

Strain in cable BE: $\varepsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2\text{ mm}}{800\text{ mm}} = 2.50\times10^{-3}$

Since $\varepsilon_{AD} < \varepsilon_Y$, $\sigma_{AD} = E\varepsilon_{AD} = (200\times10^9)(1.25\times10^{-3}) = 250\times10^6$ Pa

Since $\varepsilon_{BE} > \varepsilon_Y$, $\sigma_{BE} = \sigma_Y = 345\times10^6$ Pa

Forces: $P_{AD} = P_{CF} = A\sigma_{AD} = (28.274\times10^{-6})(250\times10^6) = 7.0685\times10^3$ N

$P_{BE} = A\sigma_{BE} = (28.274\times10^{-6})(345\times10^6) = 9.7545\times10^3$ N

For equilibrium of bar ABC $\quad P_{AD} + P_{BE} + P_{CF} - P = 0$

(a) $P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685)\times10^3$ N

$= 23.9\times10^3$ N $= 23.9$ kN ◄

(b) $\sigma_{AD} = 250\times10^6$ Pa $= 250$ MPa ◄

After unloading $\quad P = 0$

Cable BE is not taut $\quad P_{BE} = 0$

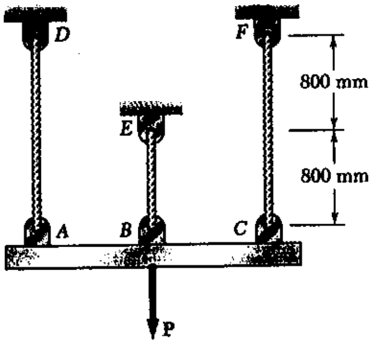
By symmetry $\quad P_{AD} = P_{CF}$

For equilibrium $\quad P_{AD} = P_{CF} = 0$

(c) Final displacement δ is controlled by the final lengths of cables AD and CF. Since these cables were never permanently deformed, the final displacement is

$$\delta = \delta_{AD} = \delta_{CF} = 0$$ ◄

PROBLEM 2.108



2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **P** is applied to the rigid bar *ABC* until the bar has moved downward a distance $\delta = 2$ mm. Knowing that the cables were initially taut, determine (*a*) the maximum value of **P**, (*b*) the maximum stress that occurs in cable *AD*, (*c*) the final displacement of the bar after the load is removed. (*Hint*: In part *c*, cable *BE* is not taut.)

2.108 Solve Prob. 2.107, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

SOLUTION

For each rod $A = \frac{\pi}{4}(0.006)^2 = 28.274 \times 10^{-6}\ m^2$

Strain at initial yielding $\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{345 \times 10^6}{200 \times 10^9} = 1.725 \times 10^{-3}$

Strain in rods AD and CF: $\varepsilon_{AD} = \varepsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2\ mm}{1600\ mm} = 1.25 \times 10^{-3}$

Strain in rod BE: $\varepsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2\ mm}{800\ mm} = 2.50 \times 10^{-3}$

Since $\varepsilon_{AD} < \varepsilon_Y$, $\sigma_{AD} = E\varepsilon_{AD} = (200 \times 10^9)(1.25 \times 10^{-3}) = 250 \times 10^6$ Pa

Since $\varepsilon_{BE} > \varepsilon_Y$, $\sigma_{BE} = \sigma_Y = 345 \times 10^6$ Pa

Forces: $P_{AD} = P_{CF} = A\sigma_{AD} = (28.274 \times 10^{-6})(250 \times 10^6) = 7.0685 \times 10^3$ N

$P_{BE} = A\sigma_{BE} = (28.274 \times 10^{-6})(345 \times 10^6) = 9.7545 \times 10^3$ N

For equilibrium of bar ABC $P_{AD} + P_{BE} + P_{CF} - P = 0$

(a) $P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685) \times 10^3$ N $= 23.9$ kN

(b) $\sigma_{AD} = 250 \times 10^6$ Pa $= 250$ MPa

Let δ' = change in displacement during unloading

$$P'_{AD} = \frac{EA}{L_{AD}}\delta' = \frac{(200 \times 10^9)(28.274 \times 10^{-6})}{1600 \times 10^{-3}}\delta' = 3.534 \times 10^6\ \delta' = P'_{CF}$$

$$P'_{BE} = \frac{EA}{L_{BE}}\delta' = \frac{(200 \times 10^9)(28.274 \times 10^{-6})}{800 \times 10^{-3}}\delta' = 7.0685 \times 10^6\ \delta'$$

For equilibrium $P' = P'_{AD} + P'_{BE} + P'_{CF} = 14.137 \times 10^6\ \delta'$

But $P - P' = 0$ $\quad P' = P = 23.89 \times 10^3$ N

$$\delta' = \frac{23.89 \times 10^3}{14.137 \times 10^6} = 1.690 \times 10^{-3}\ m$$

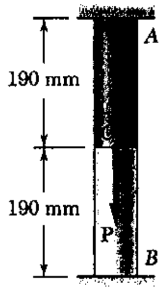
Permanent displacement of bar

$\delta_{final} = \delta_{max} - \delta' = 2 \times 10^{-3} - 1.690 \times 10^{-3} = 0.310 \times 10^{-3}$ m

$= 0.310$ mm

PROBLEM 2.109

2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion BC is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.



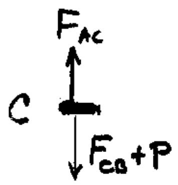
SOLUTION

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\,\varepsilon_{Y,AC} = \frac{L_{AC}\,\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\ \text{m}$$

Corresponding force $\quad F_{AC} = A\,\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6)$
$= 437.5\times10^3\ \text{N}$

$$F_{CB} = -\frac{EA\,\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\ \text{N}$$



For equilibrium of element at C

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\ \text{N}$$

Since applied load $P = 975\times10^3\ \text{N} > 875\times10^3\ \text{N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\ \text{N} = -537.5\times10^3\ \text{N}$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\ \text{m}$$
$$= 0.292\ \text{mm}$$

(b) Maximum stresses $\quad \sigma_{AB} = \sigma_{Y,AB} = 250\ \text{MPa}$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\ \text{Pa} = -307\ \text{MPa}$$

(c) Deflection and forces for unloading

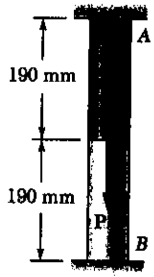
$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 487.5\times10^3\ \text{N}$$

$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^{-3}\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3} = 0.02715\times10^{-3}\ \text{m}$$
$$= 0.027\ \text{mm}$$

PROBLEM 2.110



2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm². Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion BC is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

2.110 For the composite rod of Prob. 2.109, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3$ mm and then decreased back to zero, determine, (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

SOLUTION

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding

$$\varepsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,CB} = -\frac{\sigma_{Y,BC}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(a) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^{-3} \text{ N}$$

For equilibrium of element at C $\qquad F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CB} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} = 990 \text{ kN}$$

(b) Stresses: AC $\quad \sigma_{AC} = \sigma_{Y,AC} = 250$ MPa

CB $\quad \sigma_{CB} = \dfrac{F_{CB}}{A} = -\dfrac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} = -316 \text{ MPa}$

(c) Deflection and forces for unloading

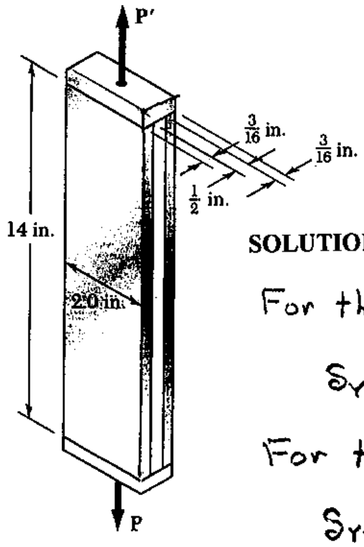
$$\delta' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} \quad \therefore \; P_{CB}' = -P_{AC}' \frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P_{AC}' - P_{CB}' = 2P_{AC}' = 990.1 \times 10^3 \text{ N} \quad \therefore \; P_{AC}' = 495.05 \times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26874 \times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm} = 0.031 \text{ mm}$$

PROBLEM 2.111



2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value δ_m = 0.04 in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel $\quad A_1 = (\tfrac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times 10^3)}{29\times 10^6} = 0.024138 \text{ in.}$$

For the tempered steel $\quad A_2 = 2(\tfrac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times 10^3)}{29\times 10^3} = 0.048276 \text{ in.}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ $\quad$ The mild steel yields. Tempered steel is elastic.

(a) Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50\times 10^3) = 50\times 10^3$ lb.

