تعداد سؤالات: تستی: **تشریحی:** ۵ **زمان آزمون (دقیقه): تستی:** **تشریحی:** ۱۲۰ **کُد سری سؤال:** یک ۱

نام درس: مقاومت مصالح۱

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بارم هر سوال ۲/۸۰ می باشد.

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$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{50-10}{200} * 10^3 = 200MPa$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{50}{200} * 10^3 = 250MPa$$

$$\delta_B = \frac{F_{BC} L_{BC}}{A_{BC} E}$$

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SOLUTION

Hooke's Law. We note that $\sigma_y = 0$. Using Eqs. (2.28) we find the strain in each of the coordinate directions.

$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{10 \times 10^6 \text{ psi}}\left[(12 \text{ ksi}) - 0 - \frac{1}{3}(20 \text{ ksi})\right] = +0.533 \times 10^{-3} \text{ in./in.}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{10 \times 10^6 \text{ psi}}\left[-\frac{1}{3}(12 \text{ ksi}) + 0 - \frac{1}{3}(20 \text{ ksi})\right] = -1.067 \times 10^{-3} \text{ in./in.}$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= \frac{1}{10 \times 10^6 \text{ psi}}\left[-\frac{1}{3}(12 \text{ ksi}) - 0 + (20 \text{ ksi})\right] = +1.600 \times 10^{-3} \text{ in./in.}$$

a. Diameter *AB*. The change in length is $\delta_{B/A} = \epsilon_x d$.

$$\delta_{B/A} = \epsilon_x d = (+0.533 \times 10^{-3} \text{ in./in.})(9 \text{ in.})$$

$$\delta_{B/A} = +4.8 \times 10^{-3} \text{ in.}$$

b. Diameter *CD*.

$$\delta_{C/D} = \epsilon_z d = (+1.600 \times 10^{-3} \text{ in./in.})(9 \text{ in.})$$

$$\delta_{C/D} = +14.4 \times 10^{-3} \text{ in.}$$

c. Thickness. Recalling that $t = \frac{3}{4}$ in., we have

$$\delta_t = \epsilon_y t = (-1.067 \times 10^{-3} \text{ in./in.})(\tfrac{3}{4} \text{ in.})$$

$$\delta_t = -0.800 \times 10^{-3} \text{ in.}$$

d. Volume of the Plate. Using Eq. (2.30), we write

$$e = \epsilon_x + \epsilon_y + \epsilon_z = (+0.533 - 1.067 + 1.600)10^{-3} = +1.067 \times 10^{-3}$$

$$\Delta V = eV = +1.067 \times 10^{-3}[(15 \text{ in.})(15 \text{ in.})(\tfrac{3}{4} \text{ in.})]\Delta V = +0.187 \times \text{in}^3$$

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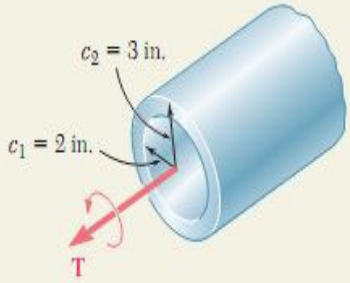
SOLUTION

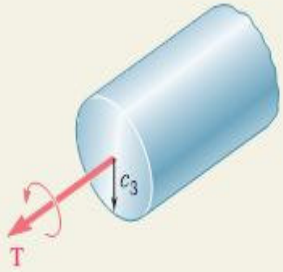


a. Hollow Shaft as Designed. For the hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(3\text{ in.})^4 - (2\text{ in.})^4] = 102.1\text{ in}^4$$

Using Eq. (3.9), we write

$$\tau_{max} = \frac{Tc_2}{J} \qquad 12\text{ ksi} = \frac{T(3\text{ in.})}{102.1\text{ in}^4} \qquad T = 408\text{ kip}\cdot\text{in.}$$



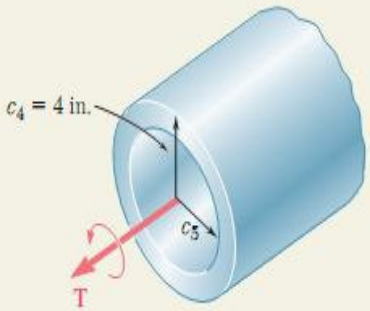
b. Solid Shaft of Equal Weight. For the shaft as designed and this solid shaft to have the same weight and length, their cross-sectional areas must be equal.

$$A_{(a)} = A_{(b)}$$

$$\pi[(3\text{ in.})^2 - (2\text{ in.})^2] = \pi c_3^2 \qquad c_3 = 2.24\text{ in.}$$

Since τ_{all} = 12 ksi, we write

$$\tau_{max} = \frac{Tc_3}{J} \qquad 12\text{ ksi} = \frac{T(2.24\text{ in.})}{\frac{\pi}{2}(2.24\text{ in.})^4} \qquad T = 211\text{ kip}\cdot\text{in.}$$



c. Hollow Shaft of 8-in. Diameter. For equal weight, the cross-sectional areas again must be equal. We determine the inside diameter of the shaft by writing

$$A_{(a)} = A_{(c)}$$

$$\pi[(3\text{ in.})^2 - (2\text{ in.})^2] = \pi[(4\text{ in.})^2 - c_5^2] \qquad c_5 = 3.317\text{ in.}$$