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times 10^3)(0.75)(0.04)}{14} = 62.14\times 10^3 \text{ lb.}$$

$$P = P_1 + P_2 = 112.14\times 10^3 \text{ lb} = 112.1 \text{ kips}$$

(b) Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times 10^3 \text{ psi} = 50 \text{ ksi}$

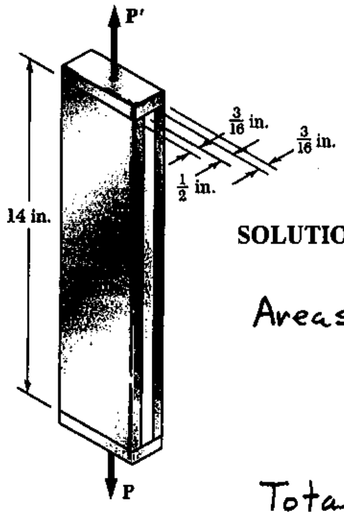
$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times 10^3}{0.75} = 82.86\times 10^3 \text{ psi} = 82.86 \text{ ksi}$$

Unloading $\quad \delta' = \frac{PL}{EA} = \frac{(112.14\times 10^3)(14)}{(29\times 10^6)(1.75)} = 0.03094 \text{ in.}$

(c) Permanent set $\quad \delta_p = \delta_m - \delta' = 0.04 - 0.03094$

$= 0.00906$ in.

PROBLEM 2.112



2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

2.112 For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

Tempered steel $A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb.}$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_1 = (1.00)(50\times10^3) = 50\times10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb.}$$

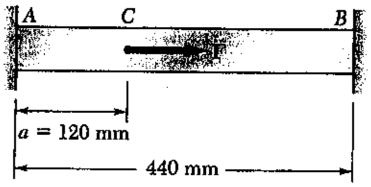
(a) $$\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times10^3)(14)}{(29\times10^6)(0.75)} = 0.03090 \text{ in.}$$

(b) $$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi} = 64 \text{ ksi}$$

Unloading $$\delta' = \frac{PL}{EA} = \frac{(98\times10^3)(14)}{(29\times10^6)(1.75)} = 0.02703 \text{ in}$$

(c) $$\delta_P = \delta_m - \delta' = 0.03090 - 0.02703 = 0.00387 \text{ in.}$$

PROBLEM 2.113



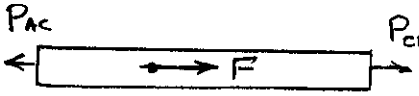
2.113 Bar *AB* has a cross-sectional area of 1200 mm² and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

SOLUTION

$$A = 1200\ \text{mm}^2 = 1200 \times 10^{-6}\ \text{m}^2$$

Force to yield portion AC: $P_{AC} = A\sigma_Y = (1200\times10^{-6})(250\times10^{6})$

$$= 300 \times 10^3\ \text{N}$$



For equilibrium $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300\times10^3 - 520\times10^3$$

$$= -220\times10^3\ \text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times10^3)(0.440-0.120)}{(200\times10^9)(1200\times10^{-6})} = 0.293333\times10^{-3}\ \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220\times10^3}{1200\times10^{-6}} = -183.333\times10^6\ \text{Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}'L_{AC}}{EA} = -\frac{P_{CB}'L_{CB}}{EA} = \frac{(F-P_{AC}')L_{CB}}{EA}$$

$$P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC}+L_{CB}} = \frac{(520\times10^3)(0.440-0.120)}{0.440} = 378.182\times10^3\ \text{N}$$

$$P_{CB}' = P_{AC}' - F = 378.182\times10^3 - 520\times10^3 = -141.818\times10^3\ \text{N}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{378.182\times10^3}{1200\times10^{-6}} = 315.152\times10^6\ \text{Pa}$$

$$\sigma_{BC}' = \frac{P_{BC}'}{A} = -\frac{141.818\times10^3}{1200\times10^{-6}} = -118.182\times10^6\ \text{Pa}$$

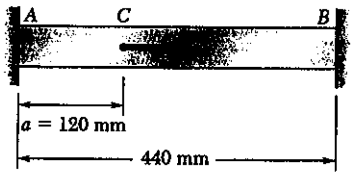
$$\delta_C' = \frac{(378.182)(0.120)}{(200\times10^9)(1200\times10^{6})} = 0.189091\times10^{-3}\ \text{m}$$

(a) $\delta_{cp} = \delta_C - \delta_C' = 0.293333\times10^{-3} - 0.189091\times10^{-3} = 0.1042\times10^{-3}\ \text{m}$
$= 0.1042\ \text{mm}$ ◄

(b) $\sigma_{AC,res} = \sigma_Y - \sigma_{AC}' = 250\times10^6 - 315.152\times10^6 = -65.2\times10^6\ \text{Pa}$
$= -65.2\ \text{MPa}$ ◄

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333\times10^6 + 118.182\times10^6 = -65.2\times10^6\ \text{Pa}$
$= -65.2\ \text{MPa}$ ◄

PROBLEM 2.114



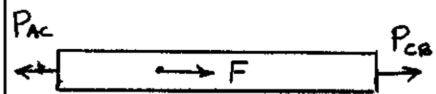
2.113 Bar *AB* has a cross-sectional area of 1200 mm² and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

2.114 Solve Prob. 2.113, assuming that $a = 180$ mm.

SOLUTION

$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$

Force to yield portion AC: $P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$
$= 300 \times 10^3 \text{ N}$



For equilibrium $\quad F + P_{CB} - P_{AC} = 0$

$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$
$= -220 \times 10^3 \text{ N}$

$$\delta_c = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.238333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta_c' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}') L_{CB}}{EA} \quad \therefore \quad P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA}\right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.180)}{0.440} = 307.273 \times 10^3 \text{ N}$$

$$P_{CB}' = P_{AC}' - F = 307.273 \times 10^3 - 520 \times 10^3 = -212.727 \times 10^3 \text{ N}$$

$$\delta_c' = \frac{(307.273 \times 10^3)(0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.230455 \times 10^{-3} \text{ m}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{307.273 \times 10^3}{1200 \times 10^{-6}} = 256.061 \times 10^6 \text{ Pa}$$

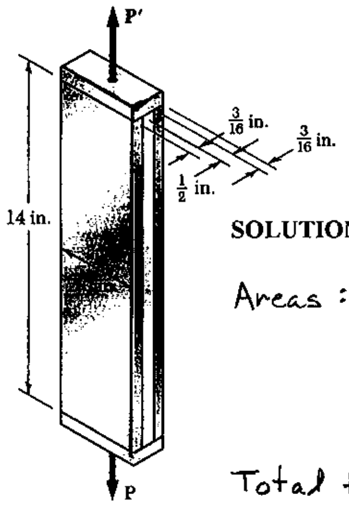
$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727 \times 10^3}{1200 \times 10^{-6}} = -177.273 \times 10^6 \text{ P}$$

(a) $\quad \delta_{cp} = \delta_c - \delta_c' = 0.238333 \times 10^{-3} - 0.230455 \times 10^{-3} = 0.00788 \times 10^{-3} \text{ m}$
$= 0.00788 \text{ mm}$ ◄

(b) $\quad \sigma_{AC,res} = \sigma_{AC} - \sigma_{AC}' = 250 \times 10^6 - 256.061 \times 10^6 = -6.06 \times 10^6 \text{ Pa}$
$= -6.06 \text{ MPa}$ ◄

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333 \times 10^6 + 177.273 \times 10^6 = -6.06 \times 10^6 \text{ Pa}$
$= -6.06 \text{ MPa}$ ◄

PROBLEM 2.115



2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

***2.115** For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

SOLUTION

Areas: Mild steel $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

Tempered steel $A_2 = (2)(\frac{3}{16})(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_Y$; therefore mild steel yields

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1 \sigma_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \qquad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi}$$

Unloading $\quad \sigma' = \dfrac{P}{A} = \dfrac{98 \times 10^3}{1.75} = 56 \times 10^3 \text{ psi}$

Residual stresses

mild steel $\quad \sigma_{1,res} = \sigma_1 - \sigma' = 50 \times 10^3 - 56 \times 10^3 = -6 \times 10^{-3} \text{ psi}$

$= -6$ ksi

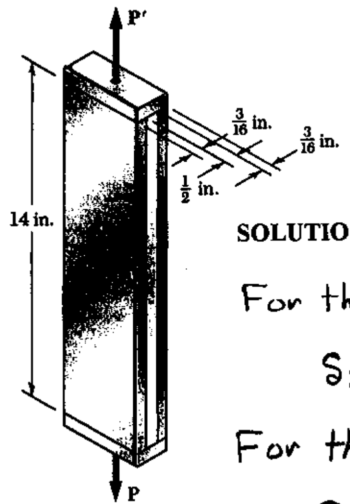
tempered steel $\quad \sigma_{2,res} = \sigma_2 - \sigma_1 = 64 \times 10^3 - 56 \times 10^3$

$= 8 \times 10^3 \text{ psi} = 8$ ksi ◄

PROBLEM 2.116

2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

***2.116** **For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.**



SOLUTION

For the mild steel $\quad A_1 = (\tfrac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in}$$

For the tempered steel $\quad A_2 = 2(\tfrac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in.}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ $\quad$ The mild steel yields. Tempered steel is elastic.

Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3$ lb.

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb.}$$

Stresses $\quad \sigma_1 = \dfrac{P_1}{A_1} = \sigma_{Y1} = 50 \times 10^3$ psi

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi}$$

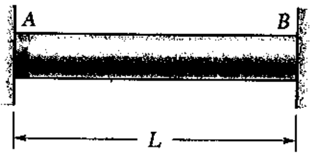
Unloading $\quad \sigma' = \dfrac{P}{A} = \dfrac{112.14}{1.75} = 64.08 \times 10^3$ psi

Residual stresses

$$\sigma_{1,res} = \sigma_1 - \sigma' = 50 \times 10^3 - 64.08 \times 10^3 = -14.08 \times 10^3 \text{ psi}$$
$$= -14.08 \text{ ksi}$$

$$\sigma_{2,res} = \sigma_2 - \sigma' = 82.86 \times 10^3 - 64.08 \times 10^3 = 18.78 \times 10^3 \text{ psi}$$
$$= 18.78 \text{ ksi}$$

PROBLEM 2.117



2.117 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 8°C. The steel is assumed to be elastoplastic with $\sigma_Y = 250$ MPa and $G = 200$ GPa. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine the stress in the bar (*a*) when the temperature is raised to 165°C, (*b*) after the temperature has returned to 8°C.

SOLUTION

Determine temperature change to cause yielding

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{250 \times 10^6}{(200 \times 10^9)(11.7 \times 10^{-6})} = 106.838\,°C$$

But $\Delta T = 165 - 8 = 157\,°C$

(a) Yielding occurs $= \quad \sigma = -\sigma_Y = -250$ MPa

Cooling $(\Delta T)' = 157°C$

$$\delta' = \delta'_P + \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

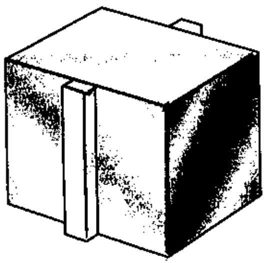
$$= -(200 \times 10^9)(11.7 \times 10^{-6})(157) = -367.38 \times 10^6 \text{ Pa}$$

(b) $\sigma_{res} = -\sigma_Y + \sigma' = -250 \times 10^6 + 367.38 \times 10^6 = 117.38 \times 10^6$ Pa

$= 117.4$ MPa

PROBLEM 2.118

2.118 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 20°C$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 100 MPa. Assume $\alpha_a = 23.6 \times 10^{-6}/°C$ for the aluminum and $\alpha_s = 11.7 \times 10^{-6}/°C$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 70$ GPa and $\sigma_Y = 100$ MPa. (*Hint*: Neglect the small stresses in the plate.)



SOLUTION

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T_Y) = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{100 \times 10^6}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 120.04°C$$

(a) $T_{2Y} = T_1 + (\Delta T)_Y = 20 + 120.04 = 140.04°C$ ◄

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{res} = -\sigma_Y$

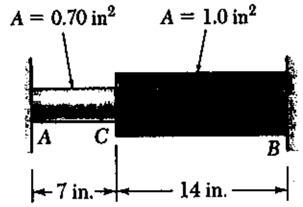
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(100 \times 10^6)}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 240.1°C$$

(b) $T_2 = T_1 + \Delta T = 20 + 240.1 = 260.1°C$ ◄

If $T_2 > 260.1°C$, the aluminum bar will most likely yield in compression.

PROBLEM 2.119

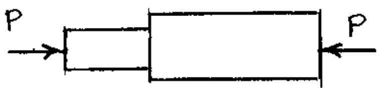


2.119 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 38°F. The steel is assumed elastoplastic, with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The temperature of both portions of the rod is then raised to 250 °F. Knowing that $\alpha = 6.5 \times 10^{-6}/°\text{F}$, determine (*a*) the stress in portion *AC*, (*b*) the deflection of point *C*.

SOLUTION

$$\delta_{B/A} = \delta_{B/A,P} + \delta_{B/A,T} = 0 \qquad \text{(constraint)}$$

Determine ΔT to cause yielding in AC.



$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} + L_{AB}\,\alpha(\Delta T) = 0$$

$$(\Delta T) = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding $\quad P = A_{AC}\sigma_Y$

$$(\Delta T)_Y = \frac{A_{AC}\sigma_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{(0.70)(36\times10^3)}{(21)(29\times10^6)(6.5\times10^{-6})}\left(\frac{7}{0.70} + \frac{14}{1.0}\right)$$

$$= 152.785\ °\text{F}$$

Actual $\Delta T = 250 - 38 = 212\ °\text{F} > (\Delta T)_Y \quad \therefore$ yielding occurs.

$$\sigma_{AC} = -\sigma_Y = -36 \text{ ksi}$$

$$P = \sigma_Y A_{AC} = (36\times10^3)(0.70) = 25.2\times10^3 \text{ lb}$$

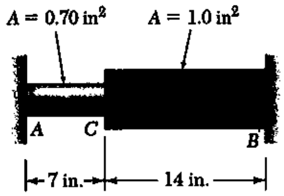
$$\delta_C = -\delta_{C/B} = \frac{PL_{CB}}{EA_{CB}} - L_{CB}\,\alpha(\Delta T)$$

$$= \frac{(25.2\times10^3)(14)}{(29\times10^6)(1.0)} - (14)(6.5\times10^{-6})(212)$$

$$= 0.012176 - 0.019292 \qquad = -0.007116 \text{ in}$$

$$\delta_C = 0.00712 \text{ in} \leftarrow$$

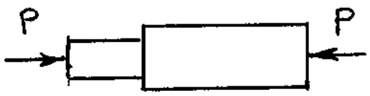
PROBLEM 2.120



2.119 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 38°F. The steel is assumed elastoplastic, with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The temperature of both portions of the rod is then raised to 250 °F. Knowing that $\alpha = 6.5 \times 10^{-6}/°F$, determine (*a*) the stress in portion *AC*, (*b*) the deflection of point *C*.

***2.120** Solve Prob. 2.119, assuming that the temperature of the rod is raised to 250 °F and then returned to 38 °F.

SOLUTION



$\delta_{B/A} = \delta_{B/A,P} + \delta_{B/A,T} = 0$ (constraint)

Determine ΔT to cause yielding in AC.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} + L_{AB}\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{P}{(21)(29\times10^6)(6.5\times10^{-6})}\left(\frac{7}{0.70} + \frac{14}{1.0}\right)$$

$= 6.0629\times10^{-3}\,P$ At yielding $P_Y = \sigma_Y A_{AC} = (36\times10^3)(0.7) = 25.2\times10^3$ lb

$(\Delta T)_Y = (6.0629\times10^{-3})(25.2\times10^3) = 152.785\,°F$

Actual $\Delta T = 250 - 38 = 212\,°F > (\Delta T)_Y$ $\therefore$ yielding occurs.

$\sigma_{AC} = -\sigma_Y = -36\times10^3$ psi

$$\delta_C = -\delta_{B/C} = \frac{PL_{CB}}{EA_{CB}} - L_{BC}\alpha(\Delta T) = \frac{(25.2\times10^3)(14)}{(29\times10^6)(1.0)} - (14)(6.5\times10^{-6})(212)$$

$= 0.012176 - 0.019292 = -0.007116$ in

Cooling $\Delta T' = 212\,°F$ $P' = \dfrac{\Delta T}{6.0629\times10^{-3}} = \dfrac{212}{6.0629\times10^{-3}}$

$$P' = \frac{\Delta T'}{6.0629\times10^{-3}} = \frac{212}{6.0629\times10^{-3}} = 34.967\times10^3 \text{ lb.}$$

(a) Residual stress in AC

$$\sigma_{AC,res} = -\sigma_Y + \frac{P'}{A_{AC}} = -36\times10^3 + \frac{34.967\times10^3}{0.7} = 13.95\times10^3 \text{ psi}$$

$= 13.95$ ksi ◀

$$\delta_C' = -\delta_{B/C}' = -\frac{P'L_{CB}}{EA_{CB}} + L_{CB}\alpha(\Delta T)$$

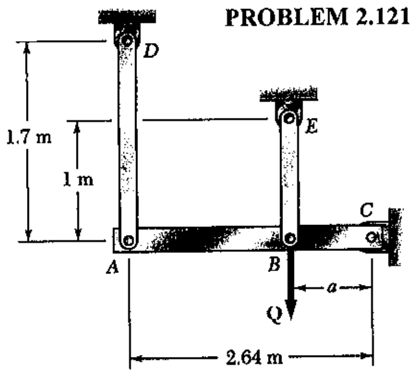
$$= -\frac{(34.967\times10^3)(14)}{(29\times10^6)(1.0)} + (14)(6.5\times10^{-6})(212)$$