For c_5 = 3.317 in. and c_4 = 4 in.,

$$J = \frac{\pi}{2}[(4\text{ in.})^4 - (3.317\text{ in.})^4] = 212\text{ in}^4$$

With τ_{all} = 12 ksi and c_4 = 4 in.,

$$\tau_{max} = \frac{Tc_4}{J} \qquad 12\text{ ksi} = \frac{T(4\text{ in.})}{212\text{ in}^4} \qquad T = 636\text{ kip}\cdot\text{in.}$$

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	A, mm^2	$\bar{y}_o, mm$	$A\bar{y}_o, mm^3$
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3

$$\bar{Y}_o = \frac{15.5\times10^3}{900} = 17.5\ mm$$

The neutral axis lies 17.5mm above the bottom.

$y_{top} = 30-17.5 = 12.5\ mm = 0.0125\ m$, $y_{bot} = -17.5\ mm = -0.0175\ m$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25\times10^3\ mm^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625\times10^3\ mm^4$$

$$I = I_1 + I_2 = 61.875\times10^3\ mm^4 = 61.875\times10^{-9}\ m^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: tension side $\quad M = \dfrac{(24\times10^6)(61.875\times10^{-9})}{0.0125} = 118.8\ N\cdot m$

Bottom: compression $\quad M = \dfrac{(30\times10^6)(61.875\times10^{-9})}{0.0175} = 106.1\ N\cdot m$

Choose smaller value $\qquad M = 106.1\ N\cdot m$

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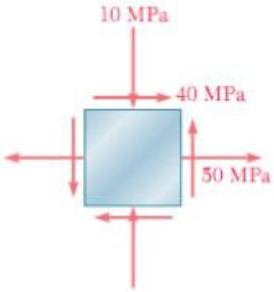

Fig. 7.11

(a) Principal Planes. Following the usual sign convention, we write the stress components as

$$\sigma_x = +50 \text{ MPa} \qquad \sigma_y = -10 \text{ MPa} \qquad \tau_{xy} = +40 \text{ MPa}$$

Substituting into Eq. (7.12), we have

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(+40)}{50-(-10)} = \frac{80}{60}$$

$$2\theta_p = 53.1° \quad \text{and} \quad 180° + 53.1° = 233.1°$$

$$\theta_p = 26.6° \quad \text{and} \quad 116.6°$$

(b) Principal Stresses. Formula (7.14) yields

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 20 \pm \sqrt{(30)^2 + (40)^2}$$

$$\sigma_{\max} = 20 + 50 = 70 \text{ MPa}$$

$$\sigma_{\min} = 20 - 50 = -30 \text{ MPa}$$

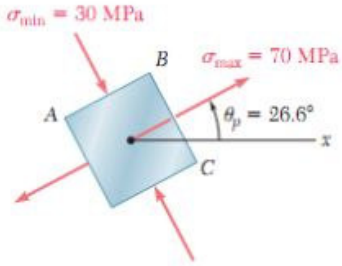

Fig. 7.12

The principal planes and principal stresses are sketched in Fig. 7.12. Making $\theta = 26.6°$ in Eq. (7.5), we check that the normal stress exerted on face BC of the element is the maximum stress:

$$\sigma_{x'} = \frac{50-10}{2} + \frac{50+10}{2}\cos 53.1° + 40 \sin 53.1°$$

$$= 20 + 30 \cos 53.1° + 40 \sin 53.1° = 70 \text{ MPa} = \sigma_{\max}$$

(c) Maximum Shearing Stress. Formula (7.16) yields

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(30)^2 + (40)^2} = 50 \text{ MPa}$$

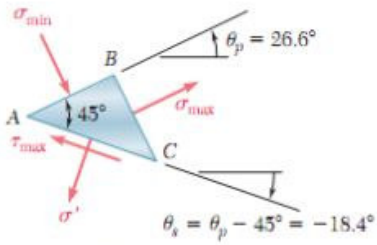

Fig. 7.13

Since $\sigma_{\max}$ and $\sigma_{\min}$ have opposite signs, the value obtained for $\tau_{\max}$ actually represents the maximum value of the shearing stress at the point considered. The orientation of the planes of maximum shearing stress and the sense of the shearing stresses are best determined by passing a section along the diagonal plane AC of the element of Fig. 7.12. Since the faces AB and BC of the element are contained in the principal planes, the diagonal plane AC must be one of the planes of maximum shearing stress (Fig. 7.13). Furthermore, the equilibrium conditions for the prismatic element ABC require that the shearing stress exerted on AC be directed as shown. The cubic element corresponding to the maximum shearing stress is shown in Fig. 7.14. The normal stress on each of the four faces of the element is given by Eq. (7.17):

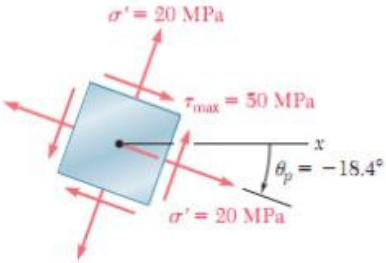

Fig. 7.14

$$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{50-10}{2} = 20 \text{ MPa}$$