$= -0.016881 + 0.019292 = 0.002411$ in

$\delta_{C,P} = \delta_C + \delta_C' = -0.007116 + 0.002411 = -0.00471$ in

0.00471 in ← ◀

PROBLEM 2.121

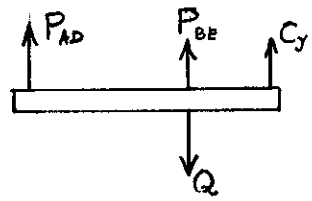


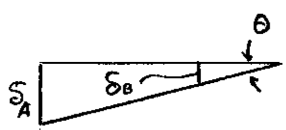
2.121 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

Statics: $\Sigma M_C = 0 \quad 0.640(Q - P_{BE}) - 2.64\, P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = a\theta = 0.640\,\theta$





Elastic Analysis:

$A = (37.5)(6) = 225\ \text{mm}^2 = 225 \times 10^{-6}\ \text{m}^2$

$P_{AD} = \dfrac{EA}{L_{AD}}\delta_A = \dfrac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$

$\sigma_{AD} = \dfrac{P_{AD}}{A} = 310.6\times10^9\,\theta$

$P_{BE} = \dfrac{EA}{L_{BE}}\delta_B = \dfrac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B$

$= (45\times10^6)(0.640\,\theta) = 28.80\times10^6\,\theta$

$\sigma_{BE} = \dfrac{P_{BE}}{A} = 128\times10^9\,\theta$

From Statics $\quad Q = P_{BE} + \dfrac{2.64}{0.640}P_{AD} = P_{BE} + 4.125\,P_{AD}$

$= [28.80\times10^6 + (4.125)(69.88\times10^6)]\theta = 317.06\times10^6\,\theta$

θ_Y at yielding of link AD $\quad \sigma_{AD} = \sigma_Y = 250\times10^6 = 310.6\times10^9\,\theta$

$\theta_Y = 804.89\times10^{-6}$

$Q_Y = (317.06\times10^6)(804.89\times10^{-6}) = 255.2\times10^3\ N$

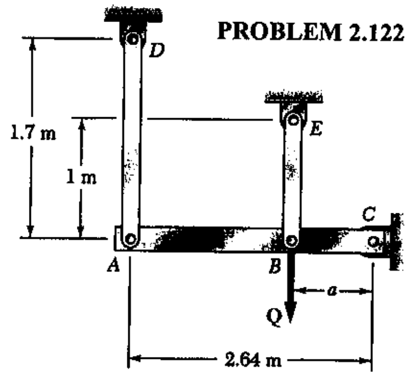
Since $Q = 260\times10^3 > Q_Y$, link AD yields. $\quad \sigma_{AD} = 250$ MPa

$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3\ N$

From Statics $\quad P_{BE} = Q - 4.125\,P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$P_{BE} = 27.97\times10^3\ N \qquad \sigma_{BE} = \dfrac{P_{BE}}{A} = \dfrac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6\ Pa$
$= 124.3$ MPa

$\delta_B = \dfrac{P_{BE}L_{BE}}{EA} = \dfrac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6}\ m$
$= 0.622$ mm

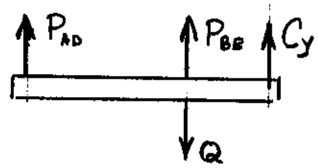


PROBLEM 2.122

2.121 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.

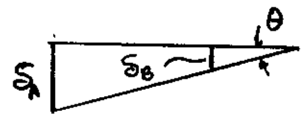
2.122 Solve Prob. 2.121, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

SOLUTION



Statics: $\Sigma M_C = 0 \quad 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = 1.76\,\theta$



Elastic Analysis

$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times10^{-6}\ \text{m}^2$

$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$

$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$

$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B = (45\times10^6)(1.76\,\theta)$

$= 79.2\times10^6\,\theta \qquad \sigma_{BE} = \frac{P_{BE}}{A} = 352\times10^9\,\theta$

From Statics $\quad Q = P_{BE} + \frac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$= [73.8\times10^6 + (1.500)(69.88\times10^6)]\,\theta = 178.62\times10^6\,\theta$

θ_Y at yielding of link BE $\quad \sigma_{BE} = \sigma_Y = 250\times10^6 = 352\times10^9\,\theta_Y$

$\theta_Y = 710.23\times10^{-6}$

$Q_Y = (178.62\times10^6)(710.23\times10^{-6}) = 126.86\times10^3$ N

Since $Q = 135\times10^3\,N > Q_Y$, link BE yields $\sigma_{BE} = \sigma_Y = 250$ MPa ◄

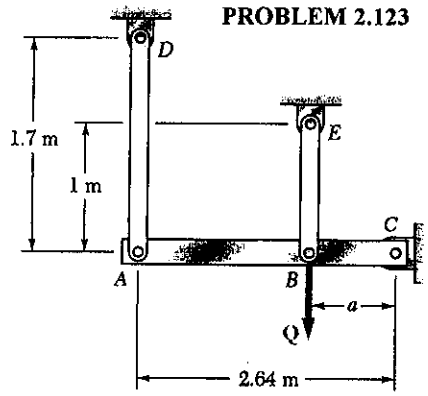
$P_{BE} = A\,\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3$ N

From Statics $\quad P_{AD} = \frac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3$ N

$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6 = 233$ MPa ◄

From elastic analysis of AD $\quad \theta = \frac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-5}$ rad

$\delta_B = 1.76\,\theta = 1.322\times10^{-3}$ m $= 1.322$ mm ◄



PROBLEM 2.123

2.121 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the ~~maximum~~ final deflection of point B.

***2.123** Solve Prob. 2.121, assuming that the magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point B the residual stress in each link. Assume that the links are braced so that they can carry compressive forces without buckling.

SOLUTION

See solution to PROBLEM 2.121 for the normal stresses in each link and the deflection of point B after loading

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa} \qquad \sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$

$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to PROBLEM 2.121 applies to the unloading

$$Q = 317.06 \times 10^6 \, \theta'$$

$$\theta' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} \doteq 310.6 \times 10^9 \, \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$

$$\sigma'_{BE} = 128 \times 10^9 \, \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$

$$\delta'_B = 0.640 \, \theta' = 524.82 \times 10^{-6} \text{ m}$$

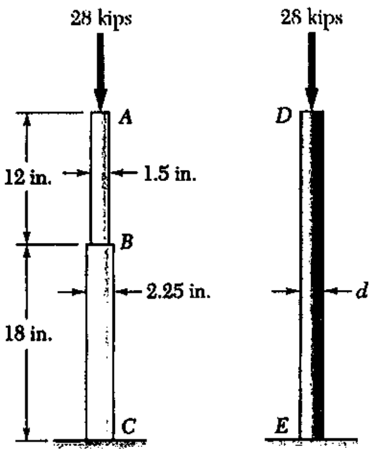
(a) Residual stresses

$$\sigma_{AD,res} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa}$$
$$= -4.70 \text{ MPa}$$

$$\sigma_{BE,res} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa}$$
$$= 19.34 \text{ MPa}$$

(b) $$\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6}$$
$$= 96.71 \times 10^{-6} \text{ m} = 0.0967 \text{ mm}$$

PROBLEM 2.124



2.124 The aluminum rod *ABC* ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions *AB* and *BC*, is to be replaced with a cylindrical steel rod *DE* ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter *d* of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28 \times 10^3}{10.1 \times 10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right)^2 = 0.031376 \text{ in}$$

Steel rod $\qquad \delta = 0.031376$ in

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28 \times 10^3)(30)}{(29 \times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{24 \times 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value $\qquad A = 1.1667 \text{ in}^2$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.6667)}{\pi}} = 1.219 \text{ in.}$$

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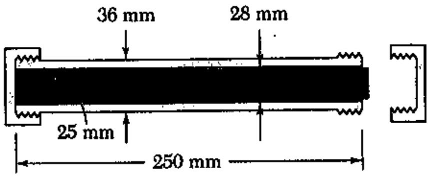
PROBLEM 2.125



2.125 A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E = 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12\times10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87\times10^{-6} \text{ m}^2$$

$$\delta_{tube} = \frac{PL}{E_{tube}A_{tube}} = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} = 8.8815\times10^{-9} P$$

$$\delta_{rod} = -\frac{PL}{E_{rod}A_{rod}} = \frac{P(0.250)}{(105\times10^6)(490.87\times10^{-6})} = -4.8505\times10^{-9} P$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375\times10^{-6} \text{ m}$$

$$\delta_{tube} = \delta^* + \delta_{rod} \quad \text{or} \quad \delta_{tube} - \delta_{rod} = \delta^*$$

$$8.8815\times10^{-9} P + 4.8505\times10^{-9} P = 375 \times10^{-6}$$

$$P = \frac{0.375\times10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308\times10^3 \text{ N}$$

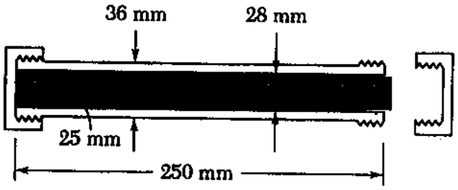
(a) $$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{27.308\cdot10^3}{402.12\times10^{-6}} = 67.9\times10^6 \text{ Pa} = 67.9 \text{ MPa}$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{27.308\times10^3}{490.87\times10^{-6}} = -55.6\times10^6 \text{ Pa} = -55.6 \text{ MPa}$$

(b) $$\delta_{tube} = (8.8815\times10^{-9})(27.308\times10^3) = 242.5\times10^{-6} \text{ m} = 0.2425 \text{ mm}$$

$$\delta_{rod} = -(4.8505\times10^{-9})(27.308\times10^3) = -132.5\times10^{-6} \text{ m} = -0.1325 \text{ mm}$$

PROBLEM 2.126



2.125 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

2.126 In Prob. 2.125, determine the average normal stress in the tube and the rod, assuming that the temperature was 15° C when the nuts were snugly fitted and that the final temperature is 55° C. (For aluminum, $\alpha = 23.6 \times 10^{6}/°C$; for brass, $\alpha = 20.9 \times 10^{6}/°C$.)

SOLUTION

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\Delta T = 55 - 15 = 40 \text{ °C}$$

$$\delta_{tube} = \frac{PL}{E_{tube} A_{tube}} + L\alpha_{tube}(\Delta T) = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} + (0.250)(23.6 \times 10^{-6})(40)$$

$$= 8.8815 \times 10^{-9} P + 236 \times 10^{-6}$$

$$\delta_{rod} = -\frac{PL}{E_{rod} A_{rod}} + L\alpha_{rod}(\Delta T) = -\frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} + (0.250)(20.9 \times 10^{-6})(40)$$

$$= -4.8505 \times 10^{-9} P + 209 \times 10^{-6}$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{tube} = \delta_{rod} + \delta^*$$

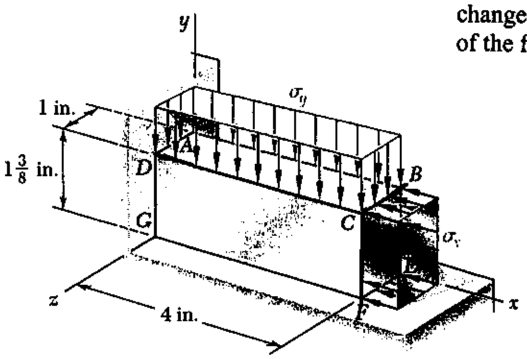
$$8.8815 \times 10^{-9} P + 236 \times 10^{-6} = -4.8505 \times 10^{-9} P + 209 \times 10^{-6} + 375 \times 10^{-6}$$

$$13.732 \times 10^{-9} P = 348 \times 10^{-6} \qquad P = 25.342 \times 10^3 \text{ N}$$

$$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{25.342 \times 10^3}{402.12 \times 10^{-6}} = 63.0 \times 10^6 \text{ Pa} = 63.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{25.342 \times 10^3}{490.87 \times 10^{-6}} = -51.6 \times 10^6 \text{ Pa} = -51.6 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.127



2.127 The block shown is made of a magnesium alloy for which $E = 6.5 \times 10^6$ psi and $\nu = 0.35$. Knowing that $\sigma_x = -20$ ksi, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

$$\delta_y = 0 \qquad \varepsilon_y = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

(a) $\sigma_y = \nu\sigma_x = (0.35)(-20\times10^3)$

$= -7\times10^3 \text{psi} = -7 \text{ ksi}$

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu(\sigma_x+\sigma_y)}{E}$

$= \frac{(0.35)(-20\times10^3 - 7\times10^3)}{6.5\times10^6} = 1.4538\times10^{-3}$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{-20\times10^3 - (0.35)(-7\times10^3)}{6.5\times10^6}$

$= -2.7\times10^{-3}$

$A_0 + \Delta A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_xL_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$

But $A_0 = L_xL_z$

$\Delta A = L_xL_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$

$= (4.0)(1.0)(1.4538\times10^{-3} - 2.7\times10^{-3} + \text{small term})$

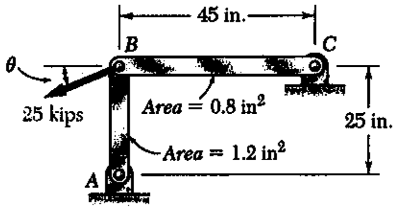
$= -4.98\times10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2$

(c) Since L_y is constant

$\Delta V = L_y(\Delta A) = (1.375)(-4.98\times10^{-3}) = -6.85\times10^{-3} \text{ in}^3$

$= -0.00685 \text{ in}^3$

PROBLEM 2.128

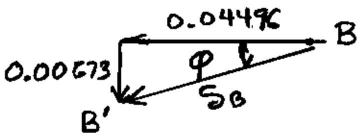


2.128 The uniform rods *AB* and *BC* are made of steel and are loaded as shown. Knowing that $E = 29 \times 10^6$ psi, determine the magnitude and direction of the deflection of point *B* when $\theta = 22°$.

SOLUTION

$$P_{BC} = P\cos\theta = (25\times10^3)\cos 22° = 23.18\times10^3 \text{ lb.}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E\,A_{BC}} = \frac{(23.18\times10^3)(45)}{(29\times10^3)(0.8)} = 0.04496 \text{ in}$$



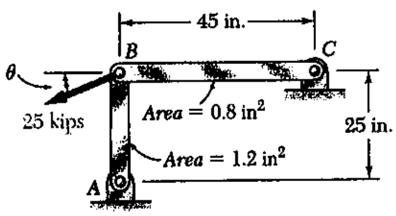
$$P_{AB} = P\sin\theta = (25\times10^3)\sin 22° = 9.365\times10^3 \text{ lb.}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E\,A_{AB}} = \frac{(9.365\times10^3)(25)}{(29\times10^6)(1.2)} = 0.00673 \text{ in.}$$

$$\tan\varphi = \frac{0.00673}{0.04496} = 0.1496 \qquad \varphi = 8.51° \ ◄$$

$$\delta = \sqrt{0.04496^2 + 0.00673^2} = 0.0455 \text{ in.} \ ◄$$

PROBLEM 2.129



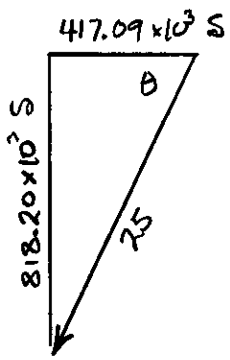
2.129 Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the value of θ for which the deflection of point *B* is down and to the left along a line forming an angle of 36° with the horizontal, (*b*) the corresponding magnitude of the deflection of *B*.

SOLUTION

$$\delta_{BC} = \delta\cos 36°$$

$$P_{BC} = \frac{E\,A_{BC}\,\delta_{BC}}{L_{BC}} = \frac{(29\times10^6)(0.8)\,\delta\cos 36°}{45}$$

$$= 417.09\times10^3\ \delta$$



$$\delta_{AB} = \delta\sin 36°$$

$$P_{AC} = \frac{E\,A_{AC}\,\delta_{AC}}{L_{AC}} = \frac{(29\times10^6)(1.2)\,\delta\sin 36°}{25}$$

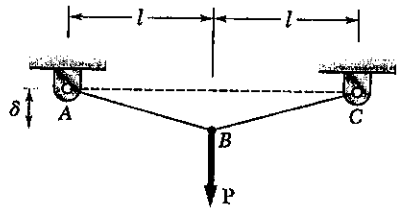
$$= 818.20\times10^3\ \delta$$

$$\tan\theta = \frac{818.20\times10^3\ \delta}{417.09\times10^3\ \delta} = 1.9617 \qquad \theta = 63.0° \ ◄$$

$$P = 25\times10^3 = \sqrt{(417.09\times10^3\ \delta)^2 + (818.20\times10^{-3}\ \delta)^2} = 918.38\times10^3\ \delta$$

$$\delta = \frac{25\times10^3}{918.38\times10^3} = 0.0272 \text{ in.} \ ◄$$

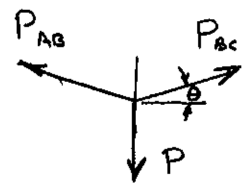
PROBLEM 2.130



2.130 The uniform wire ABC, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint B. Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta << l$, the deflection at the midpoint B is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

SOLUTION



Use approximation

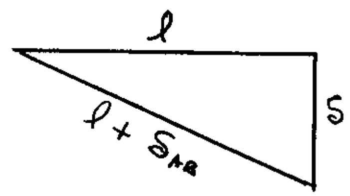
$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

Statics $\quad \Sigma F_y = 0 \qquad 2P_{AB}\sin\theta - P = 0$

$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation $\quad \delta_{AB} = \dfrac{P_{AB}\,l}{AE} = \dfrac{Pl^2}{2AE\delta}$

Deflection



From the right triangle

$$(l + \delta_{AB})^2 = l^2 + \delta^2$$

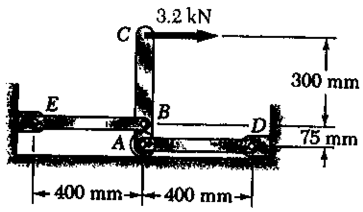
$$\delta^2 = \cancel{l^2} + 2l\,\delta_{AB} + \delta_{AB}^2 - \cancel{l^2}$$

$$= 2l\,\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\,\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

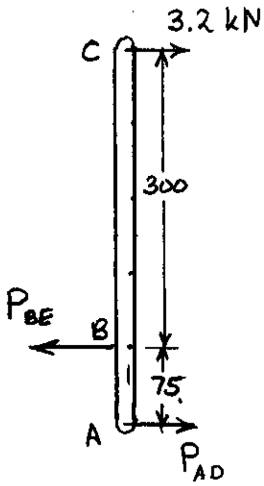
$$\delta^3 \approx \frac{Pl^3}{AE} \qquad \therefore \ \delta \approx l\sqrt[3]{\frac{P}{AE}}$$

PROBLEM 2.131



2.131 The steel bars *BE* and *AD* each have a 6×18-mm cross section. Knowing that $E = 200$ GPa, determine the deflections of points *A*, *B*, and *C* of the rigid bar *ABC*.

SOLUTION



Use rigid bar ABC as a free body

$\circlearrowleft +\ \Sigma M_B = 0 \qquad (75) P_{AD} - (300)(3.2) = 0$

$P_{AD} = 12.8 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -P_{BE} + 3.2 + P_{AD} = 0$

$P_{BE} = 16 \text{ kN}$

Deformations

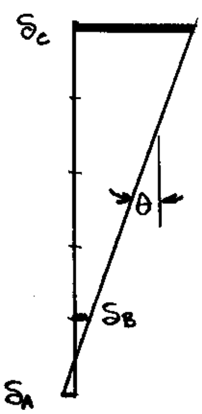
$A = (6)(18) = 108 \text{ mm}^2 = 108 \times 10^{-6} \text{ m}^2$

$\leftarrow \delta_A = \delta_{AD} = \dfrac{P_{AD} L_{AD}}{EA} = \dfrac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})}$

$= 237.04 \times 10^{-6} \text{ m} = 0.237 \text{ mm} \leftarrow$ ◀

$\leftarrow \delta_B = \delta_{BE} = \dfrac{P_{BE} L_{BE}}{EA} = \dfrac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})}$

$= 296.30 \times 10^{-6} \text{ m} = 0.296 \text{ mm} \rightarrow$ ◀



$\theta = \dfrac{\delta_A + \delta_B}{L_{AB}} = \dfrac{(237.04 + 296.30)(10^{-6})}{75 \times 10^{-3}}$

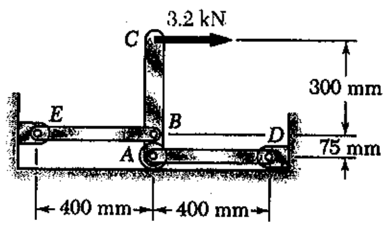
$= 7.1112 \times 10^{-3}$

$\delta_C = \delta_B + L_{BC}\,\theta$

$= 296.30 \times 10^{-6} + (300 \times 10^{-3})(7.1112 \times 10^{-3})$

$= 2.4297 \times 10^{-3} \text{ m} = 2.43 \text{ mm} \rightarrow$ ◀

PROBLEM 2.132

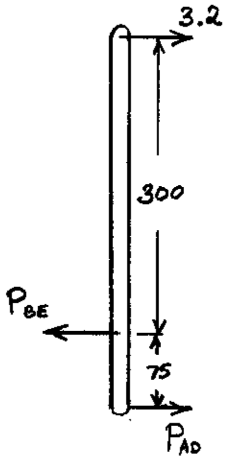


2.131 The steel bars BE and AD each have a 6×18-mm cross section. Knowing that $E = 200$ GPa, determine the deflections of points A, B, and C of the rigid bar ABC.

2.132 In Prob. 2.131, the 3.2-kN force caused point C to deflect to the right. Using $\alpha = 11.7 \times 10^{6}/°C$, determine the ($a$) the overall change in temperature that causes point C to return to its original position, (b) the corresponding total deflection of points A and B.

SOLUTION

Use rigid ABC as a free body



$$\circlearrowleft \Sigma M_B = 0 \qquad 75\,P_{AD} - (300)(3.2) = 0$$

$$P_{AD} = 12.8 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0 \qquad -P_{BE} + 3.2 + P_{AD} = 0$$

$$P_{BE} = 16 \text{ kN}$$

Deformations:

$$\leftarrow \delta_A = \delta_{AD} = \frac{P_{AD} L_{AD}}{EA} + L_{AD}\alpha(\Delta T)$$

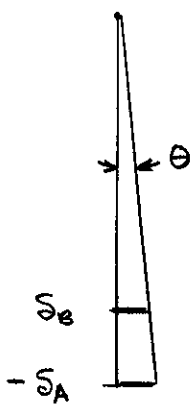
$$= \frac{(12.8\times10^3)(400\times10^{-3})}{(200\times10^9)(108\times10^{-6})} + (400\times10^{-3})(11.7\times10^{-6})(\Delta T)$$

$$= 237.04\times10^{-6} + 4.68\times10^{-6}(\Delta T)$$

$$\rightarrow \delta_B = \delta_{BE} = \frac{P_{BE} L_{BE}}{EA} + L_{BE}\alpha(\Delta T)$$

$$= \frac{(16\times10^3)(400\times10^{-3})}{(200\times10^9)(108\times10^{-6})} + (400\times10^{-3})(11.7\times10^{-6})(\Delta T)$$

$$= 296.30\times10^{-6} + 4.68\times10^{-6}(\Delta T)$$



$$\delta_C = 0 \qquad \delta_B = 0.300\,\theta \qquad -\delta_A = 0.375\,\theta$$

$$-\delta_A = \frac{0.375}{0.300}\,\delta_A = 1.25\,\delta_A$$

$$-(237.04\times10^{-6} + 4.68\times10^{-6}(\Delta T)) = (1.25)[296.30\times10^{-6} + 4.68\times10^{-6}(\Delta T)]$$

$$-10.53\times10^{-6}(\Delta T) = 607.415\times10^{-6} \quad \ldots \quad \Delta T = -57.684\ C$$

$$= -57.7\,°C$$

$$\delta_A = 237.04\times10^{-6} + (4.68\times10^{-6})(-57.684) = -32.92\times10^{-6}\text{ m}$$

$$\delta_A = 0.0329 \text{ mm} \rightarrow$$

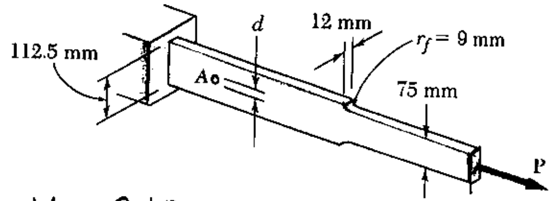
$$\delta_B = 296.30\times10^{-6} - (4.68\times10^{-6})(57.684) = +26.34\times10^{-6}\text{ m}$$

$$\delta_B = 0.0263 \text{ mm} \rightarrow$$

PROBLEM 2.133

2.133 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from 9 to 27 mm in 6-mm increments. (*a*) Determine the diameter d of the largest bit that can be used if the allowable load at the hole is not to exceed that at the fillets. (*b*) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load **P**?

SOLUTION



At the fillets, $r = 9$ mm $\quad d = 75$ mm

$D = 112.5$ mm $\qquad \frac{D}{d} = \frac{112.5}{75} = 1.5$

$\frac{r}{d} = \frac{9}{75} = 0.12 \qquad$ From Fig 2.64 b $\qquad K = 2.10$

$A_{min} = (75)(12) = 900\ mm^2 = 900\ mm^2 = 900 \times 10^{-6}\ m^2$

$$\sigma_{max} = K\frac{P_{all}}{A_{min}} = \sigma_{all} \quad \therefore \quad P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(900\times10^{-6})(145\times10^{6})}{2.10}$$

$$= 62.1\times10^{3}\ N = 62.1\ kN$$

At the hole: $\quad A_{net} = (D - 2r)t, \qquad \frac{r}{d} = \frac{r}{D-2r}$

where $D = 112.5$ mm $\qquad r =$ radius of circle $\qquad t = 12$ mm

K is taken from Fig 2.64 a

$$\sigma_{max} = K\frac{P}{A_{net}} = \sigma_{all} \quad \therefore \quad P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

Hole diam	r	$d = D - 2r$	r/d	K	A_{net}	P_{all}
9 mm	4.5 mm	103.5 mm	0.0435	2.87	$1242\times10^{-6}\ m^2$	62.7×10^{3} N
15 mm	7.5 mm	97.5 mm	0.077	2.75	$1170\times10^{-6}\ m^2$	61.7×10^{3} N
21 mm	10.5 mm	91.5 mm	0.115	2.67	$1098\times10^{-6}\ m^2$	59.6×10^{3} N
27 mm	13.5 mm	85.5 mm	0.158	2.57	$1026\times10^{-6}\ m^2$	57.9×10^{6} N

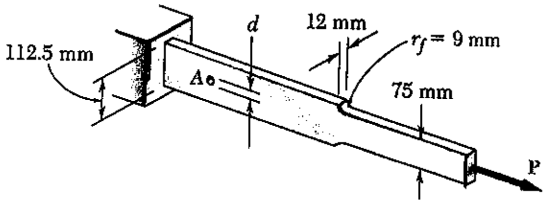
Largest hole with $P_{all} > 62$ kN is the 9 mm diameter hole. ◄

Allowable force $\quad P_{all} = 62$ kN ◄

PROBLEM 2.134

2.134 (*a*) For $P = 58$ kN and $d = 12$ mm, determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION



Maximum stress at hole

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \qquad K = 2.80$$

$$A_{net} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \qquad \frac{D}{d} = \frac{112.5}{75} = 1.50, \qquad K = 2.10$$

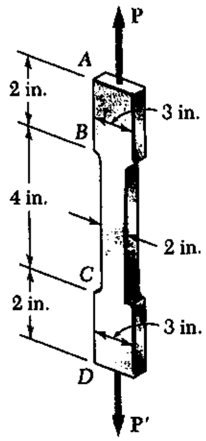
$$A_{min} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets $\qquad \sigma_{max} = 134.7$ MPa ◄

(b) Without hole $\qquad \sigma_{max} = 135.3$ MPa ◄

PROBLEM 2.135



2.135 The steel tensile specimen *ABCD* ($E = 29 \times 10^6$ psi and $\sigma_y = 50$ ksi) is loaded in tension until the maximum tensile strain is $\epsilon = 0.0025$. (*a*) Neglecting the effect of the fillets on the change in length of the specimen, determine the resulting overall length *AD* of the specimen after the load is removed. (*b*) Following the removal of the load in part (*b*), a compressive load is applied until the maximum compressive strain is $\epsilon = 0.0020$. Determine the resulting overall length *AD* after the load is removed.

SOLUTION

(a) $\varepsilon_Y = \dfrac{\sigma_Y}{E} = \dfrac{50 \times 10^3}{29 \times 10^6} = 0.001724$

$\varepsilon_{max} = 0.0025 > \varepsilon_Y$ Yielding occurs in portion BC

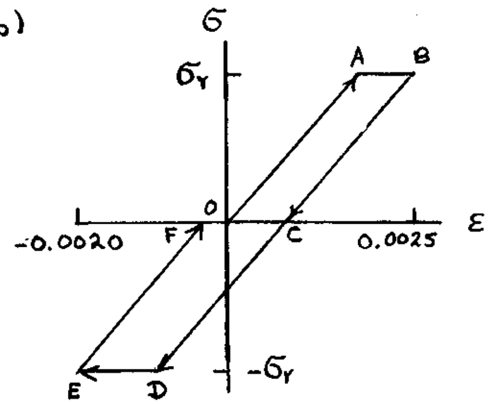
$\sigma_{BC} = \sigma_Y = 50 \times 10^3$ psi

Permanent strain in BC

$\varepsilon_{BC} = \varepsilon_{max} - \varepsilon_Y = 0.0025 - 0.001724 = 0.000776$

$\delta_{BC} = L_{BC}\,\varepsilon_B = (4)(0.000776) = 0.00310$ in. ◀

(b)



In reversed loading, at point E on stress-strain plot

$\varepsilon = -0.0020$

as given. During removal of the reversed load, the change in strain is $\sigma_Y / E = 0.001724$.

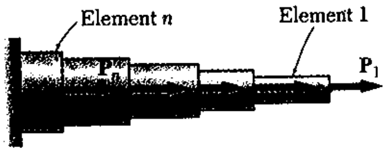
The permanent strain in BC is

$\varepsilon_{BC} = -0.0020 + 0.001724 = 0.000276$

$\delta_{BC} = L\,\varepsilon_{BC} = (4)(-0.000276) = -0.001104$ in. ◀

Note that portions AB and CD are always elastic, thus their deformations during loading and unloading do not contribute to any permanent deformation.

PROBLEM 2.C1



2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.17 and 2.18.

SOLUTION

FOR EACH ELEMENT, ENTER

L_i, A_i, E_i

COMPUTE DEFORMATION

UPDATE AXIAL LOAD $P = P + P_i$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_iE_i$$

TOTAL DEFORMATION:

UPDATE THROUGH n ELEMENTS

$$\delta = \delta + \delta_i$$

PROGRAM OUTPUT

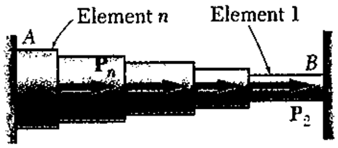
```
Problem  2.17
   Element  Stress (MPa)  Deformation (mm)

      1       19.0986          .1091
      2      -12.7324         -.0909
   Total Deformation =       .0182 mm

Problem  2.18
   Element  Stress (MPa)  Deformation (mm)

      1       98.2438         2.3391
      2       98.2438         1.4737
      3      147.3657         1.4737
   Total Deformation =      5.2865 mm
```

PROBLEM 2.C2



2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program that can be used to determine the reactions at A and B, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Prob. 2.41.

SOLUTION

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE δ_B WITH $R_B = 0$

FOR EACH ELEMENT, ENTER

L_i, A_i, E_i

UPDATE AXIAL LOAD

$P = P + P_i$

COMPUTE FOR EACH ELEMENT

$\sigma_i = P/A_i$

$\delta_i = PL_i/A_iE_i$

UPDATE TOTAL DEFORMATION

$\delta_B = \delta_B + \delta_i$

COMPUTE δ_B DUE TO UNIT LOAD AT B

UNIT $\sigma_i = 1/A_i$

UNIT $\delta_i = L_i/A_iE_i$

UPDATE TOTAL UNIT DEFORMATION

UNIT δ_B = UNIT δ_B + UNIT δ_i

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT B = ZERO

$\delta_B + R_B$ UNIT $\delta_B = 0$

SOLVING:

$R_B = -\delta_B/$UNIT δ_B

THEN:

$R_A = \Sigma P_i + R_B$

CONTINUED

PROBLEM 2.C2 CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

```
Problem  2.41
     RA =   -11.909 kips
     RB =   -20.091 kips
   Element  Stress (ksi)   Deformation (in.)

      1        12.002           -.00923
      2        -6.128           -.00589
      3        -9.687           -.00334
```

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PROBLEM 2.C3

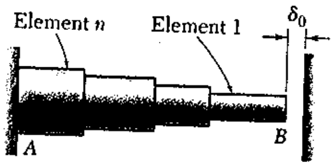


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program that can be used to determine the magnitude of the reactions at A and B, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.53, 2.54, 2.57, and 2.59.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B ASSUMING THERE IS NO SUPPORT AT B:

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = -R_B$$

FOR EACH ELEMENT

$$\sigma_i = -R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

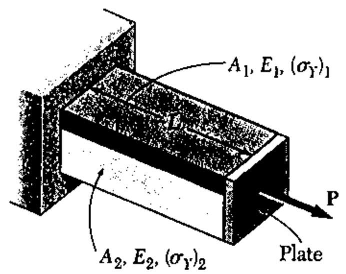
CONTINUED

PROBLEM 2.C3 CONTINUED

PROGRAM OUTPUT

```
Problem  2.53
 R =      25.837 kips

  Element     Stress (ksi)    Deform.(10*-3 in.)

     1          -21.054         -3.642
     2           -6.498          3.642

Problem  2.54
 R =     125.628 kN

  Element     Stress (MPa)    Deform.(microm)

     1          -44.432        500.104
     2          -99.972       -500.104

Problem  2.57
 R =     217.465 kN

  Element     Stress (MPa)    Deform.(microm)

     1         -144.977        242.504
     2         -120.814        257.496

Problem  2.59
 R =      61.857 kips

  Element     Stress (ksi)    Deform.(10*-3 in.)

     1          -22.092         14.410
     2          -51.547          5.590
```

PROBLEM 2.C4



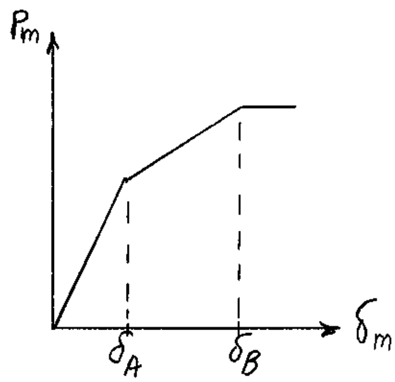
2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.109, 2.111, and 2.112.

SOLUTION

NOTE: THE FOLLOWING ASSUMES $(\sigma_Y)_1 < (\sigma_Y)_2$

DISPLACEMENT INCREMENT

$$\delta_m = 0.05\,(\sigma_Y)_2 L/E_2$$



DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1 L/E_1 \qquad \delta_B = (\sigma_Y)_2 L/E_2$$

FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$

IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)A_2E_2$$

IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1 \qquad \sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

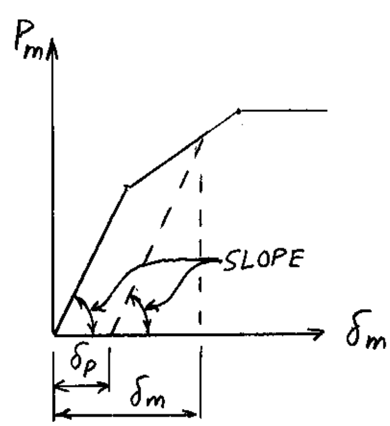
SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1E_1 + A_2E_2)/L$$

$$\delta_p = \delta_m - (P_m/\text{SLOPE})$$

$$(\sigma_1)_{res} = \sigma_1 - (E_1P_m/(L\ \text{SLOPE}))$$

$$(\sigma_2)_{res} = \sigma_2 - (E_2P_m/(L\ \text{SLOPE}))$$



CONTINUED

PROBLEM 2.C4 CONTINUED

PROGRAM OUTPUT

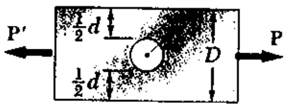
Problem 2.109

DM MUm	PM kN	SIGM(1) MPa	SIGM(2) MPa	DP MUm	SIGR(1) MPa	SIG(2) MPa
.000	.000	.000	.000	.000	.000	.000
16.387	60.375	17.250	17.250	.000	.000	.000
32.775	120.750	34.500	34.500	.000	.000	.000
49.162	181.125	51.750	51.750	.000	.000	.000
65.550	241.500	69.000	69.000	.000	.000	.000
81.938	301.875	86.250	86.250	.000	.000	.000
98.325	362.250	103.500	103.500	.000	.000	.000
114.713	422.625	120.750	120.750	.000	.000	.000
131.100	483.000	138.000	138.000	.000	.000	.000
147.487	543.375	155.250	155.250	.000	.000	.000
163.875	603.750	172.500	172.500	.000	.000	.000
180.262	664.125	189.750	189.750	.000	.000	.000
196.650	724.500	207.000	207.000	.000	.000	.000
213.037	784.875	224.250	224.250	.000	.000	.000
229.425	845.250	241.500	241.500	.000	.000	.000
245.812	890.312	250.000	258.750	4.156	-4.375	4.375
262.200	920.500	250.000	276.000	12.350	-13.000	13.000
278.587	950.687	250.000	293.250	20.544	-21.625	21.625
294.975	980.875	250.000	310.500	28.737	-30.250	30.250
311.362	1011.062	250.000	327.750	36.931	-38.875	38.875
327.750	1041.250	250.000	345.000	45.125	-47.500	47.500
344.137	1041.250	250.000	345.000	61.512	-47.500	47.500
360.525	1041.250	250.000	345.000	77.900	-47.500	47.500
376.912	1041.250	250.000	345.000	94.287	-47.500	47.500
393.300	1041.250	250.000	345.000	110.675	-47.500	47.500

Problems 2.111 and 2.112

DM 10**-3 in.	PM kips	SIGM(1) ksi	SIGM(2) ksi	DP 10**-3 in.	SIGR(1) ksi	SIG(2) ksi
.000	.000	.000	.000	.000	.000	.000
2.414	8.750	5.000	5.000	.000	.000	.000
4.828	17.500	10.000	10.000	.000	.000	.000
7.241	26.250	15.000	15.000	.000	.000	.000
9.655	35.000	20.000	20.000	.000	.000	.000
12.069	43.750	25.000	25.000	.000	.000	.000
14.483	52.500	30.000	30.000	.000	.000	.000
16.897	61.250	35.000	35.000	.000	.000	.000
19.310	70.000	40.000	40.000	.000	.000	.000
21.724	78.750	45.000	45.000	.000	.000	.000
24.138	87.500	50.000	50.000	.000	.000	.000
26.552	91.250	50.000	55.000	1.379	-2.143	2.857
28.966	95.000	50.000	60.000	2.759	-4.286	5.714
31.379	98.750	50.000	65.000	4.138	-6.429	8.571
33.793	102.500	50.000	70.000	5.517	-8.571	11.429
36.207	106.250	50.000	75.000	6.897	-10.714	14.286
38.621	110.000	50.000	80.000	8.276	-12.857	17.143
41.034	113.750	50.000	85.000	9.655	-15.000	20.000
43.448	117.500	50.000	90.000	11.034	-17.143	22.857
45.862	121.250	50.000	95.000	12.414	-19.286	25.714
48.276	125.000	50.000	100.000	13.793	-21.429	28.571
50.690	125.000	50.000	100.000	16.207	-21.429	28.571
53.103	125.000	50.000	100.000	18.621	-21.429	28.571
55.517	125.000	50.000	100.000	21.034	-21.429	28.571
57.931	125.000	50.000	100.000	23.448	-21.429	28.571

PROBLEM 2.C5



2.C5 The stress concentration factor for a flat bar with a centric hole under axial loading can be expressed as:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. (*a*) Write a computer program that can be used to determine the allowable load P for given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. (*b*) Use this program to solve Prob. 2.94.

SOLUTION

ENTER

$r,\ D,\ t,\ \sigma_{all}$

COMPUTE K

$RD = 2.0\ r/D$

$K = 3.00 - 3.13\ RD + 3.66\ RD^2 - 1.53\ RD^3$

COMPUTE AVERAGE STRESS

$\sigma_{ave} = \sigma_{all}/K$

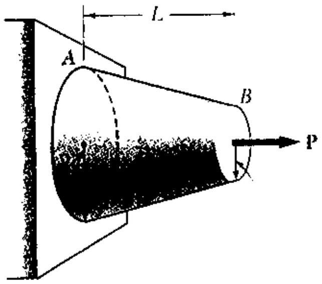
ALLOWABLE LOAD

$P_{all} = \sigma_{ave}\ (D - 2.0\,r)\,t$

PROGRAM OUTPUT

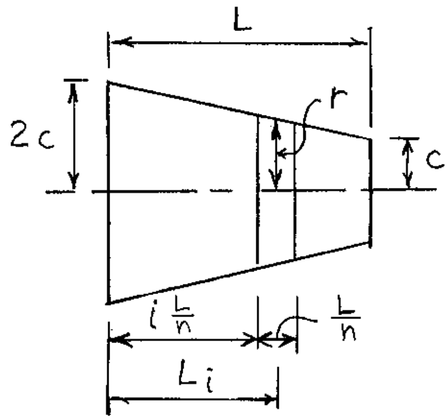
```
Problem 2.94

    Hole at A:

           K =   2.573     P =     7.773 Kips

    Hole at B:

           K =   2.159     P =     5.559 Kips
```

PROBLEM 2.C6



2.C6 A solid truncated cone is subjected to an axial force **P** as shown. Write a computer program that can be used to obtain an approximation of the elongation of the cone by replacing it by n circular cylinders of equal thickness and of radius equal to the mean radius of the portion of cone they replace. Knowing that the exact value of the elongation of the cone is $(PL)/(2\pi c^2 E)$ and using for P, L, c, and E values of your choice, determine the percentage error involved when the program is used with (*a*) $n = 6$, (*b*) $n = 12$, (*c*) $n = 60$.

SOLUTION



FOR $i = 1$ TO n:

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$

AREA:

$$A = \pi r_i^2$$

DISPLACEMENT:

$$\delta = \delta + P(L/n)/(AE)$$

EXACT DISPLACEMENT:

$$\delta_{EXACT} = PL/(2.0\pi c^2 E)$$

PERCENTAGE ERROR:

$$PERCENT = 100(\delta - \delta_{EXACT})/\delta_{EXACT}$$

PROGRAM OUTPUT

n	Approximate	Exact	Percent
6	0.15852	0.15915	-.40083
12	0.15899	0.15915	-.10100
60	0.15915	0.15915	-.00405